NASA Technical Paper 1996

March 1982

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NOMENCLATURE

C	controller gain (response feedback)
$C_{\Delta\theta}$	controller gain, due to control-rate limit
C_{θ}	controller gain, due to control-magnitude limit
f	factor defining functional dependence of r, P, and Q on j
J	quadratic performance function
k, k_z	Kalman-filter or recursive algorithm gain
L	number of parameters identified
M	a priori error variance matrix
m	a priori error variance
N	number of measurements
P	error variance matrix
p	error variance
Q	parameter variance matrix
q	parameter variance
R	measurement noise variance matrix
r	measurement noise variance
T	transfer-function matrix
t	transpose of row of T matrix; time
u	random variable describing parameter variation
v	measurement noise
W	weighting matrix
W_z	weighting matrix in performance function, on response
$W_{\Delta\theta}$	weighting matrix in performance function, on control rate
W_{θ}	weighting matrix in performance function, on control amplitude
w_z	weight in performance function
Z	matrix of response-vector measurements

z vector of harmonics of response variables

z_0 uncontrolled response level

α exponential filter parameter

Δt sampling time-step

Δz_n response increment, $z_n - z_{n-1}$

$\Delta\theta_n$ control increment, $\theta_n - \theta_{n-1}$

δ_{nm} Kronecker delta function ($\delta = 1$ if $n = m$; zero otherwise)

Θ matrix of control-vector measurements

θ vector of harmonics of control variables

θ_0 input θ required for zero response

λ eigenvalue

λ_c empirical factor in cautious controller

τ time constant

Subscripts:

j measurement number

n,m time-step

0 initial conditions

Superscript:

T transpose

Special characters:

E( ) expectation

$(\hat{\ })$ estimate

$(\bar{\ })$ a priori estimate (before measurement)

SELF-TUNING REGULATORS FOR MULTICYCLIC CONTROL OF HELICOPTER VIBRATION

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SUMMARY

A class of algorithms for the multicyclic control of helicopter vibration and loads is derived and discussed. This class is characterized by a linear, quasi-static, frequency-domain model of the helicopter response to control; identification of the helicopter model by least-squared-error or Kalman-filter methods; and a minimum variance or quadratic performance function controller. Previous research on such controllers is reviewed and related to the present work. The derivations and discussions cover the helicopter model; the identification problem, including both off-line and on-line (recursive) algorithms; the control problem, including both open-loop and closed-loop feedback; and the various regulator configurations possible within the class. Conclusions from analysis and numerical simulations of the regulators provide guidance in the design and selection of algorithms for further development, including wind-tunnel and flight tests.

INTRODUCTION

A class of algorithms for the multicyclic control of helicopter vibration and loads, currently being developed by several investigators, is characterized by (1) a linear, quasi-static, frequency-domain model of the helicopter response to control; (2) identification of the helicopter model by least-squared error or Kalman-filter methods; and (3) a minimum variance or quadratic performance function controller. Such a control system combining recursive parameter estimation with linear feedback is called a self-tuning regulator.

Figure 1 outlines the control task. It is desired to minimize airframe vibration, the loads on the rotating and nonrotating components, and possibly the power requirement of the helicopter. The control parameters available are normally the pitch angles of the rotor blades, which are positioned by actuators in the rotating or nonrotating frame. In steady-state flight, the helicopter vibratory motion is ideally periodic, with fundamental frequency Ω for components in the rotating frame and $N\Omega$ for components in the nonrotating frame (where Ω is the rotational speed of the rotor, and N is the number of blades; see Johnson (1980)). Hence the control required to alleviate the vibration and loads will be periodic, and the control system can deal with the harmonics of the input and output.

This control is referred to as multicyclic or higher-harmonic control, to distinguish it from the mean and once-per-revolution blade-pitch control (in the rotating frame) that is required to trim the helicopter. The regulator algorithm consists of parameter estimation, gain calculation, and the control feedback. Some of these steps may be performed off-line. A digital control system operating on the harmonics of the input and output is considered here. Hence, the regulator also includes transformations between the time and frequency domains, and between analog and digital representations of the signals. The present report is concerned with the regulator algorithms, so the time-frequency domain and analog-digital transformations are not considered further. Moreover, the actuators can be treated by simply including them in the helicopter model. Hence the simplified system outline in figure 2 is the basis for the present work. Only self-tuning regulators and related systems are examined in this report. Shaw (1980), McCloud (1980a,b), and Johnson (1980) review multicyclic control, including other feedback concepts, and discuss helicopter vibration in general.

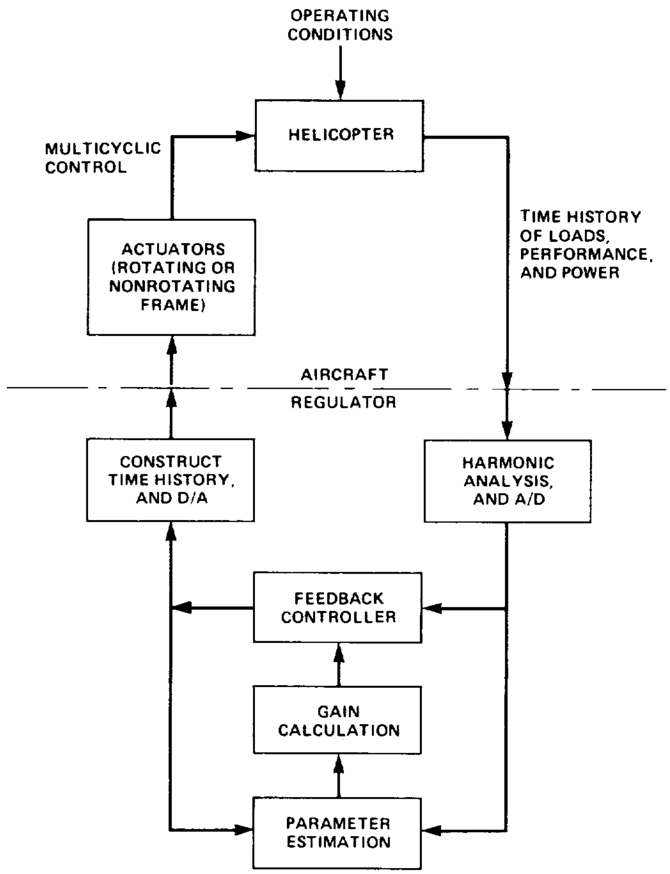

Figure 1.- Schematic of helicopter multicyclic control system.

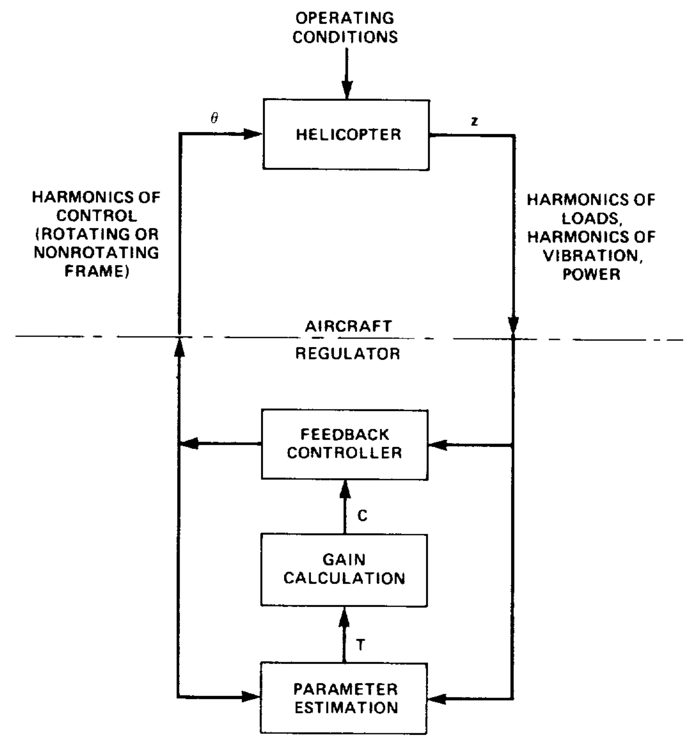

Figure 2.- Simplified schematic for a digital frequency-domain control system.

Previous Work

McCloud and Kretz (1974) and Kretz et al. (1973a,b) tested multicyclic control on a full-scale jet-flap rotor in a wing tunnel. They examined the response of the blade loads and vibration to control in the rotating frame. They introduced the concept of a linear, quasi-static representation of the rotor response (including the notation "T" for the transfer function). This transfer function representation was attributed to J.-N. Aubrun (McCloud and Kretz, 1974). The T-matrix was calculated from the wind-tunnel data by the least-squares method. Then the open-loop control required to minimize a quadratic performance function was calculated. McCloud (1975) applied this method to data obtained by theoretical analysis of a Multicyclic Controllable Twist Rotor (MCTR). He considered the reduction of both blade loads and hub shears, including the influence of the relative weights on the loads and vibration in the performance function. McCloud and Weisbrich (1978) then applied the method to data from a wind-tunnel test of a full-scale MCTR. They considered the control required to reduce both blade loads and test module acceleration, including the influence of relative weights in the performance index and the sensitivity of the control to the rotor lift. Brown and McCloud (1980) considered reduction of the test module vibration, using the data from the MCTR test again. They examined the influence of the relative weights on the various accelerometers, and the influence of weights on the control magnitude in the performance function. They examined the influence of rotor lift, propulsive force, and speed on the open-loop control.

Sissingh and Donham (1974) tested a model hingeless rotor in a wind tunnel. They measured the vibratory hub moment and vertical shear response to swashplate control. The transfer-function matrix and the control required to eliminate the vibration were then calculated by direct inversion. Powers (1978) and Wood et al. (1980) tested a model articulated rotor in a wind tunnel, measuring the response of the oscillatory shaft forces to swashplate control. They calculated the transfer-function matrix by several methods. The control required to null the hub forces was calculated by direct inversion of the T-matrix. McHugh and Shaw (1978) tested a model hingeless rotor in a wind tunnel, measuring the vibratory hub moments and vertical shear response to swashplate control. They also considered blade loads. The input required to null the hub moment was estimated by extrapolation and interpolation of the test data. The resulting control did not null two or three quantities at the same time, but it did reduce all three hub loads when tested in the wind tunnel.

Shaw and Albion (1980) also measured the response to swashplate control of a model hingeless rotor in a wind tunnel. They considered third, fourth, and fifth harmonics of the root flapwise bending, which were equivalent to the vibratory hub moments and vertical shear for this four-bladed rotor. They tested closed-loop feedback control of the loads, with the control gains obtained by direct inversion of the T-matrix. This controller was able to null the vibratory loads at one speed; the loads were reduced at higher and lower speeds, but not nulled because the pitch required exceeded the available control authority. The transient characteristics of the controller were good.

Shaw (1980) conducted a theoretical investigation of the closed-loop feedback control of vibratory vertical and in-plane hub shears. The control gains were again calculated by direct inversion of the T-matrix, and he considered the influence of errors in the estimate of the T-matrix on the stability of the controller. The system response was calculated for an abrupt maneuver (a change in speed, hence a change in the level of vibration and the true T-matrix), using a fixed-gain matrix. The controller performance was good when a single input was used to reduce the vertical shear, but was poor when two input parameters (two harmonics) were used to reduce both vertical and in-plane shear forces. The poor performance in the latter case was caused by the T-matrix changing enough to make the controller unstable. Hence a fixed-gain matrix was not acceptable; gain scheduling or on-line identification would be necessary to estimate the parameters accurately enough for satisfactory closed-loop performance. Shaw used a Kalman filter for on-line identification of the T-matrix. When applied to the case involving reduction of both vertical and in-plane shears, the control system utilizing the Kalman filter displayed good convergence and stability, confirming its ability to handle abrupt changes in parameters.

Taylor, Farrar, and Miao (1980) and Taylor et al. (1980) conducted a numerical simulation of the control of helicopter fuselage acceleration, using closed-loop feedback control and a Kalman filter for on-line identification of the T-matrix. The feedback gains were calculated to minimize a quadratic performance function. This control concept was developed by J. A. Molusis. The simulations were for a constant flight condition, and the performance of the system was studied in terms of the initial behavior after starting the regulator. The control system showed good convergence and accomplished a significant reduction in the vibration. The above authors examined the influence of the relative weights on the accelerometers in the performance function, on the update time, on the measurement noise level, and the influence of a limit on the maximum control change in one step.

Hammond (1980) tested a model articulated rotor in a wind tunnel. The response of vibratory hub moments and vertical shear to swashplate control was measured. A Kalman filter was used to identify the T-matrix and the uncontrolled vibration level. Feedback of the identified vibration level was used to minimize a quadratic performance function. Both deterministic and stochastic (or cautious) control algorithms were considered, as well as an option to identify only the vibration level, not the T-matrix. The development of these regulator algorithms was attributed to J. A. Molusis (Hammond, 1980). Test results were presented only for the cautious controller. A converged solution was reached when the controller was started with specified initial estimates. The controller reduced the vertical force significantly, and reduced the pitch moment to some extent; however, the roll-moment reduction was small. Molusis, Hammond, and Cline (1981) extended this investigation, considering feedback of vertical, longitudinal, and lateral acceleration. The controllers were tested in steady-state operating conditions, with varying wind-tunnel speed, and with collective pitch variations. The cautious controller showed good performance, with smooth operation and good tracking ability. The vertical and longitudinal vibrations were reduced significantly, but the lateral acceleration was actually increased at low speed. The deterministic controller was more erratic than the cautious controller, and the system using identification of only the uncontrolled acceleration level was not successful in reducing the vibration.

The research outlined above has established that multicyclic control can reduce, and in many cases null, helicopter vibration and loads. That most of this work has been based on experimental data reflects the difficulty of calculating vibration and simulating the system dynamics using current analytical tools. The theoretical investigations have been useful, however, and consistent with the experiments. As yet there have been no flight tests of such regulators; all of the tests have been conducted in wind tunnels. Moreover, the experimental verification of these control concepts has either been only partially successful, or has not yet been accomplished. The work discussed above can all be considered to deal with the same type of regulator, and most of the possible combinations of control and identification algorithms within this class of regulators have been examined to some extent. However, with the exception of the work of Shaw (1980) and of Molusis, Hammond, and Cline (1981), there has been little derivation or discussion of the regulator algorithms, and little direct comparison of the alternatives.

