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Advanced Space Shuttle Simulation Model

Frank B. Tatom and S. Ray Smith

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1. INTRODUCTION

The effects of atmospheric turbulence in both horizontal and near-horizontal flight, during the return of the Space Shuttle, are important for determining design, control, and "pilot-in-the-loop" effects. A non-recursive model (based on von Karman spectra) for atmospheric turbulence along the flight path of the Shuttle Orbiter has been developed which provides for simulation of instantaneous vertical and horizontal gusts at the vehicle center-of-gravity, and also for simulation of instantaneous gusts gradients. Based on this model the time series for both gusts and gust gradients have been generated and stored on a series of magnetic tapes which are entitled Shuttle Stmulation Turbulence Tapes (SSTT). The time series are designed to represent atmospheric turbulence from ground level to an altitude of 120,000 meters,

A description of the turbulence generation procedure is provided in Secti'on 2, The results of validating the simulated turbulence are described in Section 3, Conclustons and recommendations are presented in Section 4 with Section 5 containing references ctted. Appendix A contains the tabulated one-dimensional von Karman spectra while Appendix B provides a discussion of the minimum frequency stmulated. Appendices C and D present the results of spectral and statistical analyses of the SSTT. A more detailed description of the proper use of the tapes is provided elsewhere [1].

2. TURBULENCE GENERATION PROCEDURE

The non-recursive turbulence model used to generate the SSTT is based on von Karman spectra with finite upper limits corresponding to the dimensions of the Space Shuttle, relative to the scale of turbulence in the atmosphere. Because the scale of turbulence increases with altitude while the dimensions of the Space Shuttle are fixed, the finite upper limits of the von Karman spectra increase with altitude. In order to take into account the resulting spectral changes, the atmosphere, extending from ground level to 120,000 meters, was divided into six altitude bands. The subsections which follow provide a description of the development and application of the turbulence generation procedures.

2.1 BACKGROUND

The current turbulence model represents the results of the development and evaluation of several different turbulence simulation techniques. Two of the earlier techniques, which were given serious consideration, warrant further discussion.

Initial efforts involved refinement and evaluation of a turbulence model, TBMOD [2], which had been developed elsewhere [3]. This model was based on discretization of the Fourier integral representation of turbulence and was designed for use with von Karman spectra. Several problems were encountered with TBMOD, both theoretical and practical. First, from a theoretical standpoint, the assumptions used in the development of the shear (gust gradient) simulation were difficult to justify. Second, from a practical standpoint, the output of TBMOD, representing turbulent gusts, when subjected to Fast Fourier Transform (FFT) spectral analysis, did not possess the proper von Karman spectral shape. Because of such problems, further development of the model was halted.

The second simulation technique was based on digital filter theory coupled with a combined von Karman - Saffman spectral model [4]. Meromorphic functions were used to approximate the various spectra. Based on z-transform theory, recursive difference equations were derived from such approximations. Such difference equations were then used to generate

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the appropriate turbulence gusts and gust gradients. Unfortunately the recursive difference equations, in addition to being somewhat complex, proved numerically unstable and could not be used in their original recursive form [5].

A non-recursive version of the same difference equations was subsequently developed in an effort to overcome the stability problem [5]. These difference equations were also quite complex but resembled in form the output of digital filters, characterized by some impulse response function, with a white noise input. The present model, as described in subsections 2.2 through 2.4, was an outgrowth of this resemblance.

2.2 SELECTION OF ATMOSPHERIC BANDS

The standard deviations (σ_1, σ_2, σ_3) and the integral scale lengths (L_1, L_2, L_3) of atmospheric turbulence are functions of altitude, as shown in Table 2-1. Notice should be taken that the values for σ_i and L_i presented in this table are consistent with those presented in JSC 7700 [6]. Based on the variation of σ_i and L_i presented in Table 2-1, the atmosphere was divided into six altitude bands as presented in Table 2-2. Within each band, as also indicated in Table 2-2, characteristic integral scales of turbulence were selected for use in calculating the finite upper limit of the turbulence spectral model discussed in subsection 2.3.

2.3 DEVELOPMENT OF VON KARMAN SPECTRA WITH FINITE UPPER LIMITS

As developed previously [5] the basic three-dimensional von Karman relation to be integrated for the dimensionless gust spectra is,

$$\Phi_{ii}(\Omega_1,\Omega_2,\Omega_3) = \frac{55}{36a\pi^2}\frac{(\Omega^2-\Omega_i^2)}{(1+\Omega^2)^{17/6}} \tag{2-1}$$

The corresponding von Karman relation for dimensionless gust gradient spectra is

$$\Phi_{ii/jj}(\Omega_1,\Omega_2,\Omega_3) = \frac{55}{36\pi^2 a^3}\frac{\Omega_j^2(\Omega^2-\Omega_i^2)}{(1+\Omega^2)^{17/6}} \tag{2-2}$$

These three-dimensional spectral relations must be integrated over certain ranges of values of Ω_2 and Ω_3 to obtain one-dimensional spectral models $\Phi_{ii}(\Omega_1)$ and $\Phi_{ii/jj}(\Omega_1)$.

TABLE 2-1. VARIATION OF STANDARD DEVIATION AND LENGTH SCALE WITH ALTITUDE*

ALTITUDE (m)	STANDARD DEVIATION OF TURBULENCE			INTEGRAL SCALES OF TURBULENCE		
	σ_1(m/sec)	σ_2(m/sec)	σ_3(m/sec)	L_1(m)	L_2(m)	L_3(m)
10	2.31	1.67	1.15	21	11	5
20	2.58	1.98	1.46	33	19	11
30	2.75	2.20	1.71	43	28	17
40	2.88	2.36	1.89	52	35	23
50	2.98	2.49	2.05	61	42	29
60	3.07	2.61	2.19	68	49	35
70	3.15	2.71	2.32	75	56	41
80	3.22	2.81	2.43	82	63	47
90	3.28	2.89	2.54	89	69	53
100	3.33	2.97	2.64	95	75	59
200	3.72	3.53	3.38	149	134	123
304.8	3.95/4.37	3.95/4.37	3.95/4.39	196/300	190/300	192/300
400	4.39	4.39	4.39	300	300	300
500	4.39	4.39	4.39	300	300	300
600	4.39	4.39	4.39	300	300	300
700	4.39	4.39	4.39	300	300	300
762	4.39/5.70	4.39/5.70	4.39/5.70	300/533	300/533	300/533
800	5.70	5.70	5.70	533	533	533
900	5.70	5.70	5.70	533	533	533
1524	5.70/5.79	5.70/5.79	5.70/5.79	533	533	533
2000	5.79	5.79	5.79	533	533	533
3048	5.79/5.52	5.79/5.52	5.79/5.52	533	533	533
4000	5.52	5.52	5.52	533	533	533
5000	5.52	5.52	5.52	533	533	533
6096	5.52/5.27	5.52/5.27	5.52/5.27	533	533	533
7000	5.27	5.27	5.27	533	533	533
8000	5.27	5.27	5.27	533	533	533
9144	5.27/4.22	5.27/4.22	5.27/4.22	533	533	533
10000	4.22	4.22	4.22	533	533	533
20000	6.01	6.01	4.22	6691	6691	955

* Double entries for a tabulated altitude indicate a step change in standard deviation or integral scale at that altitude.

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TABLE 2-1. VARIATION OF STANDARD DEVIATION AND LENGTH SCALE WITH ALTITUDE (Continued)

RLTITUDE (m)	STANDARD DEVIATION OF TURBULENCE			INTEGRAL SCALES OF TURBULENCE		
	σ_1(m/sec)	σ_2(m/sec)	σ_3(m/sec)	L_1(m)	L_2(m)	L_3(m)
27000	7.00	7.00	4.22	20000	20000	1230
30000	8.23	8.23	4.66	23533	23533	1443
40000	12.82	12.82	6.09	36693	36693	2231
50000	18.08	18.08	7.51	51786	51786	3128
60000	23.94	23.94	8.90	68623	68623	4124
70000	30.36	30.36	10.28	87063	87063	5208
80000	37.29	37.29	11.65	106998	106998	6376
90000	44.70	44.70	13.01	128338	128338	7622
100000	52.58	52.58	14.35	151010	151010	8941
110000	60.89	60.89	15.69	174950	174950	10330
120000	69.62	69.62	17.02	200000	200000	11800

2.3.1 Upper Limits of Integration

The upper limits of integration for $j = 2$ and 3 are calculated according to the relation [7]

$$\Omega_{ij\max} = aL_i/\ell_j \qquad (j = 2,3) \qquad (2\text{-}3)$$

where

a = 1,339

L_i = integral scale of turbulence associated with the $\Phi_{ii}(\Omega_1)$ spectrum

ℓ_j = characteristic length of Space Shuttle in the jth direction

Values of L_i for the six bands are given in Table 2-2 while the characterfstic lengths, ℓ_js are presented in Table 2-3.

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TABLE 2-2. SUMMARY OF TURBULENCE PARAMETERS IN DISCRETE ALTITUDE BANDS

BAND #	LOWER LIMIT (m)	UPPER LIMIT (m)	TURBULENCE LENGTH SCALE L_i(m)			TIME INTERVAL T_i (dimensionless)			MAXIMUM FREQUENCY Ω_{i1max}(dimensionless)		
			i = 1	i = 2	i = 3	i = 1	i = 2	i = 3	i = 1	i = 2	i = 3
1	0	30	43.4	27.7	16.8	.6520	1.022	1.684	4.818	3.075	1.866
2	30	304.8	196	190	192	.1444	.1489	.1474	21.76	21.10	21.32
3	304.8	762	300	300	300	.09432	.09432	.09432	33.310	33.310	33.310
4	762	10,000	533	533	533	.05309	.05309	.05309	59.180	59.180	59.180
5	10,000	27,000	20,000	20,000	1,230	.004266	.004266	.06785	736.5	736.5	46.30
6	27,000	120,000	200,000	200,000	11,800	.003511	.003511	.05950	894.9	894.9	52.80

NOTES: i = 1 applies to u_1-gust

i = 2 applies to u_2-gust and $\partial u_2/\partial x_i$ gust gradients

i = 3 applies to u_3-gust and $\partial u_3/\partial x_1$ gust gradients

TABLE 2-3. CHARACTERISTIC DIMENSIONS OF THE SPACE SHUTTLE [8]

Characteristic Length	Magnitude (ft)	Magnitude (m)	Explanation
ℓ_1	39.56	12.06	mean aerodynamic chord
	39.05	11.9	1/2 wingspan
ℓ_3	10.95	3.34	1/2 fuselage thickness

In the case of Ω_{i1max} special consideration must be given to the *dimensional* frequencies corresponding to the *dimensionless* limits, The dimensional frequency limit satisfies the relation

$$f_{1max} = \Omega_{i1max} V/(2\pi a L_i) \tag{2-4}$$

where

V = vehicle velocity

The maximum dimensional frequency which the Space Shuttle simulators are capable of handling is 4 hertz, Thus any higher frequencies should be excluded from the simulation. For this reason the dimensionless frequency limit Ω_{i1max} must satisfy the relation

$$\Omega_{i1max} = \min(aL_i/\ell_1,\ 2\pi a L_i f_{1max}/V) \tag{2-5}$$

where

f_{1max} = 4 hertz

Values of Ω_{i1max} based on Eq (2-5) are included in Table 2-2.

2.3.2 One-Dimensional Spectra

There are six spectra of primary interest for turbulence simulation, as indicated in Table 2-4. Based on second-order numerical integration, the six corresponding three-dimensional gust and gust gradient spectral relations, as given by Eqs (2-1) and (2-2), were integrated over Ω_3 and Ω_2 (with the appropriate upper limits). The resulting one-dimensional spectra for all altitude bands are presented in Appendix A. These spectra were used in establishing the impulse response functions associated with digital filter simulation processes described in subsection 2.4.

TABLE 2-4. TYPES OF SIMULATED TURBULENCE

Type	Corresponding Spectrum	Comments
u_1	Φ_{11}	longitudinal gust
u_2	Φ_{22}	transverse gust
u_3	Φ_{33}	vertical gust
$\partial u_2/\partial x_1$	$\Phi_{22/11}$	yaw
$\partial u_3/\partial x_1$	$\Phi_{33/11}$	pitch
$\partial u_3/\partial x_2$	$\Phi_{33/22}$	roll

2.3.3 Dimensionless Energy Content

The total dimensionless energy content of each one-dimensional spectra in each altitude band was established by integrating the corresponding spectra over the appropriate finite limits, indicated in Table 2-2. The resulting energy content is presented in Table 2-5. As might be expected

TABLE 2-5. DIMENSIONLESS ENERGY CONTENT FOR GUSTS AND GUST GRADIENTS

ALTITUDE BAND	SPECTRUM					
	Φ_{11}	Φ_{22}	Φ_{33}	$\Phi_{22/11}$	$\Phi_{33/11}$	$\Phi_{33/22}$
1	.6225	.5010	.2752	.5877	.1525	.1557
2	.8595	.8560	.8383	13.147	12.171	12.308
3	.8956	.8952	.8809	24.767	22.643	22.890
4	.9298	.9296	.9197	54.123	49.527	50.060
5	.9977	.9953	.9251	1740.	41.71	95.62
6	1.000	.9973	.9363	2309.	52.08	391.6

the total dimensionless energy content of each* of the turbulent gust series is less than unity. The dimensionless energy content for each gust gradient, however, is not limited in such a manner and range as high has 391.6. For both gusts and gust gradients the total energy content increases with altitude because of similar increases in the limits of integration,

2.4 DIGITAL FILTER SIMULATION

As suggested in subsection 2.1, simulated turbulence, $Y(t)$, can be interpreted as the response or output of a control system [9] with double-sided response functions, $h(t)$, subject to an input consisting of Gaussian white noise $I(t)$. This response can be represented by the convolution integral

$$Y(t) = \int_{-\infty}^{\infty} h(\tau)\ I(t-\tau)\ d\tau \tag{2-6}$$

Based on filter theory the double-sided spectrum, $\Phi_{DY}(\Omega)$, of the simulated turbulence satisfies the relation

$$\Phi_{DY}(\Omega_1) = H(\Omega_1)H^*(\Omega_1)\Phi_{DI}(\Omega_1) \tag{2-7}$$

where $H(\Omega_1) = F[h(t)]$

$\Phi_{DI}(\Omega_1)$ = double-sided power spectrum for white noise

Generally the standard deviation of any white noise signal has a value of unity. Furthermore in most practical situations the white noise is defined** to occur over some interval extending from $-\Omega_{1max}$ to $+\Omega_{1max}$. For this case ,

$$\Phi_{DI}(\Omega_1) = \frac{1}{2\Omega_{1max}}$$

$$= \frac{T}{2\pi} \tag{2-8}$$

---

* Actually the term "energy" is not precise when dealing with gust gradients.

** The subscript, 1, normally applied to the variables Ω_{1max} and T, has been suppressed in Eqs (2-8) through (2-13) for simplicity.

where

T = time interval associated with generation process $(=\pi/\Omega_{1max})$

By substitution,

$$\Phi_{DY}(\Omega_1) = H(\Omega_1)H^*(\Omega_1)\,\frac{T}{2\pi} \tag{2-9}$$

If $H(\Omega_1)$ is limited to real values,

$$\Phi_{DY}(\Omega_1) = H^2(\Omega_1)\,\frac{T}{2\pi} \tag{2-10}$$

Rearrangement of Eq (2-10) yields

$$H(\Omega_1) = \sqrt{\frac{2\pi}{T}\,\Phi_{DY}(\Omega_1)} \tag{2-11}$$

Then based on the definition of the inverse Fourier transform, the double-sided impulse response function $h(t)$ can be expressed as

$$\begin{aligned} h(t) &= F^{-1}[H(\Omega_1)] \\ &= \int_{-\infty}^{\infty} \sqrt{\frac{2\pi}{T}\,\Phi_{DY}(\Omega_1)}\,\cos(\Omega_1 t)\,d\Omega_1 \\ &= 2\int_0^{\infty} \sqrt{\frac{2\pi}{T}\,\Phi_{DY}(\Omega_1)}\,\cos(\Omega_1 t)\,d\Omega_1 \\ &= 2\int_0^{\infty} \sqrt{\frac{2\pi}{T}\,\frac{\Phi_Y(\Omega_1)}{2}}\,\cos(\Omega_1 t)\,d\Omega_1 \\ &= 2\sqrt{\frac{\pi}{T}}\int_0^{\infty} \sqrt{\Phi_Y(\Omega_1)}\,\cos(\Omega_1 t)\,d\Omega_1 \end{aligned} \tag{2-12}$$

where

$\phi_Y(\Omega_1)$ = single-sided spectrum of $Y(t)$

The single-sided spectra tabulated in Appendix A correspond to $\Phi_Y(\Omega_1)$

The discrete version of the convolution'integral given in Eq (2-6) yields

$$Y(k) = \sum_{j=-N}^{+N} h(j)\ I(k-j)\ T \qquad (2\text{-}13)$$

where

$Y(k)$ = discrete sampled turbulence output

$h(j)$ = discrete double-sided impulse response function $h(jT)$

$I(k)$ = discrete sampled white noise input

Eq (2-13) represents* the basic, non-recursive relation for the generation of simulated turbulence . The impulse response functions $h(t)$ were evaluated by means of second-order numerical integration of Eq (2-12) using the six spectra from Appendix A. In carrying out this evaluation some maximum value of t must be established, corresponding to the value of N for Eq (2-13), and replacing the infinite limit of Eq (2-6). As discussed in Appendix B this maximum time limit determines the minimum frequency, Ω_{i1max}, for which the corresponding spectrum is accurately simulated.

The values of dimensionless time increment, T_i, used for the six altitude bands are included in Table 2-2 and are based on the values of Ω_{imax} shown in the same table. Thus the Nyquist generation frequencies Ω_{iNG} for the simulated turbulence correspond to the upper frequency limits for Ω_{i1} as computed by Eq (2-5) for each altitude band,

---

* In certain references 19,111 to correct for the "effect of digitizing" the series represented by Eq (2-13) has been divided by $\sqrt{T}$. This process can be seen to be dimensionally incorrect and actually results from the use of a white noise spectrum with unit strength instead of a strength of $T/2\pi$.

2.5 EFFECTS OF DIGITIZATION

The effects of digitization in turbulence simulation have been considered by a number of investtgators [9-12]. As a result of these studies two basic digitization effects have been generally identified.

The first effect results from the assumption of a white noise specrrum with unit *strength* instead of unit *power,* noted in subsection **2.4.** To correct for such an "effect" the proposed procedure is to divide the series approximation of the convolution integral by $\sqrt{T}$. This "effect" disappears when the white noise spectrum has unit power.

The second effect involves the *tapering* of the spectrum of simulated white noise, Φ_I', in the vicinity of the Nyquist generation frequency, Ω_{NG}. Some investigators [10,12] have considered it necessary, because of the tapering effect, to generate the simulated turbulence time series at a rate from four to ten times the rate at which the series will be sampled.

The second effect arises from the discrete processes associated with both the generation and sampling of the simulated turbulence. In the case of discrete white noise with unit variance, the time series involved is basically a train of step functions as shown in Figure 2-1. The autocorrelation function of the train of step functions depicted in Figure **2-1** can

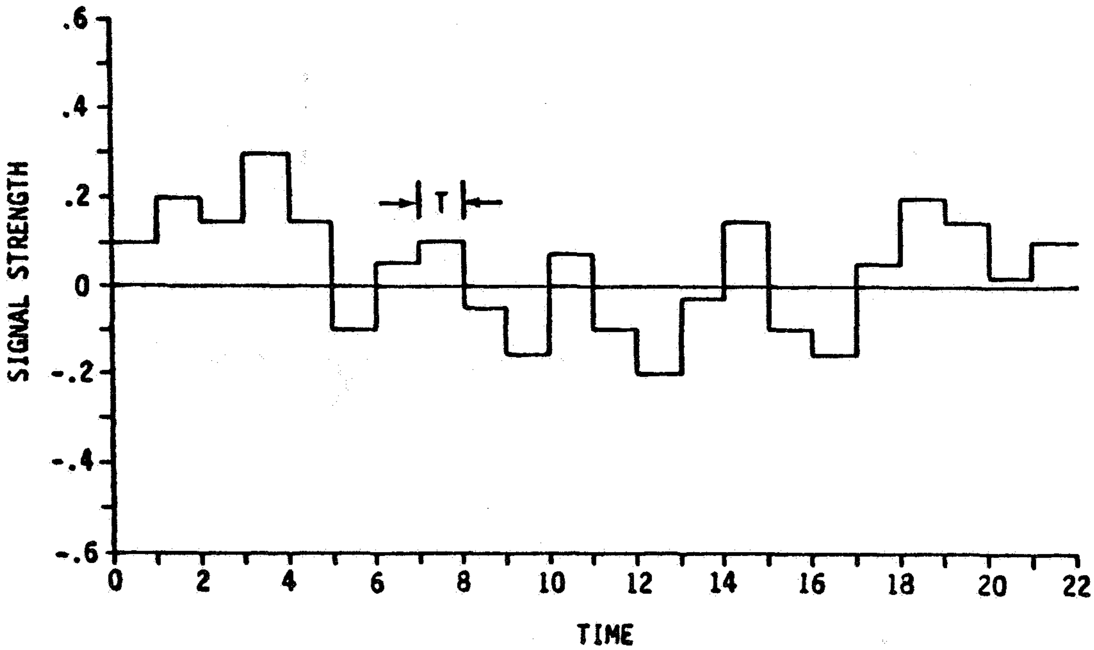

Figure **2-1.** Discrete White Noise Series be shown to be

$$R_{DI}(\tau) = \begin{cases} 1 - \frac{|\tau|}{T_G} & (|\tau| \leq T_G) \\ 0 & (|\tau| > T_G) \end{cases} \tag{2-14}$$

The corresponding double-sided power spectrum by definition is

$$\begin{aligned} \Phi'_{DI}(\Omega) &\equiv \mathcal{F}[R_{DI}(\tau)] \\ &= \frac{T_G}{2\pi} \frac{\sin^2(\Omega T_G/2)}{(\Omega T_G/2)^2} \\ &= \frac{1}{2\Omega_{NG}} \frac{\sin^2(\Omega\pi/2\Omega_{NG})}{(\Omega\pi/2\Omega_{NG})^2} \end{aligned} \tag{2-15}$$

The single-sided version of this power spectrum is shown in Figure 2-2.

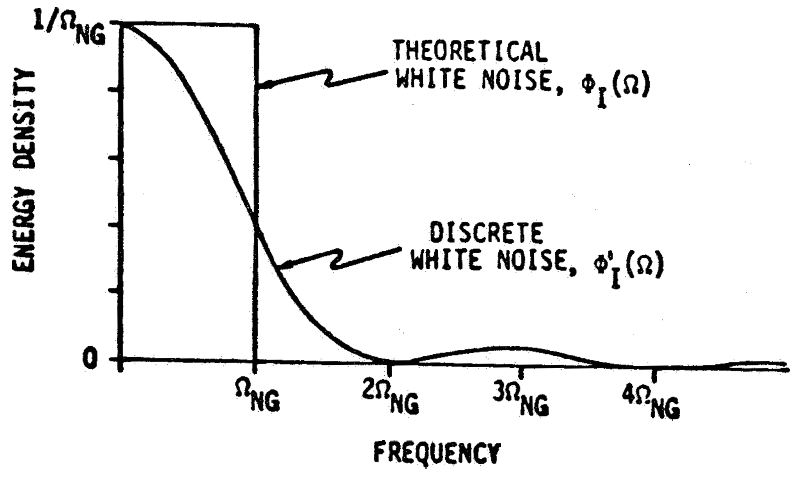

Figure 2-2. White Noise Spectra

Theoretical white noise, by definition, is characterized by a uniform power spectral 'distribution. In order to avoid infinite power, such a spectral distribution is normally restricted to the frequency band $(-\Omega_{NG} \leq \Omega \leq \Omega_{NG})$ for a double-sided spectrum. For a signal with unit power the spectral density function for such white noise is

$$\Phi_{DI}(\Omega) = \begin{cases} \dfrac{1}{2\Omega_{NG}} & (-\Omega_{NG} \leq \Omega \leq \Omega_{NG}) \\ 0 & (\Omega_{NG} < |\Omega|) \end{cases} \qquad (2\text{-}16)$$

Such a theoretical distribution in single-sided form is also shown in Figure 2-2.

It is important to note that the two power spectra shown in Figure 2-2 are both normalized and thus

$$\int_{-\infty}^{\infty} \Phi_{DI}(\Omega)d\Omega = \int_{-\infty}^{\infty} \Phi'_{DI}(\Omega)d\Omega \qquad (2\text{-}17)$$
$$= 1$$

The theoretical spectrum is basically a rectangular pulse function while the discrete spectrum is characterized by tapering, The difference between these two spectra is generally considered the basis for the second digitization effect,

The preceding descriptions of the two spectra $\Phi_I(\Omega)$ and $\Phi_{DI}(\Omega)$ are based purely on mathematical theory. To observe such spectra in reality the corresponding time series would have to be sampled with an infinitesimal sampling interval. Actually, finite sampling intervals, T_S, must be used but this finite (or discrete) sampling process results in *aliasing*. The aliased spectrum, $\Phi^+(\Omega)$, based on the finite sampling process, is related to the original spectrum according to the relation [13]

$$\Phi^+(\Omega) = \sum_{k=-\infty}^{\infty} \Phi(\Omega+2k\Omega_{NS}) \qquad (2\text{-}18)$$

where

Ω_{NS} = Nyquist sampling frequency $(=\pi/T_S)$

For the case of the discrete white noise

$$\Phi'^+_{DI}(\Omega) = \sum_{k=-\infty}^{\infty} \Phi'_{DI}(\Omega+2k\Omega_{NS})$$

$$= \frac{1}{2\Omega_{NG}} \sum_{k=-\infty}^{\infty} \frac{\sin^2[(\Omega+2k\Omega_{NS})\pi/2\Omega_{NG}]}{[(\Omega+2k\Omega_{NS})\pi/2\Omega_{NG}]^2} \qquad (2\text{-}19)$$

Numerical evaluation of this series has been carried out for Ω_{NG} = 100 with various ratios of Ω_{NS}/Ω_{NG}, including .5, 1, 2, and 4. The resulting aliased spectra are presented in Figure 2-3. It is important to note that the figure indicates that

$$\Phi'^+_{DI}(\Omega) = \left\{ \begin{array}{ll} \frac{1}{2\Omega_{NG}} & (-\Omega_{NG} \le \Omega \le \Omega_{NG}) \\ 0 & (\Omega_{NG} < |\Omega|) \end{array} \right\} (\Omega_{NG} = \Omega_{NS}) \qquad (2\text{-}20)$$

In this case, by comparision with $\Phi_{DI}(\Omega)$,

$$\Phi'^+_{DI}(\Omega) = \Phi_{DI}(\Omega) \qquad (\Omega_{NG} = \Omega_{NS}) \qquad (2\text{-}21)$$

Thus for white noise *the aliasing due to discrete sampling exactly offsets the tapering due to discrete generation when the sampling frequency equals the generation frequency.* Based on this fundamental point, it is clear that in the simulation of white noise no tapering of the spectrum occurs as long as the sampling rate equals the generation rate. Under most conditions this equality is automatically satisfied, The process of convolving the white noise with the appropriate impulse response function is also carried out in a discrete manner. The process involves selecting (or sampling) values of the white noise signal and the impulse response function at equal intervals in time and then approximating the convolution integral by a summation of products. It is important to note that the discrete sampling of both the white noise and the impulse response function normally occurs at the same rate as the generation rate for the white noise. Thus the resulting spectra for the sampled discrete white noise, as previously shown, will be uniform, According to the convolution theorem, the spectrum of the output signal equals the spectrum of the input white noise multiplied by the product of the Fourier transform of the impulse response function and its complex conjugate. Thus, as previously noted in subsection 2.4, for a continuous signal,

$$\Phi_{DY}(\Omega) = \Phi_{DI}(\Omega)H(\Omega)H^*(\Omega) \tag{2-22}$$

The Fourier transform $H(\Omega)$ for the *continuous* impulse function, $h(t)$, is

$$H(\Omega) = \mathcal{F}[h(t)]$$

$$= \sqrt{\frac{2\pi}{T} \Phi_{DY}(\Omega)} \tag{2-23}$$

The corresponding output spectrum for a discrete signal would be

$$\Phi'_{DY}(\Omega) = \Phi'_{DI}(\Omega)H'(\Omega)H'^*(\Omega) \tag{2-24}$$

The Fourier transform $H'(\Omega)$ of the *discrete* impulse response function, $h'(t)$, is

$$H'(\Omega) = \mathcal{F}[h'(t)] \tag{2-25}$$

Based on the preceding development, for cases in which the sampling frequency equals the generation frequency, any difference between the discrete turbulence spectrum and the continuous spectrum apparently originates because of some difference between $H(\Omega)$ and $H'(\Omega)$.

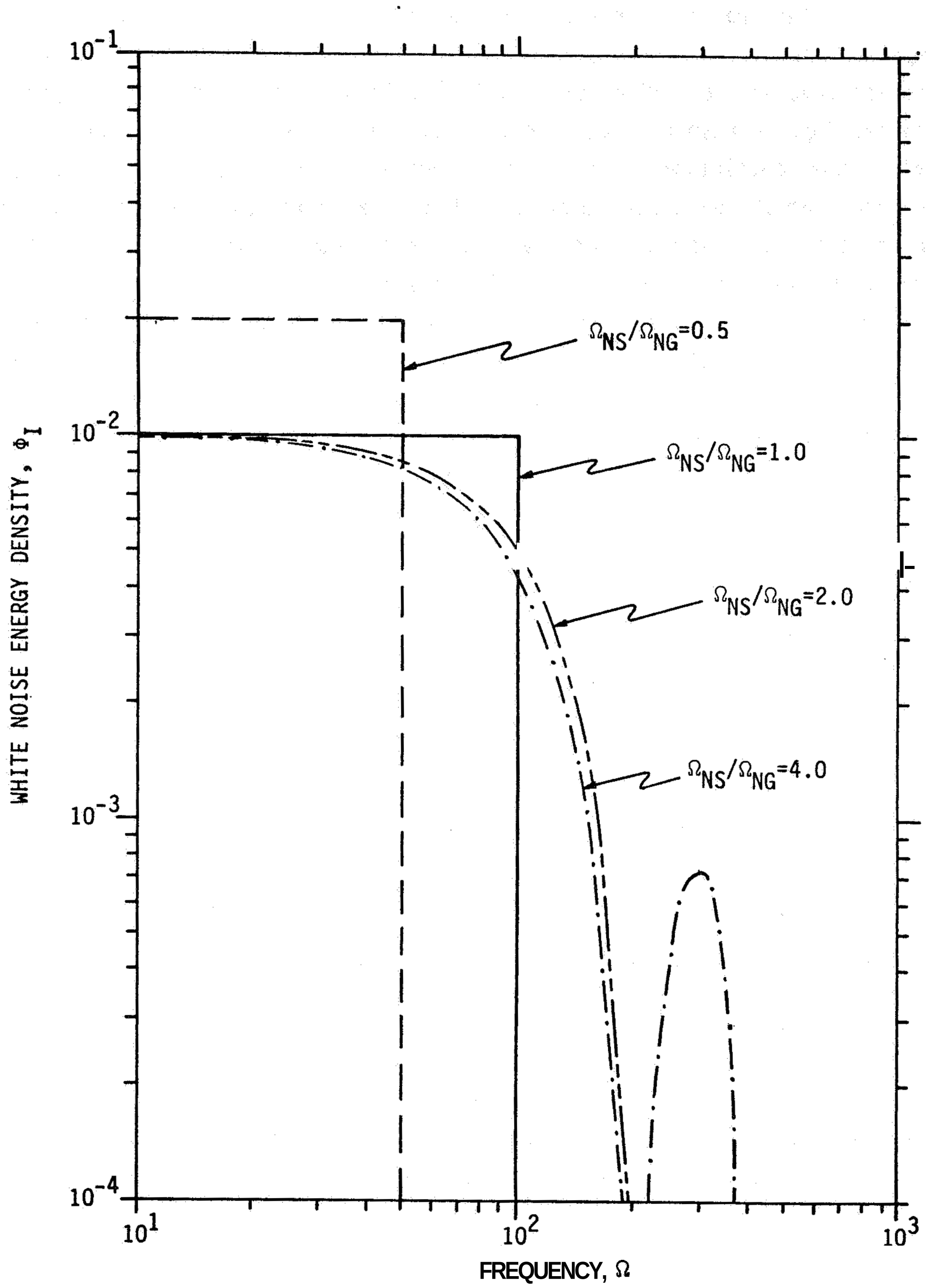

Figure 2-3. Effects of Aliasing on White Noise Spectrum

3. SIMULATED TURBULENCE TAPES

The turbulence generation procedure described in Section 2 has been used to generate six dimensionless simulated turbulence time series which are stored on magnetic tapes as summarized in Table 3-1. The appropriate procedures for using the tapes are described elsewhere [1]. Subsection 3.1 provides a description of the results of validating the tapes while subsection 3.2 presents an explanation of the process for converting from dimensionless to dimensional values.

TABLE 3-1. INDEX OF SHUTTLE SIMULATED TURBULENCE TAPES (SSTT)

Tape	Time Series	Comments
SSTT-1	u_1 - gust	longitudinal gust
SSTT-2	u_2 - gust	transverse gust
SSTT-3	u_3 - gust	vertical gust
SSTT-4	$\partial u_2/\partial x_1$ - gust gradient	yaw
SSTT-5	$\partial u_3/\partial x_1$ - gust gradient	pitch
SSTT-6	$\partial u_3/\partial x_2$ - gust gradient	roll

3.1 VALDIATION OF SIMULATED TURBULENCE

A spectral analysis of each of the dimensionless time series has been carried out by means of a Fast Fourier Transform FFT4 [14]. The results, which are presented in Appendix C, demonstrate that the simulated turbulence possesses the proper von Karman spectral characteristics.

All of the dimensionless time series have also been analyzed statistically to determine the gust and gust gradient probability density functions. As shown in Appendix D the results of these analyses indicate that both the simulated gusts and gust gradients are normally distributed, with near-zero means and standard deviations consistent with the energy content presented in Table 2-5.

3.2 CONVERSION TO DIMENSIONAL VALUES

The dimensionless time series on each tape must be converted to dimensional form before actual use in a simulation exercise. The conversion process generally involves multiplication and/or division by the appropriate turbulence parameters. For dimensionless gusts, u_i, the corresponding standard deviation, σ_i, should be used. Thus

$$u_i^* = \sigma_i u_i \tag{3-1}$$

where

u_i^* = dimensional gust

For dimensionless gust gradient, $\frac{\partial u_i}{\partial x_j}$, the parameters σ_i and L_j are used. Thus

$$\frac{\partial u_i^*}{\partial x_j^*} = \frac{\sigma_i}{L_i} \frac{\partial u_i}{\partial x_j} \tag{3-2}$$

where

$\partial u_i^*/\partial x_j^*$ = dimensional gust gradient

In the case of dimensionless time it is necessary to develop the procedures for converting both from dimensionless to dimensional form and also to dimensionless from dimensional. In proceeding from dimensionless to dimensional time the dimensionless time step, T_i, represents the basic unit to be converted. The conversion involves the vehicle velocity, V, and the turbulence scale, L_i. Thus

$$\Delta t_i^* = aL_i T_i/V \tag{3-3}$$

where

Δt_i^* = dimensional time step

It is important to note that because both L_i and V vary with altitude, the resulting dimensional time step Δt_i^* is not a constant. To obtain dimensional time, t_i, a summation process is involved as follows:

$$t_{iN}^* = \sum_{n=1}^{N} \Delta t_{in}^* \tag{3-4}$$

$$= aT_i \sum_{n=1}^{N} L_{in}/V_n$$

where

$L_{in} = L_i(Z_n)$

$V_n = V(Z_n)$

Z_n = altitude at nth step

In converting to dimensionless from dimensional time the basic unit, the dimensional time step, δt^*, will normally be a constant. The corresponding dimensionless time interval, T_{im}, will be

$$T_{im} = \frac{V_m \delta t^*}{aL_{im}} \tag{3-5}$$

The total dimensionless time, t_{iM}, will be

$$t_{iM} = \sum_{m=1}^{M} T_{im}$$

$$= \sum_{m=1}^{M} \frac{V_m \delta t^*}{aL_{im}}$$

$$= \frac{\delta t^*}{a} \sum_{m=1}^{M} V_m/L_{im} \tag{3-6}$$

The dimensionless time, t_{iM}, corresponds to some M' dimensionless time intervals, T_i, plus some fractional interval, T', as follows:

$$t_{iM} = M'T_i + T' \tag{3-7}$$

where

$$0 \leq T' \leq T_i$$

Thus the number of dimensionless time intervals, M', can be computed as

$$M' = \mathrm{Int}(t_{iM}/T_i)$$
$$= \mathrm{Int}\left(\frac{\delta t^*}{aT_i} \sum_{m=1}^{M} V_m/L_{im}\right) \tag{3-8}$$

where

Int( ) = integer value of ( )

The fractional interval, T', can be computed by the relation

$$T' = t_{iM} - M'T_i \tag{3-9}$$

The interpolation process will involve interpolating between $t_{iM'}$ and $t_{iM'+1}$ at the point t_{iM} as shown in Figure 3-1.

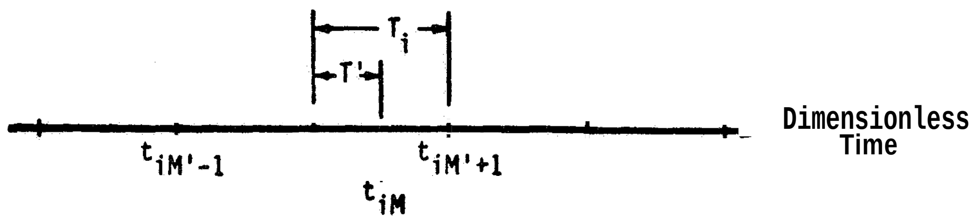

Figure 3-1. Relationship Between t_{iM}, $t_{iM'}$, and $t_{iM'+1}$

In the conversion to or from dimensional values three parameters are required: standard deviation, integral scale of turbulence, and vehicle speed. The variation of the turbulence standard deviation, σ_i, with altitude was presented in Table 2-1. The same table contains the turbulence scale, L_i, as a function of altitude. The vehicle speed, V, is a function of altitude but also may vary from one trajectory to another. Table **3-2** provides ***representative*** values of V as a function of altitude.

TABLE 3-2. VARIATION OF SHUTTLE SPEED WITH ALTITUDE [12]

ALTITUDE (m)	V (m/sec)
100	152
300	156
500	158
2000	170
4000	188
6000	200
8000	240
10000	300
20000	500
40000	1928
60000	4695
80000	7468
100000	7521
120000	7510

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4. CONCLUSIONS AND RECOMMENDATIONS

By means of a non-recursive discrete generation process, based on a von Karman spectral model with finite upper limits, dimensionless simulated turbulence time series have been developed and stored on six magnetic tapes. Longitudinal, transverse, and vertical gusts are simulated as well as the gust gradients associated with yaw, pitch, and roll, For each gust or gust gradient six separate time series (corresponding to the six altitude bands extending from ground level to 120,000 meters) have been stored on each tape.

The results of spectral analyses of each tape reveals that the simulated turbulence possesses the appropriate von Karman spectral characteristics. Statistical analyses of the tapes indicate that both the simulated gust and gust gradients are normally distributed with near-zero means, Furthermore the standard deviation of each series is constant with the theoretical energy content.

The Shuttle Simulated Turbulence Tapes (SSTT) are now ready for actual use for simulating turbulence at altitudes up to 120,000 meters,

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APPENDIX A

DIMENSIONLESS VON KARMAN SPECTRA WITH FINITE UPPER LIMITS

For each altitude band, the three-dimensional spectral model for gusts, as given by Eq (2-1), and the three-dimensional model for gust gradients, as given by Eq (2-2), have been integrated with respect to Ω_2 and Ω_3 over the finite limits calculated according to Eq (2-3). The six resulting one-dimensional spectra are presented in Tables A-1 through A-6, corresponding to Altitude Bands #1 through 6 respectively, These spectra were used in the numerical evaluation of the impulse response functions described in subsection 2.4.

