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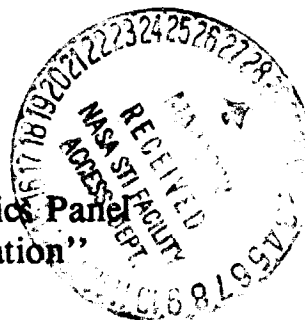
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GEOMETRY AND STARVATION EFFECTS IN HYDRODYNAMIC LUBRICATION

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SUMMARY

E-11147
 Numerical methods were used to determine the effects of lubricant starvation on the minimum film thickness under conditions of a hydrodynamic point contact. Starvation was effected by varying the fluid inlet level. The Reynolds boundary conditions were applied at the cavitation boundary and zero pressure was stipulated at the meniscus or inlet boundary. A minimum-film-thickness equation as a function of both the ratio of dimensionless load to dimensionless speed and inlet supply level was determined. By comparing the film generated under the starved inlet condition with the film generated from the fully flooded inlet, an expression for the film reduction factor was obtained. Based on this factor a starvation threshold was defined as well as a critically starved inlet. The changes in the inlet pressure buildup due to changing the available lubricant supply are presented in the form of three-dimensional isometric plots and also in the form of contour plots.

NOMENCLATURE

C_0, C_1	= least-squares coefficients
D	= difference $[(\tilde{H}_0 - H_0)/H_0] \times 100$, percent
H	= dimensionless film thickness, h/R_x
H_0	= dimensionless minimum (central) film thickness, h_0/R_x
\tilde{H}_0	= dimensionless calculated minimum (central) film thickness
H_{in}	= dimensionless fluid inlet level, h_{in}/R_x
H_{in}^*	= dimensionless fluid inlet level (onset of starvation)
h	= film thickness, m
h_0	= minimum (central) film thickness, m
I	= reduced hydrodynamic lift, dimensionless
N	= direction normal to boundary
P	= dimensionless pressure, $pR_x/\eta_0 u$
p	= pressure, N/m^2
R	= effective radius of curvature, $R_x R_y / (R_x + R_y)$, m
W/U	= ratio of dimensionless load to dimensionless speed
u	= average surface velocity in x direction, $(u_A + u_B)/2$, m/s
w	= load capacity, N
X	= dimensionless coordinate, x/R_x
x	= coordinate along rolling direction, m
Y	= dimensionless coordinate, y/R_x
y	= coordinate transverse to rolling direction, m
α	= radius ratio, R_y/R_x
β	= film reduction factor
η_0	= fluid viscosity at standard temperature and pressure, Ns/m^2
ϕ	= Archard-Cowling side-leakage factor, $1/(1 + 2/(3\alpha))$

Subscripts

cr	= critical
f	= flooded conjunction
x, y	= coordinate direction

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E-1147
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Subscripts

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The effect of starvation in a hydrodynamically lubricated conjunction can be studied by systematically reducing the inlet supply and observing the resultant pressure distribution and film thickness. This starvation effect can have a significant role in the operation of machine elements. For example, roller-end wear due to roller skewing can be a critical problem for high-speed cylindrical roller bearings. It is desirable that the hydrodynamic film generated between the roller end and the guide flange provide stiffness and damping to limit the amplitude of the roller skewing motion. However, at high rotational speeds the roller end and the flange are often subjected to a depletion in the lubricant supply due to centrifugal effects. In such cases, the minute amount of lubricant available at the roller-end-flange conjunction might well represent an example of steady-state starvation. Starvation effects in hydrodynamically lubricated contacts are important also if one wishes to calculate the rolling and sliding resistance and/or traction encountered in ball and roller bearings [1]. In another example, the effect of restricting the lubricant to a roller bearing is seen experimentally and theoretically to reduce the amount of cage and roller slip [2]. The theoretical analysis was accomplished by changing the location of the boundary where the pressure begins to buildup and noting the effect on the hydrodynamic forces. Combining this with relative velocity expressions and equilibrium equations enabled the determination of the amount of cage and roller slip.

The location of the inlet and exit boundaries as well as the respective boundary conditions to be applied has been one of the most controversial issues concerning starvation of hydrodynamic contacts. The issue of the effect of the lubricant supply on the inlet boundary condition and its consequences to incipient pressure buildup began to materialize as a result of earlier studies applied to rigid cylinders [3,4]. Lauder [5] and Tipei [6] asserted an upstream limit of the fluid film where the pressure begins to rise as governed by the Reynolds equation. This limit according to Lauder is determined by applying reverse flow boundary conditions (i.e., $u = \partial u / \partial y = 0$). Tipei locates the upstream limit as defined by the line of centers of two bounded vortices that are observed for pure rolling. Both cases have been criticized because their analyses lead to one position of pressure buildup regardless of the oil supply [7]. Dowson [8] Floberg [9,10] and most dramatically, Wedeven, Evans, and Cameron [11] provide experimental evidence supporting the idea that the location of incipient pressure rise is determined by the oil supply. Further Wedeven, et al., using a Grubin type of EHD analysis, obtained very good correlation between experiment and the theory of starvation effects by choosing the start of the pressure buildup to occur at the meniscus boundary. Oteri [12], using stream function analysis for rolling rigid cylinders, showed that incipient pressure rise occurs at the meniscus boundary even in the presence of reverse-flow conditions. In view of this work, starvation effects in machine element applications can be predicted and relied on with a greater degree of confidence.

One of the more important manifestations of lubricant starvation is the reduction in film thickness. This topic has received a good deal of attention in the literature [11, 13-21]. With the exception of [13, 19-21], these references are applicable only to elastohydrodynamic situations. Most of the work concerned with rigid contacts has been devoted to line contact applications [13,20,21]. Dalmaz and Godet [19] analyze the effect of the inlet on the film reduction factor for a sphere against a plate. However, to the authors' knowledge, an effort that parallels that of Hamrock and Dowson [14,15] for the EHD contact is absent from the rigid contact theory. In those works, an expression was determined that relates the film reduction to the inlet distance.