Scope of Present Investigation

The present paper provides a detailed derivation and discussion of the class of control algorithms characterized by a self-tuning regulator applied to a quasi-static model of the helicopter response. This work is intended to guide the selection of algorithms for further development, including wind-tunnel and flight tests. The paper discusses the helicopter model; the identification problems, including off-line and on-line algorithms; the control problem, including open-loop and closed-loop feedback; and the various regulator configurations possible within this class of control systems. The behavior and characteristics of these regulators are examined, particularly by analysis and numerical simulations of a single-input and single-output system. The conclusions about the design of the regulators are summarized, and recommendations for further work are given.

HELICOPTER MODEL

It is assumed that the helicopter can be represented by a linear, quasi-static frequency-domain model relating the output z to the input θ (see fig. 2) at time $t_n = n\ \Delta t$. Here z is a vector of the harmonics (both sine and cosine components) of the loads and vibration, in either the rotating or the nonrotating frame. Performance quantities, such as the mean power, can also be included in z. The input θ is a vector of the harmonics of the multicyclic control, in either the rotating or nonrotating frame. The sampling time-step Δt must be long enough for transients to die out and for the harmonics to be measured. Typically, this requires an interval of at least one rotor revolution. The operating condition is defined by the rotor lift, propulsive force, and forward speed (at least). It is possible to investigate the relationship of the vibration and loads to these operating condition parameters, in the same manner that the relationship to the control parameters is established (see McCloud and Kretz, 1974). For the present purpose it is only necessary to recognize that the helicopter model will depend on the operating condition.

Local and global models of the helicopter response are considered. The local model is a linearization of the response about the current control value:

$$\text{(a)}\quad z_n = z_{n-1} + T(\theta_n - \theta_{n-1})$$

or $\Delta z_n = T\Delta\theta_n$. The global model is linear over the entire range of control:

$$\text{(b)}\quad z_n = z_0 + T\theta_n$$

Note that the global model also gives $\Delta z_n = T\Delta\theta_n$; the global model implies that T is independent of θ_n, and $z_0 = z_{n-1} - T\theta_{n-1}$. Here z_0 is the uncontrolled vibration level, and in both models T is the transfer-function matrix. Three cases are distinguished for the global model, depending on the identification algorithm:

(b1) identify z_0 only

(b2) identify T only

(b3) identify both z_0 and T

If only T or only z_0 is identified, the remaining parameters must be estimated by some other means (e.g., direct measurement, off-line identification, or calculation). It is possible to measure z_0 directly by setting $\theta = 0$.

The quasi-static assumption requires that the update time Δt be long enough for transients produced in the response harmonics by the control change to die out. A linear, dynamic system could be described by an equation of the following form:

$$z_n = \sum_{i=1}^{\infty} F_{in} z_{n-i} + \sum_{i=0}^{\infty} G_{in} \theta_{n-i}$$

where F_{in} and G_{in} are matrices that describe the system. Here the variables z and θ are obtained from the data in the time-domain by a linear operator (an analog or digital filter that is equivalent to harmonic analysis for steady-state data). The coefficient matrices are in general a function of n since the helicopter in forward flight is not a time-invariant system; rather it is described by periodic coefficient differential equations. To keep the identification problem manageable, it would be desirable to maintain the time-step Δt large enough so that the dependence of the coefficient matrices on n could be neglected, and so that the summation over i could be truncated at one or two steps. Such a model for the helicopter dynamics is not considered further in the present report.

The assumption of linear response to control is expected to be reasonable, since the available experimental data imply that only a small multicyclic control amplitude (of the order of 0.5° to 1.5°) is required for vibration alleviation. The uncontrolled vibration level (z_0) is indeed a highly nonlinear function of the helicopter operating condition, and involves nonlinear aerodynamic and dynamic phenomena. Here it is only the response to control inputs that is being linearized. There is some evidence of nonlinear response to control over the required amplitude range, which may require the use of the locally linearized model defined above. McCloud (1975), in a theoretical investigation, and McCloud and Weisbrich (1978), using experimental data, found it necessary to limit the calculation of the T-matrix to low-vibration data in order to improve the accuracy of the identified linear model, which had a significant influence on the predicted open-loop control. The experimental results of McHugh and Shaw (1978) show some nonlinear dependence of the response on the magnitude of the control. A similar effect was observed in the theoretical investigation of Taylor et al. (1980). However, Shaw (1980) concluded (based on theoretical calculations) that the response was essentially linear, and Shaw and Albion (1980) concluded that their experimental results confirmed this linearity.

The large variation of the uncontrolled vibration level with speed is well established (see Johnson, 1980). McCloud and Weisbrich (1978) found a low sensitivity of the open-loop control to lift variations, by examining the calculated load alleviation over a range of lifts, using the open-loop control designed for a fixed-lift value in the middle of the range. Brown and McCloud (1980) found a weak influence of rotor lift and propulsive force on the calculated open-loop control, but a strong influence of flight speed on the T-matrix and the control. Shaw and Albion (1980) and Shaw (1980) found also that the T-matrix varied significantly with speed. The variation was large enough to make a closed-loop control system using constant gains unstable. Molusis, Hammond, and Cline (1981) found large differences in the T-matrices measured at three speeds.

IDENTIFICATION

In this section, algorithms for on-line and off-line identification of the parameters in the helicopter model are derived. The four models defined in the last section are considered:

(a) Local: $\Delta z_n = T\,\Delta\theta_n$

(b1) Global, identify z_0: $z_n - T\theta_n = z_0$

(b2) Global, identify T: $z_n - z_0 = T\theta_n$

(b3) Global, identify T and z_0: $z_n = [T \quad z_0]\begin{pmatrix}\theta_n \\ 1\end{pmatrix}$

A common notation will be used to represent all four cases:

$$z_n = T\theta_n$$

(Note that for case (b3), it is necessary to interpret "T" in this equation as z_0 and "θ" as 1.) For the case when T (or at least the estimate of T) is time-varying, the equation becomes

$$z_n = T_n\theta_n + v_n$$

including measurement noise v_n. It is assumed that there is no noise in the measurement of θ. The identification algorithms will be derived considering the jth measurement:

$$z_{jn} = \theta_n^T t_{jn} + v_{jn}$$

where t_j^T is the jth row of T. Note that z_j and v_j are scalars. Often the subscript j will be omitted, to simplify the notation.

$$z_n = \theta_n^T t_n + v_n$$

The task is to identify t from measurements of z. The measurement noise has zero mean, and variance $E(v_n v_m) = r_n \delta_{nm}$ for the jth measurement. For the local model (interpreting "z_n" as Δz_n) this representation of the noise is not correct, since noise in a measurement of z_n would contribute to both Δz_n and Δz_{n+1}. Hence, for that case successive values of v_n are correlated, with nonzero elements just above and just below the diagonal in $E(v_n v_m)$. This complication will be ignored.

The helicopter model includes cosine and sine harmonics for both input and output variables. For example, with a single input and single output, the equation is

$$\begin{pmatrix} z_{cn} \\ z_{sn} \end{pmatrix} = T \begin{pmatrix} \theta_{cn} \\ \theta_{sn} \end{pmatrix} + v_n$$

Then for a linear time-invariant system (such as a helicopter with three or more blades in hover) the transfer function relating z and θ is defined by two parameters, not four:

$$T = \begin{bmatrix} t_c & t_s \\ -t_s & t_c \end{bmatrix}$$

The equation for the helicopter response can then be rewritten in terms of the parameter vector $t^T = (t_c \quad t_s)$:

$$\begin{pmatrix} z_{cn} \\ z_{sn} \end{pmatrix} = \begin{bmatrix} \theta_{cn} & \theta_{sn} \\ -\theta_{sn} & \theta_{cn} \end{bmatrix} \begin{pmatrix} t_c \\ t_s \end{pmatrix} + v_n$$

However, such a representation complicates the recursive identification algorithms (by introducing matrices in place of vectors and scalars), because two measurements provide information about the same two parameters. Moreover, in forward flight, the helicopter is described by periodic coefficient differential equations, which implies that all four elements in this T-matrix will be different. Consequently the distinction between the cosine and sine harmonics is not introduced here.

Least-Squares Method

Off-line identification can be done by the method of least squares (Mendel, 1973; Goodwin and Payne, 1977). Off-line identification implies constant parameters. Also, the local model is not appropriate since it associates successive measurements. A set of N measurements is made, using a prescribed schedule of independent control inputs. The number of measurements N (the dimension of z_j, defined below) must be greater than the number of parameters to be identified L (the dimension of t_j). Consider the sum of the squares of errors:

$$S = \sum_{n=1}^{N} (z_{jn} - \theta_n^T t_j)^2 = (z_j - \Theta t_j)^T (z_j - \Theta t_j)$$

where the vector z_j and matrix Θ are defined as

$$z_j = \begin{pmatrix} \vdots \\ z_{jn} \\ \vdots \end{pmatrix} \qquad \Theta = \begin{bmatrix} \vdots \\ \theta_n^T \\ \vdots \end{bmatrix}$$

The solution that minimizes S is the least-squares estimate:

$$\hat{t}_j = (\Theta^T\Theta)^{-1}\Theta^T z_j$$

or

$$\hat{t}_j^T = z_j^T\Theta(\Theta^T\Theta)^{-1}$$

Putting the rows together again gives

$$\hat{T} = Z\Theta(\Theta^T\Theta)^{-1}$$

where

$$Z = \begin{bmatrix} \vdots \\ z_j^T \\ \vdots \end{bmatrix} = [\ldots \; z_n \; \ldots]$$

Note that $\hat{t}$ is a linear estimate, that is, a linear function of the data Z.

It is assumed that the measurement noise v_n is stationary, with zero mean, and is uncorrelated at different times $[E(v_n v_m) = r\,\delta_{nm}]$. There is no noise in the measurement of θ. It follows then that the least-squares estimate is unbiased, $E(\hat{t}_j) = t_j$, and the error-variance is

$$P = E(\hat{t}_j - t_j)(\hat{t}_j - t_j)^T = r(\Theta^T\Theta)^{-1}$$

An unbiased estimate of r is

$$\hat{r} = (z_j - \Theta\hat{t}_j)^T(z_j - \Theta\hat{t}_j)/(N - L)$$

With this type of measurement noise, the least-squares estimate is equivalent to the unbiased minimum error-variance estimate, so it has the minimum error-variance of all linear, unbiased estimators.

Generalized Least-Squares Method

The generalized least-squares estimate (Mendel, 1973) is obtained by minimizing the weighted sum of squares:

$$S_w = (z_j - \Theta t_j)^T W(z_j - \Theta t_j)$$

The solution is

$$\hat{t}_j = (\Theta^T W\Theta)^{-1}\Theta^T W z_j$$

The matrix W can be used to introduce weights based on the level of θ or z, for example to emphasize the measurements at low vibration levels in order to improve the identification in the vicinity of the optimum response.

If the measurement noise has zero mean and variance $E(v_n v_m) = R$, then the generalized least-squares estimate with $W = R^{-1}$ is equivalent to the unbiased minimum variance estimate. Hence, when the noise is not stationary or is correlated, the weighting matrix is chosen to emphasize the more precise data. Using $W = R^{-1}$, the error variance is

$$P = (\Theta^T R^{-1}\Theta)^{-1}$$

If, in addition, the noise has a normal probability distribution, then the minimum-variance estimate is equivalent to the maximum-likelihood estimate.

Recursive Parameter Identification

Recursive algorithms will be used for on-line identification from a sequence of measurements of the response to control. These algorithms can be used when the parameters are constant, or when they vary with time; either global or local models of the helicopter can be used. The algorithms are still derived for the jth measurements, but the subscripts j are omitted to simplify the notation. Since the parameters may be time-varying now, the equation of the helicopter model is

$$z_n = \theta_n^T t_n + v_n$$

Recursive Generalized Least Squares

A recursive form of the generalized least-squares estimate can be used for on-line parameter identification (Mendel, 1973; Goodwin and Payne, 1977). The weighted sum of squares

$$S_w = \sum_{n=1}^{N} (z_n - \theta_n^T t_n)^2 w_n$$

is to be minimized. The solution was given above. Here the weighting matrix is diagonal, and, furthermore, the notation $w_n = 1/r_n$ will be used (where r_n can be interpreted as the noise variance). The error matrix is defined as

$$P_N = (\Theta_N^T W_N \Theta_N)^{-1}$$

The effect of adding one more measurement, z_{N+1}, is obtained by applying the matrix inversion formula to

$$P_{N+1} = (P_N^{-1} + \theta_{N+1} w_{N+1} \theta_{N+1}^T)^{-1}$$

The result is the recursive algorithm

$$\hat{t}_{n+1} = \hat{t}_n + k_{n+1}(z_{n+1} - \theta_{n+1}^T \hat{t}_n)$$

where the gain vector k_{n+1} is obtained from

$$P_{n+1} = P_n - P_n \theta_{n+1} \theta_{n+1}^T P_n / (r_{n+1} + \theta_{n+1}^T P_n \theta_{n+1})$$

$$k_{n+1} = P_{n+1} \theta_{n+1} / r_{n+1}$$

This is the estimate for the jth measurement. In general there will be a different weight r for each measurement, hence a different solution for P and k. Let us assume, however, that the time behavior of r_n is the same for all measurements, that is, that r_{jn} for the jth measurement is a product of a function of time (n) and a function of the measurement (j). It is assumed that the starting value for P has the same variation. Hence

$$r_{jn} = f_j r_n$$

$$P_{j0} = f_j P_0$$

It follows that the solution for P has the same behavior:

$$P_{jn} = f_j P_n$$

and that the solution for k_{n+1} is independent of f_j, that is, the same for all measurements. The equations given above can thus be solved for P_n, which is multiplied by f_j to get the true variance. With the same k_{n+1} for each measurement, the rows

$$\hat{t}^T_{n+1} = \hat{t}^T_n + (z_{n+1} - \hat{t}^T_n \theta_{n+1}) k^T_{n+1}$$

can be combined to form

$$\hat{T}_{n+1} = \hat{T}_n + (z_{n+1} - \hat{T}_n \theta_{n+1}) k^T_{n+1}$$

(where here z is the vector of all measurements). So the entire matrix $\hat{T}$ can be identified in a single step, with k_{n+1} calculated only once.

