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# Numerical Modeling of Three-Dimensional Confined Flows

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# **Numerical Modeling of Three-Dimensional Confined Flows**

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## 1. INTRODUCTION

Viscous flows have been calculated for a long time by separating the flow into a boundary layer and a potential flow region. The boundary layer then is calculated by using integral methods and the potential flow by using inviscid flow equations. This approach does not require large computers, and even can be carried out by use of hand calculations. This approach, however, has its limitations. For example, in calculating flow through nozzles, channels, and diffusers this approach is limited to the region near the entrance. As the boundary layer thickens, the flow cannot be meaningfully separated into a potential flow region and a boundary layer region. Further, this approach cannot describe the corner effects, such as the secondary flows, or adequately describe flows in which the body forces are nonuniform in the cross-stream plane (such as magnetohydrodynamic channel flows).

In the past two decades, with the advent of high speed computers, many numerical methods to compute velocity and temperature fields without separating the flow into a boundary layer and a potential flow region have been developed. In this paper, a new approach to calculate three-dimensional compressible viscous flows is presented. The approach is an extension of the author's previous work [1] to three-dimensional flows. The method, as presented in this paper, is for "parabolic" flows through rectangular ducts, and its possible extensions to other types of three-dimensional flows is under investigation. "Parabolic" or "boundary layer" flows are characterized by the existence of a predominant flow direction along which

downstream conditions have a negligible influence on the upstream conditions. This assumption allows use of a marching integration procedure. A lucid discussion of the parabolic flow assumption is given by Caretto et al [2], and Patankar and Spalding [3].

Methods developed so far to compute three-dimensional viscous flows are, among others, those given in [2-10]. These methods differ in their finite difference approximations, and whether the flow is assumed to be parabolic or not. From among these methods, those proposed for calculation of confined flows differ further in the procedure used to calculate the pressure field. These methods are discussed here briefly. Included in the discussion is the method of Harlow and Welsh [11]. Although this method is for two-dimensional confined transient viscous flows, it has some features in common with the methods developed later for computation of three-dimensional parabolic flows. The method of Chorin [4] calculates incompressible elliptic flows. Finite difference equations are obtained by using the leap-frog method for the time and convective terms, and Duffort-Frankel for the diffusion terms. The pressure field is calculated by coupling the incompressible continuity equation to the momentum equations via an artificial variable density and an artificial equation of state. Miller [5], on the basis of the work of Chorin, computed steady incompressible three-dimensional elliptic flows through rectangular ducts. In the method of Harlow and Welsh [11] the finite difference equations are obtained by using forward in time and center in space differencing. The pressure field is calculated from an elliptic equation obtained from the continuity equation and the divergence of the momentum equations. The method uses a "staggered" grid in which the velocities are defined at the cell boundaries and pressure at the cell centers. A similar type of staggered grid is used in the cross-stream plane, later, in [2,3,6,8] for the computation of three-dimensional flows. In these methods the cross-

stream velocities are specified at the cell boundaries and the axial velocity, along with all the other variables, at the cell centers. Methods [2,3,6,7] calculate three-dimensional confined parabolic flows. In these methods the pressure field in the cross-stream plane is calculated from an elliptic pressure equation obtained from the continuity equation and the cross-stream momentum equations. The axial, along the duct axis, pressure gradient is calculated to insure conservation of mass along the duct axis. In these methods Caretto et al. [2], and Patankar and Spalding [3] obtain finite difference equations by using the control volume approach; Emery et al. [6] use Dufort-Frankel for the axial velocity momentum equation and implicit differencing for the cross-stream velocity momentum equations; Roberts and Forrester [7] use one-sided upstream differencing in the flow direction and second order differencing in the other two directions. Partap and Spalding [8] extend the method of Patankar and Spalding to "partially parabolic" flows. Methods of Nash [9] and Wang [10] are for external (pressure field specified) three-dimensional parabolic flows; Nash uses forward explicit differencing, and Wang uses Crank-Nicolson type differencing scheme.

All the methods described in the preceding paragraph use Eulerian system. Flow field is computed by calculating the velocity components  $u_x$ ,  $u_y$ , and  $u_z$  along a set of spatial grid points fixed in advance. In contrast, in the present approach, the flow field is computed by calculating the streamwise (not axial) velocity,  $u$ , along a set of chosen streamlines. Streamlines are identified by a pair of indices,  $(i,j)$ , such that the coordinates of the streamline  $(i,j)$  in the cross-stream plane are  $Y_i$  and  $Z_j$ . Coordinates of the streamlines in the cross-stream plane are calculated as part of the flow computations. The dependent variables in the present approach are  $u_{i,j}$ , the streamwise velocities, and  $(Y_i, Z_j)$ , the coordinates

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of the streamlines. To calculate the streamwise velocities streamtubes are constructed around the individual streamlines such that the sum total of all the streamtubes fills the duct. One such streamtube, constructed around the streamline  $(i,j)$ , is shown in Fig. 1.1. Finite difference equations for  $u_{i,j}$  and  $h_{i,j}$ , velocity and enthalpy along the streamline  $(i,j)$ , are now obtained by applying Euler's momentum theorem and the first law of thermodynamics directly to the flow in the streamtube between  $x$  and  $x + \Delta x$ . After the streamwise velocities and densities have been calculated at  $x + \Delta x$ , the streamline coordinates at  $x + \Delta x$  are calculated next by using mass conservation. From  $(Y_i, Z_j)$  and  $(Y_i^+, Z_j^+)$ , coordinates of the streamline  $(i,j)$  at  $x$  and  $x + \Delta x$ , one can calculate the slope of the streamline  $(i,j)$ , and then using this slope decompose  $u_{i,j}$  into its components  $u_{i,j}^x$ ,  $u_{i,j}^y$ , and  $u_{i,j}^z$ .

Strictly speaking, the present approach is not Lagrangian, as one does not follow the individual fluid particles, but rather follows the flow through individual streamtubes. This approach, however, seems closer to the Lagrangian than the Eulerian flow description. The advantage of the present approach is that one needs to solve only one set of finite difference equations--for the streamwise velocity  $u$ --rather than three sets of equations for  $u_x$ ,  $u_y$ , and  $u_z$ . The disadvantage is that extra computations are required to calculate streamline coordinates.





## 2. FINITE DIFFERENCE EQUATIONS

### 2.1 Background

Consider the flow through a rectangular duct with its axis along the x-direction. Flow in the duct is partitioned into a finite number of streamtubes. To define the various streamtubes lines  $Y_2, Y_3, \dots, Y_i, \dots, Y_I$  are drawn parallel to the z-axis (left duct wall), and lines  $Z_2, Z_3, \dots, Z_j, \dots, Z_J$  parallel to the y-axis (lower duct wall), as shown in Fig. 2.1. Lines  $Y_1$  and  $Z_1$  (not shown in Fig. 2.1) are drawn through the middle of the lower wall and the middle of the left wall, respectively. The lower and left walls are also referred to as  $Z_1$  and  $Y_1$  lines. Line  $Y_{i+\frac{1}{2}}$  is drawn in the middle of  $Y_i$  and  $Y_{i+1}$ , lines  $Y_{i-\frac{1}{2}}$ ,  $Z_{j+\frac{1}{2}}$ , and  $Z_{j-\frac{1}{2}}$  are drawn similarly. Various streamtubes, called streams for short, are now defined as follows.

Stream (2,2): flow bounded by the left wall, the lower wall,  $Z_{2+\frac{1}{2}}$ , and  $Y_{2+\frac{1}{2}}$ .

Stream (i,2):  $i = 3, I$ : flow bounded by the lower wall,  $Z_{2+\frac{1}{2}}$ ,  $Y_{i-\frac{1}{2}}$ , and  $Y_{i+\frac{1}{2}}$ .

Stream (2,j):  $j = 3, J$ : flow bounded by the left wall,  $Y_{2+\frac{1}{2}}$ ,  $Z_{j-\frac{1}{2}}$ , and  $Z_{j+\frac{1}{2}}$ .

Stream (i,j):  $i = 3, I$ ;  $j = 3, J$ : flow bounded by  $Y_{i-\frac{1}{2}}$ ,  $Y_{i+\frac{1}{2}}$ ,  $Z_{j-\frac{1}{2}}$ , and  $Z_{j+\frac{1}{2}}$ .

Streamline (i,j) is defined as the streamline passing through the intersection of  $Y_i$  and  $Z_j$ .

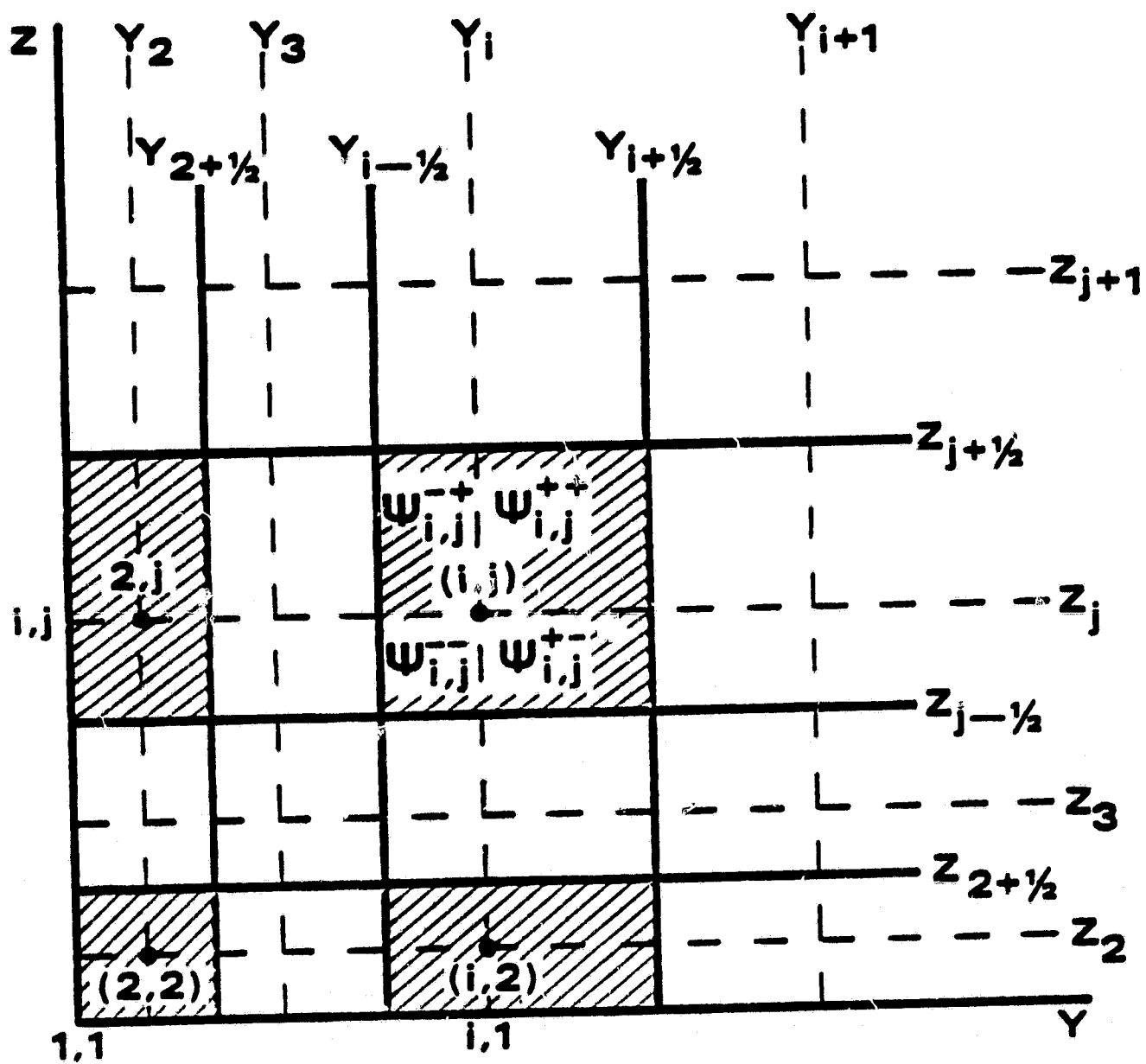


Fig. 2.1. Definition of the various streams.

Other nomenclature is as follows:

$\Psi_{i,j}^{00}$ : mass flow rate through stream (i,j).

The mass flow rate  $\Psi_{i,j}^{00}$  is partitioned further into four parts  $\Psi_{i,j}^{++}$ ,  $\Psi_{i,j}^{-+}$ ,  $\Psi_{i,j}^{--}$ , and  $\Psi_{i,j}^{+-}$ ; mass flow rates through the upper-right, upper-left, lower-left, and lower-right quadrants, respectively (see Fig. 2.1).

$$\Psi_{i,j}^{+0} \equiv \Psi_{i,j}^{++} + \Psi_{i,j}^{+-}$$

$$\Psi_{i,j}^{-0} \equiv \Psi_{i,j}^{-+} + \Psi_{i,j}^{--}$$

$$\Psi_{i,j}^{0+} \equiv \Psi_{i,j}^{++} + \Psi_{i,j}^{-+}$$

$$\Psi_{i,j}^{0-} \equiv \Psi_{i,j}^{--} + \Psi_{i,j}^{+-}$$

$A_{i,j}^X$ : cross-sectional area of the stream (i,j).

To apply conservation principles control volumes are constructed for each stream using a distance  $\Delta x$  along x. Control volume for the stream (i,j) is shown in Fig. 2.2.

$A_j^{SI}$ ;  $A_i^{SJ}$ : surface areas of the control volume normal to the y and z axis respectively.

$u_{i,j}$ ;  $T_{i,j}$ ;  $h_{i,j}$ ;  $\rho_{i,j}$ ;  $c_{i,j}$ : velocity, temperature, enthalpy, density, and constant pressure specific heat along the streamline (i,j) at x.

$u_{i,j}^+$ , etc.: variables along the streamline (i,j) at  $x + \Delta x$ .

$\bar{u}_{i,j}$ : average of  $u_{i,j}$  and  $u_{i,j}^+$ .

$u_{i,j}^x$ ;  $u_{i,j}^y$ ;  $u_{i,j}^z$ : components of  $\bar{u}_{i,j}$  along the x, y, and z directions.

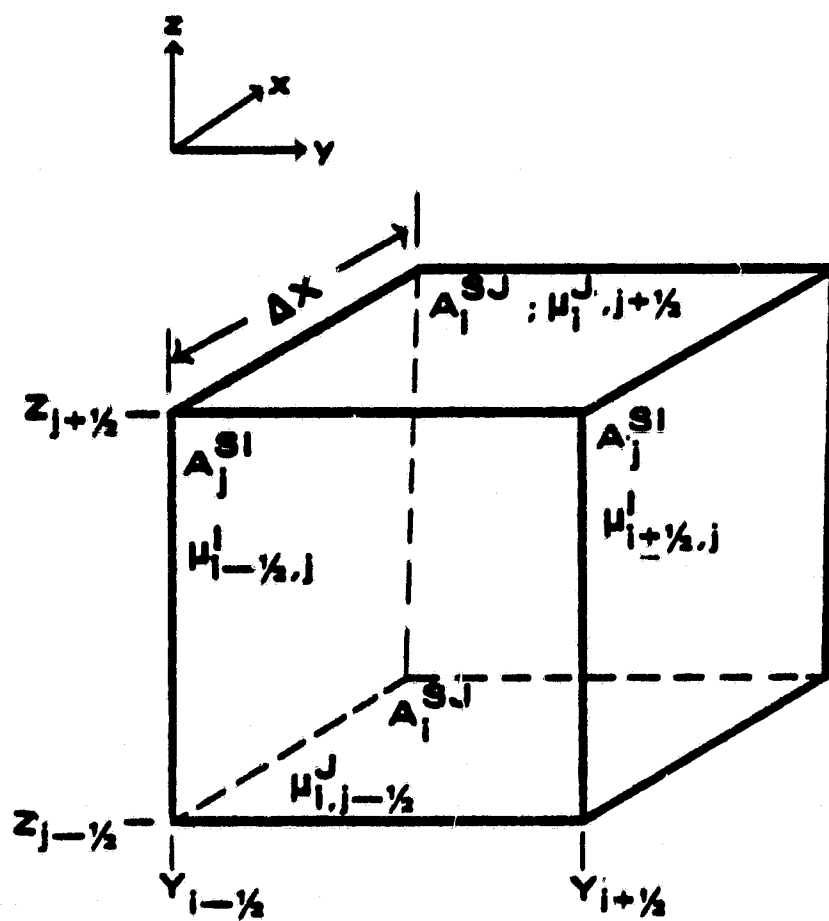


Fig. 2.2. Control volume for the stream  $(i, j)$ .

$Y_i; Z_j$ : coordinates of the streamline (i,j) in the cross-section plane at  $x$ .

$Y_i^+; Z_j^+$ : coordinates of the streamline (i,j) at  $x + \Delta x$ .

$$s_{i,j} \equiv \frac{1}{2} u_{i,j}^2$$

$$s_{i,j}^+ \equiv \frac{1}{2} u_{i,j}^{+2}$$

$\mu_{i+\frac{1}{2},j}^I; \mu_{i-\frac{1}{2},j}^I; \mu_{i,j+\frac{1}{2}}^J; \mu_{i,j-\frac{1}{2}}^J$ : effective viscosities, including the turbulent contributions, for the momentum transfer across the surfaces of the control volume, as shown in Fig. 2.2, evaluated using the information at  $x$ .

$\mu_{i+\frac{1}{2},j}^{I+}$ , etc.: effective viscosities evaluated using the information at  $x + \Delta x$ .

$K_{i+\frac{1}{2},j}^I$ , etc.: effective thermal conductivities, defined similarly as the effective viscosities.

$$V_{i+1,j}^I \equiv \frac{\mu_{i+\frac{1}{2},j}^I A_j^{SI}}{|Y_{i+1} - Y_i|}; \quad V_{i,j+1}^J \equiv \frac{\mu_{i,j+\frac{1}{2}}^J A_i^{SJ}}{|Z_{j+1} - Z_j|}$$

$$H_{i+1,j}^I \equiv \frac{K_{i+\frac{1}{2},j}^I A_j^{SI}}{C_{i,j} |Y_{i+1} - Y_i|}; \quad H_{i,j+1}^J \equiv \frac{K_{i,j+\frac{1}{2}}^J A_i^{SJ}}{C_{i,j} |Z_{j+1} - Z_j|}$$

Corresponding  $V$ 's and  $H$ 's with superscript  $+$  are defined in terms of  $\mu^+$ ,  $k^+$ ,  $\gamma^+$ , and  $Z^+$ .

In estimating momentum and energy fluxes through a streamtube  $u$  and  $h$  are assumed, in general, to vary linearly between the neighboring streamlines. Between the walls, however, and the adjacent streamlines (streamlines with  $i$  or  $j$  equal to 2 in Fig. 2.1), a nonlinear variation of  $u$  and  $h$  is allowed.