The current study is a resumption of a previous rigid-contact analysis [22] to extend validity for the minimum-film-thickness equation derived there over a wider range of film thicknesses as well as to include the effects of starvation in this equation. The start of the pressure buildup as determined by the Reynolds equation is assumed to occur at the inlet meniscus. The location of the cavitation boundary was determined by applying the Reynolds boundary conditions as discussed in previous work [22]. The study applies to a wide range of geometries (i.e., from a ball-on-plate configuration to a ball in a conforming groove). Seventy-four cases were used to numerically determine (1) an equation relating minimum film thickness with the fluid inlet level as well as with the dimensionless load-speed ratio and geometry, (2) an equation predicting the onset of starvation, and (3) an equation predicting the onset of a critically starved conjunction. The resulting equations are valid for dimensionless minimum film thicknesses H_0 ranging from 5.0×10^{-5} to 1.0×10^{-3} . Further, contour isobar plots and three-dimensional isometric pressure plots are presented.

Numerical Procedure

The hydrodynamic effects on the central film thickness between two rigid solids in lubricated rolling and/or sliding contact are analyzed under conditions of lubricant starvation. The effects of starvation are determined by systematically decreasing the fluid inlet level. The Reynolds boundary conditions are applied at the cavitation boundary, and zero pressure is stipulated at the meniscus or inlet boundary. The lubricant is assumed to be an incompressible Newtonian fluid under laminar, isothermal, isoviscous, and steady-state conditions. The numerical approach follows that of a previous investigation [22]. There, a fully flooded film profile was specified and a pressure distribution satisfying the Reynolds equation was determined for a given speed, viscosity, and geometry. The analysis treats the two rigid bodies as having parallel principal axes of inertia. This enables one to make a simplifying transformation to an equivalent system of a rigid solid near a plane separated by a lubricant film (Fig. 1).

Relevant equations. - The same dimensionless expressions are used here as in [22], that is,

$$X = x/R_x, Y = y/R_x, H = h/R_x \quad (1)$$

also

$$p = pR_x/\eta_0 u, \alpha = R_y/R_x$$

The Reynolds equation

$$\frac{\partial}{\partial X} \left(\mu^3 \frac{\partial P}{\partial X} \right) + \frac{\partial}{\partial Y} \left(H^3 \frac{\partial P}{\partial Y} \right) = 12 \frac{\partial H}{\partial X} \quad (2)$$

is the governing equation within the conjunction.

We recognize that, when the inlet supply levels are increased to values much greater than the minimum film thickness, calculations as governed by the Reynolds equation are inherently in error far from the center of contact. The reason is that the Reynolds equation neglects curvature of the fluid film. Dowson [23] has pointed out that the errors involved in using this equation to determine the buildup of pressure in such regions are negligible. The predicted pressures are themselves so very much smaller than the effective load-carrying pressures in the region of closest approach of the solids. The dimensionless film thickness equation is given as

$$H = H_0 + 1 - \sqrt{1 - X^2} + \alpha \left[1 - \sqrt{1 - (Y/\alpha)^2} \right] \quad (3)$$

where H is bounded above by the dimensionless fluid inlet level H_{in} and below by the dimensionless minimum film thickness H_0 (i.e., $H_0 \leq H \leq H_{in}$, e.g., (Fig. 1)). The fluid inlet level H_{in} is made dimensionless so that, if the inlet is completely filled, $H_{in} = 1$ and the conjunction is said to be fully flooded.

The Reynolds boundary conditions are used, that is, $P = \partial P / \partial N = 0$ at the cavitation boundary and $P = 0$ at the inlet boundary ($H = H_{in}$). A pressure distribution that satisfies these boundary conditions is then determined numerically by finite differencing with Gauss-Seidel successive over relaxation method. With this algorithm we are able to generate pressure distributions for given contact geometry, speed, viscosity, film thickness, and fluid inlet level.

Effect of inlet on pressure. - Figure 2 graphically portrays how a pressure distribution is affected by the lubricant supply for a dimensionless minimum film thickness of 1×10^{-4} . The two views, (a) and (b), are for a fully flooded contact ($H_{in} = 1.00$) and a starved contact ($H_{in} = 0.001$). The starvation effect on the pressure distribution might be considered rather moderate. That is to say, the effect of reducing the fluid inlet level has essentially had no effect on the magnitude of the peak pressure. Only the size of the load bearing region has been affected. The load bearing region is defined by the meniscus and the cavitation boundary.

We repeat this process of decreasing the fluid inlet level for a given minimum film thickness for a wide range of minimum film thickness values. Figure 3 illustrates a situation in which the starvation is more severe. The minimum film thickness here is ten times as thick as what is indicated in Fig. 2. Note that the pressure distribution of Fig. 3 is not as localized as in Fig. 2. Consequently decreasing the size of the load bearing region has a more noticeable effect on the load capacity. Starvation is more evident here since we also see that the peak pressure is noticeably reduced from the fully flooded condition. This sort of behavior takes place if the fluid inlet level is of the same order of magnitude as the minimum film thickness.

Generalized film thickness formula. - Now we've seen how starvation effects the pressure distribution for two different minimum film thickness values. In this analysis, we repeated this determination for minimum film thickness values ranging from 10^{-3} to 10^{-5} . In all, 74 different cases were analyzed to determine a minimum film thickness equation as a function of dimensionless load to speed ratio (W/U) and fluid inlet level.

The integration of the pressure distribution can be used in relating the hydrodynamic effects (i.e., load, speed, and viscosity) to the minimum (central) film thickness for a given fluid inlet level. In general, for a given H_0 and u the load capacity w and/or the dimensionless load-speed ratio is determined as follows:

$$w = \eta_0 u R_x \iint P \, dX \, dY \quad (4)$$

or

$$W/U = \iint P \, dX \, dY$$

where $W/U = w\eta_0 u R_x$.