If $r_n = 1$ (or any other constant), the recursive least-squares algorithm is obtained. The solution will be the same as that from the batch least-squares algorithm. The recursive implementation might be useful in order to track the estimates and error as the data are acquired. Eventually the old data dominate (k approaches zero), however, so the recursive least-squares algorithm is not appropriate with time-varying parameters.

Exponential Window

A recursive estimate applicable to the case of time-varying parameters can be obtained using an exponential window for the weighting function (Goodwin and Payne, 1977). By setting $r_n = \alpha^n$, where $0 < \alpha < 1$, the current data are emphasized. Since r_n is continuously decreasing, it is best to solve for the gain k_{n+1} in terms of $P^*_n = P_n/\alpha^n$:

$$P^*_{n+1} = \alpha^{-1}[P^*_n - P^*_n \theta_{n+1} \theta^T_{n+1} P^*_n / (\alpha + \theta^T_{n+1} P^*_n \theta_{n+1})]$$

$$k_{n+1} = P^*_{n+1} \theta_{n+1}$$

This algorithm can be obtained directly by minimizing the sum:

$$S^*_{N+1} = \alpha S^*_N + (z_{N+1} - \theta^T_{N+1} t)^2$$

Kalman-Filter Identification

A Kalman filter can be used for on-line identification of time-varying parameters (Bryson and Ho, 1969; Sage and Melsa, 1971). The equation for the jth measurement is again

$$z_n = \theta^T_n t_n + v_n$$

where the measurement noise has zero mean, variance $E(v_n v_m) = r_n \delta_{nm}$, and Gaussian probability distribution. The variation of the parameters will be modeled as a random process:

$$t_{n+1} = t_n + u_n$$

where u_n is a random variable with zero mean, variance $E(u_n u_m^T) = Q_n \delta_{mn}$, and Gaussian probability distribution. This equation implies that it is known that t varies, and that the order of the change in one time-step can be estimated; but no information is available about the specific dynamics governing the variation of t. The minimum error-variance estimate of t_n is then obtained from a Kalman filter:

$$\hat{t}_n = \hat{t}_{n-1} + k_n(z_n - \theta_n^T \hat{t}_{n-1})$$

where

$$M_n = P_{n-1} + Q_{n-1}$$

$$P_n = (M_n^{-1} + \theta_n \theta_n^T / r_n)^{-1}$$

$$= M_n - M_n \theta_n \theta_n^T M_n / (r_n + \theta_n^T M_n \theta_n)$$

$$= (I - k_n \theta_n^T) M_n (I - k_n \theta_n^T)^T + k_n k_n^T r_n$$

$$k_n = P_n \theta_n / r_n$$

$$= M_n \theta_n / (r_n + \theta_n^T M_n \theta_n)$$

Here M_n is the variance of the error in the estimate of t_n before the measurement, and P_n is the variance after the measurement. Note that P depends on the control input θ, but not on the measurement z; and no matrix inversion is required, because t_n is related to only one measured variable. The Kalman filter can be considered a time-variant dynamic system with state $\hat{t}$:

$$\hat{t}_n = (I - k_n \theta_n^T)\hat{t}_{n-1} + k_n z_n$$

$$= (I - k_n \theta_n^T)\hat{t}_{n-1} + k_n \theta_n^T t_n + k_n v_n$$

where

$$I - k_n \theta_n^T = I - P_n \theta_n \theta_n^T / r_n = P_n M_n^{-1}$$

If there are no process dynamics ($Q_n = 0$), the Kalman filter is equivalent to the generalized least-squares algorithm with $w_n = 1/r_n$ (for minimum error-variance).

The variances Q_n, r_n, and P_0 are different for the various measurements. It will be assumed that Q and r have the same time variation for all measurements, and that Q, r, and P_0 are proportional to the same function f_j:

$$r_{jn} = f_j r_n$$

$$Q_{jn} = f_j Q_n$$

$$P_{j0} = f_j P_0$$

Then it follows that $P_{jn} = f_j P_n$ and $M_{jn} = f_j M_n$; and k_n is the same for all the measurements. With the same gains, the rows can be combined now to form

$$\hat{T}_n = \hat{T}_{n-1} + (z_n - \hat{T}_{n-1}\theta_n)k_n^T$$

(here z_n is the vector of all the measurements). So the entire matrix $\hat{T}$ is identified in a single step, with P_n and k_n calculated only once. The basis of this result is the assumption that the ratio of the parameter and measurement noise variances, Q_{jn}/r_{jn}, is the same for every measurement. There is no reason to expect this assumption to be true, although it may be consistent with the accuracy of the knowledge of Q and r. The great reduction in computation is the strongest argument of accepting the assumption.

The Kalman-filter algorithm is completed by the specification of Q_n and r_n. Shaw (1980) discusses the choice of these parameters in terms of the noise sources for the helicopter. Taylor et al. (1980) used $Q/T \triangleq 0.001$, and

$$r_{n+1} = r_n(J_{n+1}/J_n)$$

with minimum and maximum limits on r_n, and $r_0/z_0^2 \triangleq 0.3$. Here J is a quadratic performance function, so this equation keeps the noise-to-signal ratio approximately constant. The generalized least-squares algorithm is obtained for $Q = 0$.

In summary, the Kalman-filter algorithms for the local and global helicopter models are as follows:

(a) $$\hat{T}_n = \hat{T}_{n-1} + [z_n - z_{n-1} - \hat{T}_{n-1}(\theta_n - \theta_{n-1})]k_n^T$$

(b1) $$\hat{z}_{0_n} = \hat{z}_{0_{n-1}} + \left(z_n - T\theta_n - \hat{z}_{0_{n-1}}\right)k_n$$

(b2) $$\hat{T}_n = \hat{T}_{n-1} + (z_n - z_0 - \hat{T}_{n-1}\theta_n)k_n^T$$

(b3) $$\left[\hat{T}_n \hat{z}_{0_n}\right] = \left[\hat{T}_{n-1}\hat{z}_{0_{n-1}}\right] + \left(z_n - \hat{z}_{0_{n-1}} - \hat{T}_{n-1}\theta_n\right)k_n^T$$

and "θ" in the calculation of P_n and k_n becomes $\Delta\theta$ for (a), 1 for (b1), θ for (b2), and $(\theta^T 1)^T$ for (b3). For the last case, in which only z_0 is identified, the gain calculation is simplified since "θ" = 1 and all quantities are scalars. In this case, k_n is determined solely by Q_n and r_n, independent of the control input θ.

Indentifiability

Estimation of parameters with closed-loop control can introduce identifiability questions. The equation for recursive identification of T and z_0 is

$$\begin{pmatrix} \hat{t}_n \\ \hat{z}_{0_n} \end{pmatrix} = \begin{pmatrix} \hat{t}_{n-1} \\ \hat{z}_{0_{n-1}} \end{pmatrix} + \begin{pmatrix} k_n \\ k_{z_n} \end{pmatrix} (z_n - \hat{z}_{0_{n-1}} - \theta_n^T \hat{t}_{n-1})$$

$$= \begin{bmatrix} I - k_n \theta_n^T & -k_n \\ -k_{z_n} \theta_n^T & 1 - k_{z_n} \end{bmatrix} \begin{pmatrix} \hat{t}_{n-1} \\ \hat{z}_{0_{n-1}} \end{pmatrix} + \begin{pmatrix} k_n \\ k_{z_n} \end{pmatrix} z_n$$

The on-line identification algorithm will be used with closed-loop control of the system. Hence, in the steady-state limit (if it exists), the control θ_n will approach a constant, and so the Kalman gains will also be constant. Consider the dynamics of the above equation when θ, k, and k_z are constants. The steady-state solution is

$$z_n = z_0 + \theta^T t = \hat{z}_0 + \theta^T \hat{t}$$

and the eigenvalues λ are obtained from

$$\begin{bmatrix} I - k\theta^T - \lambda I & -k \\ -k_z \theta^T & 1 - k_z - \lambda \end{bmatrix} \begin{pmatrix} \hat{t} \\ \hat{z}_0 \end{pmatrix} = 0$$

Note that $\lambda = 1$ is an eigenvalue. For

$$\begin{bmatrix} -k\theta^T & -k \\ -k_z \theta^T & -k_z \end{bmatrix} \begin{pmatrix} \hat{t} \\ \hat{z}_0 \end{pmatrix} = 0$$

each row of the matrix is proportional to $(\theta^T \; 1)$, so the determinant is zero as required. The eigenvector is the solution of the equation

$$\hat{z}_0 + \theta^T \hat{t} = 0$$

Thus, there are undamped modes in the identified solution. The problem is that in the steady state (with θ constant), the algorithm is trying to identify more than one parameter, using only one measurement. The eigenvector of this undamped mode has components such that $\hat{z}_0 + \theta^T \hat{t} = 0$, so it will not influence the identification and prediction of z. Since the objective is to minimize z, it may be expected that the steady-state controlled response will remain acceptable. An error in the estimate of t may adversely affect the stability of the system, however.

If only z_0 is identified, then the number of parameters equals the number of measurements always. If only t is identified, there will again be an eigenvalue $\lambda = 1$ (with corresponding eigenvector such that $\theta^T \hat{t} = 0$) if there is more than one parameter in t, that is, if there is more than one control variable. In the helicopter problem, the multicyclic control variables include both cosine and sine harmonics, so there will always be at least two elements in t.

Difficulties must be expected in the identification with a closed-loop system, if the number of parameters to be identified is greater than the number of measurements. Astrom et al. (1977) suggest two ways to handle this problem. The first approach is to eliminate or reduce the redundancy by constraining the values of some parameters. Constraining some elements of t is possible (the Kalman-filter algorithm need not be changed), or only z_0 can be identified. The second approach is to ignore the problem, since the estimate of the parameters is an intermediate step and errors are acceptable if the closed-loop performance of the system is good.

In the present problem, the identifiability is also improved if the parameters do vary with time. Then the varying control input provides the independent measurements needed to obtain information about all the parameters. Note also that when the number of measurements equals the number of controls (T a square matrix), the helicopter equation can be written as

$$z = z_0 + T\theta = T(\theta - \theta_0)$$

It is desired to identify θ_0 in this case, so $z = 0$ can be achieved with closed-loop control. The matrix T must be known accurately enough for acceptable convergence and stability. In this case identifying θ_0 is equivalent to identifying z_0.

A related problem occurs when the local model of the helicopter is used. The recursive identification algorithm is

$$\hat{t}_n = \hat{t}_{n-1} + k_n(z_n - z_{n-1} - \Delta\theta_n^T\hat{t}_{n-1}) = [I - k_n\Delta\theta_n^T]\hat{t}_{n-1} + k_n(z_n - z_{n-1})$$

When operating under closed-loop control, $\Delta\theta$ should approach zero in the steady state (if it exists). In the limit of small $\Delta\theta$, the Kalman gain k_n approaches a finite constant. Hence $k_n \Delta\theta_n^T$ is small and all the eigenvalues are near 1. The implication is that identifiability problems may also be expected with the local model, when $\Delta\theta$ is small.

Starting Recursive Algorithms

The usual starting procedure for these recursive algorithms involves setting P_0 to a very large value, and $\hat{T}_0$ to some initial estimate (see Mendel, 1973; Goodwin and Payne, 1977). Note that the limit $P_0^{-1} = 0$ gives

$$P_1^{-1} = (P_0 + Q_0)^{-1} + \theta_1\theta_1^T/r_1 = \theta_1\theta_1^T/r_1$$

which could actually be used when only one parameter is being identified. With two or more parameters to be identified, $\theta_1\theta_1^T$ is a singular matrix, and matrix inversion is undesirable in any case. This result does show, however, that large P_0 is to be interpreted as $P_0 \gg r/\theta^2$. Taylor et al. (1980) used $P_0/T^2 \triangleq 10$; as a result $P_0/r = 100$, which indeed was much larger than θ^{-2}.

Some special procedures may be appropriate for the start of the algorithm. Molusis et al. (1981) used a prescribed set of initial control values followed by off-line least-squares identification to obtain initial values of z_0, T, and P. A stricter than normal limit on the magnitude or rate of change of the control could be used initially. In particular, the use of a time lag in the controller will prevent the implementation of a large, incorrect control at the start, if the initial estimate of T is too much smaller than the actual value. In general, care must be taken in the use of the identified parameters until new measurements are obtained to update the estimates.

Numerical Implementation

Implementation of the identification algorithms requires procedures designed to minimize the numerical errors. Goodwin and Payne (1977) discuss such procedures for batch least-squares identification. Goodwin and Payne (1977), and Anderson and Moore (1979) discuss procedures for recursive identification (specifically, the square-root algorithm).

Two forms of the equations for P and k have been given (see Gelb, 1974; Anderson and Moore, 1979). The second equation for P is less sensitive to numerical errors, and the second equation for k is more appropriate if r is small. Note that not all combinations of the two equations are possible: the first equation for P can be followed by either k equation; or the second equation for k can be used, followed by either P equation. In addition, the fact that P is symmetric can be used to reduce the required computation, and also to reduce the numerical errors.

CONTROL

The control algorithms will be based on the minimization of a performance index J that is a quadratic function of the input and output variables. This function will depend on the input and output at the nth time-step, and perhaps at past times, but not on the future values. If all the parameters in the model are known, a deterministic controller is obtained. With unknown, estimated parameters, the certainty-equivalence principle may be applied (without regard to the question of the validity of the principle): the deterministic control solution is used with the estimated parameter values. Alternatively, a cautious controller (Wittenmark, 1975) can be obtained by minimizing the expected value of the performance function.

Such control systems are called passive-adaptive or nondual controllers (Wittenmark, 1975; Goodwin and Payne, 1977). The performance index does not consider that future measurements will be made, so it ignores the possibility of learning from the measurements. A dual or active-adaptive controller (Wittenmark, 1975; Goodwin and Payne, 1977) actively probes the system to reduce the parameter errors. The control is used for learning, to improve the parameter estimates, but the improvement is achieved at the expense of short-term deterioration of the closed-loop performance.