TABLE A-1. DIMENSIONLESS SPECTRUM FOR ALTITUDE BAND # 1

i=1		i=2			i=3			
DIMENSION-LESS WAVE NUMBER Ω_1	DIMENSIONLESS SPECTRUM Φ_{11}	DIMENSION-LESS WAVE NUMBER Ω_1	DIMENSIONLESS SPECTRUM Φ_{22}	$\Phi_{22/11}$	DIMENSION-LESS WAVE NUMBER Ω_1	DIMENSIONLESS SPECTRUM Φ_{33}	$\Phi_{33/11}$	$\Phi_{33/22}$
0.00000	.44171	0.00000	.21638	0.0000	0.00000	.14709	0.0000	.10454
.01000	.44167	.01000	.21640	.12070E-04	.01000	.14711	.82051E-05	.10454
.02000	.44156	.02000	.21646	.48293E-04	.02000	.14717	.32834E-04	.10454
.03000	.44136	.03000	.21656	.10871E-03	.03000	.14728	.73930E-04	.10453
.04000	.44108	.04000	.21670	.19339E-03	.04000	.14742	.13156E-03	.10452
.05000	.44072	.05000	.21688	.30242E-03	.05000	.14761	.20582E-03	.10451
.06000	.44029	.06000	.21710	.43591E-03	.06000	.14783	.29684E-03	.10450
.07000	.43978	.07000	.21735	.59402E-03	.07000	.14810	.40475E-03	.10449
.08000	.43919	.09000	.21765	.77691E-03	.09000	.14840	.52974E-03	.10447
.09000	.43852	.09000	.21797	.98475E-03	.09000	.14875	.67200E-03	.10445
.10000	.43778	.10000	.21834	.12178E-02	.10000	.14912	.83173E-03	.10443
.19000	.42785	.19000	.22295	.44890E-02	.19000	.15395	.30997E-02	.10413
.28000	.41270	.28000	.22910	.10018E-01	.28000	.16046	.70165E-02	.10355
.37000	.39348	.37000	.23541	.17975E-01	.37000	.16726	.12771E-01	.10262
.46000	.37147	.46000	.24056	.28391E-01	.46000	.17305	.20423E-01	.10127
.55000	.34786	.55000	.24364	.41107E-01	.55000	.17692	.29850E-01	.99460E-01
.64000	.32369	.64000	.24419	.55787E-01	.64000	.17841	.40757E-01	.97177E-01
.73000	.29978	.73000	.24217	.71978E-01	.73000	.17745	.52742E-01	.94454E-01
.82000	.27671	.82000	.23780	.89131E-01	.82000	.17423	.65360E-01	.91349E-01
.91000	.25485	.91000	.23148	.10692	.91000	.16928	.78187E-01	.87935E-01
1.00000	.23441	1.00000	.22368	.12476	1.00000	.16290	.90859E-01	.84293E-01
1.38186	.16426	1.20755	.20235	.16457	1.08653	.15586	.10263	.80650E-01
1.76373	.11685	1.41510	.17955	.20054	1.17305	.14827	.11379	.76934E-01
2.14559	.84980E-01	1.62264	.15772	.23161	1.25958	.14039	.12423	.73201E-01
2.52745	.63167E-01	1.83019	.13792	.25766	1.34611	.13246	.13387	.69498E-01
2.90931	.47895E-01	2.03774	.12048	.27903	1.43264	.12463	.14267	.65865E-01
3.29117	.36960E-01	2.24529	.10535	.29623	1.51916	.11702	.15063	.62331E-01
3.67304	.28969E-01	2.45284	.92322E-01	.30980	1.60569	.10971	.15776	.58919E-01
4.05490	.23021E-01	2.66039	.81130E-01	.32026	1.69222	.10274	.16410	.55643E-01
4.43676	.18521E-01	2.86793	.71517E-01	.32808	1.77875	.96161E-01	.16969	52514E-01
4.81862	.15066E-01	3.07548	.63248E-01	.33366	1.86527	.89967E-01	.17453	49538E-01

TABLE A-2. DIMENSIONLESS SPECTRUM FOR ALTITUDE BAND # 2

i=1		i=2			i=3			
DIMENSION-LESS WAVE NUMBER	DIMENSIONLESS SPECTRUM	DIMENSION-LESS WAVE NUMBER	DIMENSIONLESS SPECTRUM		DIMENSION-LESS WAVE NUMBER	DIMENSIONLESS SPECTRUM		
Ω_1	ϕ_{11}	Ω_1	ϕ_{22}	$\phi_{22/11}$	Ω_1	ϕ_{33}	$\phi_{33/11}$	$\phi_{33/22}$
0.00000	.47130	0.00000	.23623	0.0000	0.00000	.23492	0.0000	1.4485
.01000	.47126	.01000	.23625	.13177E-04	.01000	.23494	.13104E-04	1.4484
.02000	.47114	.02000	.23632	.58722E-04	.02000	.23500	.52429E-04	1.4484
.03000	.47094	.03000	.23642	.11858E-03	.03000	.23510	.11802E-03	1.4484
.04000	.47066	.04000	.23656	.21110E-03	.04000	.23524	.20993E-03	1.4483
.05000	.47030	.05000	.23674	.33010E-03	.05000	.23543	.32827E-03	1.4483
.06000	.46986	.06000	.23696	.47579E-03	.06000	.23565	.47315E-03	1.4482
.07000	.46934	.07000	.23722	.64831E-03	.07000	.23590	.64472E-03	1.4481
.08000	.46875	.08000	.23751	.84783E-03	.08000	.23620	.84314E-03	1.4480
.09000	.46808	.09000	.23785	.107<5E-02	.09000	.23653	.10686E-02	1.4479
.10000	.46733	.10000	.23821	.13286E-02	.10000	.23690	.13213E-02	1.4478
.19000	.45731	.19000	.24289	.48905E-02	.19000	.24158	.48641E-02	1.4461
.28000	.44205	.28000	.24914	.10894E-01	.28000	.24783	.10837E-01	1.4433
.37000	.42269	.37000	.25555	.19512E-01	.37000	.25423	.19412E-01	1.4393
.46000	.40053	.46000	.26079	.30778E-01	.46000	.25948	.30624E-01	1.4341
.55000	.37678	.55000	.26395	.44133E-01	.55000	.26264	.44312E-01	1.4276
.64000	.35250	.64000	.26455	.60137E-01	.64000	.26324	.60138E-01	1.4198
.73000	.32848	.73000	.26254	.78073E-01	.73000	.26123	.77645E-01	1.4110
.82000	.30531	.82000	.25817	.96823E-01	.82000	.25687	.96333E-01	1.4010
.91000	.28336	.91000	.25185	.11632	.91000	.25055	.11572	1.3902
1.00000	.26282	1.00000	.24404	.13611	1.00000	.24274	.13539	1.3785
1.90000	.13051	1.90000	.15244	.30693	1.90000	.15117	.30438	.42978
2.80000	.74892E-01	2.80000	.95050E-01	.41563	2.80000	.93328E-01	.41028	.38314
3.70000	.48151E-01	3.70000	.63922E-01	.48809	3.70000	.62751E-01	.47914	.34244
4.60000	.33430E-01	4.60000	.45837E-01	.54097	4.60000	.44718E-01	.52776	.30684
5.50000	.24468E-01	5.50000	.34499E-01	.58207	5.50000	.33429E-01	.56401	.27562
6.40000	.18602E-01	6.40000	.26931E-01	.61520	6.40000	.25906E-01	.59182	.24817
7.30000	.14550E-01	7.30000	.21622E-01	.64267	7.30000	.20637E-01	.61339	.22400
8.20000	.11636E-01	8.20000	.17747E-01	.66558	8.20000	.16301E-01	.63009	.20265
9.10000	.94722E-02	9.10000	.14828E-01	.68486	9.10000	.13919E-01	.64286	.18376
10.00000	.78233E-02	10.00000	.12570E-01	.70107	10.00000	.11697E-01	.65240	.16700

TABLE A-2. DIMENSIONLESS SPECTRUM FOR ALTITUDE BAND # 2 (continued)

i=1		i=2			i=3			
DIMENSIONLESS WAVE NUMBER Ω_1	DIMENSIONLESS SPECTRUM Φ_{11}	DIMENSIONLESS WAVE NUMBER Ω_1	DIMENSIONLESS SPECTRUM Φ_{22}	$\Phi_{22/11}$	DIMENSIONLESS WAVE NUMBER Ω_1	DIMENSIONLESS SPECTRUM Φ_{33}	$\Phi_{33/11}$	$\Phi_{33/22}$
11.17616	.62035E-02	11.10954	.10423E-01	.71748	11.13174	.95590E-02	.66066	1.2419
12.35231	.50019E-02	12.21907	.87718E-02	.73047	12.26348	.79344E-02	.66555	1.1067
13.52846	.[illegible]0891E-02	13.32861	.74735E-02	.74052	13.39522	.66722E-02	.66774	.98783
14.70461	.33820E-02	14.43814	.64331E-02	.74796	14.52696	.56732E-02	.66776	.88476
15.88077	.28252E-02	15.54768	.55859E-02	.75312	15.65870	.48701E-02	.66602	.79506
17.05692	.23807E-02	16.65722	.48867E-02	.75624	16.79045	.42156E-02	.66287	.71653
18.23307	.202[illegible]E-02	17.76675	.43030E-02	.75757	17.92219	.36760E-02	.65857	.64736
19.40922	.17297E-02	18.87629	.38107E-02	.75731	19.05393	.32265E-02	.65334	.58615
20.58537	.14875E-02	19.98582	.33919E-02	.75566	20.18567	.28486E-02	.64737	.53180
21.76152	.12872E-02	21.09536	.30329E-02	.75279	21.31741	.25283E-02	.64081	.48338

TABLE A-3. DIMENSIONLESS SPECTRUM FOR ALTITUDE BAND # 3

i=1		i=2			i=3			
DIMENSION-LESS WAVE NUMBER	DIMENSIONLESS SPECTRUM	DIMENSION-LESS WAVE NUMBER	DIMENSIONLESS SPECTRUM		DIMENSION-LESS WAVE NUMBER	DIMENSIONLESS SPECTRUM		
Ω_1	Φ_{11}	Ω_1	Φ_{22}	$\Phi_{22/11}$	Ω_1	Φ_{33}	$\Phi_{33/11}$	$\Phi_{33/22}$
0.00000	.47307	0.00000	.23698	0.0000	0.00000	.23610	0.0000	1.8735
.01000	.47303	.01000	.23700	.13213E-04	.01000	.23612	.13169E-04	1.8735
.02000	.47291	.02000	.23706	.52887E-04	.02000	.23618	.52691E-04	1.8735
.03000	.47271	.03000	.23716	.11905E-03	.03000	.23628	.11861E-03	1.8735
.04000	.47243	.04000	.23730	.21177E-03	.04000	.23642	.21098E-03	1.8734
.05000	.47207	.05000	.23748	.33114E-03	.05000	.23660	.32991E-03	1.8734
.06000	.47164	.06000	.23770	.47728E-03	.06000	.23682	.47551E-03	1.8733
.07000	.47112	.07000	.23796	.65034E-03	.07000	.23708	.64793E-03	1.8732
.08000	.47052	.08000	.23826	.85047E-03	.08000	.23737	.84733E-03	1.8731
.09000	.46985	.09000	.23859	.10779E-02	.09000	.23771	.10739E-02	1.8730
.10000	.46910	.10000	.23896	.13328E-02	.10000	.23807	.13279E-02	1.8729
.19000	.45909	.19000	.24363	.49055E-02	.19000	.24275	.48878E-02	1.8712
.28000	.44382	.28000	.24938	.10927E-01	.28000	.24900	.10888E-01	1.8683
.37000	.42447	.37000	.25629	.19569E-01	.37000	.25541	.19502E-01	1.8643
.46000	.40230	.46000	.26153	.30866E-01	.46000	.26065	.30762E-01	1.8589
.55000	.37855	.55000	.26469	.44658E-01	.55000	.26381	.44510E-01	1.8524
.64000	.35426	.64000	.26529	.60606E-01	.64000	.26441	.60406E-01	1.8446
.73000	.33025	.73000	.26328	.78253E-01	.73000	.26241	.77995E-01	1.8356
.82000	.30707	.82000	.25891	.97099E-01	.82000	.25804	.96774E-01	1.8255
.91000	.28512	.91000	.25259	.11666	.91000	.25172	.11626	1.8146
1.00000	.26458	1.00000	.24477	.13652	1.00000	.24391	.13604	1.8028
1.90000	.13222	1.90000	.15316	.30838	1.90000	.15235	.30675	1.6640
2.80000	.76534E-01	2.80000	.95746E-01	.41867	2.80000	.94999E-01	.41541	1.5254
3.70000	.49712E-01	3.70000	.64531E-01	.49319	3.70000	.63915E-01	.48803	1.4019
4.60000	.34905E-01	4.60000	.46476E-01	.54851	4.60000	.45866E-01	.54131	1.2930
5.50000	.25860E-01	5.50000	.35110E-01	.59237	5.50000	.34555E-01	.58301	1.1965
6.40000	.19915E-01	6.40000	.27517E-01	.62864	6.40000	.27002E-01	.61688	1.1101
7.30000	.15792E-01	7.30000	.22187E-01	.65945	7.30000	.21699E-01	.64496	1.0323
8.20000	.12812E-01	8.20000	.18295E-01	.68612	8.20000	.17825E-01	.66848	.96174
9.10000	.10588E-01	9.10000	.15363E-01	.70955	9.10000	.14902E-01	.68827	.89746
10.00000	.88940E-02	10.00000	.13095E-01	.73035	10.00000	.12639E-01	.70496	.83868

TABLE A-3. DIMENSIONLESS SPECTRUM FOR ALTITUDE BAND # 3 (continued)

i=1		i=2			i=3			
DIMENSIONLESS WAVE NUMBER Ω_1	DIMENSIONLESS SPECTRUM Φ_{11}	DIMENSIONLESS WAVE NUMBER Ω_1	DIMENSIONLESS SPECTRUM Φ_{22}	$\Phi_{22/11}$	DIMENSIONLESS WAVE NUMBER Ω_1	DIMENSIONLESS SPECTRUM Φ_{33}	$\Phi_{33/11}$	$\Phi_{33/22}$
12.33085	.59522E-02	12.33085	.91368E-02	.77485	12.33085	.86903E-02	.73698	.70768
14.66170	.42188E-02	14.66170	.67485E-02	.80912	14.66170	.63163E-02	.75731	.60124
16.99254	.31087E-02	16.99254	.51879E-02	.83551	16.99254	.47784E-02	.76956	.51377
19.32339	.23566E-02	19.32339	.41069E-02	.85530	19.32339	.37255E-02	.77588	.44127
21.65423	.18260E-02	21.65423	.33244E-02	.86943	21.65423	.29733E-02	.77762	.38079
23.98508	.14401E-02	23.98508	.27383E-02	.87863	23.98508	.24177E-02	.77575	.33006
26.31592	.11528E-02	26.31592	.22875E-02	.88358	26.31592	.19961E-02	.77102	.28731
28.64677	.93463E-03	28.64677	.19333E-02	.88488	28.64677	.16693E-02	.76404	.25114
30.97761	.76627E-03	30.97761	.16499E-02	.88309	30.97761	.14113E-02	.75533	.22040
33.30846	.63450E-03	33.30846	.14200E-02	.87869	33.30846	.12045E-02	.74533	.19419

TABLE A-4. DIMENSIONLESS SPECTRUM FOR ALTITUDE BAND # 4

i=1		i=2			i=3			
DIMENSIONLESS WAVE NUMBER Ω_1	DIMENSIONLESS SPECTRUM Φ_{11}	DIMENSIONLESS WAVE NUMBER Ω_1	DIMENSIONLESS SPECTRUM Φ_{22}	$\Phi_{22/11}$	DIMENSIONLESS WAVE NUMBER Ω_1	DIMENSIONLESS SPECTRUM Φ_{33}	$\Phi_{33/11}$	$\Phi_{33/22}$
0.00000	.47403	0.00000	.23725	0.0000	0.00000	.23678	0.0000	2.5189
.01000	.47399	.01000	.23727	.13234E-04	.01000	.23680	.13207E-04	2.5189
.02000	.47387	.02000	.23733	.52949E-04	.02000	.23686	.52843E-04	2.5189
.03000	.47367	.03000	.23744	.11919E-03	.03000	.23696	.11895E-03	2.5188
.04000	.47339	.04000	.23759	.21201E-03	.04000	.23710	.21159E-03	2.5188
.05000	.47303	.05000	.23776	.33152E-03	.05000	.23728	.33086E-03	2.5187
.06000	.47259	.06000	.23798	.47784E-03	.06000	.23750	.47688E-03	2.5187
.07000	.47208	.07000	.23824	.65109E-03	.07000	.23776	.64979E-03	2.5186
.08000	.47148	.08000	.23853	.85146E-03	.08000	.23805	.84976E-03	2.5185
.09000	.47081	.09000	.23886	.10791E-02	.09000	.23839	.10770E-02	2.5184
.10000	.47006	.10000	.23923	.13343E-02	.10000	.23875	.13317E-02	2.5182
.19000	.46005	.19000	.24391	.49111E-02	.19000	.24343	.49014E-02	2.5165
.28000	.44478	.28000	.25016	.10939E-01	.28000	.24968	.10918E-01	2.5136
.37000	.42542	.37000	.25656	.19590E-01	.37000	.25609	.19554E-01	2.5096
.46000	.40326	.46000	.26181	.30899E-01	.46000	.26133	.30843E-01	2.5042
.55000	.37951	.55000	.26496	.44704E-01	.55000	.26449	.44625E-01	2.4976
.64000	.35522	.64000	.26556	.60669E-01	.64000	.26509	.60561E-01	2.4897
.73000	.33120	.73000	.26356	.78336E-01	.73000	.26309	.78196E-01	2.4807
.82000	.30803	.82000	.25919	.97204E-01	.82000	.25872	.97028E-01	2.4706
.91000	.28607	.91000	.25287	.11679	.91000	.25240	.11658	2.4595
1.00000	.26553	1.00000	.24505	.13663	1.00000	.24459	.13642	2.4476
1.90000	.13316	1.90000	.15344	.30895	1.90000	.15301	.30809	2.3076
2.80000	.77440E-01	2.80000	.96030E-01	.41992	2.80000	.95655E-01	.41828	2.1670
3.70000	.50590E-01	3.70000	.64876E-01	.49537	3.70000	.64556E-01	.49292	2.0408
4.60000	.35757E-01	4.60000	.46762E-01	.55188	4.60000	.46492E-01	.54870	1.9286
5.50000	.26687E-01	5.50000	.35394E-01	.59716	5.50000	.35166E-01	.59331	1.8281
6.40000	.20721E-01	6.40000	.27797E-01	.63503	6.40000	.27599E-01	.63052	1.7372
7.30000	.16579E-01	7.30000	.22463E-01	.66765	7.30000	.22284E-01	.66234	1.6543
8.20000	.13584E-01	8.20000	.18567E-01	.69632	8.20000	.18398E-01	.68999	1.5782
9.10000	.11346E-01	9.10000	.15631E-01	.72195	9.10000	.15465E-01	.71430	1.5077
10.00000	.96272E-02	10.00000	.13361E-01	.74519	10.00000	.13194E-01	.73587	1.4423

TABLE A-4. DIMENSIONLESS SPECTRUM FOR ALTITUDE BAND #4 (continued)

i=1		i=2			i=3			
DIMENSIONLESS WAVE NUMBER	DIMENSIONLESS SPECTRUM	DIMENSIONLESS WAVE NUMBER	DIMENSIONLESS SPECTRUM		DIMENSIONLESS WAVE NUMBER	DIMENSIONLESS SPECTRUM		
Ω_1	Φ_{11}	Ω_1	Φ_{22}	$\Phi_{22/11}$	Ω_1	Φ_{33}	$\Phi_{33/11}$	$\Phi_{33/22}$
14.91780	.47319E-02	14.91780	.68101E-02	.84529	14.91780	.66213E-02	.82185	1.1506
19.83561	.27884E-02	19.83561	.41833E-02	.91802	19.83561	.39942E-02	.87653	.93474
24.75341	.18076E-02	24.75341	.28455E-02	.97244	24.75341	.26687E-02	.91204	.76806
29.67122	.12440E-02	29.67122	.20620E-02	1.0125	29.67122	.19004E-02	.93314	.63641
34.58902	.89257E-03	34.58902	.15598E-02	1.0408	34.58902	.14135E-02	.94319	.53107
39.50682	.66065E-03	39.50682	.12170E-02	1.0594	39.50682	.10854E-02	.94489	.44603
44.42462	.50109E-03	44.42462	.97201E-03	1.0699	44.42462	.85433E-03	.94040	.37691
49.34242	.38773E-03	49.34242	.79069E-03	1.0737	49.34242	.68586E-03	.93136	.32040
54.26022	.30509E-03	54.26022	.65279E-03	1.0719	54.26022	.55968E-03	.91905	.27394
59.17802	.24355E-03	59.17802	.54558E-03	1.0657	59.17802	.46304E-03	.90444	.23553

TABLE A-5. DIMENSIONLESS SPECTRUM FOR ALTITUDE BAND # 5

i=1		i=2			i=3			
DIMENSION-LESS WAVE NUMBER Ω_1	DIMENSIONLESS SPECTRUM Φ_{11}	DIMENSION-LESS WAVE NUMBER Ω_1	DIMENSIONLESS SPECTRUM Φ_{22}	$\Phi_{22/11}$	DIMENSION-LESS WAVE NUMBER Ω_1	DIMENSIONLESS SPECTRUM Φ_{33}	$\Phi_{33/11}$	$\Phi_{33/22}$
0.00000	.47517	0.00000	.23758	0.0000	0.00000	.23748	0.0000	3.7129
.01000	.47513	.01000	.23760	.13252E-04	.01000	.23750	.13246E-04	3.7129
.02000	.47501	.02000	.23766	.53023E-04	.02000	.23756	52999E-04	3.7129
.03000	.47481	.03000	.23777	.11935E-03	.03000	.23766	11930E-03	3.7128
.04000	.47453	.04000	.23791	.21231E-03	.04000	.23780	.21221E-03	3.7128
.05000	.47417	.05000	.23809	.33198E-03	.05000	.23798	[illegible]3183E-03	3.7127
.06000	.47373	.06000	.23831	.47950E-03	.06000	.23820	.47828E-03	3.7127
.07000	.47321	.07000	.23857	.65200E-03	.07000	.23846	.65170E-03	3.7126
.08000	.47262	.08000	.23886	.85264E-03	.08000	.23876	.85226E-03	3.7125
.09000	.47195	.09000	.23919	.10806E-02	.09000	.23909	10801E-02	3.7124
.10000	.47120	.10000	.23956	.13362E-02	.10000	.23946	.13356E-02	3.7122
.19000	.46118	.19000	.24424	.49177E-02	.19000	.24413	.49156E-02	3.7105
.28000	.44591	.28000	.25049	.10953E-01	.28000	.25038	.10949E-01	3.7076
.37000	.42656	.37000	.25689	.19615E-01	.37000	.25679	19607E-01	3.7035
.46000	.40439	.46000	.26214	.30938E-01	.46000	.26203	30925E-01	3.6981
.55000	.38064	.55000	.26530	.44760E-01	.55000	.26519	.44742E-01	3.6915
.64000	.35635	.64000	.26590	.60745E-01	.64000	.26579	[illegible]0720E-01	3.6835
.73000	.33232	.73000	.26389	.78434E-01	.73000	.26378	78[illegible]03E-01	3.6744
.82000	.30915	.82000	.25952	.97328E-01	.82000	.25941	972[illegible]8E-01	3.6643
.91000	.28719	.91000	.25320	.11695	.91000	.25309	11[illegible]90	3.6531
1.00000	.26664	1.00000	.24539	.13686	1.00000	.24528	.13680	3.6411
1.90000	.13421	1.90000	.15378	.30964	1.90000	.15368	.30942	3.4999
2.80000	.78413E-01	2.80000	.96383E-01	.42146	2.80000	.96276E-01	.42099	3.3575
3.70000	.51470E-01	3.70000	.65236E-01	.49812	3.70000	.65129E-01	.49730	3.2290
4.60000	.36545E-01	4.60000	.47121E-01	.55612	4.60000	.47014E-01	.55486	3.1143
5.50000	.27395E-01	5.50000	.35746E-01	.60310	5.50000	.35639E-01	.60129	3.0113
6.40000	.21364E-01	6.40000	.28134E-01	.64274	6.40000	.28028E-01	64030	2.9178
7.30000	.17173E-01	7.30000	.22781E-01	.67709	7.30000	.22674E-01	.67392	2.8323
8.20000	.14143E-01	8.20000	.18863E-01	.70741	8.20000	.18756E-01	[illegible]03[illegible]0	2.7534
9.10000	.11882E-01	9.10000	.15904E-01	.73457	9.10000	.15797E-01	.72964	2.6801
10.00000	.10150E-01	10.00000	.13612E-01	.75923	10.00000	.13506E-01	.75328	2.6116

TABLE A-5. DIMENSIONLESS SPECTRUM FOR ALTITUDE BAND #5 (continued)

i=1		i=2			i=3			
DIMENSION-LESS WAVE NUMBER	DIMENSIONLESS SPECTRUM	DIMENSION-LESS WAVE NUMBER	DIMENSIONLESS SPECTRUM		DIMENSION-LESS WAVE NUMBER	DIMENSIONLESS SPECTRUM		
Ω_1	ϕ_{11}	Ω_1	ϕ_{22}	$\phi_{22/11}$	Ω_1	ϕ_{33}	$\phi_{33/11}$	$\phi_{33/22}$
19.00000	.35256E-02	19.00000	.46754E-02	.94138	19.00000	.45700E-02	.92017	2.0899
28.00001	.18 20E-02	28.00001	.24617E-02	1.0765	28.00001	.23585E-02	1.0313	1.7286
37.00001	11733E-02	37.00001	.15535E-02	1.1862	37.00001	.1[illegible]531E-02	1.1095	1.4530
46.00001	81 2[illegible]5E-03	46.00001	.10839E-02	1.2793	46.00001	.98684E-03	1.1647	1.2335
55.00001	.60468E-03	55.00001	.80633E-03	1.3604	55.00001	71299E-03	1.2030	1.0545
64.00002	46794E-03	64.00002	.62698E-03	1.4324	64.00002	53764E-03	1.2283	.90651
73.00002	37409E-03	73.00002	.50358E-03	1.4968	73.00002	41833E-03	1.2434	.78297
82.00002	30680E-03	82.00002	.41460E-03	1.5549	82.00002	.33340E-03	1.2504	.67918
91.00002	.25687E-03	91.00002	.34810E-03	1.6078	91.00002	.27084E-03	1.2509	.59154
100.00002	21879E-03	100.00002	.29698E-03	1.6564	100.00002	.22347E-03	1.2464	.51721
190.00003	[illegible]4430E-04	190.00003	.10056E-03	2.0248	103.65648	20[illegible]48E-03	1.2434	.49029
280.00006	38592E-04	280.00006	.52567E-04	2.2986	107.31294	193 [illegible]E-03	1.2398	.46505
370.00006	23822E-04	370.00006	.32946E-04	2.5156	110.96941	.17991E-03	1.2357	.44138
460.00006	16216E-04	460.00006	.22822E-04	2.6935	114.62587	16798E-03	1.2310	.41918
550.00012	11742E-04	550.00012	.16851E-04	2.8430	118.28233	15711E-03	1.2260	.39833
640.00012	.88702E-05	640.00012	.13003E-04	2.9705	121.93880	14718E-03	1.2206	.37874
730.00012	69113E-05	730.00012	.10363E-04	3.0300	125.59526	13808E-03	1.2148	.36032
820.00012	55136E-05	820.00012	.84644E-05	3.1744	129.25174	1297[illegible]E-03	1.2088	.34301
910.00012	44814E-05	910.00012	.70491E-05	3.2558	132.90820	.12205E-03	1.2025	.32671
1000.00010	.36980E-05	1000.00010	.59628E-05	3.3257	136.56467	.11497E-03	1.1959	.31137
1000.00020	.36980E-05	1000.00020	.59628E-05	3.3257				
1122.05660	.29062E-05	1122.05660	.48484E-05	3.4046				
1244.11300	.23262E-05	1244.11300	.40164E-05	3.4673				
1366.16940	.18901E-05	1366.16940	.33773E-05	3.5158				
1488.22580	.15551E-05	1488 225[illegible]0	.28750E-05	3.5515				
1610.28220	.12931E-05	1610.28220	.24726E-05	3.5760				
1732.33860	.10852E-05	1732 33860	.21450E-05	3.5903				
1854.39500	.91821E-06	1[illegible]5[illegible] 39500	.18748E-05	3.5957				
1976.45140	.[illegible]82[illegible]5E-06	1976.45140	.16492E-05	3.5932				
2098.50780	.67131E-06	2098.50780	.14591E-05	3.5838				
2220.56400	.57933E-06	2220.56400	.12974E-05	3.5682				

TABLE A-6. DIMENSIONLESS SPECTRUM FOR ALTITUDE BAND # 6

i=1		i=2			i=3			
DIMENSION-LESS WAVE NUMBER Ω_1	DIMENSIONLESS SPECTRUM Φ_{11}	DIMENSION-LESS WAVE NUMBER Ω_1	DIMENSIONLESS SPECTRUM Φ_{22}	$\Phi_{22/11}$	DIMENSION-LESS WAVE NUMBER Ω_1	DIMENSIONLESS SPECTRUM Φ_{33}	$\Phi_{33/11}$	$\Phi_{33/22}$
0.00000	.47517	0.00000	.23758	0.0000	0.00000	.23758	0.0000	9.2415
.01000	.47513	.01000	.23760	.13252E-04	.01000	.23760	.13252E-04	9.2415
.02000	.47501	.02000	23767	.53023E-04	.02000	.23766	53022E-04	9.2415
.03000	.47481	.03000	Z3777	.11935E-03	.03000	.23776	.11935E-03	9.2414
.04000	.47453	.04000	.23791	.21231E-03	.04000	.23791	.21231E-03	9.2414
.05000	.47417	.05000	23809	.33198E-03	.05000	.23809	.33198E-03	9.2413
.06000	.47373	.06000	.23831	.47850E-03	.06000	.23831	.47849E-03	9.2413
.07000	.47322	.07000	.23857	.65200E-03	.07000	.23856	.65199E-03	9.2412
.08000	.47262	.08000	.23886	.85264E-03	.08000	.23886	85264E-03	9.2411
.09000	.47195	.09000	.23920	.10806E-02	.09000	.23919	10806E-02	9.2410
.10000	.47120	.10000	.23956	.13362E-02	.10000	.23956	.13361E-02	9.2408
.19000	.46118	.19000	.24424	.49177E-02	.19000	.24424	.49177E-02	9.2391
.28000	.44592	.28000	.25049	.10953E-01	.28000	.25049	1095[illegible]E-01	9.2362
.37000	.42656	.37000	.25689	.19615E-01	.37000	.25689	1[illegible]615E-01	9.2321
.46000	.40439	.46000	26214	.30938E-01	.46000	.26214	30937E-01	9.2267
.55000	.38064	.55000	26530	.44760E-01	.55000	.26529	44760E-01	9.2200
.64000	.35635	.64000	.26590	.60745E-01	.64000	.26589	.60744E-01	9.2121
.73000	.33233	.73000	26389	.78435E-01	.73000	.26389	.78434E-01	9.2030
.82000	30915	.82000	.25952	.97329E-01	.82000	.25952	.97328E-01	9.1928
.91000	.28719	.91000	.25320	.11695	.91000	.25320	.11694	9.1816
1.00000	.26665	1.00000	24[illegible]39	.13686	1.00000	.24538	1368[illegible]	9.1697
1.90000	.13421	1.90000	.15378	.30964	1.90000	.15378	.30964	9.0282
2.80000	.78414E-01	2.80000	96334E-01	.42146	2.80000	.96382E-01	42145	8.8855
3.70000	.51471E-01	3.70000	.65237E-01	.49812	3.70000	.65234E-01	.49810	8.7566
4.60000	.36547E-01	4.60000	471[illegible]2E-01	.55613	4.60000	.47119E-01	5[illegible]610	8.6413
5.50000	.27396E-01	5.50000	.35746E-01	.60311	5.50000	.35744E-01	60307	8.5376
6.40000	.21365E-01	6.40000	.28135E-01	.64276	6.40000	.28133E-01	.64270	8.4433
7.30000	.17175E-01	7.30000	.22781E-01	.67711	7.30000	.22779E-01	6[illegible]04	8.3568
8.20000	.14145E-01	8.20000	1[illegible]863E-01	.70743	8.20000	.18861E-01	[illegible]073[illegible]	8.2769
9.10000	.11884E-01	9.10000	15305E-01	.73459	9.10000	.15902E-01	.73448	8.2024
10.00000	.10152E-01	10.00000	[illegible]36[illegible]E-01	.75926	10.00000	.13611E-01	75913	8.1326

TABLE A-6. DIMENSIONLESS SPECTRUM FOR ALTITUDE BAND #6 (continued)

i=1		i=2			i=3			
DIMENSIONLESS WAVE NUMBER	DIMENSIONLESS SPECTRUM	DIMENSIONLESS WAVE NUMBER	DIMENSIONLESS SPECTRUM		DIMENSIONLESS WAVE NUMBER	DIMENSIONLESS SPECTRUM		
Ω_1	Φ_{11}	Ω_1	Φ_{22}	$\Phi_{22/11}$	Ω_1	Φ_{33}	$\Phi_{33/11}$	$\Phi_{33/22}$
19.00000	.35273E-02	19.00000	.46760E-02	.94149	19.00000	.98722E-04	.94100	7.5915
28.00001	.18637E-02	28.00001	.24623E-02	1.0767	28.00001	.50774E-04	1.0756	7.1997
37.00001	.11750E-02	37.00001	.15541E-02	1.1866	37.00001	.31210E-04	1.1848	6.8838
46.00001	.81798E-03	46.00001	.10845E-02	1.2799	46.00001	.21153E-04	1.2770	6.6155
55.00001	.60640E-03	55.00001	.80690E-03	1.3614	55.00001	.15252E-04	1.3573	6.3806
64.00002	.46966E-03	64.00002	.62756E-03	1.4337	64.00002	.11478E-04	1.4281	6.1709
73.00002	.37581E-03	73.00002	.50415E-03	1.4985	73.00002	.89135E-05	1.4912	5.9809
82.00002	.30851E-03	82.00002	.41517E-03	1.5570	82.00002	.70905E-05	1.5479	5.8068
91.00002	.25858E-03	91.00002	.34867E-03	1.6104	91.00002	.57493E-05	1.5991	5.6460
100.00002	.22050E-03	100.00002	.29755E-03	1.6596	100.00002	.47355E-05	1.6460	5.4962
190.00003	.76128E-04	190.00003	.10113E-03	2.0362	190.00003	.46735E-02	1.9877	4.3655
280.00006	.40158E-04	280.00006	.53133E-04	2.3234	280.00006	.24598E-02	2.2202	3.5890
370.00006	.25293E-04	370.00006	.33505E-04	2.5583	370.00006	.15516E-02	2.3831	2.9997
460.00006	.17593E-04	460.00006	.23369E-04	2.7580	460.00006	.10821E-02	2.4964	2.5328
550.00012	.13031E-04	550.00012	.17380E-04	2.9324	550.00012	.80445E-03	2.5733	2.1540
640.00012	.10083E-04	640.00012	.13513E-04	3.0870	640.00012	.62511E-03	2.6223	1.8424
730.00012	.80609E-05	730.00012	.10852E-04	3.2255	730.00012	.50171E-03	2.6493	1.5838
820.00012	.66106E-05	820.00012	.89342E-05	3.3506	820.00012	.41273E-03	2.6591	1.3677
910.00012	.55347E-05	910.00012	.75009E-05	3.4645	910.00012	.34623E-03	2.6555	1.1863
1000.00010	.47141E-05	1000.00010	.63991E-05	3.5691	1000.00010	.29511E-03	2.6412	1.0332
1000.00020	.47141E-05	1000.00020	.63991E-05	3.5691	1000.00020	.29511E-03	2.6412	1.0332
1900.00050	.16047E-05	1900.00050	.21666E-05	4.3624	1031.01340	.44433E-05	2.6344	.98605
2800.00100	.83145E-06	2800.00100	.11325E-05	4.9523	1062.02660	.41753E-05	2.6266	.94155
3700.00150	.51323E-06	3700.00150	.70980E-06	5.4198	1093.03980	.39288E-05	2.6180	.89947
4600.00200	.34936E-06	4600.00200	.49170E-06	5.8030	1124.05300	.37019E-05	2.6088	.85969
5500.00200	.25297E-06	5500.00200	.36304E-06	6.1252	1155.06620	.34924E-05	2.5988	.82204
6400.00200	.19110E-06	6400.00200	.28014E-06	6.3998	1186.07930	.32987E-05	2.5883	.78641
7300.00200	.14890E-06	7300.00200	.22326E-06	6.6358	1217.09250	.31194E-05	2.5772	.75267
8200.00200	.11879E-06	8200.00200	.18236E-06	6.8391	1248.10570	.29530E-05	2.5657	.72070
9100.00200	.96550E-07	9100.00200	.15187E-06	7.0144	1279.11890	.27984E-05	2.5537	.69041
10000.00200	.79671E-07	10000.00200	.12846E-06	7.1651	1310.13210	.26546E-05	2.5414	.66168

TABLE A-6. DIMENSIONLESS SPECTRUM FOR ALTITUDE BAND # 6 (concluded)

i=1		i=2			i=3			
DIMENSION-LESS WAVE NUMBER Ω_1	DIMENSIONLESS SPECTRUM Φ_{11}	DIMENSION-LESS WAVE NUMBER Ω_1	DIMENSIONLESS SPECTRUM Φ_{22}	$\Phi_{22/11}$	DIMENSION-LESS WAVE NUMBER Ω_1	DIMENSIONLESS SPECTRUM Φ_{33}	$\Phi_{33/11}$	$\Phi_{33/22}$
11220.566	.62612E-07	11220.566	.10446E-06	7.3351				
12441.131	.50118E-07	12441.131	.86531E-07	7.4701				
13661.695	.40722E-07	13661.695	.72762E-07	7.5745				
14882.260	.33503E-07	14882.260	.61940E-07	7.6515				
16102.824	.27859E-07	16102.824	.53270E-07	7.7042				
17323.387	.23381E-07	17323.387	.46213E-07	7.7351				
18543.949	.19782E-07	18543.949	.40390E-07	7.7468				
19764.512	.16860E-07	19764.512	.35531E-07	7.7414				
20985.074	.14463E-07	20985.074	.31435E-07	7.7210				
22205.637	.12481E-07	22205.637	.27952E-07	7.6875				

APPENDIX B

ESTABLISHMENT OF LOWER FREQUENCY LIMITS

The maximum time limit, t_{max}, for which the impulse response function is computed, determines the minimum frequency, Ω_{i1min}, for which the corresponding spectrum is accurately simulated according to the relation

$$\Omega_{i1min} = \pi/t_{max} \tag{B-1}$$

The simulated turbulence may contain lower frequencies, depending on the total length of the time series, but the shape of the actual spectrum for any frequencies less than Ω_{i1min} will not in general match the theoretical spectral shape. To verify this point for the $\Phi_{33/11}$ spectrum two impulse response functions were generated, the first extending out to a t_{max} of 42.10573 and the second extending out to a t_{max} of 85.895689. Two separate turbulence time series were then generated, one for each impulse function. The results of spectral analysis of these two time series is presented in Figures B-1 and B-2. As indicated in each figure, the observed spectrum tends to drift away from the theoretical spectrum for frequencies below π/t_{max}. The impulse response function with the larger t_{max} produces a time series whose spectral shape matches the theoretical spectrum to the lower frequency.