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18-4 Kapitza [27], using half Sommerfeld boundary conditions derived the following relationship:

$$\frac{W}{U} = \frac{8\phi L \sqrt{2\alpha}}{\sqrt{H_0}} \quad (5)$$

where $L = \pi/2$ half-Sommerfeld B.C. (Ref. 27)

$$L = 0.131 \tan^{-1} (\alpha/2) + 1.683 \text{ Reynolds B.C. (Ref. 22)}$$

In a previous work [22], we found that if the problem is considered using Reynolds boundary conditions, then L required the above modification. But even then the equation was valid only if the pressure distribution was very localized in the case of thin films (i.e. $5 \times 10^{-5} \leq H_0 \leq 10^{-4}$). Consider, for example, a ball in rolling motion that is loaded against a flat plate. For a load-speed ratio W/U of 340, the numerically determined value (i.e., as determined by finite difference analysis) of the dimensionless film thickness is 10^{-3} (Table I). Equation (5) predicts the dimensionless film thickness to be 1.22×10^{-3} , which is in error of the numerical value by 22 percent. Consequently, we wish to revise Eq. (5) so that it is valid for the thicker films as well. This revised equation should reduce to Eq. (5) in the limit for thin films. Furthermore, the revised dimensionless film thickness should be expressed in such a way as to easily include the effects of starvation. This would then enable us to present one general expression to be presented for the dimensionless film thickness that can be used for the full range of film thicknesses for a starved conjunction as well as for a fully flooded conjunction. After the numerical analysis for each case was complete, the several curve fits or regression curves of H_0 on W/U were considered that would be consistent with these above requirements. The most suitable curve fit considered was in the form of a more general linear equation, that is,

$$\frac{1}{\sqrt{H_0}} = C_1 \frac{W}{U} + C_0 \quad (6)$$

Generalized film thickness formula (applicable to starved as well as fully flooded conjunctions). - The effect of lubricant starvation on the hydrodynamic film thickness was observed by varying the fluid inlet level to the contact and noting the effect on load capacity for five different film thicknesses for the ball-on-plate contact (i.e., $\alpha = 1.00$). In addition to the fully flooded data, 55 computer-generated data points were used to arrive at a family of equations having the form given in Eq. (6). An equation for each fluid inlet level was determined by performing a linear regression by the method of least squares. Table II lists for each fluid inlet level the values of the coefficients C_1 and C_0 , the coefficient of determination r^2 , and D_{\max} , the maximum percentage of error D defined as

$$D = \frac{\bar{H}_0 - H_0}{H_0} \times 100 \quad (7)$$

Note that C_1 remains essentially unchanged, and can be determined from

$$C_1 = \frac{1}{\phi L (128\alpha)^{1/2}} \quad (8)$$

Furthermore, all of the effect of starvation is described in the value of C_0 . An expression for the coefficient C_0 as a function of the fluid inlet level H_{in} would enable a determination of a generalized minimum-film-thickness formula that applies to starved as well as fully flooded conditions. A close examination of the variation of C_0 with H_{in} in Table II reveals that $C_0 \approx (2/H_{in})^{1/2}$ for the severely starved situations. As H_{in} approaches 1, C_0 approaches a value very nearly equal to e (i.e., the base of the natural systems of logarithms, 2.718). This suggests $e^{H_{in}}$ as a modulating factor. Further considerations for the nearly flooded inlet levels show that

$$C_0 e^{-H_{in}} \approx [(2 - H_{in})/H_{in}]^{1/2}$$

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Finally for the full range of values, C_0 varies with H_{in} as follows:

$$C_0 = 1.11 e^{H_{in}} \left(\frac{2 - H_{in}}{H_{in}} \right)^{1/2} \quad (9)$$

Thus, our generalized minimum film thickness formula in terms of geometry (i.e., radius ratio α), load-speed ratio W/U , and fluid inlet level H_{in} , can be written as

$$\bar{H}_0 = \left[\frac{W/U}{\phi L (128\alpha)^{1/2}} + 1.11 \left(\frac{2 - H_{in}}{H_{in}} \right)^{1/2} e^{H_{in}} \right]^{-2} \quad (10)$$

The measure of agreement between the calculated and input values of H_0 is represented by the value of D (Eq. (7)) and presented in Table 1. Table I shows that, when the fluid inlet level is of the same order of magnitude as the minimum film thickness, the error that results from using Eq. (10) becomes larger. However, Eq. (10) can confidently be used for the full range of minimum film thickness if the fluid inlet level is such that $0.004 \leq H_{in} \leq 1.000$. For very thin films (i.e., $H_0 \leq 10^{-4}$) Eq. (10) can be useful throughout the full range of fluid inlet levels that were investigated (i.e., $0.001 \leq H_{in} \leq 1.000$). Note that the film thickness formula is intended to be used only for a range of speeds and loads in which piezoviscous and deformation effects are negligible. It was determined that these effects can be significant for minimum film thicknesses less than 5.0×10^{-5} .

Note also from Table I that excellent agreement is obtained for the near-line-contact applications (i.e., $\alpha = 36.54$) even though these data were not used in the determination of the above Eq. (10). Most of the predictions by Eq. (10) are within 3 percent of the numerically determined values and do not exceed 6 percent for any case.

Equation (10) is equally valid for the fully flooded conjunction as well as for the starved conjunction. That is, if we set $H_{in} = 1.00$, then Eq. (10) reduces to

$$\tilde{H}_{0,f} = \left(\frac{W/U}{\phi L \sqrt{128\alpha}} + 3.02 \right)^{-2} \quad (11)$$

Equation (11) should be used in place of Eq. (5) whenever a calculation is made for a fully flooded conjunction since it is valid for a broader range of film thicknesses (i.e., 5×10^{-5} to 1×10^{-3}). Table I shows that the error for the full range of minimum film thicknesses investigated (for $H_{in} = 1$) is less than 1 percent for $\alpha = 1$. For the near-line-contact geometry (i.e., $\alpha = 36.54$) the error does not exceed 3.36 percent.

Reduction in minimum film thickness. - It is now possible to determine the reduction in minimum film thickness from the fully flooded value if the fluid inlet level is known. This can be done by inserting Eq. (11) into Eq. (10).

$$\tilde{H}_0 = \left\{ \frac{1}{\sqrt{\tilde{H}_{0,f}}} + 3.02 \left[\sqrt{\frac{2 - H_{in}}{H_{in}}} e^{(H_{in}-1)} - 1 \right] \right\}^{-2} \quad (12)$$

Dividing both sides of the equation by H_0 gives

$$\beta \equiv \frac{\tilde{H}_0}{H_0} = \left\{ 1 + 3.02 \sqrt{\tilde{H}_{0,f}} \left[\sqrt{\frac{2 - H_{in}}{H_{in}}} e^{(H_{in}-1)} - 1 \right] \right\}^{-2} \quad (13)$$

where β is the reduction in minimum film thickness due to starvation.

