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IMPLEMENTATION OF STRUCTURAL RESPONSE SENSITIVITY CALCULATIONS IN A LARGE-SCALE FINITE-ELEMENT ANALYSIS SYSTEM

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IMPLEMENTATION OF STRUCTURAL RESPONSE SENSITIVITY CALCULATIONS IN A LARGE-SCALE FINITE-ELEMENT ANALYSIS SYSTEM

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Abstract

The methodology used to implement structural sensitivity calculations into a major, general-purpose finite-element analysis system (SPAR) is described. This implementation includes a generalized method for specifying element cross-sectional dimensions as design variables that can be used in analytically calculating derivatives of output quantities from static stress, vibration, and buckling analyses for both membrane and bending elements. Limited sample results for static displacements and stresses are presented to indicate the advantages of analytically calculating response derivatives compared to finite difference methods. Continuing developments to implement these procedures into an enhanced version of SPAR are also discussed.

Introduction

General-purpose finite-element structural analysis programs are widely used in a variety of design applications to calculate response quantities such as displacements, stresses, vibration frequencies, buckling loads, and mode shapes to assess the integrity of a proposed structure. [1,2,3] Until recently the dominant objective of these finite element structural analysis programs has been limited to the accurate prediction of such structural behavior for a given design: the designer still relies on manual evaluation of design modifications based on engineering judgment. However, in the structural design context, the objective of structural analysis should be broadened to include calculating estimates of the sensitivity of structural response to changes in the design to provide a formal approach to guide design modifications. This recommendation is based on the fact that structural sensitivities can be calculated in a straight-forward manner using finite difference methods or by using a more accurate and efficient approach involving analytical gradients or derivatives. [4-9] The established analytical methods and user needs indicate the feasibility and desirability of including the additional capability to calculate analytical structural derivatives as a standard option in general-purpose, finite-element analysis programs. This sensitivity information has several important uses such as: (1) a structural designer-computer interaction to determine sensitivity of a design to changes in structural properties; (2) adjusting properties of a finite element model to obtain correlation between analysis and test; (3) approximating structural analysis using first order Taylor series expansion; and (4) calculating constraint gradients needed in optimization algorithms. Analytical derivatives have been incorporated into structural optimization systems but such systems often lack generality and/or have limited analysis capability. [10] Self-contained additions have been made to the computer code of the large-scale, general-purpose finite element analysis system SPAR[3], to include sensitivity calculations for static stress and flutter analyses as described in Ref. 11. In Ref. 11, a linear relationship is specified between design variables and the parameters describing the structural model. The present paper describes an alternate approach for implementing techniques which analytically calculate structural response derivatives into SPAR. This approach uses innovative, specially-constructed sequences of input instructions to perform the desired calculations with existing analytical procedures in SPAR. The unique features of SPAR which facilitate this approach are discussed along with the generalized method used to allow a nonlinear relation between design variables and structural model parameters. The equations for the structural response derivatives and methods of solution are well documented in the literature but are summarized herein as a basis for describing their implementation into SPAR. [4-9] Limited sample calculations are presented for static displacements and stresses to indicate the advantages of analytically calculating derivatives compared to finite difference methods. Also, on-going and planned developments for analytical derivative calculation are discussed emphasizing the anticipated benefits of implementing these procedures in an enhanced version of SPAR.

SPAR Characteristics

The capability to calculate analytical derivatives is implemented primarily by making innovative use of existing features of SPAR with required supplemental computations performed by user written subroutines in separate programs. The SPAR system is used in a "black-box" mode in that required computations are invoked with standard SPAR input commands, hence, no internal coding changes are needed.

Modular Organization

The organization of the SPAR analysis system is shown schematically in figure 1. This system is composed of a group of individual modules called "processors" which are used in a logical sequence to perform a desired analysis. Each processor is designed for a limited, yet distinct and complete function and is referred to by a unique name as shown at the top of figure 1. The functions of each of the SPAR processors are given in table 1. For conventional analysis, the processors TAB through KG read user input, form element matrices, and assemble element matrices into system matrices which represent the overall stiffness and mass of the structure. Equations for static stress analysis are solved using

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processors INV through PSF. The EIG processor is used for eigensolutions and AUS and DCU provide general input and matrix arithmetic capability.

A high degree of modularity is provided in SPAR since all data communication between processors is handled through a data base complex which contains the information generated by or used by other processors during a computer run. A set of data handling utilities transfers data between the processors in central memory of the computer and the data base complex on auxiliary storage. [12] These utilities may be used in auxiliary programs to communicate data to the SPAR processors.

Flexibility of User Input

User input to the system is shown in the upper left portion of figure 1 as executive control commands ([XQT followed by the name of the processor to be executed) and related processor input data located sequentially on the input file. A high degree of user flexibility is provided since processors can be called for execution in any logical sequence. The basis for the calculation of structural response derivatives described herein is the development of input sequences to form and solve the structural sensitivity equations.

Coupling SPAR With Auxiliary Programs

The modular organization and flexible user input facilitate the use of SPAR analytical procedures for "nonstandard" purposes. Such usage, as in the implementation of structural sensitivity calculations, often requires coupling of SPAR to auxiliary programs as illustrated in figure 2. Sequences of SPAR input commands for invoking processors required to perform desired calculations can be prepared manually or generated by auxiliary programs. Calculations outside the capabilities of the SPAR processors can be performed in external programs which use the SPAR data handling utilities to communicate with the data base complex. Computational sequences involving combined and/or repeated executions of both SPAR and external programs can be specified using the computer operating system control language as shown at the top of figure 2. Specific programs and procedures used to implement the structural response sensitivity calculations are presented in subsequent sections of this paper.

