A METHOD TO MODEL LATENT HEAT

FOR TRANSIENT ANALYSIS USING NASTRAN

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SUMMARY

A sample heat transfer analysis is shown which includes the heat of fusion. The method can be used to analyze a system with nonconstant specific heat. The enthalpy is introduced as an independent degree of freedom at each node. The user input consists of a curve of temperature as a function of enthalpy, which may include a constant temperature phase change. The basic NASTRAN heat transfer capability is used to model the effects of latent heat with existing direct matrix output (DMI) and nonlinear load (NØLIN) data cards. Although some user care is required, the numerical stability of the integration is quite good when the given recommendations are followed. The theoretical equations used and the NASTRAN techniques are shown in the paper.

INTRODUCTION

The problem of heat transfer with latent heat or nonconstant specific heat is one of current interest. Methods based upon introduction of enthalpy[1] and upon a very high specific heat in a small temperature range[2] have been published. These methods can be implemented in either finite difference or finite element formulations. The numerical stability of the transient integration must be analyzed, since failure in this area makes the method very expensive, due to excessive time steps required.

Values are given in SI Units.

A   area, m^2

B   capacity matrix, $J/^{\circ}C$

c   specific heat, $J/kg^{\circ}C$

h   enthalpy, J/kg

K   conduction matrix, $J/s^{\circ}C$

L   latent heat, $h_{\ell} - h_s$, J/kg

M   mass, kg

N   nonlinear function

N_e number of finite elements

P   thermal load, J/s

T   temperature, $^{\circ}C$

t   time, s

x   coordinate, m

β Newmark beta parameter

γ stability parameter

κ thermal conductivity, $J/s\ m\ ^{\circ}C$

λ eigenvalue, growth factor

ρ mass density, k/m^3

Subscripts:

ℓ liquidus

s   solidus

w   wall

n   time step

MATHEMATICAL ANALYSIS

The derivation will be made for one-dimensional heat conduction, however the method is general. The temperature distribution is found from solution of the diffusion equation

$$\kappa \frac{\partial^2 T}{\partial x^2} = \rho c \frac{\partial T}{\partial t} \tag{1}$$

When this equation is analyzed by finite element techniques, it is written

$$[B]\{\dot{T}\} + [K]\{T\} = \{P\} \tag{2}$$

where the terms of the B (capacitances) matrix are $\rho c A \Delta x$ and the terms of the K (finite element conductivities) matrix are $\Delta \kappa / \Delta x$.

In the general case the conductivity and the heat capacity are not constant. In the case of phase change at constant temperature (see fig. 1) the method fails since the specific heat is effectively infinite. Introduction of the enthalpy gives two simultaneous equations

$$[M]\{\dot{h}\} + [K]\{T\} = \{P\} \tag{3}$$

$$T = N \ (h) \tag{4}$$

A Newmark beta numerical integration scheme is used, where the velocity terms are replaced by

$$\dot{h} = \left(h_{n+1} - h_n\right)/\Delta t \tag{5}$$

and the constant terms by

$$T = \beta \ T_{n+1} + (1-\beta) \ T_n \tag{6}$$

This parameter β is a stability parameter. If $\beta = 0$, the method is called foreward differencing, and $\beta = 1$ corresponds to backward differencing. For β greater than 0.5, integration of equation (2) is numerically stable for any mesh size and time step. Integration of (3) and (4) shows that there is a tendency to instability at large time steps.

The basic method of stabilizing the equations is to evaluate the terms at the advanced time (t_{n+1}) rather than the time (t_n). We replace equation (4) by

$$T - h/c_{min} = N(h) - h/c_{min} \qquad (7)$$

where c_{min} is the minimum specific heat. The terms on the left side of (7) are evaluated as shown in (6), while the terms on the right are evaluated at t_n. An entirely equivalent method to (7) is

$$T - \left(\gamma \Delta t / c_{min}\right) \dot{h} = N (h) \qquad (8)$$

If (8) is derived from (7), the parameter γ would be β; however, we shall treat γ as an independent stability parameter.