Performance Function

The quadratic performance function to be used is

$$J = z_n^T W_z z_n + \theta_n^T W_\theta \theta_n + \Delta\theta_n^T W_{\Delta\theta}\, \Delta\theta_n$$

where $\Delta\theta_n = \theta_n - \theta_{n-1}$. The vectors θ and z contain the harmonics of the input and output. Typically the weighting matrices are diagonal, and have the same value for all harmonics of a particular quantity. Then J is a weighted sum of the mean squares of the vibration, loads, and control. The matrix W_θ constrains the amplitude of the control, while $W_{\Delta\theta}$ constrains the rate of change of the control.

To simplify the specification of the weighting matrices, they can be written as follows:

$$W_z = D_z$$

$$W_\theta = D_\theta / g$$

$$W_{\Delta\theta} = D_{\Delta\theta}\tau$$

where D_z, D_θ, and $D_{\Delta\theta}$ are diagonal matrices. The factor g can be interpreted as the gain of the control loop, and τ defines the time-constant of the control lag. The output vector z may include terms from several transducers, so D_z can have different weights for each transducer (the values of the weights must also account for different units). Often the weights are the same for all harmonics of a particular measurement. The matrices D_θ and $D_{\Delta\theta}$ may also have different weights (for example, to reflect different limits on the collective and cyclic actuators), and the weights may increase with harmonic number to account for the actuator frequency response characteristics.

Deterministic Controller

The control required to alleviate the helicopter vibration is found by substituting for z_n in the performance function, using the helicopter model, and then solving for θ_n that minimizes J. Both the global and local models of the helicopter can be written as

$$z_n = z_{n-1} + T(\theta_n - \theta_{n-1})$$

Substituting for z_n and setting $\partial J/\partial\theta_{jn} = 0$ (for each component in the vector θ_n), gives a set of equations that can be solved for θ_n. The result is

$$\theta_n = Cz_{n-1} + (C_{\Delta\theta} - CT)\theta_{n-1}$$

or

$$\Delta\theta_n = Cz_{n-1} - C_\theta\theta_{n-1}$$

where

$$C = -DT^TW_z$$

$$C_\theta = DW_\theta$$

$$C_{\Delta\theta} = DW_{\Delta\theta}$$

$$D = (T^TW_zT + W_\theta + W_{\Delta\theta})^{-1}$$

For the global model, a second form of the solution can be obtained by substituting $z_{n-1} = z_0 + T\theta_{n-1}$. The result is

$$\theta_n = Cz_0 + C_{\Delta\theta}\theta_{n-1}$$

or

$$\Delta\theta_n = Cz_0 - (C_\theta - CT)\theta_{n-1}$$

This equation defines an open-loop control determined by the uncontrolled response level (z_0). The preceding result defines a closed-loop control obtained by feedback of the measured response (z_{n-1}). The open-loop case is applicable only with the global model of the helicopter.

When $W_\theta = 0$, the solution reduces to $\Delta\theta_n = Cz_{n-1}$; and for $W_{\Delta\theta} = 0$, it reduces to $\theta_n = Cz_0$. If both W_θ and $W_{\Delta\theta}$ are zero, then $CT = -I$. Finally, if $W_\theta = W_{\Delta\theta} = 0$, and the number of controls equals the number of measurements (so T is a square matrix) then $C = -T^{-1}$ (regardless of W_z).

From the above derivation it follows that the gains used to calculate θ_n should be based on the parameters ($\hat{T}$ and $\hat{z}_0$) at the nth step. However, the recursive identification algorithm obtains the estimate $\hat{T}_n$ only after the measurement z_n, which is a result of the θ_n input, is obtained. The best that can be done is to use the a priori estimate $\bar{T}_n$, which is the prediction of the parameter before the measurement is made. The model used here for the variation of the parameters gives simply $\bar{T}_n = \hat{T}_{n-1}$. Thus, the control gains are evaluated using the parameters identified at the (n - 1)th time-step.

The performance function used results in proportional control (feedback of z_{n-1} or z_0). An integral-proportional controller can be obtained by adding the term

$$y_n^T W_y y_n$$

to J, where y is the integral of z:

$$y_n = y_{n-1} + z_n$$

The control to minimize J then includes feedback of y_{n-1} as well (see Molusis et al., 1981). Such integral control is not considered further in the present paper.

Controller Dynamics

To examine the dynamics of the system using closed-loop control, substitute $z_{n-1} = z_0 + T\theta_{n-1} + v_{n-1}$ into the control law based on identified parameters:

$$\begin{aligned}\theta_n &= [C(T - \hat{T}) + C_{\Delta\theta}]\theta_{n-1} + C(z_0 + v_{n-1}) \\ &= D\{[\hat{T}^T W_z(\hat{T} - T) + W_{\Delta\theta}]\theta_{n-1} - \hat{T}^T W_z(z_0 + v_{n-1})\}\end{aligned}$$

where

$$D = (\hat{T}^T W_z \hat{T} + W_\theta + W_{\Delta\theta})^{-1}$$

The stability is determined by the eigenvalues of the matrix

$$D(\hat{T}^T W_z(\hat{T} - T) + W_{\Delta\theta})$$

The steady-state solution is

$$\theta = -D^*\hat{T}^T W_z z_0$$

$$z = (I - TD^*\hat{T}^T W_z)z_0$$

where

$$D^* = (\hat{T}^T W_z T + W_\theta)^{-1}$$

The dynamic response is determined by the estimation error $(\hat{T} - T)$ and the rate limit $(W_{\Delta\theta})$. If there is no estimation error and $W_{\Delta\theta} = 0$, the steady-state solution is reached in a single step:

$$\theta_n = -DT^T W_z(z_0 + v_{n-1})$$

for all n. If $W_\theta = 0$ and T is a square matrix, the steady-state solution is the direct inverse:

$$\theta = -T^{-1}z_0$$

$$z = 0$$

(regardless of the estimation error or $W_{\Delta\theta}$). Note however that the system always responds to the measurement noise.

Using the identified parameters, the open-loop control is:

$$\theta_n = D(-\hat{T}W_z\hat{z}_0 + W_{\Delta\theta}\theta_{n-1})$$

The stability is determined by the eigenvalues of the matrix $DW_{\Delta\theta}$. The steady-state solution is

$$\theta = -D^+\hat{T}^T W_z\hat{z}_0$$

$$z = z_0 - TD^+\hat{T}^T W_z\hat{z}_0$$

where

$$D^+ = (\hat{T}^T W_z\hat{T} + W_\theta)^{-1}$$

The transient response is due solely to the rate limit $W_{\Delta\theta}$. If $W_{\Delta\theta} = 0$, the steady-state solution is reached in a single step. If $W_\theta = 0$ and T is a square matrix, the steady-state solution is

$$\theta = -\hat{T}^{-1}\hat{z}_0$$

$$z = z_0 - T\hat{T}^{-1}\hat{z}_0$$

which depends on the estimation errors.

When the open-loop control is used with on-line identification, it is necessary to consider the effect of the identification on the controller performance. The steady-state solution (if it exists) of the Kalman filter is

$$z = z_0 + T\theta = \hat{z}_0 + \hat{T}\theta$$

(whether z_0, or T, or both are identified). Hence the steady-state control response will be

$$\theta = -(\hat{T}^T W_z \hat{T} + W_\theta)^{-1} \hat{T}^T W_z [z_0 + (T - \hat{T})\theta]$$

or

$$\theta = -D^* \hat{T}^T W_z z_0$$

$$z = (I - TD^* \hat{T}^T W_z) z_0$$

which is identical to the result for closed-loop control (z_{n-1} feedback). Specifically, the control system produces $z = 0$ if $W_\theta = 0$, and T is a square matrix, even though the control is based on $\hat{z}_0$ (which includes estimation errors). Hence, in terms of the steady-state performance, the open-loop and closed-loop controllers are identical when used with on-line parameter identification.

On-line identification influences the regulator dynamics by changing the parameter estimates at each step. It is assumed that the parameters vary slowly enough that the identification and control problems can be separated in the design of the regulator. In fact, however, recursive identification produces a nonlinear feedback control system, because of the dependence of the Kalman-gain k on θ, the dependence of the controller gains on $\hat{T}$, and the appearance of the combination $\hat{T}\theta$ (since the parameter estimates as well as θ are dynamic variables). On-line identification also makes the open-loop case a feedback control system, since $\hat{T}$ and $\hat{z}_0$ then depend on the measurement z_n. However, the combination of open-loop control with on-line identification of only z_0 is still a linear system, since in this case the Kalman gain is independent of the system control or response.

Controller Gain Update

With a new estimate of T, it is necessary to reevaluate the control gains, which requires

$$D_n = (\hat{T}_n^T W_z \hat{T}_n + W_\theta + W_{\Delta\theta})^{-1}$$

The Kalman filter gives the parameter update in the form

$$\hat{T}_n = \hat{T}_{n-1} + ek^T$$

where e and k are vectors. Matrix inversion can be avoided by using the relationship

$$(A + BC^T)^{-1} = A^{-1} - A^{-1}B(I + C^T A^{-1} B)^{-1} C^T A^{-1}$$

(see Sage and Melsa, 1971). Writing

$$D_n^{-1} = \hat{T}_{n-1}^T W_z \hat{T}_{n-1} + W_\theta + W_{\Delta\theta} + k\left[\left(\hat{T}_{n-1}^T + \frac{1}{2} ke^T\right) W_z e\right]^T + \left[\left(\hat{T}_{n-1}^T + \frac{1}{2} ke^T\right) W_z e\right] k^T$$

$$= D_{n-1}^{-1} + bc^T + cb^T$$

there follows

$$D_n = (A^{-1} + cb^T)^{-1} = A - Acb^T A/(1 + b^T Ac)$$

$$A = (D_{n-1}^{-1} + bc^T)^{-1} = D_{n-1} - D_{n-1} bc^T D_{n-1}/(1 + c^T D_{n-1} b)$$

Cautious Controller

A cautious controller, which accounts for the parameter uncertainties, can be obtained using the expected value of the performance function (see Wittenmark, 1975; Molusis et al., 1981):

$$J = E\left[\sum_j w_{z_j} z_{jn}^2\right] + \theta_n^T W_\theta \theta_n + \Delta\theta_n^T W_{\Delta\theta} \Delta\theta_n$$

It is assumed here that W_z is diagonal, and θ is deterministic. For the case of closed-loop control (z_{n-1} feedback), there follows:

$$E\left[\sum_j w_{z_j} z_{jn}^2\right] = E\left[\sum_j w_{z_j}\left(z_{j_{n-1}} + \Delta\theta_n^T t_{jn}\right)^2\right]$$

$$= \sum_j w_{z_j}\left(z_{j_{n-1}} + \Delta\theta_n^T \hat{t}_{jn}\right)^2 + \Delta\theta_n^T\left(\sum_j w_{z_j} P_{jn}\right)\Delta\theta_n$$

using $E(t) = \hat{t}$ and $E(tt^T) = \hat{t}\hat{t}^T + P$. It is the actual value of z_n that is to be minimized, so the measurement noise is not included in the above expression. The parameter error variance matrix P is given by the Kalman filter (or it is calculated directly for off-line identification). However, P_n is not available until after the solution for θ_n is obtained. To avoid a very complex equation for θ_n, the a priori estimate error

$$M_n = P_{n-1} + Q_{n-1}$$

will be used. This is consistent with the use of the a priori parameter estimates ($\bar{t}_n = \hat{t}_{n-1}$) in the controller gains. Hence the performance function becomes

$$J = z_n^T W_z z_n + \theta_n^T W_\theta \theta_n + \Delta\theta_n^T (W_{\Delta\theta} + \sum_j w_{z_j} M_{jn}) \Delta\theta_n$$

The solution is identical to that for the deterministic controller, using the identified values of the parameters, and with $W_{\Delta\theta}$ replaced by

$$W_{\Delta\theta} + \sum_j w_{z_j} M_{jn} = W_{\Delta\theta} + M_n \left(\sum_j w_{z_j} f_j \right) \lambda_c$$

since $M_{jn} = f_j M_n$. The factor λ_c is introduced for an empirical modification of the controller if desired.

For the case of open-loop control (z_0 feedback), there follows

$$E\left[\sum_j w_{z_j} z_{jn}^2\right] = E \sum_j \left[w_{z_j} (z_{j0} + \theta_n^T t_{jn})^2\right]$$

$$= \sum_j w_{z_j} (\hat{z}_{j0} + \theta_n^T \hat{t}_{jn})^2 + \sum_j w_{z_j} (M_{zz} + 2\theta_n^T M_{tz} + \theta_n^T M_{tt} \theta_n)$$

where

$$M = \begin{bmatrix} M_{tt} & M_{tz} \\ M_{tz}^T & M_{zz} \end{bmatrix}$$

So the performance function becomes

$$J = z_n^T W_z z_n + \theta_n^T W_\theta \theta_n + (\theta_n^T \ 1) \left(\sum_j w_{z_j} M_{jn} \right) \begin{pmatrix} \theta_n \\ 1 \end{pmatrix} + \Delta\theta_n^T W_{\Delta\theta} \Delta\theta_n$$

The solution for the control that minimizes J is then

$$\theta_n = C z_0 + C_{\Delta\theta} \theta_{n-1} + c_0$$

The gain matrices C and $C_{\Delta\theta}$ are the same as for the deterministic controller, using the identified values of the parameters and with W_θ replaced by

$$W_\theta + (M_{tt})_n \left(\sum_j w_{z_j} f_j \right) \lambda_c$$

The new constant term is

$$c_0 = -D (M_{tz})_n \left(\sum_j w_{z_j} f \right) \lambda_c$$

If only the T-matrix is identified, the c_0 term is not present. If only z_0 is identified, the cautious controller is not applicable (since it is identical to the deterministic controller then).