Based on various characteristics of the Space Shuttle simulators, a minimum frequency, f_{1min}, of .04 hertz was established for generating simulated turbulence. The dimensional frequency, f_{1min}, is related to the dimensionless frequency, Ω_{i1min}, according to the relation

$$\Omega_{i1min} = 2\pi a L_i \, f_{1min}/V \tag{B-2}$$

By substitution and rearrangement,

$$t_{max} = V/(2aL_i \, f_{1min}) \tag{B-3}$$

To satisfy the requirement for a minimum simulation frequency of .04 hertz, values of each impulse response function were computed for 100 dimensionless time intervals. As shown in Table B-1, by using a constant number of time

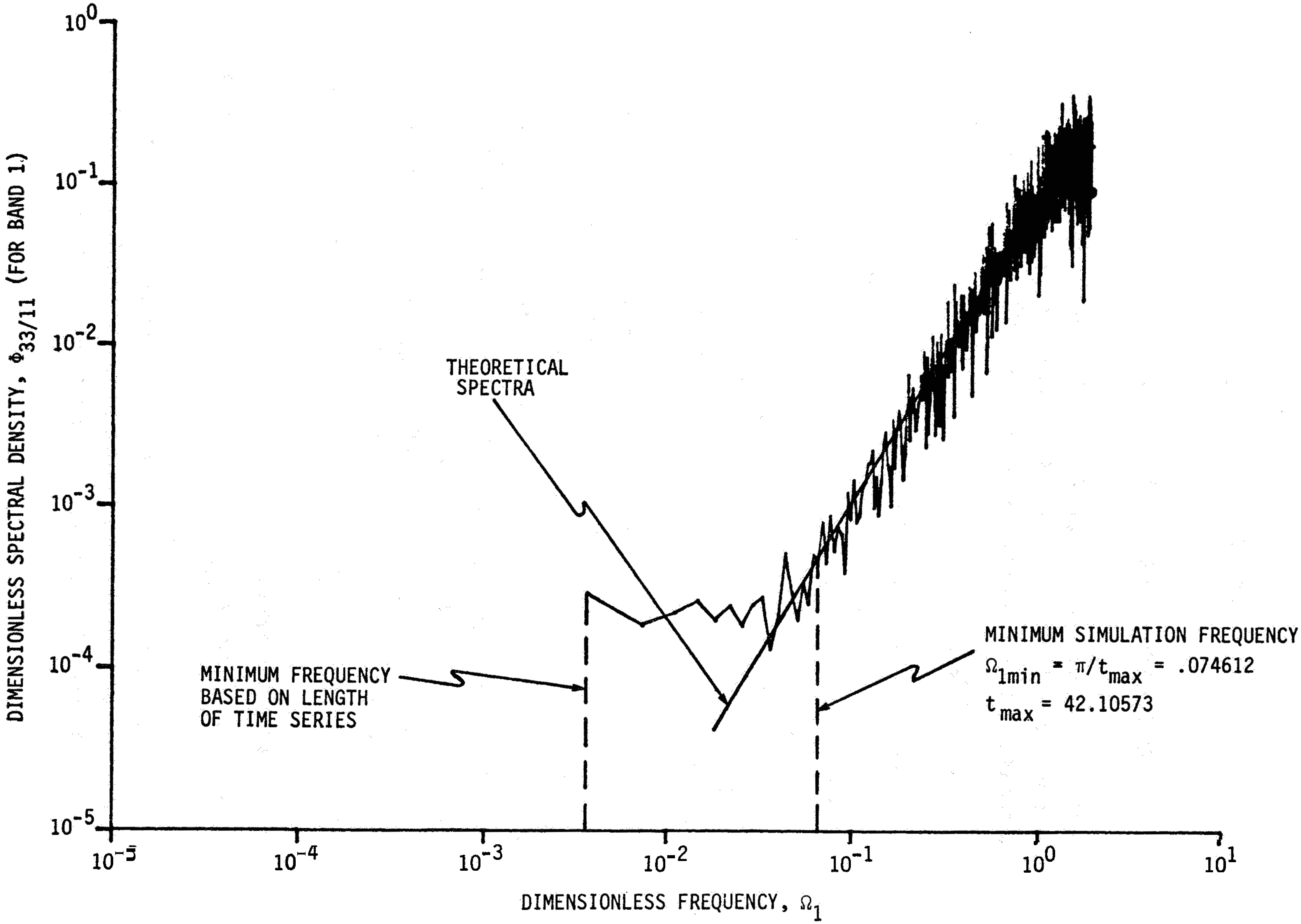

Figure B-1. Spectrum of $\Phi_{33/11}$ for Impulse Response Function with t_{max} = 42.10573

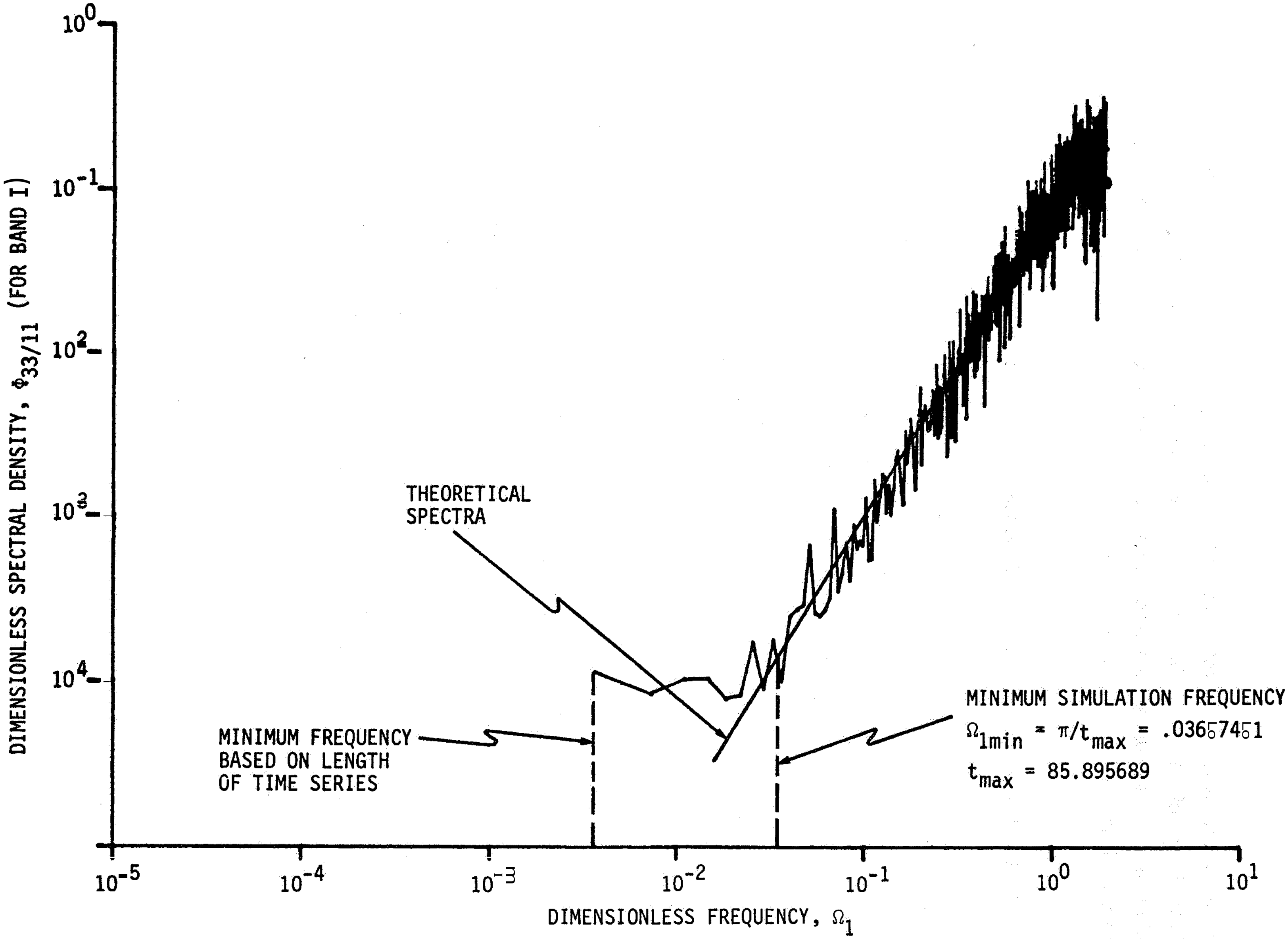

Figure B-2. Spectrum of $\Phi_{33/11}$ for Impulse Response Function with t_{max} = 85.895 89 intervals, the actual minimum simulation frequency varies from .0174 hertz in Band 1 to .04 hertz in Band 6. Thus, at the lower altitudes the turbulence simulation will actually be valid for frequencies somewhat lower than the required minimum as given by Eq (B-2).

TABLE B-1. DIMENSIONAL AND DIMENSIONLESS MINIMUM FREQUENCY LIMITS

			MINIMUM SIMULATION FREQUENCY LIMITS	
			DIMENSIONAL	DIMENSIONLESS
i	SPECTRA	BAND	f_{1min} (hertz)	Ω_{i1min}
1	Φ_{11}	1	.0174	.04818
		2	.0212	.2176
		3	.0211	.3331
		4	.0396	.5918
		5	.04	7.365
		6	.04	8.949
2	Φ_{22}, $\Phi_{22/11}$	1	.0174	.03075
		2	.0206	.2110
		3	.0211	.3331
		4	.0396	.5918
		5	.04	7.365
		6	.04	8.949
3	Φ_{33}, $\Phi_{33/11}$, $\Phi_{33/22}$	1	.0174	.01865
		2	.0208	.2132
		3	.0211	.3331
		4	.0396	.5918
		5	.04	.4630
		6	.04	.5280

APPENDIX C

SPECTRAL ANALYSIS OF SIMULATED TURBULENCE

By means of a Fast Fourier Transform [14] spectral analyses of all simulated turbulence have been performed*. The results are presented in dimensionless form in Figures C-1 through C-36. Table C-1 provides a summary of these figures. Also included in each figure is the theoretical von Karman spectra. The agreement between the theoretical spectra and the computed spectra is quite satisfactory.

TABLE C-1. MATRIX OF SPECTRAL ANALYSIS FIGURES

SERIES TYPE	ALTITUDE BAND					
	1	2	3	4	5	6
u_1	C-1	C-2	C-3	C-4	C-5	C-6
u_2	C-7	C-8	C-9	C-10	C-11	C-12
u_3	C-13	C-14	C-15	C-16	C-17	C-18
$\partial u_2/\partial x_1$	C-19	C-20	C-21	C-22	C-23	C-24
$\partial u_3/\partial x_1$	C-25	C-26	C-27	C-28	C-29	C-30
$\partial u_3/\partial x_2$	C-31	C-32	C-33	C-34	C-35	C-36

*The spectral analysis involved the first 4096 terms of each time series except for bands 5 and 6 for the u_1 and u_2 gusts. For these cases 8192 terms were used.