Thus, the mass, momentum, and energy flux through the streams adjacent to the walls are expressed using coefficients  $\gamma$ ,  $\alpha$ ,  $\beta$ , and  $\delta$  defined as follows (see Appendix A for further discussion):

$$\Psi_{2,2}^{--} \equiv \int_0^{Z_2} \int_0^{Y_2} \rho u dy dz \equiv \gamma_1^Z \rho u_{2,2} Y_2 Z_2$$

$$\int_0^{Z_2} \int_0^{Y_2} \rho u u dy dz \equiv \alpha_1^Z \Psi_{2,2}^{--} u_{2,2}$$

$$\int_0^{Z_2} \int_0^{Y_2} \rho u \frac{1}{2} u^2 dy dz \equiv \delta_1^Z \Psi_{2,2}^{--} \frac{1}{2} u_{2,2}^2$$

$$\int_0^{Z_2} \int_0^{Y_2} \rho u h dy dz \equiv \beta_1^Z \Psi_{2,2}^{--} h_{2,2}$$

The following relations are for the lower quadrants of the streams along the lower duct wall. Formulae for the lower-left quadrants are obtained from these relations by substituting  $\Psi_{i,2}^{--}$  for  $\Psi_{1,2}$ ,

$$\Delta Y = \frac{Y_i - Y_{i-1}}{2}, \quad \bar{u} = \frac{3u_{i,2} + u_{i-1,2}}{4},$$

$$\bar{h} = \frac{3h_{i,2} + h_{i-1,2}}{4}, \quad \text{and} \quad \overline{\frac{1}{2}u^2} = \frac{3\frac{1}{2}u_{i,2}^2 + \frac{1}{2}u_{i-1,2}^2}{4}$$

and for the lower-right quadrants by substituting  $\Psi_{i,2}^{+-}$  for  $\Psi_{i,2}$ ,

$$\Delta Y = \frac{Y_{i+1} - Y_i}{2}, \quad \bar{u} = \frac{3u_{i,2} + u_{i+1,2}}{4},$$

$$\bar{h} = \frac{3h_{i,2} + h_{i+1,2}}{4}, \quad \text{and} \quad \overline{\frac{1}{2}u^2} = \frac{3\frac{1}{2}u_{i,2}^2 + \frac{1}{2}u_{i+1,2}^2}{4},$$

$$\psi_{1,2} = \Delta Y \int_0^{Z_2} \rho u \, dz \equiv \gamma_1^Z \rho \bar{u} \Delta Y Z_2 ,$$

$$\Delta Y \int_0^{Z_2} \rho u u \, dz \equiv \alpha_1^Z \psi_{1,2} \bar{u} ,$$

$$\Delta Y \int_0^{Z_2} \rho u \frac{1}{2} u^2 \, dz \equiv \delta_1^Z \psi_{1,2} \overline{\frac{1}{2} u^2} ,$$

$$\Delta Y \int_0^{Z_2} \rho u h \, dz \equiv \beta_1^Z \psi_{1,2} \bar{h} .$$

Coefficients  $\gamma_j^Y$ ,  $\alpha_j^Y$ ,  $\delta_j^Y$ , and  $\beta_j^Y$ , that take into account the non-linear variations normal to the left wall, are defined similarly.

## 2.2 Finite Difference Equations for $u_{i,j}$

Applying Euler's momentum theorem to stream (i,j) between  $x$  and  $x + \Delta x$  one obtains,

$$\text{Momentum flux out} \Big|_{\text{at } x + \Delta x} - \text{Momentum flux in} \Big|_{\text{at } x}$$

$$= \text{Pressure forces} + \text{Viscous forces} + F_{i,j} \text{ (Body forces)}$$

(2-1)

For the streams (i,j) with  $i$  and  $j > 2$ , various terms in Eq. (2-1) are estimated as follows.

Momentum flux (in or out) through the stream is calculated by summing the momentum fluxes through each of the four quadrants (see Fig. 2.1).

Momentum flux through any one quadrant is estimated as equal to the mass flow



rate through that quadrant times the average velocity, which is obtained by double Taylor series expansion of  $u_{i,j}$  around  $i,j$ , for that quadrant. One thus gets:

$$\begin{aligned} & \text{Momentum flux out} \Big|_{\text{at } x + \Delta x} - \text{Momentum flux in} \Big|_{\text{at } x} = \\ & \left[ \frac{\psi_{i,j}^{++}}{4} (2u_{i,j}^+ + u_{i+1,j}^+ + u_{i,j+1}^+) + \frac{\psi_{i,j}^{-+}}{4} (2u_{i,j}^+ + u_{i,j+1}^+ + u_{i-1,j}^+) \right. \\ & \left. + \frac{\psi_{i,j}^{--}}{4} (2u_{i,j}^+ + u_{i-1,j}^+ + u_{i,j-1}^+) + \frac{\psi_{i,j}^{+-}}{4} (2u_{i,j}^+ + u_{i,j-1}^+ + u_{i+1,j}^+) \right] \\ & - [u^+ + u]. \end{aligned} \quad (2-2)$$

Symbol  $[u^+ + u]$  denotes the quantities in the preceding bracket  $[ ]$  with  $u^+$  replaced by  $u$ . This symbol is used similarly later on.

In calculating pressure and viscous forces, it is assumed that the streamline inclinations with respect to the duct axis are small; that is,  $u^y$  and  $u^z$  are small compared with  $u^x$ . Thus, in calculating pressure force  $A_{i,j}^x$ , the cross-sectional area of the streamtube  $(i,j)$  projected onto the Y-Z plane, is used. In calculating viscous forces, the velocity gradient is based on  $\Delta n_x$  (see Fig. 1.1), the distance between the neighboring streamlines projected onto the Y-Z plane. Pressure force on stream  $(i,j)$  is estimated as

$$= -DP A_{i,j}^x \quad (2-3)$$

where DP is the pressure difference between  $x$  and  $x + \Delta x$  and is assumed to be the same for all the streams. Viscous force on a surface between  $x$  and  $x + \Delta x$  is approximated by the average of the viscous forces evaluated by using

the effective viscosities and the velocity gradients at  $x$  and  $x + \Delta x$ . One thus obtains for the viscous forces on the stream  $(i,j)$  (see Appendix A),

$$\begin{aligned} \text{Viscous Forces} = & \frac{1}{2} \left[ v_{i+1,j}^{I+} (u_{i+1,j}^+ - u_{i,j}^+) + v_{i,j+1}^{J+} (u_{i,j+1}^+ - u_{i,j}^+) \right. \\ & \left. - v_{i,j}^{I+} (u_{i,j}^+ - u_{i-1,j}^+) - v_{i,j}^{J+} (u_{i,j}^+ - u_{i,j-1}^+) \right] \\ & + \frac{1}{2} [v^+, u^+ \rightarrow v, u]. \end{aligned} \quad (2-4)$$

For streams along the walls, momentum fluxes are expressed using the non-linear correction factors introduced in section 2.1. Viscous forces exerted by the lower and the left wall on the adjacent streams are, respectively, set equal to

$$v_{i,2}^J u_{i,2} \quad \text{and} \quad v_{2,j}^I u_{j,2}$$

Calculation of  $v_{i,2}^J$  and  $v_{2,j}^I$  is discussed in Appendix A.

After the various estimates in Eq. (2-1) have been substituted, all the terms involving  $u_{i,j}^+$  are collected on the left-hand side, and the resulting equation is organized in the form:

$$-A_{i,j}^I u_{i+1,j}^+ - C_{i,j}^I u_{i-1,j}^+ + B_{i,j} u_{i,j}^+ - A_{i,j}^J u_{i,j+1}^+ - C_{i,j}^J u_{i,j-1}^+ = D_{i,j} \quad (2-5)$$

Coefficients A, B, C, and D for the various streams are given next.

NOTE: In equations that follow all the  $\Psi$ 's, except  $\Psi_{2,2}^{--}$ , are  $\frac{1}{2}$  times the  $\Psi$ 's defined in section 2.1, and the  $V$ 's are  $\frac{1}{2}$  times the  $V$ 's defined earlier.

Equations (2-6):

Coefficients for stream (2,2)

$$A_{2,2}^I = -\psi_{2,2}^{++} - \alpha_2^z \psi_{2,2}^{+-} + v_{3,2}^{J+}$$

$$C_{2,2}^I = 0$$

$$B_{2,2} = 2\psi_{2,2}^{++} + 3\alpha_2^y \psi_{2,2}^{-+} + \alpha_1^z \psi_{2,2}^{--} + 3\alpha_2^z \psi_{2,2}^{+-} + v_{3,2}^{I+} + v_{2,3}^{J+} + v_{2,2}^{I+} + v_{2,2}^{J+}$$

$$A_{2,2}^J = -\psi_{2,2}^{++} - \alpha_2^y \psi_{2,2}^{-+} + v_{2,3}^{J+}$$

$$C_{2,2}^J = 0$$

$$D_{2,2} = -DPA_{2,2}^x + F_{2,2}$$

$$+ u_{3,2} \left\{ \psi_{2,2}^{++} + \alpha_2^z \psi_{2,2}^{+-} + v_{3,2}^I \right\}$$

$$+ u_{2,2} \left\{ 2\psi_{2,2}^{++} + 3\alpha_2^y \psi_{2,2}^{-+} + \alpha_1^z \psi_{2,2}^{--} + 3\alpha_2^z \psi_{2,2}^{+-} \right. \\ \left. - v_{3,2}^I - v_{2,3}^J - v_{2,2}^I - v_{2,2}^J \right\}$$

$$+ u_{2,3} \left( \psi_{2,2}^{++} + \alpha_2^y \psi_{2,2}^{-+} + v_{2,3}^J \right)$$

Coefficients for stream (i,2),  $i > 2$ .

$$A_{i,2}^I = -\psi_{i,2}^{++} - \alpha_i^z \psi_{i,2}^{+-} + v_{i+1,2}^{I+}$$

$$C_{i,2}^I = -\psi_{i,2}^{-+} - \alpha_i^z \psi_{i,2}^{--} + v_{i,2}^{I+}$$

$$B_{i,2} = 2\psi_{i,2}^{0+} + 3\alpha_i^z \psi_{i,2}^{0-} + v_{i+1,2}^{I+} + v_{i,3}^{J+} + v_{i,2}^{I+} + v_{i,2}^{J+}$$

$$A_{i,2}^J = -\psi_{i,2}^{0+} + v_{i,3}^{J+}$$

$$C_{i,2}^J = 0$$

$$D_{i,2} = -DPA_2^x + F_{i,2}$$

$$+ u_{i+1,2} \left( \psi_{i,2}^{++} + \alpha_i^z \psi_{i,2}^{+-} + v_{i+1,2}^I \right)$$

$$+ u_{i-1,2} \left( \psi_{i,2}^{-+} + \alpha_i^z \psi_{i,2}^{--} + v_{i,2}^I \right)$$

$$+ u_{i,2} \left( 2\psi_{i,2}^{0+} + 3\alpha_i^z \psi_{i,2}^{0-} + v_{i+1,2}^{I+} + v_{i,3}^{J+} + v_{i,2}^{I+} + v_{i,2}^{J+} \right)$$

$$+ u_{i,3} \left( \psi_{i,2}^{0+} + v_{i,3}^J \right)$$

Coefficients for stream (2,j),  $j > 2$ .

$$A_{2,j}^I = -\psi_{2,j}^{+0} + v_{3,j}^{I+}$$

$$C_{2,j}^I = 0$$

$$B_{2,j} = 2\psi_{2,j}^{+0} + 3\alpha_j^y \psi_{1,j}^{-0} + v_{3,j}^{I+} + v_{2,j+1}^{J+} + v_{2,j}^{I+} + v_{2,j}^{J+}$$

$$A_{2,j}^J = -\psi_{2,j}^{++} - \alpha_j^y \psi_{2,j}^{-+} + v_{2,j+1}^{J+}$$

$$C_{2,j}^J = -\alpha_j^y \psi_{2,j}^{--} - \psi_{2,j}^{+-} + v_{2,j}^{J+}$$

$$D_{1,j} = -DPA_2^x + F_{2,j}$$

$$+ u_{3,j} \left( \psi_{2,j}^{+0} + v_{3,j}^{I+} \right)$$

$$+ u_{2,j} \left( 2\psi_{2,j}^{+0} + 3\alpha_j^y \psi_{2,j}^{-0} - v_{3,j}^{I+} - v_{2,j+1}^{J+} - v_{2,j}^{I+} - v_{2,j}^{J+} \right)$$

$$+ u_{2,j+1} \left( \psi_{2,j}^{++} + \alpha_j^y \psi_{2,j}^{-+} + v_{2,j+1}^{J+} \right)$$

$$+ u_{2,j-1} \left( \alpha_j^y \psi_{2,j}^{--} + \psi_{2,j}^{+-} + v_{2,j}^{J+} \right)$$

Coefficients for stream (i,j), i and j > 2.

$$A_{i,j}^I = -\psi_{i,j}^{+0} + v_{i+1,j}^{I+}$$

$$C_{i,j}^I = -\psi_{i,j}^{-0} + v_{i,j}^{I+}$$

$$B_{i,j} = \psi_{i,j}^{00} + v_{i+1,j}^{I+} + v_{i,j+1}^{J+} + v_{i,j}^{I+} + v_{i,j}^{J+}$$

$$A_{i,j}^J = -\psi_{i,j}^{0+} + v_{i,j+1}^{J+}$$

$$C_{i,j}^J = -\psi_{i,j}^{0-} + v_{i,j}^{J+}$$

$$D_{i,j} = -DPA_{i,j}^X + F_{i,j}$$

$$+ u_{i+1,j} \left( \psi_{i,j}^{+0} + v_{i+1,j}^I \right) + u_{i-1,j} \left( \psi_{i,j}^{-0} + v_{i,j}^I \right)$$

$$+ u_{i,j} \left( \psi_{i,j}^{00} - v_{i+1,j}^I - v_{i,j+1}^J - v_{i,j}^I - v_{i,j}^J \right)$$

$$+ u_{i,j+1} \left( \psi_{i,j}^{0+} + v_{i,j+1}^J \right) + u_{i,j-1} \left( \psi_{i,j}^{0-} + v_{i,j}^J \right)$$

### 2.3 Finite Difference Equations for $h_{i,j}$

Applying first law of thermodynamics to the stream (i,j) between x and  $x + \Delta x$ , one obtains,

$$\text{Energy flux out} \Big|_{\text{at } x + \Delta x} - \text{Energy flux in} \Big|_{\text{at } x} =$$

$$\begin{aligned} & \text{Heat added by conduction} + \text{Work done by the viscous forces} \\ & + Q_{i,j}(\text{Internal heat generation}) + W_{i,j}(\text{Work done by the body forces}) \end{aligned} \quad (2-7)$$

For the streams (i,j) with i and  $j > 2$ , various terms in Eq. (2-7) are estimated as follows:

The energy flux terms are estimated in the same way as the momentum flux terms and can be obtained by substituting  $(h+s)$  for  $u$  in Eq. (2-2).

Heat conducted across the top surface of the control volume (see Fig. 2.2) is estimated as

$$= \frac{1}{2} \left[ H_{i,j+1}^{J+} (h_{i,j+1}^+ - h_{i,j}^+) \right] + \frac{1}{2} [H^+, h^+ + H, h] \quad (2-8)$$

Work done by the viscous forces on the top surface is taken to be

$$\begin{aligned} & = \frac{1}{2} \left[ V_{i,j+1}^{J+} (u_{i,j+1}^+ - u_{i,j}^+) \left( \frac{u_{i,j+1}^+ + u_{i,j}^+}{2} \right) \right] + \frac{1}{2} [V^+ u^+ + V u] \\ & = \frac{1}{2} \left[ V_{i,j+1}^{J+} (s_{i,j+1}^+ - s_{i,j}^+) \right] + \frac{1}{2} [V^+ s^+ + V s] \end{aligned} \quad (2-9)$$

Work done by the viscous forces and the heat conducted across the other three surfaces of the control volume are estimated similarly.

After the various estimates in Eq. (2-7) have been substituted, all the terms involving  $h_{i,j}^+$  are collected on the left-hand side and the resulting equation is organized in the form:

$$-A_{i,j}^I h_{i+1,j}^+ - C_{i,j}^I h_{i-1,j}^+ + B_{i,j} h_{i,j+1}^+ - A_{i,j}^J h_{i,j+1}^+ - C_{i,j}^J h_{i,j-1}^+ = D_{i,j} \quad (2-10)$$

Coefficients A, B, C, and D for the various streams are given next.

NOTE: In equations that follow all the  $\Psi$ 's, except  $\Psi_{2,2}^{--}$ , are  $\frac{1}{2}$  times the  $\Psi$ 's defined in section 2.1, and the H's are  $\frac{1}{2}$  times the H's defined earlier.



# Equations (2-11)

Coefficients for stream (2,2)

$$A_{2,2}^I = -\psi_{2,2}^{++} - \beta_2^Z \psi_{2,2}^{+-} + H_{3,2}^{I+}$$

$$C_{2,2}^I = 0$$

$$B_{2,2} = 2\psi_{2,2}^{++} + 3\beta_2^Y \psi_{2,2}^{-+} + \beta_1^Z \psi_{2,2}^{--} + 3\beta_2^Z \psi_{2,2}^{+-} + H_{3,2}^{I+} + H_{2,3}^{J+} + H_{2,2}^{I+} + H_{2,2}^{J+}$$

$$A_{2,2}^J = -\psi_{2,2}^{++} - \beta_2^Y \psi_{2,2}^{-+} + H_{2,3}^{J+}$$

$$C_{2,2}^J = 0$$

$$D_{2,2} = Q_{2,2} + W_{2,2}$$

$$+ h_{3,2} \left( \psi_{2,2}^{++} + \beta_2^Z \psi_{2,2}^{+-} + H_{3,2}^I \right)$$

$$+ h_{2,2} \left( 2\psi_{2,2}^{++} + 3\beta_2^Y \psi_{2,2}^{-+} + \beta_1^Z \psi_{2,2}^{--} + 3\beta_2^Z \psi_{2,2}^{+-} - H_{3,2}^I - H_{2,3}^J - H_{2,2}^I - H_{2,2}^J \right)$$

$$+ h_{2,3} \left( \psi_{2,2}^{++} + \beta_2^Y \psi_{2,2}^{-+} + H_{2,3}^J \right)$$

$$+ s_{3,2} \left( \psi_{2,2}^{++} + \delta_2^Z \psi_{2,2}^{+-} + V_{3,2}^I \right)$$

$$+ s_{2,2} \left( 2\psi_{2,2}^{++} + 3\delta_2^Y \psi_{2,2}^{-+} + \delta_1^Z \psi_{2,2}^{--} + 3\delta_2^Z \psi_{2,2}^{+-} - V_{3,2}^I - V_{2,3}^J - V_{2,2}^I - V_{2,2}^J \right)$$

$$+ s_{2,3} \left( \psi_{2,2}^{++} + \delta_2^Y \psi_{2,2}^{-+} + V_{2,3}^J \right) + h_{1,2} H_{2,2}^I + h_{2,1} H_{2,2}^J$$

$$+ h_{1,2} H_{2,2}^{I+} + h_{2,1} H_{2,2}^{J+} + s_{3,2}^+ \left( -\psi_{2,2}^{++} - \delta_2^Z \psi_{2,2}^{+-} + V_{3,2}^{I+} \right)$$

$$+ s_{2,2}^+ \left( 2\psi_{2,2}^{++} + 3\delta_2^Y \psi_{2,2}^{-+} + \delta_1^Z \psi_{2,2}^{--} + 3\delta_2^Z \psi_{2,2}^{+-} + V_{3,2}^{I+} + V_{2,3}^{J+} + V_{2,2}^{I+} + V_{2,2}^{J+} \right)$$

$$+ s_{2,3}^+ \left( -\psi_{2,2}^{++} - \delta_2^Y \psi_{2,2}^{-+} + V_{2,2}^{J+} \right)$$

Coefficients for stream (1,2),  $i > 2$ .