In summary, the cautious controller introduces for the closed-loop case a constraint on the rate of change of the control (an effective value of $W_{\Delta\theta}$) proportional to the current parameter error-variance. For the open-loop case, a constraint on control magnitude (effective W_θ) is introduced, and an offset (c_0) if z_0 is being identified. These are the changes required to obtain the minimum expected value of the mean-squared response when the parameter estimates are in error.

Dual Controllers

Optimal dual controller solutions are generally too complex to be practical (Wittenmark, 1975), but there has been work on suboptimal solutions that introduce learning in some fashion. There are very simple approaches, such as adding a perturbation signal to the cautious controller, or constraining the minimum control level (Wittenmark, 1975). A dual controller is defined by the minimization of a multistep performance function:

$$J_D = \sum_{n=1}^{N} J_n$$

where J_n is the one-step function used above. The solution for the general stochastic problem is not practical for the present purposes. Molusis et al. (1981) obtained an approximate solution by linearizing J_D about the one-step control law. The resulting dual controller is similar to a reduction of the weight W_z in a cautious controller (Molusis et al., 1981). Another approach is to add a term to J_n that is a function of the estimation error (Wittenmark, 1975):

$$J_D = J_n + \lambda_D G(P_n)$$

for example, $G = \ln|P_n|$ (Goodwin and Payne, 1977). Alternatively, J_n can be minimized subject to an additional constraint, such as $\mathrm{Tr}\, P_n^{-1} \geq \lambda$ (Wittenmark, 1975).

A simple learning controller can be obtained by using $G = -|M_n|/|P_n|$, where M_n is the estimation error before the measurement, and P_n is the error after the measurement (Goodwin and Payne, 1977). From the Kalman-filter equations it follows that

$$|M_n|/|P_n| = 1 + \theta_n^T M_n \theta_n / r_n$$

so the performance function becomes

$$J_D = J_n - \lambda_D \theta_n^T M_n \theta_n / r_n$$

Recall that "θ_n" is interpreted as $\Delta\theta_n$ for the local model, or as $(\theta_n^T\ 1)^T$, if both T and z_0 are being identified. This performance function has the same form as that for the cautious controller, hence the same solution is obtained, with now

$$\lambda_c = -\lambda_D \Big/ \left(r_n \sum_j w_{z_j} f_j \right)$$

Hence, $\lambda_c = 1$ for the cautious controller, and $\lambda_c < 0$ for a learning controller. The learning controller introduces an effective contribution to W_θ or $W_{\Delta\theta}$ equal to $(-\lambda_D M_n / r_n)$, so the constraint on the control is reduced by an amount proportional to the parameter error-variance. There is also an offset term (c_0) if z_0 is being identified. This learning controller is convenient since it leads to a quadratic form for J_D. However, a large and negative effective value of W_θ or $W_{\Delta\theta}$ can produce a divergence of the control.

REGULATORS

A controller combining recursive parameter estimation and linear feedback is called a self-tuning regulator (Astrom et al., 1977; Astrom and Wittenmark, 1973; Goodwin and Payne, 1977). The most common configuration consists of recursive least-squares parameter estimation and a minimum variance controller. Self-tuning regulators have been developed for systems with constant parameters, but there have also been applications to systems with slowly varying parameters (Astrom et al., 1977). Usually the system model includes the dynamic behavior. Other estimation methods (e.g., exponential window, extended least squares, or maximum likelihood) and other controllers (e.g., quadratic cost function) have been used (Astrom et al., 1977). The self-tuning regulator and the plant form a time-varying, nonlinear, stochastic system. Of concern are the stability of the system, the convergence of the regulator, and identifiability when the system operates closed-loop (Astrom et al., 1977). There are some theoretical results for the least-squares identification and minimum-variance controller combination, but primarily these issues have been investigated by simulation.

Classification

In the present work, self-tuning regulators are constructed for the alleviation of helicopter vibration using multicyclic control. The most general case involves recursive identification, using a Kalman filter, and a quadratic performance function controller. There are two fundamental options for the identification: the use of either an invariable algorithm or an adaptive algorithm. For the invariable algorithm, the constant parameters are identified off-line by applying the least-squares method to a succession of independent input and output measurements. For the adaptive algorithm, the parameters are recursively identified on-line, using a Kalman filter (or possibly the exponentially weighted generalized least-squares method).

There are also two fundamental options for the controller: open-loop or closed-loop algorithms. For the open-loop algorithm, the control is based on the uncontrolled vibration level z_0 (identified either on-line or off-line). For the closed-loop algorithm, the control is based on feedback of the measured vibration, z_{n-1}.

Hence, there are four regulator options. The invariable open-loop regulator (Fig. 3) consists of off-line identification of T and z_0, calculation of the controller gain, and then application of the fixed control $\theta = Cz_0$ ($W_{\Delta\theta} = 0$ must be used). This is the simplest and most stable option, but it may be expected to have poor performance if the parameters vary or are estimated incorrectly.

The invariable closed-loop regulator (fig. 4) consists of off-line identification of the parameters and calculation of the fixed-gain matrix, and feedback of the control based on the measured response. The feedback improves the steady-state performance, although it also means the control will respond to measurement noise. The loop closure introduces concern about dynamic stability, particularly with estimation errors or varying parameters.

The adaptive open-loop regulator (fig. 5) consists of on-line identification of the parameters and calculation of the gain matrix, with control based on the identified value of z_0. The performance of the system is expected to be increased by use of on-line identification to reduce the parameter errors.

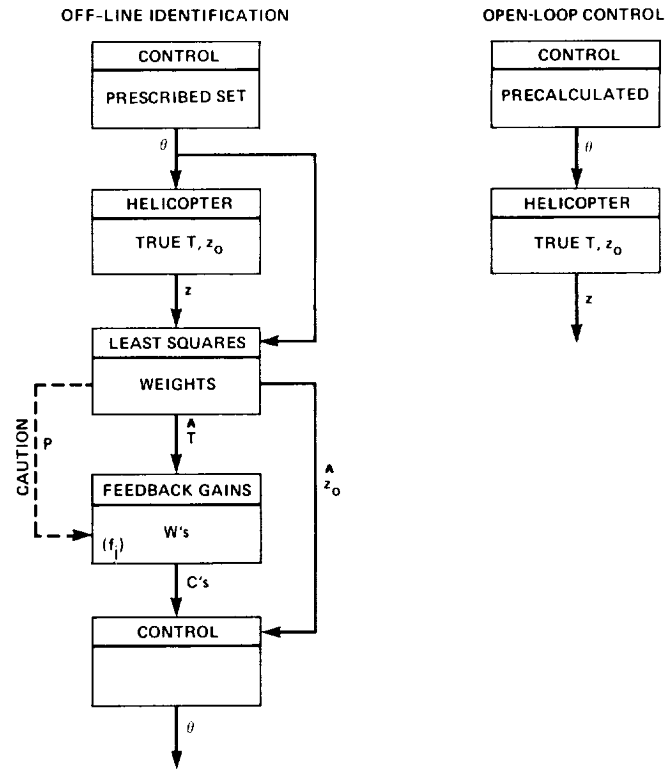

Figure 3.- Invariable open-loop regulator.

The adaptive closed-loop regulator (fig. 6) consists of on-line identification of the parameters and calculation of the gain matrix, with feedback of the control based on the measured response. Note that the Kalman-filter gains can be calculated in the time interval between application of θ_n to the helicopter and the measurement of the resulting z_n. This regulator, using both feedback control and on-line parameter identification to improve the system performance, is the most complex option. Figure 6 illustrates identification using the global model of the system. The regulator using the local model has the same form, except that z_0 is not identified.

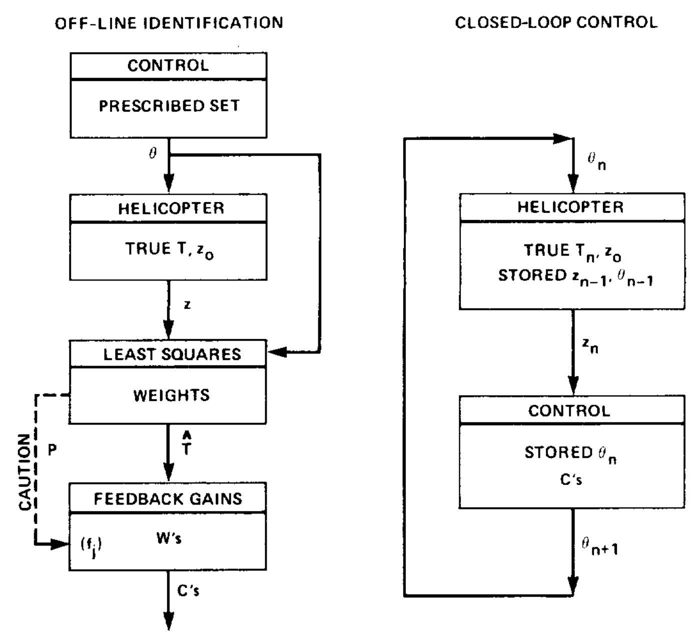

Figure 4.- Invariable closed-loop regulator.

Several models of the helicopter have been discussed, particularly with regard to the identification algorithms; however, not all of these models are appropriate for each regulator. With off-line identification, it is not possible to identify only z_0, since the least-squares method is required to identify T. Off-line identification of only T by the least-squares method is possible, since z_0 can be measured directly by setting $\theta = 0$. For the adaptive open-loop regulator, the global model with only T identified is not appropriate, because the controller requires z_0. On-line identification of only z_0 (with off-line calculation of T) is feasible for the adaptive open-loop regulator. For the

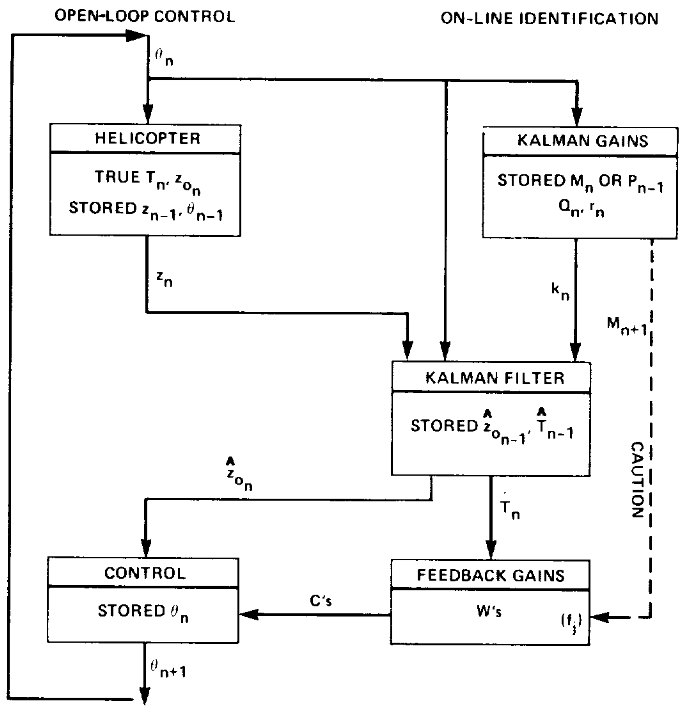

Figure 5.- Adaptive open-loop regulator.

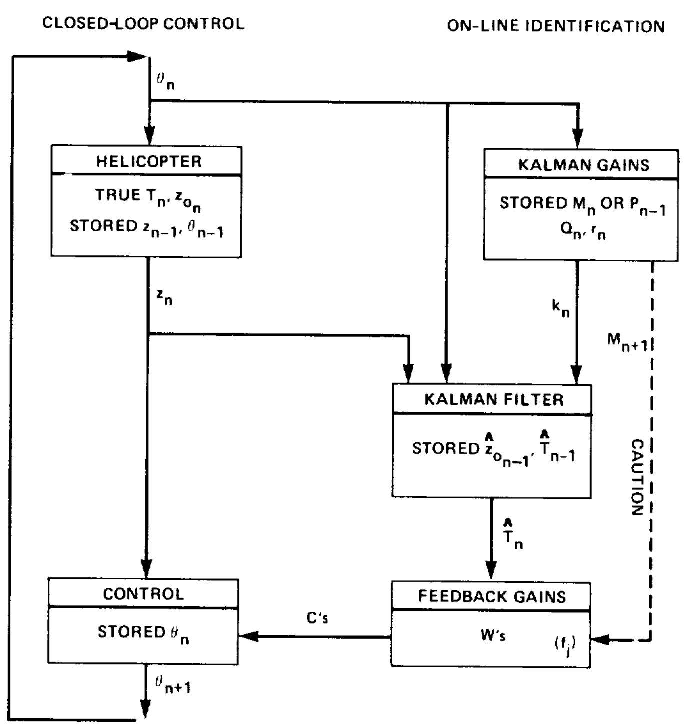

Figure 6.- Adaptive closed-loop regulator.

adaptive closed-loop regulator, on-line identification of only T (with off-line calculation of z_0) is possible. The global model with on-line identification of only z_0 is not appropriate for the adaptive closed-loop regulator, since the system then reduces to the invariable closed-loop option. Finally, the local model is not appropriate with off-line identification, and it cannot be used for the adaptive open-loop regulator (which requires z_0).

In summary, the global model with identification of both z_0 and T can be used with all four regulator options. Identifying only T is possible for all options, except the adaptive open-loop (z_0 must still be estimated, of course, by direct measurement for the invariable options or by off-line identification for the adaptive option). On-line identification of only z_0 can be used for the adaptive open-loop regulator, and the local model can be used for the adaptive closed-loop regulator.

Previous Work

Previous work has been reported on the application of each of these four regulator options. Table 1 summarizes these investigations (see also the discussion of previous work in the Introduction). The work has been both experimental and theoretical. The identification techniques considered include the off-line least-squares method; recursive estimation by a Kalman filter; and direct inversion of the measured data (when the number of independent control inputs equals the number of controls). The controller gains have been determined by (1) minimizing a quadratic performance function, sometimes with weights on the measurements and constraints on the control; (2) minimizing the expected value of the performance function (the cautious controller); and (3) by direct inversion of the T-matrix (when the number of measurements equals the number of control variables).