RESULTS AND DISCUSSION

Effect of starvation on pressure distribution. - The discussion of lubricant starvation can be facilitated by focusing on one of the simplest geometric arrangements (i.e., a ball rolling and/or sliding against a flat plate) as shown in Fig. 2. The figure compares the pressure distribution determined numerically for the fully-flooded inlet with the most severely starved inlet ($H_{in} = 0.001$). The comparison is made for a constant minimum film thickness (i.e., $H_0 = 1.0 \times 10^{-4}$). Note that the pressure peak built up in the starved inlet is only slightly smaller than that of the fully flooded inlet. However, the area of pressure build-up is considerably smaller, and so the starved inlet is unable to support as much load for a given film thickness as the fully-flooded inlet.

Figure 3 provides the same sort of comparison but for a thicker minimum film (i.e., $H_0 = 1.0 \times 10^{-3}$). The significant difference between the two figures is that the starved inlet for the thicker film has a more pronounced effect on the pressure peak. The fluid inlet level ($H_{in} = 0.002$) for the thicker film represents a relatively more highly starved inlet since H_{in} is of the order of H_0 in this case. The other feature to be noticed in comparing Figs. 2 and 3 is that the pressure distribution is more evenly spread out for the thicker film. Thus, changes in the meniscus (or integration domain) are going to have a more noticeable effect on the load-carrying capacity. Note also that because of the boundary conditions the integration domain takes on a "kidney-shaped" appearance. This is more clearly shown in the isobaric contour plot shown in Fig. 4.

Minimum film thickness equation. - Thus far we have compared the pressure build up in a severely starved inlet with that in a fully flooded inlet for two minimum film thicknesses. Our investigation, however, included several fluid inlet levels for a variety of minimum film thicknesses. The results are summarized in Table I.

A generalized minimum film thickness formula (Eq. (10)) was derived from the results of Table I. Figure 5 indicates how well the equation represents the computer generated data in the table. It was not possible to display all these results in Fig. 5. However, the figure is representative of the overall results. The equation fits the data quite well except when the fluid inlet level is of the same order of magnitude as the minimum film thickness. Based on the discussion concerning peak pressure, it would seem that the formula holds well for those cases in which the degree of starvation is such that peak pressure is not significantly reduced.

Of course, it would be most desirable to compare the data in Table I with experimental data. To the authors' knowledge, the only available experimental data were obtained by Dalmaz and Godet [25]. To compare Eq. (10) with experiment, the data from [25] were replotted in Fig. 6. The experimental data were taken under lightly loaded (rigid contact), isoviscous conditions for pure sliding of a ball on a plate. The fluid inlet level in these experiments was reported to be 1 millimeter. The ball diameter was 30 millimeters; consequently the dimensionless fluid inlet level H_{in} was 0.067. The experimental data were presented as a plot of the dimensionless parameters H_0/WG versus $U/Wl.5G^{0.5}$. The materials parameter G in the plot was included so as to accommodate the elastohydrodynamic range in a more general way.

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Here, we wish to compare our hydrodynamic starvation theory only with the hydrodynamic results of Dalmaz and Godet. To do this, the ordinate and abscissa were properly scaled (assuming a reasonable value of $WG = 4.5 \times 10^{-4}$) so that the minimum film thickness could be plotted against the dimensionless load-speed ratio as shown in Fig. 6. The solid line in Fig. 6 is a plot of Eq. (10) for $\alpha = 1$ and $H_{in} = 0.067$. The dashed line represents a previous theory [22] in which the reduced inlet level due to starvation was not considered. The present theory (Eq. (10)) shows better agreement with the experimental results than our previous theory [22] for lower values of W/U . This is because under conditions of low load and/or high speed, the pressures become more significant away from the center of the contact. Consequently, neglecting the size of the inlet introduces an increasing amount of error as W/U is decreased.

Although consideration of the size of the inlet domain improves the agreement of the theory with experiment for the thicker films, still further improvement is possible. It is believed that the effects of reverse flow in the inlet must be included in the theory to obtain better agreement. If reverse flow is considered, not all the available lubricant determined by the fluid inlet level will pass through the contact. The hydrodynamic contact would essentially see this as a reduction in supply from what actually is there. In other words, the inlet is more severely starved than we have taken into account. From Fig. 5, we see that increasing the severity of starvation has more effect on the load capacity for thicker films (i.e., $H_0 = 10^{-3}$) than it does for thinner films (i.e., $H_0 = 10^{-4}$). Consequently, reverse-flow considerations should improve agreement between theory and experiment for the thicker films while still maintaining good agreement in the thin-film range.

Lubricant film thickness reduction factor and onset of starvation. - Of practical importance to lubricant starvation is the reduction in minimum film thickness from the fully flooded value. Equation (13) is a derived expression for β in terms of the fluid inlet level and the fully flooded film thickness (also given by Eq. (11)). Figure 7 is a plot of β as a function of the fluid inlet level H_{in} for several values of $H_{0,f}$. It is of interest to determine a fully flooded-starved boundary (i.e., that fluid inlet level after which any further decrease causes a significant reduction in the film thickness). Hamrock and Dowson [14] determined this boundary for elastohydrodynamic (EHD) applications upon satisfying the following condition:

$$1 - \beta \Big|_{H_{in} = H_{in}^*} = 0.03 \quad (14)$$

The value of 0.03 was used in Eq. (14) since it was ascertained that the data in Table I were accurate to only ± 3 percent.

Thus, for a given value of $H_{0,f}$, one can solve for a value of H_{in}^* that satisfies Eq. (14). A suitable relationship between H_{in}^* and $H_{0,f}$ can be obtained by generating a table of values (e.g., Table III) and fitting a power curve by the method of least squares. This gives

$$H_{in}^* = 4.11(H_{0,f})^{0.36} \quad (15)$$

Thus, we have an equation that determines the onset of starvation. That is, for $H_{in} > H_{in}^*$ a fully flooded condition exists, whereas for $H_{in} < H_{in}^*$ a starved condition exists.

