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An Adaptive Tracking Observer For Failure-Detection Systems

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TABLE OF CONTENTS

SUMMARY 1

I. INTRODUCTION 1

II. FAILURE-DETECTION SYSTEMS (FDS) BASED ON OBSERVERS 3

III. THE "MISMATCHING" EFFECTS OF ANALYTICAL-REDUNDANT FDS ON THE OBSERVER FALSE-ALARM RATE (FAR). 6

IV. AN ADAPTIVE, PARAMETER-TRACKING OBSERVER ALGORITHM 7

V. CONDITIONS FOR CONVERGENCE AND STABILITY. 9

VI. DISCUSSION OF SIMULATION RESULTS 12

CONCLUSIONS 14

APPENDIX: PROOF OF STABILITY THEOREM 15

REFERENCES 20

FIGURES 22

AN ADAPTIVE TRACKING OBSERVER FOR FAILURE-DETECTION SYSTEMS

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SUMMARY

The design problem of adaptive observers for failure-detection purposes, applied to linear, constant and variable parameters, multi-input, multi-output systems is considered here. It is shown that, in order to keep the observer's (or Kalman filter) false-alarm rate (FAR) under a certain specified value, it is necessary to have an acceptable proper matching between the observer (or KF) model and the system parameters. An adaptive observer algorithm is introduced here in order to maintain the desired system-observer model matching, despite initial mismatching and/or system parameter variations. Only a properly designed adaptive observer is able to detect abrupt changes in the system (actuator, sensor failures, etc.) with adequate reliability and FAR. Conditions for convergence for the adaptive process are obtained, leading to a simple adaptive law (algorithm) with the possibility of an a priori choice of fixed adaptive gains. Simulation results show good tracking performance with small observer output errors and accurate and fast parameter identification, in both deterministic and stochastic cases.

I. INTRODUCTION

The use of the analytical redundancy approach for failure detection in complex, dynamic control systems is by now widely accepted as a viable concept for redundancy management (refs. 1-5). Besides an appreciable saving in cost, volume, and weight, the analytical failure-detection systems have to provide at least the same high performances as the classical voting systems based on simple threshold examinations and some crude decision logic. In aeronautical designs, in particular for flight-control purposes, figures such as 10^{-4} to 10^{-5} for MAP (mission abort probability) per flight hour, associated with false-alarm rates such as 10^{-3} to 10^{-4}, are rather common requirements (ref. 1) imposed by the necessity of operational needs.

In order to compete successfully with the triple and quadruple redundant systems based exclusively on voting schemes, the analytical-redundant failure-detection systems have to exhibit some basic features such as:

a. simplicity and fault-tolerant properties in both the software conception and the hardware implementation;

b. high reliability and high probability of failure detection;

c. low false-alarm rates, despite external disturbances such as wind gusts, abrupt maneuvering (in flight-control systems), instrumentation noise, and, in some cases, process noise;

d. ability to determine, as precise and rapidly as possible, the failure source, the extent of the failure, and in some cases, the time of occurrence;

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e. ability to reorganize and readjust itself after a major failure occurred;

f. in addition to abrupt failures detection (mainly for sensor and actuator failures), the analytical-redundancy schemes have to handle the problem of soft-failures detection, such as the detection of biases and/or scale factor changes in the instrumentation, some degradations in actuator performances, etc.

Two analytical concepts are mainly used, in particular, in guidance and flight control, for analytical failure-detection purposes:

a. Linear observers (refs. 1-3)(full- and reduced-order) in which the error between the measured output and the reconstructed one, e.g., the so-called residual errors $\underline{\varepsilon}(t)$, are tested for failure assessment. The gains of those observers are determined such that $\underline{\varepsilon}(t)$ will reveal the occurrence of a specific failure.

b. Kalman filters (ref. 3-8) where the innovation sequence $\underline{\nu}(t)$ is tested for (1) unbiasedness and (2) whiteness (orthogonality condition test).

In both cases, research results published in technical papers are based on the assumption that the dynamic system has fixed and known parameters (refs. 1, 2, 3, 9, 10).

Associated with the failure-detection function provided by the observers (or by Kalman filters), are the decision algorithms which are used, in particular for soft-failures detection and failures-extent assessment. Most of the decision algorithms, such as:

a. SLRT (Sequential likelihood ratio test) for mean values and functional compatibility (refs. 8, 11, 12, and 13).

b. GLR (generalized likelihood ratio) approach (refs. 4 to 7).

c. Recursive GLR (refs. 7, 8, 12), etc.

assume (with the exception of ref. 7) that the dynamic system is known and constant.