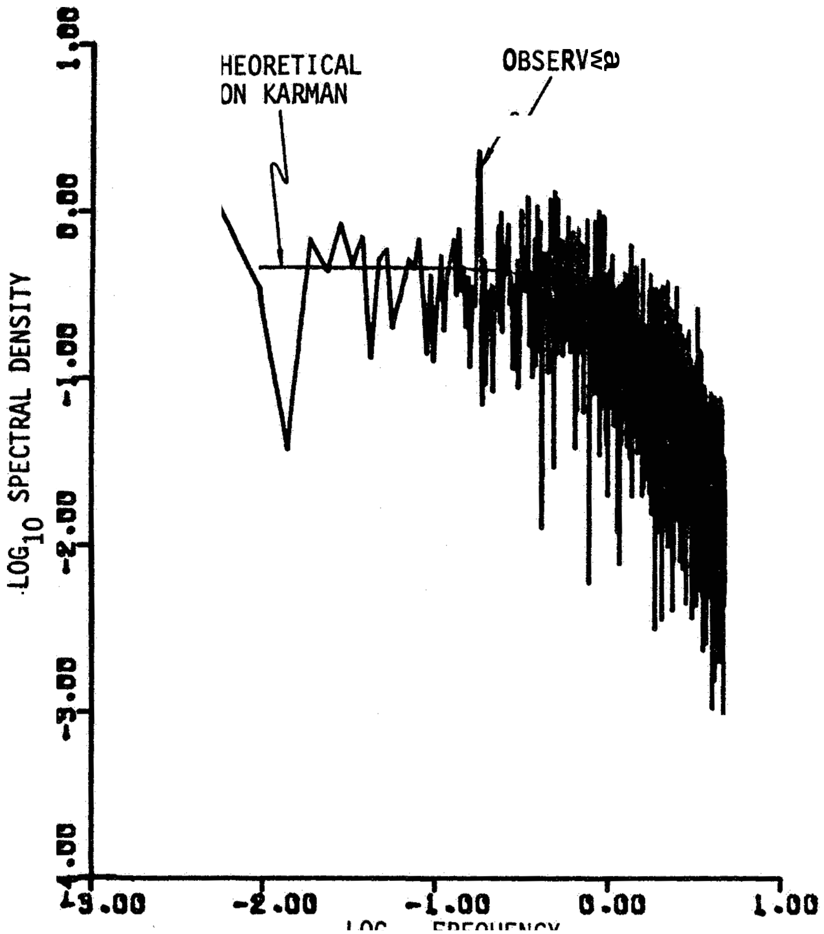

Figure C-1. u_1 - Gust Spectrum, Altitude Band #1

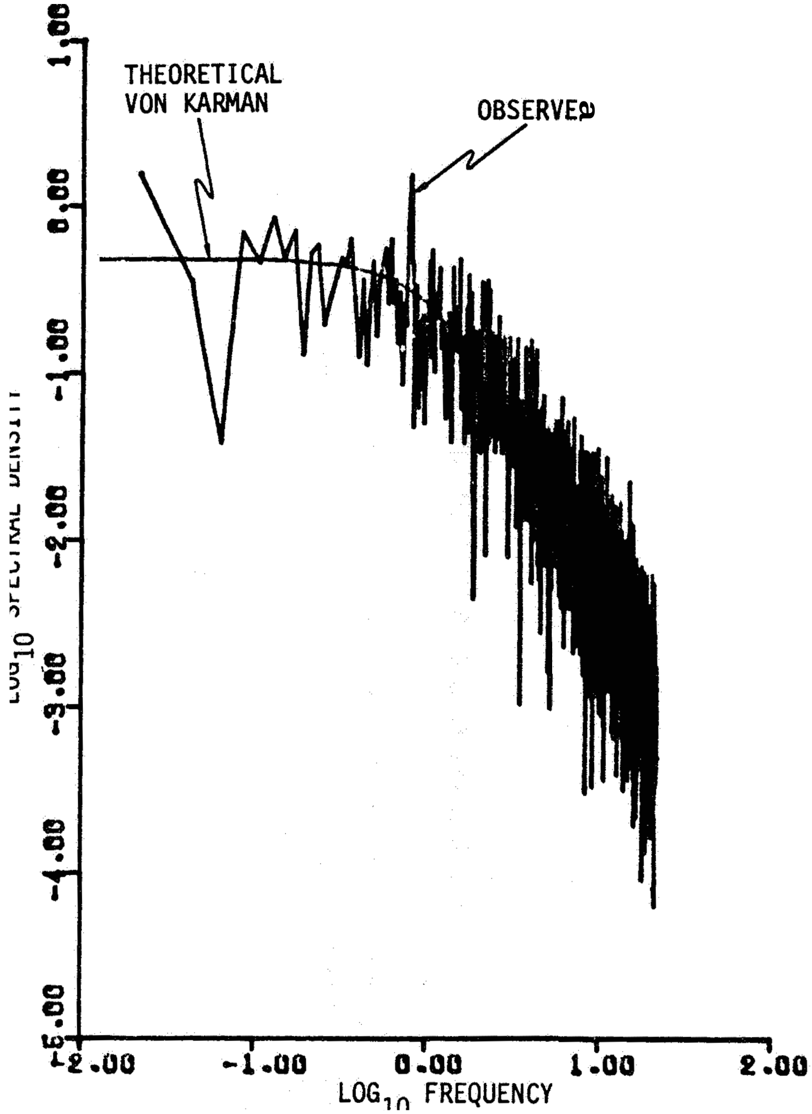

Figure C-2. u_1 - Gust Spectrum, Altitude Band #2

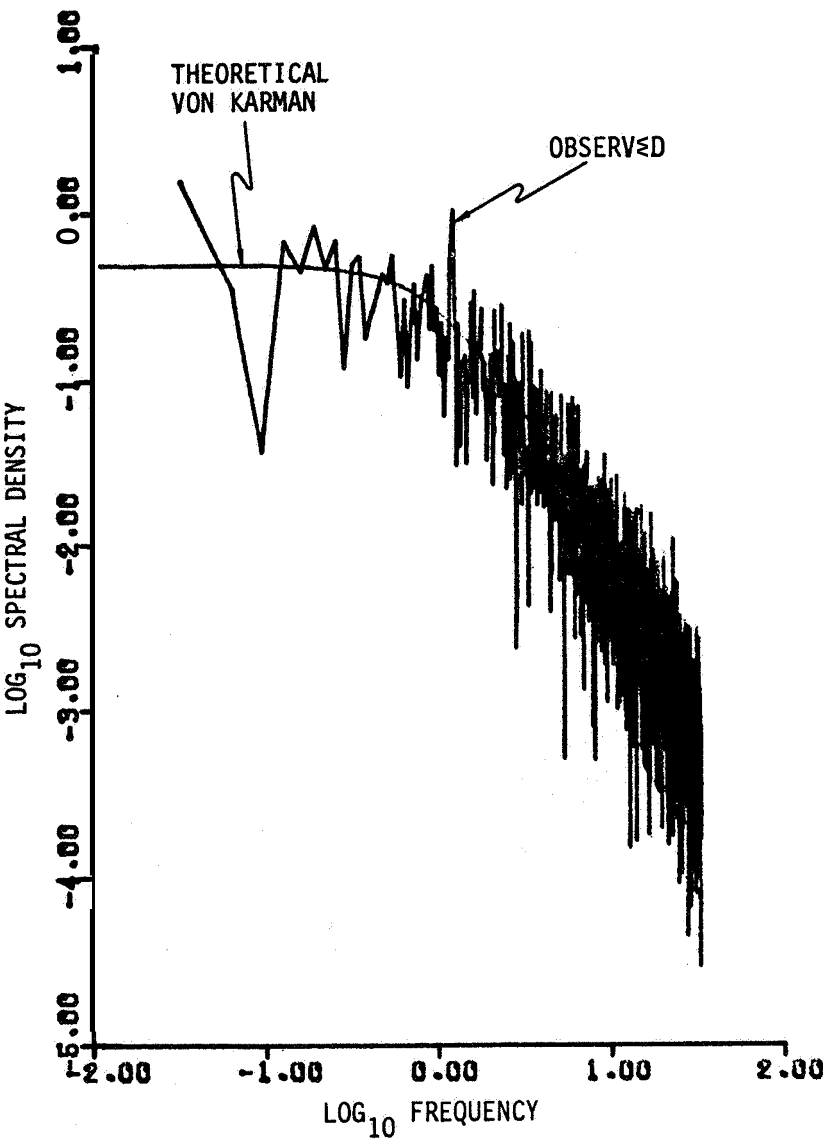

Figure C-3. u_1 - Gust Spectrum, Altitude Band #3

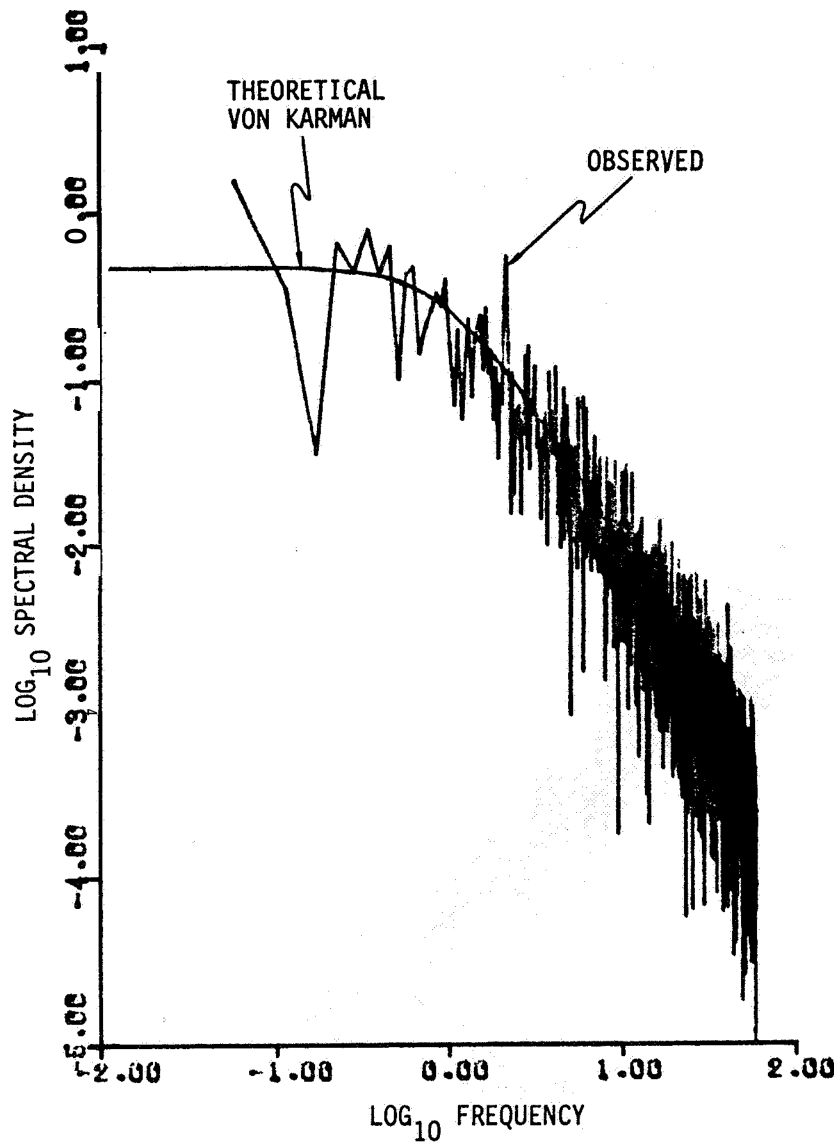

Figure C-4. w_1 - Gust Spectrum, Altitude Band #4

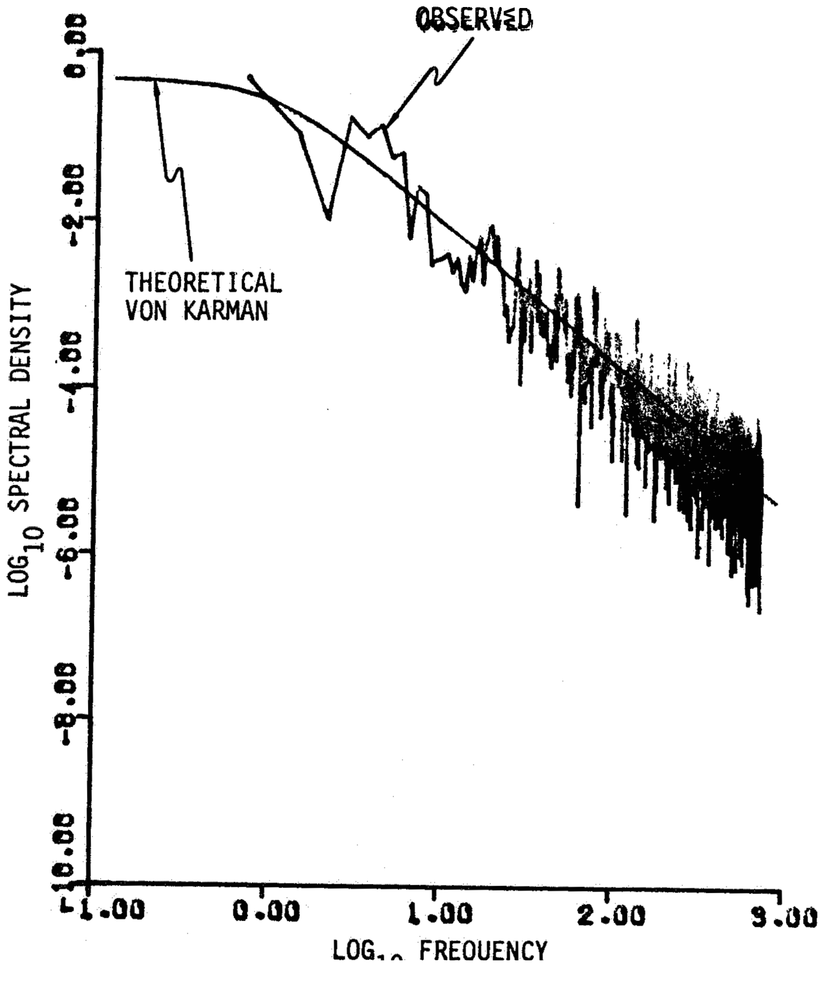

Figure C-5. u_1 - Gust Spectrum, Altitude Band #5

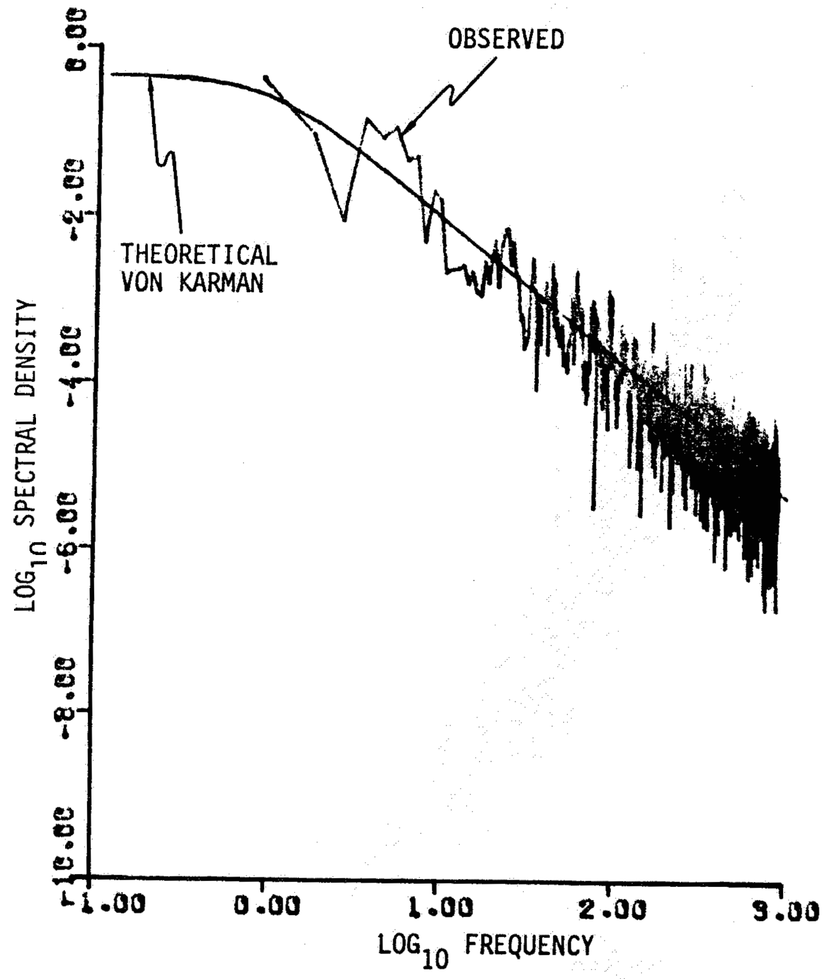

Figure C-6. u_1 - Gust Spectrum, Altitude Bamd #6

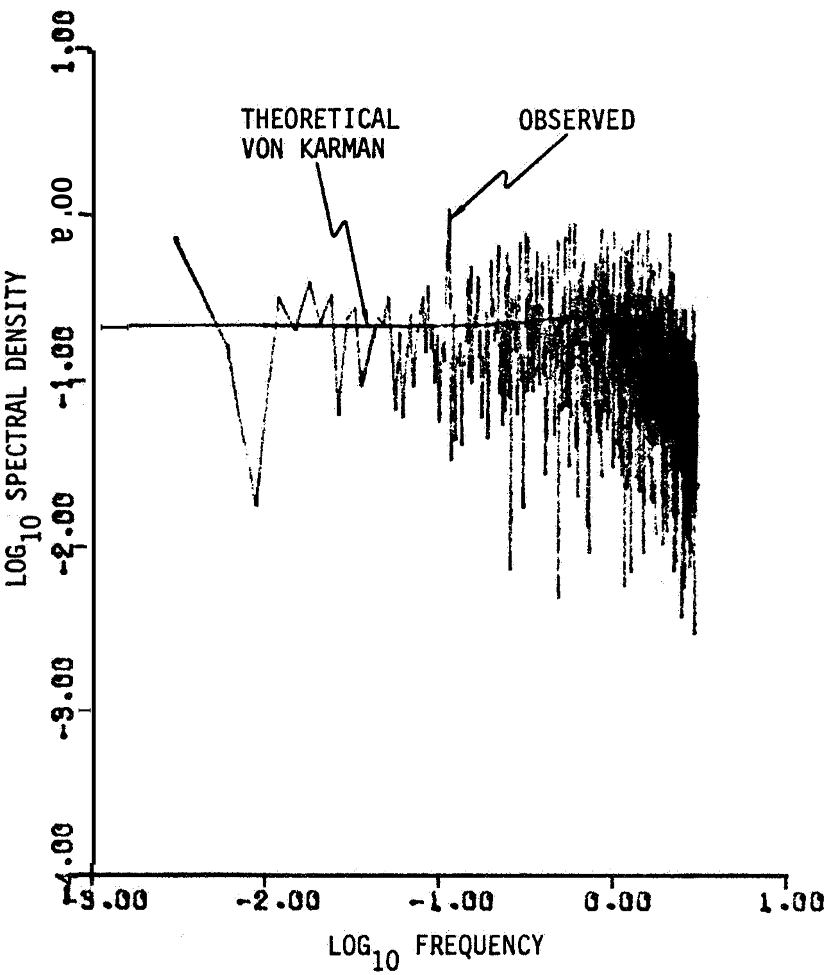

Figure C-7. u_2 - Gust Spectrum, Altitude Band #1

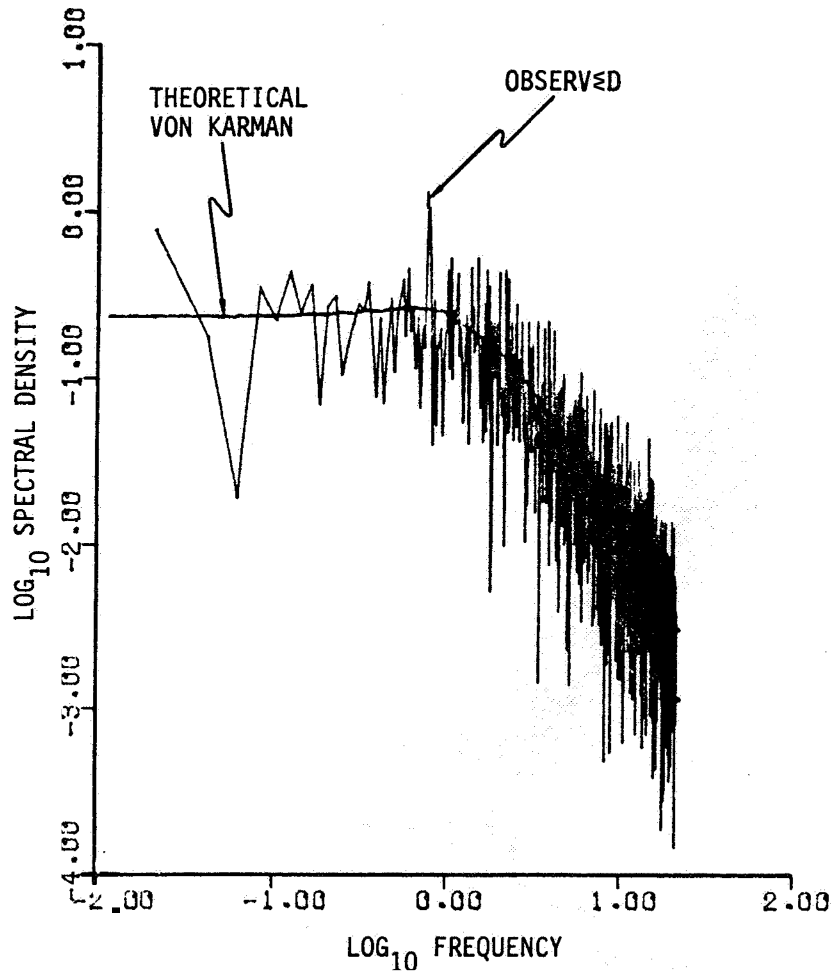

Figure C-8. w_2 - Gust Spectrum, Altitude Band #2

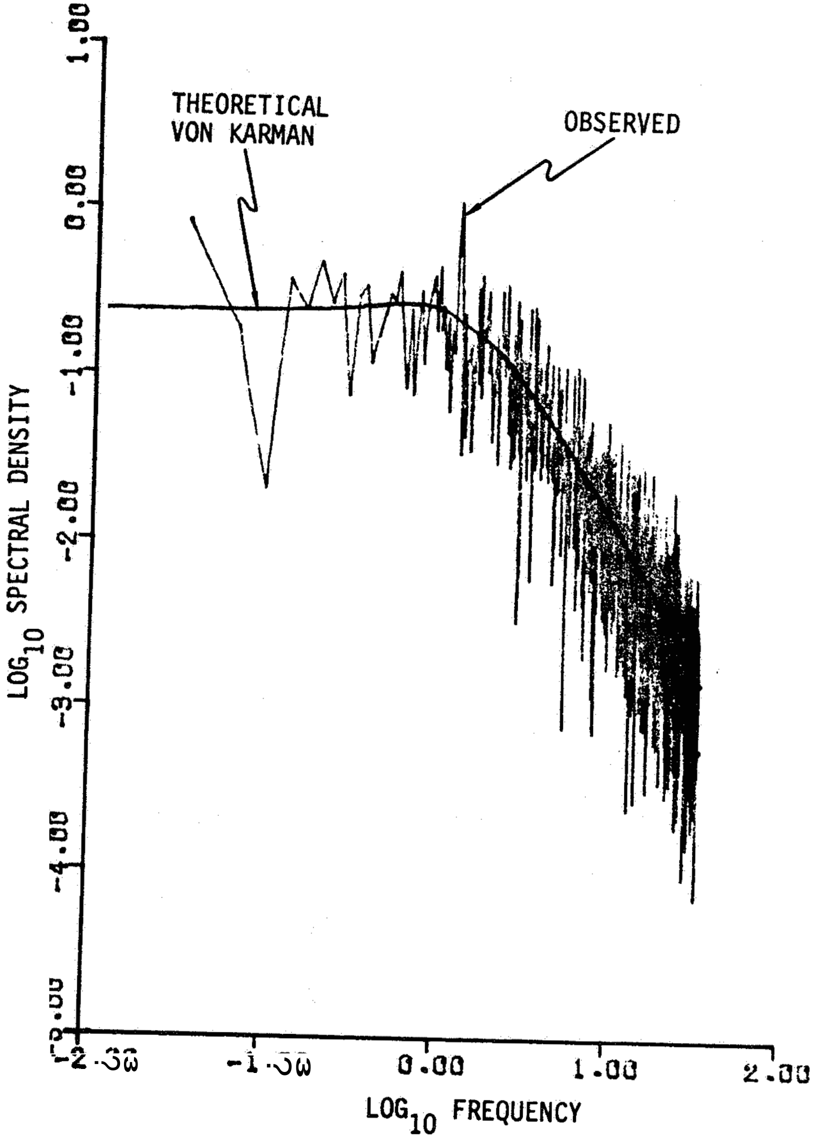

Figure C-9. u_2 - Gust Spectrum, Altitude Band #3

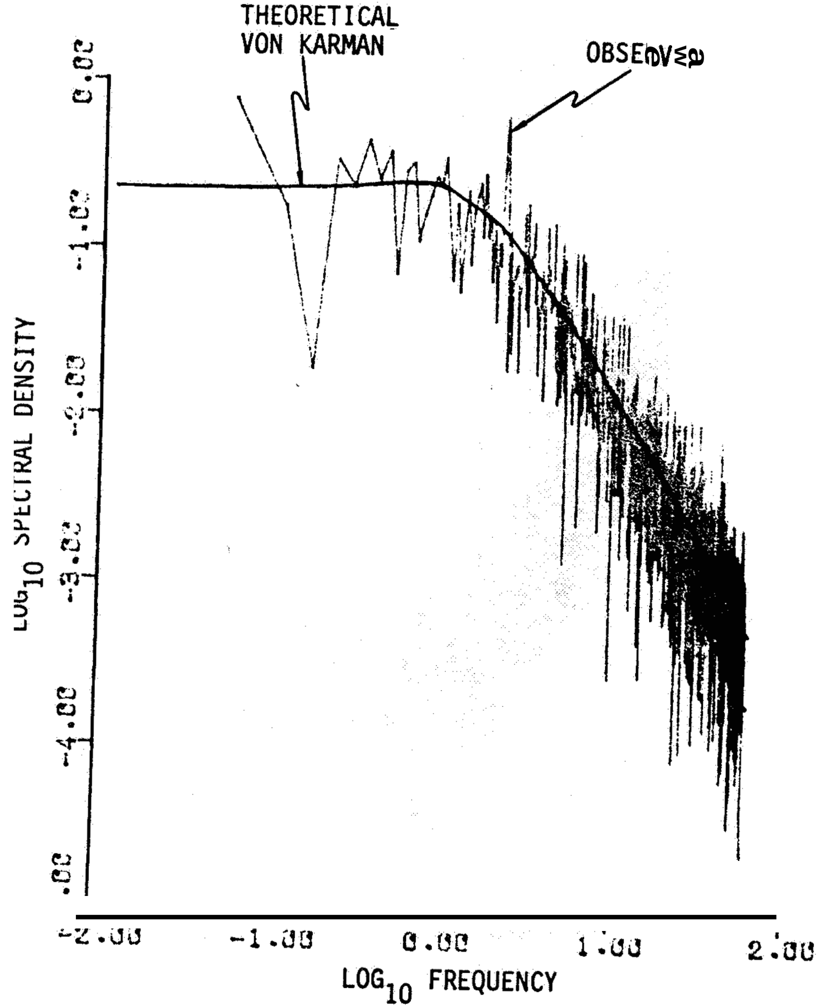

Figure C-10. u_2 - Gust Spectrum, Altitude Band #4

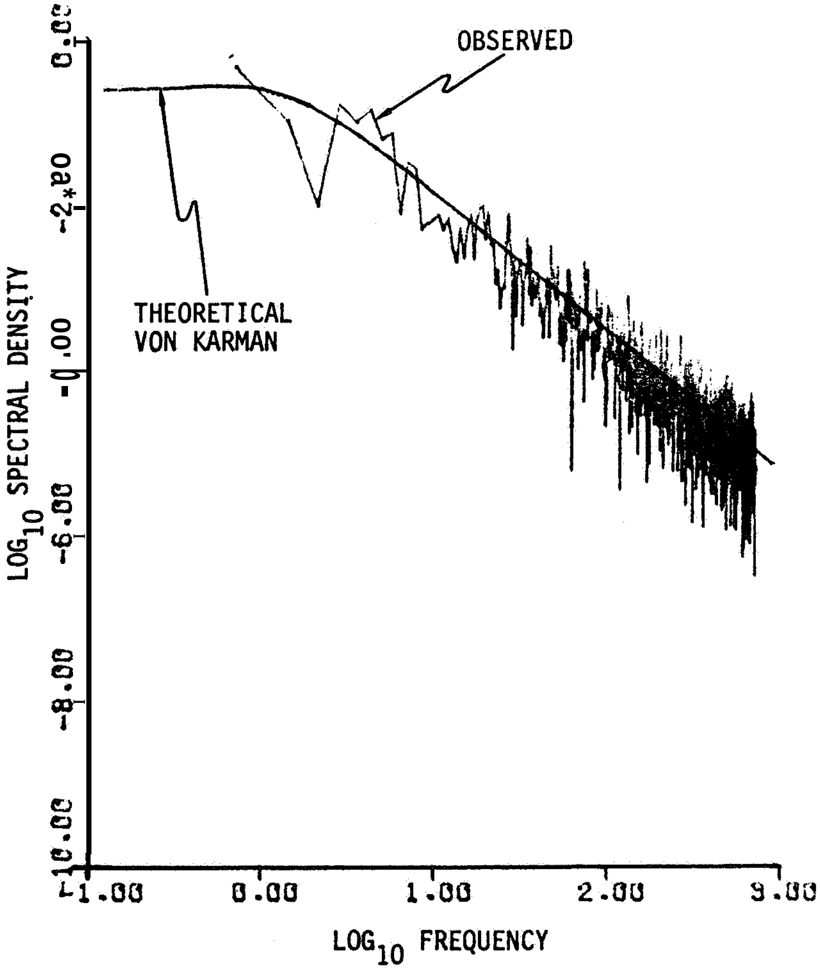

Figure C-11. u_2 - Gust Spectrum, Altitude Band #5

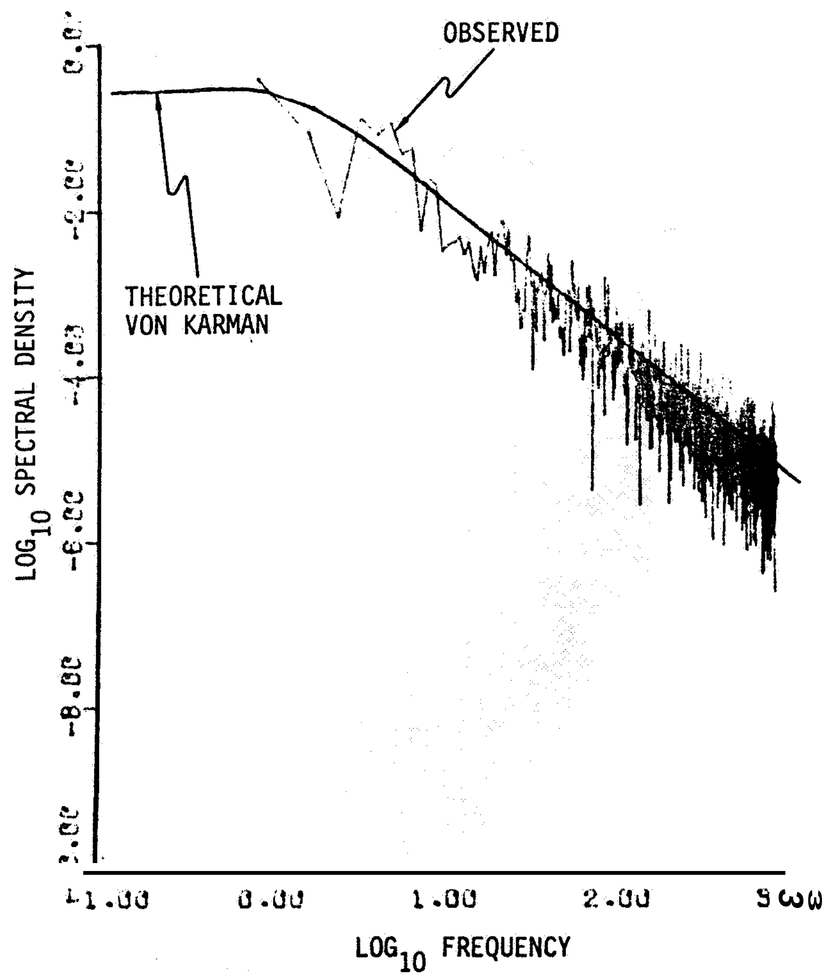

Figure C-12. u_2 - Gust Spectrum, Altitude Band #6

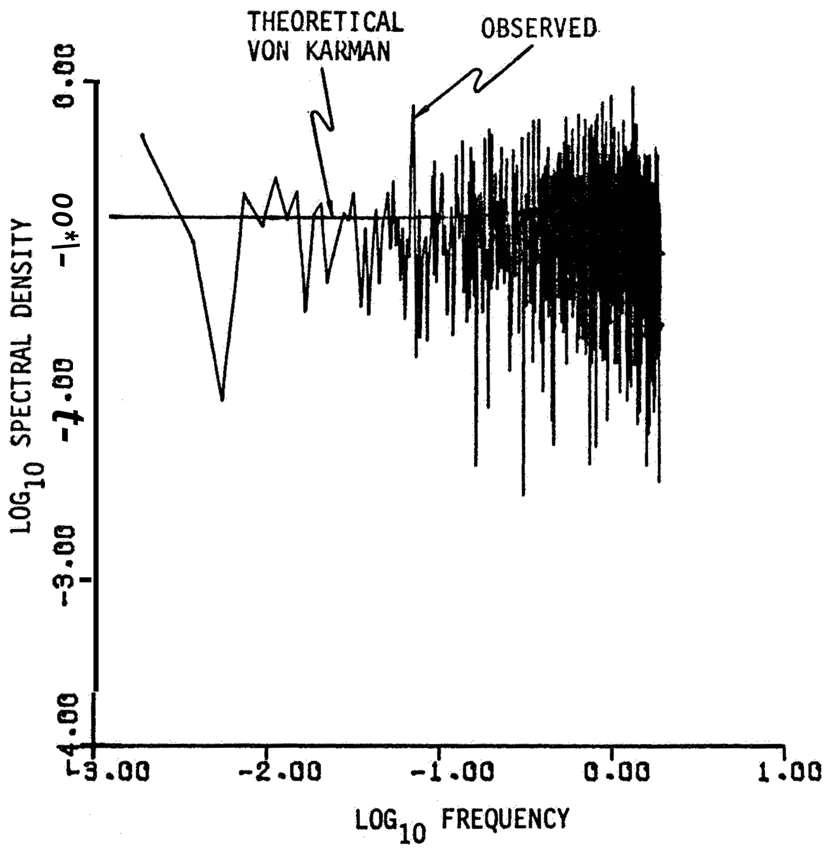

Figure C-13. u_3 - Gust Spectrum, Altitude Band #1

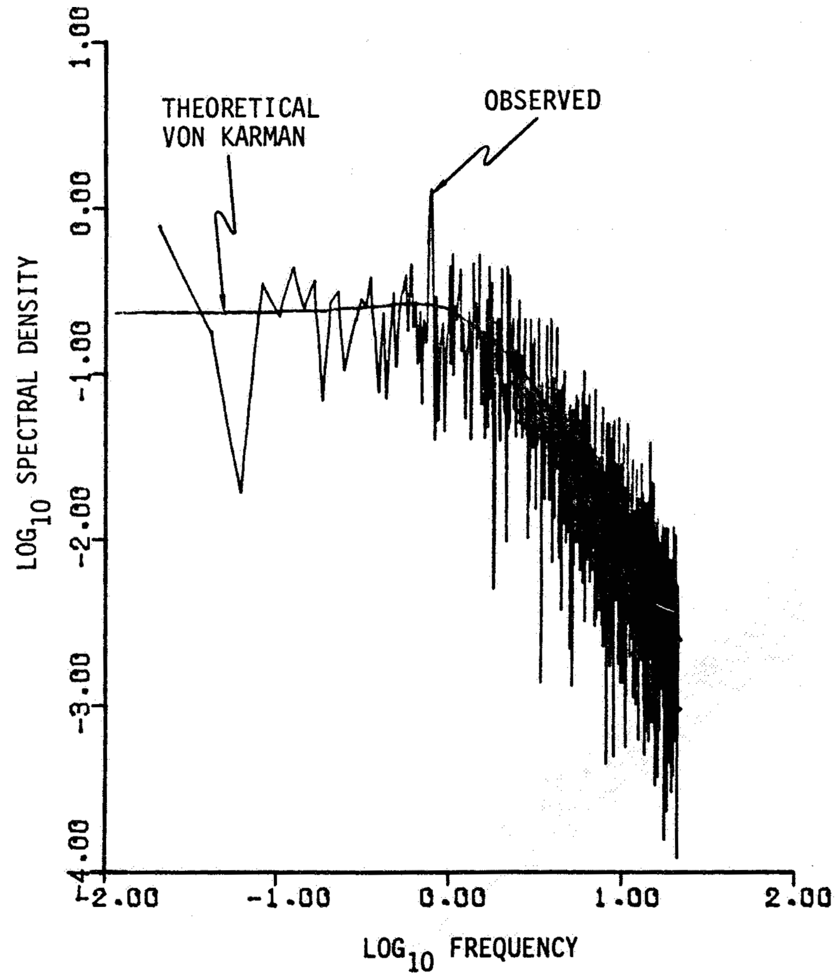

Figure C-14. u_3 - Gust Spectrum, Altitude Band #2

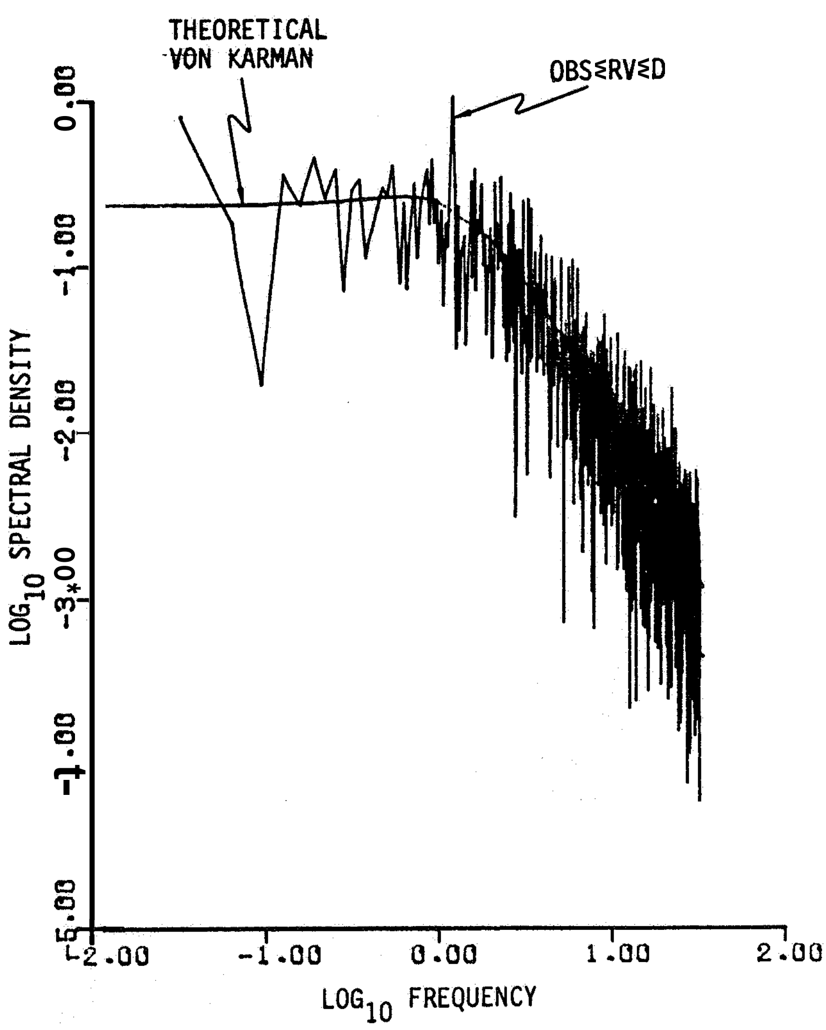

Figure C-15. u_3 - Gust Spectrum, Altitude Band #3

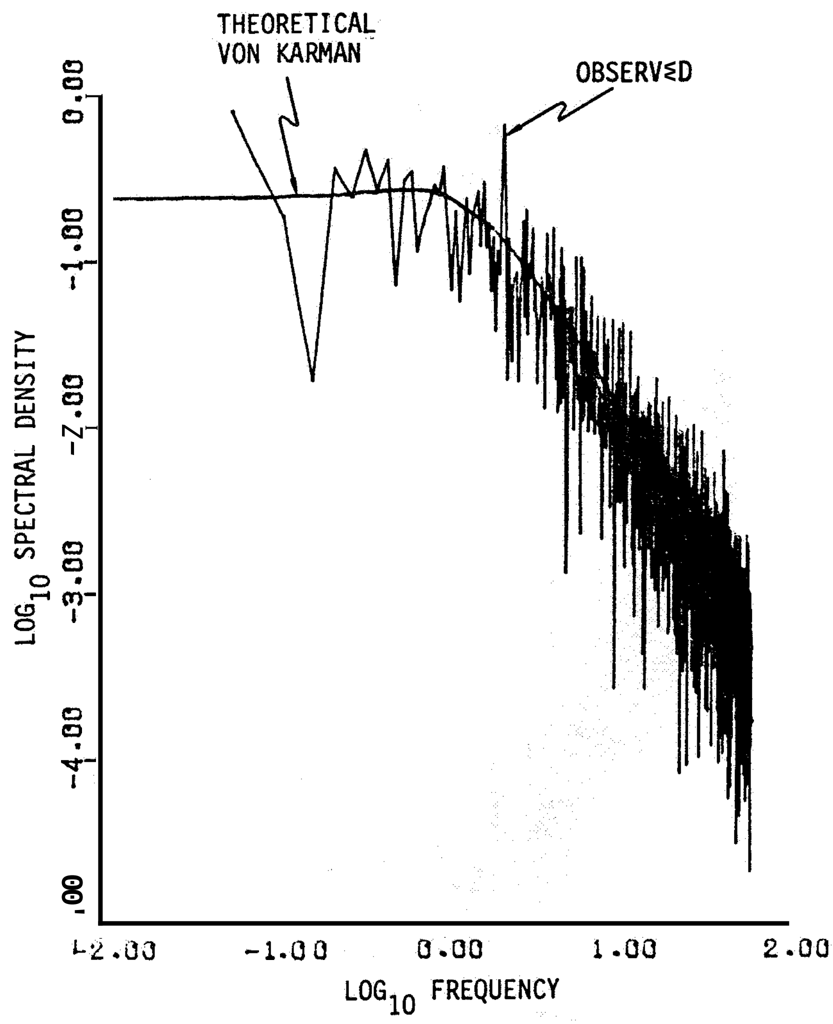

Figure C-16 u_3 - Gust Spectrum, Altitude Band #4

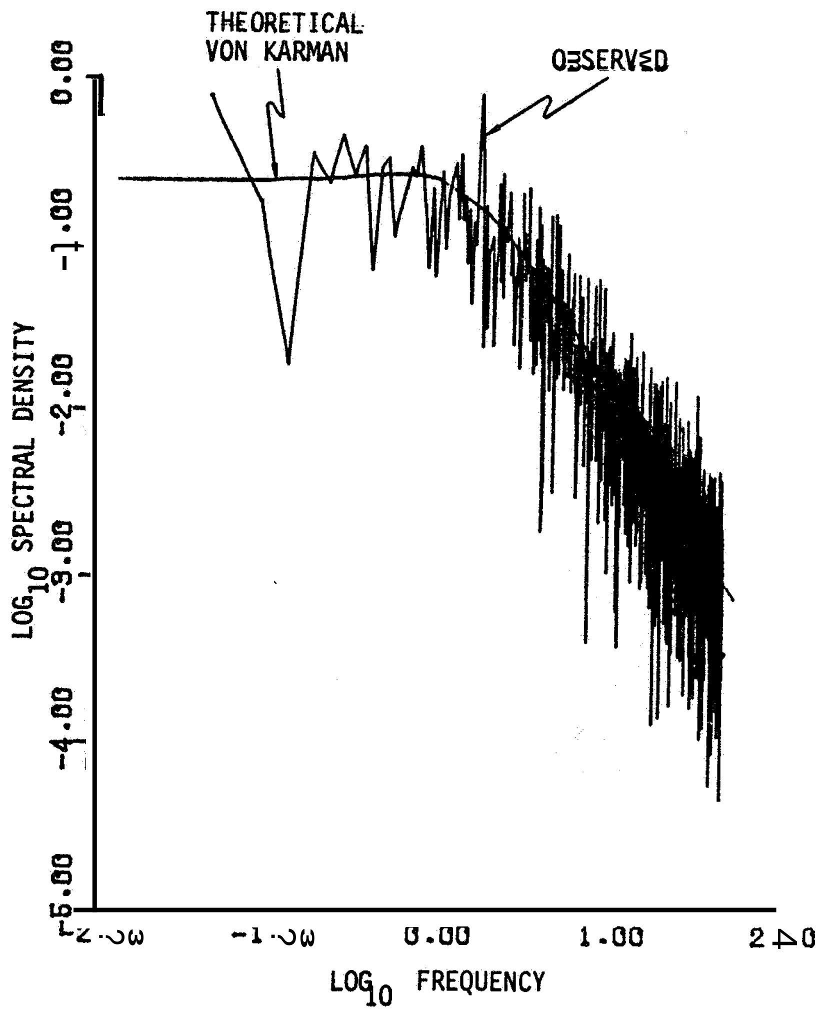

Figure C-17 ω_3 - Gust Spectrum, Altitude Band #5

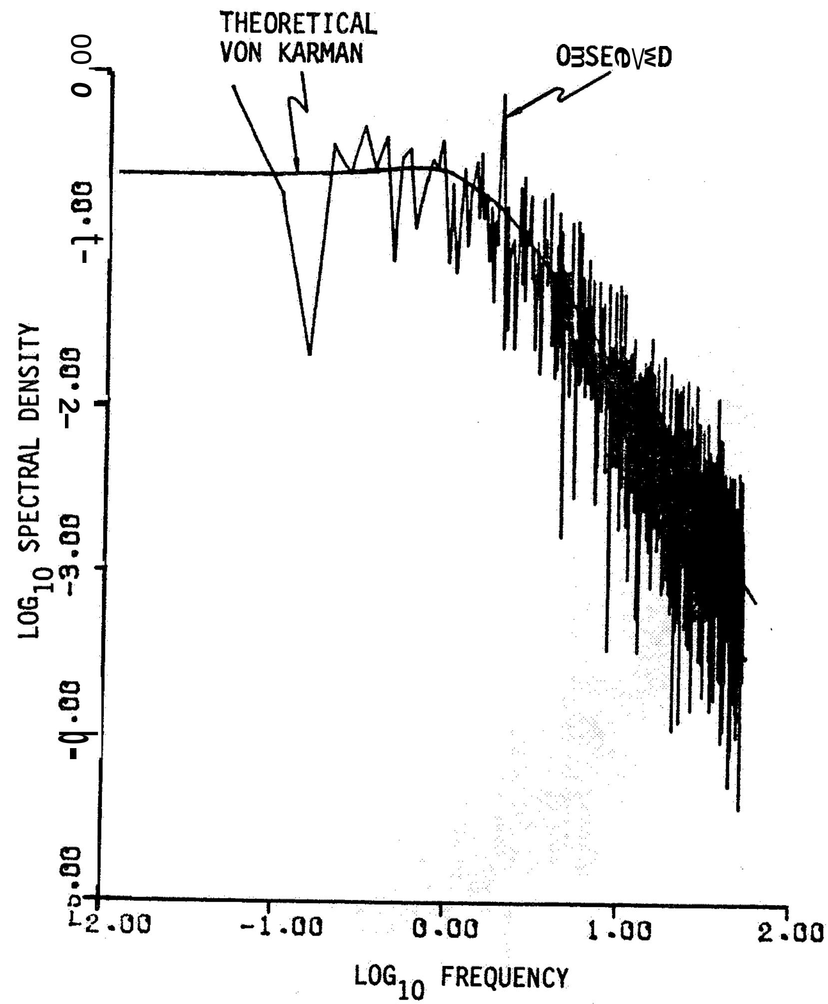

Figure C-18. u_3 - Gust Spectrum, Altitude Band #6

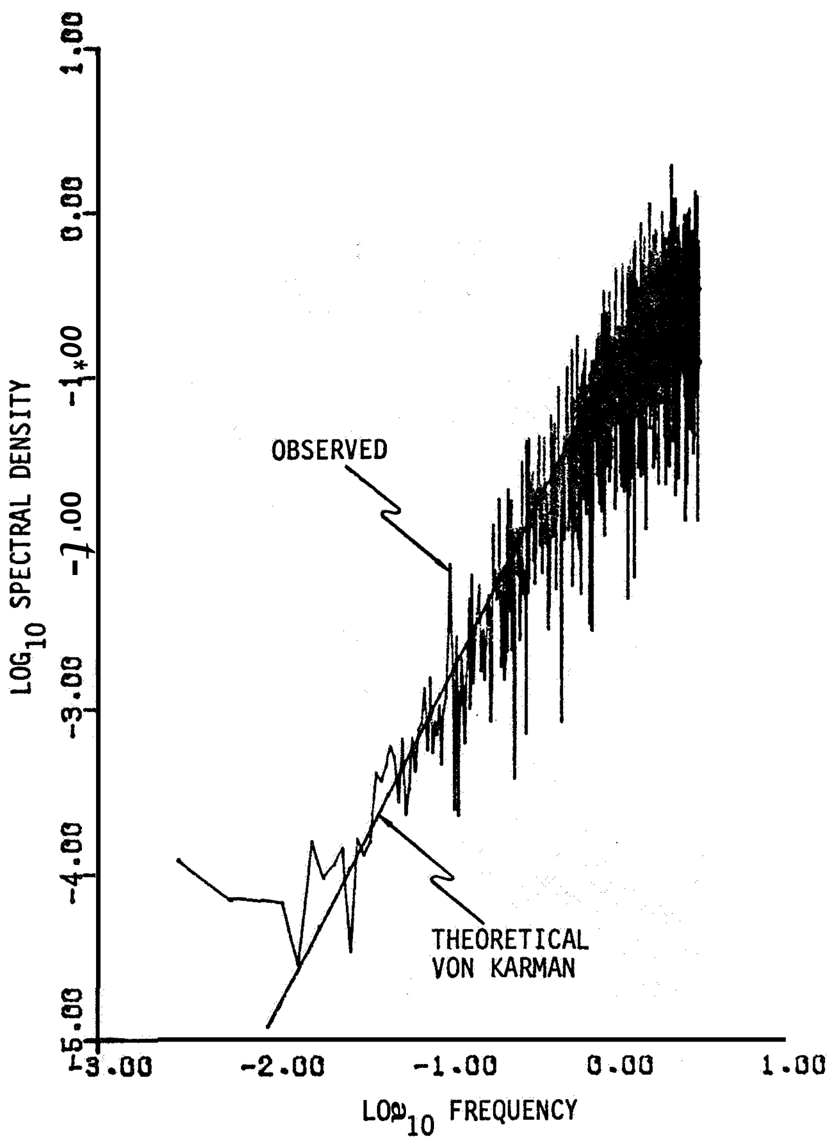

Figure C-19. $\partial u_2/\partial x_1$ - Gust Gradient Spectrum, Altitude Band #1

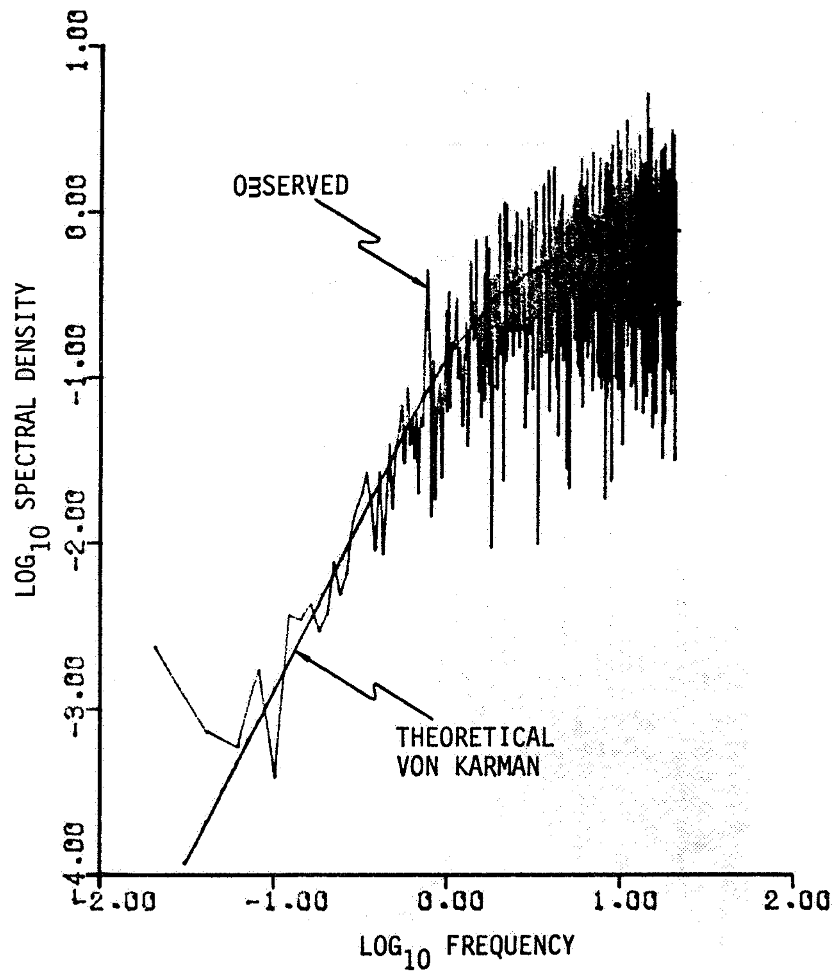

Figure C-20. $\partial u_2/\partial x_1$ - Gust Gradient Spectrum, Altitude Band #2

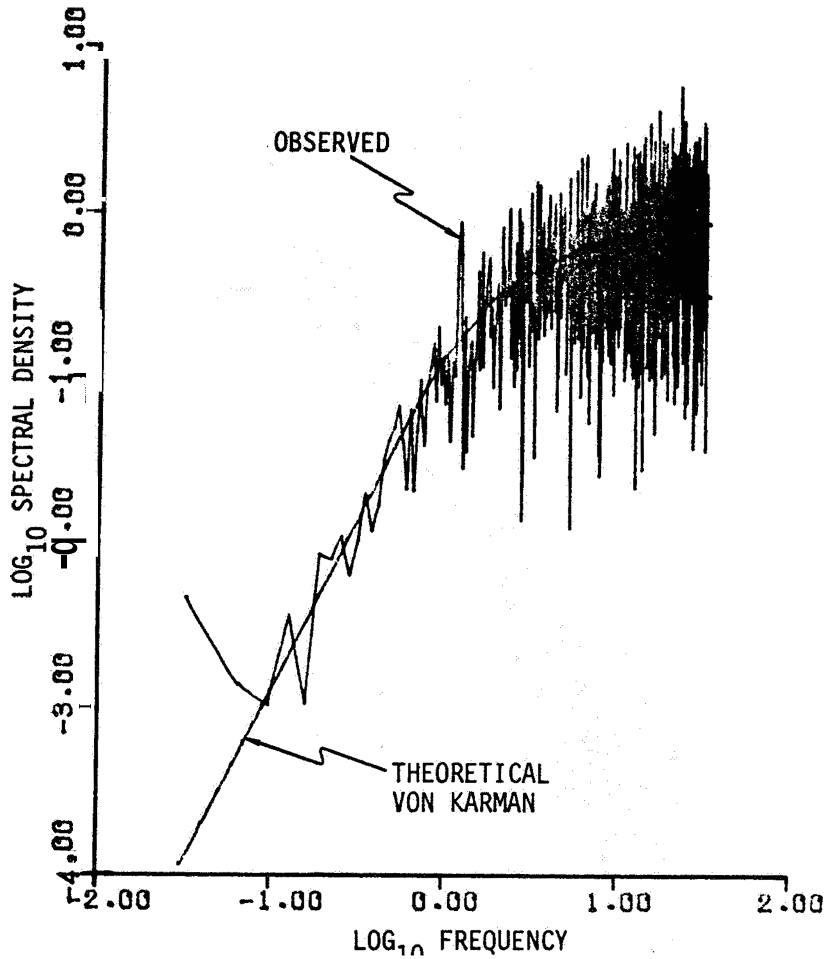

Figure C-21. $\partial u_2/\partial x_1$ - Gust Gradient Spectrum, Altitude Band #3

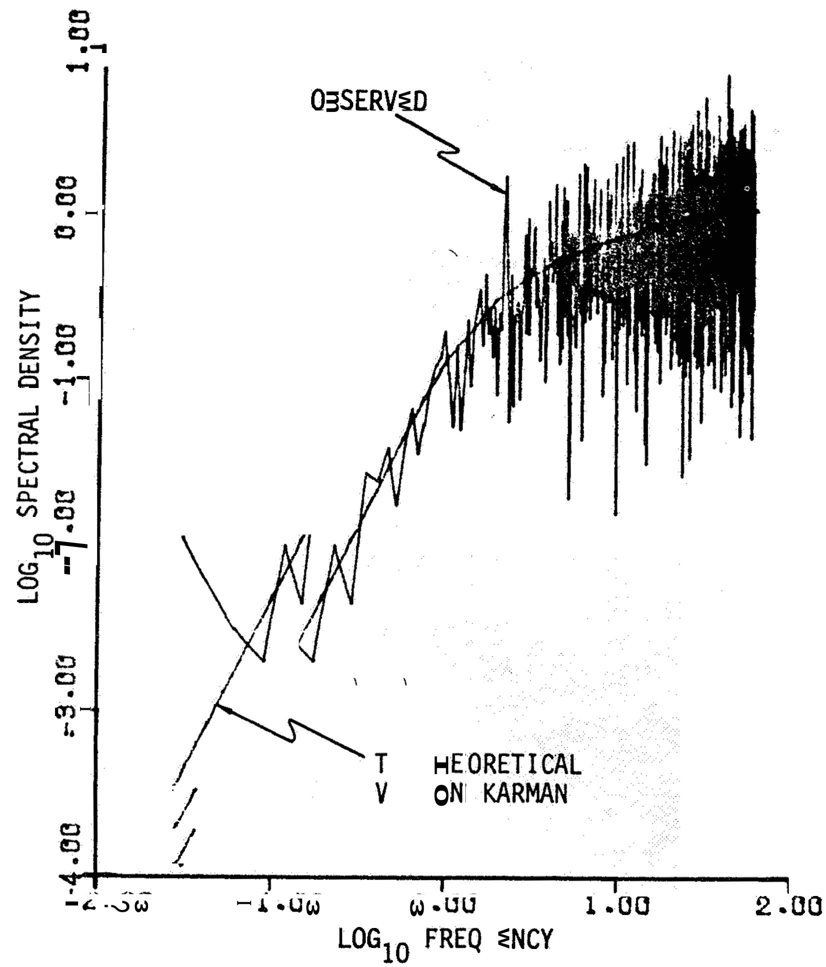

Figure C-22. $\partial u_2/\partial x_1$ - Gust Gradient Spectrum, Altitude Band #4

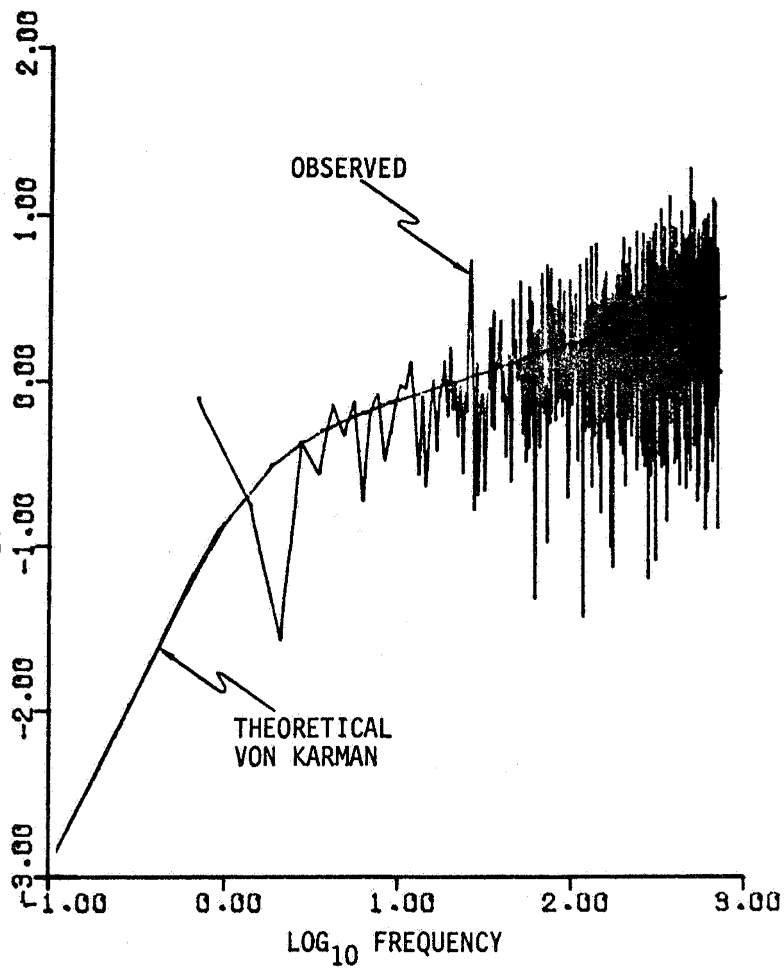

Figure C-23. $\partial u_2/\partial x_1$ - Gust Gradient Spectrum, Altitude Band #5

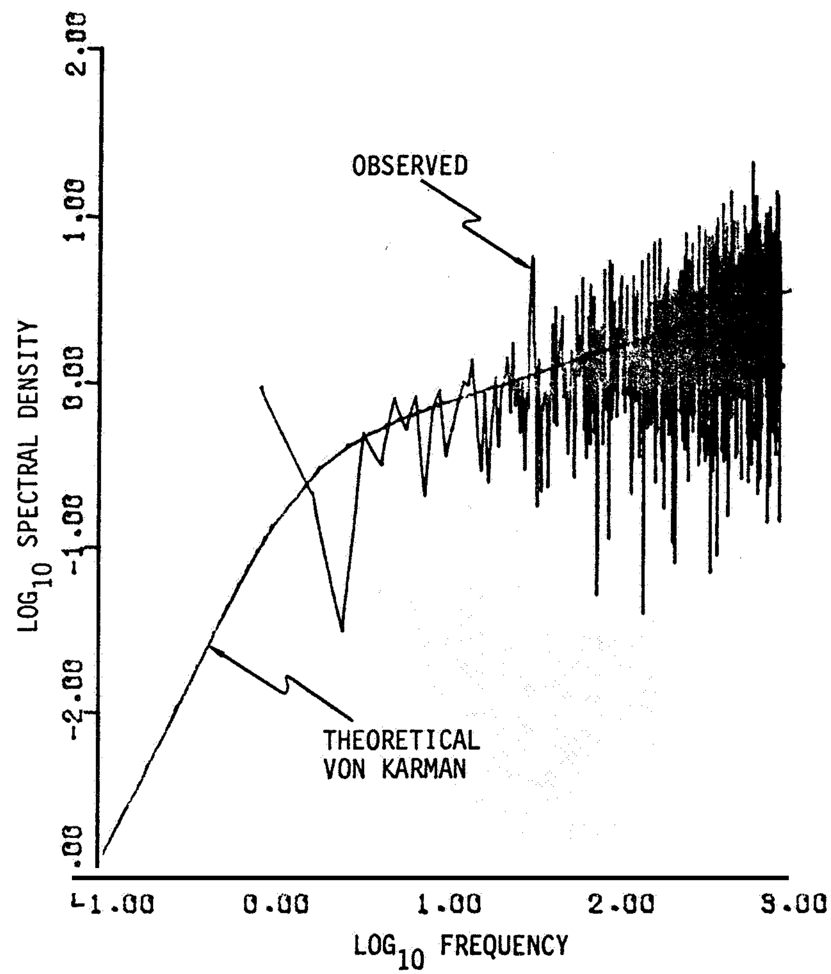

Figure C-24. $\partial u_2/\partial x_1$ - Gust Gradient Spectrum, Altitude Band #6

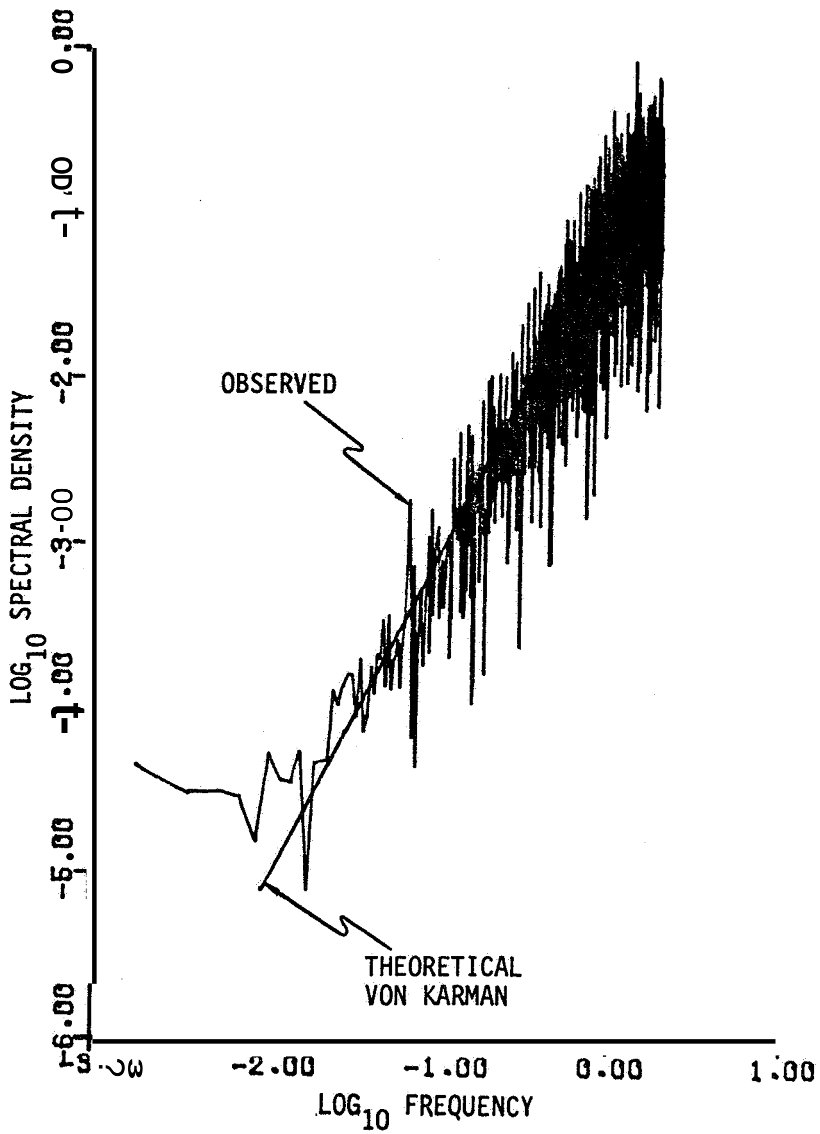

Figure C-25 $\partial u_3/\partial x_1$ - Gust Gradient Spectrum, Altitude Band #1

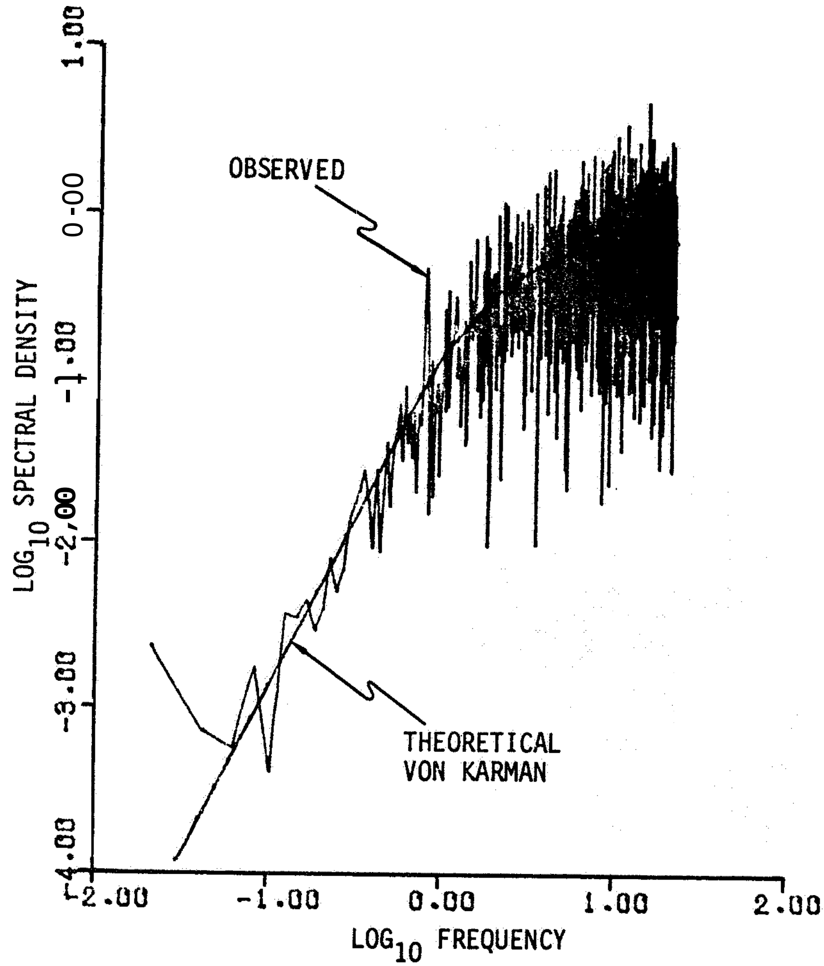

Figure C-26. $\partial u_3/\partial x_1$ - Gust Gradient Spectrum, Altitude Band #2

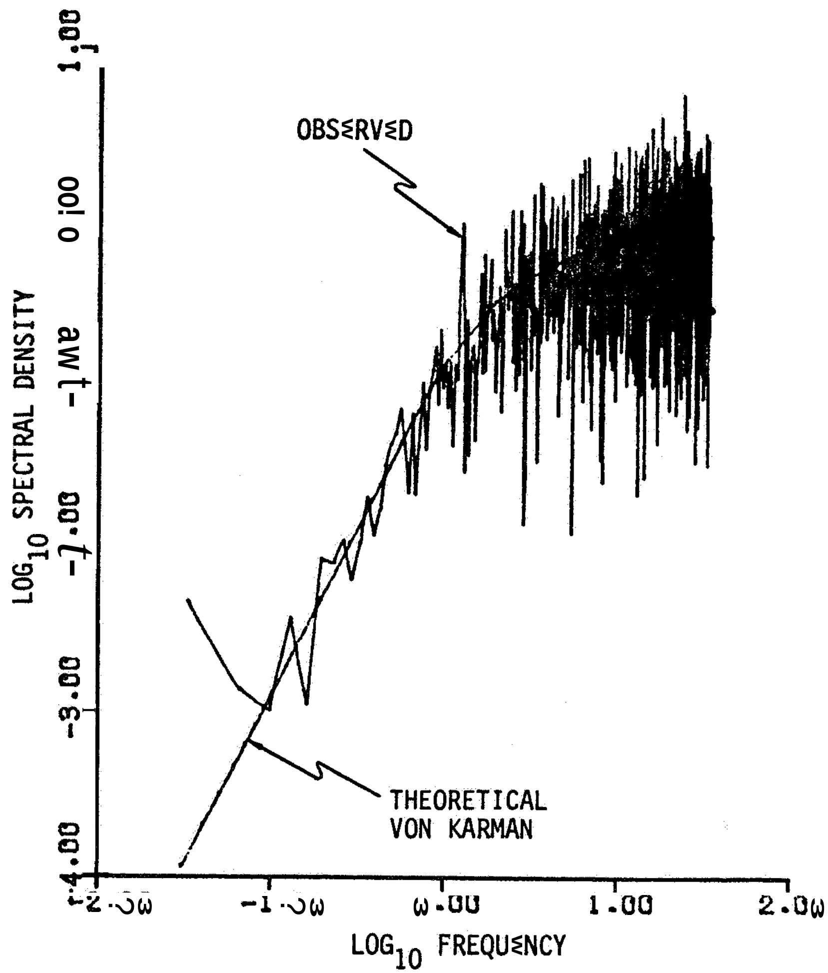

Figure C-27. $\partial u_3/\partial x_1$ - Gust Gradient Spectrum, Altitude Band #3

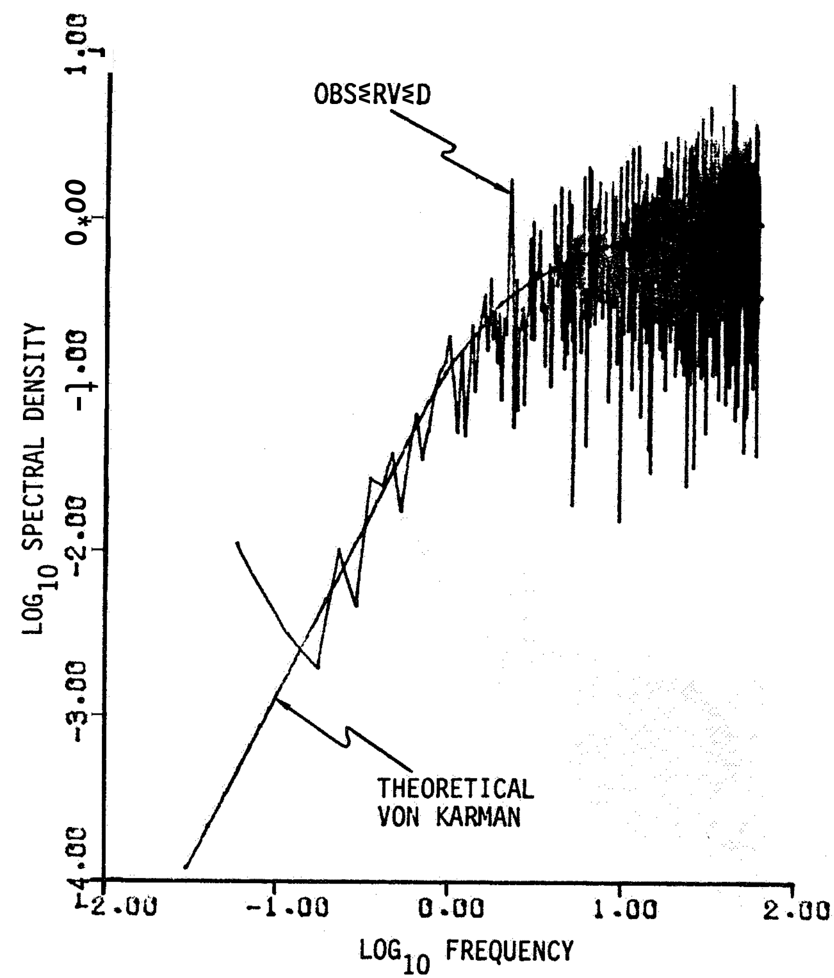

Figure C-28. $\partial u_3/\partial x_1$ - Gust Gradient Spectrum, Altitude Band #4