$$A_{i,2}^I = -\psi_{i,2}^{++} - \beta_i^Z \psi_{i,2}^{+-} + H_{i+1,2}^{I+}$$

$$C_{i,2}^I = -\psi_{i,2}^{-+} - \beta_i^Z \psi_{i,2}^{--} + H_{i,2}^{I+}$$

$$B_{i,2} = 2\psi_{i,2}^{0+} + 3\beta_i^Z \psi_{i,2}^{0-} + H_{i+1,2}^{I+} + H_{i,3}^{J+} + H_{i,2}^{I+} + H_{i,2}^{J+}$$

$$A_{i,2}^J = -\psi_{i,2}^{0+} + H_{i,3}^{J+}$$

$$C_{i,2}^J = 0$$

$$D_{i,2} = Q_{i,2} + W_{i,2}$$

$$+ h_{i+1,2} \left( \psi_{i,2}^{++} + \beta_i^Z \psi_{i,2}^{+-} + H_{i+1,2}^I \right) + h_{i-1,2} \left( \psi_{i,2}^{-+} + \beta_i^Z \psi_{i,2}^{--} + H_{i,2}^I \right)$$

$$+ h_{i,3} \left( 2\psi_{i,2}^{0+} + 3\beta_i^Z \psi_{i,2}^{0-} - H_{i+1,2}^I - H_{i,3}^J - H_{i,2}^I - H_{i,2}^J \right)$$

$$+ h_{i,2} \left( \psi_{i,2}^{0+} + H_{i,3}^J \right)$$

$$+ s_{i+1,2} \left( \psi_{i,2}^{++} + \delta_i^Z \psi_{i,2}^{+-} + V_{i+1,2}^I \right) + s_{i-1,2} \left( \psi_{i,2}^{-+} + \delta_i^Z \psi_{i,2}^{--} + V_{i,2}^I \right)$$

$$+ s_{i,2} \left( 2\psi_{i,2}^{0+} + 3\delta_i^Z \psi_{i,2}^{0-} - V_{i+1,2}^I - V_{i,3}^J - V_{i,2}^I - V_{i,2}^J \right)$$

$$+ s_{i,3} \left( \psi_{i,2}^{0+} + V_{i,3}^J \right) + h_{i,1} H_{i,2}^J + h_{i,1} H_{i,2}^{J+}$$

$$+ s_{i+1,2}^+ \left( -\psi_{i,2}^{++} - \delta_i^Z \psi_{i,2}^{+-} + V_{i+1,2}^{I+} \right) + s_{i-1,2}^+ \left( -\psi_{i,2}^{-+} - \delta_i^Z \psi_{i,2}^{--} + V_{i,2}^{I+} \right)$$

$$+ s_{i,2}^+ \left( 2\psi_{i,2}^{0+} + 3\delta_i^Z \psi_{i,2}^{0-} + V_{i+1,2}^{I+} + V_{i,3}^{J+} + V_{i,2}^{I+} + V_{i,2}^{J+} \right)$$

$$+ s_{i,3}^+ \left( -\psi_{i,2}^{0+} + V_{i,3}^{J+} \right)$$

Coefficients for stream (2,j),  $j > 2$ .

$$A_{2,j}^I = -\psi_{2,j}^{+0} + H_{3,j}^{I+}$$

$$C_{2,j}^I = 0$$

$$B_{2,j} = 2\psi_{2,j}^{+0} + 3\beta_j^y \psi_{2,j}^{-0} + H_{3,j}^{I+} + H_{2,j+1}^{J+} + H_{2,j}^{I+} + H_{2,j}^{J+}$$

$$A_{2,j}^J = -\psi_{2,j}^{++} - \beta_j^y \psi_{2,j}^{-+} + H_{2,j+1}^{J+}$$

$$C_{2,j}^J = -\beta_j^y \psi_{2,j}^{--} - \psi_{2,j}^{+-} + H_{2,j}^{J+}$$

$$D_{1,j} = Q_{2,j} + W_{2,j}$$

$$+ h_{3,j} \left( \psi_{2,j}^{+0} + H_{3,j}^I \right)$$

$$+ h_{2,j} \left( 2\psi_{2,j}^{+0} + 3\beta_j^y \psi_{2,j}^{-0} - H_{3,j}^I - H_{2,j+1}^J - H_{2,j}^I - H_{2,j}^J \right)$$

$$+ h_{2,j+1} \left( \psi_{2,j}^{++} + \beta_j^y \psi_{2,j}^{-+} + H_{2,j+1}^J \right) + h_{2,j-1} \left( \beta_j^y \psi_{2,j}^{--} + \psi_{2,j}^{+-} + H_{2,j}^J \right)$$

$$+ s_{3,j} \left( \psi_{2,j}^{+0} + V_{3,j}^I \right)$$

$$+ s_{2,j} \left( 2\psi_{2,j}^{+0} + 3\delta_j^y \psi_{2,j}^{-0} - V_{3,j}^I - V_{2,j+1}^J - V_{2,j}^I - V_{2,j}^J \right)$$

$$+ s_{2,j+1} \left( \psi_{2,j}^{++} + \delta_j^y \psi_{2,j}^{-+} + V_{2,j+1}^J \right) + s_{2,j-1} \left( \delta_j^y \psi_{2,j}^{--} + \psi_{2,j}^{+-} + V_{2,j}^J \right)$$

$$+ h_{1,j} H_{2,j}^{I+} + h_{1,j} H_{2,j}^{I+}$$

$$+ s_{3,j}^+ \left( -\psi_{2,j}^{+0} + V_{3,j}^{I+} \right)$$

$$- s_{2,j}^+ \left( 2\psi_{2,j}^{+0} + 3\delta_j^y \psi_{2,j}^{-0} + V_{3,j}^{I+} + V_{2,j+1}^{J+} + V_{2,j}^I + V_{2,j}^J \right)$$

$$+ s_{2,j+1}^+ \left( -\psi_{2,j}^{++} - \delta_j^y \psi_{2,j}^{-+} + V_{2,j+1}^{J+} \right) + s_{2,j-1}^+ \left( -\delta_j^y \psi_{2,j}^{--} - \psi_{2,j}^{+-} + V_{2,j}^{J+} \right)$$

Coefficients for stream (i,j), i and j > 2.

$$A_{i,j}^I = -\psi_{i,j}^{+0} + H_{i+1,j}^{I+}$$

$$C_{i,j}^I = -\psi_{i,j}^{-0} + H_{i,j}^{I+}$$

$$B_{i,j} = \psi_{i,j}^{00} + H_{i+1,j}^{I+} + H_{i,j+1}^{J+} + H_{i,j}^{I+} + H_{i,j}^{J+}$$

$$A_{i,j}^J = -\psi_{i,j}^{0+} + H_{i,j+1}^{J+}$$

$$C_{i,j}^J = -\psi_{i,j}^{0-} + H_{i,j}^{J+}$$

$$D_{i,j} = Q_{i,j} + W_{i,j}$$

$$\begin{aligned} & + h_{i+1,j} \left( \psi_{i,j}^{+0} + H_{i+1,j}^I \right) + h_{i-1,j} \left( \psi_{i,j}^{-0} + H_{i,j}^I \right) \\ & + h_{i,j} \left( \psi_{i,j}^{00} - H_{i+1,j}^I - H_{i,j+1}^J - H_{i,j}^I - H_{i,j}^J \right) \\ & + h_{i,j+1} \left( \psi_{i,j}^{0+} + H_{i,j+1}^J \right) + h_{i,j-1} \left( \psi_{i,j}^{0-} + H_{i,j}^J \right) \\ & + s_{i+1,j} \left( \psi_{i,j}^{+0} + V_{i+1,j}^I \right) + s_{i-1,j} \left( \psi_{i,j}^{-0} + V_{i,j}^I \right) \\ & + s_{i,j} \left( \psi_{i,j}^{00} - V_{i+1,j}^I - V_{i,j+1}^J - V_{i,j}^I - V_{i,j}^J \right) \\ & + s_{i,j+1} \left( \psi_{i,j}^{0+} + V_{i,j+1}^J \right) + s_{i,j-1} \left( \psi_{i,j}^{0-} + V_{i,j}^J \right) \\ & + s_{i+1,j}^+ \left( -\psi_{i,j}^{+0} + V_{i+1,j}^{I+} \right) + s_{i-1,j}^+ \left( -\psi_{i,j}^{-0} + V_{i,j}^{I+} \right) \\ & - s_{i,j}^+ \left( \psi_{i,j}^{00} + V_{i+1,j}^{I+} + V_{i,j+1}^{J+} + V_{i,j}^{I+} + V_{i,j}^{J+} \right) \\ & + s_{i,j+1}^+ \left( -\psi_{i,j}^{0+} + V_{i,j+1}^{J+} \right) + s_{i,j-1}^+ \left( -\psi_{i,j}^{0-} + V_{i,j}^{J+} \right) \end{aligned}$$

## 2.4 Decomposition of $u$ and the Secondary Flows

Computation of secondary flows, defined as the velocity components normal to the main flow, is discussed in this section. After  $Y_i^+$  and  $Z_j^+$  have been calculated, the streamwise velocity  $\bar{u}_{i,j}$ , the average of  $u_{i,j}$  and  $u_{i,j}^+$ , then can be decomposed into  $u_{i,j}^x$ ,  $u_{i,j}^y$ , and  $u_{i,j}^z$ , by using the formulae,

$$\frac{u_{i,j}^x}{\bar{u}_{i,j}} = \frac{\Delta X}{\Delta r} \quad , \quad \frac{u_{i,j}^y}{\bar{u}_{i,j}} = \frac{\Delta Y}{\Delta r} \quad , \quad \text{and} \quad \frac{u_{i,j}^z}{\bar{u}_{i,j}} = \frac{\Delta Z}{\Delta r} \quad (2-12)$$

where  $\Delta x$  is the integration step,

$$\Delta Y = Y_i^+ - Y_i \quad , \quad \Delta Z = Z_j^+ - Z_j \quad , \quad \text{and} \quad \Delta r^2 = \Delta X^2 + \Delta Y^2 + \Delta Z^2$$

The calculation of  $Y_i^+$  and  $Z_j^+$  as discussed in Appendix A is based on the conservation of mass alone. Velocity components  $u^y$  and  $u^z$ , obtained from  $Y_i^+$  and  $Z_j^+$  thus calculated, are the secondary flows associated with the flow development in the ducts--may the flow be laminar or turbulent. In turbulent flows through rectangular ducts, besides the primary shear stresses which are in the streamwise direction, there are also shear stresses in the plane normal to the streamwise direction (see, for example, Launder and Spalding [12]). Force due to these stresses will produce bending of the streamlines in the  $z$ - $x$  and  $z$ - $y$  planes, and thus displace  $Y_i^+$  and  $Z_j^+$  beyond that displaced by the conservation of mass. For example, force per unit area  $\Delta\tau_{yz}$ , due to the cross-stream shear stress  $\tau_{yz}$  distribution, will bend the streamline  $(i,j)$  according to the formula,

$$\frac{\rho_{i,j} \bar{u}_{i,j}^2}{R_{i,j}^z} = \frac{\Delta\tau_{yz}}{Y_{i+\frac{1}{2}} - Y_{i-\frac{1}{2}}} \quad (2-13)$$

where  $R_{i,j}$  is the radius of curvature in the  $z$ - $x$  plane. With  $R_{i,j}^z$  (approximated as a straight line for small  $\Delta x$ ) known one can calculate displacement of  $Z_j^+$  (and similarly of  $Y_i^+$ ) caused by the cross-stream turbulent shear stresses. Contributions to  $u^x$  and  $u^y$ , as calculated from Eq. (2-12), by this additional displacement of the streamlines in the cross-stream plane represents, as viewed from the present computational approach, the secondary flows associated with turbulent flow through noncircular ducts. It should be noted that the turbulent shear stresses in the cross-stream plane do not enter into the finite difference equations for the streamwise velocity presented in section 2.2. These equations are valid for laminar and turbulent flows as long as the streamwise shear stresses, appearing in the equations via  $V$ 's, are modeled appropriately.

### 3. SOLUTION TECHNIQUE

A methodology to solve the momentum and energy finite difference equations given in section 2 is presented in this section. This is the methodology used for the sample flow computations presented in section 4. Equations (2-5) and (2-10) are of the general form:

$$-A_{i,j}^I X_{i+1,j}^+ - C_{i,j}^I X_{i-1,j}^+ + B_{i,j} X_{i,j}^+ - A_{i,j+1}^J X_{i,j+1}^+ - C_{i,j}^J X_{i,j-1}^+ = D_{i,j} \quad (3-1)$$

Equations (3-1) constitute a set of coupled nonlinear algebraic equations. The solution technique discussed is for flow through a rectangular duct, with the duct cross-section specified along the axis. In this case, the pressure drop,  $DP$ , that appears in  $D_{i,j}$  for the velocity equations, is not known in advance and is to be calculated as a part of the computations. Equations are solved iteratively by using two iteration loops, one placed inside the other. The outer loop iterates on the unknown  $DP$ , and the inner loop solves the nonlinear algebraic equations, given a value for  $DP$ , iteratively.

The following steps are for one integration step from  $x$  to  $x + \Delta x$ .

1. Assign a value to  $DP$ . At the beginning of the outer iteration loop, the value assigned is based on the upstream value of  $DP$ . For the subsequent iterations,  $DP$  is assigned a value as discussed in Step 7.
2. Calculate coefficients  $A$ ,  $B$ ,  $C$ , and  $D$  for the velocity equations, Eqs. (2-5), evaluating  $V$ 's by using the information at  $x$  for the first iteration, and the information at  $x + \Delta x$  obtained in the previous iteration for the subsequent iterations.

3. Once A, B, C, and D are known Eqs. (2-5) constitute a pentadiagonal set of linear algebraic equations. These equations are solved by using alternate direction implicit method.
4. Repeat steps 2 and 3 for the enthalpy equations, Eqs. (2-10).
5. With the newly calculated values of dynamic and thermodynamic variables at  $x + \Delta x$ , calculate  $Y_i^+$  and  $Z_j^+$  by using the formulae given in Appendix A.
6. Check newly calculated  $u_{i,j}^+$  and  $h_{i,j}^+$  for convergence. If the convergence check is met, proceed to the next step; otherwise repeat the steps starting from 2.
7. Check  $Y_i^+$  and  $Z_j^+$ , calculated duct half width and half height for the value of DP assigned in Step 1, against the duct dimensions specified at  $x + \Delta x$ . If the check is within the desired accuracy, the computation of flow from  $x$  to  $x + \Delta x$  is complete. If not, DP is adjusted, on the basis of the difference between the specified and the calculated duct dimensions, and the calculations started again at Step 1, with the new value of DP. The formulae used to adjust DP for the succeeding iteration is the one given in [1]. This formula, based on the difference between the calculated and specified duct cross-sections, is applicable to both two and three dimensional flow computations.

Spacing between the streamlines in the cross-stream plane changes from  $x$  to  $x + \Delta x$ . As a result, at the beginning of each new integration step, streamtubes are defined anew in accordance with the definitions given in section 2.1.



#### 4. SAMPLE FLOW COMPUTATIONS

The approach, presented in this paper, to compute three-dimensional flows by calculating velocity and enthalpy along the streamlines, was checked by making three sample flow computations. All the computations are for laminar flow through rectangular ducts. Computation of laminar flows is free from the uncertainties associated with the modeling of turbulent shear stresses in three-dimensional flows and, thus, provides a clearer assessment of the numerical method itself.

All the computations are for the lower-left quarter of the duct. Each sample flow computation was carried out using one hundred streamlines; this corresponds to, including the grid points along the walls, a  $11 \times 11$  grid in the  $y$ - $z$  plane. Streamtubes around these streamlines were redefined at the beginning of each new integration step, as discussed in Section 3. In the present computations, however, the same set of streamlines was used for the entire length of the duct. The integration step along the duct,  $\Delta x$ , was selected for each sample flow computation so that the locations of the computer output will match the locations of the reported experimental data; the integration step varied from one-tenth to one-twenty-fifth of the equivalent duct diameter. Velocity distribution at the inlet in all the sample flows was taken to be uniform across the duct cross-section. The following nomenclature is used in the figures in this section:

—•—•—•—: Dots along the velocity distribution curves represent the streamline locations along which the flow was computed.

U: Axial velocity.

$U_0$ : U at the inlet.

$U_c$ : Centerline velocity.

W;H: Duct half width and half height, respectively.

D: Equivalent diameter ( $= 4WH/(W+H)$ ).

X: Distance along the duct measured from the inlet.

Y and Z: Distances normal to x measured from the lower-left duct corner.

Re: Reynolds number ( $= U_0 D/\nu$ , where  $\nu$  is the kinematic viscosity).

Pr: Prandtl number ( $= 0.72$ ).

Gr: Graetz number ( $= Pr Re/x$ ).

Num: Logarithmic mean Nusselt number

$$\left( = \frac{Gr}{4} \log \left( \frac{T_{center} - T_{wall}}{T_{bulk} - T_{wall}} \right) \right).$$

$T_{bulk}$ : Bulk temperature.

In Figs. 4.1a, 4.1b, 4.1c, and 4.1d the results for flow through a square-sectioned duct are shown. Calculations are compared with the experimental data of Goldstein and Kreid [13] (Figs. 4.1a, 4.1b, and 4.1c) and Beavers et al. [14] (Fig. 4.1d). Agreement between the theoretical and experimental results is good. In Figs. 4.2a, 4.2b, 4.2c, and 4.2d the results for flow through a rectangular duct of 2:1 aspect ratio are shown. Calculations are compared with the experimental data of Sparrow et al. [15]. Once again agreement between the theoretical and experimental results is satisfactory.

In the two flow computations just described only the finite difference equations for velocity were solved. Density was considered constant and, thus, the energy equation was bypassed. To check the finite difference equations for enthalpy a sample flow with heat transfer to the walls through a rectangular duct of 2:1 aspect ratio was computed. The inlet temperature and velocity distributions were taken to be uniform across the duct cross-section.

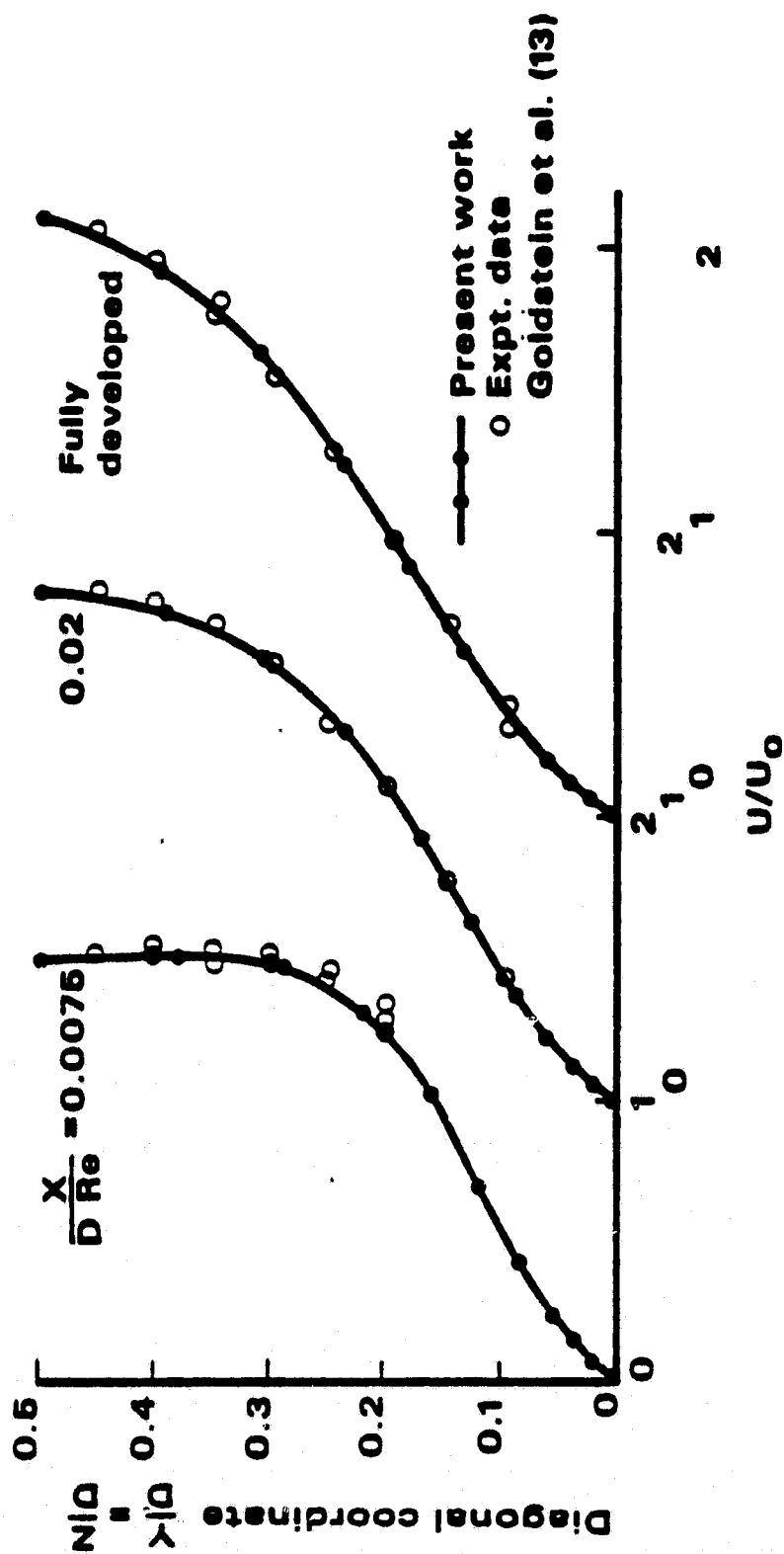


Fig. 4.1a. Development of the velocity profile in a square-sectioned duct--diagonal plane.