In order to be of practical value in applications and to provide reliable systems, the major concern of failure detection and analytical redundancy theory is to combine in a judicious manner, the state estimation capability considering noise, with an adequately high failure-detection capability.

As will be shown later in this report, it is absolutely necessary that, when using either observers or Kalman filters, those devices be "matched" to the dynamic system in order to obtain low observer output errors and, therefore, low false-alarm rates. A good matching will also provide adequate properties to the decision algorithms in order to assess the time, the place, and the extent of the failure without errors.

At this point it is worth remarking that when the plant parameter variations are themselves due to some kind of failures, the matching of the observer to the plant may unintentionally "cover up" these failures. For this reason, it is expected that a complete failure-detection system would include also some on-line parameter identification procedure in order to support the failure-detection algorithm. We will not elaborate more on this topic here, since it is beyond the purpose of this paper.

A short overview of observers and/or Kalman filters for failure-detection purposes and the effect of "mismatching" conditions is presented in sections II and III of this paper as an introductory motivation for the adaptive observer design.

In sections IV and V, an algorithm for adaptive and tracking observer design is presented, together with the appropriate conditions for convergence and stability. The proof for the necessary conditions for convergence and stability is given in the Appendix.

Simulation results for deterministic and stochastic multi-input, multi-output, linear, constant and time-varying systems, are presented and discussed.

The last section of this report contains concluding remarks and offers some suggestions for further study and research.

II. FAILURE-DETECTION SYSTEMS (FDS) BASED ON OBSERVERS

As pointed out in the introduction, various analytical redundant schemes for FDS are based on the utilization of observers of full or reduced order (refs. 1-5, 10, 13). The present section gives a short presentation of some basic notions related to the observer theory for the sake of completeness. We shall assume first the following mathematical model for the linear dynamic system under consideration:

$$\begin{aligned} \dot{\underline{x}}(t) &= A\,\underline{x}(t) + B\,\underline{u}(t) \\ \underline{y}(t) &= C\,\underline{x}(t) \end{aligned} \tag{1}$$

where $\underline{x}(t)$ is the (n x 1) state vector and $\underline{y}(t)$ is the (m x 1) measurement vector, with $m \leq n$. The system is assumed both completely controllable and observable. The well-known observer model ("matched" case) (ref. 9) is described by equation (2):

$$\dot{\hat{\underline{x}}}(t) = A\,\hat{\underline{x}}(t) + K[\underline{y}(t) - C\,\hat{\underline{x}}(t)] + B.\underline{u}(t) \tag{2}$$

where $\hat{\underline{x}}(t)$ is the (n x 1) estimated (or reconstructed) state vector, and K is a fixed-gain matrix, (n x m), with constant entries. This model does not take into consideration various external perturbations and noises that affect the observer output and can cause high false-alarm rates. The observer error, $\underline{e}(t)$ (residual), is defined by equation (3):

$$\underline{e}(t) \triangleq \underline{x}(t) - \hat{\underline{x}}(t) \tag{3}$$

and the observer output error (output residual) is defined as:

$$\underline{\varepsilon}(t) \triangleq \underline{y}(t) - \hat{\underline{y}}(t) \tag{4}$$

The output residual vector $\underline{\varepsilon}(t)$ is the quantity used for failure detection (FD) and assessment. A block-diagram of a FD scheme with an observer is presented in figure 1.

From equations (1) to (3), the following differential equation is obtained:

$$\dot{\underline{e}}(t) = (A - KC)\ \underline{e}(t) \tag{5}$$

One method of choosing the gain matrix K is to place the eigenvalues of the matrix (A-KC) so that all of them have negative real parts (refs. 9,10). Under these conditions, the observer will be stable and, as $t \to \infty$, $\underline{e}(t)$ and $\underline{\varepsilon}(t)$ will go to zero. Therefore, after a short initial transient, the estimated state $\hat{\underline{x}}(t)$ will follow $\underline{x}(t)$ such that $\hat{\underline{x}}(t) \cong \underline{x}(t)$, $\forall\, t \in [t_0, \infty]$, although the only measurable vector is $\underline{y}(t)$. A second method of choosing K is to enhance the observer's probability of failure detection. After the transient has died out, and if a hard failure of one of the actuators or sensors occurs at $t = T_0$, then a jump in $\underline{\varepsilon}(t)$ will be observed at T_0, and the vector $\varepsilon(t) \neq 0$, for all $t > T_0$ (see fig. 2).