The invariable open-loop regulator has received the most attention. Although these investigations have been based primarily on experimental data, there has been

TABLE 1.- SUMMARY OF PAST WORK ON REGULATORS FOR HELICOPTER VIBRATION ALLEVIATION

Regulator type	Reference	Investigation	Identification	Controller	Notes
Invariable open loop	Kretz et al. (1973a,b)	Experiment	Least squares	$J = z^T z$	
	McCloud and Kretz (1974)	Experiment	Least squares	$J = z^T z$	
	McCloud (1975)	Theory	Least squares	$J = z^T W_z z$	
	McCloud and Weisbrich (1978)	Experiment	Least squares	$J = z^T W_z z$	
	Brown and McCloud (1980)	Experiment	Least squares	$J = z^T W_z z + \theta^T W_\theta \theta$	
	Sissingh and Donham (1974)	Experiment	Direct inverse	Direct inverse	
	Powers (1978)	Experiment	Various methods	Direct inverse	
	Wood et al. (1980)	Experiment	Various methods	Direct inverse	
Invariable closed loop	Shaw and Albion (1980)	Experiment	Direct inverse	Direct inverse	$\Delta t = 2$ rev
	Shaw (1980)	Theory	Direct inverse	Direct inverse	
Adaptive open loop	Hammond (1980)	Experiment	Kalman filter, T and z_0	$J = z^T W_z z$, caution	$\Delta t = 1$ rev, W_θ[a]
	Molusis et al. (1981)	Experiment	Kalman filter, T and z_0; or only z_0	$J = z^T W_z z$, caution	Rate limit, W_θ[a]
Adaptive closed loop	Shaw (1980)	Theory	Kalman filter, local T	Direct inverse	
	Taylor, Farrar, and Miao (1980) Taylor et al. (1980)	Theory	Kalman filter, local T	$J = z^T W_z z$	$\Delta t = 1$-2 rev, rate limit, $W_{\Delta\theta}$[a]

[a]Included in the theoretical development, but not used in applications.

no experimental confirmation of the regulator performance when the open-loop control is applied. The open-loop control required has been calculated using the measured T-matrix, and the response resulting when this control is applied has been predicted. The investigations have considered blade loads (McCloud and Kretz, 1974; Kretz et al., 1973a,b; McCloud, 1975; McCloud and Weisbrich, 1978), hub reactions (McCloud, 1975; Sissingh and Donham, 1974; Powers, 1978; Wood, Powers, and Hammond, 1980), and accelerations (McCloud and Weisbrich, 1978; Brown and McCloud, 1980).

There has been both experimental and theoretical work on the invariable closed-loop regulator, including some direct experimental verification (Shaw and Albion, 1980). The vibration was successfully nulled at one speed, but not at higher or lower speeds, because the control authority was exceeded. The transient characteristics of the controller were good.

The performance of the adaptive open-loop regulator has been tested in terms of its starting response, ability to track speed variations, and behavior with collective changes (Molusis et al., 1981). There has been partially successful experimental verification of the vibration reduction using this regulator (Hammond, 1980; Molusis et al., 1981). Three quantities were controlled, using the three swashplate inputs, but only two were significantly reduced; the roll moment (Hammond, 1980) or lateral acceleration (Molusis et al., 1981) was only slightly reduced, or was even increased. Molusis et al. (1981) considered six controllers in the adaptive open-loop class: deterministic, cautious, and dual forms with both T and z_0 identified; and a deterministic form with only z_0 identified, including perturbation and proportional-integral feedback variants. In their terminology, the first three controllers were called adaptive and the last three were called gain-scheduled (since the gains depend only on T). The perturbation variant should be identical to the basic controller with only z_0 identified, since the T-matrix perturbation was not included in the controller-gain calculation. The cautious controller was most satisfactory, achieving the minimum vibration within a few iterations after starting, with smooth control variations and no indication of drift in steady conditions. The controller tracked well with velocity changes. The deterministic controller was more erratic in operation; introduction of a rate limit produced smooth control variations, but the system was then considered too sluggish. The regulators with only z_0 being identified were not successful in reducing the vibration (it was conjectured that the failure was due to nonlinearity in the T-matrix).

The performance of the adaptive closed-loop regulator has been examined in terms of the starting response (Taylor et al., 1980) and the response to abrupt changes in the parameters (Shaw, 1980). There has been no experimental verification of this regulator. Only the local model has been considered. Taylor et al. (1980) found that a limit on the maximum control increment was needed for smooth operation ($|\Delta\theta| < 0.1°$ in a single step, for each harmonic). They also examined the influence of the time-step and measurement noise on the system performance. An update interval of two revolutions produced somewhat smoother response, but the system converged faster with a one revolution increment. With a measurement noise level above 20% of the uncontrolled response, the regulator did not converge. Taylor et al. (1980) actually identified z_{n-1} as well as T, which leads to inconsistencies. They considered the helicopter model in the form

$$z_n = [T \quad z_{n-1}] \begin{pmatrix} \Delta\theta_n \\ 1 \end{pmatrix}$$

Hence at the $(n - 1)$th step, z_{n-1} was a measurement; at the nth step, z_{n-1} became a parameter. Furthermore, the model of the parameter variation used with the Kalman filter gives $z_{n-1} = z_{n-2} + u_{z_{n-2}}$, which is not consistent with the helicopter model, $z_{n-1} = z_{n-2} + T(\theta_{n-1} - \theta_{n-2})$.

ANALYSIS AND SIMULATION

The regulators that have been defined in the preceding sections must be analyzed in more detail, in order to further develop useful designs for helicopter vibration alleviation. Of concern regarding the on-line identification are the transient behavior and convergence; identifiability; and the selection of the parameters in the algorithm. Of concern for the controller are the interpretation and selection of the weights in the performance function; the possible use of cautious or dual controllers; and the stability and steady-state performance of the controlled system, including the effects of measurement noise, parameter-estimation errors, and nonlinear or time-varying parameters. Some of these issues will be examined here by considering a system with only one measurement and one control. The actual helicopter problem involves sine and cosine components for each harmonic, and usually at least three harmonics or variables for the input and output. The single-input and single-output case is useful however, because of the simplifications that result from dealing with scalar equations. The identification proceeds by rows in fact, so with a single input there actually is a scalar equation in the identification algorithm for each measurement. The weighting matrix W_z then is used to balance the control of the various output variables (the specific influence of W_z depends on the elements of T). For the general multi-input and multi-output case, it is necessary to deal directly with the matrix equation given above.

The characteristics of the regulators will be analyzed in the following sections by examining the equations for the case of a system with only one measurement and one control. In addition, numerical simulations were performed for several of the cases. The general behavior exhibited in the numerical simulations will be described, although it is not considered appropriate to present quantitative results from simulations of such a simple system.

Open-Loop Control

For open-loop control, involving feedback of the uncontrolled vibration level, the deterministic controller is

$$\theta_n = C\hat{z}_0 + C_{\Delta\theta}\theta_{n-1} = (-\hat{T}W_z\hat{z}_0 + W_{\Delta\theta}\theta_{n-1})/(\hat{T}^2W_z + W_\theta + W_{\Delta\theta})$$

With a single output, W_z is not relevant; it is retained however, to aid in the interpretation of the parameters for the multivariate case. For the invariable open-loop regulator, $W_{\Delta\theta}$ must be zero; for the adaptive open-loop regulator, $\hat{T}$ and $\hat{z}_0$ are the estimates at the $(n - 1)$th step. The stability of this controller is determined by the eigenvalue

$$\lambda = W_{\Delta\theta}/(\hat{T}^2W_z + W_\theta + W_{\Delta\theta})$$

The corresponding time-constant is

$$\tau = \Delta t\lambda/(1 - \lambda) = \Delta t W_{\Delta\theta}/(\hat{T}^2 W_z + W_\theta)$$

so the time lag is directly proportional to $W_{\Delta\theta}/T^2 W_z$. The time lag is reduced by W_θ, but

$$\frac{\tau/\Delta t}{\theta_{ss}/\hat{\theta}_0} = \frac{W_{\Delta\theta}}{\hat{T}^2 W_z}$$

where θ_{ss} is the steady-state response (given below). So the rate limit, which is proportional to θ_{ss}/τ, is independent of W_θ.

The steady-state limit of the controller is

$$\theta_n = -\hat{T}W_z\hat{z}_0/(\hat{T}^2 W_z + W_\theta)$$

(which is reached immediately if $W_{\Delta\theta} = 0$). Now define θ_0 as the solution of $z = z_0 + T\theta = 0$, and let $T_0 = T$ at $\theta = \theta_0$. Similarly, $\hat{\theta}_0$ is the solution of the equation using the estimated parameters, $z = \hat{z}_0 + \hat{T}\theta = 0$; hence $\hat{\theta}_0 = -\hat{z}_0/\hat{T}$. Then the steady-state control is

$$\theta/\hat{\theta}_0 = 1/(1 + W_\theta/\hat{T}^2 W_z)$$

and the system response is

$$z/z_0 = (1 - T\hat{z}_0/\hat{T}z_0 + W_\theta/\hat{T}^2 W_z)/(1 + W_\theta/\hat{T}^2 W_z)$$

The response in the presence of parameter errors depends on both $\hat{T}$ and $\hat{z}_0$, which may have either canceling or reinforcing effects. Hence, it is clearer to write the response in terms of the error in the estimate of θ_0:

$$z/z_0 = (1 - T\hat{\theta}_0/T_0\theta_0 + W_\theta/\hat{T}^2 W_z)/(1 + W_\theta/\hat{T}^2 W_z)$$

With no estimation errors, the result is

$$z/z_0 = (W_\theta/T^2 W_z)/(1 + W_\theta/T^2 W_z) = 1 - \theta/\hat{\theta}_0$$

Note that a feedback control law of the form $\theta = -Kz$ gives $z/z_0 = 1/(1 + KT)$, which implies $KT = T^2 W_z/W_\theta$. Thus, W_θ may be interpreted as the inverse of the gain.

Closed-Loop Control

For closed-loop control, involving feedback of the measured vibration, the deterministic controller is

$$\begin{aligned}\theta_n &= Cz_{n-1} + (I - C_\theta)\theta_{n-1} \\ &= [-\hat{T}W_z z_{n-1} + (\hat{T}^2 W_z + W_{\Delta\theta})\theta_{n-1}]/(\hat{T}^2 W_z + W_\theta + W_{\Delta\theta})\end{aligned}$$

where $\hat{T}$ is the estimate at the (n - 1)th step. To examine the closed-loop performance, substitute $z_{n-1} = z_0 + T\theta_{n-1} + v_{n-1}$, to obtain

$$\theta_n = \frac{1}{\Delta}\{-\hat{T}W_z z_0 + [\hat{T}(\hat{T} - T)W_z + W_{\Delta\theta}]\theta_{n-1} - \hat{T}W_z v_{n-1}\}$$

where

$$\Delta = T^2 W_z + W_\theta + W_{\Delta\theta}$$

It is assumed that the parameters vary slowly enough to be considered constant for the present purposes. Nonlinear behavior of the real system will be allowed however, so $T = T(\theta_{n-1})$. Note there are no dynamics in the helicopter model, since the quasi-static assumption gives a model of the form $z_n = f(\theta_n)$. The dynamics are introduced by the control law. An equation for the response is obtained by substituting $\theta_n = (z_n - z_0)/T$ in the controller equation, and including the measurement noise in z_{n-1}. For a linear system the result is

$$z_n = \frac{1}{\Delta}\{W_\theta z_0 + [\hat{T}(\hat{T} - T)W_z + W_{\Delta\theta}]z_{n-1} - T\hat{T}W_z v_{n-1}\}$$

Here z is the true response of the system, without the measurement noise.

For the ideal case, a linear system with no estimation errors, these equations reduce to

$$\theta_n = \frac{1}{\Delta}(-TW_z z_0 + W_{\Delta\theta}\theta_{n-1} - TW_z v_{n-1})$$

$$z_n = \frac{1}{\Delta}(W_\theta z_0 + W_{\Delta\theta}z_{n-1} - T^2 W_z v_{n-1})$$

The eigenvalue is

$$\lambda = W_{\Delta\theta}/(T^2 W_z + W_\theta + W_{\Delta\theta})$$

and the steady-state solution is

$$\theta/\theta_0 = 1/(1 + W_\theta/T^2 W_z)$$

$$z/z_0 = (W_\theta/T^2 W_z)/(1 + W_\theta/T^2 W_z)$$

If the system starts with $\theta_1 = 0$, then $z_1 = z_0$ and

$$\theta_2 = -TW_z z_0/(T^2 W_z + W_\theta + W_{\Delta\theta})$$

$$z_2 = z_0(W_\theta + W_{\Delta\theta})/(T^2 W_z + W_\theta + W_{\Delta\theta})$$

It is observed that the eigenvalue is the same as in the open-loop case. The time lag is again determined by $W_{\Delta\theta}$. This ideal system is always stable ($|\lambda| < 1$). The steady-state solution is also the same as that for the open-loop case (for no estimation errors); hence, the interpretation of W_θ is the same. If $W_{\Delta\theta} = 0$, the steady-state solution is reached immediately (at $n = 2$). The steady-state response to the measurement noise is

$$\sigma_\theta^2/r = (TW_z/\Delta)^2/[1 - (W_{\Delta\theta}/\Delta)^2]$$

$$= (TW_z)^2/[(T^2W_z + W_\theta)(T^2W_z + W_\theta + 2W_{\Delta\theta})]$$

and $\sigma_z = T\sigma_\theta$. Here σ_θ^2 and σ_z^2 are the mean-squared responses of θ_n and z_n to the noise v_n, which is a Gaussian random variable with zero mean and variance $E(v^2) = r$. These results can be written

$$(\sigma_\theta^2/\theta^2)/(r/z_0^2) = (T^2W_z + W_\theta)/(T^2W_z + W_\theta + 2W_{\Delta\theta})$$

$$(\sigma_z^2/z_0^2)/(r/z_0^2) = (\theta/\theta_0)^2(T^2W_z + W_\theta)/(T^2W_z + W_\theta + 2W_{\Delta\theta})$$

which are both order one. The rate limit $W_{\Delta\theta}$ reduces the response to measurement noise.