Stability analysis requires a long derivation and will not be done here. The two basic steps are linearization and modal analysis. The equations are linearized by replacing the nonlinear curve by a linear fit. The mode analysis replaces a multi-degree-of-freedom problem with a series of two-degree-of-freedom problems. The short wave length modes are most unstable. Introduce a growth factor

$$T_{n+1} = \lambda \, T_n \qquad (9)$$

$$h_{n+1} = \lambda \, h_n \qquad (10)$$

There are two roots to the characteristic equation

$$1 - \beta^{-1} \quad \text{and}$$

$$\lambda = 1 - \left(\frac{\rho c L^2}{4 N_e^2 \kappa \Delta t} + \gamma \frac{c}{c_{min}} \right)^{-1} \qquad (11)$$

Since $-1 < \lambda < +1$ for stability,

$$\beta > 1/2 \qquad (12)$$

and

$$\frac{\rho c\ L^2}{4N_e^2 \kappa \Delta t} + \gamma\ \frac{c}{c_{min}} > 1/2 \qquad (13)$$

Thus $\beta = \gamma = 0.55$ gives good stability for any Δt. For $\Delta t < \rho c L^2/2N_e^2\kappa$, the solution is stable without the need of γ, but this Δt is usually much smaller than needed for accuracy.

NASTRAN RESULTS

The analytic solution given in reference (1) was chosen, since an exact solution allows analysis of accuracy. This is a one-dimensional model. The initial condition is ice at freezing temperature. Starting at time zero, the end is heated to a constant temperature. The end temperature is chosen so the enthalpy change after melting has the same value as the latent heat. The melting proceeds until such time that the water-ice interface has moved 1.24 meters, and then the temperature profile is examined. The model is 1.3 meters long and has either 26 or 130 elements. This arrangement was chosen to allow comparison of results (see Table I).

CONCLUSIONS

The NASTRAN results are of good accuracy. By using the stability factor, the accuracy has not been degraded. These results can easily be adapted to other geometries with the finite element method. No changes were required in NASTRAN to solve problems with latent heat.

REFERENCES

1. Wood, A.S., Ritchie, S.I.M., and Bell, G.E.; "An Efficient Implementation of the Enthalpy Method"; International Journal for Numerical Methods in Engineering, Vol. 17, pp. 301-305; 1981.

2. Morgan, K., Lewis, R.W., and Zienkiewicz, O.C.; "An Improved Algorithm for Heat Conduction Problems with Phase Change"; International Journal for Numerical Methods in Engineering, Vol. 12, pp. 1191-1195; 1978.

TABLE I. COMPARISON OF METHODS

Dimensionless Temperature at the Final Time

$$(T - T_s) / (T_w - T_s)$$

All results are for $\beta = \gamma = 0.55$

$\Delta x = 0.05$ m (26 elements)

Method Number of Steps		Exact	NASTRAN 800	NASTRAN 200	Ref 1 800	Ref 1 200
Distance	0.2	0.8184	0.8184	0.8181	0.8207	0.8242
from hot	0.6	.4695	.4694	.4686	.4758	.4871
end, m.	1.2	.0252	.0321	.0230	.0335	.0612

$\Delta x = 0.01$ m (130 elements)