Structural Modification

The first step in calculating response derivatives for an existing structure is to identify what modifications to the finite element model are of interest. These modifications are defined in terms of design variables such as the area or thickness of structural members. The first part of this section describes the definition of design variables and their relationship to the structural model which must be established. The second part of this section discusses a procedure to produce derivatives of the overall stiffness and mass matrices with respect to the design variables needed in the calculation of structural response derivatives.

Design Variables Definition

The user specifies structural modifications using design variables, v_i, which in turn must be related to parameters which define the structural model. Typical parameters which define the structural model are section properties or mass properties of finite elements in the structure. Herein, a structural definition parameter, P_j, will be defined to be any parameter which has a linear relationship to the stiffness and mass matrices of individual finite elements in the structural model. Sometimes design variables, v_i, are defined to be identical or in a one-to-one relationship with the structural definition parameters, P_j. For example, a design variable may be the area of an axial member and the areas of all axial finite elements in the structure are defined to be equal to that design variable. The relationship between v_i and P_j can be more complicated, for example, when design variable linking is used or bending elements are considered.

A method for handling a general, nonlinear relationship between the structural design variables and structural definition parameters is presented in Refs. 13 and 14. This generalized method of design variable definition requires only that derivatives of P_j with respect to v_i exist since they are used in the calculation of the stiffness and mass matrix derivatives. An example of the nonlinearity occurs if the moment of inertia per unit width of a plate, $I_1 = t^3/12$, is a structural parameter and if the plate thickness, t, is used as a design variable, the derivative $\partial P_j/\partial v_i$ is given by $t^2/4$. In another case where the thickness of a membrane element, t, is the structural parameter and a reciprocal design variable, $1/t$ is used, the derivative, $\partial P_j/\partial v_i$ is given by $-t^2$. Also, the derivative, $\partial P_j/\partial v_i$ includes design variable linking functions when the coefficients of such functions are used as design variables.

Stiffness and Mass Matrix Derivatives

The derivatives of the stiffness, [K], and mass, [M], matrices for the structural model with respect to the design variables, v_i, are needed in the calculation of structural response derivatives. These derivatives are expressed by chain differentiation as

$$\frac{\partial [K]}{\partial v_i} = \sum_{j=1}^{NP} \frac{\partial [K]}{\partial P_j} \frac{\partial P_j}{\partial v_i} \qquad (1)$$

and

$$\frac{\partial [M]}{\partial v_i} = \sum_{j=1}^{NP} \frac{\partial [M]}{\partial P_j} \frac{\partial P_j}{\partial v_i} \qquad (2)$$

where NP is the number of structural definition parameters. [13] Since [K] and [M] are linear functions of P_j, the matrix portions of the products in equations (1) and (2), $\partial [K]/\partial P_j$ and $\partial [M]/\partial P_j$, are constant.

These terms can be calculated and saved for subsequent use in an iterative optimization procedure. The scalars $\partial P_j/\partial v_i$ are not necessarily constant and must be calculated for particular values of v_i. Formulation by the user of equations 1 and 2 for a particular application permits consideration of a wide variety of structural design problems.

An illustrative example is a beam whose structural definition parameters are the cross-sectional area A and moments of inertia I_1, I_2, and J_0. The stiffness matrix derivative is given by the equation shown at the top of figure 3. The structural definition parameters are also given for a channel section as nonlinear functions of the cross-sectional dimensions. The derivatives of the structural parameters at a particular value of the design variable are obtained by another chain differentiation expression as shown for I_1 at the bottom of figure 3. In general any of the individual dimensions may be specified as a design variable.

Implementation Procedure

The implementation procedure used to calculate stiffness and mass matrix derivatives is identical and is shown schematically in figure 4 for the stiffness matrix. Special sequences of SPAR input commands are executed to form the desired derivatives as indicated at the bottom of the figure. To automate this procedure, auxiliary computer programs are used to generate the SPAR input sequences. This input can become lengthy for large numbers of design variables, v_i, and related structural definition parameters, P_j.

The initial step in this process is to form the terms, $\partial[K]/\partial P_j$ and $\partial[M]/\partial P_j$, as shown at the left of figure 4. These terms are formed by generating stiffness and mass matrices for unit values of the various structural parameters, P_j. An example sequence of required SPAR input is shown in table 2 for the stiffness parameter, I_1, of some beam elements. Repeated sets of input, all similar to the example given in table 2, are required; each set corresponds to a structural parameter for a group of elements that undergo the same change in value of structural parameter when a design variable is changed.

In TAB (figure 1) only the unit values of structural parameters need be input since tables of joint locations, material properties, etc. can be used from the data generated during the initial definition of the model. The option to specify beam properties in terms of stiffness parameters is used for setting I_1 to unity and all other stiffness parameters to zero as shown in table 2. The group of elements corresponding to a particular structural parameter are input to the ELD processor. Processors E and EKS generate the unit elemental stiffness matrices, and processors K, or M assemble the desired derivative matrices $\partial[K]/\partial P_j$ and $\partial[M]/\partial P_j$. These matrices are saved in the SPAR data base complex for use in subsequent calculations.

The next step, shown at the right of figure 4, is to evaluate the scalar terms $\partial P_j/\partial v_i$ and to combine them with $\partial[K]/\partial P_j$ and $\partial[M]/\partial P_j$ as shown in equations 1 and 2 to form the stiffness and mass matrix derivatives. The terms $\partial P_j/\partial v_i$ are dependent on the design variable definitions used for a particular application. In general, these terms represent the evaluation of nonlinear expressions at a particular value of the design variable, v_i, and would require reevaluation during each iteration of a design process. These nonlinear expressions are evaluated in user supplied subroutines in an auxiliary program and the particular numerical values are included in input commands to the AUS processor in SPAR where the products and summations indicated in equations 1 and 2 are performed. An example sequence of SPAR input commands to calculate $\partial[K]/\partial v_i$ and $\partial[M]/\partial v_i$ for a set of beam elements is given in table 3. These derivatives are stored in the SPAR data base complex under unique names for retrieval and are used in subsequent calculations of structural response derivatives.