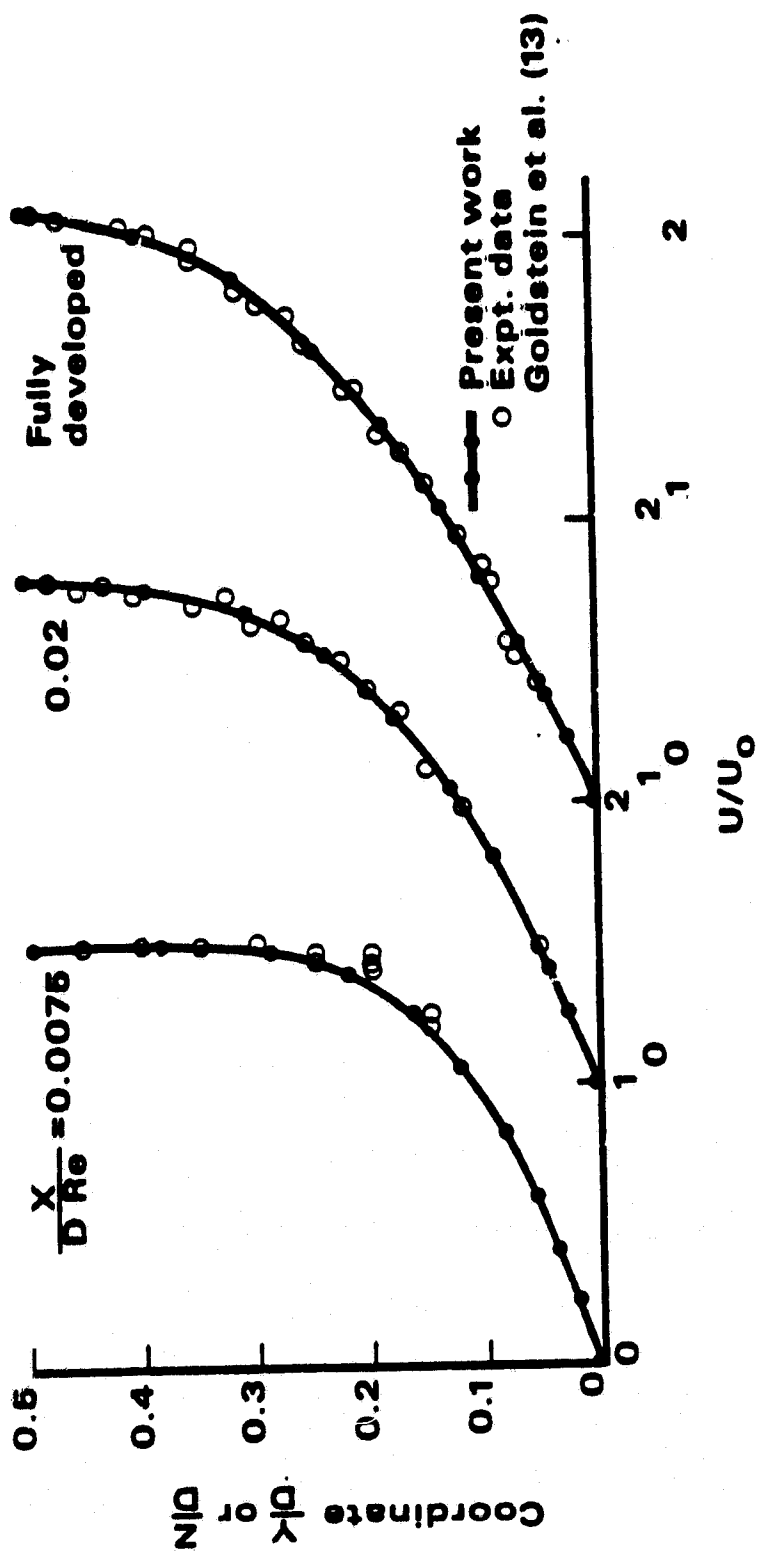


Fig. 4.1b. Development of the velocity profile in a square-sectioned duct--central plane.

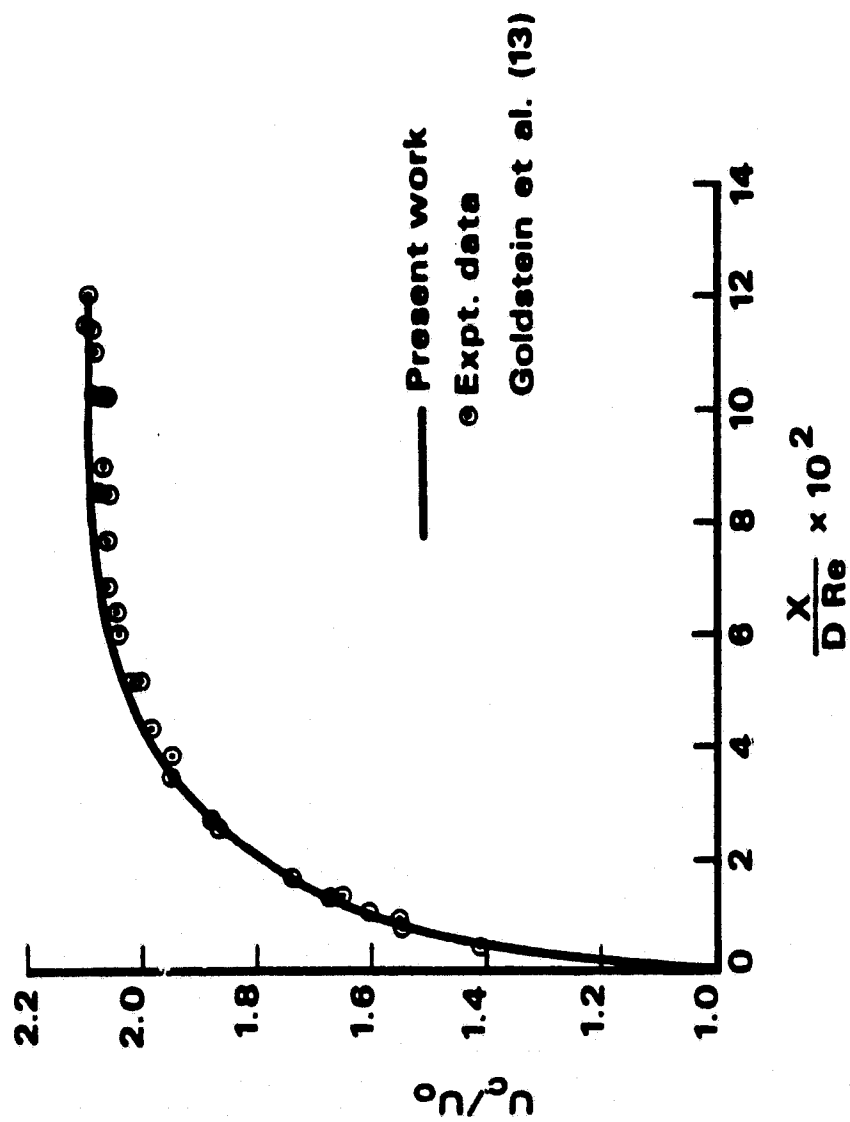


Fig. 4.1c. Center line velocity development in a square-sectioned duct.

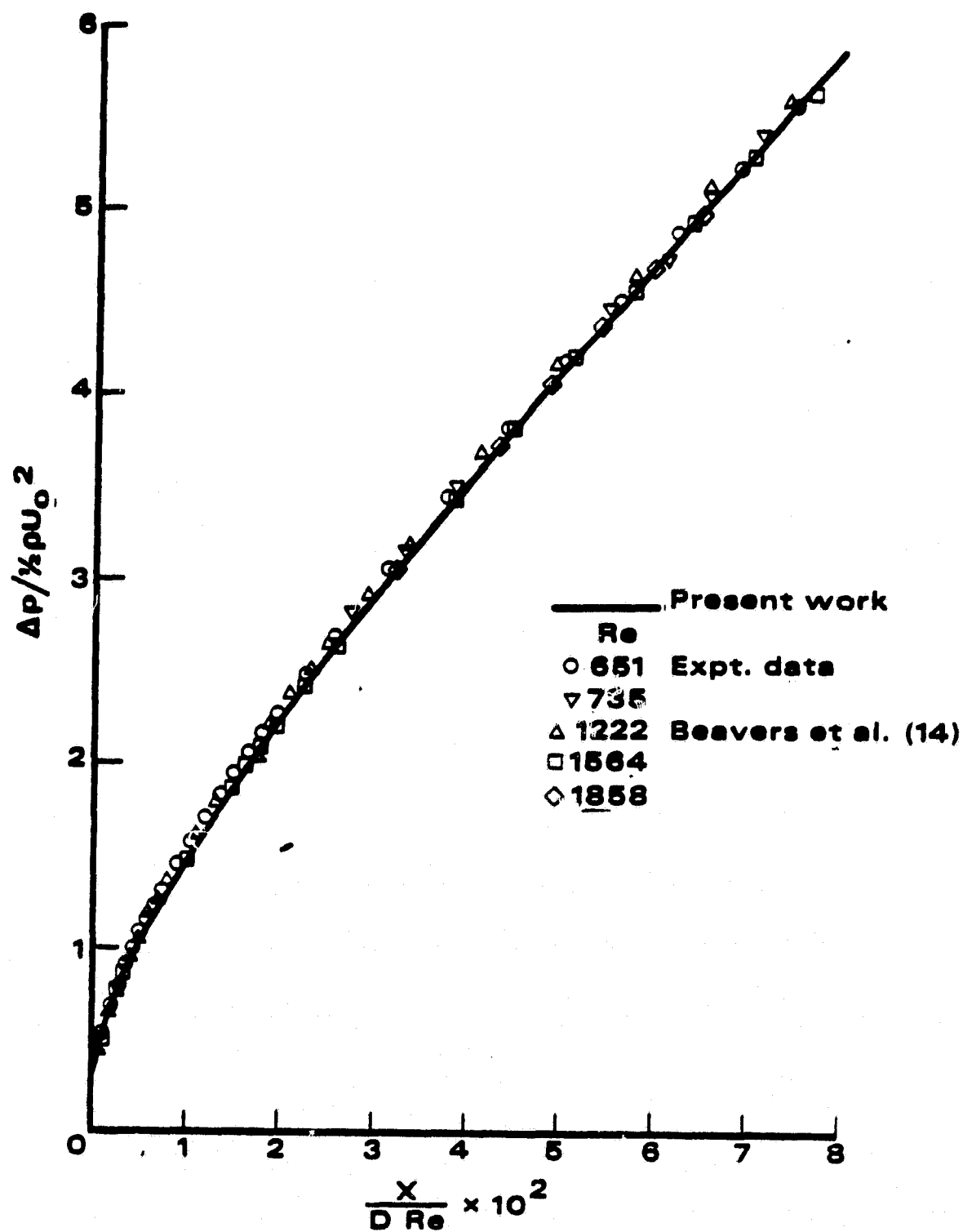


Fig. 4.1d. Pressure drop in a square-sectioned duct.

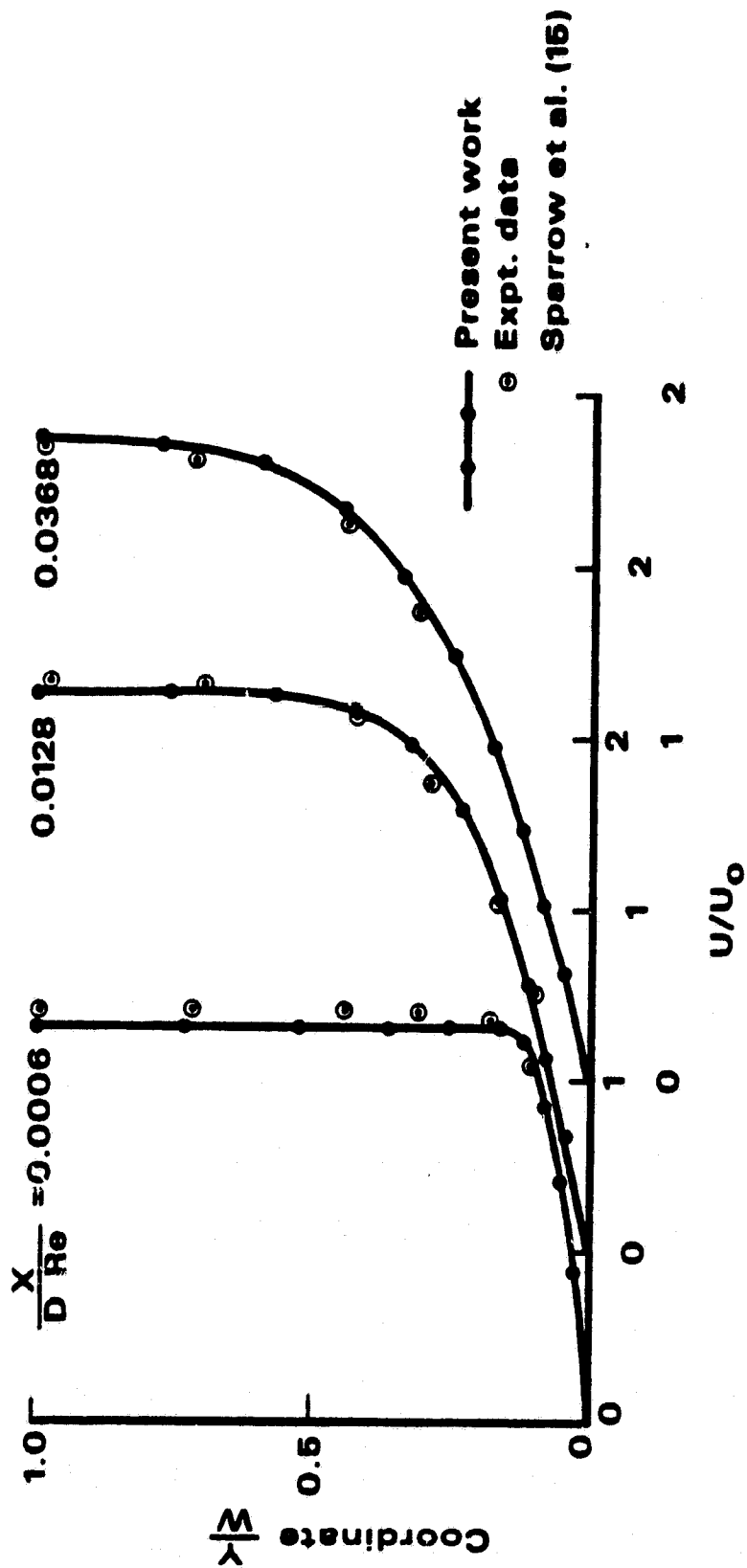


Fig. 4.2a. Development of velocity profile in rectangular duct of 2:1 aspect ratio--across long side.

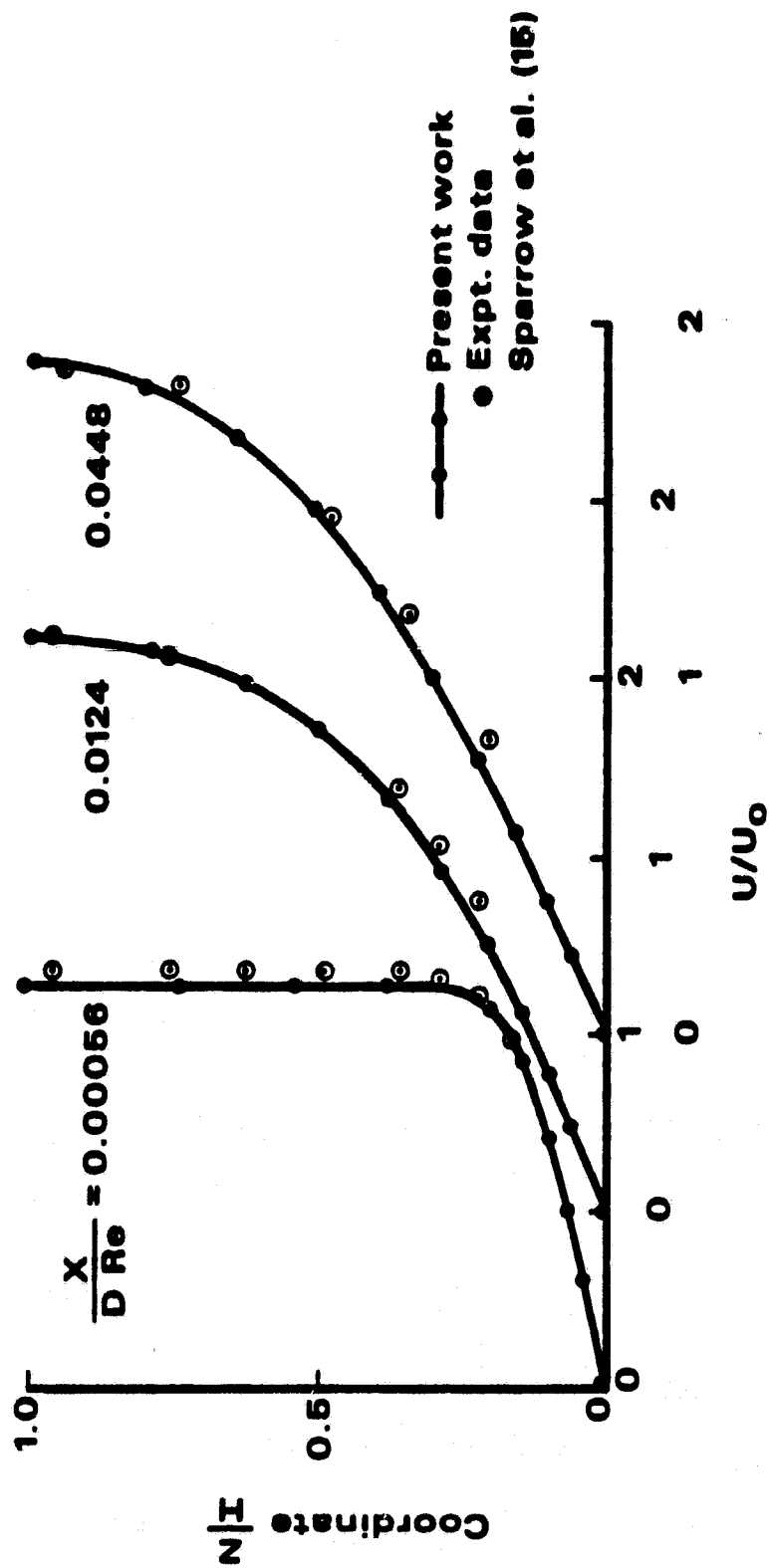


Fig. 4.2b. Development of velocity profile in rectangular duct of 2:1 aspect ratio--across short side.