Critically starved inlet. - In certain bearing applications, the power loss resulting from churning of the oil may be higher than the power loss resulting from friction of the bearing alone [26]. These power losses can be minimized by reducing the lubricant supply until a loss in film thickness causes the friction losses to increase. According to the results shown in Fig. 7, the fluid inlet level can be decreased substantially without adversely affecting the minimum film thickness. Consequently, it might be advantageous to operate the bearing with a lubricant supply just sufficient to preclude any drastic reductions in minimum film thickness, as seen in the figure. Such a critical fluid inlet level might well be defined to occur at the knee of the curve, that is,

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$$\left. \frac{d(\rho)}{dH_{in}} \right|_{H_{in} = H_{in,cr}} = 1 \quad (16)$$

Solutions to this expression for several values of minimum film thickness are listed in Table III. A power curve fit by the method of least squares gives

$$H_{in,cr} = 0.875(H_{O,f})^{0.2977} \quad (17)$$

The regression coefficient for this fit was determined to be 0.9999, indicating an extremely good fit.

CONCLUDING REMARKS

Numerical methods were used to determine the effects of lubricant starvation on the minimum film thickness under conditions of hydrodynamic point contact. Starvation was effected by varying the fluid inlet level. The Reynolds boundary conditions were applied at the cavitation boundary, and zero pressure was stipulated at the meniscus or inlet boundary. The analysis is considered valid for a range of speeds and loads for which thermal, piezoviscous, and deformation effects are negligible. It can be applied to a wide range of geometries (i.e., from a ball-on-plate configuration to a ball in a conforming groove). Seventy-four cases were used to numerically determine

(1) A generalized expression for the minimum film thickness as a function of dimensionless load-speed ratio, geometry, and fluid inlet level (Eq. (10)). The expression should be applied for film thicknesses in the range $1.0 \times 10^{-3} > H_0 > 1.0 \times 10^{-4}$ and for fluid inlet levels of $0.004 \leq H_{in} \leq 1.00$. For $5 \times 10^{-5} < H_0 < 10^{-4}$ the equation can be applied for a fluid inlet level of $0.001 \leq H_{in} \leq 1.00$.

(2) A film thickness reduction factor (Eq. (13)) expressed as a function of the degree of starvation (or fluid inlet level) for a given fully flooded film thickness value.

(3) An equation (Eq. (15)) that determines the onset of starvation.

(4) An equation (Eq. (17)) that determines a critically starved contact. Contour bar plots and three-dimensional isometric plots also presented.

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TABLE I* - COMPARISON OF DIMENSIONLESS FILM THICKNESS VALUES FROM EQUATION (10)
WITH DIMENSIONLESS FILM THICKNESS VALUES INPUT TO COMPUTER

Dimensionless fluid inlet level, H_{in}	Dimensionless minimum film thickness, H_0 ; radius ratio, α															
	$H_0 = 1 \times 10^{-3}$; $\alpha = 1.00$				$H_0 = 5 \times 10^{-4}$; $\alpha = 1.00$				$H_0 = 1 \times 10^{-4}$; $\alpha = 1.00$							
	Load-speed ratio, M/U	Equation (10) H_0	Error, D, percent	Load-speed ratio, M/U	Equation (10) H_0	Error, D, percent	Load-speed ratio, M/U	Equation (10) H_0	Error, D, percent	Load-speed ratio, M/U	Equation (10) H_0	Error, D, percent				
1.000	339.57	0.9948×10^{-3}	-0.52	3646.70	0.9843×10^{-3}	-1.57	495.54	4.9644×10^{-4}	-0.71	1153.59	0.9906×10^{-4}	-0.94				
.750	339.07	.9965	-0.35				495.03	4.9703	-0.59	1153.05	.9912	-0.88				
.500	336.70	1.0005	+0.05				492.68	4.9842	-0.32	1150.64	.9925	-0.75				
.250	328.75	1.0050	+0.50				484.57	5.0029	+0.06	1142.56	.9941	-0.59				
.150	318.46	1.0121	+1.21				474.73	5.0194	+0.39	1132.68	.9956	-0.44				
.070	295.67	1.0253	+2.53				453.65	5.0326	+0.65	1106.41	1.0056	+0.56				
.035	265.79	1.0361	+3.61				421.79	5.1078	+2.16	1077.99	1.0064	+0.64				
.020	234.65	1.0345	+3.45				387.20	5.1703	+3.41	1044.01	1.0107	+1.07				
.010	184.47	1.0142	+1.42				332.44	5.1886	+3.77	982.81	1.0235	+2.35				
.004	89.74	.9481	-5.19				232.77	5.0363	+0.73	862.58	1.0462	+4.62				
.002	25.96	.7170	-28.31				127.97	4.7331	-5.34	735.62	1.0563	+5.63				
.001	4.54	.3990	-46.80				37.09	3.5855	-28.29	567.75	1.0489	+4.89				
Dimensionless fluid inlet level, H_{in}	$H_0 = 1 \times 10^{-4}$; $\alpha = 36.54$				$H_0 = 5 \times 10^{-5}$; $\alpha = 1.00$				$H_0 = 1 \times 10^{-5}$; $\alpha = 1.00$				$H_0 = 1 \times 10^{-5}$; $\alpha = 36.54$			
1.000	12 430.93	0.9728×10^{-4}	-2.71	1646.74	4.9495×10^{-5}	-1.00	3706.19	1.0007×10^{-5}	+0.07	40 269.42	0.9664×10^{-5}	-3.36				
.750	12 428.08	.9730	-2.71	1646.28	4.9514	-0.97	3705.65	1.0009	+0.09							
.500	12 403.78	.9740	-2.60	1643.92	4.9558	-0.88	3703.29	1.0013	+0.13							
.250	12 373.80	.9746	-2.54	1635.68	4.9674	-0.75	3695.02	1.0019	+0.19							
.150	12 224.02	.9752	-2.48	1626.06	4.9662	-0.68	3685.42	1.0022	+0.22							
.070	12 042.07	.9698	-3.02	1606.17	4.9633	-0.73	3665.43	1.0020	+0.20							
.035	11 552.18	.9994	-0.06	1570.61	5.0085	-0.17	3648.45	.9961	-0.39							
.020	11 169.70	1.0069	+0.69	1536.34	5.0256	+0.51	3613.91	.9978	-0.22							
.010	10 503.93	1.0216	+2.16	1474.84	5.0721	+1.44	3552.69	1.0017	+0.17							
.004	7 869.48	1.0422	+4.22	1372.16	5.0430	+0.86	3447.13	1.0005	+0.05							
.002	6 127.10	1.0535	+5.35	1221.52	5.2245	+4.49	3294.44	1.0176	+1.76							
.001		1.0378	+3.78	1041.53	5.2763	+5.53	3108.58	1.0253	+2.53							

* (From ref. 28).