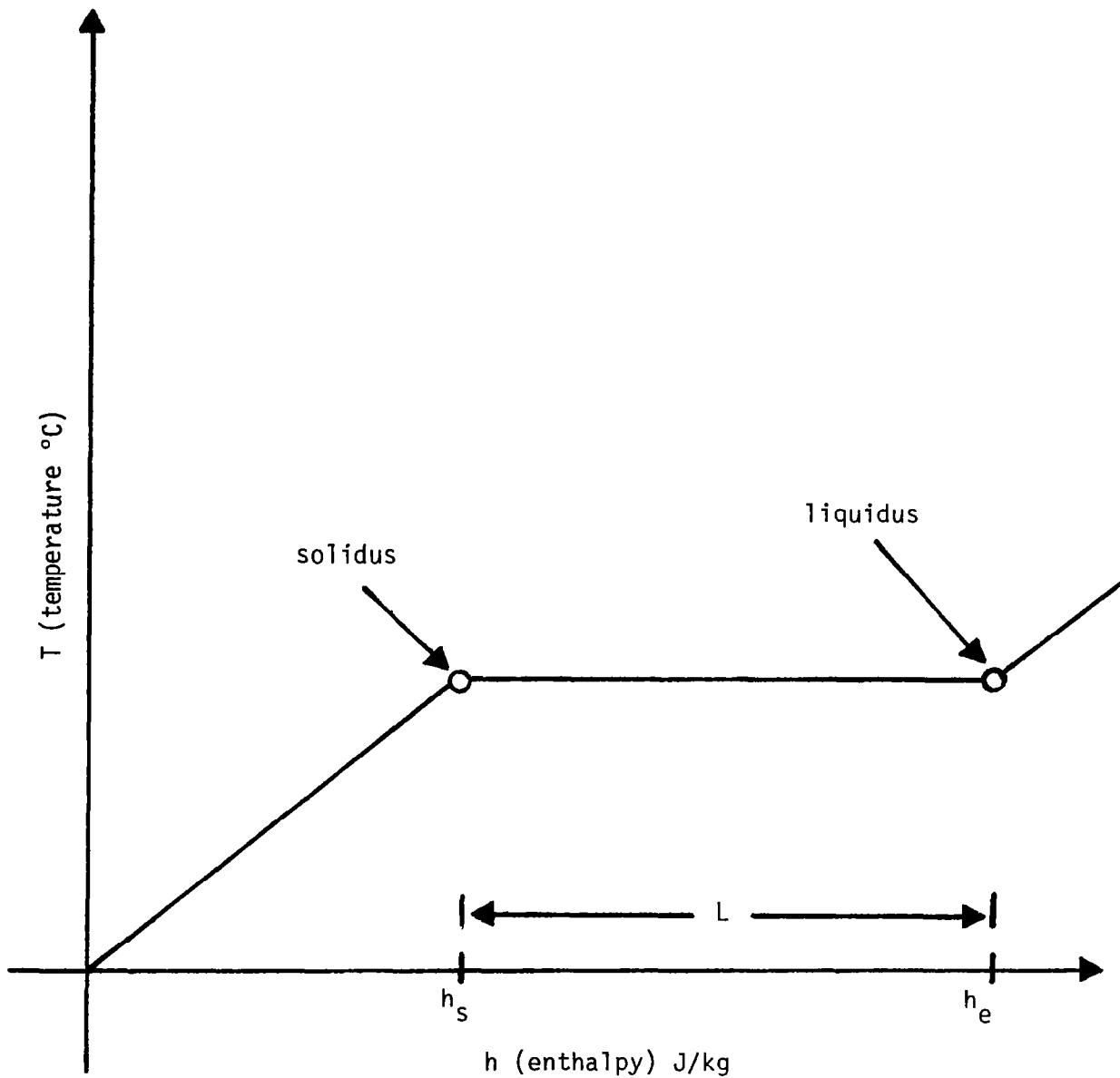| Method Number of Steps		NASTRAN 1600	NASTRAN 800	NASTRAN 200	Ref 1 20000	Ref 1 5000
Distance	0.2	0.8184	0.8184	0.8181	0.8189	0.8195
from hot	0.6	.4694	.4693	.4686	.4707	.4726
end, m.	1.2	.0256	.0263	.0273	.0262	.0316

Figure 1.- Latent heat; typical curve of temperature as a function
of enthalpy, showing change of phase. The specific heat,
$c = dh/dT$, becomes infinite.