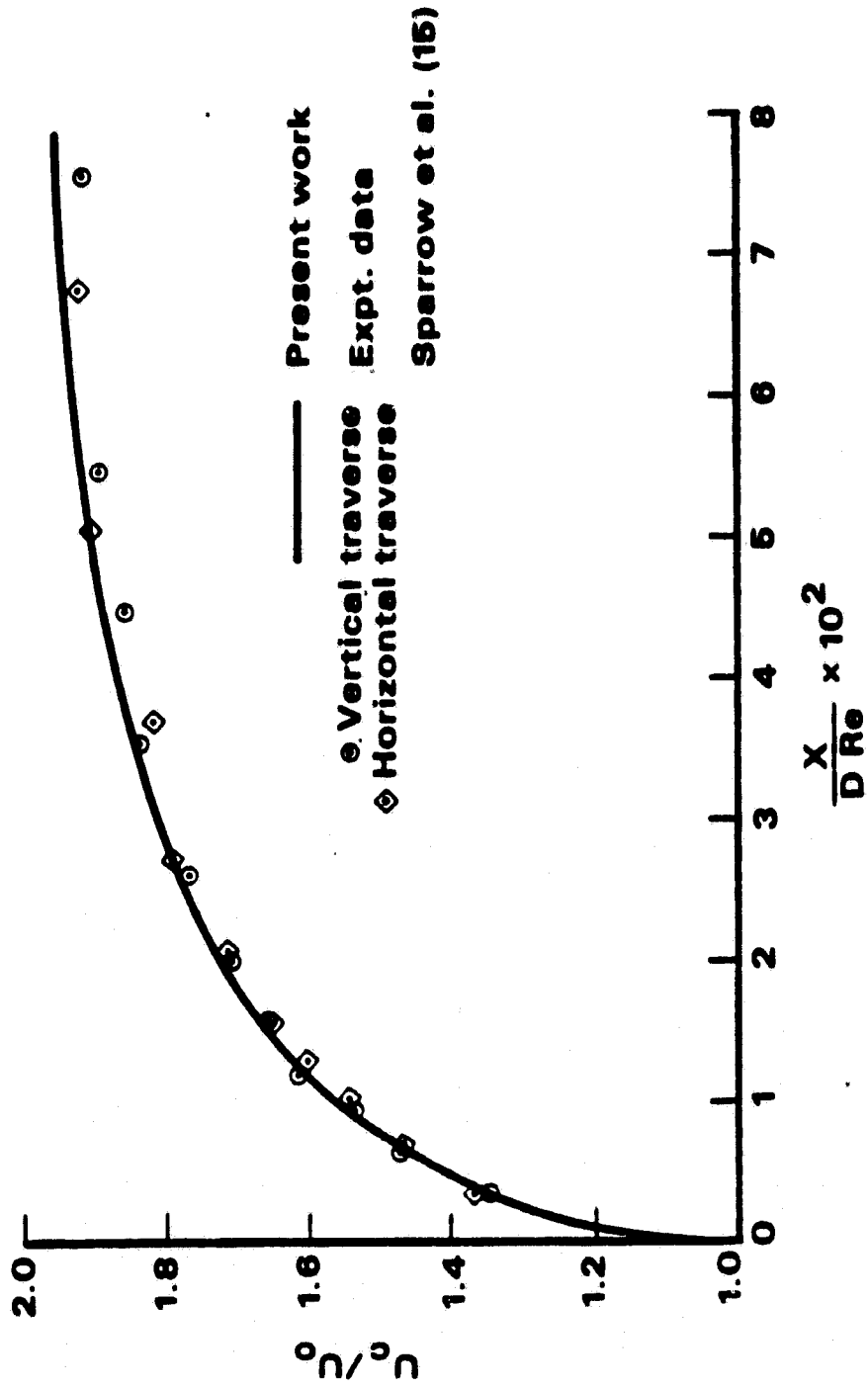


Fig. 4.2c. Center line velocity development in rectangular duct of 2:1 aspect ratio.

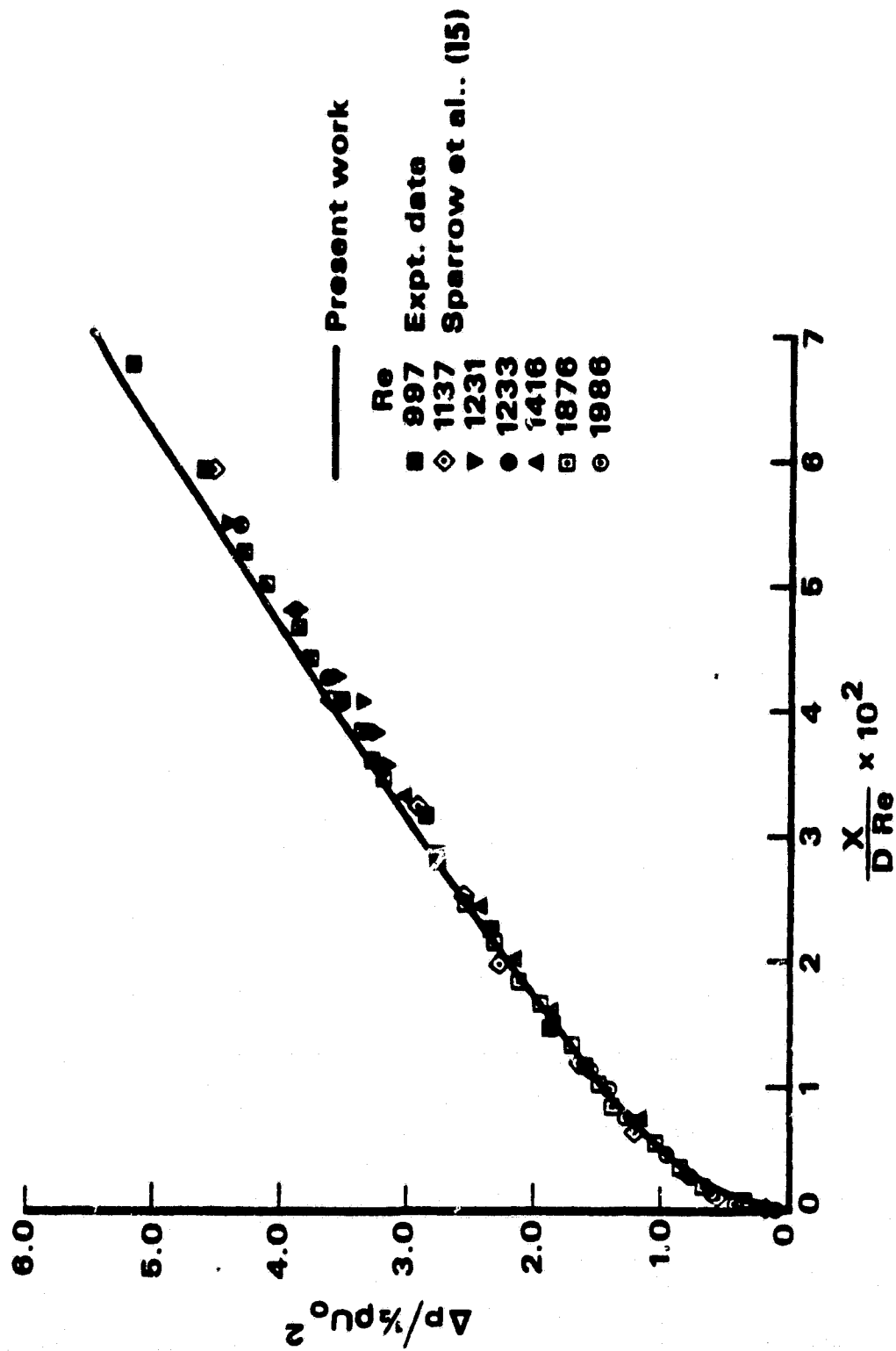


Fig. 4.2d. Pressure drop in rectangular duct of 2:1 aspect ratio.

The walls were maintained at a constant temperature of 100 degrees Celsius below the inlet flow temperature. The velocity and enthalpy finite difference equations were solved together as discussed in Section 3. Calculations were carried out using constant specific heat, perfect gas behavior, and the Prandtl number equal to 0.72. Results of the computations are shown in Fig. 4.3, and are compared with the calculations of Wibulwas as reported in [16] Table 52, p. 220.

To illustrate the velocity field in the cross-stream plane, in Fig. 4.4 are plotted the streamline locations in the lower-left quarter of the duct for air flow through a  $3/2$  meter by  $3/4$  meter rectangular duct (second sample flow computation) at two locations along the duct. The beginnings and the ends of the arrows represent, respectively, the streamline locations at 1 meter and 5 meters from the duct inlet. Bulk velocity through the duct is 0.15 cms/sec.

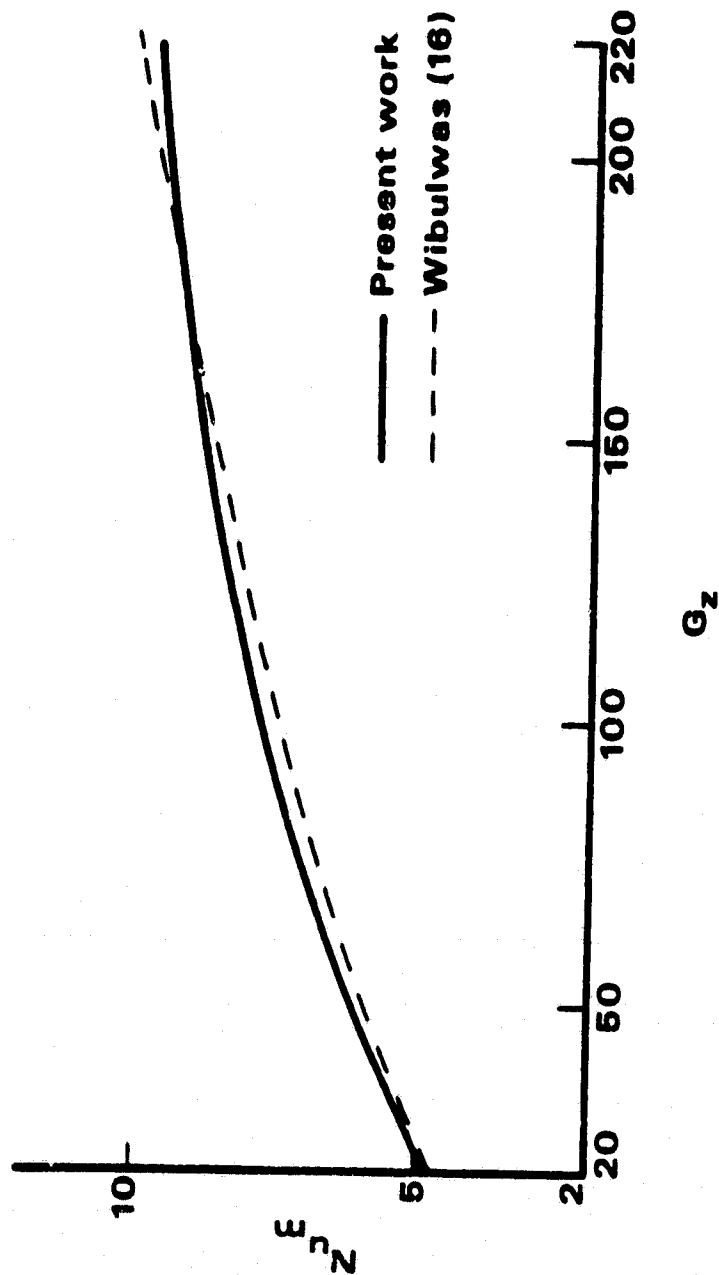


Fig. 4.3. Variation of logarithmic mean Nusselt number with Graetz number for flow through a rectangular duct with constant wall temperature.

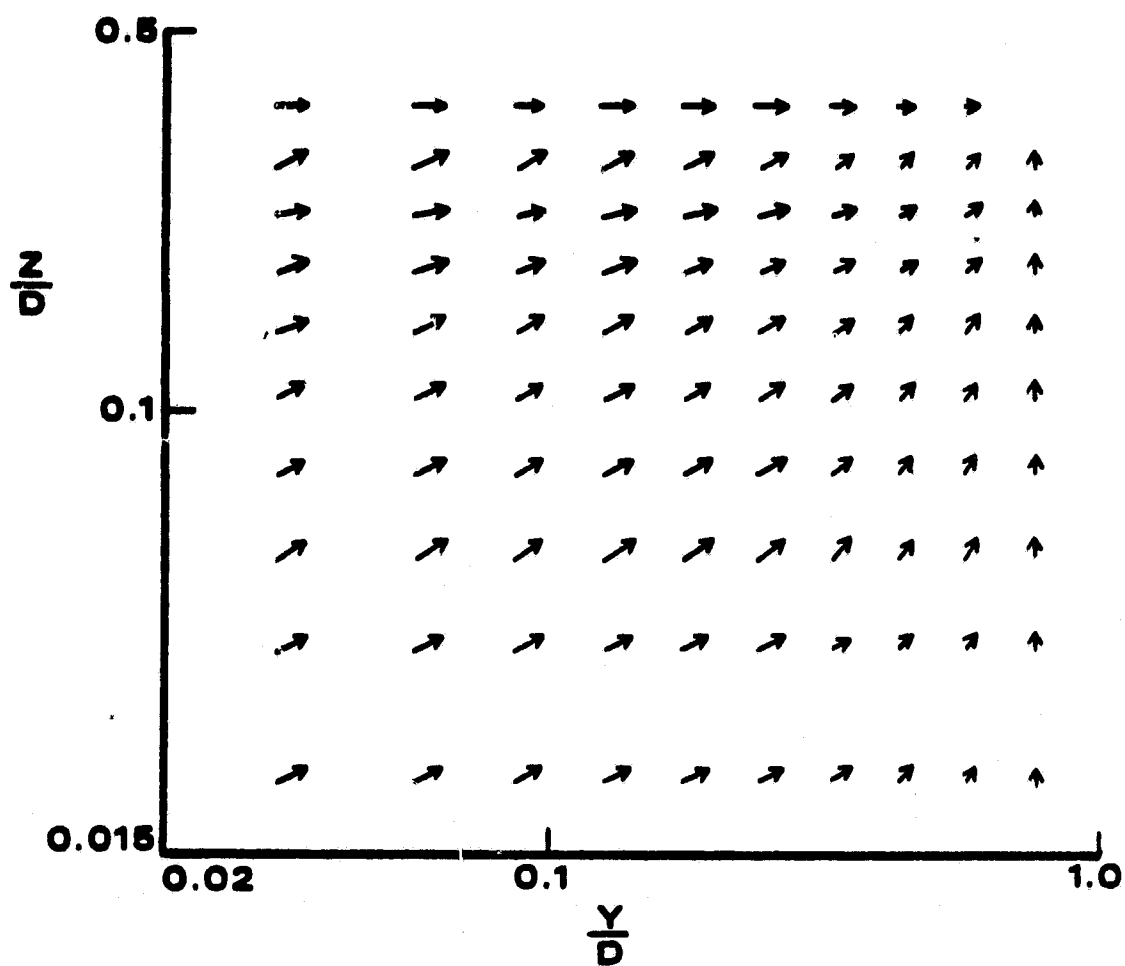


Fig. 4.4. Motion of the streamlines in the cross-stream plane-- figure is for the lower-left quarter of the duct.

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## APPENDIX A

In this appendix formulae which supplement the finite difference equations given in Section 2 are presented. The lower-left corner of the duct is taken as the origin of the y-z plane.

### A.1 Mass Flow Rates

$$\text{Let } R_{i,j} = \rho_{i,j} u_{i,j}$$

Formulae for Stream (2,2):

$$\psi_{2,2}^{--} = \gamma_1^z R_{2,2} Y_2 Z_2$$

$$\psi_{2,2}^{++} = \frac{2R_{2,2} + R_{3,2} + R_{2,3}}{4} \left( \frac{Y_3 - Y_2}{2} \right) \left( \frac{Z_3 - Z_2}{2} \right)$$

$$\psi_{2,2}^{-+} = \gamma_2^y \frac{3R_{2,2} + R_{2,3}}{4} \left( Y_2 \right) \left( \frac{Z_3 - Z_2}{2} \right)$$

$$\psi_{2,2}^{+-} = \gamma_2^z \frac{3R_{2,2} + R_{3,2}}{4} \left( \frac{Y_3 - Y_2}{2} \right) (Z_2)$$

Formulae for Stream (i,2),  $i > 2$ :

$$\psi_{i,2}^{++} = \frac{2R_{i,2} + R_{i+1,2} + R_{i,3}}{4} \left( \frac{Y_{i+1} - Y_i}{2} \right) \left( \frac{Z_3 - Z_2}{2} \right)$$

$$\psi_{i,2}^{-+} = \frac{2R_{i,2} + R_{i,3} + R_{i-1,2}}{4} \left( \frac{Y_i - Y_{i-1}}{2} \right) \left( \frac{Z_3 - Z_2}{2} \right)$$

$$\psi_{i,2}^{--} = \gamma_i^z \frac{3R_{i,2} + R_{i-1,2}}{4} \left( \frac{Y_i - Y_{i-1}}{2} \right) (Z_2)$$

$$\psi_{i,2}^{+-} = \gamma_i^z \frac{3R_{i,2} + R_{i+1,2}}{4} \left( \frac{Y_{i+1} - Y_i}{2} \right) (Z_2)$$

Formulae for Stream (2,j),  $j > 2$ :

$$\psi_{2,j}^{++} = \frac{2R_{2,j} + R_{3,j} + R_{2,j+1}}{4} \left( \frac{Y_3 - Y_2}{2} \right) \left( \frac{Z_{j+1} - Z_j}{2} \right)$$

$$\psi_{2,j}^{-+} = \gamma_j^y \frac{3R_{2,j} + R_{2,j+1}}{4} \left( Y_2 \right) \left( \frac{Z_{j+1} - Z_j}{2} \right)$$

$$\psi_{2,j}^{--} = \gamma_j^y \frac{3R_{2,j} + R_{2,j-1}}{4} \left( Y_2 \right) \left( \frac{Z_j - Z_{j-1}}{2} \right)$$

$$\psi_{2,j}^{+-} = \frac{2R_{2,j} + R_{2,j-1} + R_{3,j}}{4} \left( \frac{Y_3 - Y_2}{2} \right) \left( \frac{Z_j - Z_{j-1}}{2} \right)$$

Formulae for Stream (i,j),  $i$  and  $j > 2$ :

$$\psi_{i,j}^{++} = \frac{2R_{i,j} + R_{i+1,j} + R_{i,j+1}}{4} \left( \frac{Y_{i+1} - Y_i}{2} \right) \left( \frac{Z_{j+1} - Z_j}{2} \right)$$

$$\psi_{i,j}^{-+} = \frac{2R_{i,j} + R_{i,j+1} + R_{i-1,j}}{4} \left( \frac{Y_i - Y_{i-1}}{2} \right) \left( \frac{Z_{j+1} - Z_j}{2} \right)$$

$$\psi_{i,j}^{--} = \frac{2R_{i,j} + R_{i-1,j} + R_{i,j-1}}{4} \left( \frac{Y_i - Y_{i-1}}{2} \right) \left( \frac{Z_j - Z_{j-1}}{2} \right)$$

$$\psi_{i,j}^{+-} = \frac{2R_{i,j} + R_{i,j-1} + R_{i+1,j}}{4} \left( \frac{Y_{i+1} - Y_i}{2} \right) \left( \frac{Z_j - Z_{j-1}}{2} \right)$$

## A.2 Surface and Cross-Sectional Areas

$$A_2^{SJ} = \left( \frac{Y_3 - Y_2}{2} + Y_2 \right) \Delta x$$

$$A_i^{SJ} = \left( \frac{Y_{i+1} - Y_{i-1}}{2} \right) \Delta x$$

$$A_2^{SI} = \left( \frac{Z_3 - Z_2}{2} + Z_2 \right) \Delta x$$

$$A_j^{SI} = \left( \frac{Z_{j+1} - Z_{j-1}}{2} \right) \Delta x$$

$$A_{i,j}^x = \frac{A_i^{SJ} A_j^{SI}}{\Delta x^2}$$

### A.3 Streamline Coordinates $Y_i^+, Z_j^+$

After the velocities and densities have been calculated at  $x + \Delta x$ , the following procedure is used to calculate the streamline coordinates at  $x + \Delta x$ .

$$\text{Let } R_{i,j}^+ = \rho_{i,j}^+ u_{i,j}^+, \text{ and}$$

$$\begin{aligned} G_{i,j} = & \frac{\psi_{i,j}^{--}}{2R_{i,j}^+ + R_{i-1,j}^+ + R_{i,j-1}^+} + \frac{\psi_{i-1,j}^{+-}}{2R_{i-1,j}^+ + R_{i,j}^+ + R_{i-1,j-1}^+} \\ & + \frac{\psi_{i-1,j-1}^{++}}{2R_{i-1,j-1}^+ + R_{i,j-1}^+ + R_{i-1,j}^+} + \frac{\psi_{i,j-1}^{-+}}{2R_{i,j-1}^+ + R_{i-1,j-1}^+ + R_{i,j}^+} \end{aligned}$$

For a given value of  $j$ , by summing the mass flow rates through the streamtubes with  $i = 2$  to  $i = I$ , one obtains for  $j = 2$ ,

$$Z_2^+ Y_1^+ = \frac{\psi_{2,2}^{--}}{\gamma_1^z R_{2,2}^+} + 16 \sum_{i=3}^I \left\{ \frac{\psi_{i-1,2}^{+-}}{\gamma_{i-1}^z (3R_{i-1,2}^+ + R_{i,2}^+)} + \frac{\psi_{i,2}^{--}}{\gamma_1^z (3R_{i,2}^+ + R_{i-1,2}^+)} \right\} \quad (A-1)$$



and for  $j > 2$ ,

$$\begin{aligned} (Z_j^+ - Z_{j-1}^+) Y_I^+ &= \frac{16 \Psi_{2,j}^{--}}{\gamma_j^y (3R_{2,j}^+ + R_{2,j-1}^+)} + \frac{16 \Psi_{2,j-1}^{--}}{\gamma_{j-1}^y (3R_{2,j-1}^+ + R_{2,j}^+)} \\ &+ 16 \sum_{i=3}^I G_{i,j} \end{aligned} \quad (A-2)$$

By adding Eq. (A-1) to Eqs. (A-2) for  $j = 3$  to  $j = J$ , one gets

$$Z_J^+ Y_I^+ = \sum_I, \quad (A-3)$$

where  $\sum_I$  represents the sum of the right-hand sides. By using the duct aspect ratio, specified at  $x + \Delta x$ , Eq. (A-3) can be solved for  $Y_I^+$ . With  $Y_I^+$  known, Eq. (A-1) and Eqs. (A-2) can now be solved for  $Z_2^+, Z_3^+, \dots, Z_J^+$ . The coordinates  $Y_2^+, Y_3^+, \dots, Y_I^+$  are calculated similarly.