This procedure for generating the mass and stiffness matrix derivatives is satisfactory for cases in which many elements are governed by the same design variable and a few matrices $\partial[K]/\partial P_j$ are formed for many elements at a time and are multiplied by a single factor $\partial P_j/\partial v_i$. However, this procedure is inefficient when, in the most extreme case, $\partial[K]/\partial P_j$ must be formed for all elements one at a time and each subsequently multiplied by a different value of $\partial P_j/\partial v_i$. Capabilities of a recent, enhanced version of SPAR that offer an alternate, improved method for generating the stiffness and mass matrix derivatives which overcomes this difficulty are discussed in the section entitled "Continuing Developments."

Static Analysis

Methodology for calculating derivatives of displacements and stresses from a static analysis are presented in this section. The equations are obtained by taking analytical derivatives, with respect to specified design variables, of the governing structural response equations. [4-9]

Derivative Equations

The matrix equation for static displacements, $\{x\}$, is

$$[K]\{x\} = \{F\} \qquad (3)$$

where $[K]$ is the overall stiffness matrix and $\{F\}$ is a vector of applied loads. Equation 3 is differentiated with respect to a design variable v_i giving

$$[K]\frac{\partial\{x\}}{\partial v_i} = -\frac{\partial[K]}{\partial v_i}\{x\} + \frac{\partial\{F\}}{\partial v_i} \qquad (4)$$

The applied load $\{F\}$ is usually independent of the design variables and $\partial\{F\}/\partial v_i$ is taken to be zero herein. The remaining quantity on the right hand side of equation 4 is obtained by multiplying $\partial[K]/\partial v_i$ from equation 1 by the displacement vector $\{x\}$. Equation 4 can be solved by the same solution algorithm used for

equation 3, taking advantage of the fact that [K] is available in factored form from the solution of equation 3. The procedure described above generates derivatives for all displacement components but requires the solution of equation 4 for each design variable.

A procedure with increased efficiency has been implemented for cases where derivatives of a selected subset of displacements are required. [15] Equation 4 is rewritten as

$$\frac{\partial\{x\}}{\partial v_i} = [K]^{-1}\left\{-\frac{\partial[K]}{\partial v_i}\{x\} + \frac{\partial\{F\}}{\partial v_i}\right\} \qquad (5)$$

and advantage is taken of the fact that selected rows or columns of $[K]^{-1}$ are formed by calculating displacements, $\{q\}$, due to unit loads, $\{U\}$, applied at the selected degrees of freedom. A significant computational saving results when the subset of displacement derivatives is small compared to the number of design variables, since the number of required solutions to the equation $[K]\{q\} = \{U\}$ is equal to the number of displacements of interest. The desired derivatives are then obtained from

$$\frac{\partial\{x\}}{\partial v_i} = q^T\left\{-\frac{\partial[K]}{\partial v_i}\{x\} + \frac{\partial\{F\}}{\partial v_i}\right\} \qquad (6)$$

Element stresses $\{\sigma\}$ are related to the joint displacements by the equation

$$\{\sigma\} = [S]\{x\} \qquad (7)$$

and the general expression for stress derivatives is

$$\frac{\partial\{\sigma\}}{\partial v_i} = [S]\frac{\partial\{x\}}{\partial v_i} + \frac{\partial[S]}{\partial v_i}\{x\} \qquad (8)$$

For membrane elements, the matrices of the stress-displacement relationships, [S], are independent of the design variables and $\partial[S]/\partial v_i$ is zero. Therefore, stress derivatives can be calculated by simply repeating the stress calculation procedure, equation 7, with the displacement derivatives $\partial\{x\}/\partial v_i$ replacing the displacements $\{x\}$.

For bending elements, the stress-displacement relationships are dependent on the element cross section geometry and this dependence must be included in the stress derivative calculations. As an example, the stresses for a beam are given by

$$\sigma_p = \frac{F_3}{A} + \frac{M_1}{I_1}y_{p2} - \frac{M_2}{I_2}y_{p1} \qquad (9)$$

where p refers to a specified point of the beam cross section. F_3 is the longitudinal force acting at the centroid of the cross sectional area, A; and M_1, M_2, I_1, and I_2 are applied moments and area moments of inertia about

the principal axes of the beam cross section. The distance of the specified point from the principal axes are given by y_{p2} and y_{p1}. Differentiating equation 9 with respect to a design variable, v_i, gives

$$\frac{\partial\sigma_p}{\partial v_i} = \left\{F_3\left[-\frac{\partial A}{\partial v_i}\left(\frac{1}{A^2}\right)\right] + \frac{\partial F_3}{\partial v_i}\left(\frac{1}{A}\right)\right\} +$$
$$\left\{M_1\left[\frac{I_1\frac{\partial y_{p2}}{\partial v_i} - y_{p2}\frac{\partial I_1}{\partial v_i}}{I_1^2}\right] + \frac{\partial M_1}{\partial v_i}\left(\frac{y_{p2}}{I_1}\right)\right\} -$$
$$\left\{M_2\left[\frac{I_2\frac{\partial y_{p1}}{\partial v_i} - y_{p1}\frac{\partial I_2}{\partial v_i}}{I_2^2}\right] + \frac{\partial M_2}{\partial v_i}\left(\frac{y_{p1}}{I_2}\right)\right\} \qquad (10)$$

The terms in the square brackets, [ ], can be calculated using geometric properties of the beam cross section and are multiplied by F_3, M_1, and M_2 from a static analysis of the original structure. Evaluation of the force and moment derivatives $\partial F_3/\partial v_i$, $\partial M_1/\partial v_i$, and $\partial M_2/\partial v_i$ is performed by repeating the procedure for calculating element forces and moments with the displacement derivatives $\partial\{x\}/\partial v_i$ replacing the displacements $\{x\}$ and these derivatives are subsequently multiplied by the geometric terms as shown in equation 10.