At this value of M/U was determined using $H_0 = 7.5 \times 10^{-4}$, since $H_0 = H_{in}$ results in no load capacity.

TABLE II* - CONSTANTS APPEARING IN FILM THICKNESS EQUATION
(EQ. (6)) FOR EACH FLUID INLET LEVEL

Dimensionless fluid inlet level, H_{in}	Least-squares coefficients		Coefficient of determination, r^2	Maximum percentage of error in film thickness determination (eq. (7)), U_{max}
	C_1	C_0		
1.000	0.08455	2.6511	0.99999	+1.67
.750	.08455	2.6931	↓	+1.67
.500	.08455	2.8941		+1.67
.250	.08456	3.5720		+1.61
.150	.08454	4.4226		+1.78
.070	.08447	6.3850	↓	+1.68
.035	.08414	9.2635		1.00000
.020	.08418	12.0388	1.00000	-1.06
.010	.08440	16.6239	.99999	-3.51
.004	.08452	25.2892	.99990	-7.47
.002	.08630	33.5023	.99934	-21.72
.001	.08878	43.2636	.99737	-30.07

*(From ref. 28)

TABLE III* - DIMENSIONLESS FLUID INLET VALUES THAT DETERMINE STARVED - FULLY FLOODED BOUNDARY AND CRITICALLY STARVED BOUNDARY FOR SEVERAL VALUES OF MINIMUM FILM THICKNESS FOR A FLOODED CONJUNCTION

Dimensionless fluid inlet values	Dimensionless minimum film thickness, for a flooded conjunction, $H_{0,f}$						
	5×10^{-5}	7.5×10^{-5}	1×10^{-4}	2.5×10^{-4}	5×10^{-4}	7.5×10^{-4}	1×10^{-3}
H_{in}^*	0.107	0.130	0.148	0.212	0.265	0.297	0.320
$H_{in,cr}$.046	.052	.056	.074	.091	.103	.112

*(From ref. 28)

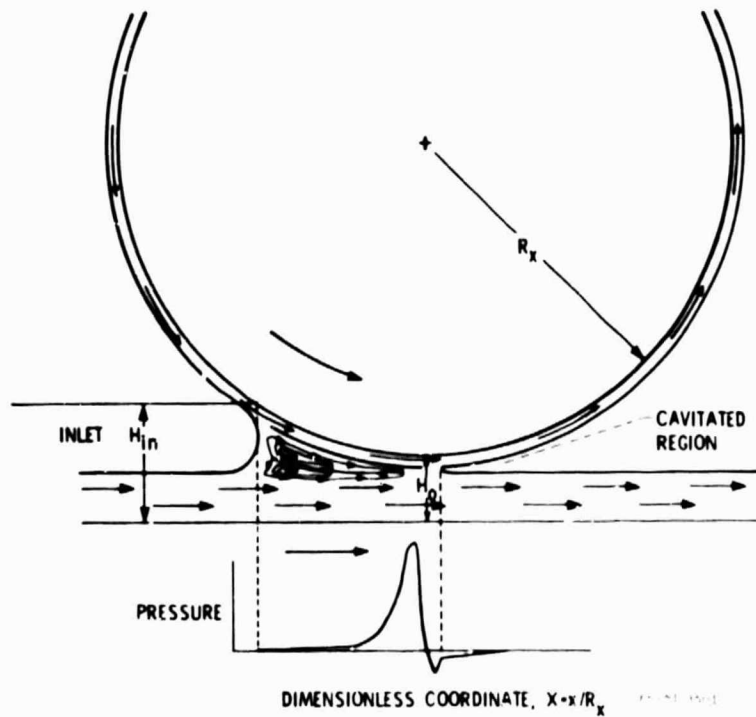
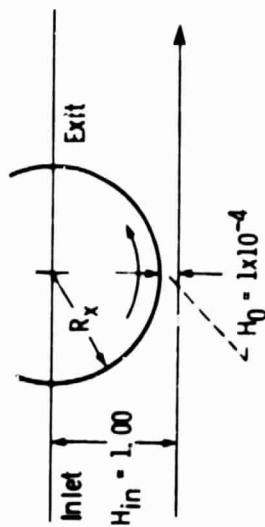
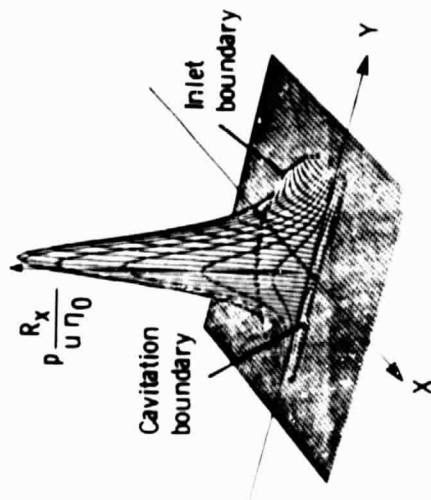
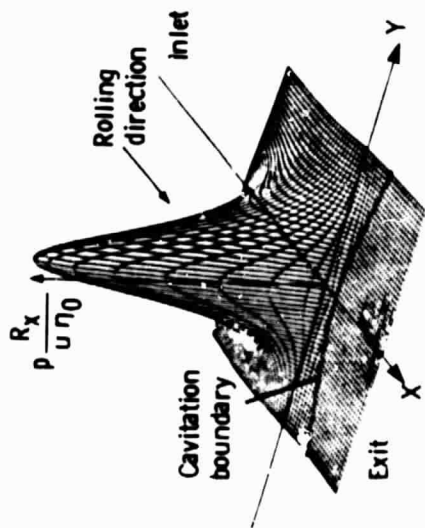
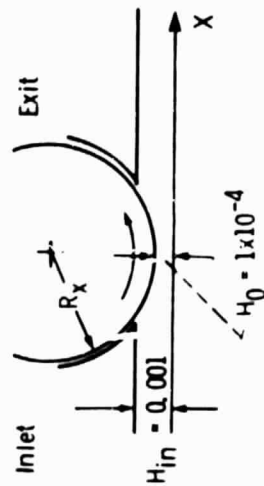


Figure 1. - Depiction of lubricant flow for a rolling/sliding contact and corresponding pressure build-up.