In order to better illustrate the second method, let us examine the case of an actuator failure (i-th actuator), and the possibility to enhance the detection of this event. From equations (1 to 3), one obtains the following result:

$$\underline{e}(t) = (A - KC)\ \underline{e} + \underline{b}_i \cdot u_i \tag{6}$$

where $\underline{b}_i$ is the i-th column of the time-invariant matrix B, and u_i is the i-th control of the system. The solution of equation (6) is given by:

$$\underline{e}(t) = \exp\left[(A-KC)(t-T_0)\right] \cdot \underline{e}(T_0) + \left\{ \int_{T_0}^{t} \exp\left[(A-KC)(\tau - T_0)\right] u_i(\tau)\, d\tau \right\} \underline{b}_i \ . \tag{7}$$

The first term is negligible in both the deterministic and the stochastic cases, since we assume that the failure occurs at some time T_0 during the system's operation, after the initial transient has died-out. Let us assume that the effects of measurement noise and other perturbations on $\underline{\varepsilon}(t)$ are small. Therefore, the term containing the abrupt failure information is the second one. Choosing (for C = I):

$$(A - KC) \triangleq -\ I \cdot \frac{1}{T} \tag{8}$$

where I is the (n x n) identity matrix and T is a convenient, arbitrarily chosen time constant (ref. 10), one gets:

$$\underline{e}^T(t) \cong \underline{b}_i^T \int_{T_0}^{t} \exp\ -\left[\frac{(\tau - T_0)}{T}\right] \cdot u_i(t)\, dt \tag{9}$$

$$\forall\ t > T_0$$

Therefore, the error vector $\underline{e}(t)$ will point in a specific direction in the E^n space, e.g., the direction defined by $\underline{b}_i$, associated with the failure of the

i-th actuator. Since the only access one has to the system is by measuring the vector $\underline{\varepsilon}(t)$, the measured residual will point in the direction of $C\underline{b}_i$. Since the matrix C is known a priori, and, by assuming no-failures in the measurement set-up, one is thus able to infer from $\underline{\varepsilon}(t)$, when rank C = n, what actuator failed.

By a similar treatment one is able to show how sensor failures can be detected, but, in this case, $\underline{e}(t)$ lies in a two-dimensional plane. In such a case, it is proposed to use two (or more) observers, so that the detection of the failed sensor will be feasible, simple, and unique. By processing the information of (at least) two observers in an optimal way, a failure direction will be determined, even in the presence of measurement noise. Besides the possibility of enhancing the detectability of certain specific failures in a unique way, the analytical redundancy FDS based on the use of observers leads also to important hardware savings.

As shown schematically in figure 3, a substantial saving in measurement instrumentation (a saving of from three up to six sensors in this case) can be obtained by the use of three simple comparators and only two observers (in software) in this particular failure-detection system. The logic table from figure 3 shows that there is a unique condition for every sensor failure that may occur, and therefore every failure source is uniquely determined.

When the measurement noise cannot be neglected, it is possible to take for K, instead of the values obtained from equation (8), the steady-state value of the Kalman filter optimal gain matrix K^*. In this case, given an i-th actuator failure, for instance, the vector $\underline{e}(t)$ will not point in the $\underline{b}_i$ direction, but rather will be contained in a particular subspace, defined a priori, since the matrix $(A - K^*C)$ is given. By measuring the observer's output residual, together with the utilization of an appropriate decision algorithm, we can also determine the nature of the failure and the time of occurrence in this case.

If the Kalman optimal gain is used in the Analytical Redundant and Failure-Detection System and the filter is matched exactly to the dynamic system parameters, the innovation vector sequence $\underline{\nu}_k$ is tested for detection of abrupt failures. For no-failure condition, the following condtions must exist:

$$E[\underline{\nu}_k] = 0 \qquad \forall k = 1,2, \ldots \tag{10a}$$

$$E[\underline{\nu}_i \underline{\nu}_j^T] = 0 \qquad \forall i \neq j = 1,2, \ldots \tag{10b}$$

An additional test to perform is the test for the "orthogonality" between the innovation vector and the (optimal) filter estimates. For detection probability enhancement, it may be wiser to use another gain matrix instead of K^*, but in this case one cannot use the innovations sequence test in a simple way. Finally, it is worthwhile to point out again that in principle any matched observer or Kalman filter, in association with an appropriate decision algorithm, carries the necessary information that enables one to detect and infer sensors, actuators, and other system failures.

In the next paragraph, an analysis is carried out to show the influence of non-matching conditions on the false-alarm rate of failure-detection systems. This condition occurs when the observer, or KF parameters, do not match or track the dynamic system parameters but are time varying, as in navigation and flight-control systems.