Implementation Procedure

A static stress analysis of the original structural model is performed as the first step in calculating displacement and stress gradients. The SPAR processors TAB through PSF are used for these calculations. The resulting displacement vectors for all applied load cases are multiplied, using processor AUS, by all previously generated stiffness matrix derivatives to form the set of pseudo-load vectors shown on the right side of equation 4. These pseudo-load vectors are given names corresponding to a set of applied forces with unique load set numbers. Two options exist for solving for the displacement gradients. In option I, for use when the number of desired displacement gradient components is larger than the number of design variables, the SSOL processor is used to solve equation 4 directly. In option II, for use when the number of design variables is larger than the number of desired displacement gradient components, the SSOL processor is used to calculate displacements $\{q\}$ due to unit loads at the selected degrees of freedom. The displacement derivatives shown in equation 6 are then formed in AUS by multiplying $\{q\}^T$ by the pseudo-load vectors.

Required stress derivative information for membrane elements is generated using the GSF processor with the displacement gradients specified as input in place of the actual displacements as indicated in equation 8. For beam elements, an auxiliary program which uses SPAR data directly as illustrated at the right of figure 2 is used to evaluate the stress derivatives. This program retrieves previously generated forces and moments and their derivatives

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from the SPAR data base complex and combines them with values produced by subroutines which evaluate the geometric terms given in equation 10. The SPAR data handling utilities are used in the auxiliary program to store the resulting beam stress derivatives in the SPAR data base complex.

Vibration and Buckling Analysis

Methodology for calculating derivatives of eigenvalues and eigenvectors is presented in this section. The equations for vibration analysis are presented and modifications required for buckling analysis are indicated.

Derivative Equations

The matrix equation for vibration analysis is

$$[K - \omega^2 M]\{\phi\} = 0 \tag{11}$$

where ω^2 and $\{\phi\}$ are sets of eigenvalues and eigenvectors corresponding to the natural frequencies and mode shapes of the structural model. The normalization of the modes with respect to the mass matrix is given by

$$\{\phi\}^T [M]\{\phi\} = [I] \tag{12}$$

Differentiating equation 11 with respect to the design variable v_i gives

$$[K - \omega^2 M]\frac{\partial\{\phi\}}{\partial v_i} = \frac{\partial\omega^2}{\partial v_i}[M]\{\phi\} - \frac{\partial[K]}{\partial v_i}\{\phi\} + \omega^2 \frac{\partial[M]}{\partial v_i}\{\phi\} \tag{13}$$

and the corresponding differentiation of equation 12 gives

$$2\{\phi\}^T [M]\frac{\partial\{\phi\}}{\partial v_i} = -\{\phi\}^T \frac{\partial[M]}{\partial v_i}\{\phi\} \tag{14}$$

If only derivatives of the frequency are required, equation 13 is premultiplied by $\{\phi\}^T$, and when it is recognized that $\{\phi\}^T[K-\omega^2 M]$ equals 0.0 and $\{\phi\}^T \partial\omega^2/\partial v_i [M]\{\phi\}$ equals $\partial\omega^2/\partial v_i$ the equation becomes

$$\frac{\partial\omega^2}{\partial v_i} = \{\phi\}^T \frac{\partial[K]}{\partial v_i}\{\phi\} - \omega^2 \{\phi\}^T \frac{\partial[M]}{\partial v_i}\{\phi\} \tag{15}$$

Several methods exist to solve for the derivatives of the natural vibration modes $\partial\phi/\partial v_i$.[4,7,8] The method presented in Ref. 8 combines equations 13 and 14 in the form

$$\begin{bmatrix} K - \omega^2 M & -M\phi \\ \\ -\phi^T M & 0 \end{bmatrix} \begin{Bmatrix} \frac{\partial\phi}{\partial v_i} \\ \\ \frac{\partial\omega^2}{\partial v_i} \end{Bmatrix} = \begin{Bmatrix} -\frac{\partial K}{\partial v_i}\phi + \omega^2 \frac{\partial M}{\partial v_i}\phi \\ \\ \frac{1}{2}\phi^T \frac{\partial M}{\partial v_i}\phi \end{Bmatrix} \tag{16}$$

The mode and frequency derivatives are both obtained by the solution of equation 16. This method requires the formation of the matrix on the left side of equation 16 which has an additional row and column and hence a different topology than the original global stiffness and mass matrices. Since the SPAR system handles the global matrices in a special sparse format, the addition of a row and column would require the judicious use of fictitious elements with properties selected to give the desired terms in the equations.

An approximate method for calculating the eigenvector derivatives, described in Ref. 4, is available in SPAR as part of the SM processor which is used to modify the properties of structural models to correlate analytical and test results as discussed in Ref. 16. In this method, the eigenvector derivatives are approximated as a linear combination of a subset of eigenvectors of the original structure. The derivative of the jth eigenvector is expressed as

$$\frac{\partial\{\phi_j\}}{\partial v_i} = \sum_{k=1}^{n} a_{ijk}\{\phi_k\} \tag{17}$$

The contribution of the kth eigenvector is determined by substituting the expression for $\partial\{\phi_j\}/\partial v_i$ into equation 13 and premultiplying by $\{\phi_k\}^T$ to give

$$a_{ijk} = \{\phi_k\}^T \left[\frac{\partial[K]}{\partial v_i} - \omega_j^2 \frac{\partial[M]}{\partial v_i}\right]\{\phi_j\}/(\omega_j^2 - \omega_k^2) \text{ for } k \neq j \tag{18}$$

and by a similar substitution into equation 14 to give

$$a_{ijk} = -\frac{1}{2}\{\phi_j\}^T \frac{\partial[M]}{\partial v_i}\{\phi_j\} \text{ for } k = j \tag{19}$$

The disadvantage of this method is the uncertainty of how many eigenvectors are required to give an adequate representation of the eigenvector derivatives.