#### A.4 Mass, Momentum, and Energy Correction Factors

For laminar flow, the streamlines adjacent to the duct walls can be chosen near enough to the walls so that the velocity and enthalpy distributions between the walls and the adjacent streamlines can be approximated by linear functions. In this case  $V_{i,2}^J$  and  $V_{2,j}^I$ , introduced in Section 2.2, are given by the general expressions for  $V$ 's given in Section 2.1, and the correction factors  $\gamma$ ,  $\alpha$ , and  $\delta$  take on the following values:

$$\gamma_1^z = \frac{1}{4} , \quad \gamma_1^z = \gamma_j^y = \frac{1}{2} ,$$

$$\alpha_1^z = \frac{4}{9} , \quad \alpha_1^z = \alpha_j^y = \frac{2}{3} ,$$

$$\delta_1^z = \frac{1}{4} , \quad \delta_1^z = \delta_j^y = \frac{1}{2} .$$

These are the values for  $\gamma$ ,  $\alpha$ , and  $\delta$  used in the sample flow computations given in Section 4. The finite difference equations for the enthalpy were solved by using the wall temperature as the reference temperature; for this case the  $\beta$ 's become equal to the corresponding  $\alpha$ 's.

For turbulent flow the correction factors  $\gamma$ ,  $\alpha$ ,  $\delta$ , and  $\beta$ , and  $V_{i,2}^J$  and  $V_{2,j}^I$  will depend on the turbulence model chosen for the near wall region in three-dimensional flows. Their calculation can be carried out by using three-dimensional extension of the procedure given in [1] for the case of two-dimensional flows.

#### A.5 Viscous Forces on Stream (i,j)

The viscous forces exerted on the stream (i,j) by the neighboring streams are calculated as follows. The force exerted on the stream (i,j) by the stream (i,j+1), call it  $F(\text{top})$ , is given by

$$F(\text{top}) = A \mu \frac{\partial u}{\partial z} , \quad (\text{A-4})$$

where  $A$  is the interface area between the streams (i,j) and (i,j+1), and  $\mu$  is the effective viscosity at this interface. By substituting the appropriate symbols for  $A$  and  $\mu$  defined in Section 2.1 (see Fig. 2.2), and by approximating the velocity gradient by

$$\frac{u_{i,j+1} - u_{i,j}}{z_{j+1} - z_j},$$

one obtains from Eq. (A-4)

$$F(\text{top}) = A_i^{SJ} \mu_{i,j+1}^J \frac{u_{i,j+1} - u_{i,j}}{z_{j+1} - z_j} \quad (\text{A-5})$$

By using the definition of  $V_{i,j+1}^J$ , Eq. (A-5) is rewritten as,

$$F(\text{top}) = V_{i,j+1}^J (u_{i,j+1} - u_{i,j}) \quad (\text{A-6})$$

Similarly one obtains for the viscous force exerted by the stream (i,j) on the stream (i,j-1), call it F(bottom),

$$F(\text{bottom}) = V_{i,j}^J (u_{i,j} - u_{i,j-1}) \quad (\text{A-7})$$

The net force on the stream (i,j) by the neighboring streams (i,j+1) and (i,j-1) is thus given by,

$$F(\text{top}) - F(\text{bottom}) = V_{i,j+1}^J (u_{i,j+1} - u_{i,j}) - V_{i,j}^J (u_{i,j} - u_{i,j-1}) \quad (\text{A-8})$$

By adding to Eq. (A-8) a similar expression for the net force exerted by the streams (i+1,j) and (i-1,j), one obtains the required relation for the viscous forces exerted on the stream (i,j) by the neighboring streams.

## APPENDIX B

### B.1 Introduction

In this appendix the listing of a computer code which calculates three-dimensional compressible laminar viscous flow through the lower-left quarter of a rectangular duct is presented. If the flow is not symmetric with respect to the center-planes of the duct, the code can be easily modified to compute the flow across the whole duct. The code is organized along the lines of the author's code [17] for two-dimensional flows.

The code consists of MAIN program and the following six subroutines:

- Subroutine START; Assigns values to the streamline coordinates, and the velocity and temperature distributions, at the duct inlet.
- Subroutine XSAREA; Calculates cross-sectional and surface areas of the streamtubes.
- Subroutine SIPSIM; Calculates mass flow rates through the streamtubes.
- Subroutine ZDZYDY; Calculates the new streamline coordinates at the end of an integration step.
- Subroutine VSCSTY; Calculates viscosity.
- Subroutine PDSOLV; Solves a penta-diagonal set of linear algebraic equations.

Subsection B.2 contains the computer listings for the MAIN program and the subroutines.

## B.2 Listings of the MAIN Program and the Subroutines

The following FORTRAN names are used in the MAIN program.

$U(I,J)$  = Velocity along the streamline  $(I,J)$ .

$T(I,J)$  = Temperature along the streamline  $(I,J)$ .

$H(I,J)$  = Enthalpy along the streamline  $(I,J)$ .

$\text{RHO}(I,J)$  = Density along the streamline  $(I,J)$ .

$\text{USH}(I,J) = U^2(I,J)/2$ .

$Y(I); Z(J)$  = Coordinates of the streamline  $(I,J)$ .

$\text{DZ}(J) = Z(J) - Z(J-1)$ .

$\text{DY}(I) = Y(I) - Y(I-1)$ .

$\text{NI}; \text{NJ}$  = Specifies the number of streamlines. Total number of streamlines used is equal to  $(\text{NI}-1)$  times  $(\text{NJ}-1)$ , and corresponds to a  $\text{NI} \times \text{NJ}$  grid.

$\text{NI1} = \text{NI}-1$

$\text{NI2} = \text{NI}-2$

$\text{NIP} = \text{NI}+1$

$\text{NJ1} = \text{NJ}-1$

$\text{NJ2} = \text{NJ}-2$

$\text{NJP} = \text{NJ}+1$

$\text{NBLZ}; \text{NBLY}$  = Specifies the number of streamlines assigned to the boundary layer at the inlet.

$\text{DX}$  = Size of the integration step along the duct axis.

$\text{SIPP}(I,J), \text{SIMP}(I,J), \text{SIMM}(I,J)$ , etc. = Mass flow rates through the various quadrants of the stream  $(I,J)$ . The last two letters in the FORTRAN name SIXX are to be interpreted as;  $P = +$ ,  $M = -$ , and  $O = 0$ ; thus,

$$\text{SIPM}(I,J) = \psi_{i,j}^{+-}$$

$\text{AX}(I,J)$  = Cross-sectional area of the stream  $(I,J)$ .

$\text{ASJ}(I), \text{ASI}(J)$  = Surface areas of the stream  $(I,J)$ .

$ALPHAY(J) = \alpha_j^Y$ ; see Section 2.1.

Other  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$ 's, from Section 2.1, are named similarly.

$AI(I,J) = A_{i,j}^I$  used in the finite difference equations for the velocity or enthalpy.

Other coefficients appearing in the velocity and the enthalpy finite difference equations are named similarly.

Other FORTRAN names appearing in the program are either defined in Section B.1 or defined via assignment statements in the program itself.

Following are some explanatory notes on the various subroutines:

**Subroutine START;** The subroutine assumes  $NI = NJ$ . The grid along  $J$  is set up along the lines discussed in Ref. [17]. The grid along  $I$  (the lower wall) is scaled from the grid along  $J$  using the duct aspect ratio,  $ASPR$ .

**Subroutines XSAREA, SIPSIM, and ZDZYDY;** FORTRAN statements in these subroutines are almost direct translations of the formulae given in Appendix A.

**Subroutine VSCSTY;** In its present form, this subroutine supplies a constant value laminar viscosity to the MAIN program. To extend the code to calculate turbulent flows this subroutine will require a major change.

**Subroutine PDSOLV;** This subroutine solves a set of linear penta-diagonal algebraic equations by using the alternating direction implicit method. The boundary conditions used are (a) the function is zero at the walls, and (b) the function is symmetric about the centerplanes of the duct.

The data is put in via assignment statements in the MAIN program. The input statements have been placed at the very beginning of the program and are organized into the following four groups:

a) Dynamic and thermodynamic data;

UCNTR = Duct centerline velocity

TCNTR = Duct centerline temperature

TWALLZ = Left wall temperature

TWALLY = Lower wall temperature

P = Pressure

CP = Constant pressure specific heat

GASK = Ratio of the specific heats

GASR = Gas constant

PRNDL = Prandtl number

RENUM = Reynolds number

DPDX = Pressure gradient guess at the duct entrance

b) Geometric duct data;

HEIGHT = Half duct height

WIDTH = Half duct width

HSLOPE, WSLOPE = Slopes of the duct walls defined as,

WIDTH at  $x + \Delta x$  = WIDTH at  $x$  + WSLOPE  $\cdot \Delta x$ ,

HEIGHT at  $x + \Delta x$  = HEIGHT at  $x$  + HSLOPE  $\cdot \Delta x$ .

XEND = Length of the duct

c) Convergence criteria, grid size, etc.

DPDTL = Tolerance within which the axial pressure gradient is to be calculated

DXFAC = Specifies the integration step;  $DX = (HEIGHT + WIDTH)/2 \cdot DXFAC$

MAXIT = Maximum number of iterations allowed; if this number is exceeded the program will stop and write out a message

NJ, NI = Number of grid points in the cross-stream plane

NPRNT = Governs the output frequency

d) Data for VSCSTY and START subroutines;

EXPU, EXPV = Exponents for the velocity distributions in the boundary layers along the lower wall and the left wall respectively.

VZERO, TZERO = Optional parameters for calculating laminar viscosity.

The program prints results at specified intervals along the duct axis.

The output consists of,

EX = Distance along the duct measured from the duct entrance.

HEIGHT, WIDTH = Duct half width and half height at EX.

ITRN = Number of iterations required for the last integration step.

DPDX = Pressure gradient.

PREC = Pressure recovery.

Z(J), Y(I) = Coordinates of the streamline (I,J) in the cross-stream plane at EX.

TAUY(I), TAUZ(J) = Shear stress distributions along the lower wall and the left wall, respectively.

QDOTY(I), QDOTZ(J) = Distributions of heat flux to the walls along the lower wall and the left wall, respectively.

TAUX = Average wall shear stress at x.

QDOTX = Average wall heat flux at x.

U(I,J) = Velocity along the streamline (I,J).

T(I,J) = Temperature along the streamline (I,J).

The results for U(I,J) and T(I,J) are printed in a matrix form.



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C
C*****NEMCO-U3-B1*****
C
COMMON/COM1/U(25,25),T(25,25),H(25,25),RHO(25,25),USH(25,25)
COMMON/COM2/Z(25),OZ(25),Y(25),OY(25)
COMMON/COM3/NJ,NJ1,NJ2,NJP,NI,NI1,NI2,NIP,NBLZ,NBLY,DX
COMMON/COM4/SIPP(25,25),SIMP(25,25),SINH(25,25),SIPM(25,25),FLORAT
L,SIPQ(25,25),SINO(25,25),SIOP(25,25),SION(25,25),SIOO(25,25)
COMMON/COM5/UCNTR,TCNTR,THALLZ,THALLY,P,RINV,VZERO,TZERO,EXPV,EXPU
COMMON/COM6/AX(25,25),ASJ(25),ASI(25)
COMMON/COM7/ALPHAY(25),BETAY(25),DELTAY(25),GAMMAY(25),ALPHAZ(25),
LBETAZ(25),DELTAZ(25),GAMMAZ(25),VISCJ(25,25),VISCJ(25,25)
COMMON/COM8/HEIGHT,WIDTH,WSLOPE,WSLOPE,OLDA,ASPR
DIMENSION AI(25,25),CI(25,25),BB(25,25),AJ(25,25),CJ(25,25),DD(25,
125),VFJ(25,25),VFI(25,25),HTJ(25,25),HTI(25,25),UU(25,25),
2DU(25,25),UN(25,25),OT(25,25),TT(25,25),
3TAUY(20),TAUZ(20),CGTY(20),QOOTZ(20),HH(25,25)
C
C**DATA INPUT *****
C**DYNAMIC AND THERMODYNAMIC DATA*****
UCNTR=0.0015
TCNTR=100.0
THALLZ=0.0
THALLY=0.0
P=71000.0
CP=1003.5
GASK=1.4
GASR=287.0
PRNOL=0.72
RENUM=100.0
OPDX=-0.000075
C GEOMETRIC DUCT DATA*****
HEIGHT=3./8.
WIDTH=3./4.
ASPR=WIDTH/HEIGHT
WSLOPE=0.0
HSLOPE=0.0
XEND=0.5
C**CONVERGENCE CRITERIA,GRID SIZE,AND OTHER DATA*****
DPOTL=0.01
ACLNT=0.00001
DXFAC=5.0
MAXIT=150
NJ=11
NI=11
NPRNT=1
C**DATA FOR VISCSTY AND SATART SUBROUTINE*****
VZERO=0.0000386

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TZERO=860.3
EXPV=.7
EXPU=.7
C
C===== INITIALIZATION FOR LAMINAR FLOWS =====
DO 20 J=1,NJ
  BETAY(J)=2./3.
  DELTAY(J)=0.5
  GAMMAY(J)=0.5
20 ALPHAY(J)=2./3.
DO 21 I=1,NI
  BETAZ(I)=2./3.
  DELTAZ(I)=0.5
  GAMMAZ(I)=0.5
21 ALPHAZ(I)=2./3.
  DELTAZ(I)=0.25
  BETAZ(I)=4./9.
  GAMMAZ(I)=0.25
  ALPHAZ(I)=4./9.
C== END LAMINAR FLOW INITIALIZATION.=====
C==INITIALIZATIONS=====
XAMP=2.0
ISTEP=0
ITRN=0
NPI=NPRINT-1
NJP=NJ+1
NJ1=NJ-1
NJ2=NJ-2
NIP=NI+1
NI1=NI-1
NI2=NI-2
DO 50 J=1,NJ
50 CI(2,J)=0.
DO 52 I=1,NI
52 CJ(I,2)=0.
  OXF=(HEIGHT+WIDTH)/2.0/OXFAC
  OX=OXF/512.
  UXF=OXF-ACENT
  EX=-OX
  UP=OPDX*OX
  PRLINV=1./PRNDL
  CPINV=1./CP
  RINV=1./GASR
  GASKIN=1./GASK
  SNDSPO=(GASK*GASR*TCNTR)**0.5
  PINL=P
  OPL=0.
  OP2=0.

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DX1=0.
OLDA=HEIGHT*WIDTH
CALL START
WRITE(6,141) (Z(J),J=2,NJ)
WRITE(6,142) (Y(I),I=2,NI)
CALL XSAREA(1)
CALL VSCSTY(1)
CALL SIPSIM(0)
CALL ZDZYDY
PRFAC=0.5*RHO(NI,NJ)*UCNTR*UCNTR
OPTOT=-OP
DO 53 N=1,20
VFJ(N,2)=0.
VF(12,N)=0.
HTJ(N,2)=0.
53 HTI(2,N)=0.
C
C**AXIAL CALCULATIONS*****
100 ISTEP=ISTEP+1
EX=EX+OX
OPTOT=OPTOT+OP
P=P+OPTOT
CALL VSCSTY(0)
NPI=NPI+1
IF(XAMP.GT.1) NPI=NPRNT
IF(NPI.LT.NPRNT) GO TO 200
NPI=0
DPOX=OP/OX
PREC=OPTOT/PRFAC
TAU=0.
QDOT=0.
DO 111 I=2,NI
TAUY(I)=2.*VFJ(I,2)*U(I,2)
QDOTY(I)=2.*HTJ(I,2)*(H(I,2)-H(I,1))
TAU=TAU+TAUY(I)
111 QDOT=QDOT+QDOTY(I)
DO 112 J=2,NJ
TAUZ(J)=2.*VFI(2,J)*U(2,J)
QDOTZ(J)=2.*HTI(2,J)*(H(2,J)-H(1,J))
TAU=TAU+TAUZ(J)
112 QDOT=QDOT+QDOTZ(J)
TAUX=TAU/(HEIGHT+WIDTH)/OX
QDOTX=QDOT/(HEIGHT+WIDTH)/OX
WRITE(6,130) EX,HEIGHT,WIDTH,ITRN,DPOX,PREC
130 FORMAT(1H0,'DIS. ALONG X=',F7.4,'** HEIGHT=',F7.4,'*** WIDTH=',
1 F7.4,'*** ITERATIONS=',I3,'*** DPOX=',E11.4,'*** PREC=',E11.4)
WRITE(6,131) TAUX,QDOTX
131 FORMAT(1H,'TAUX=',E11.4,'*** QDOTX=',E11.4)

```

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```
WRITE(6,141) (Z(J),J=2,NJ)
WRITE(6,142) (Y(I),I=2,NI)
WRITE(6,144) (TAUY(I),I=2,NI)
WRITE(6,145) (TAUZ(J),J=2,NJ)
WRITE(6,146) (QDOTY(I),I=2,NI)
WRITE(6,147) (QDOTZ(J),J=2,NJ)
DO 135 JREV=1,NJ1
J=NJP-JREV
135 WRITE(6,143) J, (U(I,J),I=2,NI)
DO 136 JREV=1,NJ1
J=NJP-JREV
136 WRITE(6,148) J, (T(I,J),I=2,NI)
141 FORMAT(1H0,'Z',8X,1P10E11.4/(10X,1P10E11.4))
142 FORMAT(1H,'Y',8X,1P10E11.4/(10X,1P10E11.4))
143 FORMAT(1H,'U',J=1,12,2X,1P10E11.4/(10X,1P10E11.4))
144 FORMAT(1H,'TAUY',5X,1P10E11.4/(10X,1P10E11.4))
145 FORMAT(1H,'TAUZ',5X,1P10E11.4/(10X,1P10E11.4))
146 FORMAT(1H,'QDOTY',4X,1P10E11.4/(10X,1P10E11.4))
147 FORMAT(1H,'QDOTZ',4X,1P10E11.4/(10X,1P10E11.4))
148 FORMAT(1H,'T',J=1,12,2X,1P10E11.4/(10X,1P10E11.4))
IF (EX.GT.XEND) GO TO 3000
```