III. THE "MISMATCHING" EFFECTS OF ANALYTICAL-REDUNDANT FDS ON THE OBSERVER FALSE-ALARM RATE (FAR)

Various methods presented in many references dealing with analytical redundancy system (refs. 1-13) are based on the assumption that the observers (or KF) used in connection with the FD systems are matched to the parameters of the dynamic system. A notable exception is Willsky's method (ref. 7), wherein an attempt is made to provide for some adaptive features together with the solution of the failure-detection problem. In aeronautical engineering applications of FDS and analytical redundancy concepts there is a particular interest taking into consideration the unavoidable plant parameter variation caused by large dynamic pressure variations encountered by flying in different flight conditions. In references 4, 5, and 6, such changes are indeed taken into account, but the solutions proposed are complex. It will be shown in the sequel what the effects are on the FAR caused by mismatching between the actual plant and the analytic observer (or FK).

First, the mismatched observer case will be treated, and we shall assume that the analytical implementation of the observer is according to the following observer model:

$$\dot{\hat{\underline{x}}}(t) = (A + \Delta A)\,\hat{\underline{x}}(t) + K[y(t) - C\,\hat{\underline{x}}(t)] + (B + \Delta B)\,\underline{u}(t) \tag{11}$$

Accordingly, the observer residual error will be the solution of the following linear differential equation:

$$\dot{\underline{e}}(t) = (A - KC)\,\underline{e}(t) - \Delta A.\hat{\underline{x}}(t) - \Delta B.\underline{u}(t) \tag{12}$$

where ΔA and ΔB represent the difference between the parameters of the real plant and those of the observer. It is easy to see that the last two terms in equation (12) will cause a high residual $\underline{e}(t)$, even after the initial transient died out. The large value of $\underline{e}(t)$ is directly responsible for an unacceptable high FAR. Acceptable values of FAR will be obtained only for observers that are matched to the plant dynamics. Using design methods based on the "robust observer" approach will not be of much use, because this approach will lead to insensitive observers with respect to failure detection. Therefore, it is easy to see the need for adaptive observers that can track the plant parameter variations in FDS applications. Also, in the Kalman filter case, a notable change in the basic characteristics of the innovation sequence will be caused by mismatching conditions. Let us define the dynamic system (plant) equation by:

$$\dot{\underline{x}}(t) = A\underline{x}(t) + B\underline{u}(t) + \Gamma\underline{w}(t) \tag{13}$$

where $\underline{w}(t)$ is the $(q \times 1)$ noise input vector assumed white and Gaussian. The measurement vector $\underline{y}(t)$, $(n \times 1)$, is contaminated by white noise $\underline{n}(t)$, with $E[\underline{n}] = 0$ and $E[\underline{n}(t)\underline{n}^T(s)] = Q_1\delta(t - s)$

$$\underline{y}(t) = C.\underline{x}(t) + \underline{n}(t) \tag{14}$$

Assume, for simplification, only plant-parameter variations causing the following mismatching conditions:

$$\tilde{A} \triangleq A + \Delta A$$

$$\tilde{K} \triangleq K + \Delta K \tag{15}$$

where $\tilde{A}$; $\tilde{K}$ are the matrices used in the Kalman filter implementation. The equation of the Kalman filter is given by:

$$\dot{\hat{\underline{x}}}(t) = \tilde{A}.\hat{\underline{x}}(t) + \tilde{K}[\underline{y}(t) - C\,\hat{\underline{x}}(t)] \tag{16}$$

Define $\hat{\underline{x}}(t)$ as the best estimate for the ideal matching condition and $\Delta\underline{x}(t)$ the change in the estimate due to mismatching:

$$\tilde{\underline{x}}(t) \triangleq \hat{\underline{x}}(t) + \Delta\underline{x}(t) \tag{17}$$

Denote also $\tilde{\underline{\nu}}(t)$ as the innovation vector for the mismatched system and $\underline{\nu}(t)$ as the innovation of the ideal-matched KF-system. Based on linearity property, one can write:

$$\tilde{\underline{\nu}}(t) \triangleq \underline{\nu}(t) + \Delta\underline{\nu}(t) \tag{18}$$

From equations (14), (17), and (18) one obtains:

$$\tilde{\underline{\nu}}(t) = \underline{\nu}(t) - C.\Delta\underline{x}(t) \tag{19}$$

where $\Delta\underline{x}(t)$ is the solution of the following differential equation:

$$\dot{\Delta\underline{x}}(t) = (A - KC)\,\Delta\underline{x}(t) + \Delta K \cdot \tilde{\underline{\nu}}(t) + \Delta A \cdot \tilde{\underline{x}}(t) \tag{20}$$

It is clear from equations (19) and (20), and also shown explicitly in figure 4, that the stochastic process $\tilde{\underline{\nu}}(t)$, which is the actual innovation vector, will be a colored noise process with $E[\tilde{\underline{\nu}}(t)] \neq 0$. Therefore, no adequate test can be made on $\tilde{\underline{\nu}}(t)$ in order to detect a failure in a reliable way, e.g., with very low, admissible FAR.