The method for calculating eigenvector derivatives that is described in Ref. 7 has been implemented using sequences of SPAR input commands for performing the required calculations. In this method, the right side of equation 13 is evaluated and treated as a pseudo-load vector. A direct solution of equation 13 is not possible since $[K - \omega^2 M]$ is a singular matrix. As discussed in Ref. 7, if the value of one component of $\partial\{\phi\}/\partial v_i$ is fixed, the remaining components can then be calculated yielding a solution $\{P\}$ to equation 13. Since the eigenvector $\{\phi\}$ is the homogeneous solution of equation 13, the following expression is also a solution

$$\frac{\partial\{\phi_j\}}{\partial v_i} = \{P\} + C\{\phi\} \tag{20}$$

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The value of the multiplier, C, for which equation 20 also satisfies equation 14 is given by

$$C = -\{\phi_j\}^T [M]\{P\} - \frac{1}{2}\{\phi_j\}^T \frac{\partial[M]}{\partial v_i}\{\phi_j\} \qquad (21)$$

The equations for buckling response derivatives are similar, with the geometric stiffness matrix replacing the mass matrix.

Implementation Procedure

The procedure for calculating vibration response derivatives, described in Ref. 7, was implemented by manually preparing the required SPAR input commands to demonstrate the method. Computer programs for automatic generation of this input similar to those for static analysis derivatives have not been developed. Use is made of the static analysis procedures, discussed in previous sections, to generate the stiffness and mass matrices, [K] and [M], and their derivatives, $\partial[K]/\partial v_i$ and $\partial[M]/\partial v_i$. The EIG processor in SPAR is used to calculate eigenvalues, ω^2, and eigenvectors, $\{\phi\}$, for the original structure. These terms are combined, using the AUS processor, to form the frequency derivatives shown in equation 15. The right side of equation 13 and the matrix $[K-\omega^2 M]$ in sparse format is also formed using the AUS processor. A solution, $\{P\}$, to equation 13 is calculated using the INV and SSOL processors with one component of $\{P\}$ set to zero using a fictitious boundary constraint. The component corresponding to the largest component of $\{\phi_j\}$ is selected following the recommendation of Ref. 7 based on numerical accuracy considerations. Finally, the required vibration mode derivatives are calculated using the AUS processor to perform the matrix algebra for evaluating equations 21 and 20, respectively. The entire procedure must be repeated for each eigenvector derivative.

In the calculation of buckling response derivatives, similar steps are performed with the KG processor used to calculate the differential stiffness matrix $[K_g]$. The stress derivatives, obtained from a previously described procedure are input to the KG processor to form the $\partial[K_g]/\partial v_i$ matrix that is required in the buckling derivative equations.

Sample Calculations

The analytical gradient capabilities that are implemented in SPAR were used to calculate derivatives for comparison with similar results from finite difference methods. The use of analytical gradients provides improved computational efficiency and eliminates the uncertainty of numerical accuracy associated with selection of a finite difference perturbation to give desired convergence.

Model Description

The finite element structural model of a stiffened cylinder containing a rectangular

cutout, as shown in figure 5, is used to illustrate the calculation of static displacement and stress derivatives. Results are calculated for three different levels of model refinement to assess the effect of number of degrees-of-freedom on the required computational time. The 337 degree-of-freedom cylinder model, referred to as Model 1, is stiffened by five rings equally spaced along its length and sixteen stringers equally spaced around its circumference as illustrated in the figure. The rings and stringers are modeled by beam and rod elements respectively. Rectangular panels between rings and stringers are modeled by membrane plate elements. The cutout is located between the second and fourth rings and encompasses a 90° segment of the cylinder's circumference. The translational degrees of freedom are constrained at all points on one end of the cylinder. The level of modeling is refined for Models 2 and 3 by dividing the skin panels with additional rings and stringers to give the total number shown on figure 5. Two sets of concentrated forces are applied at the unsupported end of the cylinder as shown in the figure.

Three design variables are considered: (1) an area which governs all stringer rod elements; (2) a thickness for all membrane plates; (3) a scale factor on the dimensions of a specified channel cross-section used for all beams in the rings. Hence, each design variable affects many elements in the same manner and results in a simplified calculation of the derivatives, $\partial[K]/\partial P_j$.

Numerical Results

The computation times required for each of the major steps in calculating stress and displacement derivatives for the two load cases are shown in figure 6 for the three levels of model refinement. Performing a static analysis with processors INV and SSOL is the step that requires the largest amount of time. This time is used primarily by INV to factor the stiffness matrix. However, only a single static analysis is needed rather than multiple analyses for perturbations of all the design variables one at a time as required by the finite difference method. The times shown in figure 6 are CPU (central processing unit) seconds on a CDC Cyber 173 computer when three design variables and two loading cases are considered.

The relative computer time (finite difference method divided by analytical gradient method) is shown in figure 7. The results for the case of three design variables are extrapolated to show the effect of a larger number of design variables. This extrapolation is based on appropriate scaling of the times shown in figure 6 to reflect the number of times each step in the process must be repeated for additional design variables. The relative times and hence benefits increase with the number of degrees of freedom. Benefits of the analytical gradient method also increase with an increase in the number of design variables, especially for the most refined model. These results were obtained using the procedure which calculates displacement derivatives for all degrees of freedom and stress derivatives for all elements.

Use In Optimization Procedures

The methods used to form and solve the analytical derivative equations are organized for efficient use in structural optimization systems. [17] The major steps in an optimization procedure which includes displacement and stress constraints are shown schematically in figure 8. All steps required to compute response sensitivities or gradients are shown in the left and center of the figure. The optimization loop to update the design variables is indicated at the right of the figure. The steps or operations shown in the solid boxes are performed by executing SPAR with previously described input sequences. Auxiliary programs are used to perform the operations in the dashed boxes including calculation of $\partial P_j/\partial v_i$ using problem dependent subroutines and corresponding subroutines needed to calculate the beam stress derivatives. The formation of the linear derivatives $\partial[K]/\partial P_j$ is not included in the optimization loop since it is performed as a preprocessor operation and need not be repeated.