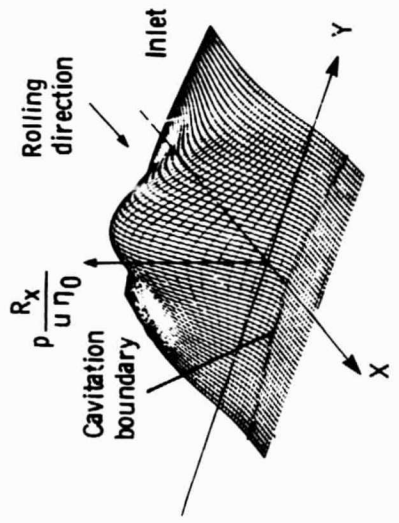


(a) Fully flooded condition: dimensionless fluid inlet level $H_{in} = 1.00$.

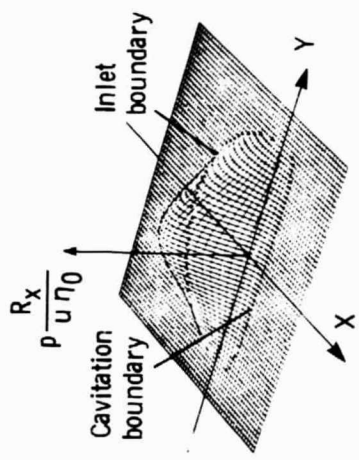


(b) Starved condition: dimensionless fluid inlet level $H_{in} = 0.001$.

Figure 2 - Three-dimensional representation of pressure distribution, comparing starved with fully flooded conjunction for dimensionless minimum film thickness $H_0 = 1 \times 10^{-4}$.



(a) Fully flooded condition: dimensionless fluid inlet level $H_{in} = 1.00$.



(b) Starved condition: dimensionless fluid inlet level $H_{in} = 0.002$.

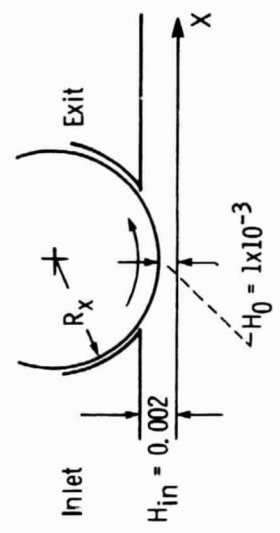
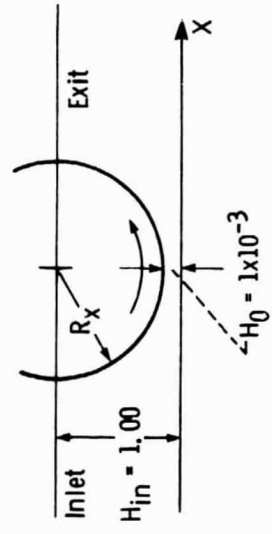


Figure 3. - Three-dimensional representation of pressure distribution, comparing starved with fully flooded conjunction for dimensionless minimum film thickness $H_0 = 1 \times 10^{-3}$.

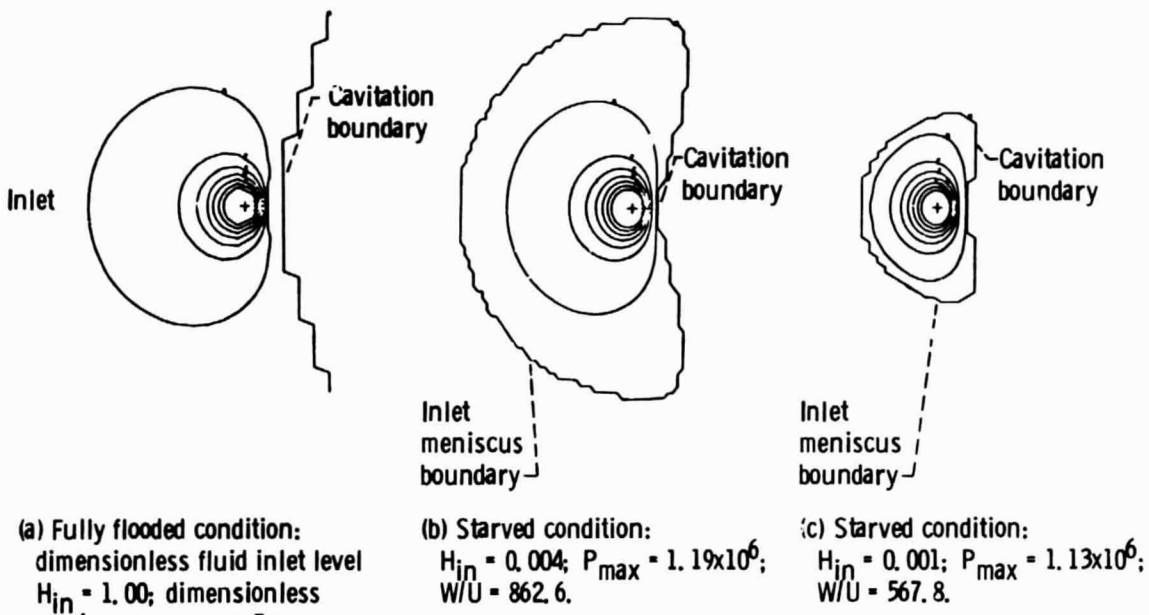


Figure 4. - Isobaric contour plots for three fluid inlet levels for dimensionless minimum film thickness $H_0 = 1 \times 10^{-4}$.