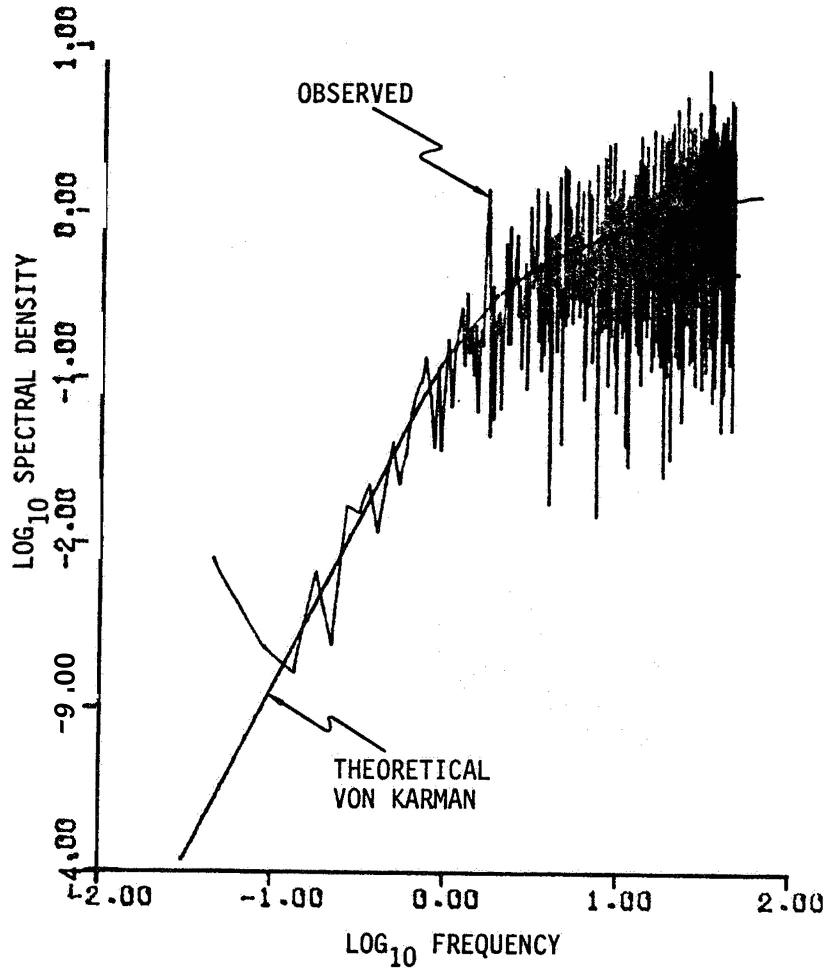

Figure C-29. $\partial u_3/\partial x_1$ - Gust Gradient Spectrum, Altitude Band #5

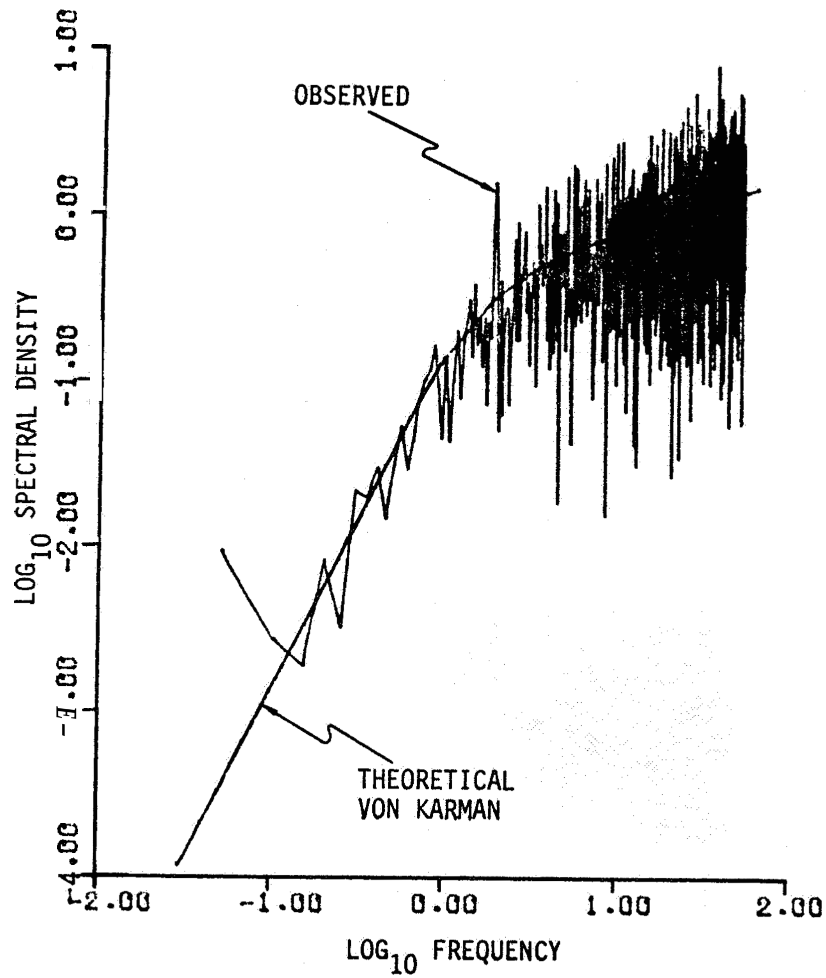

Figure C-30. $\partial u_3/\partial x_1$ - Gust Gradient Spectrum, Altitude Band #6

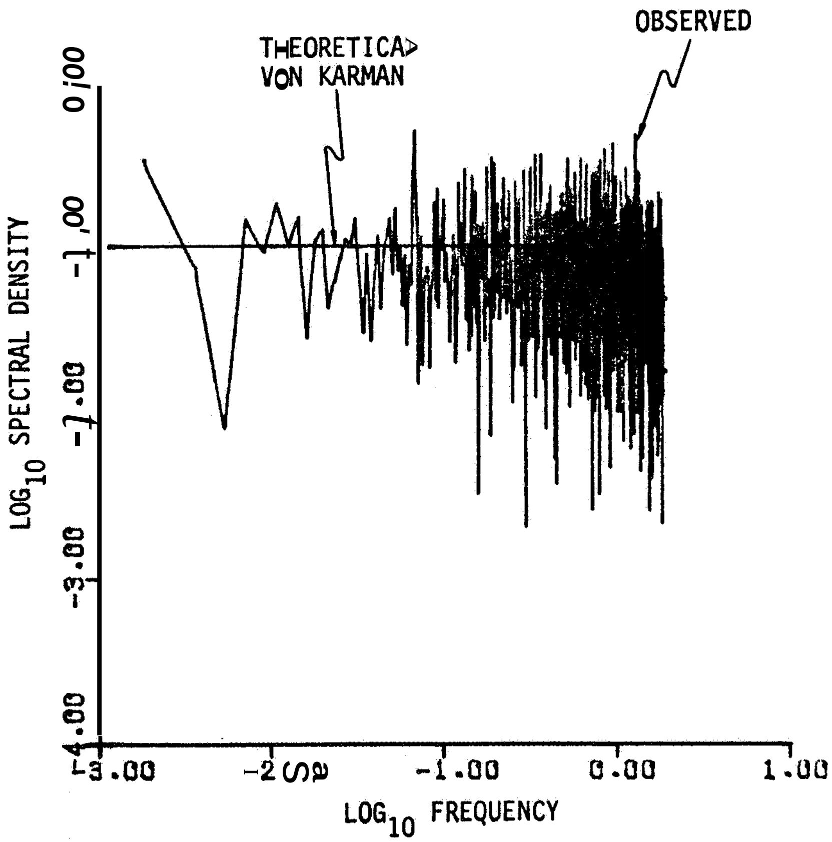

Figure C-31. $\partial u_3/\partial x_2$ - Gust Gradient Spectrum, Altitude Band #1

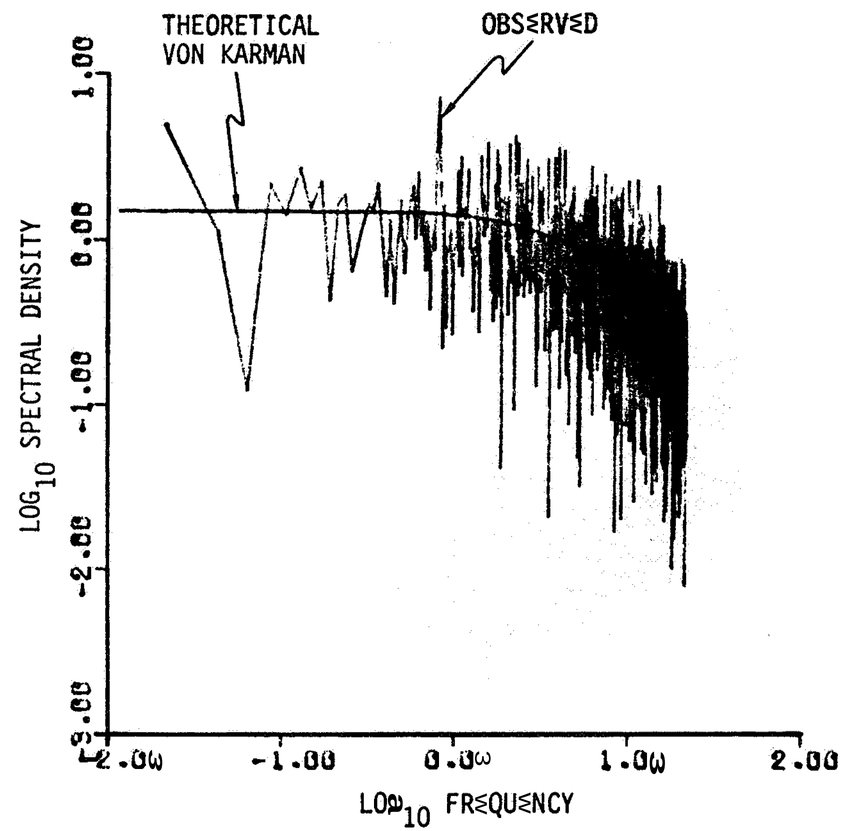

Figure C-32. $\partial u_3/\partial x_2$ - Gust Gradient Spectrum, Altitude Band #2

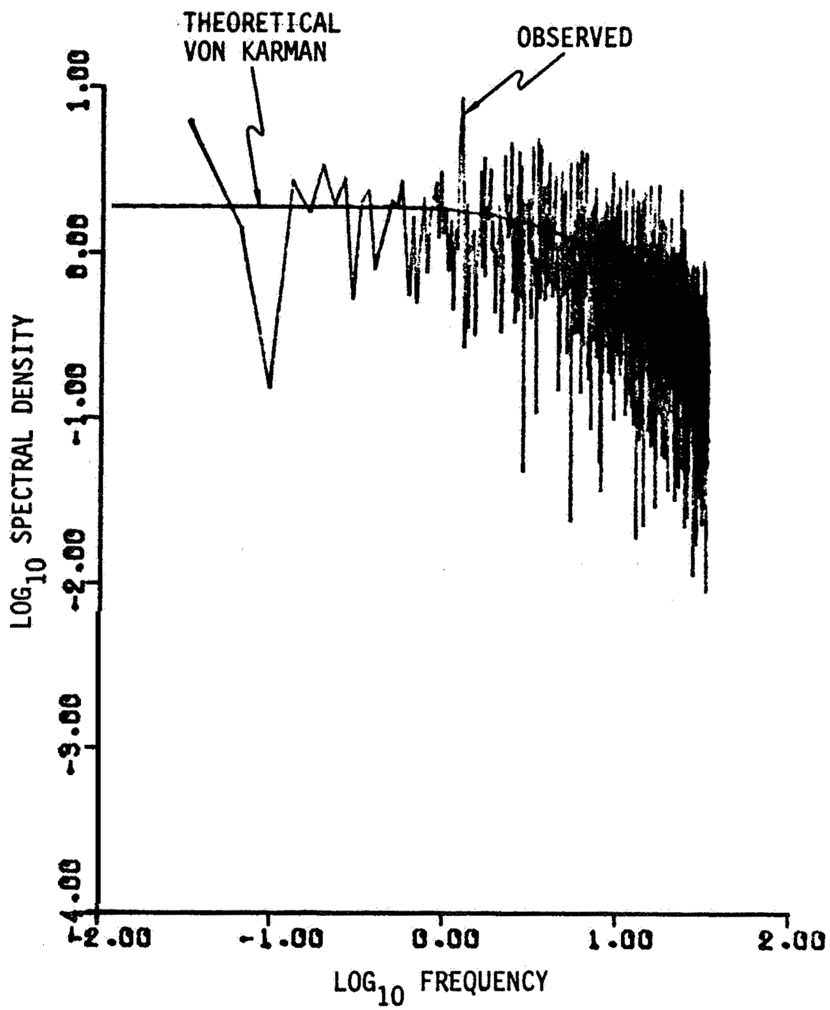

Figure C-33. $\partial u_3/\partial x_2$ - Gust Gradient Spectrum, Altitude Band #3

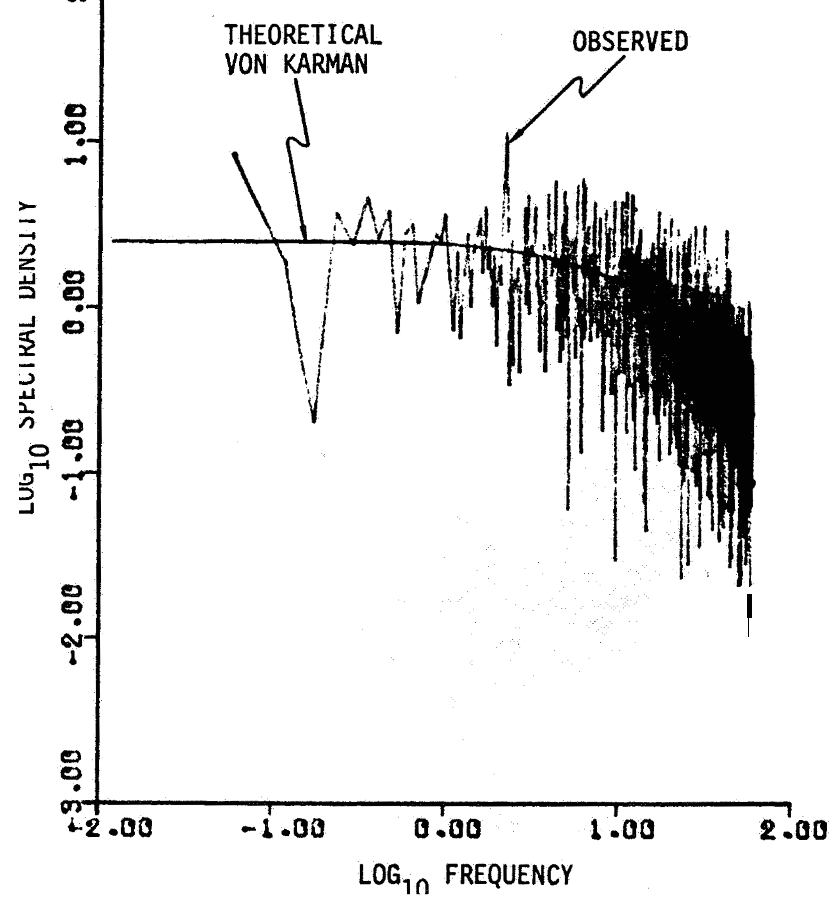

Figure C-34. $\partial u_3/\partial x_2$ - Gust Gradient Spectrum, Altitude Band #4

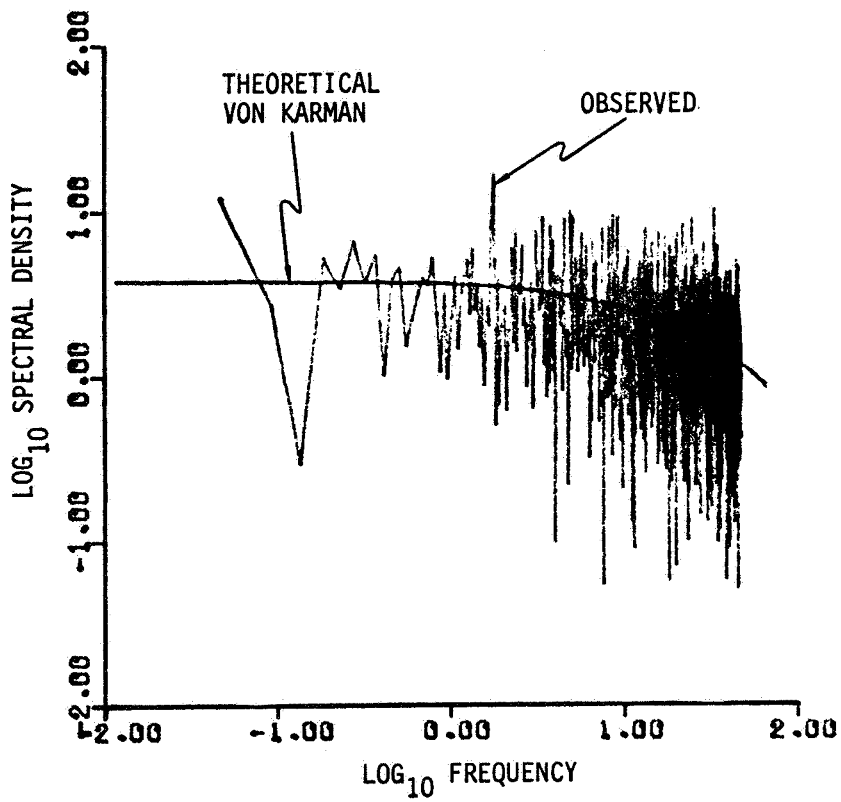

Figure C-35. $\partial u_3/\partial x_2$ - Gust Gradient Spectrum, Altitude Band #5

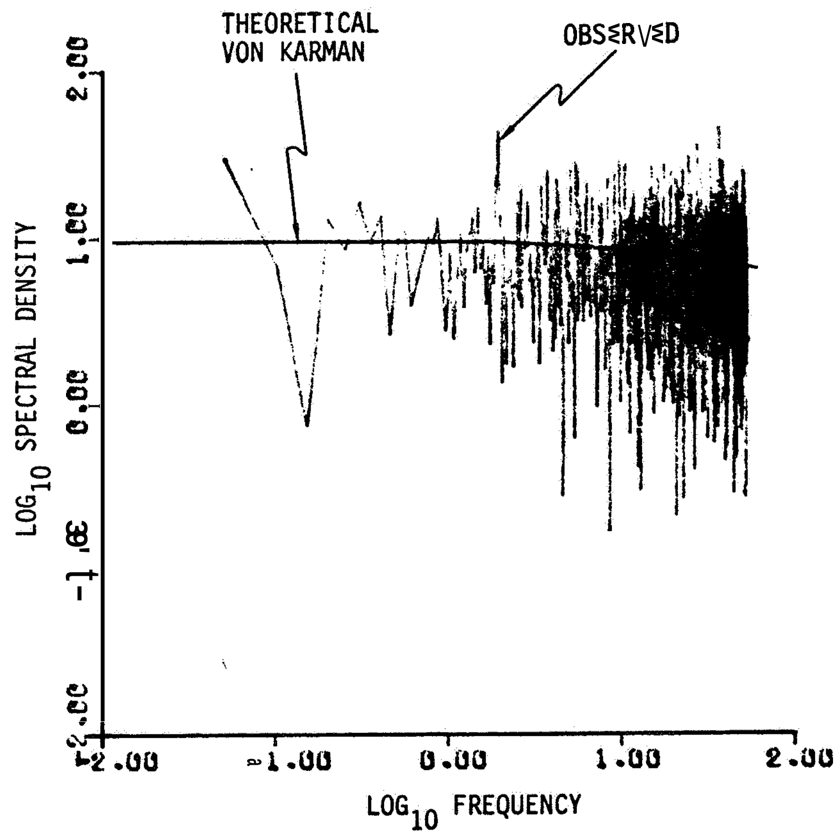

Figure C-36. $\partial u_3/\partial x_2$ - Gust Gradient Spectrum, Altitude Band #6

APPENDIX D

STATISTICAL ANALYSIS OF SIMULATED TURBULENCE

By means of standard statistical analysis procedures each of the SSTT has been analyzed to determine its[*] mean value, standard deviation, and probability density distribution. The resulting mean values are presented in Table D-1 while Table D-2 contains the resulting standard deviations. As expected all mean values were near zero. The standard deviations represent the square root of the energy content. The ratio of the theoretical energy content (from Table 2-5) to the square of the corresponding standard deviation (from Table D-2) is presented in Table D-3. The agreement appears quite satisfactory.

The gust and gust gradient probability density distributions are presented in Figures D-1 through D-36 in accordance with Table D-4. In each figure the corresponding theoretical normal distribution is also presented. The results indicate that both the gust and gust gradient time series are very close to normal distributions.

TABLE D-1. MEAN VALUE OF GUST AND GUST GRADIENTS

SERIES TYPE	ALTITUDE BAND					
	1	2	3	4	5	6
u_1	-.019295	-.042050	-.051852	-.088142	-.0455	-.0464
u_2	-.010671	-.029576	-.0371	-.049431	-.0428	-.0441
u_3	-.006806	-.029652	-.037043	-.049370	-,043788	-.046637
$\partial u_2/\partial x_1$	-.000002	-.001572	-.002794	-.005628	-.172152	-.206385
$\partial u_3/\partial x_2$	-.000001	-.001591	-.002798	-.005649	-.004293	-,004893
$\partial u_3/\partial x_1$	-.005760	-.073072	-.103823	-,160303	-.171448	-.288178

[*] The statistical analysis involved the first 4096 terms of each time series except for bands 5 and 6 for the u_1 and u_2 gusts. For these cases 8192 terms were used.

TABLE D-2. STANDARD DEVIATION OF GUST AND GUST GRADIENTS

SERIES TYPE	ALTITUDE BAND					
	1	2	3	4	5	6
u_1	.788959	.927098	.946351	.964271	.99888	.99996
u_2	.707845	.925201	,94619	.964152	.99764	.99863
u_3	.524571	,915606	.938552	-958985	.961845	.967651
$\partial u_2/\partial x_1$	.766627	3.625937	4.976618	7.356808	41.717292	48.052734
$\partial u_3/\partial x_2$	.390512	3.488677	4.758426	7.037516	6.458092	7.216648
$\partial u_3/\partial x_1$	.394539	3.508280	4.784378	7.075349	9.778736	19.790119

TABLE D-3. RATIO OF THE THEORETICAL ENERGY CONTENT* TO THE SQUARE OF THE OBSERVED STANDARD DEVIATION†

SERIES TYPE	ALTITUDE BAND					
	1	2	3	4	5	6
u_1	1.0001	1.0000	1.0000	1.0000	.9999	1.0001
u_2	.9999	1.0000	.9999	1.0000	1.0000	1 0000
u_3	1.0001	1.0000	1.0000	1.0001	1.0000	.9999
$\partial u_2/\partial x_1$	1.0000	1.0000	1.0000	1.0000	.9998	1.0000
$\partial u_3/\partial x_2$	1.0000	1.0000	1.0000	1.0000	1.0001	1.0000
$\partial u_3/\partial x_1$	1.0002	1.0000	1.0000	1.0000	1.0000	.9999

* Theoretical energy content taken from Table 2-3.

† Observed standard deviation taken from Table D-2.

TABLE D-4. MATRIX OF STATISTICAL ANALYSIS FIGURES

SERIES TYPE	ALTITUDE BAND					
	1	2	3	4	5	6
u_1	D-1	D-2	D-3	D-4	D-5	D-6
u_2	D-7	D-8	D-9	D-10	D-11	D-12
u_3	D-13	D-14	D-15	D-16	D-17	D-18
$\partial u_2/\partial x_1$	D-19	D-20	D-21	D-22	D-23	D-24
$\partial u_3/\partial x_1$	D-25	D-26	D-27	D-28	D-29	D-30
$\partial u_3/\partial x_2$	D-31	D-32	D-33	D-34	D-35	0-36