Continuing Developments

A recently enhanced version of SPAR, called EAL (Engineering Analysis Language)[18] permits improvements in the implementation of the structural response sensitivity calculations. The existing procedures for gradient calculation which have been discussed herein are being implemented in EAL. Current efforts and plans for this work are discussed in this section.

The enhancements in EAL which make the improvements in gradient calculation capability possible are shown in table 4. Facilities are provided for addition of new processors to allow all computational procedures which were previously handled in auxiliary programs to be integrated into the EAL system with all data communication through a common data base complex. The addition of new processors in the EAL system is similar to the approach taken in Ref. 11.

Sequences of input instructions can be stored as data sets in the EAL data base complex and retrieved by name for execution. This feature provides a more unified implementation of the gradient capability than defining the input sequences for various steps in the gradient calculations as external files to be handled by the computer operating system. Also, looping and branching statements available in EAL permit repetitive execution of a sequence of such steps for calculation of vibration mode derivatives for each natural frequency or for steps in an iterative optimization procedure.

A processor called LSK is available in EAL to assemble a partial system stiffness matrix which contains contributions from elemental stiffness matrices for a user-selected set of elements. Use of this processor simplifies the calculation of the stiffness matrix derivatives $\partial[K]/\partial v_i$. The terms $\partial[K]/\partial P_j$ are formed for all elements at once by generating individual stiffness matrices with unit values of the structural parameters. The LSK processor can be used subsequently to scale and assemble these matrices to produce the stiffness matrix derivatives $\partial[K]/\partial v_i$ as shown

in equation 1. A table is input to LSK to specify which subset of element matrices are to be included during an execution and another table is input to specify a multiplying factor to be applied to the stiffness matrix of each element as it is being assembled. The first table can be used to simplify the general specification of design variable linking with the values in the second table corresponding to the $\partial P_j/\partial v_i$ terms in equation 1. Another approach which has proved to be effective is to use the LSK processor to retrieve element stiffness matrices needed to calculate the stiffness matrix derivatives using finite difference methods. The finite difference calculation of $\partial[K]/\partial v_i$ allows any structural model definition input parameter to be used as a design variable.

Implementation of procedures using EAL to produce gradients of structural response with respect to changes in joint locations of the model has been investigated. Finite difference methods are used to calculate the stiffness and mass matrix derivatives, $\partial[K]/\partial v_i$ and $\partial[M]/\partial v_i$, and the remainder of the steps are the same as those where member sizes are designated as design variables. The finite difference perturbation of joint locations is expedited by use of parametric definition of model geometry which is available in EAL. Variables, whose values are set at execution time, are used in the input instructions along with the joint mesh generation capabilities of EAL to define the parametric models. For example, all joint locations in a wing structure could be changed by changing the variable defining sweep angle.

Continued utilization of enhancements available in EAL is leading to an integrated, large-scale capability to calculate structural response sensitivities for a variety of design variables.

Concluding Remarks

The methodology used to implement structural sensitivity calculations into a major, general-purpose finite element analysis system (SPAR) was described. This implementation includes a generalized method for specifying element cross-sectional dimensions as design variables that can be used in analytically calculating derivatives of output quantities from static stress, vibration, and buckling analyses for both membrane and bending elements. Limited sample results for static displacements and stresses were presented to indicate the advantages of analytically calculating response derivatives compared to finite difference methods. Continuing developments to implement these procedures into an enhanced version of SPAR were also discussed.

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Table 1 SPAR processors

Name	Function
TAB	Creates data sets containing tables of joint locations, section properties, material constants, etc.
ELD	Defines the finite elements making up the model.
E	Generates sets of information for each element including connected joint numbers, geometrical data, material and section property data.
EKS	Adds the stiffness and stress matrices for each element to the set of information produced by the E processor.
TOPO	Analyzes element interconnection topology and creates data sets used to assemble and factor the system mass and stiffness matrices.
K	Assembles the unconstrained system stiffness matrix in a sparse format.
M	Assembles the unconstrained system mass matrix in a sparse format.
KG	Assembles the unconstrained system initial-stress (geometric) stiffness matrix in a sparse format.
INV	Factors the assembled system matrices.
EQNF	Computes equivalent joint loading associated with thermal, dislocational, and pressure loading.
SSOL	Computes displacements and reactions due to loading applied at the joints.
GSF	Generates element stresses and internal loads.
PSF	Prints the information generated by the GSF processor.
EIG	Solves linear vibration and bifurcation buckling eigenproblems.
SM	Modifies structural model definition parameters to cause vibration modes and frequencies to approach target values specified by the user.
AUS	Performs an array of matrix arithmetic functions and is used in construction, editing, and modification of data sets.
DCU	Performs an array of data management functions including display of table of contents, data transfer between libraries, changing data set names, printing data sets, and transferring data between libraries and sequential files.