For a linear system with estimation errors, the eigenvalue is

$$\lambda = [\hat{T}(\hat{T} - T)W_z + W_{\Delta\theta}]/(T^2W_z + W_\theta + W_{\Delta\theta})$$

$$= (1 - T/\hat{T} + W_{\Delta\theta}/\hat{T}^2W_z)/[1 + (W_\theta + W_{\Delta\theta})/\hat{T}^2W_z]$$

For stability, $|\lambda| < 1$, it is thus necessary that

$$-W_\theta/\hat{T}^2W_z < T/\hat{T} < 2 + (W_\theta + 2W_{\Delta\theta})/\hat{T}^2W_z$$

If $W_\theta = W_{\Delta\theta} = 0$, the criterion is that $0 < T/\hat{T} < 2$; that is, the estimated value of T must have the same sign and at least 50% the magnitude of the true value. It is found that this result is unchanged by the addition of the steady-state Kalman-filter dynamics. The control constraints W_θ and $W_{\Delta\theta}$ improve the stability range. The steady-state solution is now

$$\theta = (-z_0/T)/(1 + W_\theta/T\hat{T}W_z)$$

$$z = z_0(W_\theta/T\hat{T}W_z)/(1 + W_\theta/T\hat{T}W_z)$$

So with feedback of z_{n-1}, the steady-state response is not sensitive to estimation errors, except for a change in the influence of W_θ. Specifically, with $W_\theta = 0$, the null response $z = 0$ is always achieved. Feedback of the measured vibration introduces a response to measurement noise however, and estimation errors do influence the system stability. The stability range is large, but on-line identification is still likely to be required for the helicopter problem.

For a nonlinear system, $T(\theta_{n-1})$ must be used in the equation for θ_n. The steady-state solution is still given by the above equations. Now an explicit equation for the control is not possible, since T depends on θ; but still, $z = 0$ is achieved if $W_\theta = 0$. Convergence and stability of the nonlinear equation for θ_n are difficult to examine analytically, but the local stability can be calculated.

Consider the nonlinear equation $\theta_n = a + b(\theta_{n-1})\theta_{n-1}$, and a solution θ_n^* of this equation for the specified initial conditions. For a perturbation about this solution, $\theta_n = \theta_n^* + \delta\theta_n$, the linearized equation is

$$\delta\theta_n = (b + \theta\ \partial b/\partial\theta)\delta\theta_{n-1}$$

where the coefficient is evaluated at $\theta = \theta_{n-1}^*$. Hence the eigenvalue is

$$\lambda = b + \theta b' = \frac{1}{\Delta}\ [\hat{T}(\hat{T} - T - \theta T')W_z + W_{\Delta\theta}]$$

for the present problem, and the stability criterion becomes

$$-W_\theta/\hat{T}^2W_z < (T + \theta T')/\hat{T} < 2 + (W_\theta + 2W_{\Delta\theta})/\hat{T}^2W_z$$

So the estimate $\hat{T}$ must be close to $T + \theta T'$, which is just the local slope of the response $z(\theta)$.

Caution and Learning

Accounting for uncertain parameters in the performance function has the effect for open-loop control of replacing W_θ with $(W_\theta + \lambda_c M_{tt} W_z)$, and introducing the offset

$$c_0 = -\lambda_c D M_{tz} W_z$$

if z_0 is identified. For the cautious controller, $\lambda_c = 1$. The open-loop control algorithm than becomes

$$\theta_n = (-\hat{T}W_z\hat{z}_0 - M_{tz}W_z + W_{\Delta\theta}\theta_{n-1})/(\hat{T}^2W_z + W_\theta + W_{\Delta\theta} + M_{tt}W_z)$$

which has the steady-state solution

$$\theta/\hat{\theta}_0 = (1 + M_{tz}/\hat{T}\hat{z}_0)/(1 + W_\theta/\hat{T}^2W_z + M_{tt}/\hat{T}^2)$$

$$\frac{z}{z_0} = \frac{1 - (T\hat{\theta}_0/T_0\theta_0)(1 + M_{tz}/\hat{T}\hat{z}_0) + W_\theta/\hat{T}^2W_z + M_{tt}/\hat{T}^2}{1 + W_\theta/\hat{T}^2W_z + M_{tt}/T^2}$$

With parameter errors there is a shift in the control required to obtain the minimum expected value of J. The control is always decreased by an error in $\hat{T}$, while M_{tz} can produce a control change in either direction.

For closed-loop control, $W_{\Delta\theta}$ is replaced by $(W_{\Delta\theta} + \lambda_c M_{tt} W_z)$, where again $\lambda_c = 1$ for the cautious controller. The control algorithm becomes

$$\theta_n = [-\hat{T}W_z z_{n-1} + (\hat{T}^2W_z + W_{\Delta\theta} + M_{tt}W_z)\theta_{n-1}]/(\hat{T}^2W_z + W_\theta + W_{\Delta\theta} + M_{tt}W_z)$$

The steady-state solution is not influenced by the caution in this case, but the response to measurement noise will be decreased and the stability range increased.

The effectiveness of the caution is determined by the normalized squared errors, $M_{tt}/\hat{T}^2$ and $M_{zt}/\hat{T}\hat{z}_0$. For example, an rms error of 20% gives $M_{tt}/\hat{T}^2 = 0.04$, which would produce only small changes in θ and z. To have a significant influence of the caution, $M_{tt}/\hat{T}^2$ must be order one, which implies an extremely large estimation error. It is probable that the regulator performance would be unacceptable with such large errors. The recursive identification algorithm is often started with extremely large values of P_0 (typically $P_0/T^2 \doteq 10$). The caution would then be active at the start and would have the effect of limiting or smoothing the control until the parameters are estimated well. However, W_θ or $W_{\Delta\theta}$ could be used instead, with the same effect. Also, the identifiability problem with a closed-loop system can lead to large estimation errors for individual parameters, which would be reflected in large values of the error variance P (although P/T^2 could still be small). The caution could become effective in this case.

Recall that a form of learning controller can be obtained by using $\lambda_c < 0$. The control diverges at $\lambda_c M_{tt}/\hat{T}^2 = -1$ (if $W_\theta = W_{\Delta\theta} = 0$). For the learning to be effective however, the magnitude of $\lambda_c M_{tt}/\hat{T}^2$ must be order one. Hence this learning controller operates near the divergence, and with a much reduced stability range for the closed-loop control. These characteristics make this type of dual controller unacceptable.

Adaptive Identification

To examine the characteristics of the recursive parameter identification, consider the algorithm for the identification of T in the global helicopter model:

$$\hat{T}_n = \hat{T}_{n-1} + k_n(z_n - z_0 - \hat{T}_{n-1}\theta_n)$$

where

$$m_n = p_{n-1} + q_{n-1}$$

$$p_n^{-1} = m_n^{-1} + \theta_n^2/r_n \;, \quad \text{or} \quad p_n = m_n r_n/(r_n + m_n\theta_n^2)$$

$$k_n = p_n\theta_n/r_n = m_n\theta_n/(r_n + m_n\theta_n^2)$$

The generalized least-squares solution is obtained by setting $q = 0$:

$$p_n^{-1} = p_{n-1}^{-1} + \theta_n^2/r_n = p_0^{-1} + \sum_{i=1}^{n} \theta_i^2/r_i \cong \sum_{i=1}^{n} \theta_i^2/r_i$$

$$k_n = \frac{\theta_n/r_n}{\sum_{i=1}^{n} \theta_i^2/r_i}$$

For the least-squares case, r_n = constant; therefore, $k_n = \theta_n/\Sigma\theta_i^2$ approaches zero as n increases. This algorithm is not appropriate for recursive identification because it shuts off. As data are acquired, each new piece of information is worth less and less compared with the accumulated knowledge. In contrast, by using the exponential window method, each succeeding measurement is weighted more highly. With $r_n = \alpha^n (0 < \alpha < 1)$, the solution for $p_n^* = p_n/\alpha^n$ is

$$1/p_n^* = \alpha/p_{n-1}^* + \theta_n^2 = \alpha^n/p_0 + \sum_{i=1}^{n} \theta_i^2\alpha^{n-i} \cong \sum_{i=0}^{n-1} \theta_{n-i}^2\alpha^i$$

$$k_n = p_n^*\theta_n = \frac{\theta_n}{\sum_{i=0}^{n-1} \theta_{n-i}^2\alpha^i}$$

Note that

$$\sum_{i=0}^{n-1} \alpha^i = \frac{1 - \alpha^n}{1 - \alpha}$$

so in the steady-state limit, when θ is constant for the closed-loop system, $p_n^* \cong \theta^2/(1 - \alpha)$ and $k \cong (1 - \alpha)/\theta$. In numerical simulations of the exponential-window filter, good results were obtained when α was small.

The Kalman filter is obtained for $q > 0$. In the limit of no measurement noise, $r = 0$, the solution is $p_n = 0$ and $k_n = 1/\theta_n$ (which is independent of q). This result reflects too much confidence in the latest measurement, but it is useful for comparison with the general solutions. The properties of the Kalman filter can be examined in the steady-state limit (disregarding the question of whether the steady state exists). The system is controlled, so θ is constant in that limit. Hence the equation for p has the solution

$$p/q = (1/2)[-1 + (1 + 4r/\theta^2q)^{1/2}]$$

and $k\theta = p\theta^2/r = (p/q)(\theta^2q/r)$. For small r/θ^2q then:

$$p/q \cong r/\theta^2q - (r/\theta^2q)^2$$

$$k\theta \cong 1 - r/\theta^2q$$

and for large r/θ^2q:

$$p/q \cong (r/\theta^2q)^{1/2}$$

$$k\theta \cong 1/(r/\theta^2q)^{1/2}$$

Note that $0 < p/q < r/\theta^2q$ for finite and nonzero r/θ^2q, so $0 < k\theta < 1$ or $|k| < 1/|\theta|$. Hence the Kalman-gain k is always smaller than the solution for zero measurement noise ($r = 0$).

When $z_n = z_0 + t\theta_n + v_n$ is substituted, the Kalman filter becomes

$$\hat{T}_n = (1 - k_n\theta_n)\hat{T}_{n-1} + k_n\theta_nT + k_nv_n$$

The dynamics of this equation will be examined separately, although in fact the filter and controller form a coupled system. Note that in general $(1 - k_n\theta_n) = p_n/m_n$, which is positive and less than 1. In the steady-state limit, $k_n\theta_n$ is constant, so the filter equation is time-invariant with eigenvalue $\lambda = 1 - k\theta$. Recall $0 < k\theta < 1$, so $0 < \lambda < 1$; the filter is stable. For small $r/\theta^2 q$

$$\lambda \cong r/\theta^2 q \cong 0$$

and for large $r/\theta^2 q$

$$\lambda \cong 1 - 1/(r/\theta^2 q)^{1/2} \cong 1$$

Now $\lambda \cong 0$ implies immediate convergence, $\hat{T}_n = T$; and $\lambda \cong 1$ implies no convergence at all, $\hat{T}_n = \hat{T}_{n-1}$. Hence a large value of q is better than a large value of r. The limit $q/r = 0$ is the least-squares algorithm, so the tracking ability will be poor for large $r/\theta^2 q$. The converged solution of the filter is $\hat{T} = T$, unless $\lambda = 1$ exactly. The filter response to the measurement noise is

$$\sigma_t^2/q = (r/\theta^2 q)/(1 + 4r/\theta^2 q)^{1/2}$$

The mean-square response of the identified parameter, σ_t^2, is large for large values of $r/\theta^2 q$.

In general, the solution for small values of $\varepsilon = r/\theta^2 q$ is

$$p_n/q_{n-1} \cong 1/(1 + \theta_n^2 q_{n-1}/r_n) + O(\varepsilon^3)$$

$$\cong r_n/\theta_n^2 q_{n-1} + O(\varepsilon^2)$$

So p/q is order ε small always. This result gives the steady-state solution and the limit $r = 0$ properly too. If the solution is started with $1/p_0 = 0$, then $p_1 = r_1/\theta_1^2$. So p_n immediately has the order required for the small $r/\theta^2 q$ solution.

The general solution for large values of $r/\theta^2 q$ is

$$p_n/q_{n-1} \cong (p_{n-1}/q_{n-1} + 1) - (\theta_n^2 q_{n-1}/r_n)(p_{n-1}/q_{n-1} + 1)^2 + O(\pi^3\varepsilon^2)$$

where $\pi = p/q$ and $\varepsilon = \theta^2 q/r$. This equation is valid only if $|\pi| < \varepsilon^{-1}$, that is, if p/q is smaller than order ε^{-1}. Indeed, this equation eventually gives p/q order $\varepsilon^{-1/2}$, which is the proper steady-state solution also. If p/q is order ε^{-1}, however, the solution to lowest order is

$$q_{n-1}/p_n \cong q_{n-1}/p_{n-1} + \theta_n^2 q_{n-1}/r_n + O(\pi^{-2})$$

Again, $1/p_0 \cong 0$ gives $p_1 = r_1/\theta_1^2$. So this order ε^{-1} solution is encountered at the start of the algorithm, and whenever p/q becomes too large. If the order ε^{-1} solution proceeds for N steps, then

$$q_{n-1}/p_n = \sum_{N \text{ steps}} \theta_i^2 q_{i-1}/r_i \doteq N\,\varepsilon$$

So eventually p/q becomes smaller than order ε^{-1}, and the order $\varepsilon^{-1/2}$ solution is encountered. The number of steps that the solution remains order ε^{-1} is inversely proportional to ε. In the limit $\varepsilon = 0$, the order ε^{-1} solution is always valid; in fact, the equation above is the exact solution for $\varepsilon = q\theta^2/r = 0$ (namely, the least-squares limit).