In conclusion, in order to obtain adequate FAR in a FD system, it is absolutely mandatory to have a good matching between the observer's (or KF) model parameters and the parameters of the dynamic, real plant.

IV. AN ADAPTIVE, PARAMETER-TRACKING OBSERVER ALGORITHM

This section develops an algorithm for an adaptive observer design. The adaptation law provides also for parameter identification and tracking. The adaptive observer design problem was treated previously in the technical literature, and a number of algorithms were proposed. In reference 14, a scheme for simultaneous estimation of states and parameters for single-input, single-output linear, constant-parameter systems is presented. The proposed algorithm is based on a particular canonical form and makes use of Kalman filter equations. But, it can be shown that such canonical forms cannot be obtained in the general case. In reference 15, an adaptive observer for single-input, single-output linear systems was discussed. The algorithm introduced in reference 15 makes use of some additional filters and is intended for the

constant parameter case. In references 16 to 22, various schemes and algorithms of adaptive observers were presented. Those algorithms are mainly variations of a certain integral adaptation law, making use of the solution of sensitivity functions differential equations and/or the so-called state-variable filters. In addition, a common feature of those algorithms is the time-varying adaptive gain used in the adaptive law. Those features lead, necessarily, to a quite complex algorithm and a large computational effort. In reference 23 an algorithm for parameter identification was introduced and in principle, offers also the possibility for the implementation of an adaptive observer.

The approach presented in this paper is based on a simple, yet effective, adaptive law (algorithm) for linear, possibly time-varying multi-input, multi-output systems, which makes use of a priori determined adaptive gains and does not require solution of additional differential equations. Therefore, the computational effort fits the practical needs and objectives for real-time, on-line simple adaptive observers for failure-detection systems.

As shown previously in equations (11) and (12), the model of mismatched observer leads to an augmented observer output residual $\underline{\varepsilon}(t)$, which is given by $\underline{\varepsilon} = C\,\underline{e}(t)$, where $\underline{e}(t)$ will be the solution of the differential equation:

$$\dot{\underline{e}}(t) = (A - KC)\,\underline{e}(t) - \Delta A.\hat{\underline{x}}(t) - \Delta B.\underline{u}(t) \tag{21}$$

In order to compensate for ΔA and ΔB, it is proposed here to change the entries of the observer matrices A_0 and B_0 according to the following adaptation laws (algorithm):

$$\Delta A_0 = M\,\underline{\varepsilon}(t)\,\hat{\underline{x}}^T(t) \tag{22a}$$

$$\Delta B_0 = N\,\underline{\varepsilon}(t)\,\underline{u}^T(t) \tag{22b}$$

or, in the discrete case:

$$\Delta A_0(k) = M\,\underline{\varepsilon}(k)\,\hat{\underline{x}}^T(k) \tag{23a}$$

$$\Delta B_0(k) = N\,\underline{\varepsilon}(k)\,\underline{u}^T(k) \tag{23b}$$

with:

$$k = 1,2,\ldots n$$

The algorithm (22) is based on measurable values, such as the observer output (the estimated state) $-\hat{\underline{x}}(t)$, the plant (and the observer) input $\underline{u}(t)$, and $\underline{\varepsilon}(t)$ the observer output residual vector. The matrices $M(n \times m)$ and $N(n \times m)$ are to be chosen in such a way, that convergence and good tracking are provided. As shown in the next two paragraphs, the adaptive algorithm introduced here makes possible:

- maintaining a low value of the observer output residual error, in spite of plant parameter variations
- fast adaptation of the observer parameters to those of the dynamic plant
- tracking of the varying dynamic plant parameters by the adaptive observer parameters.

Substituting equation (22) into equation (21), one gets:

$$\dot{\underline{e}}(t) = (A - KC)\,\underline{e}(t) - \|\hat{\underline{x}}\|^2 MC\,\underline{e}(t) - \|\underline{u}\|^2 NC\,\underline{e}(t) \tag{24}$$

Equation (24) can be put in the more compact form:

$$\dot{\underline{e}}(t) = [(A - KC) - \|\hat{\underline{x}}\|^2 MC - \|\underline{u}\|^2 NC]\underline{e}(t) \tag{25}$$