Table 2 SPAR input sequence to form $\partial[K]/\partial P_j$ for beam elements using a unit moment of inertia

```
[XQT TAB
UPDATE=1
BC
2 1.0
BA
DSY 2 1.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0          Define unit value for beam moment of
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0            inertia, I₁.
DSY 3 0.0 0.0 1.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
DSY 4 0.0 0.0 0.0 0.0 1.0 0.0 0.0 0.0 0.0 0.0 0.0        Define unit values of other beam
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0            definition parameters.
DSY 5 0.0 0.0 0.0 0.0 0.0 1.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

[XQT ELD
E21
NSECT= 2
NREF=1
1 2 2 16 2 16 $
49 50 2 16 2 16$
33 34$
34 35$
39 40$
40 41$
41 42$
42 43$                                                Form stiffness matrix derivative ∂[K]/∂I₁
43 44$                                                for the beam elements in the rings of
44 45$                                                model 1 of the cylinder shown in figure 5.
45 46$
46 47$
47 48$
48 33$
[XQT E
[XQT EKS
[XQT TOPO
RESET MAXSUB=2500
[XQT K
COPY 1,2 K SPAR 25 0                                   Store ∂[K]/∂I₁ in SPAR data complex for use
CHANGE 2,K SPAR 25 0,DKIX SPAR 25 2                    in subsequent calculations.
```

9

Table 3 Description of SPAR input sequence to form $\partial[K]/\partial v_i$ and $\partial[M]/\partial v_i$ for the beam elements designated in Table 2

The following equations are evaluated:

$$\frac{\partial[K]}{\partial v_i} = \frac{\partial[K]}{\partial A}\frac{\partial A}{\partial v_i} + \frac{\partial[K]}{\partial I_1}\frac{\partial I_1}{\partial v_i} + \frac{\partial[K]}{\partial I_2}\frac{\partial I_2}{\partial v_i} + \frac{\partial[K]}{\partial J_0}\frac{\partial J_0}{\partial v_i}$$

$$\frac{\partial[M]}{\partial v_i} = \frac{\partial[M]}{\partial A}\frac{\partial A}{\partial v_i} + \frac{\partial[M]}{\partial I_1}\frac{\partial I_1}{\partial v_i} + \frac{\partial[M]}{\partial I_2}\frac{\partial I_2}{\partial v_i} + \frac{\partial[M]}{\partial J_0}\frac{\partial J_0}{\partial v_i}$$

using

```
[XQT AUS
  DEFINE A1=DKDA SPAR 25 2
  DEFINE A2=DKIX SPAR 25 2  }    Designate A2 as ∂K/∂I₁
  DEFINE A3=DKIY SPAR 25 2
  DEFINE A4=DKJO SPAR 25 2
  DEFINE B1=DMDA DIAG 0 2
  DEFINE B2=DMIX DIAG 0 2
  DEFINE B3=DMIY DIAG 0 2       ∂I₁
  DEFINE B4=DMJO DIAG 0 2      ─── ∂vᵢ
   S1=SUM( .10000+01 A1, (.31119+03)A2)
   S2=SUM( .23383+02 A3, .26667+01 A4)
  DKDV SPAR 25 2=SUM(S1,S2)
   T1=SUM( .10000+01 B1, .31119+03 B2)
   T2=SUM( .23383+02 B3, .26667+01 B4)
  DMDV DIAG 0 2=SUM(T1,T2)
```

where the matrix terms $\partial[K]/\partial P_j$ are defined as data previously stored in the data complex and the scalars in the SUM statements are the result of evaluating $\partial P_j/\partial v_i$ for a particular design variable as illustrated for P_j being I_1.

Table 4 Enhancements in EAL which provide improvements in structural response derivative calculations

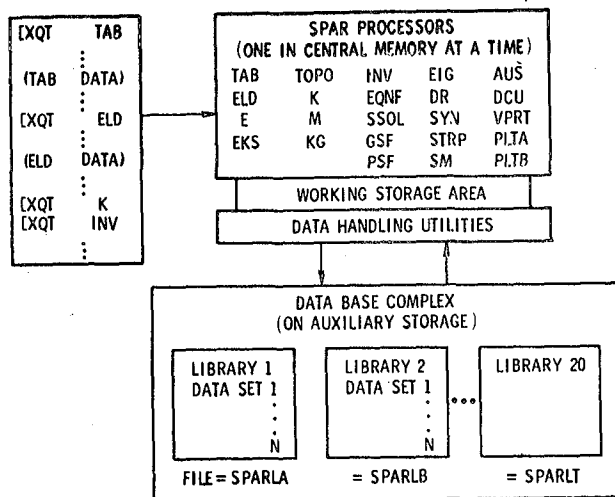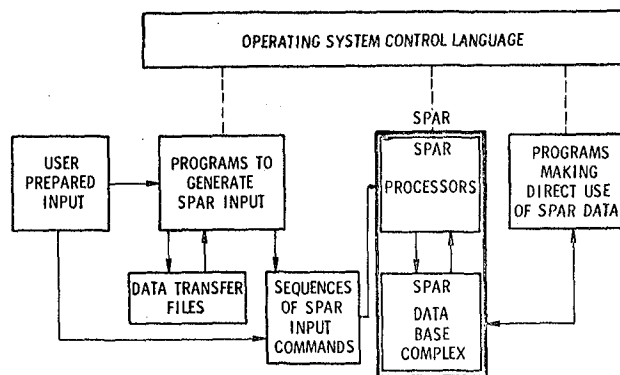| Enhancements	Improvements
Facilities for adding new processors	Integrated system with all data communicated internally through data base
Input sequences stored in and called from data base	Replace use of computer operating system control language
Looping and branching commands in input sequences	
LSK processor to assemble stiffness matrices of selected elements	Simplify existing method of calculating $\partial[K]/\partial v_i$
	Finite-difference calculation of $\partial[K]/\partial v_i$
Input variables whose values are set at execution time	Parametric definition of model geometry

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Fig. 1 Organization of SPAR analysis system.

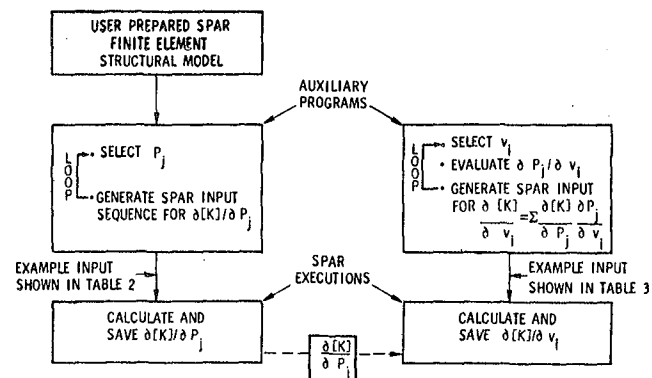Fig. 2 Coupling of SPAR and auxiliary programs for nonstandard calculations.