C

C== RESETTING DYNAMIC, THERMO, AND GEOM. VARIABLES FOR NEXT ITERATION=====

```
200 ITRN=1
X1=DX1
X2=OX+DX1
SUMX=X1+X2
SUMXX=X1*X1+X2*X2
SUMP=OP+OP1+OP2
SUMXP=X1*OP1+X2*OP
DEN=3.*SUMXX-SUMX*SUMX
JPSLOP=(3.*SUMXP-SUMP*SUMX)/DEN
OPZERO=(SUMP-SUMXX-SUMX*SUMXP)/DEN
DX1=OX
OP2=OP1
OP1=OP
IF (ISTEP.LE.2) GO TO 220
IF (DX.GE.OXF) XAMP=1.0
UX=OX*XAMP
IF (ISTEP.LT.4) GO TO 220
OP=OPSLUP*(X2+OX)+OPZERO
220 PSTART=P
OPDAMP=0.5*EXP(-0.115*EX/HEIGHT)
UOLD=U(2,2)
CALL SIPS IN(0)
CALL XSARE(0)
ZYFAC=FLORAT*U(NI,NJ)/(QLDA-FLORAT*U(NI,NJ)*GASKIN/P)
ZYTLRN=OP1-OPOTL*QLDA/ZYFAC
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UTLRN=OPI*OPDTL*CLDA/FLORAT
IF (ZYLNR.LT.ACLMT*OLDA) ZYLNR=ACLMT*OLDA
IF (UTLRN.LT.ACLMT*UOLD) UTLRN=ACLMT*UOLD

```

C

C\*\*\*\* EVALUATION OF R.H.S. OF MOM. AND ENERGY EQS. \*\*\*\*\*

```

DO 300 J=2,NJ
DO 300 I=2,NI
VFJ(I,J)=0.5*ASJ(I)*VISCJ(I,J)/DZ(J)
VFI(I,J)=0.5*ASI(J)*VISC(I,J)/DY(I)
HTI(I,J)=VFI(I,J)*PRLINV
300 HTJ(I,J)=VFJ(I,J)*PRLINV
DO 301 J=2,NJ
HTI(NIP,J)=HTI(NI,J)
301 VFI(NIP,J)=VFI(NI,J)
DO 302 I=2,NI
HTJ(I,NJP)=HTJ(I,NJ)
302 VFJ(I,NJP)=VFJ(I,NJ)
DU(2,2)=U(3,2)*(SIPP(2,2)+ALPHAZ(2)*SIMP(2,2)+VFI(3,2))+U(2,2)*
1(2.*SIPP(2,2)+3.*ALPHAY(2)*SIMP(2,2)+3.*ALPHAZ(2)*SIMP(2,2)+
2ALPHAZ(1)*SIMP(2,2)-VFJ(2,2)-VFI(2,2)-VFJ(2,3)-VFI(3,2))+
3*U(2,3)*(SIPP(2,2)+ALPHAY(2)*SIMP(2,2)+VFJ(2,3))
DO 310 I=3,NI
310 DU(I,2)=U(I-1,2)*(SIMP(I,2)+ALPHAZ(I)*SIMP(I,2)+VFI(I,2))+
1U(I+1,2)*(SIPP(I,2)+ALPHAZ(I)*SIMP(I,2)+VFI(I+1,2))+
2U(I,2)*(2.*SIOP(I,2)+3.*ALPHAZ(I)*SIMP(I,2)-VFI(I+1,2)-VFJ(I,3)-
3VFI(I,2)-VFJ(I,2))+U(I,3)*(SIOP(I,2)+VFJ(I,3))
DO 311 J=3,NJ
311 DU(2,J)=U(3,J)*(SIPO(2,J)+VFI(3,J))+U(2,J)*(2.*SIPO(2,J)+
13.*ALPHAY(J)*SIPO(2,J)-VFJ(2,J)-VFJ(2,J+1)-VFI(2,J)-VFJ(2,J))+
2U(2,J+1)*(SIPP(2,J)+ALPHAY(J)*SIMP(2,J)+VFJ(2,J+1))+
3U(2,J-1)*(ALPHAY(J)*SIMP(2,J)+SIMP(2,J)+VFJ(2,J))
DO 312 J=3,NJ
DO 312 I=3,NI
312 DU(I,J)=U(I+1,J)*(SIPO(I,J)+VFI(I+1,J))+U(I-1,J)*(SIPO(I,J)+
1VFI(I,J))+U(I,J)*(SIPO(I,J)-VFJ(I+1,J)-VFJ(I,J+1)-VFI(I,J)-
2VFJ(I,J))+U(I,J+1)*(SIOP(I,J)+VFJ(I,J+1))+U(I,J-1)*(SIOP(I,J)+
3VFJ(I,J))
DO 315 J=2,NJ
DO 315 I=2,NI
315 DU(I,J)=DU(I,J)-DP*AX(I,J)

```

C

```

OT(2,2)=H(3,2)*(SIPP(2,2)+BETAZ(2)*SIMP(2,2)+HTI(3,2))+H(2,2)*
1(2.*SIPP(2,2)+3.*BETAY(2)*SIMP(2,2)+BETAZ(1)*SIMP(2,2)+3.*BETAZ(2)*
2SIMP(2,2)-HTI(3,2)-HTJ(2,3)-HTI(2,2)-HTJ(2,2))+H(2,3)*(SIPP(2,2)+
3BETAY(2)*SIMP(2,2)+HTJ(2,3))+USH(3,2)*(SIPP(2,2)+DELTAZ(2)+
4SIMP(2,2)+VFI(3,2))+USH(2,2)*(2.*SIPP(2,2)+3.*DELTAZ(2)*SIMP(2,2)+
5DELTAZ(1)*SIMP(2,2)+3.*DELTAZ(2)*SIMP(2,2)-VFI(3,2)-VFJ(2,3)-
6-VFI(2,2)-VFJ(2,2))+USH(2,3)*(SIPP(2,2)+DELTAZ(2)*SIMP(2,2)

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7 +VFJ(2,3)) +H(1,2)*HTI(2,2)+H(2,1)*HTJ(2,2)
DO 321 I=3,N1
321 DT(I,2)=H(I+1,2)*(SIPP(1,2)+BETAZ(1)*SIMP(1,2)+HTI(I+1,2)) +
  H(I-1,2)*(SIMP(1,2)+BETAZ(1)*SIMP(1,2)+HTI(I,2))+ H(I,2)*(2.*
  2SIOP(1,2)+3.*BETAZ(1)*SIOM(1,2)-HTI(I+1,2)-HTJ(I,3)-HTI(I,2)-
  3HTJ(I,2)) +H(I,3)*(SIOP(1,2)+HTJ(I,3)) +USH(I+1,2)*(SIPP(1,2)+
  4DELTAZ(1)*SIMP(1,2)+VFI(I+1,2)) +USH(I-1,2)*(SIMP(1,2)+DELTAZ(1)*
  5SIMP(1,2)+VFI(I,2)) +USH(I,2)*(2.*SIOP(1,2)+3.*DELTAZ(1)*SIOM(1,2)
  6-VFI(I+1,2)-VFJ(I,3)-VFI(I,2)-VFJ(I,2)) +USH(I,3)*(SIOP(1,2)+
  7VFJ(I,3)) +H(I,1)*HTJ(I,2)
DO 322 J=3,NJ
322 DT(2,J)=H(3,J)*(SIPO(2,J)+HTI(3,J)) +H(2,J)*(2.*SIPO(2,J)+3.*
  1BETAY(J)*SIOM(2,J)-HTI(3,J)-HTJ(2,J+1)-HTI(2,J)-HTJ(2,J))+
  2H(2,J+1)*(SIPP(2,J)+BETAY(J)*SIMP(2,J)+HTJ(2,J+1))+ H(2,J-1)*
  3(BETAY(J)*SIMP(2,J)+SIMP(2,J)+HTJ(2,J)) +USH(3,J)*SIPO(2,J)+
  4VFI(3,J)+USH(2,J)*(2.*SIPO(2,J)+3.*DELTAZ(J)*SIOM(2,J)-
  5VFI(3,J)-VFJ(2,J+1)-VFI(2,J)-VFJ(2,J)) +USH(2,J+1)*(SIPP(2,J)+
  6DELTAZ(J)*SIMP(2,J)+VFI(2,J+1)) +USH(2,J-1)*(DELTAZ(J)*
  7SIMP(2,J)+SIMP(2,J)+VFJ(2,J))+ H(1,J)*HTI(2,J)
DO 323 J=3,NJ
DO 323 I=3,N1
323 DT(I,J)=H(I+1,J)*(SIPO(I,J)+HTI(I+1,J)) +H(I-1,J)*(SIOM(I,J)+
  1HTI(I,J))+ H(I,J)*(SIOP(I,J)-HTI(I+1,J)-HTJ(I,J+1)-HTI(I,J)-
  2HTJ(I,J))+ H(I,J+1)*(SIOP(I,J)+HTJ(I,J+1))+ H(I,J-1)*(SIOM(I,J)+
  3HTJ(I,J)) +USH(I+1,J)*(SIPO(I,J)+VFI(I+1,J)) +USH(I-1,J)*
  4SIOM(I,J)+VFI(I,J)) +USH(I,J)*(SIOP(I,J)-VFI(I+1,J)-VFJ(I,J+1)-
  5VFI(I,J)-VFJ(I,J)) +USH(I,J+1)*(SIOP(I,J)+VFJ(I,J+1)) +USH(I,J-1)*
  6(SIOM(I,J)+VFJ(I,J))
C==ITERATIVE CALCULATIONS FOR NEW U ,H ,Z *****
C==CALCULATIONS OF NEW VELOCITIES*****
GO TO 500
400 CONTINUE
DO 410 J=2,NJ
DO 410 I=2,N1
VFJ(I,J)=0.5*ASJ(I)*VISCJ(I,J)/OZ(J)
VFI(I,J)=0.5*ASJ(I)*VISCJ(I,J)/OY(I)
HTI(I,J)=VFI(I,J)*PRLINV
410 HTJ(I,J)=VFJ(I,J)*PRLINV
DO 411 J=2,NJ
HTI(NIP,J)=HTI(NI,J)
411 VFI(NIP,J)=VFI(NI,J)
DO 412 I=2,N1
HTJ(I,NJP)=HTJ(I,NJ)
412 VFJ(I,NJP)=VFJ(I,NJ)
500 CONTINUE
A(12,2)=-SIPP(2,2)-ALPHAZ(2)*SIMP(2,2)+VFI(3,2)
3B(2,2)=2.*SIPP(2,2)+3.*ALPHAY(2)*SIMP(2,2)+3.*ALPHAZ(2)*SIMP(2,2)
1+ALPHAZ(1)*SIMP(2,2)+VFJ(2,2)+VFI(2,2)+VFJ(2,3)+VFI(3,2)

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      AJ(2,2) = -SIPP(2,2) - ALPHAY(2) * SIMP(2,2) + VFJ(2,3)
      DO 515 I=3,NI
      AI(1,2) = -SIPP(1,2) - ALPHAZ(1) * SIMP(1,2) + VFI(1+1,2)
      CI(1,2) = -SIMP(1,2) - ALPHAZ(1) * SIMM(1,2) + VFI(1,2)
      BB(1,2) = 2. * SLOP(1,2) + 3. * ALPHAZ(1) * SIMM(1,2) + VFI(1+1,2) +
      1 VFJ(1,3) + VFI(1,2) + VFJ(1,2)
      AJ(1,2) = -SLOP(1,2) + VFJ(1,3)
515  CONTINUE
      DO 516 J=3,NJ
      AI(2,J) = -SLOP(2,J) + VFI(3,J)
      BB(2,J) = 2. * SLOP(2,J) + 3. * ALPHAY(J) * SIMM(2,J) + VFI(3,J) +
      1 VFJ(2,J+1) + VFI(2,J) + VFJ(2,J)
      AJ(2,J) = -SIPP(2,J) - ALPHAY(J) * SIMP(2,J) + VFJ(2,J+1)
      CJ(2,J) = -ALPHAY(J) * SIMM(2,J) - SIMP(2,J) + VFJ(2,J)
516  CONTINUE
      DO 520 J=3,NJ
      DO 520 I=3,NI
      AI(1,J) = -SLOP(1,J) + VFI(1+1,J)
      CI(1,J) = -SIMM(1,J) + VFI(1,J)
      BB(1,J) = SLOP(1,J) + VFI(1+1,J) + VFJ(1,J+1) + VFI(1,J) + VFJ(1,J)
      AJ(1,J) = -SLOP(1,J) + VFJ(1,J+1)
      CJ(1,J) = -SIMM(1,J) + VFJ(1,J)
520  CONTINUE
      CALL POSOLV(AI,CI,BB,AJ,CJ,DD,U,UU)
      IF (ITRN.GT.1) GO TO 340
      DO 530 J=2,NJ
      DO 530 I=2,NI
530  U(I,J) = UU(I,J)
540  DO 550 J=2,NJ
      DO 550 I=2,NI
550  U(I,J) = 0.5 * (U(I,J) + UU(I,J))
      DO 560 J=2,NJ
560  U(NIP,J) = U(NI1,J)
      DO 570 I=2,NI
570  U(I,NJP) = U(I,NJ1)
      U(NIP,NJP) = U(NI1,NJ1)
C
C**CALCULATION OF NEW ENTHALPIES*****
      DO 510 J=2,NJP
      DO 510 I=2,NIP
510  USH(I,J) = 0.5 * U(I,J) * U(I,J)
      AI(2,2) = -SIPP(2,2) - BETAZ(2) * SIMP(2,2) + HTI(3,2)
      BB(2,2) = 2. * SIPP(2,2) + 3. * BETAY(2) * SIMP(2,2) + BETAZ(1) * SIMM(2,2) +
      13. * BETAZ(2) * SIMP(2,2) + HTI(3,2) + HTJ(2,3) + HTI(2,2) + HTJ(2,2)
      AJ(2,2) = -SIPP(2,2) - BETAY(2) * SIMP(2,2) + HTJ(2,3)
      TT(2,2) = UT(2,2) + H(1,2) * HTI(2,2) + H(2,1) * HTJ(2,2) + USH(3,2) * (-
      1SIPP(2,2) - DELTAZ(2) * SIMP(2,2) + VFI(3,2)) - USH(2,2) * (2. * SIPP(2,2) +
      23. * DELTAY(2) * SIMP(2,2) + DELTAZ(1) * SIMM(2,2) + 3. * DELTAZ(2) * SIMP(2,2) +

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3VF(1,2)+VFJ(2,3)+VF(1,2)+VFJ(2,2)+ USH(2,3)*(-SIPP(2,2)-
+DELTA(2)*SIMP(2,2)+VFJ(2,2))
DO 615 I=3,N1
AI(1,2)=-SIPP(1,2)-BETAY(1)*SIMP(1,2)+HTI(1+1,2)
CI(1,2)=-SIMP(1,2)-BETAY(1)*SIMP(1,2)+HTI(1,2)
BB(1,2)=2.*SICP(1,2)+3.*BETAY(1)*SICM(1,2)+HTI(1+1,2)+HTJ(1,3)+
1HTI(1,2)+HTJ(1,2)
AJ(1,2)=-SIPP(1,2)+HTJ(1,3)
TT(1,2)=OT(1,2)+H(1,1)+HTJ(1,2)+ USH(1+1,2)*(-SIPP(1,2)-DELTA(1)*
1SIMP(1,2)+VF(1+1,2))+ USH(1-1,2)*(-SIMP(1,2)-DELTA(1)*SIMP(1,2)+
2VF(1,2))- USH(1,2)*(2.*SIPP(1,2)+3.*DELTA(1)*SICM(1,2)+
3VF(1+1,2)+VFJ(1,3)+VF(1,2)+VFJ(1,2))+USH(1,3)*(-SIPP(1,2)+
4VFJ(1,3))
615 CONTINUE
DO 616 J=3,NJ
AI(2,J)=-SIPP(2,J)+HTI(3,J)
BB(2,J)=2.*SIPP(2,J)+3.*BETAY(J)*SIMP(2,J)+HTI(3,J)+HTJ(2,J+1)+
1HTI(2,J)+HTJ(2,J)
AJ(2,J)=-SIPP(2,J)-BETAY(J)*SIMP(2,J)+HTJ(2,J+1)
CJ(2,J)=-BETAY(J)*SIMP(2,J)-SIMP(2,J)+HTJ(2,J)
TT(2,J)=OT(2,J)+H(1,J)+HTI(2,J)+ USH(3,J)*(-SIPP(2,J)+VF(3,J))-
1USH(2,J)*(2.*SIPP(2,J)+3.*DELTA(J)*SIMP(2,J)+VF(3,J)+VFJ(2,J+1)+
2VF(2,J)+VFJ(2,J))+ USH(2,J+1)*(-SIPP(2,J)-DELTA(J)*SIMP(2,J)+
3VFJ(2,J+1))+ USH(2,J-1)*(-DELTA(J)*SIMP(2,J)-SIMP(2,J)+
4VFJ(2,J))
616 CONTINUE
DO 620 J=2,NJ
DO 620 I=2,N1
AI(1,J)=-SIPP(1,J)+HTI(1+1,J)
CI(1,J)=-SIMP(1,J)+HTI(1,J)
BB(1,J)=SICP(1,J)+HTI(1+1,J)+HTJ(1,J+1)+HTI(1,J)+HTJ(1,J)
AJ(1,J)=-SIPP(1,J)+HTJ(1,J+1)
CJ(1,J)=-SICM(1,J)+HTJ(1,J)
TT(1,J)=OT(1,J)+ USH(1+1,J)*(-SIPP(1,J)+VF(1+1,J))+USH(1-1,J)*
1(-SIMP(1,J)+VF(1,J))-USH(1,J)*(SICP(1,J)+VF(1+1,J)+VFJ(1,J+1)+
2VF(1,J)+VFJ(1,J))+USH(1,J+1)*(-SIPP(1,J)+VFJ(1,J+1))+USH(1,J-1)*
3(-SICM(1,J)+VFJ(1,J))
620 CONTINUE
DO 621 J=2,NJP
DO 621 I=2,NIP
621 H(1,J)=H(1,J)-H(1,1)
CALL POSOLV(AI,CI,BB,AJ,CJ,TT,H,HH)
IF (ITRN.JT.1) GO TO 640
DO 630 J=2,NJ
DO 630 I=2,N1
630 H(1,J)=HH(1,J)
640 DO 650 J=2,NJ
DO 650 I=2,N1

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650 H(I,J)=0.5*(H(I,J)+HH(I,J))+H(1,1)
    DO 660 J=2,NJ
660 H(NIP,J)=H(NI1,J)
    DO 670 I=2,NI
670 H(I,NJP)=H(I,NJ1)
    H(NIP,NJP)=H(NI1,NJ1)
    DO 1100 J=2,NJP
    DO 1100 I=2,NIP
1100 T(I,J)=H(I,J)*CPINV
C
C== CONVERGENCE CHECKS=====
    CALL ZOZYDY
    ZCOR=Z(NJ)-HEIGHT
    YCOR=Y(NI)-WIDTH
    UCHEK=U(2,2)-UCLO
    UCHEK=ABS(UCHEK)
    ZYCHEK=ABS(WIDTH*ZCOR)+ABS(HEIGHT*YCOR)
    IF(UCHEK.GT.UTLRN)GO TO 950
    IF(ZYCHEK.LT.ZYTLRN) GO TO 100
    UPCOR=-(ZCOR*WIDTH+YCOR*HEIGHT)*ZYFAC/OLDA
    OPCOR=UPCOR*OPDAMP
    OP=OP+OPCOR
    DO 940 J=2,NJ
    DO 940 I=2,NI
940 UO(I,J)=OU(I,J)-OP*AX(I,J)
950 IF(ITRN.GT.MAXIT) GO TO 2100
    UOLD=U(2,2)
    CALL VSCSTY(0)
    ITRN=ITRN+1
    GO TO 400
2100 WRITE(6,2101)
2101 FORMAT(//,IX,'**VELOCITY-PRESSURE ITERATIONS EXCEEDED THE LIMIT SET
      1 BY MAXIT=====')
3000 CONTINUE
    STOP
    END