a

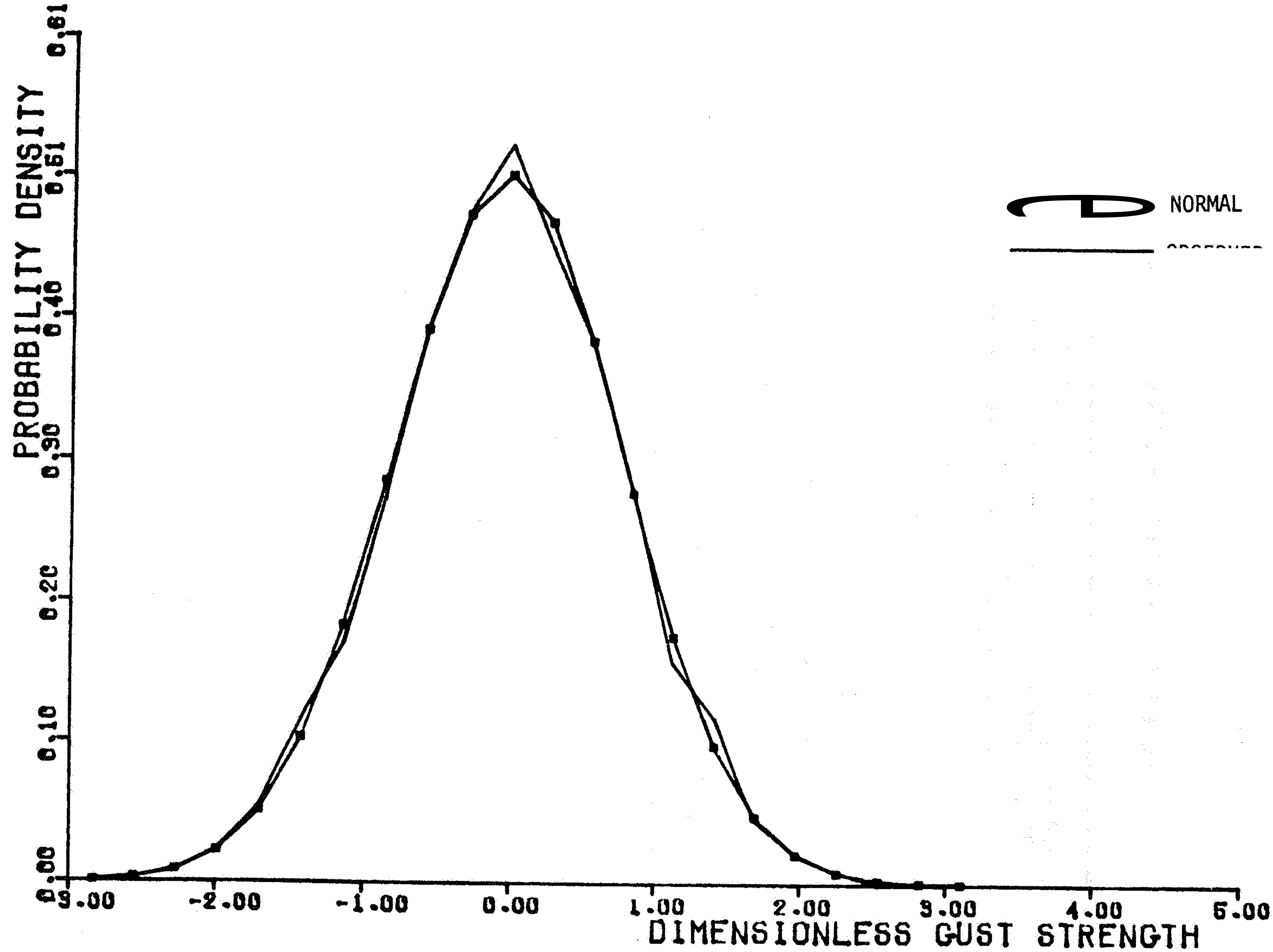

Figure D-1. u_1 - Gust Probability Density Distribution, Altitude Band #1

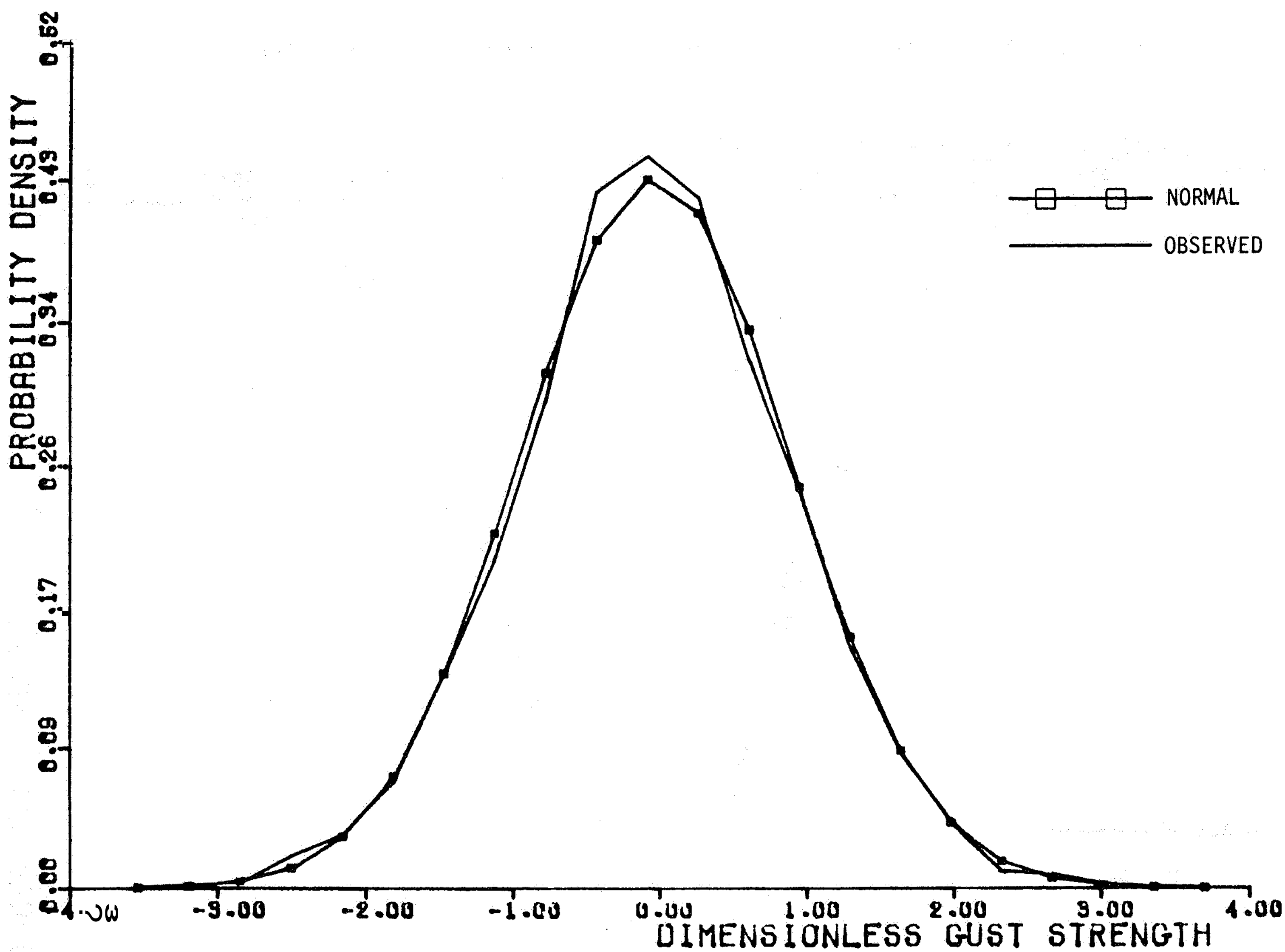

Figure D-2 u_1 - Gust Probability Density Distribution, Altitude Band #2

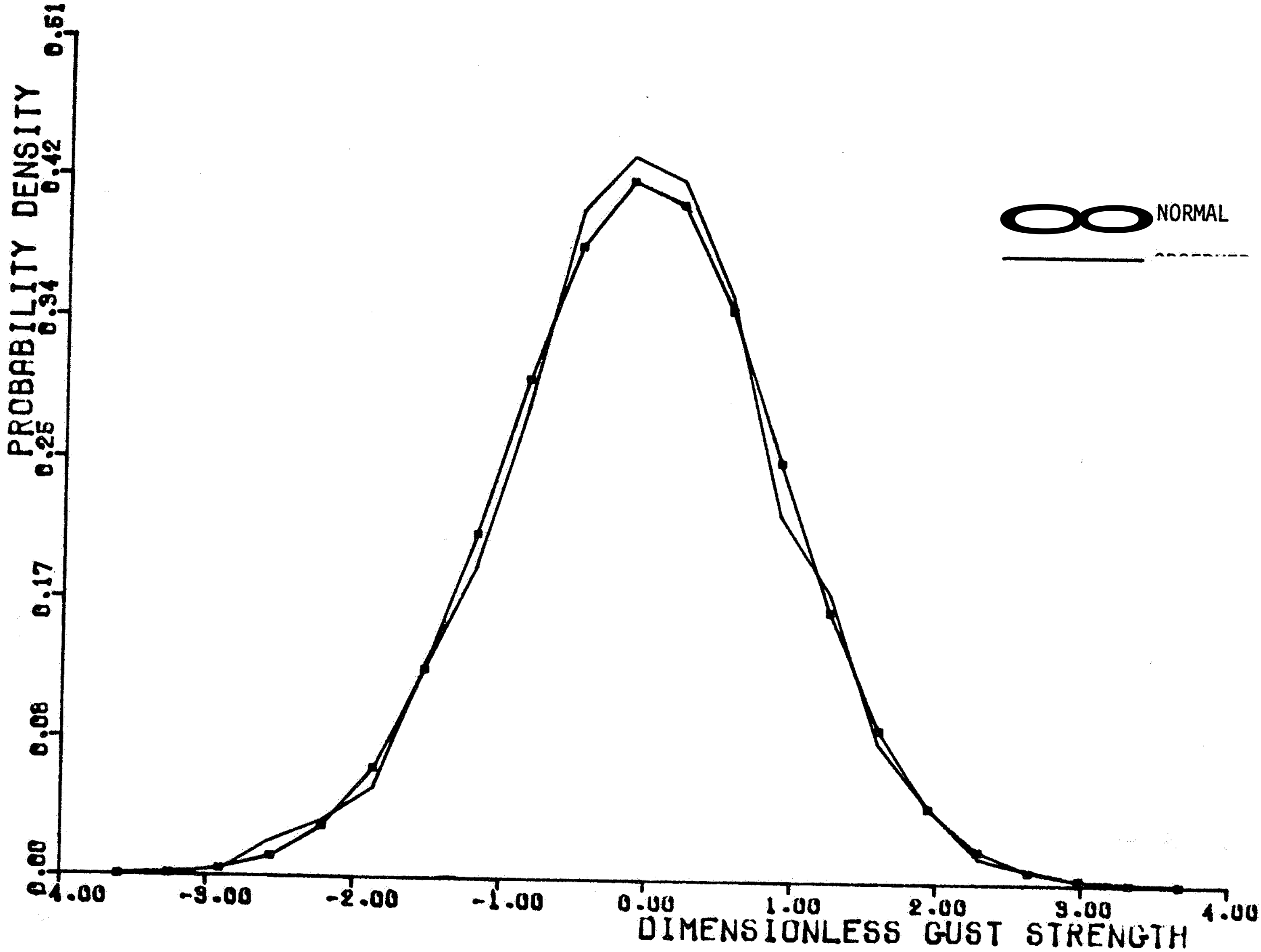

Figure D-3. u_1 - Gust Probability Density Distribution, Altitude Band #3

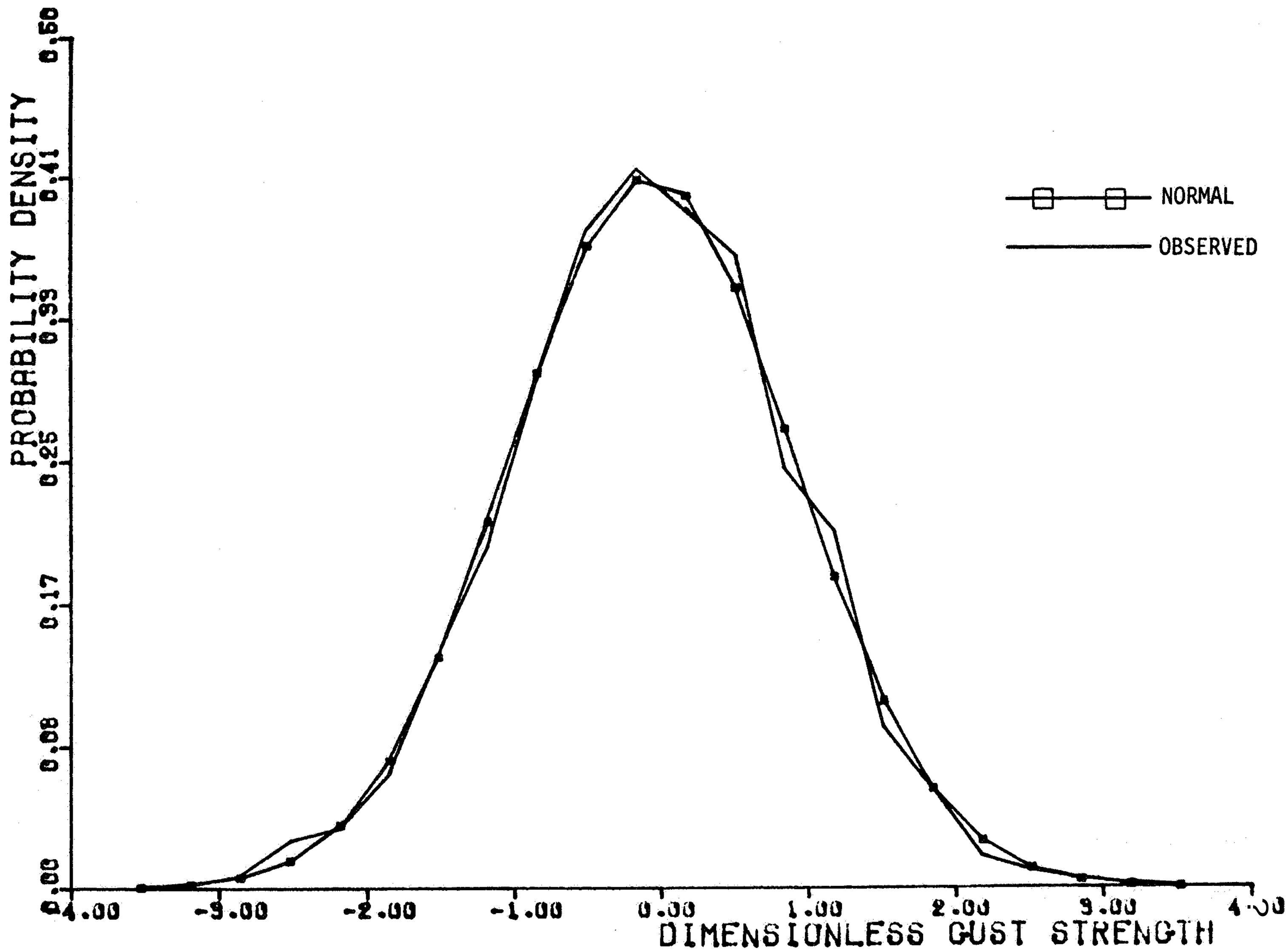

Figure D-4. u_1 - Gust Probability Density Distribution, Altitude Band #4

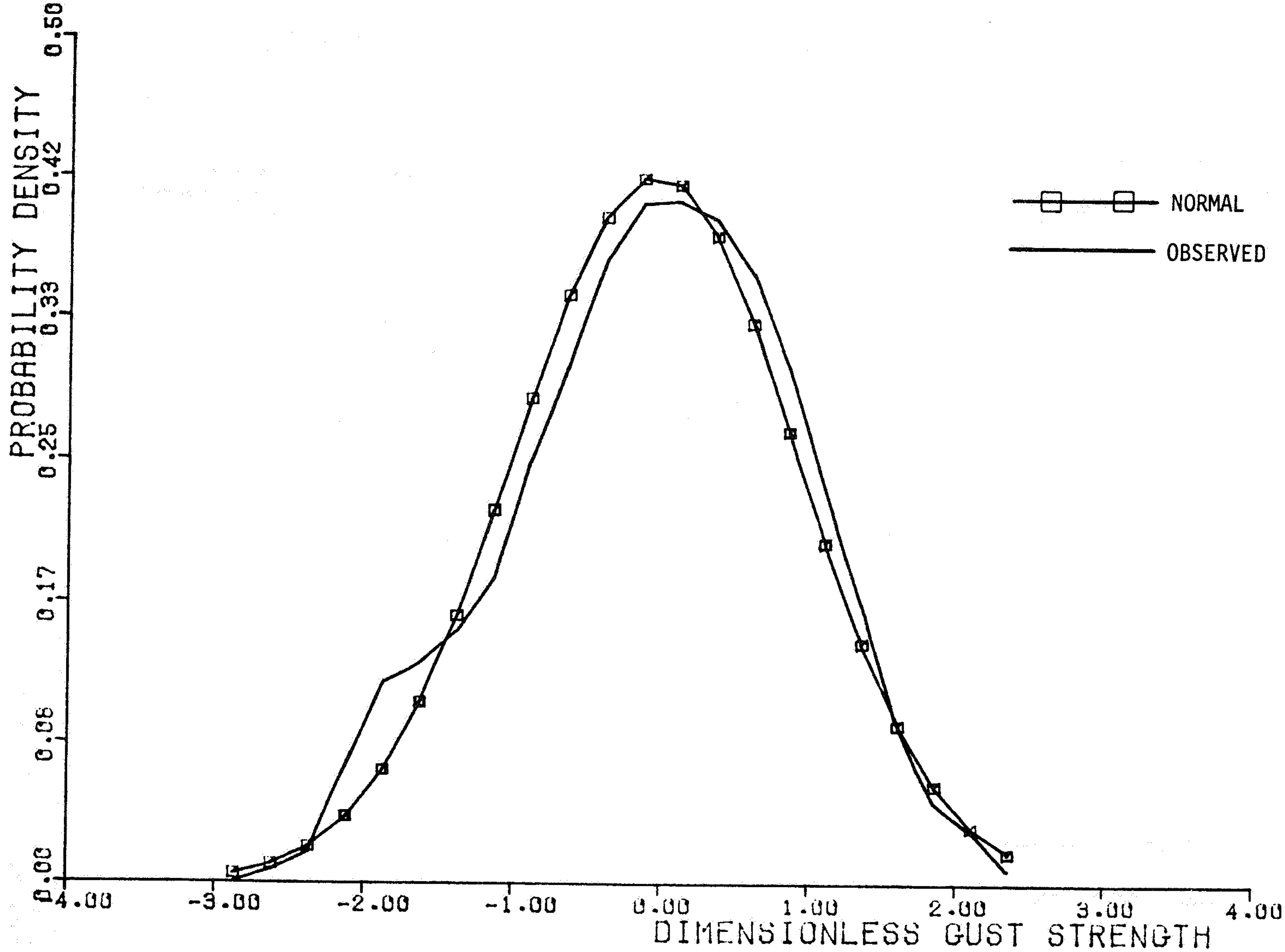

Figure D-5. u_1 - Gust Probability Density Distribution, Altitude Band #5

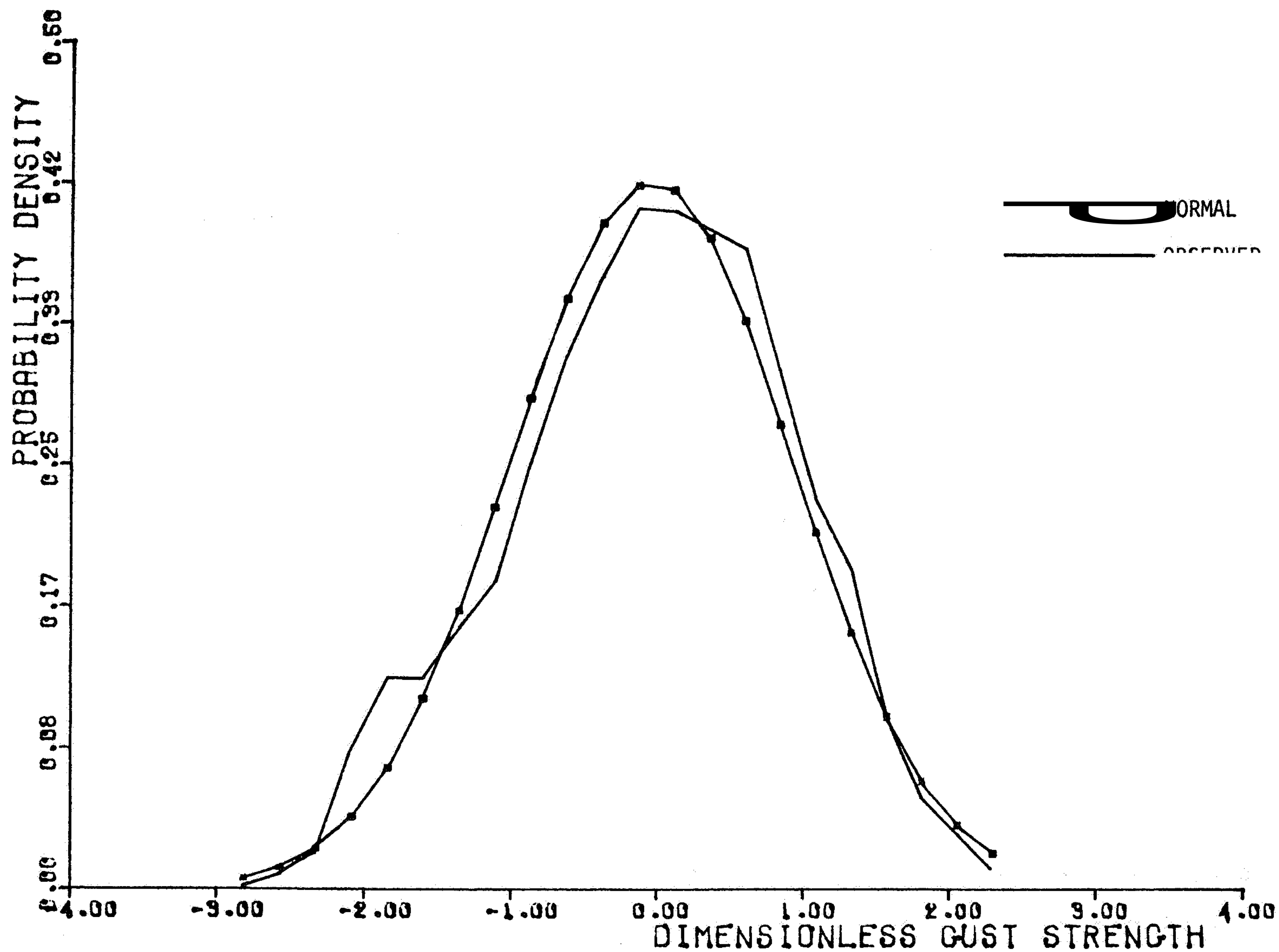

Figure D-6. u_1 - Gust Probability Density Distribution, Altitude Band #6

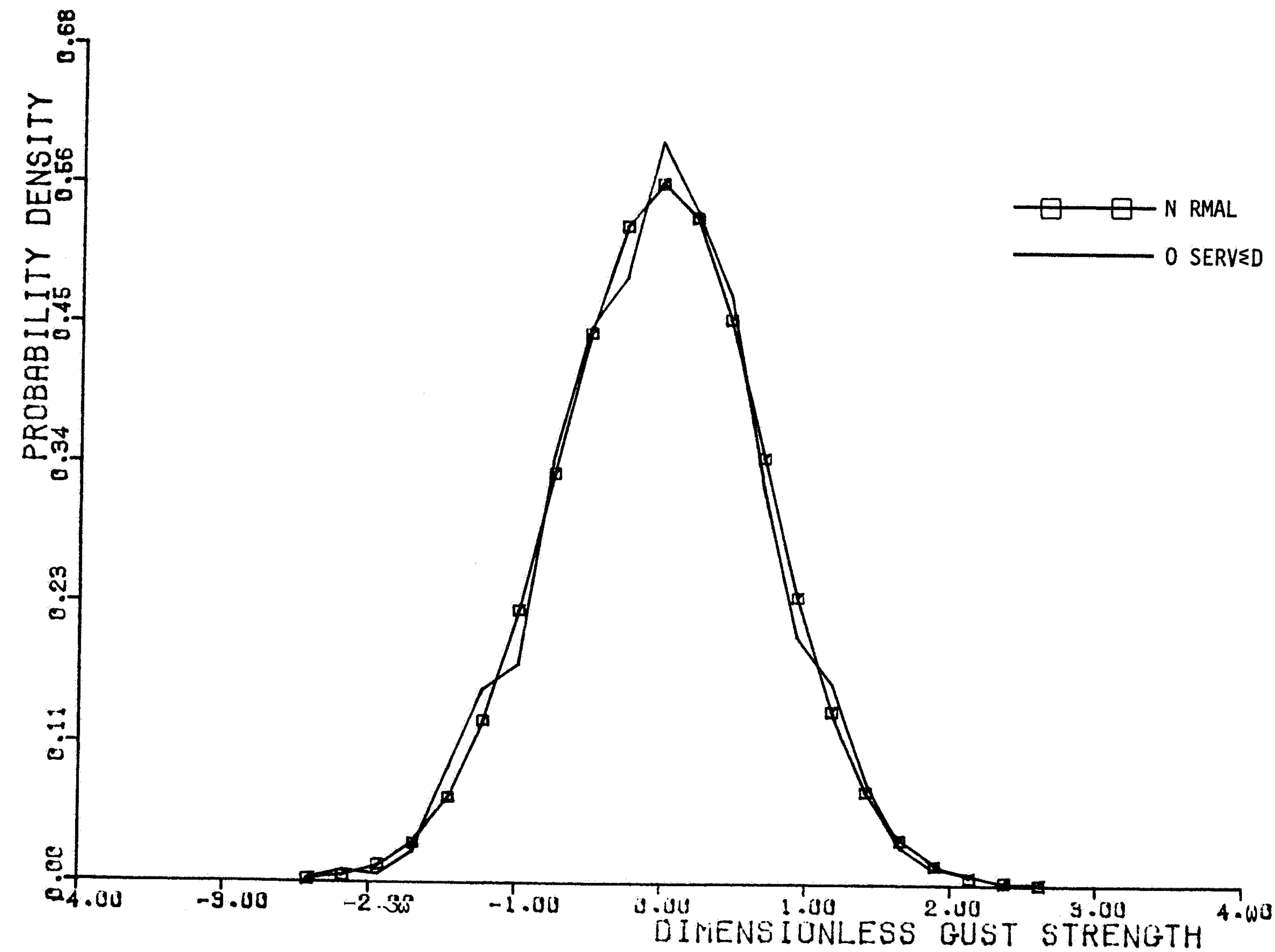

Figure D-7 u_2 - Gust Probability Density Distribution, Altitude Band #1

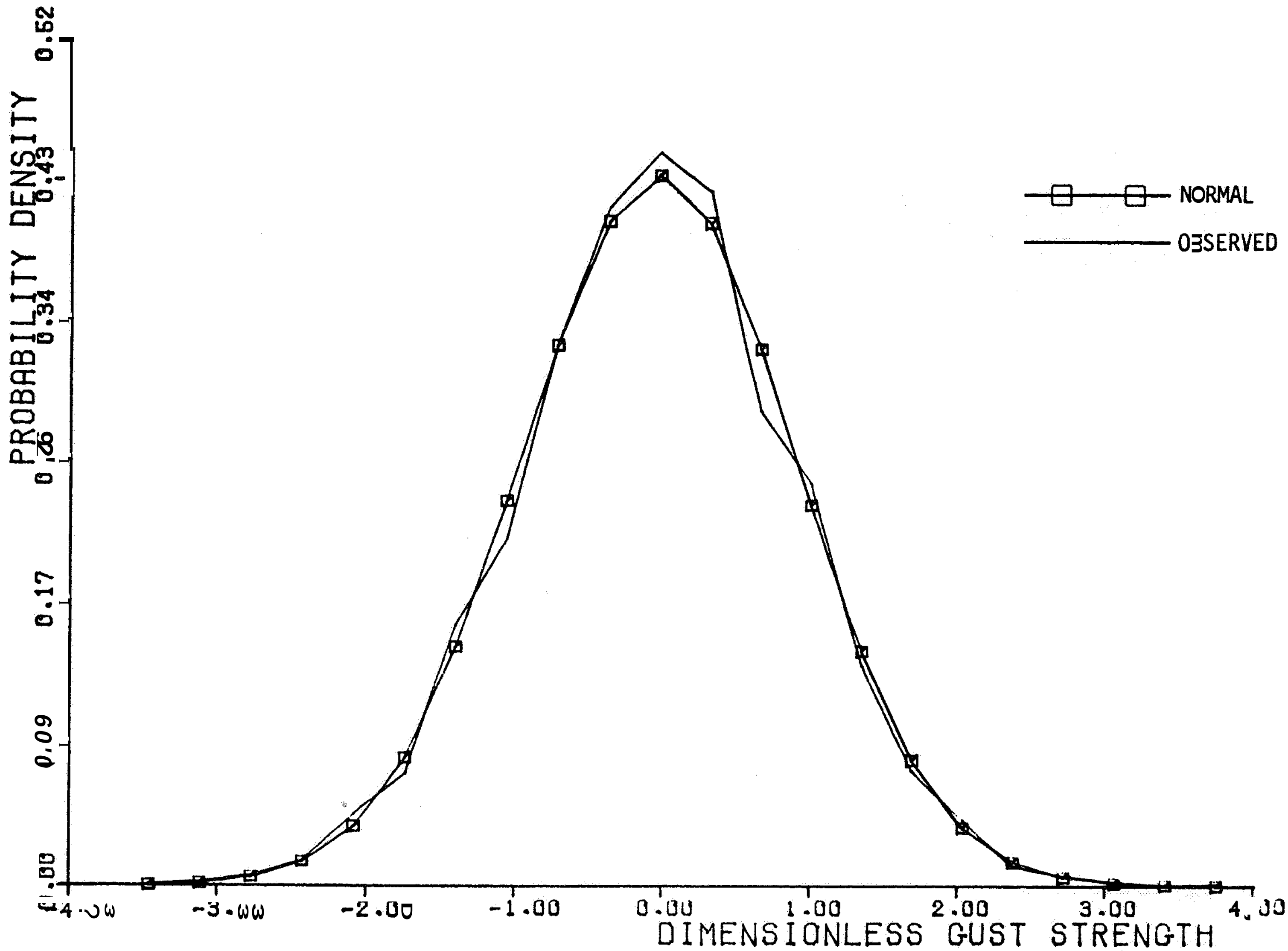

Figure D-8. u_2 - Gust Probability Density Distribution, Altitude Band #2

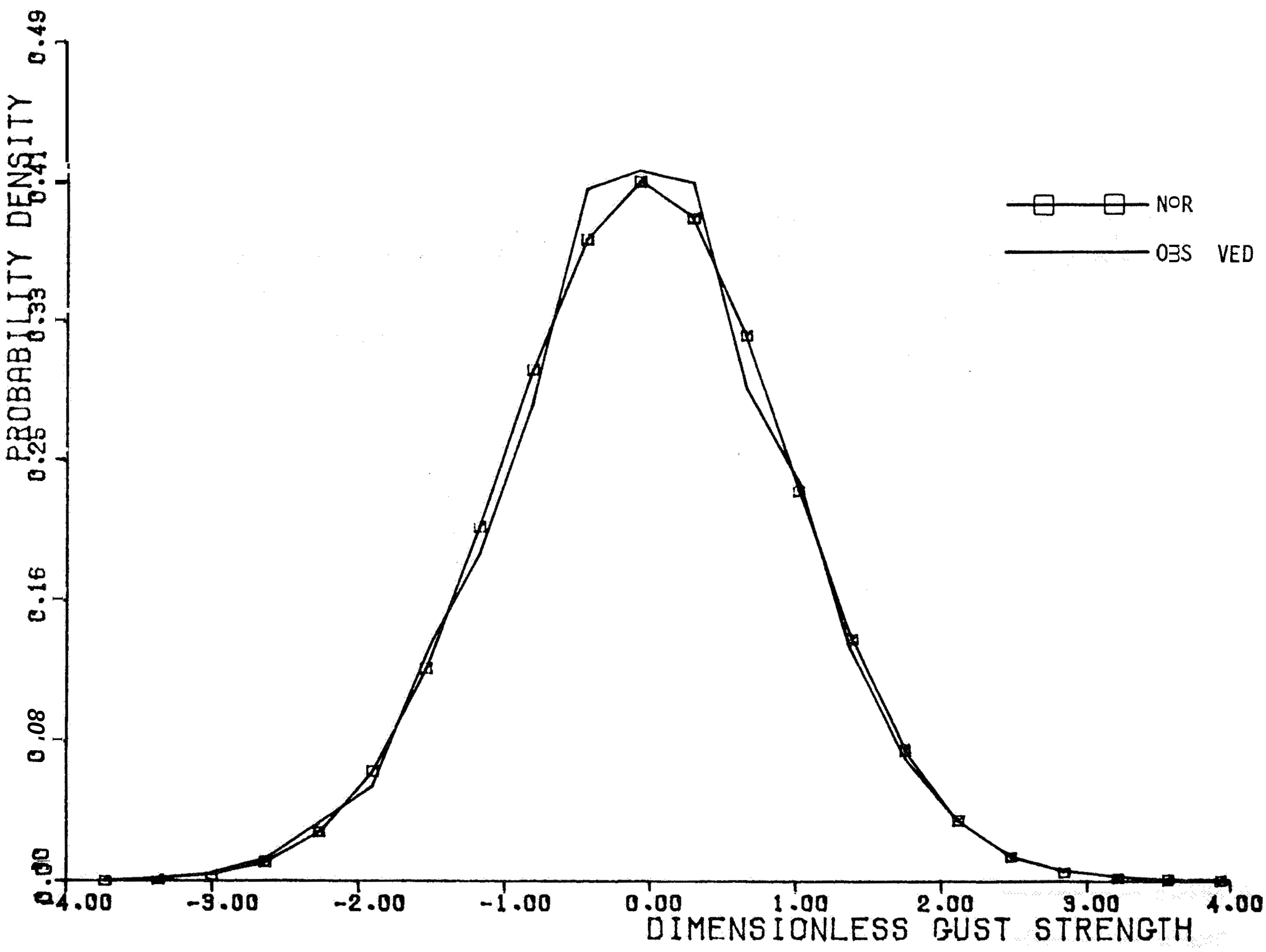

Figure D-9. ω_2 - Gust Probability Density Distribution, Altitude Band #3

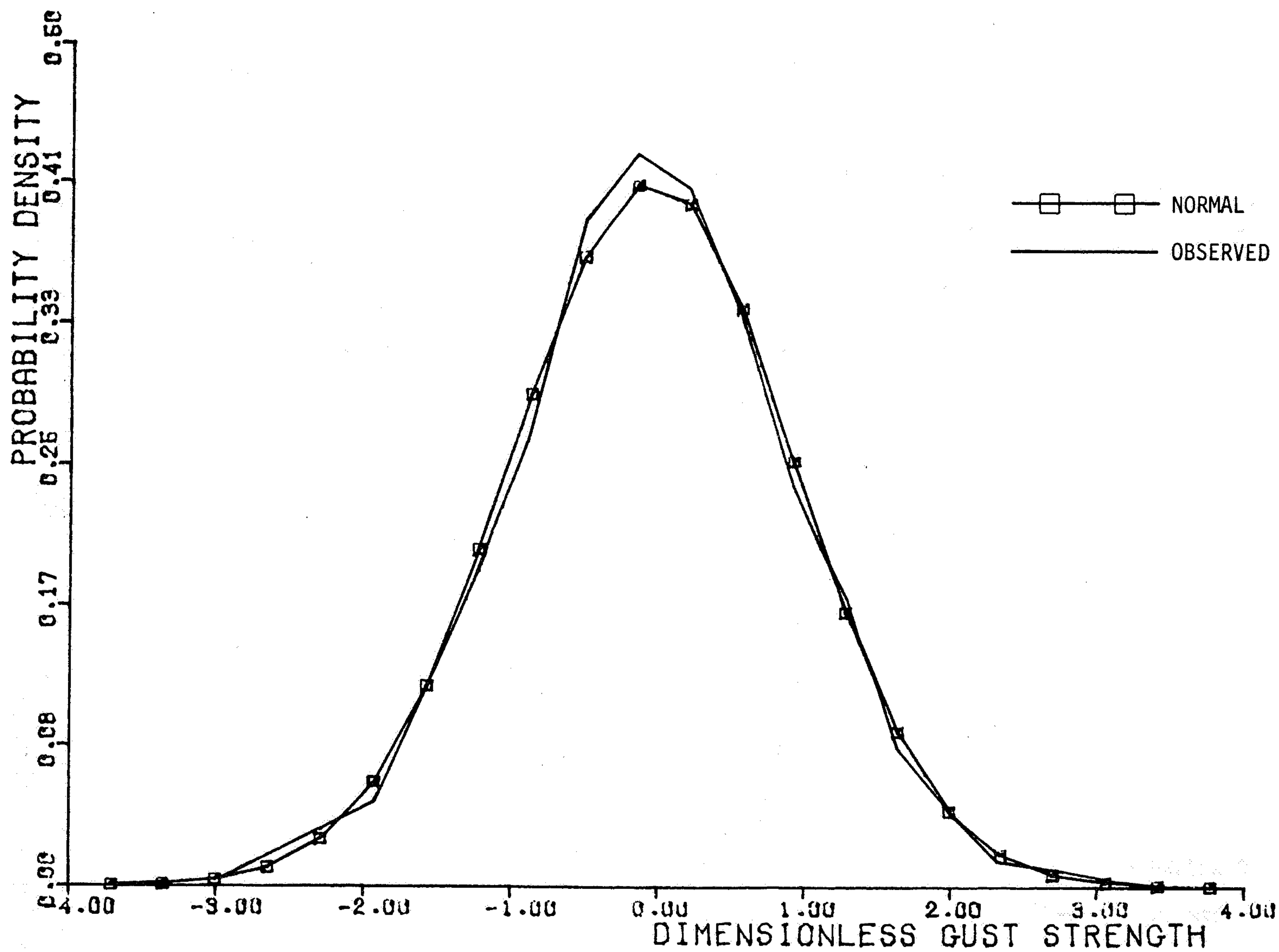

Figure D-10. u_2 - Gust Probability Density Distribution, Altitude Band #4

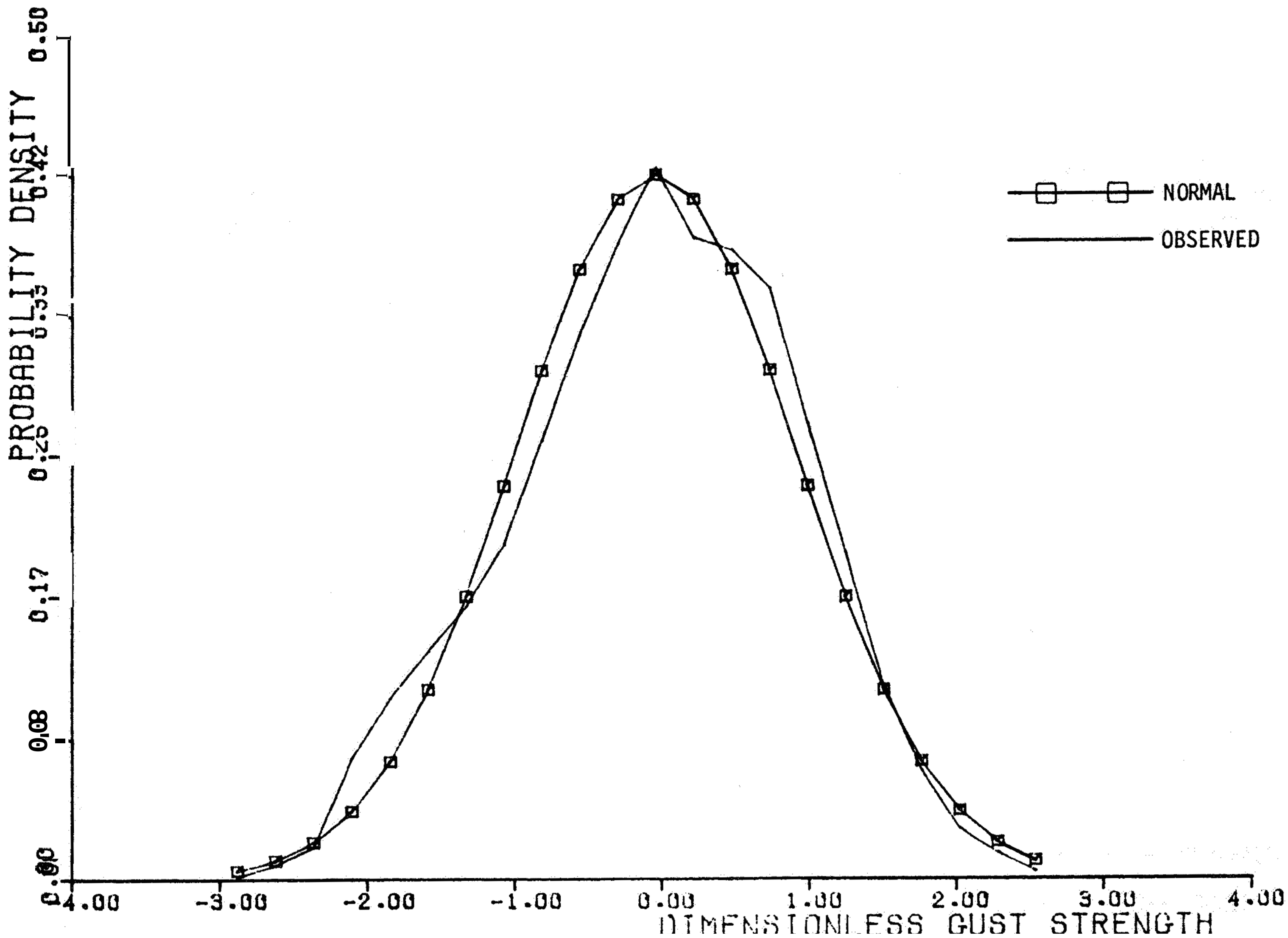

Figure D-11. u_2 - Gust Probability Density Distribution, Altitude Band #5

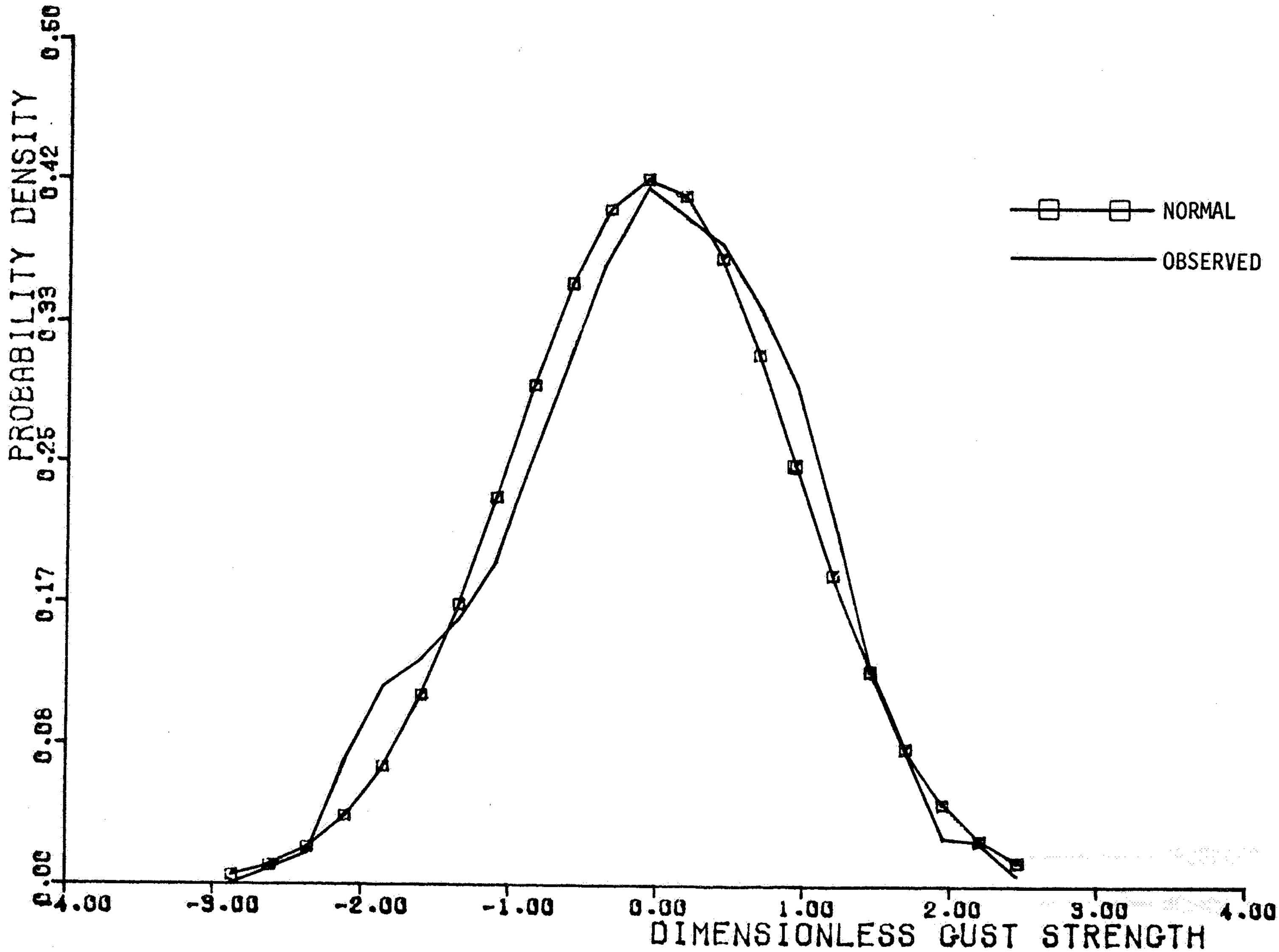

Figure D-12 u_2 - Gust Probability Density Distribution, Altitude Band #6

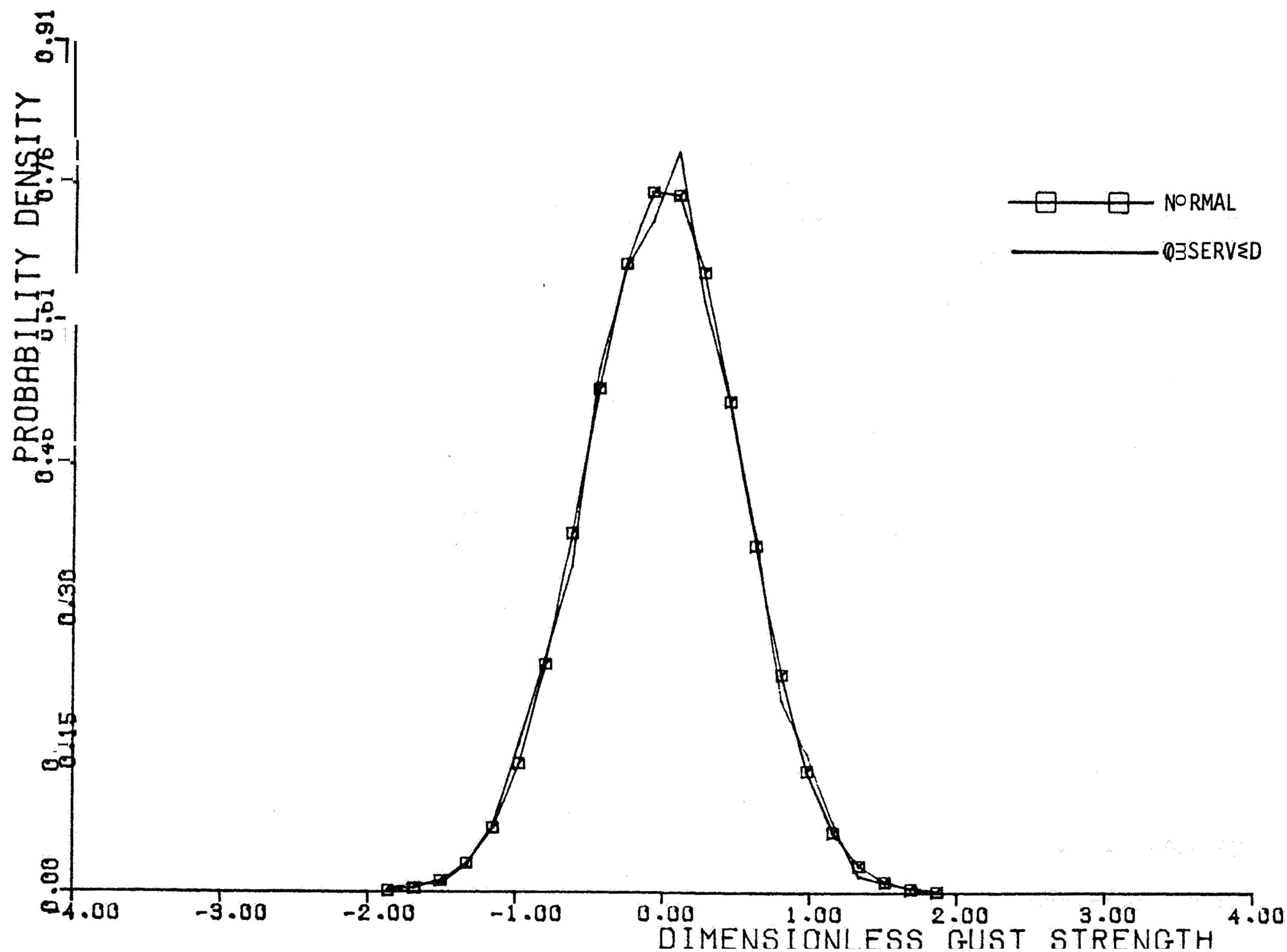

Figure D-13 u_3 - Gust Probability Density Distribution, Altitude Band #1

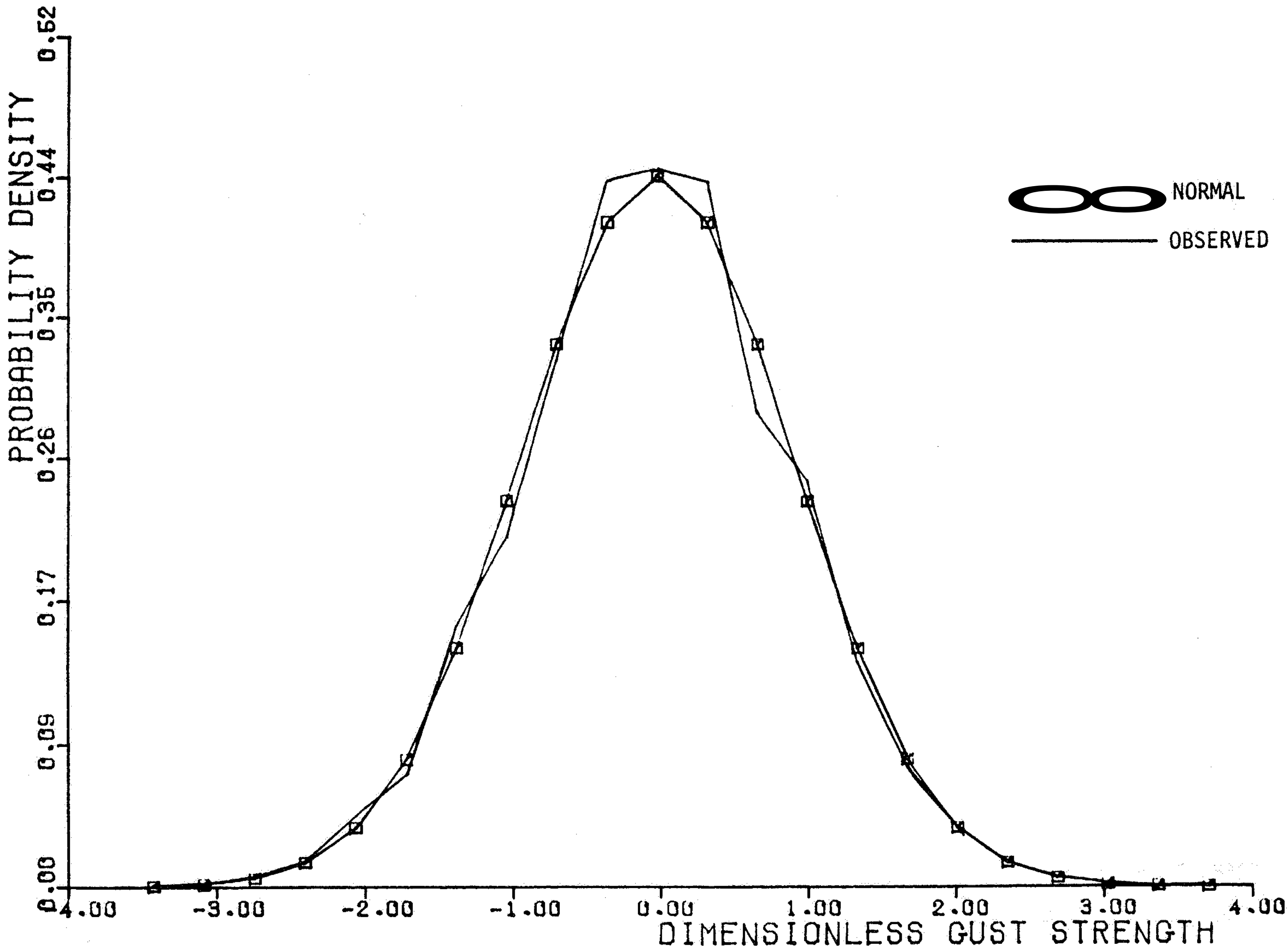

Figure D-14. u_3 - Gust Probability Density Distribution, Altitude Band #2