In order to obtain for the time-varying, nonlinear differential equation an asymptotically stable in the large (ASIL) solution, several approaches can be taken. The first approach is a heuristic one; although the matrix included in the square bracket is time-varying because of the time-dependent positive scalars $\|\hat{\underline{x}}\|^2$ and $\|\underline{u}\|^2$, it is conjectured here, that by an appropriate choice of M and N, based on a priori knowledge of $\underline{x}(t)$ and $\underline{u}(t)$, the adaptive algorithm (22) can be made asymptotically convergent. Loosely speaking, the M and N matrices allow us to locate the eigenvalues of this square matrix so that all of them will have negative real values, providing us with the result: $\underline{e}(t) \to 0$ as $t \to \infty$. A second way to obtain an ASIL solution for equation (25) is to make use of a version of Perron's Theorem (refs. 24 and 25) and to determine, accordingly, the entries of the gain matrices M and N. A more appropriate way to obtain a convergent adaptive law is to determine the gain matrices M and N by making use of Lyapunov's second method (ref. 26) and this approach will be presented in the next paragraph.

In figure 5, a schematic block diagram of the adaptive observer is presented, pointing out the simplicity of the adaptive law and the fact that this algorithm makes use of only accessible measurable functions.

If measurement noise is to be taken into account, the filter gain matrix K and the adaptive gain matrices M and N will have to meet requirements in addition to those imposed by the appropriate convergency conditions. In this case, a trade-off is to be made in the choice of M and N, between fast parameter-tracking requirements and minimal noise susceptibility. Finally, the gain matrices M and N, particularly the gain matrix K, have to be chosen such that the observer sensitivity for failure detection will be maximal. This topic which is of considerable importance, is the subject of further research and is not discussed herein.

To summarize, besides the necessary convergency conditions, the triplet {K,M,N} is to be judiciously determined by taking into account such considerations as:

(1) minimum parameter alignment time (rate of convergence), (2) fast tracking capabilities, (3) minimum noise susceptibility for minimal FAR, and (4) maximum sensitivity for high-probability failures detections.

V. CONDITIONS FOR CONVERGENCE AND STABILITY

In the previous section, a procedure for the choice of M and N matrices based on a heuristical approach was discussed briefly. Here, a procedure to determine the matrices M and N, based on Lyapunov's theorem for asymptotic stability, will be developed. It will be shown that for a system described by a differential equation such as (25), and having a general form such as equation (26):

$$\dot{\underline{e}}(t) = W(\underline{e},t)\ \underline{e}(t) \tag{26}$$

where:

$$W(\underline{e},t) \triangleq [(A - KC) - \|\hat{\underline{x}}\|^2\ MC - \|\underline{u}\|^2\ NC] \tag{27}$$

the solution is uniformly asymptotically stable in the large, about the zero solution $\underline{e}(t) = 0$, which is the equilibrium point, if the entries of the matrix $W(\underline{e},t)$ satisfy certain requirements, provided by some inequality conditions. Let's consider the following positive definite scalar quadratic function $V(\underline{e})$ as a candidate for a Lyapunov function:

$$V(\underline{e}) = \underline{e}^T Q \underline{e} \tag{28}$$

with Q an arbitrary, constant, diagonal, positive definite matrix, such that:

$$V(\underline{e}) \begin{cases} = 0 & \text{if } \underline{e} = 0 \\ > 0 & \forall\, \underline{e} \neq 0, \quad \forall t \end{cases} \tag{29}$$

In addition, equation (28) provides us with:

$$\lim_{\|e\| \to \infty} V(\underline{e}) = \infty \tag{30}$$

In order to obtain ASIL conditions for the system in equation (26), in addition to conditions in equations (29) and (30), it is necessary that $\dot{V} \triangleq dV/dt$ meet the following condition

$$\dot{V}(\underline{e}) < 0 \quad , \quad \forall\, \underline{e} \neq 0 \quad , \quad \forall t \tag{31}$$

We will now proceed to obtain the necessary conditions to be fulfilled by $W(\underline{e},t)$ in order to satisfy conditions in equations (29) to (31). If those conditions are satisfied, than $V(e)$ from equation (28) will be an adequate Lyapunov function for the system in equation (26), and the ASIL property will be obtained.

From equations (26) and (28), we get the following expression for $\dot{V}$:

$$\dot{V} = \underline{e}^T [QW(\underline{e},t) + W^T(\underline{e},t)Q]\underline{e} \tag{32}$$

In order to satisfy the condition in equation (31), the matrix $P \triangleq [QW + W^T Q]$ has to be negative-definite (ref. 26). The symmetric matrix P is a function of the triplet $\{K,M,N\}$ and depends also on $Q, \underline{u}(t)$ and $\underline{e}(t)$. We shall proceed further to seek the necessary conditions for the elements P_{ij} of P such that $\dot{V} < 0$. By expanding the quadratic form given in equation (32) the following expression for $\dot{V}$ is found:

$$\dot{V} = \sum_{\substack{i=1 \\ (i \neq j)}}^{n} \sum_{j=1}^{n} [q_{ii} w_{ii} e_i^2 + (q_{ii} w_{ij} + q_{jj} w_{ji}) e_i e_j + q_{jj} w_{jj} e_j^2] \tag{33}$$

where q_{ij} and w_{ij} are the elements of the matrix Q and W, respectively, and we take $i \neq j$ in the crossterms of equation (33).