For the global model, it is anticipated that

$$\theta^2 q/r = (\theta^2/\theta_0^2)(q/T^2)/(r/z_0^2)$$

will be order one for typical values of q and r. For the identification of z_0, the corresponding parameter

$$q/r = (q/z_0^2)/(r/z_0^2)$$

should also be order one. With the local model however, "θ" is interpreted as $\Delta\theta$, and

$$\Delta\theta^2 q/r = (\Delta\theta^2/\theta_0^2)(q/T^2)/(r/z_0^2) \triangleq (\Delta\theta/\theta_0)^2$$

will be small. Hence, difficulties may be anticipated with identification of the local model. In numerical simulations of this case, the estimate of the local T diverged in the presence of measurement noise (the filter shuts off without measurement noise). The controlled response remained acceptable however. The system was stable because the estimate of T was greater than the true value. The divergence of the estimate was not eliminated by the use of $W_{\Delta\theta}$, a large value of q, a limit on the minimum magnitude of $\Delta\theta$, or caution. The estimate did not diverge when T varied with time, although the tracking ability of the filter was not as good as when the global model was used.

Identifiability

When both z_0 and T are identified, the Kalman filter is

$$\begin{pmatrix} T_n \\ \hat{z}_{0n} \end{pmatrix} = \begin{bmatrix} 1 - k\theta_n & -k \\ -k_z\theta_n & 1 - k_z \end{bmatrix} \begin{pmatrix} \hat{T}_{n-1} \\ \hat{z}_{0n-1} \end{pmatrix} + \begin{pmatrix} k \\ k_z \end{pmatrix} z_n$$

In the steady state, where θ and the Kalman gains are constants, the eigenvalues of this equation are $\lambda = 1$ and $\lambda = 1 - k_z - k\theta$, with corresponding eigenvector matrix

$$\begin{pmatrix} T \\ z_0 \end{pmatrix} = \begin{bmatrix} -1 & k \\ \theta & k_z \end{bmatrix}$$

As discussed above, the $\lambda = 1$ eigenvalue reflects the problem of identifying two parameters with one measurement for a closed-loop system. In numerical simulations of the above equations, the identified parameters diverged in the presence of measurement noise, with $\hat{z}_0 + \hat{T}\theta$ remaining near the correct value. The estimated value of T was large, so the controller was stable, and the controlled response remained acceptable. The use of $W_{\Delta\theta}$ or caution did not improve the identifiability. The estimates did not diverge when the parameters varied with time, although the undamped $(\hat{z}_0 + \hat{T}\theta)$ mode was still evident.

Some of the phenomena encountered by Molusis et al. (1981) in tests of an adaptive open-loop regulator are consistent with the use of a cautious controller in the context of the identifiability problems with a closed-loop system. They were identifying 42 parameters, using six measurements, so a large parameter variance P would be expected in steady-state conditions. Direct evidence of the identifiability problem is provided by the fact the T-matrix obtained by on-line identification was not the same as that obtained by off-line identification, and by the observation that the T-matrix identified on-line was not repeatable. A large value of P would make the caution effective, which is consistent with the fact that the cautious controller was smoother than the deterministic controller, and with the observation that the weighting matrix W_z influenced the solution (Hammond, 1980) (those are the influences of the effective control weight W_θ due to caution). That the roll moment or lateral acceleration was not reduced could also be due to a large effective value of W_θ. The observation that during speed changes the controller actually maintained the vibration below the levels obtained in steady conditions is consistent with reduced caution resulting from improved identifiability when the parameters vary. There are however other factors, such as nonlinearity, that might account for some of these effects. The argument is also contradicted by the statement that the same stabilized condition was approached with the deterministic controller as with the cautious controller (Hammond, 1980).

Adaptive Open-Loop Regulator

For the combination of open-loop control and on-line identification, the coupled controller and filter dynamics must be considered in order to analyze the regulator performance. The steady-state solution (if it exists) of the Kalman filter is

$$z = z_0 + T\theta = \hat{z}_0 + \hat{T}\theta$$

In this result, either z_0 or T may be estimated on-line, and the other parameter obtained from off-line calculations; or both may be identified on-line. A linear system is assumed, but T may be the local derivative of $z(\theta)$. The Kalman-gains influence the convergence of the filter, but not the steady-state solution. Using the open-loop control solution, the steady-state response of the regulator is then

$$\theta = [-\hat{T}W_z/(\hat{T}^2W_z + W_\theta)][z_0 + (T - \hat{T})\theta]$$

$$= (-z_0/T)/(1 + W_\theta/T\hat{T}W_z)$$

$$z = z_0(W_\theta/T\hat{T}W_z)/(1 + W_\theta/T\hat{T}W_z)$$

which is the same as the closed-loop control result. The response depends on the actual response z_0 rather than the estimate $\hat{z}_0$, because of the on-line identification. Recall that the open-loop response depends on the error in $\hat{\theta}_0$. From $z_0 + T\theta = \hat{z}_0 + \hat{T}\theta$ it follows that

$$T/\hat{T} = (z_0/\hat{z}_0)(1 + W_\theta/\hat{T}^2W_z) - W_\theta/\hat{T}^2W_z$$

$$z_0/\hat{z}_0 = (T/\hat{T} + W_\theta/\hat{T}^2W_z)/(1 + W_\theta/\hat{T}^2W_z)$$

and so

$$1 - \hat{\theta}_0/\theta_0 = 1 - T\hat{z}_0/\hat{T}z_0 = (\hat{z}_0/z_0 - 1)(W_\theta/\hat{T}^2W_z)$$

$$= (1 - T/\hat{T})(W_\theta/\hat{T}^2W_z)/(T/\hat{T} + W_\theta/\hat{T}^2W_z)$$

which is zero regardless of the parameter errors if $W_\theta = 0$. The filter identifies one or both parameters such that the measured value of z will be predicted correctly. Hence the parameter errors must compensate in the vicinity of this value of the response. With a controlled system the response will be small, so the error in the estimate of θ_0 (where $z = 0$) should be much smaller than the error in the individual parameter estimates. Thus the regulator has good closed-loop performance.

When only z_0 is identified, the equations for the filter and controller are

$$\hat{z}_{0_n} = \hat{z}_{0_{n-1}} + k_n(z_n - \hat{z}_{0_{n-1}} - \hat{T}\theta)$$

$$\theta_n = C\hat{z}_{0_{n-1}} + C_{\Delta\theta}\theta_{n-1}$$

This system is linear. Although k_n varies with time initially, it depends only on q_n and r_n, not on θ_n in this case. Assuming that q_n and r_n are constant, it is appropriate to use the steady-state solution for k_n. Substituting for $z_n = z_0 + T\theta_n + v_n$ gives the equations

$$\begin{pmatrix} \hat{z}_{0_n} \\ \theta_n \end{pmatrix} = \begin{bmatrix} 1 - k + k(T - \hat{T})C & k(T - \hat{T})C_{\Delta\theta} \\ C & C_{\Delta\theta} \end{bmatrix} \begin{pmatrix} \hat{z}_{0_{n-1}} \\ \theta_{n-1} \end{pmatrix} + \begin{pmatrix} k(z_0 + v_n) \\ 0 \end{pmatrix}$$

In the ideal case, with no error in the estimate of T, the filter is decoupled from the controller. The estimate $\hat{z}_{0_n}$ follows $z_0 + v_n$ (actually measured as $z_n - \hat{T}\theta_n$) with a first-order lag ($\lambda = 1 - k$). Then the control θ_n follows $\hat{z}_{0n}$ with a lag due to $W_{\Delta\theta}$ ($\lambda = C_{\Delta\theta}$), and the response z_n is determined by θ_n. The steady-state response of the filter to the measurement noise is

$$\sigma_0^2/r = k/(2 - k)$$

where σ_0^2 is the mean-squared response of $\hat{z}_{0n}$ to the noise v_n, which is a Gaussian random variable with zero mean and variance $E(v^2) = r$. Then the mean-squared response of the control θ_n is

$$\frac{\sigma_\theta^2}{r} = \left[\frac{C^2}{1 - C_{\Delta\theta}^2}\right]\left[\frac{1 + (1 - k)C_{\Delta\theta}}{1 - (1 - k)C_{\Delta\theta}}\right]\left[\frac{k}{2 - k}\right]$$

The first factor in σ_θ^2 identical to the response of the closed-loop controller to measurement noise. The product of the last two factors is always less than 1, so the adaptive open-loop regulator has a smaller response to noise than does the closed loop controller. The eigenvalues of the coupled system are the solutions of

$$\lambda^2 - [1 - k + k(T - \hat{T})C + C_{\Delta\theta}]\lambda + (1 - k)C_{\Delta\theta} = 0$$

If $\hat{T} = T$, the filter and controller are decoupled and the solutions are $\lambda = 1 - k$ and $\lambda = C_{\Delta\theta}$. In general, the criterion for stability ($|\lambda| < 1$) is

$$-W_\theta/\hat{T}^2 W_z < T/\hat{T} < 1 + [(2 - k)/k][1 + (W_\theta + 2W_{\Delta\theta})/\hat{T}^2 W_z]$$

If $W_\theta = W_{\Delta\theta} = 0$, the system is stable if $0 < T/\hat{T} < 2/k$. Since $k < 1$, the stability range is larger than for the closed-loop controller.

In numerical simulations of the adaptive open-loop regulator, good performance was obtained when only z_0 was identified, although the ability to handle a system with a time-varying value of T was not very good. In wind-tunnel tests however, Molusis et al. (1981) found that the regulator using identification of z_0 only was not successful in reducing the vibration.

CONCLUSIONS

Self-tuning regulators for the multicyclic control of helicopter vibration have been examined. The topics considered have included the selection of the parameters for the identification and control algorithms; the best combination of identification, control, and helicopter model options for the regulator; and the regulator performance, involving steady-state response, stability, convergence, and identifiability.

Regarding the identification algorithms, it is concluded that the values of the parameter variance and measurement noise variance must be correctly chosen for proper performance of the regulators. The use of the Kalman filter is preferred to the exponential-weighted generalized least squares, since the former is more flexible and the theory provides a guide for the choice of the parameters.

Regarding the controller algorithms, the analysis has provided guidelines for the choice of the parameters. The use of the largest value of $W_{\Delta\theta}$ that does not make the response too sluggish is appropriate, to improve the transient response, the sensitivity to measurement noise, and the sensitivity of the stability to parameter errors. The rate limit $W_{\Delta\theta}$ should always be used during the start of the recursive identification, and a small value of $W_{\Delta\theta}$ can be used to avoid the possibility of control divergence, should the estimated T-matrix be too small. The control magnitude constraint W_θ may not be too useful, since it limits the control relative to the ideal value θ_0 rather than relative to an absolute value. Absolute limits on the control magnitude are probably better applied by uniformly reducing the elements of θ before the command signals are sent to the actuators. The benefits of the cautious controller can probably be obtained more directly by the appropriate choices of W_θ and $W_{\Delta\theta}$ (which could be temporarily increased in special circumstances such as the start of the identification algorithm). The use of a dual controller does not appear necessary, since the identifiability problems encountered are not accompanied by poor controller performance.

Regarding the regulator algorithms, it is concluded that the following options (in order of preference based on simplicity) are potentially applicable to the control of helicopter vibration.

1. Invariable open-loop regulator: This is the simplest option. It has no stability problems. This regulator will be satisfactory only if there are no significant parameter errors.

2. Invariable closed-loop regulator: This option provides good performance if the measurement noise is not too large. It will be satisfactory if the parameters do not vary so much that the system becomes unstable.

3. Adaptive open-loop regulator, with only the uncontrolled vibration level identified: This is the simplest adaptive option. It has no identifiability problems, and may provide good performance and stability. This regulator will be satisfactory if it can provide good control, particularly with time-varying parameters.

4. Adaptive open-loop or closed-loop regulator, with all parameters identified: This option probably has the best performance with time-varying parameters. The open-loop and closed-loop cases are nearly indistinguishable in terms of system performance. This regulator will be satisfactory if the identifiability problems do not degrade the performance, and if the helicopter response is not too nonlinear.

5. Adaptive closed-loop regulator, with the local helicopter model: This option is probably most suitable for very nonlinear systems; however, there are identifiability problems.

The theoretical and experimental work to date concerning these regulators suggests that an adaptive system will be needed. Hence, it is anticipated that one of the last three options will be required for helicopter vibration control.

RECOMMENDATIONS

Further work on self-tuning regulators for the multicyclic control of helicopter vibration is required in the following topics.

1. The conclusions of the present report must be verified or modified for the multivariable case. The implications of the use of cosine and sine components in the input and measurements must be investigated.

2. Techniques for the numerical implementation of the identification and control algorithms must be developed.

3. The regulator development can be extended; for example, dynamics can be included in the helicopter model, and dual controllers could be considered further.

4. The regulator designs must be examined more thoroughly in terms of the specific characteristics of the helicopter, including the selection of the control and measurement variables; off-design performance or the performance with time-varying parameters; the influence of noise and nonlinearities; and the influence of the regulator on the helicopter trim and on the stability augmentation system.

Finally, the regulators require further experimental development and experimental confirmation of their performance in order to complete the development.

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1. Report No. NASA TP-1996	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle SELF-TUNING REGULATORS FOR MULTICYCLIC CONTROL OF HELICOPTER VIBRATION		5. Report Date March 1982
		6. Performing Organization Code
7. Author(s) Wayne Johnson		8. Performing Organization Report No. A-8719
9. Performing Organization Name and Address Ames Research Center, NASA Moffett Field, Calif. 94035		10. Work Unit No. 505-42-21
		11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		13. Type of Report and Period Covered Technical Paper
		14. Sponsoring Agency Code

15. Supplementary Notes

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16. Abstract

A class of algorithms for the multicyclic control of helicopter vibration and loads is derived and discussed. This class is characterized by a linear, quasi-static, frequency-domain model of the helicopter response to control; identification of the helicopter model by least-squared-error or Kalman-filter methods; and a minimum variance or quadratic performance function controller. Previous research on such controllers is reviewed and related to the present work. The derivations and discussions cover the helicopter model; the identification problem, including both off-line and on-line (recursive) algorithms; the control problem, including both open-loop and closed-loop feedback; and the various regulator configurations possible within this class. Conclusions from analysis and numerical simulations of the regulators provide guidance in the design and selection of algorithms for further development, including wind-tunnel and flight tests.

17. Key Words (Suggested by Author(s)) Helicopter; Vibration control; Self-tuning regulator		18. Distribution Statement Unclassified - Unlimited; Subject Category 08	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 51	22. Price* A04

*For sale by the National Technical Information Service, Springfield, Virginia 22161

NASA-Langley, 1982