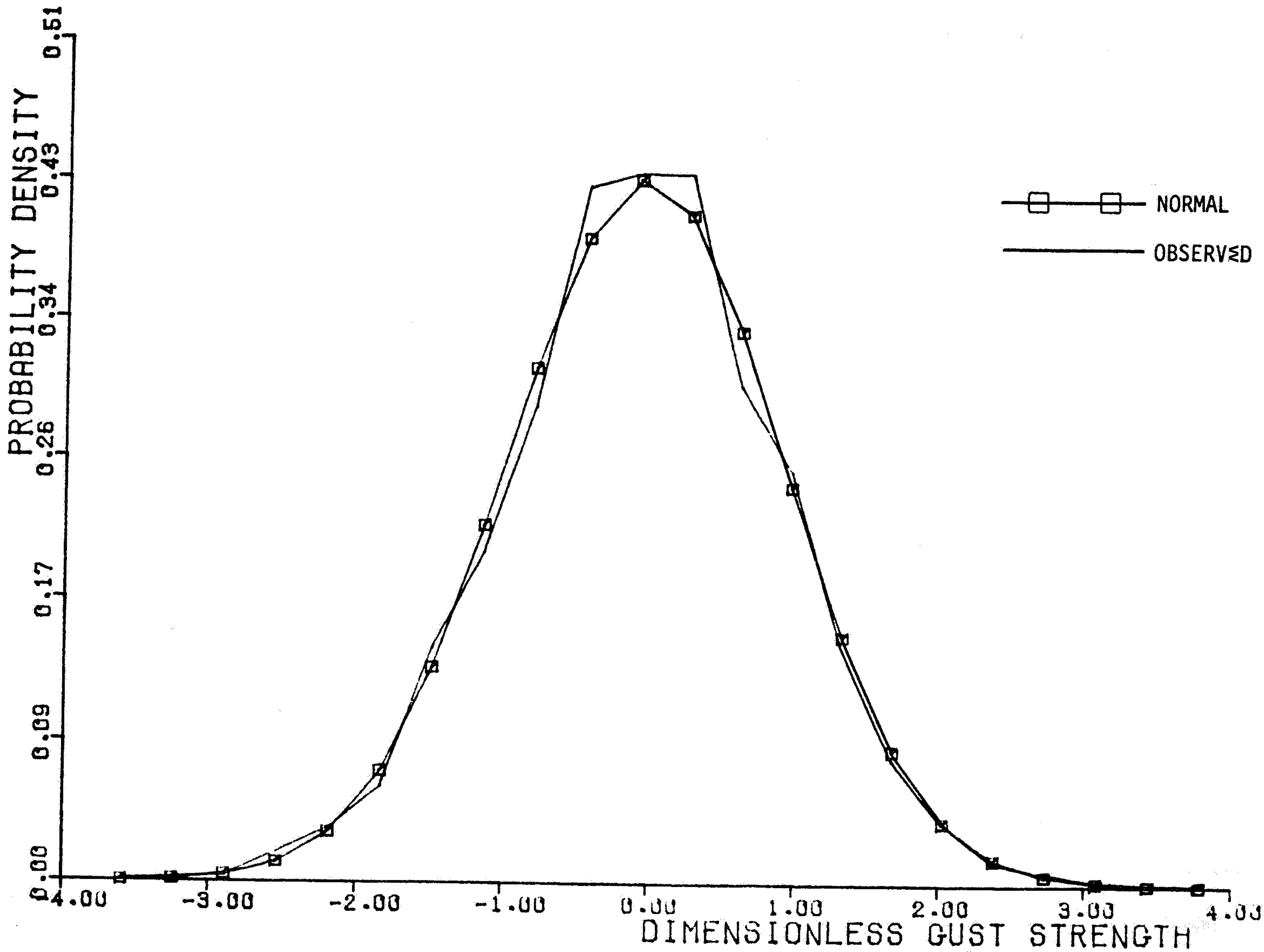

Figure D-15 u_3 - Gust Probability Density Distribution, Altitude Band #3

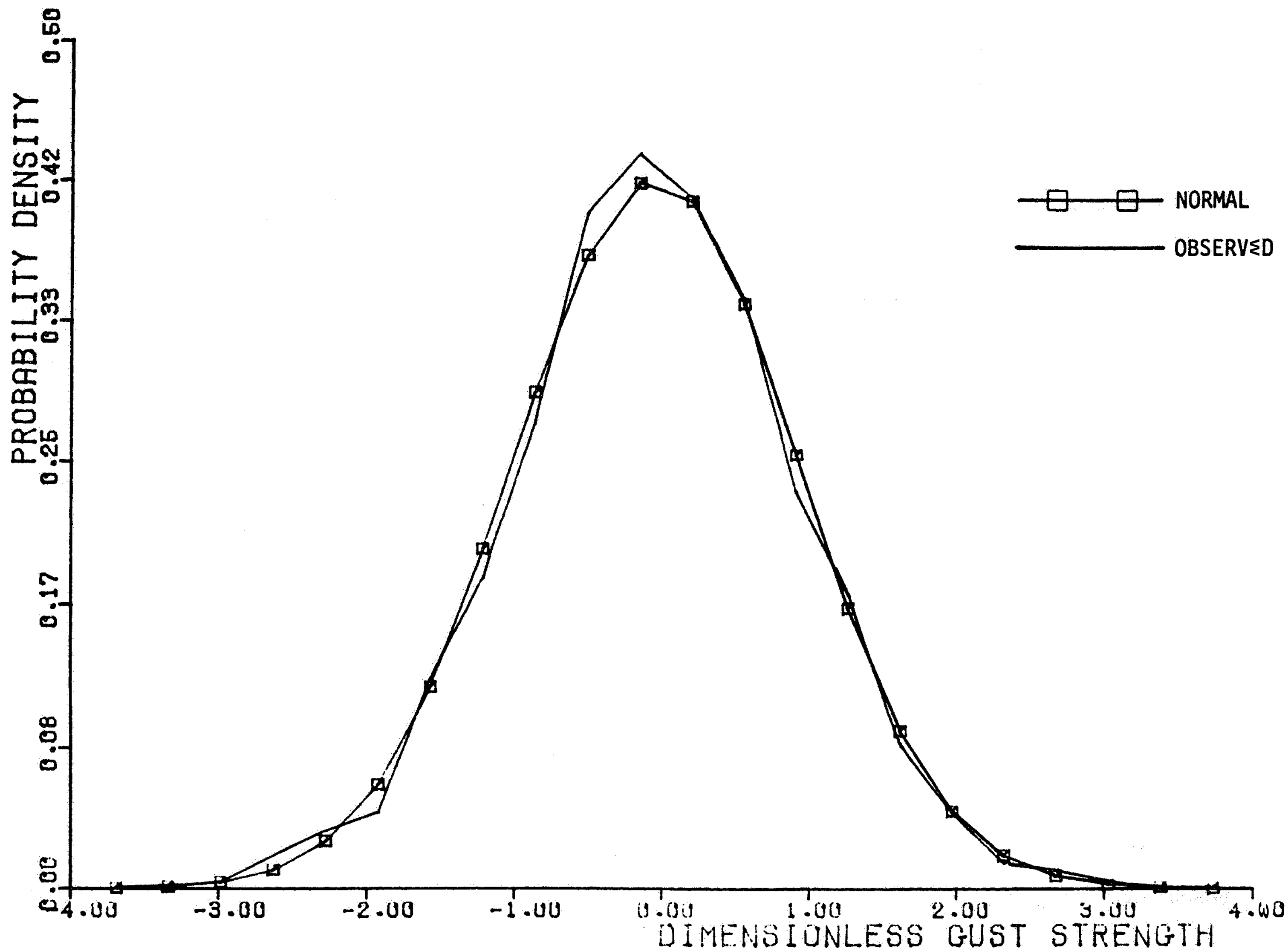

Figure D-16. u_3 - Gust Probability Density Distribution, Altitude Band #4

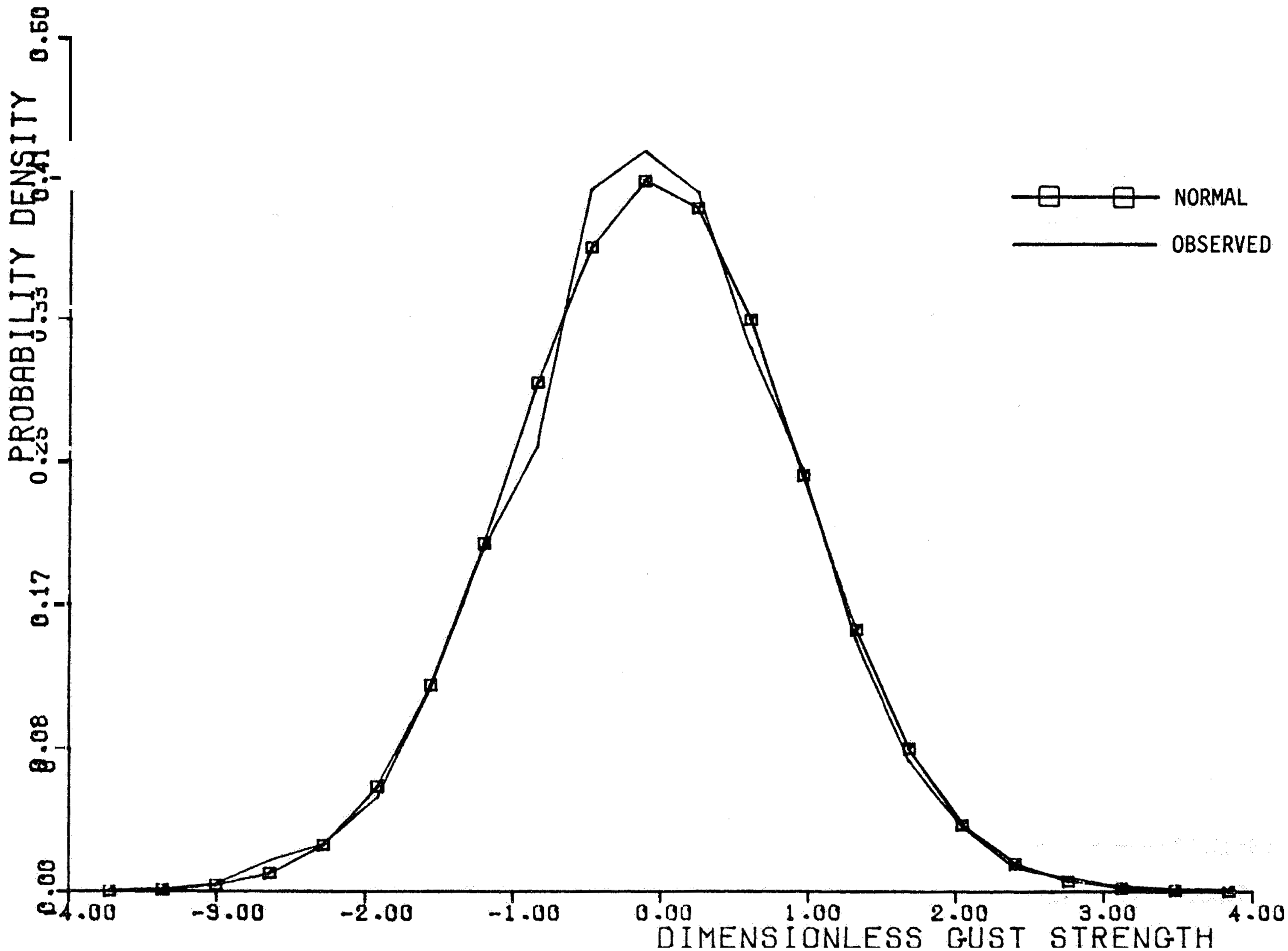

Figure D-17. u_3 - Gust Probability Density Distribution, Altitude Band #5

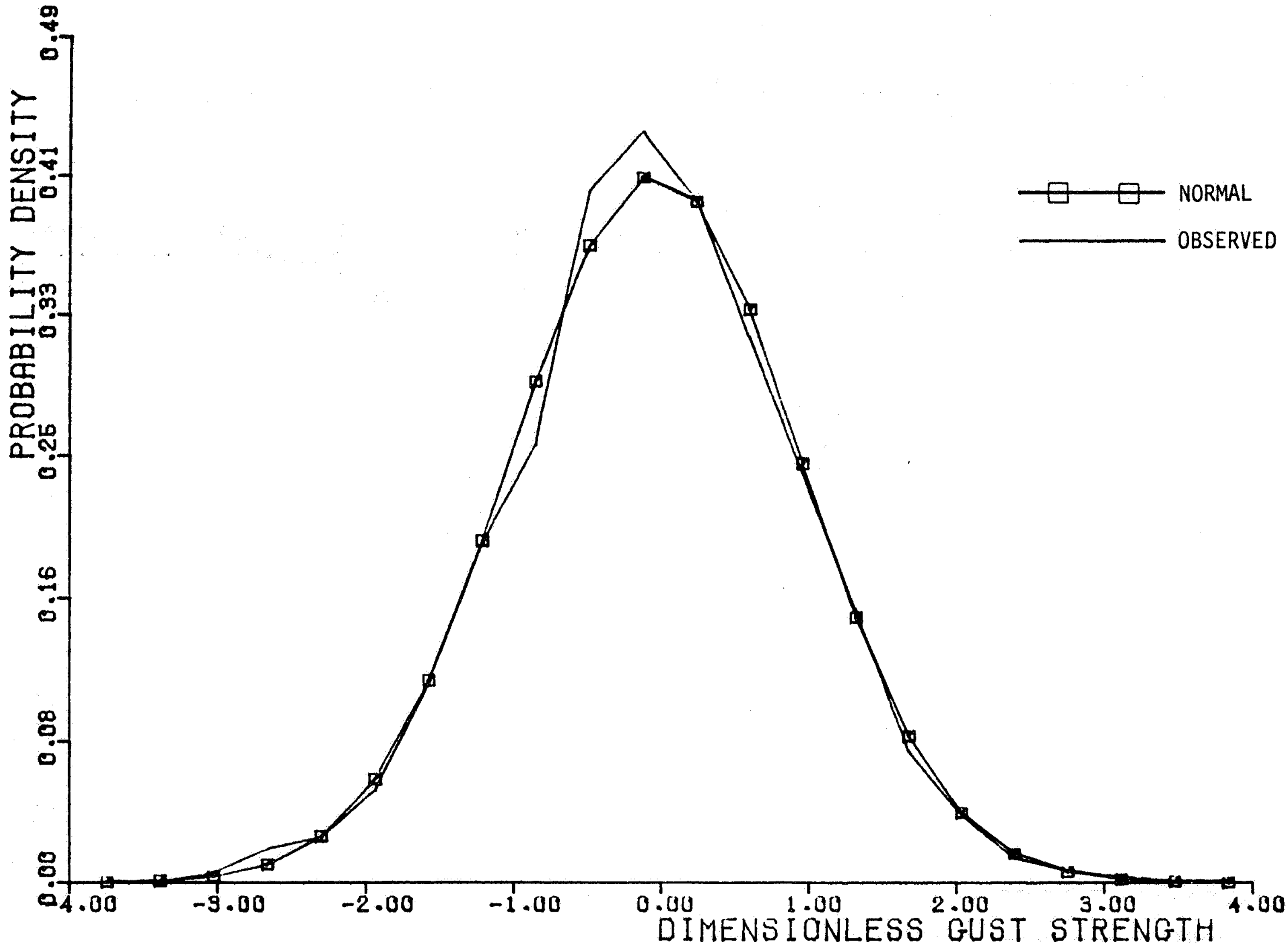

Figure D-18 ω_3 - Gust Probability Density Distribution, Altitude Band #6

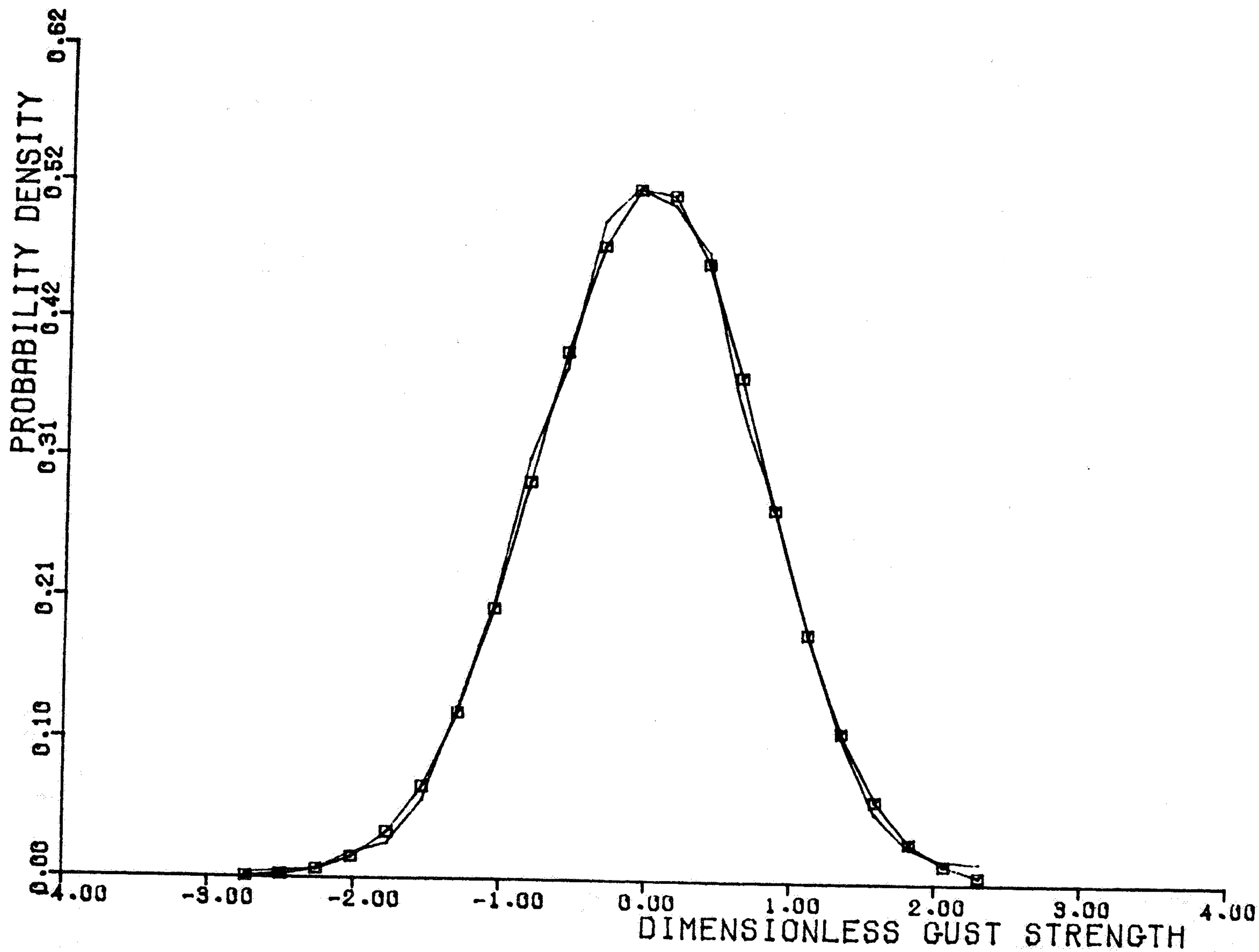

Figure D-19. $\partial u_2/\partial x_1$ - Gust Gradient Probability Density Distribution, Altitude Band #1

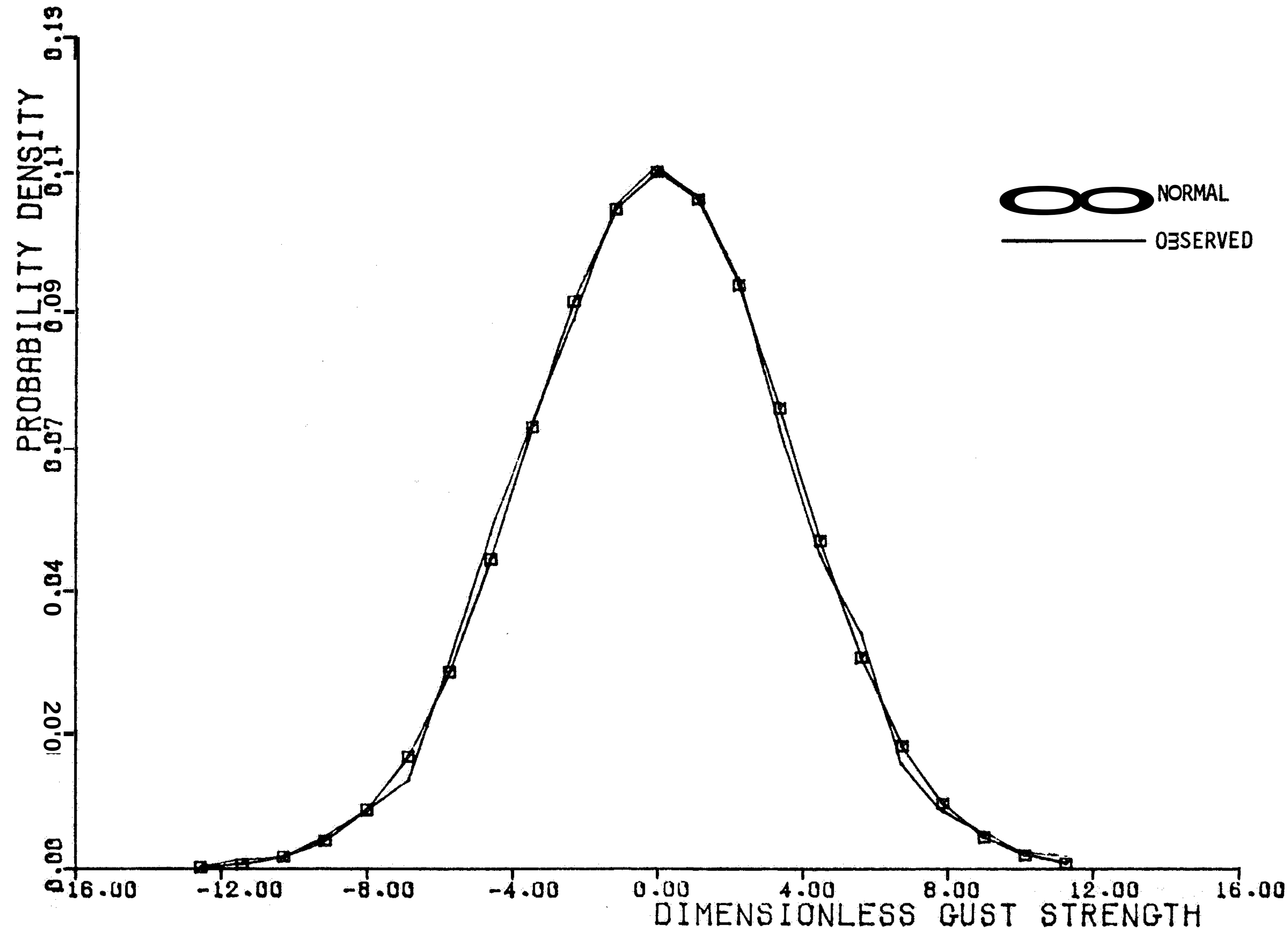

Figure D-20. $\partial u_2/\partial x_1$ - Gust Gradient Probability Density Distribution, Altitude Band #2

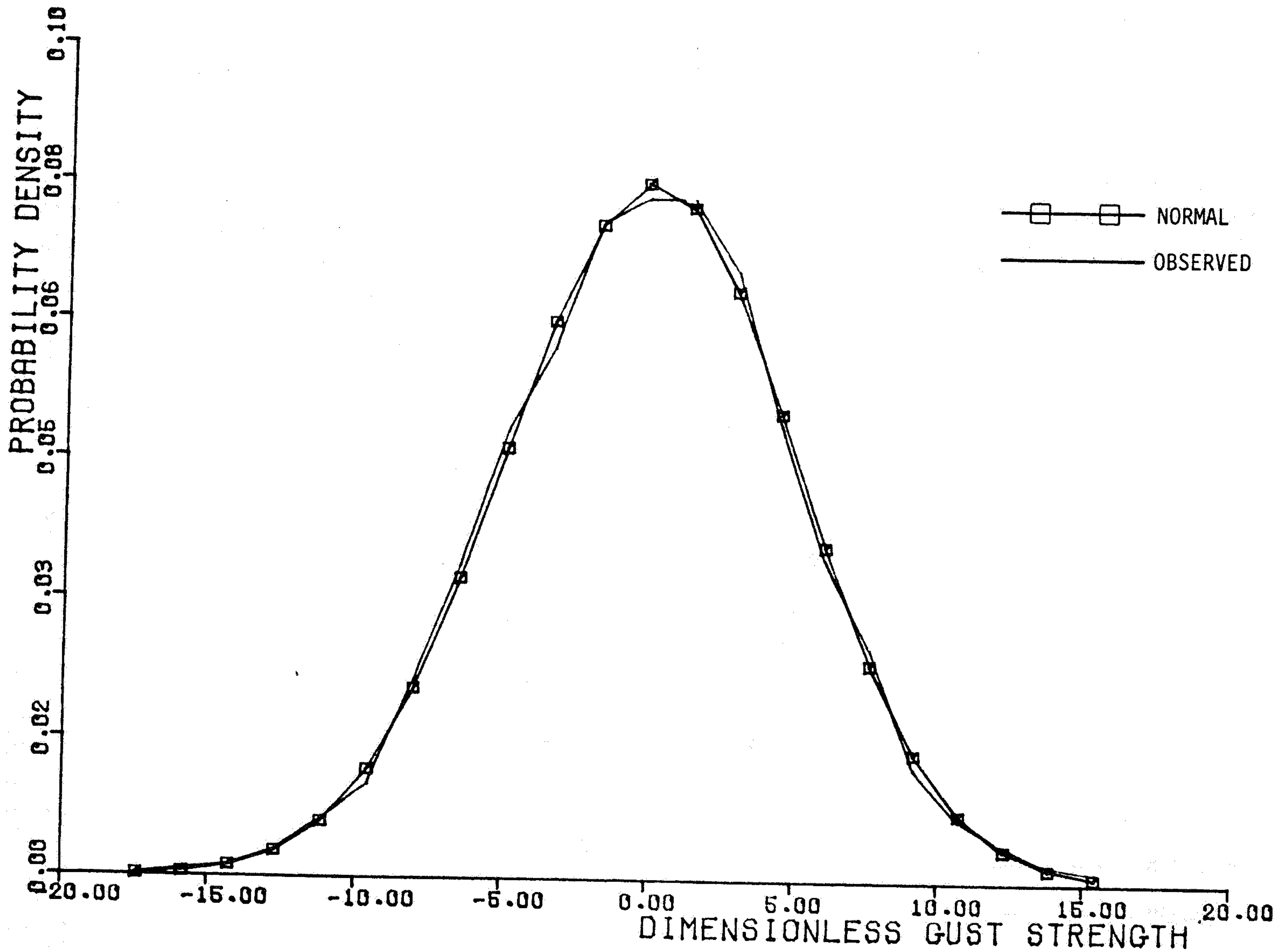

Figure D-21. $\partial u_2/\partial x_1$ - Gust Gradient Probability Density Distribution, Altitude Band #3

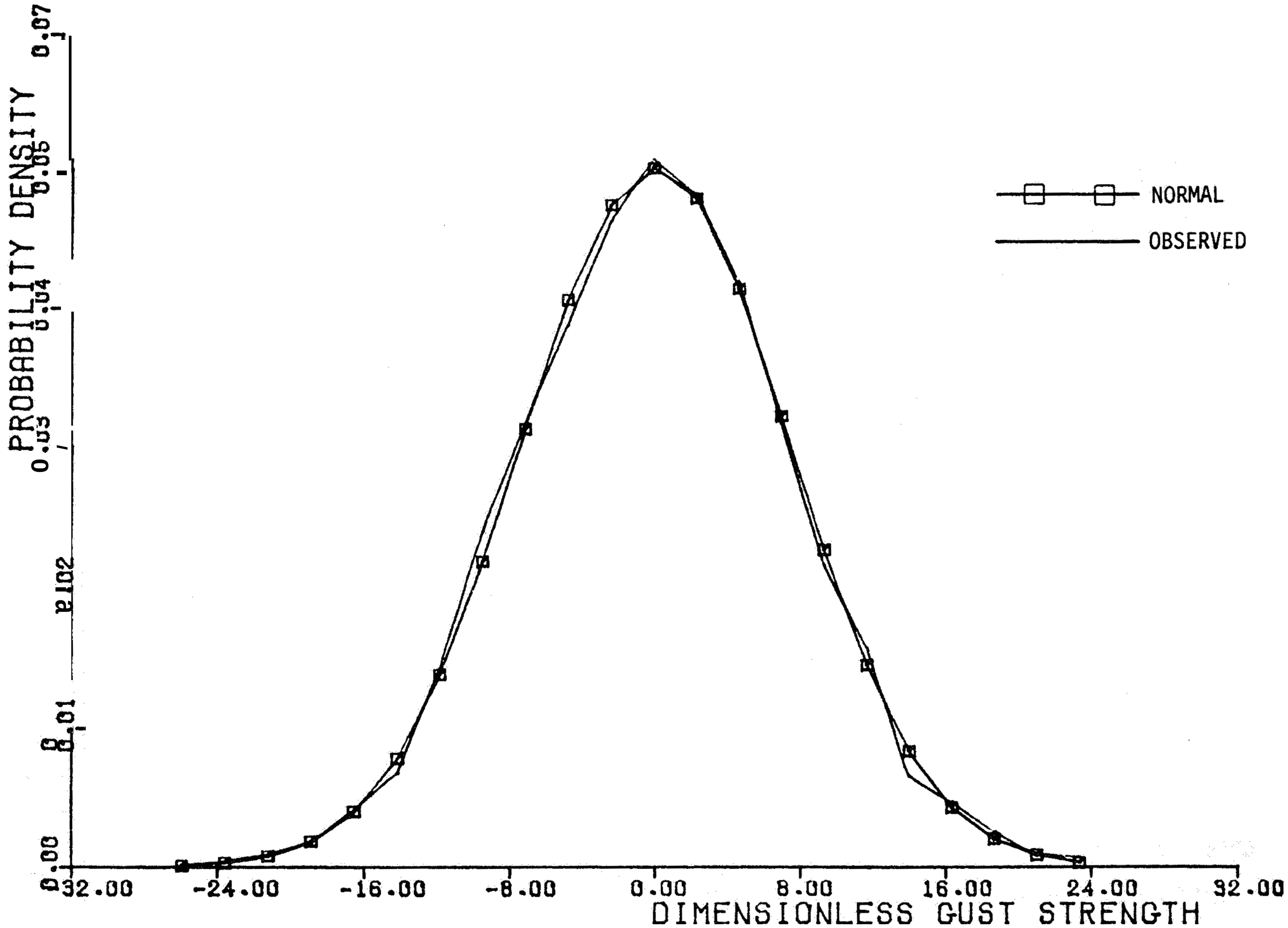

Figure D-22. $\partial u_2/\partial x_1$ - Gust Gradient Probability Density Distribution, Altitude Band #4

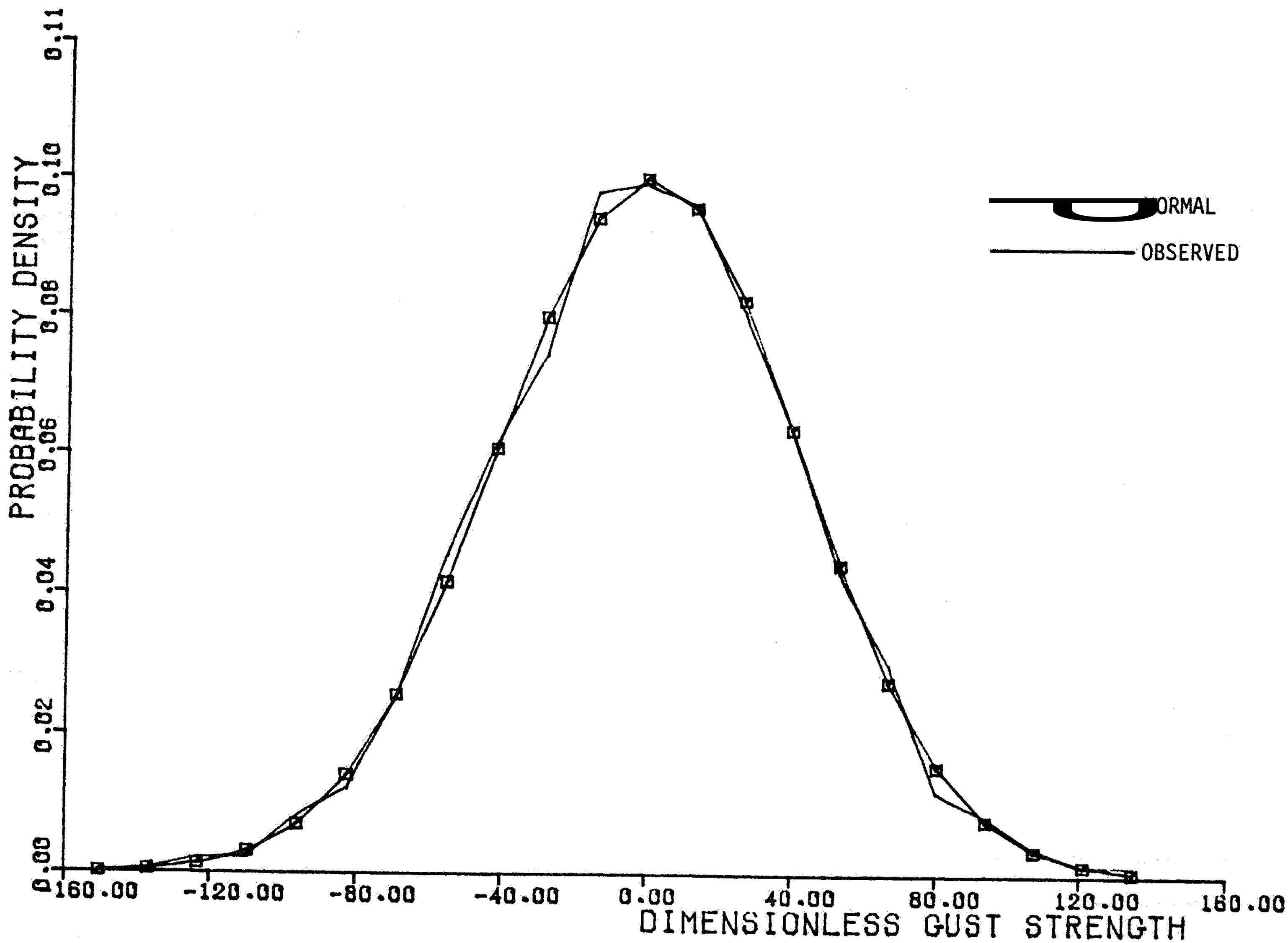

Figure D-23 $\partial u_2/\partial x_1$ - Gust Gradient Probability Density Distribution, Altitude Band #5

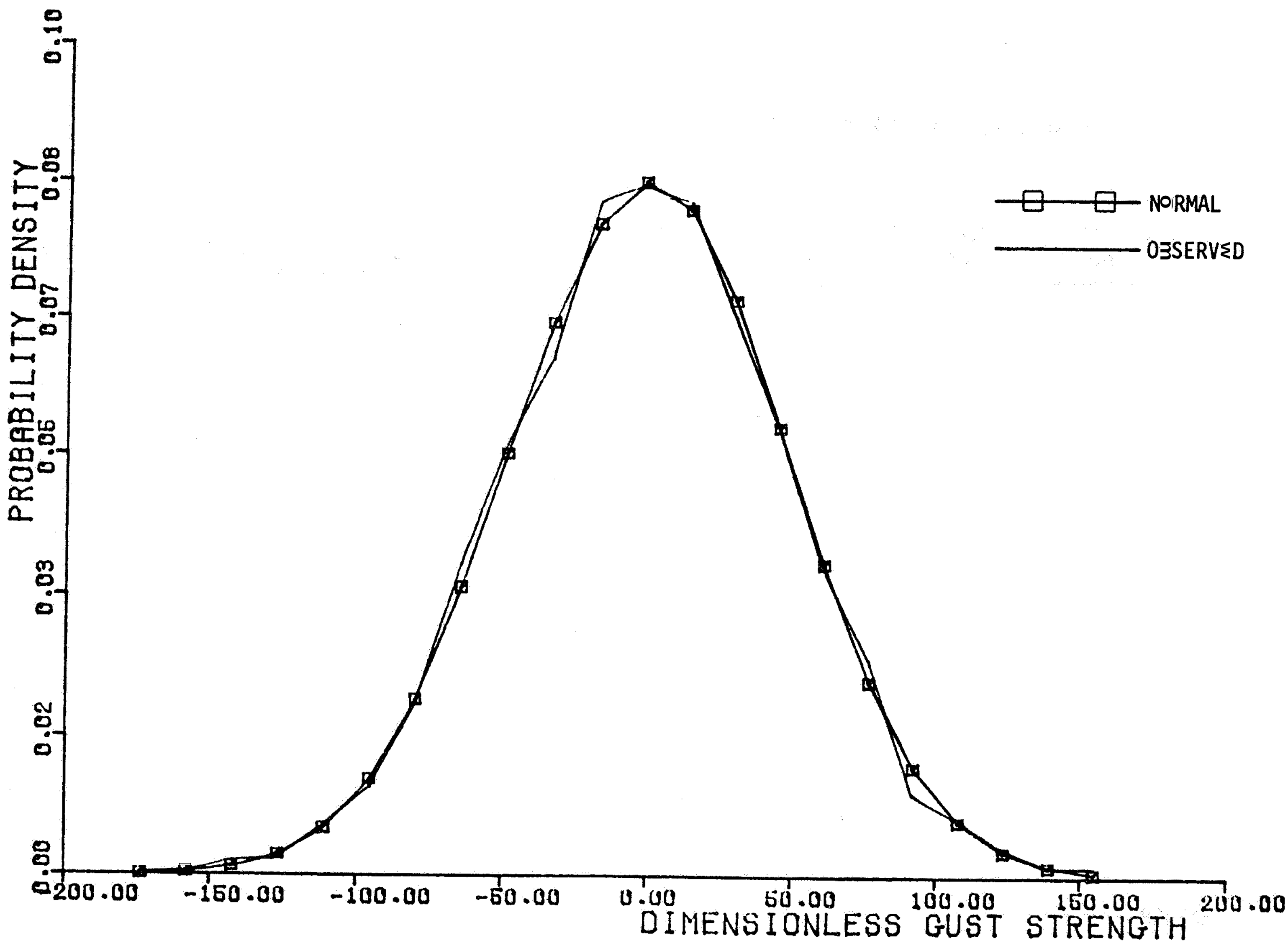

Figure D-24. $\partial u_2/\partial x_1$ - Gust Gradient Probability Density Distribution, Altitude Band #6

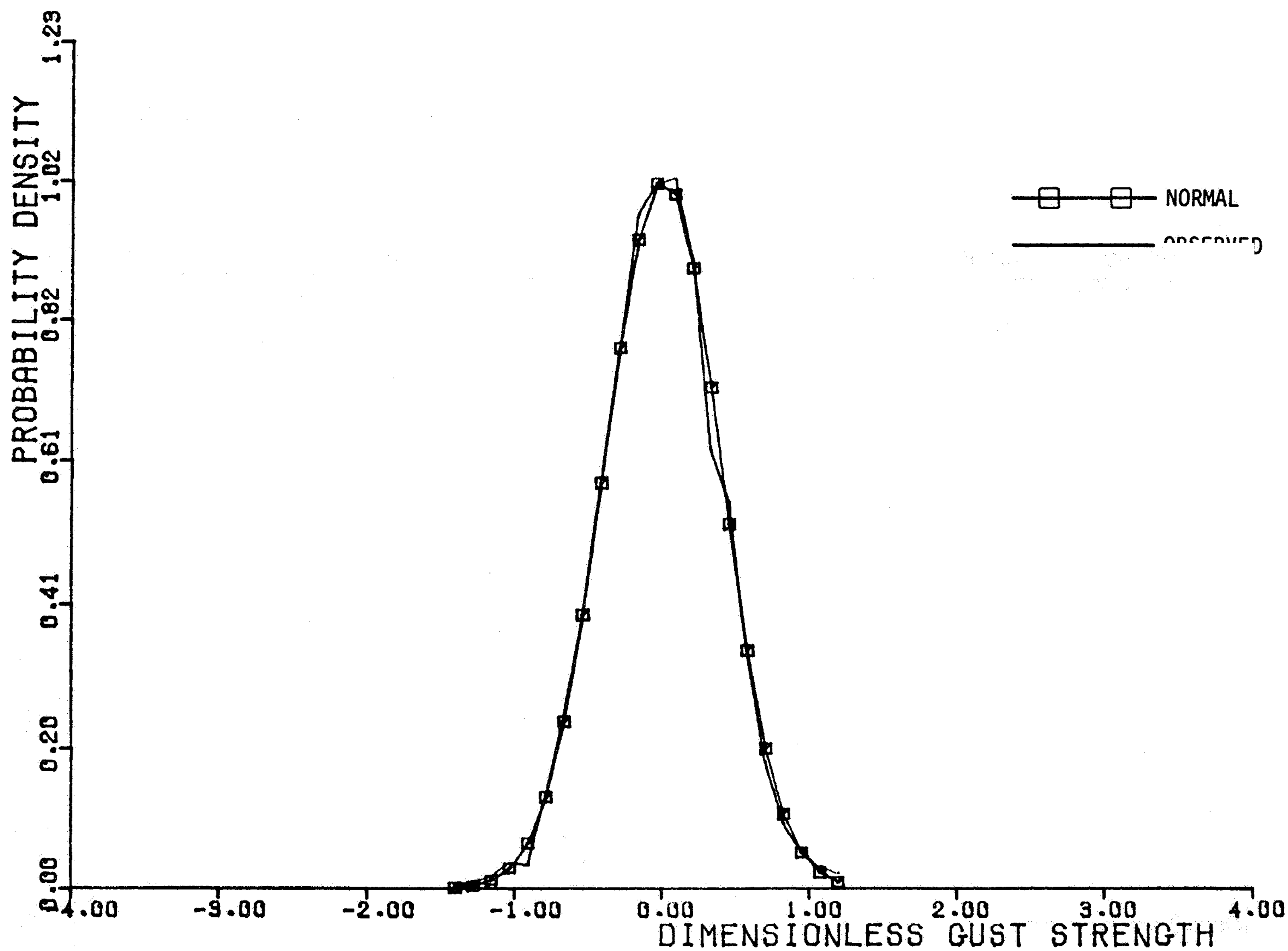

Figure D-25. $\partial u_3/\partial x_1$ - Gust Gradient Probability Density Distribution, Altitude Band #1

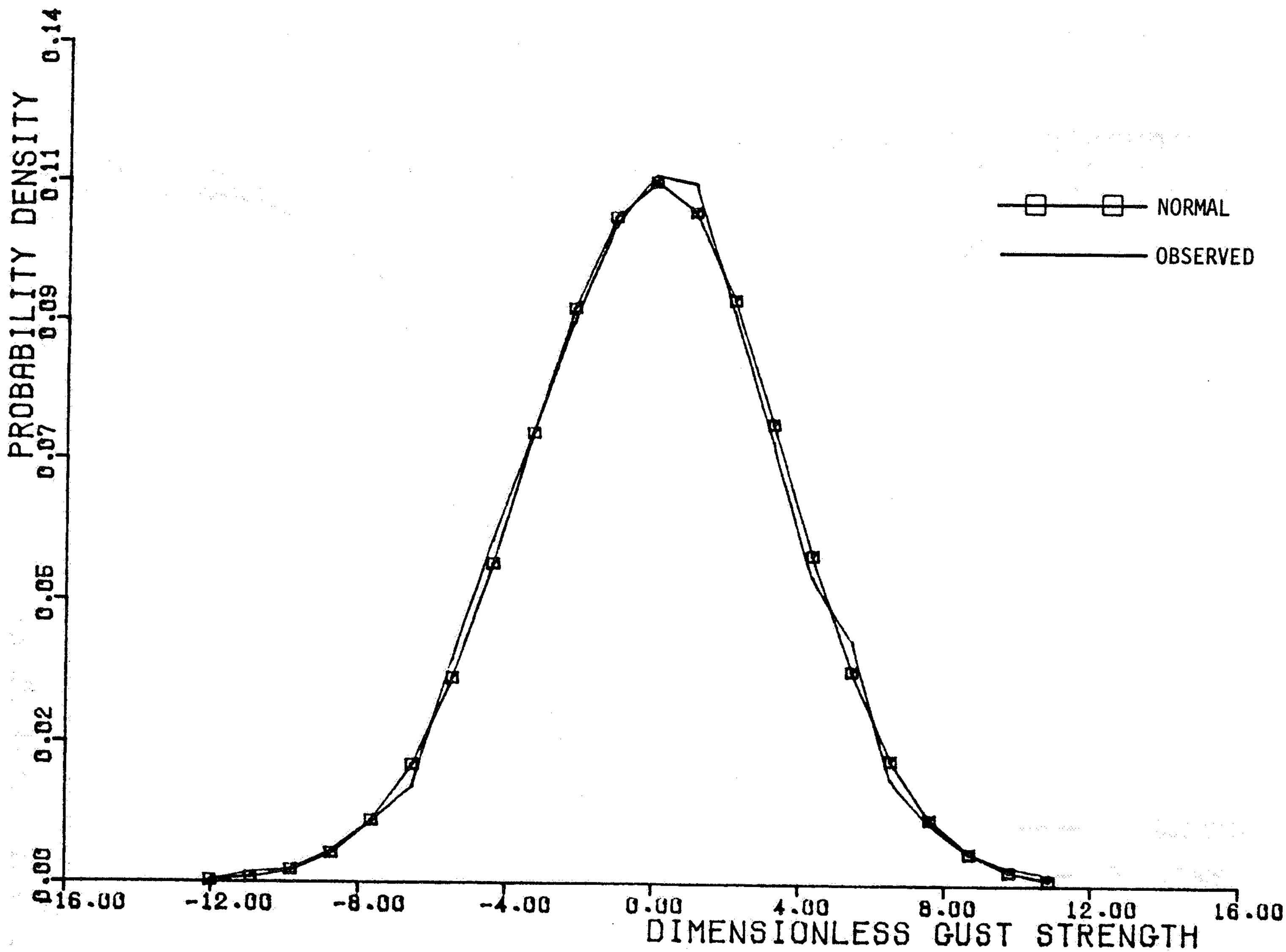

Figure D-26. $\partial u_3/\partial x_1$ - Gust Gradient Probability Density Distribution, Altitude Band #2