In order to obtain appropriate conditions for convergence and ASIL stability of the adaptation algorithm from equation (22), it is necessary that the conditions established in the following theorem hold.

Theorem

For the time-varying system in equation (27):

$$W(\underline{e},t) = [A - KC - \|\underline{\hat{x}}\|^2 MC - \|\underline{u}\|^2 NC]$$

to be asymptotically stable in the large, about the singular stable point $\underline{e} = 0$, the following conditions are to be satisfied:

$$V > 0 \quad , \quad \forall \underline{e} \neq 0 \quad , \quad \forall t \tag{34a}$$

$$q_{ii}w_{ii} \leq - C < 0 \qquad i = 1,2,\ldots n \tag{34b}$$

$$q_{jj}w_{jj} \leq - C < 0 \qquad j = 1,2,\ldots n \tag{34c}$$

$$\sqrt{q_{ii}w_{ii}q_{jj}w_{jj}} \geq \frac{(q_{ii}w_{ij} + q_{jj}w_{ji})}{2} \tag{34d}$$

$i = 1,2,\ldots n$

$j = 1,2,\ldots n$ ($i \neq j$ in the crossterms)

If the conditions of the theorem are satisfied, it is guaranteed that the time derivative of the Lyapunov function will be negative definite everywhere in the n-dimensional vector space E^n spanned by $\underline{e}$, i.e.,

$$\dot{V} < 0 \quad , \quad \forall \underline{e} \neq 0 \quad , \quad \forall t \tag{35}$$

the function V(e) being, therefore, an admissible Lyapunov function for the system in equation (27).

The conditions established in (34) are not difficult to meet, since the values of C, q_{ij}, and those of the gains m_{ij} and n_{ij} (contained in w_{ij}) can be arbitrarily chosen. The proof of the theorem is given in the Appendix; it is also shown that if the conditions given in (34b), (34c), and (34d) are satisfied, the value of the function $\dot{V}$ will be:

$$\dot{V} \leq - C \sum_{i=1}^{n} \sum_{j=i}^{n} [e_i^2 + e_j^2] < 0 \tag{36}$$

From equation (36), it is easy to see that, by an appropriate choice of the matrix Q and of the constant C, it is possible to accelerate the convergence rate of the adaptation process. But as pointed out before, a trade is to be made between high convergence rate and susceptibility toward possible existing measurement noise.

It is worthwhile to remark here that similar conditions to those in equation (34) can be obtained by applying Sylvester's theorem for negative definiteness directly to the system matrix P. This alternative approach is not explicitly shown in this paper, since the establishment of the ASIL conditions following this approach is associated with a lengthy and tedious algebraic manipulation.

VI. DISCUSSION OF SIMULATION RESULTS

In order to illustrate the utilization of the adaptive observer algorithm introduced in this paper, the results of two examples are shown in the sequel.

(a) First Example: The adaptation process of two observer parameters a_o and b_o is shown in figure 6. The observer parameters are converging, respectively, toward the two system parameters $a_n = 1.0$ and $b_n = 2.0$, of the following second- order system:

$$\dot{x}_1 = -a_n x_1 + x_2$$

$$\dot{x}_2 = b_n u$$

with one output measurement:

$$y = x_1$$

The initial values of the observer parameters were $a_o(o) = 1.5$ and $b_o = 1.5$. The observer poles were placed at: $s_{1,2} = -5 \pm j5$. The input was of a persistently exciting type:

$$u(t) = \sin \pi t + \frac{1}{3} \cos t$$

One can see in figure 6 the simultaneous transients, due to mismatching in the observer initial conditions $[x_1(o) = 1.0;\ \hat{x}_1(o) = 0.8;\ x_2(o) = 0;\ \hat{x}_2(o) = 1.0]$ and to parameter mismatching. The absolute value of the observer output error becomes less than 4×10^{-3} after 3.5 sec. The two observer parameters a_o and b_o converged to the true parameter values a_n and b_n within 95% accuracy, after 8 sec. (160 steps). The values of m and n, according to conditions in equation (34), were chosen 10 and 5, respectively.