```

# ORIGINAL FORM OF PROGRAM

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START

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SUBROUTINE START
COMMON/COM1/U(25,25),T(25,25),H(25,25),RHC(25,25),USH(25,25)
COMMON/COM2/Z(25),DZ(25),Y(25),DY(25)
COMMON/COM3/NJ,NJ1,NJ2,NJP,NI,NI1,NI2,NIP,NBLZ,NBLY,DX
COMMON/COM5/UCNTR,TCNTR,TWALLZ,TWALLY,P,AINV,VZERO,TZERO,EXPV,EXPU
COMMON/COM6/AX(25,25),ASJ(25),ASI(25)
COMMON/COM8/HEIGHT,WIDTH,HSLOPE,VSLOPE,OLDA,ASPR
CP=1003.9
ZSL=0.1
VISKIN=VZERO*(300.0/TZERO)**EXPV/(P*AINV/300.0)
FNJ=FLCAT(NJ)
UZZZ=1200.*VISKIN
DO 10 I=1,NIP
  RHO(I,1)=1.0
  T(I,1)=TWALLZ
  H(I,1)=TWALLZ*CP
  USH(I,1)=0.0
10 U(I,1)=0.0
DO 15 J=1,NJP
  RHO(1,J)=1.0
  T(1,J)=TWALLY
  H(1,J)=TWALLY*CP
  USH(1,J)=0.0
15 U(1,J)=0.0
  Z(1)=0.0
  DZ(1)=0.0
  UZZ=(UCNTR**EXPU*UZZZ/ZSL)**(1./(1.+EXPU))
  UZ2=(10.00**EXPU*UZZZ/ZSL)**(1./(1.+EXPU))
  Z(2)=UZZZ/UZZ
  DZ(2)=Z(2)-Z(1)
  EXPG=ALOG(HEIGHT/Z(2)-1.)/ALOG(FNJ/3.)
  DO 20 J=3,NJ
    FJ=FLCAT(J)
    Z(J)=Z(2)+(HEIGHT-Z(2))*(FJ/FNJ)**EXPG
20 DZ(J)=Z(J)-Z(J-1)
  UC 30 I=1,NI
    Y(I)=Z(I)*ASPR
30 DY(I)=DZ(I)*ASPR
  DO 150 I=2,NIP
    DO 150 J=2,NJP
      RHO(I,J)=1.0
      T(I,J)=TCNTR
      H(I,J)=TCNTR*CP
      USH(I,J)=0.5*UCNTR*UCNTR
150 U(I,J)=UCNTR
  RETURN
END

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SUBROUTINE XSAREA(NP)
COMMON/COM1/U(25,25),T(25,25),H(25,25),AHO(25,25),JSH(25,25)
COMMON/COM2/Z(25),OZ(25),Y(25),OY(25)
COMMON/COM3/NJ,NJ1,NJ2,NJP,NI,N11,N12,N1P,NBLZ,NBLV,OX
COMMON/COM4/AX(25,25),ASJ(25),ASI(25)
COMMON/COM5/HEIGHT,WIDTH,WSLOPE,WSLOPE,OLDA,ASPR

```

C

C== CALCULATION OF NEW DUCT CROSS-SECTION =====

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HEIGHT=HEIGHT+WSLOPE*OX
WIDTH=WIDTH+WSLOPE*OX
OLCA=HEIGHT*WIDTH

```

C== SURFACE AREAS FOR OX=1 =====

ASJ(2)=(Y(3)+Y(2))\*0.5

DO 10 I=3,N11

10 ASJ(I)=(Y(I+1)-Y(I-1))\*0.5

ASJ(N1)=(Y(N11)-Y(N11))

ASI(2)=(Z(3)+Z(2))\*0.5

DO 20 J=3,NJ1

20 ASI(J)=(Z(J+1)-Z(J-1))\*0.5

ASI(NJ1)=(Z(NJ1)-Z(NJ1))

C== CROSS-SECTIONAL AREAS =====

DO 30 J=2,NJ

DO 50 I=2,N1

50 AX(I,J)=ASJ(I)\*ASI(J)

C== SURFACE AREAS =====

DO 40 I=2,N1

40 ASJ(I)=ASJ(I)\*OX

DO 65 J=2,NJ

65 ASI(J)=ASI(J)\*OX

RETURN

END

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SUBROUTINE SIPSIM(NIP)
COMMON/COM1/U(25,25),T(25,25),H(25,25),RHO(25,25),USH(25,25)
COMMON/COM2/Z(25),UZ(25),Y(25),OY(25)
COMMON/COM3/NJ,NJ1,NJ2,NJP,NI,N11,N12,N1P,NBLZ,NBLY,OX
COMMON/COM4/SIPP(25,25),SIMP(25,25),SIMM(25,25),SIPM(25,25),FLORAT
1,SIPG(25,25),SING(25,25),SINP(25,25),SIGM(25,25),SIGO(25,25)
COMMON/COM5/UCNTR,TCNTR,TWALLZ,TWALLY,P,RINV,VZERO,TZERO,EXPV,EXPU
COMMON/COM6/AX(25,25),ASJ(25),ASI(25)
COMMON/COM7/ALPHAY(25),BETAY(25),DELTAY(25),GAMMAY(25),ALPHAZ(25),
1,BETAZ(25),DELTAZ(25),GAMMAZ(25),VISCJ(25,25),VISCJ(25,25)
COMMON/COM8/HEIGHT,WIDTH,HSLOPE,WSLOPE,OLDA,ASPR
DIMENSION RU(25,25)
OZ(NJP)=OZ(NJ)
OY(NIP)=OY(NI)
FFF=1./64.
HOZ2=OZ(2)/32.
HOY2=OY(2)/32.
C= SIPS, EXCEPT SIMM(2,2), REDUCED BY 4 TO ABSORB 1/4 APPEARING IN FOF=
DO 10 J=2,NJP
DO 10 I=2,NIP
10 RU(I,J)=RHO(I,J)*U(I,J)
SIMM(2,2)=GAMMAZ(1)*RU(2,2)+Y(2)*Z(2)
SIPP(2,2)=(2.*RU(2,2)+RU(3,2)+RU(2,3))*OY(3)*OZ(3)*FFF
SIMP(2,2)=GAMMAY(2)*(3.*RU(2,2)+RU(2,3))*HOY2*OZ(3)
SIPM(2,2)=GAMMAZ(2)*(3.*RU(2,2)+RU(3,2))*OY(3)*HOZ2
FLORAT=SIPM(2,2)+SIPP(2,2)+SIMP(2,2)+SIMM(2,2)*C.25
DO 20 I=3,NI
SIPP(I,2)=(2.*RU(I,2)+RU(I+1,2)+RU(I,3))*OY(I+1)*OZ(3)*FFF
SIMP(I,2)=(2.*RU(I,2)+RU(I,3)+RU(I-1,2))*OY(I)*OZ(3)*FFF
SIMM(I,2)=GAMMAZ(1)*(3.*RU(I,2)+RU(I-1,2))*OY(I)*HOZ2
SIPM(I,2)=GAMMAZ(1)*(3.*RU(I,2)+RU(I+1,2))*OY(I+1)*HOZ2
20 FLORAT=FLORAT+SIPP(I,2)+SIMP(I,2)+SIMM(I,2)+SIPM(I,2)
FLORAT=FLORAT-SIPM(NI,2)-SIPP(NI,2)
DO 30 J=3,NJ
SIPP(2,J)=(2.*RU(2,J)+RU(3,J)+RU(2,J+1))*OY(3)*OZ(J+1)*FFF
SIMP(2,J)=GAMMAY(J)*(3.*RU(2,J)+RU(2,J+1))*HOY2*OZ(J+1)
SIMM(2,J)=GAMMAY(J)*(3.*RU(2,J)+RU(2,J-1))*HOY2*OZ(J)
SIPM(2,J)=(2.*RU(2,J)+RU(2,J-1)+RU(3,J))*OY(3)*OZ(J)*FFF
30 FLORAT=FLORAT+SIPP(2,J)+SIMP(2,J)+SIMM(2,J)+SIPM(2,J)
FLORAT=FLORAT-SIMP(2,NJ)-SIPP(2,NJ)
DO 40 J=3,NJ
DO 39 I=3,NI
SIPP(I,J)=(2.*RU(I,J)+RU(I+1,J)+RU(I,J+1))*OZ(J+1)*OY(I+1)*FFF
SIMP(I,J)=(2.*RU(I,J)+RU(I-1,J)+RU(I,J+1))*OZ(J+1)*OY(I)*FFF
SIMM(I,J)=(2.*RU(I,J)+RU(I-1,J)+RU(I,J-1))*OZ(J)*OY(I)*FFF
SIPM(I,J)=(2.*RU(I,J)+RU(I+1,J)+RU(I,J-1))*OZ(J)*OY(I+1)*FFF
39 FLORAT=FLORAT+SIPP(I,J)+SIMP(I,J)+SIMM(I,J)+SIPM(I,J)
40 FLORAT=FLORAT-SIPP(NI,J)-SIPM(NI,J)

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CU 50 I=3,N1
50 FLORAT=FLORAT-SIMP(I,NJ)-SIPP(I,NJ)
   FLORAT=FLORAT+SIPP(N1,NJ)
   FLORAT=FLORAT*4.
   DO 55 J=2,NJ
   DO 55 I=2,N1
   SIPO(I,J)=SIPP(I,J)+SIPM(I,J)
   SIMO(I,J)=SIMP(I,J)+SIMM(I,J)
   SICP(I,J)=SIPP(I,J)+SIMP(I,J)
   SION(I,J)=SIMM(I,J)+SIPM(I,J)
55 SIGU(I,J)=2.*(SIPO(I,J)+SIMO(I,J))
   IF(NP.NE.1) GC TO 100
   WRITE(6,63) FLORAT
63 FORMAT(1H, '====FLORAT=', LPE11.4)
   DO 70 JREV=1,NJ1
   J=NJP-JREV
   WRITE(6,65) J, (SIMP(I,J), SIPP(I,J), I=2,N1)
70 WRITE(6,65) J, (SIMM(I,J), SIPM(I,J), I=2,N1)
65 FORMAT(1H, ' J=', I2, 3X, LPE11.4/(11X, LPE11.4))
100 RETURN
   END

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SUBROUTINE ZDZYDY
COMMON/COM1/U(25,25),T(25,25),H(25,25),RHO(25,25),USH(25,25)
COMMON/COM2/Z(25),OZ(25),Y(25),OY(25)
COMMON/COM3/NJ,NJ1,NJ2,NJP,NI,NI1,NI2,NIP,NOLZ,NOLY,DX
COMMON/COM4/SIPP(25,25),SIMP(25,25),SIM(25,25),SIPM(25,25),FLORAT
1,SIPO(25,25),SIMO(25,25),SIOP(25,25),SIOM(25,25),SIOD(25,25)
COMMON/COM5/UCNTR,TENR,TWALLZ,TWALLY,P,RINV,VZERO,TZERO,EXPV,EXPU
COMMON/COM6/AX(25,25),ASJ(25),ASI(25)
COMMON/COM7/ALPHAY(25),BETAY(25),DELTAY(25),GAMMAY(25),ALPHAZ(25),
1,BETAZ(25),DELTAZ(25),GAMMAZ(25),VISC(25,25),VISCJ(25,25)
COMMON/COM8/HEIGHT,WIDTH,SLOPE,WSLOPE,OLDA,ASPR
DIMENSION RU(25,25),G(25,25)
DO 10 J=2,NJ
DO 10 I=2,NI
10 RU(I,J)=RHO(I,J)*U(I,J)
SUM=0.0
DO 20 I=3,NI
20 SUM=SUM+SIMP(I-1,2)/(3.*RU(I-1,2)+RU(I,2))/GAMMAZ(I-1)+SIM(I,2)/
1(3.*RU(I,2)+RU(I-1,2))/GAMMAZ(I)
OZ(2)=SIM(2,2)/GAMMAZ(1)/RU(2,2)+16.*SUM
DO 25 I=3,NI
DO 25 J=3,NJ
25 G(I,J)=SIM(I,J)/(2.*RU(I,J)+RU(I-1,J)+RU(I,J-1))+SIMP(I-1,J)/
1(2.*RU(I-1,J)+RU(I,J)+RU(I-1,J-1))+SIPP(I-1,J-1)/(2.*RU(I-1,J-1)+R
2U(I,J-1)+RU(I-1,J))+SIMP(I,J-1)/(2.*RU(I,J-1)+RU(I-1,J-1)+RU(I,J))
DO 35 J=3,NJ
SUM=0.0
DO 30 I=3,NI
30 SUM=SUM+G(I,J)
SUM=SUM+SIMP(2,J)/(3.*RU(2,J)+RU(2,J-1))/GAMMAY(J)+SIMP(2,J-1)/
1(3.*RU(2,J-1)+RU(2,J))/GAMMAY(J-1)
35 OZ(J)=SUM+16.
SUM=0.
DO 40 J=2,NJ
40 SUM=SUM+OZ(J)
YNI=1./(SUM+ASPR)+0.5
DO 45 J=2,NJ
OZ(J)=OZ(J)+YNI
45 Z(J)=Z(J-1)+OZ(J)
SUM=0.
DO 47 J=3,NJ
47 SUM=SUM+SIMP(2,J-1)/(3.*RU(2,J-1)+RU(2,J))/GAMMAY(J-1)+SIM(2,J)/
1(3.*RU(2,J)+RU(2,J-1))/GAMMAZ(J)
OY(2)=SIM(2,2)/GAMMAZ(1)/RU(2,2)+16.*SUM
DO 55 I=3,NI
SUM=0.
DO 50 J=3,NJ
50 SUM=SUM+G(I,J)

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      SUM=SUM+SIMM(I,2)/(3.*RU(I,2)+RU(I-1,2))/GAMMAZ(I)+SPH(I-1,2)/
1  (3.*RU(I-1,2)+RU(I,2))/GAMMAZ(I-1)
55 OY(I)=SUM*16.
      SUM=0.
      DO 60 I=2,NI
60 SUM=SUM+OY(I)
      ZNI=1./(SUM/ASPR)**0.5
      DO 65 I=2,NI
      OY(I)=OY(I)*ZNI
65 Y(I)=Y(I-1)+OY(I)
      RETURN
      END

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SUBROUTINE VSCSTY(NP)
COMMON/COM1/PU(25,25),T(25,25),H(25,25),PHO(25,25),USH(25,25)
COMMON/COM2/MJ,NJ1,NJ2,NJP,NI,N11,N12,N1P,NB12,NBLY,DX
COMMON/COM7/ALPHAY(25),BETAY(25),DELTAY(25),GAMMAY(25),ALPHAZ(25),
1BETAZ(25),DELTAZ(25),GAMMAZ(25),VISC1(25,25),VISCJ(25,25)
HVAL1=H(1,1)
DO 40 J=1,NJ
DO 40 I=1,NI
VISC1(I,J)=0.000015
40 VISCJ(I,J)=0.000015
RETURN
END
```



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SUBROUTINE POSCLV(AI,CI,BB,AJ,CJ,OO,X,XX)
COMMON/COM3/NJ,NJ1,NJ2,NJP,NI,NI1,NI2,NIP,NBLZ,NBLY,OX
DIMENSION AI(25,25),CI(25,25),BB(25,25),AJ(25,25),CJ(25,25)
1 ,OO(25,25),X(25,25),XX(25,25),O(25),E(25),F(25),B(25)
C** ROUTINE SOLVES THE PENTAOTAGCNAL EGS. BY A-D-I METHOD *****
DO 50 I=1,NIP
DO 50 J=1,NJP
50 XX(I,J)=X(I,J)
DO 300 NN=2,NJ
J=NN
DO 100 I=2,NI
100 O(I)=OO(I,J)+XX(I,J-1)*CJ(I,J)+XX(I,J+1)*AJ(I,J)
E(2)=AI(2,J)/BB(2,J)
F(2)=O(2)/BB(2,J)
DO 110 I=3,NI1
DEN=BB(I,J)-CI(I,J)*O(I-1)
E(I)=AI(I,J)/DEN
110 F(I)=(O(I)+CI(I,J)*F(I-1))/DEN
XX(NI,J)=(O(NI)+F(NI1)*(AI(NI,J)+CI(NI,J)))/(BB(NI,J)-E(NI1)*
1 (AI(NI,J)+CI(NI,J)))
DO 120 N=1,NI2
I=NI-N
120 XX(I,J)=E(I)+XX(I+1,J)+F(I)
I=NN
DO 200 J=2,NJ
200 O(J)=OO(I,J)+XX(I-1,J)*CI(I,J)+XX(I+1,J)*AJ(I,J)
E(2)=AJ(I,2)/BB(I,2)
F(2)=O(2)/BB(I,2)
DO 210 J=3,NJ1
DEN=BB(I,J)-CJ(I,J)*O(J-1)
E(J)=AJ(I,J)/DEN
210 F(J)=(O(J)+CJ(I,J)*F(J-1))/DEN
XX(I,NJ)=(O(NJ)+F(NJ1)*(AJ(I,NJ)+CJ(I,NJ)))/(BB(I,NJ)-E(NJ1)*
1 (AJ(I,NJ)+CJ(I,NJ)))
DO 220 N=1,NJ2
J=NJ-N
220 XX(I,J)=E(J)+XX(I,J+1)+F(J)
300 CONTINUE
RETURN
END

```

## REFERENCES

1. M.S. Greywall, "Streamwise computation of duct flows," Computer Methods in Applied Mechanics and Engineering 21 (1980) 231-247.
2. L.S. Caretto, R.M. Curr, and D.B. Spalding, "Two numerical methods for three-dimensional boundary layers," Computer Methods in Applied Mechanics and Engineering 1 (1972) 39-57.
3. S.V. Patankar and D.B. Spalding, "A calculation procedure for heat, mass, and momentum transfer in three-dimensional parabolic flows," International Journal of Heat and Mass Transfer 15 (1972) 1787-1806.
4. A.J. Chorin, "A numerical method for solving incompressible viscous flow problems," Journal of Computational Physics 2 (1967) 12-26.
5. J.A. Miller, "Laminar incompressible flow in the entrance region of ducts of arbitrary cross section," J. Engr. for Power, Trans. ASME 93 (1971) 113-118.
6. A.F. Emery, P.K. Neighbors, and F.B. Gessner, "The numerical prediction of developing turbulent flow and heat transfer in a square duct," Jr. Heat Transfer, Trans. ASME 102 (1980) 51-57.
7. D.W. Roberts and C.K. Forester, "Parabolic procedure for flows in ducts with arbitrary cross sections," AIAA Journal 17 (1979) 33-40.
8. V.S. Pratap and D.B. Spalding, "Fluid flow and heat transfer in three-dimensional duct flows," International Journal of Heat and Mass Transfer 19 (1976) 1183-1188.
9. J.F. Nash, "The calculation of three-dimensional turbulent boundary layers in incompressible flow," Journal of Fluid Mechanics 37 (1969) 625-642.
10. K.C. Wang, "Three-dimensional boundary layer near the plane of symmetry of a spheroid at incidence," Journal of Fluid Mechanics 43 (1970) 187-209.
11. F.H. Harlow and J.E. Welsh, "Numerical calculation of time-dependent viscous incompressible flow of fluid with free surface," Physics of Fluids 8 (1965) 2182-2189.
12. B.E. Launder and D.B. Spalding, "The numerical computation of turbulent flows," Computer Methods in Applied Mechanics and Engineering 3 (1974) 269-289.

13. R.J. Goldstein and D.K. Kreid, "Measurement of laminar flow development in a square duct using a Laser-Doppler flowmeter," J. App. Mech. Trans. ASME 34 (1967) 813-818.
14. G.S. Beavers, E.M. Sparrow, and R.A. Magnuson, "Experiments on hydrodynamically developing flow in rectangular ducts of arbitrary aspect ratio," International Journal of Heat and Mass Transfer 13 (1970) 689-702.
15. E.M. Sparrow, C.W. Hixon, and G. Shavit, "Experiments on laminar flow development in rectangular ducts," J. Basis Engg. Trans. ASME 89 (1967) 116-124.
16. R.K. Shah and A.L. London, Laminar Flow Forced Convection in Ducts, Academic Press, New York, 1978.
17. M.S. Greywall, Numerical Modeling of Two-Dimensional Confined Flows, Wichita State University, Mech. Engr. Dept., Rep. ME-MG79-1.