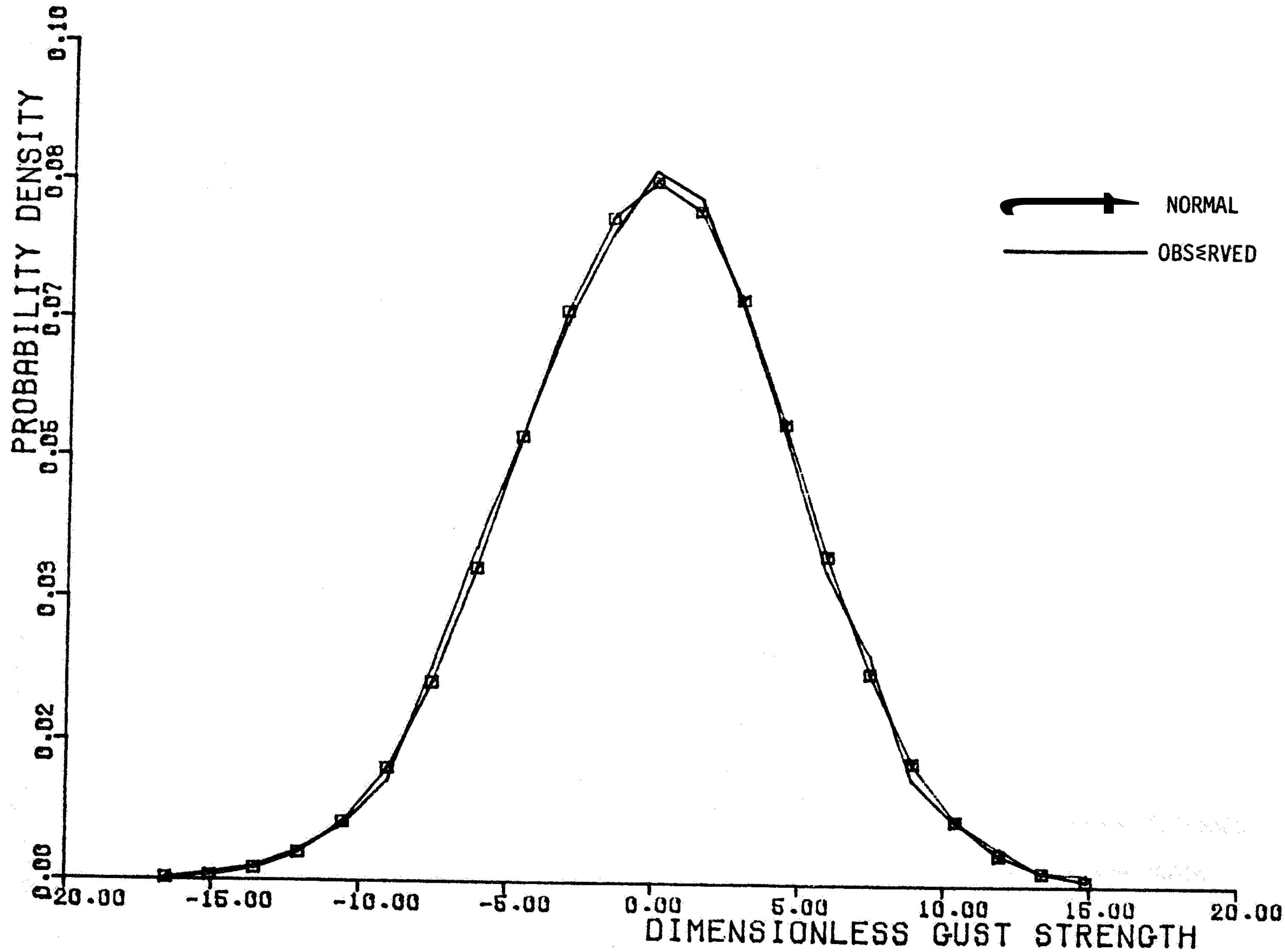

Figure D-27 $\partial u_3/\partial x_1$ - Gust Gradient Probability Density Distribution, Altitude Band #3

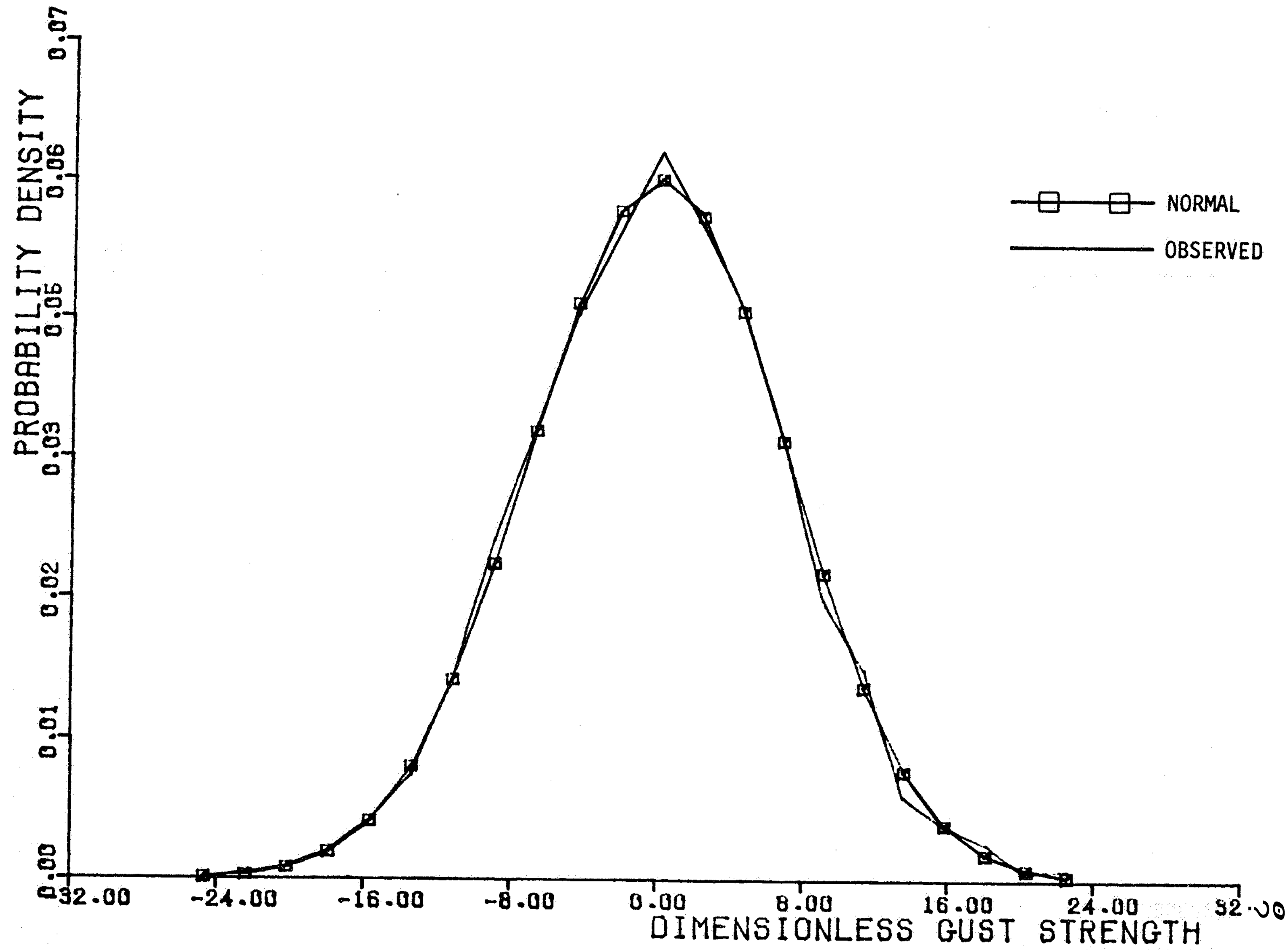

Figure D-28. $\partial u_3/\partial x_1$ - Gust Gradient Probability Density Distribution, Altitude Band #4

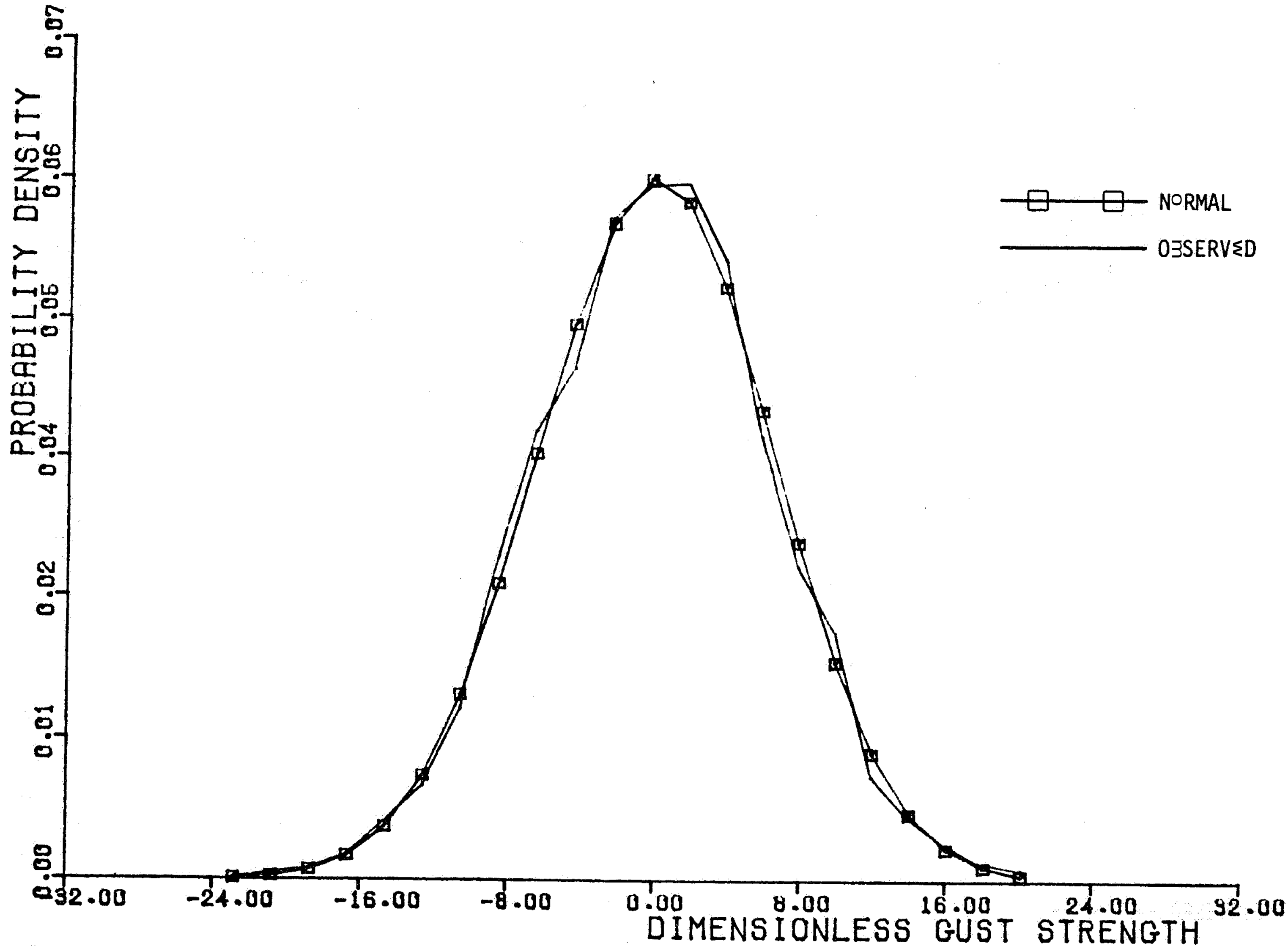

Figure D-29 $\partial w_3/\partial x_1$ - Gust Gradient Probability Density Distribution, Altitude Band #5

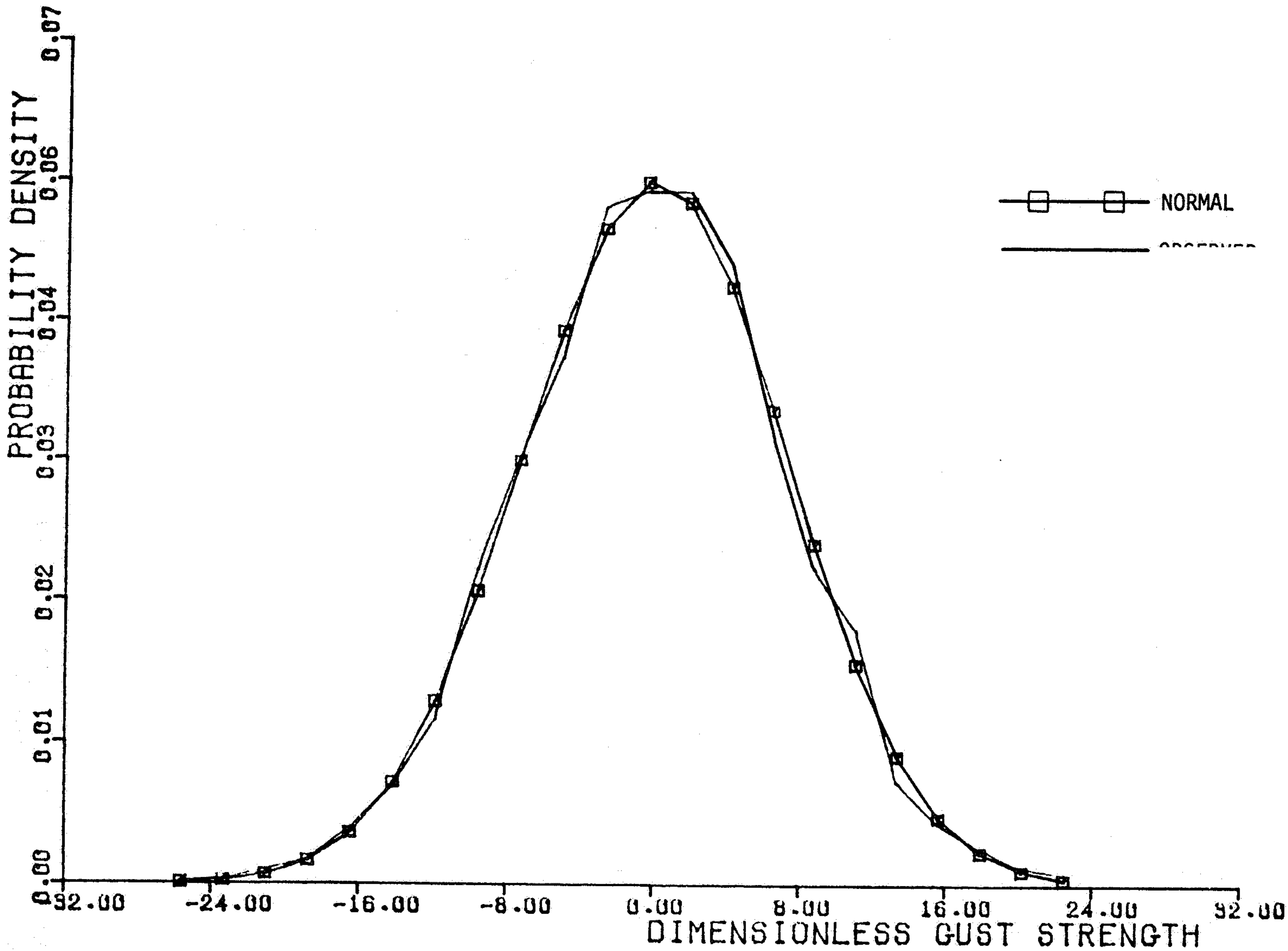

Figure D-30 $\partial w_3/\partial x_1$ - Gust Gradient Probability Density Distribution Altitude Band #6

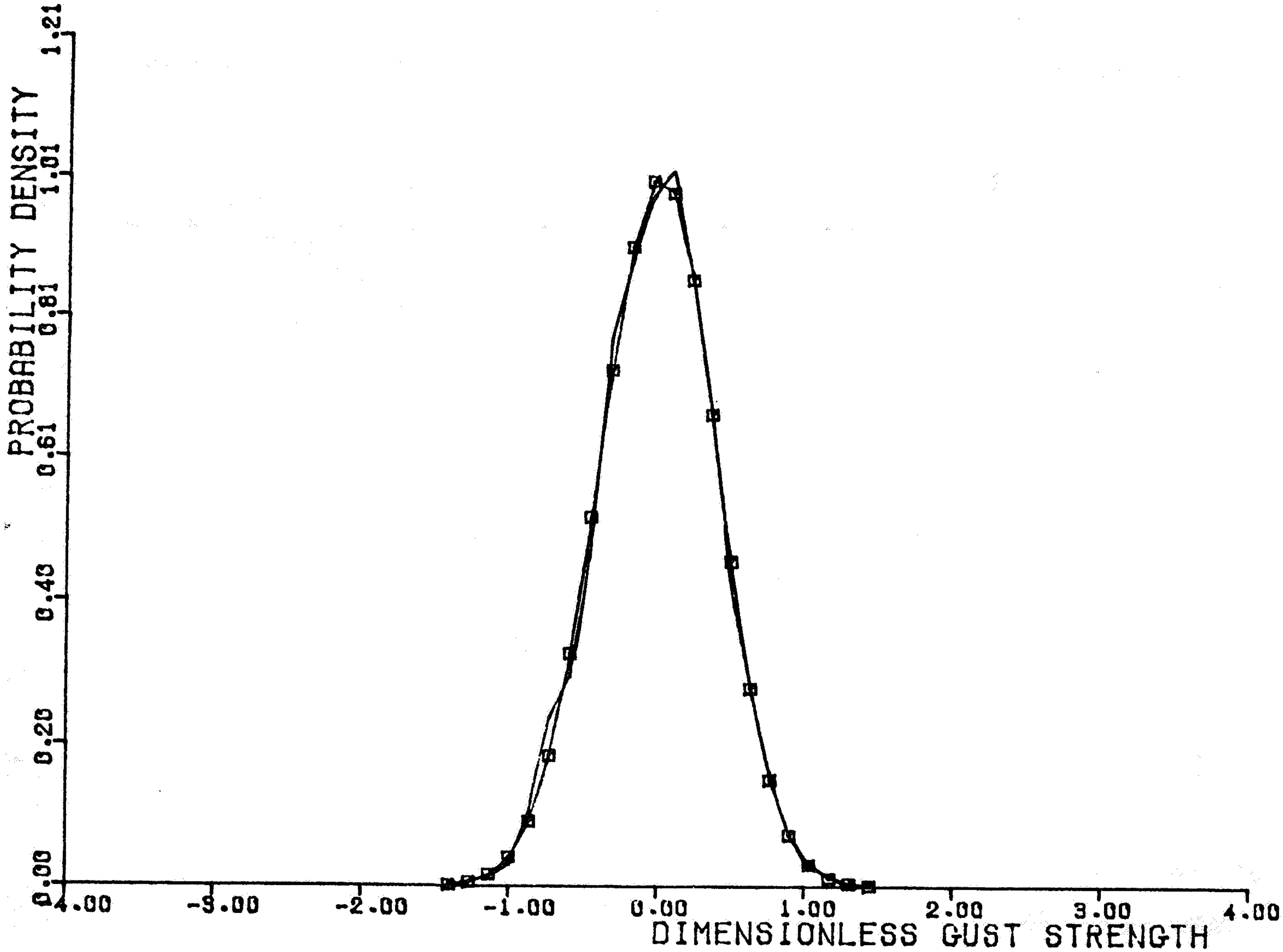

Figure D-31. $\partial u_3/\partial x_2$ - Gust Gradiemt Probability Density Distribution, Altitude Band #1

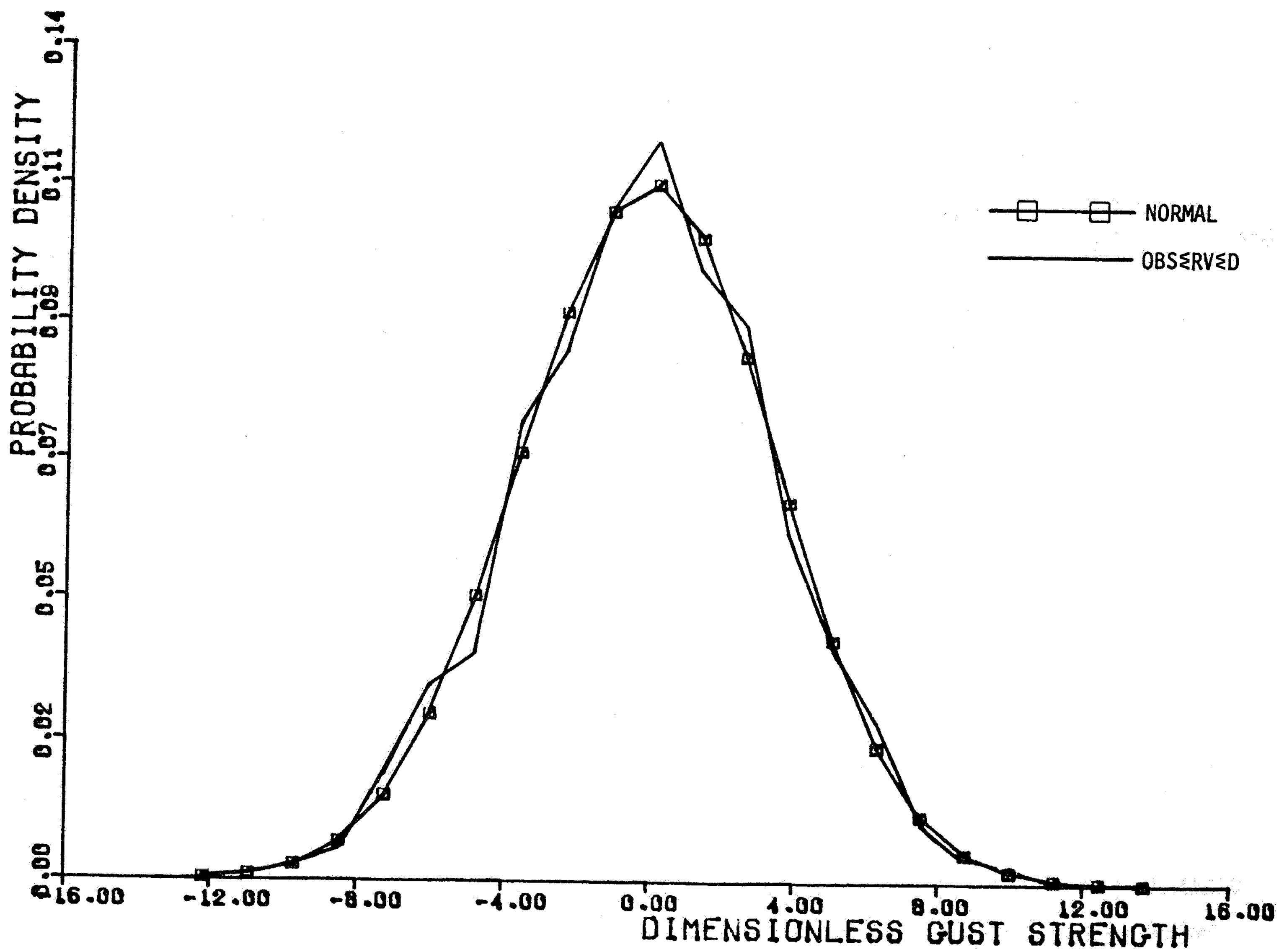

Figure D-32. $\partial u_3/\partial x_2$ - Gust Gradient Probability Density Distribution, Altitude Band #2

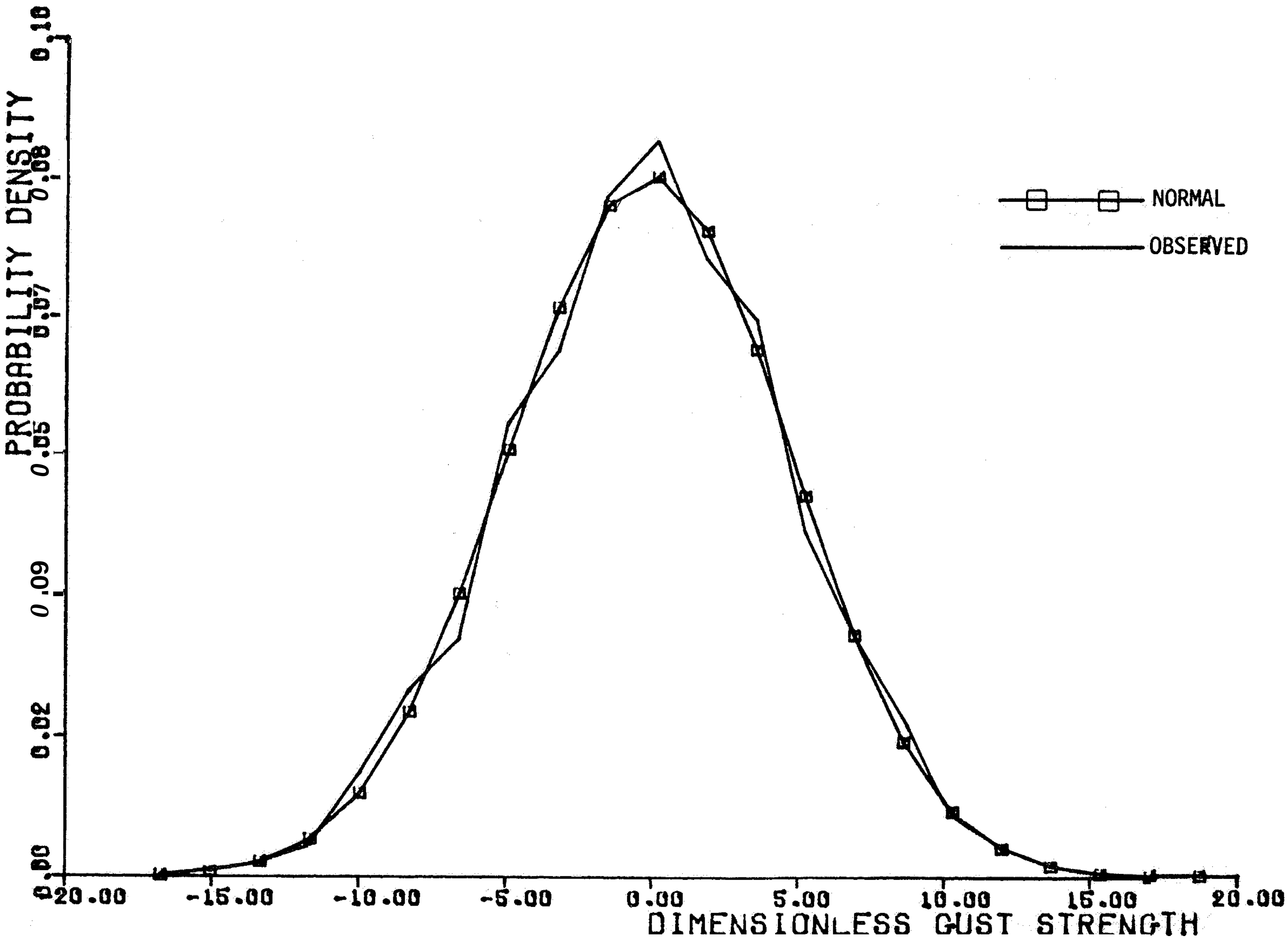

Figure D-33 $\partial u_3/\partial x_2$ - Gust Gradient Probability Density Distribution Altitude Band #3

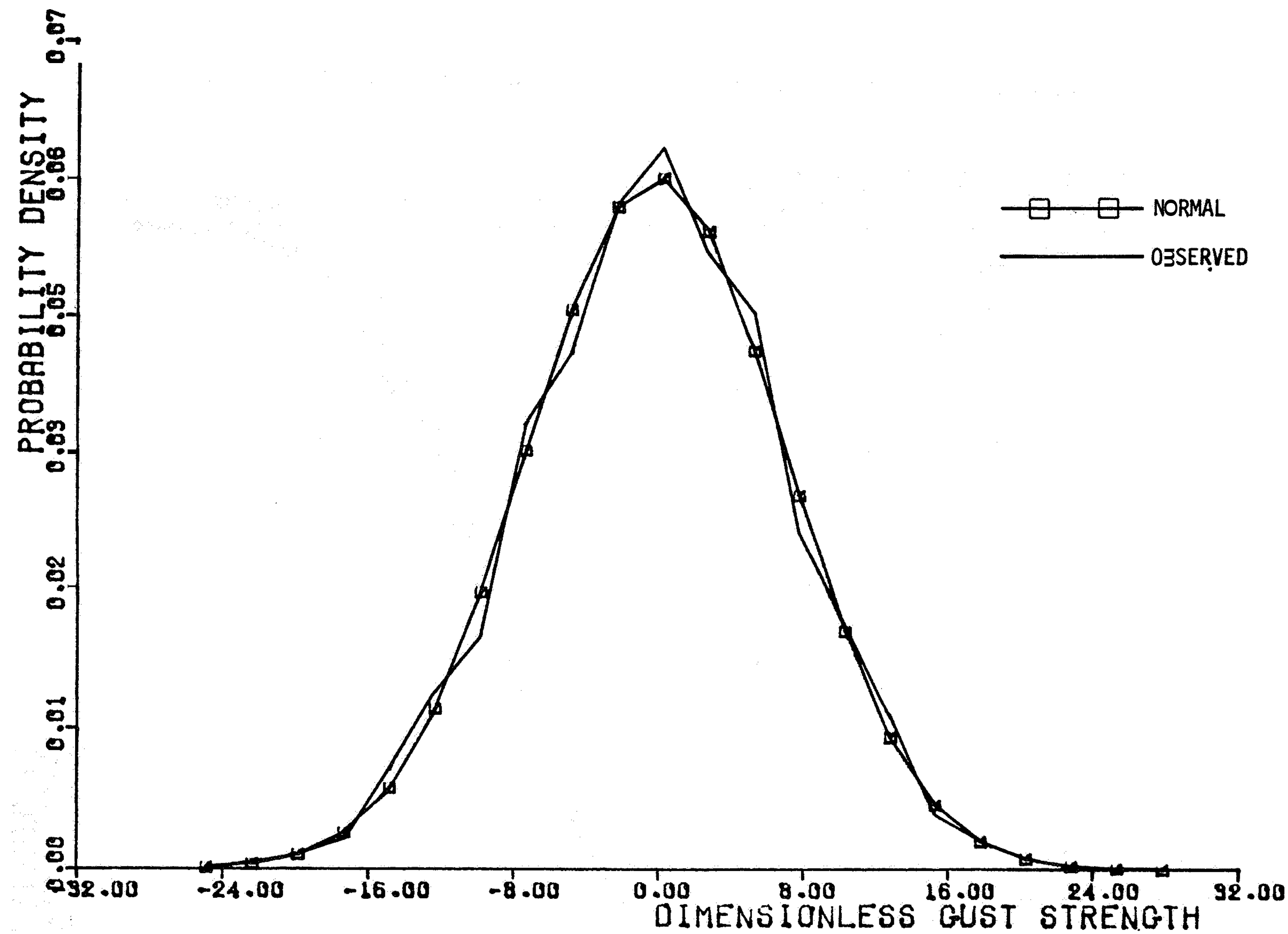

Figure D-34. $\partial u_3/\partial x_2$ - Gust Gradient Probability Density Distribution, Altitude Band #4

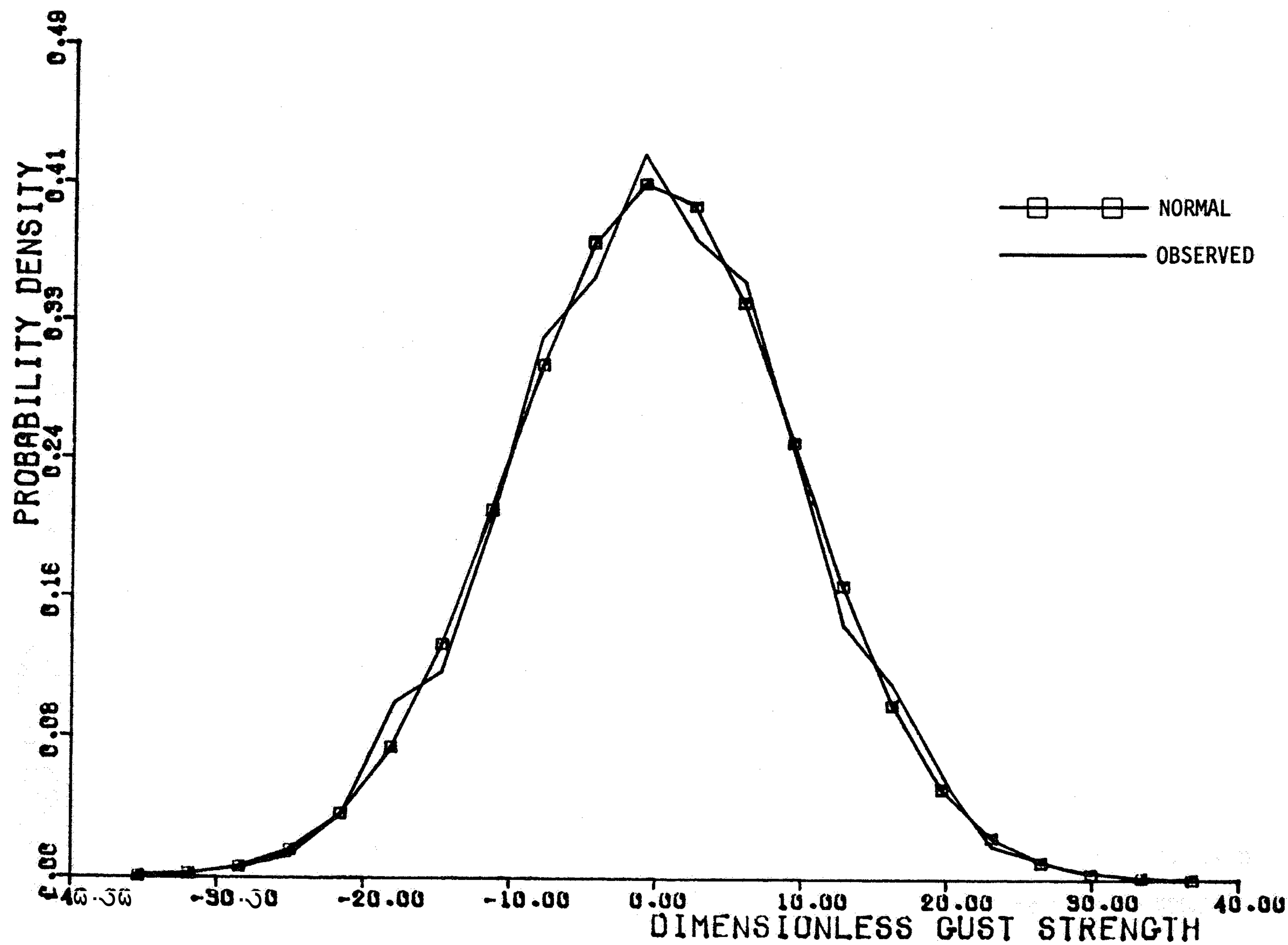

Figure D-35 $\partial u_3/\partial x_2$ - Gust Gradient Probability Density Distribution, Altitude Band #5

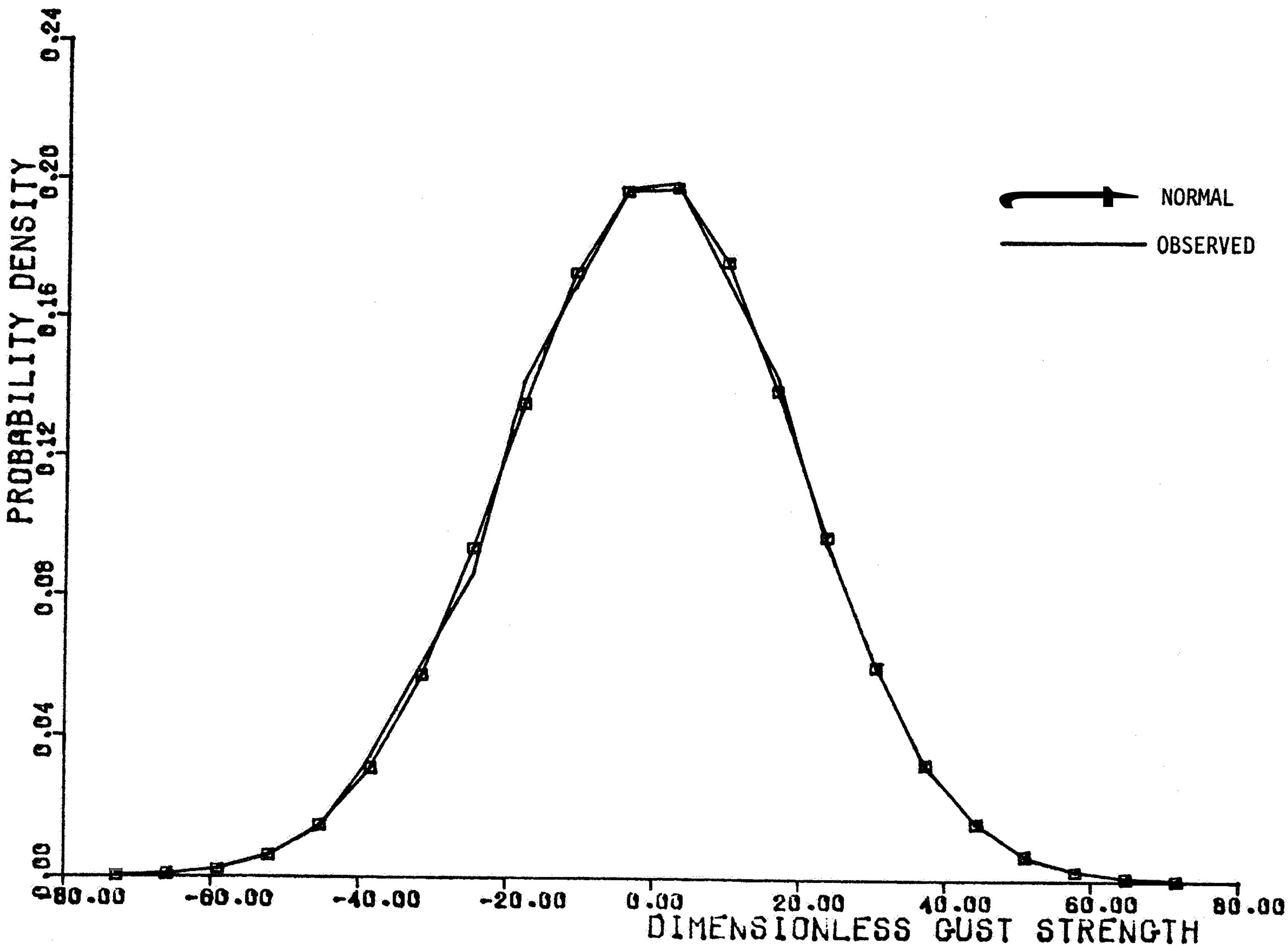

Figure D-36. $\partial u_3/\partial x_2$ - Gust Gradient Probability Density Distribution, Altitude Band #6

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16. ABSTRACT

The effects of atmospheric turbulence in horizontal and near-horizontal flight during the return of the Space Shuttle are important for determining design, control, and pilot-in-the-loop effects. A nonrecursive model (based on von Karman spectra) for atmospheric turbulence along the flight path of the Shuttle Orbiter has been developed which provides for simulation of instantaneous vertical and horizontal gusts at the vehicle center-of-gravity and also for simulation of instantaneous gust gradients. Based on this model the time series for gusts and gust gradients have been generated and stored on a series of magnetic tapes which are entitled Shuttle Simulation Turbulence Tapes (SSTT). The time series are designed to represent atmospheric turbulence from ground level to an altitude of 120,000 meters.

A description of the turbulence generation procedure is provided, the results of validating the simulated turbulence are described, and conclusions and recommendations are presented. Appendices provide tabulated one-dimensional von Karman spectra, a discussion of the minimum frequency simulated, and the results of spectral and statistical analyses of the SSTT.

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