Dimensionless fluid inlet level,
 H_{in}

- 1.000
- ◇ .035
- ◐ .010
- ◑ .004
- △ .002
- ◓ .001
- Eq. (10)

$$\tilde{H}_0 = \left[\frac{W/U}{\phi L (128a)^{1/2}} + 1.11 \left(\frac{2 - H_{in}}{H_{in}} \right)^{1/2} e^{H_{in}} \right]^{-2}$$

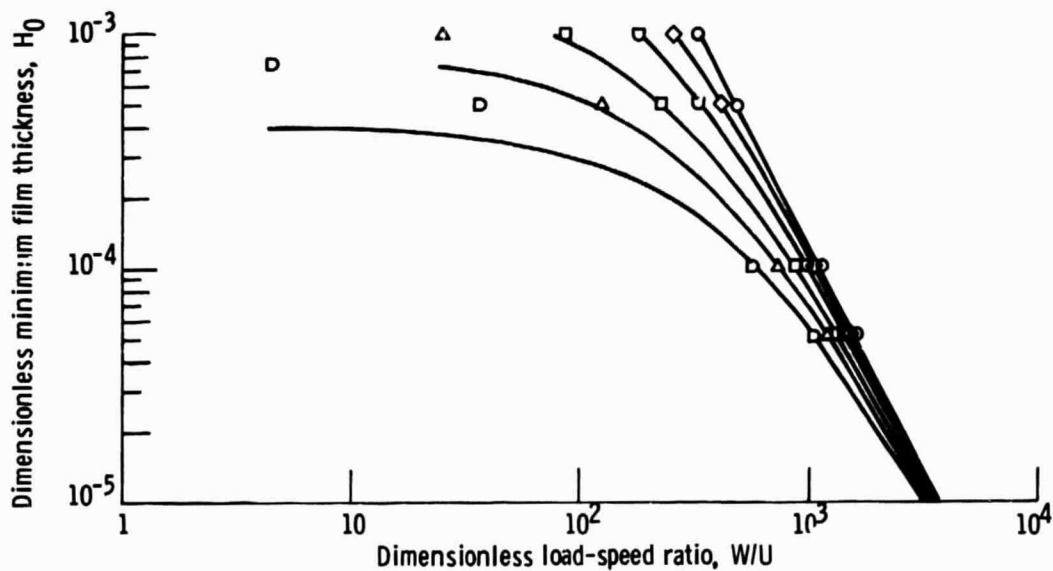


Figure 5. - Comparison of dimensionless minimum film thickness equation (eq. (10)) with computer-generated data as a function of dimensionless load-speed ratio for several values of dimensionless fluid inlet level.

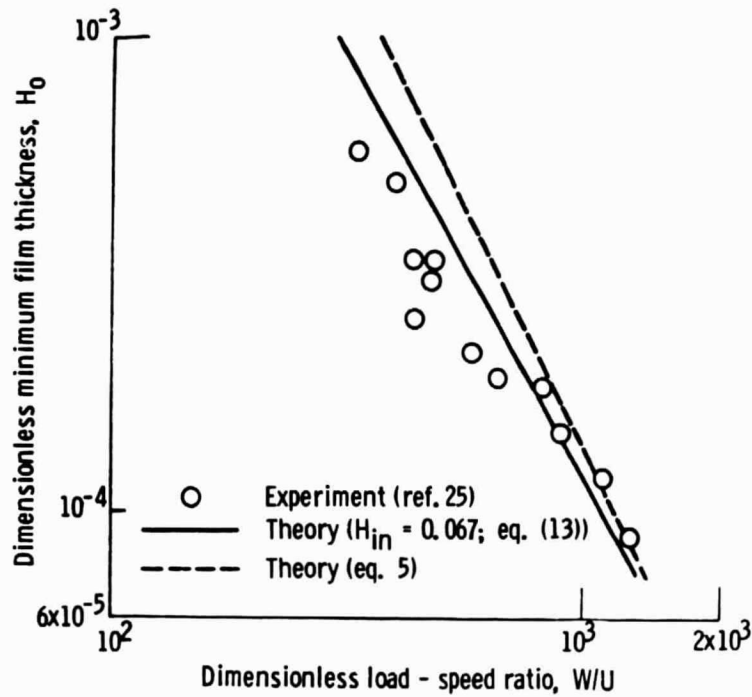


Figure 6. - Theoretical and experimental results.

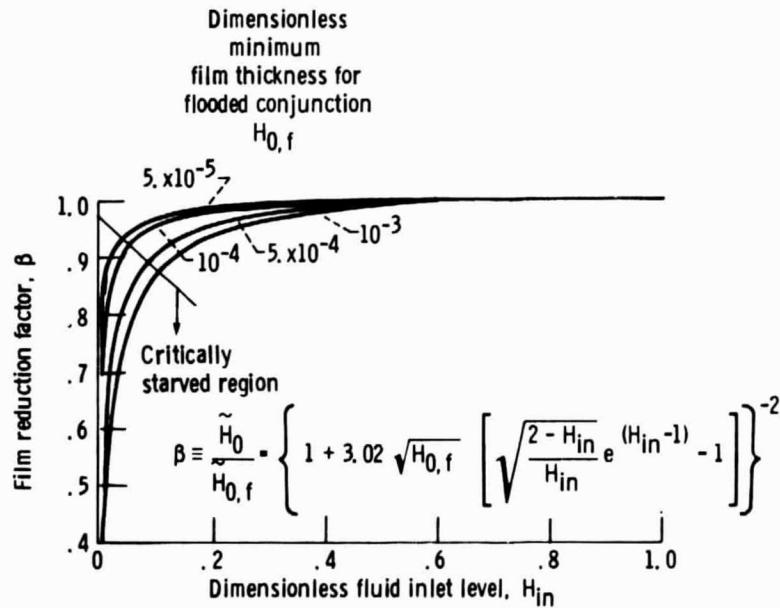


Figure 7. - Minimum film thickness reduction factor as a function of fluid inlet level for several values of dimensionless minimum film thickness for flooded conjunction.