STIFFNESS MATRIX DERIVATIVE

$$\frac{\partial [K]}{\partial v} = \frac{\partial [K]}{\partial A}\frac{\partial A}{\partial v} + \frac{\partial [K]}{\partial I_1}\frac{\partial I_1}{\partial v} + \frac{\partial [K]}{\partial I_2}\frac{\partial I_2}{\partial v} + \frac{\partial [K]}{\partial J_0}\frac{\partial J_0}{\partial v}$$

STRUCTURAL DEFINITION PARAMETERS

$$A = (2B_1 + B_2)t$$

$$I_1 = \frac{B_1 (B_2 + 2t)^3}{12} - \frac{(B_1 - t) (B_2)^3}{12}$$

$$I_2 = \frac{2(t)B_1^3}{12} + 2tB_1\left(\frac{B_1}{2} - c\right)^2 + \frac{B_2 t^3}{12} + B_2 t \left(c - \frac{t}{2}\right)^2$$

$$J_0 = \frac{1}{3}(2B_1 + B_2) t^3$$

WHERE: $c = \left(B_1^2 t + \frac{B_2 t^2}{2}\right)\left(\frac{1}{A}\right)$

MOMENT OF INERTIA DERIVATIVE:
$$\frac{\partial I_1}{\partial v} = \frac{\partial I_1}{\partial B_1}\frac{\partial B_1}{\partial v} + \frac{\partial I_1}{\partial B_2}\frac{\partial B_2}{\partial v} + \frac{\partial I_1}{\partial t}\frac{\partial t}{\partial v}$$

Fig. 3 Stiffness matrix derivative expressions for channel beam cross-sectional changes.

Fig. 4 Implementation procedure used to calculate stiffness matrix derivatives.

MODEL	RINGS	STRINGERS	D.O.F.
1	5	16	337
2	9	32	1239
3	17	64	4723

Fig. 5 Example structural model of stiffened cylinder with cutout.

MODEL	DEGREES OF FREEDOM	TIME, sec				
		MODEL DEFINITION	FORM $\frac{\partial K}{\partial P}$, $\frac{\partial K}{\partial v}$	PERFORM STATIC ANALYSIS	CALCULATE DERIVATIVES	TOTAL
1	337	2	17	26	23	68
2	1239	7	66	204	90	367
3	4723	25	497	2483	430	3435

• TIME IN CPU SECONDS ON CDC CYBER 173 COMPUTER.
• THREE DESIGN VARIABLES USED.
• TWO LOADING CASES CONSIDERED.

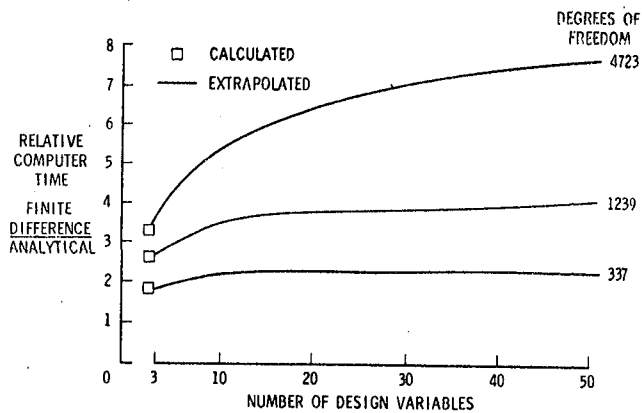Fig. 6 Computation time for steps required to calculate static displacement and stress derivatives.

Fig. 7 Relative computer time for calculation of static derivatives using finite difference and analytical gradient methods.
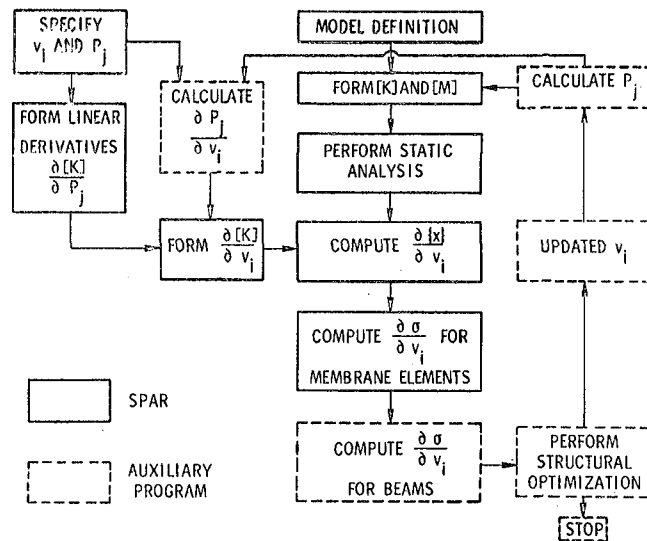

Fig. 8 Implementation of analytical structural response derivatives in a structural optimization procedure.

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15. Supplementary Notes

16. Abstract

The methodology used to implement structural sensitivity calculations into a major, general-purpose finite element analysis system (SPAR) is described. This implementation includes a generalized method for specifying element cross-sectional dimensions as design variables that can be used in analytically calculating derivatives of output quantities from static stress, vibration, and buckling analyses for both membrane and bending elements. Limited sample results for static displacements and stresses are presented to indicate the advantages of analytically calculating response derivatives compared to finite difference methods. Continuing developments to implement these procedures into an enhanced version of SPAR are also discussed.

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