In figure 7, for the same system as above, the system parameters were varied deliberately as follows:

$$a_n = \begin{cases} 1.0 & \text{for } 0 \le t \le 1 \text{ sec} \\ 1.0 + 0.2(t - 1) & \text{for } 1 < t \le 12 \text{ sec} \\ 3.2 & \text{for } t > 12 \text{ sec} \end{cases}$$

$$b_n = \begin{cases} 2.0 & \text{for } 0 \le t \le 4 \text{ sec} \\ 2.0 + 0.2(t - 4) & \text{for } 4 < t \le 12 \text{ sec} \\ 3.6 & \text{for } t > 12 \text{ sec} \end{cases}$$

In the tracking of a_o and b_o, following $a_n(t)$ and $b_n(t)$, as shown in figure 7, we can observe some time lag and a characteristic frequency of the adaptive loop of about 0.5 Hz. During the tracking phase, the observer output error was less than 2.5×10^{-2}. One second after the end of the parameter variations, the observer output error was less than 4×10^{-3}, and the accuracy of the parameter identification was better than 90%. The accuracy in parameter identification while in steady state was of the order of 98-99%.

(b) Second Example: In figure 8 the simultaneous adaptation process of three observer parameters a_o, b_o, and c_o is shown. Those three parameters converge, respectively, toward the nominal system parameters values: $a_n = 1.0$, $b_n = 1.0$, and $c_n = 3.0$, of the third-order system:

$$\dot{x}_1 = -a_n x_1 + x_2$$

$$\dot{x}_2 = x_3 - c_n x_1$$

$$\dot{x}_3 = b_n u$$

with two output measurements:

$$y_1 = x_1$$

$$y_2 = x_2$$

The starting values of the observer parameters were: $a_o(o) = 0.5$, $b_o(o) = 0.5$, $c_o(0) = 2.0$. The observer poles were placed at $s_1 = -10$ and $s_{2,3} = -8 \pm j8$. The following mismatching conditions in the initial values were used:

$$x_1(o) = 1.0 \qquad \hat{x}_1(o) = 0.5$$

$$x_2(o) = 0 \qquad \hat{x}_2(o) = 1.0$$

$$x_3(o) = 0 \qquad \hat{x}_3(o) = 1.0$$

The norm of the observer output error vector dropped to less than 5×10^{-2}, after 2.75 sec. The norm of the error vector in the parameter identification was less than 10%, after 10 sec. The values of the adaptation gains were all chosen as unity. Observe in figure 8 that after a very short observer transient, one can see a smooth and uniform convergence of the triplet (a_o, b_o, c_o) to the (a_n, b_n, c_n) values. In figure 9 the adaptive tracking of parameters a_o and b_o, following rapid variations in a_n and b_n, as in the first example, is shown. After $t = 12$ sec, when the variations of a_n and b_n stopped, the norm of the error vector in the observer parameter identification (with respect to the corresponding system parameters) dropped to less than 10%, within 2.5 sec. The observer output error norm was less than 10^{-2} within 1.4 sec, and during the tracking period the error norm was less than 2×10^{-2}, this fact being the dominant property that one asks for in the application of observers in failure-detection systems.

The same adaptive observer was simulated under various output measurement noises. In figure 10, the effect of 10%, white, output measurement noise in y_1 is

represented. Although the noise value was chosen deliberately (unpractically) high, the effect on the adaptation process was rather minor. It is clear from figure 10 that the same adaptive algorithm could be used in adaptive Kalman filters in a more efficient and simple way than other proposed methods.

CONCLUSIONS

It is shown in this paper that for a useful and proper utilization of observers and/or Kalman filters for the purpose of the failure detections in linear systems, it is necessary to adapt the observer (or the Kalman filter) to the parameters of the dynamic system. If this is not done, it is shown that the mismatching conditions may cause prohibitive false-alarm rates.

An algorithm for a tracking-adaptive observer for multi-input, multi-output linear systems is introduced, and conditions for convergence and asymptotic stability were developed. Those conditions are established on an a priori base, such that the use of the algorithm is simple and effective. In the examples shown, in both deterministic and stochastic cases, the adaptive law exhibited satisfactory accuracy and tracking capabilities by maintaining a very low observer output error and, simultaneously, identifying the system paramters in an accurate manner.

An important topic for additional research is the development of an adequate synthesis technique for the optimal choice of the matrices K, M, and N in order to maintain low false-alarm rates associated with high sensitivity of failure detection in a stochastic environment.

Another topic for further research is the possible reorganization of the adaptive observer (or KF) after a major failure has occurred. An alternate way is to design robust adaptive observers for failure-detection systems which are able, without structural change, to survive an abrupt and major failure in the system, but still exhibit high sensitviity and sufficiently low false-alarm rate.

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APPENDIX

Proof of Stability Theorem

In this appendix, a proof of the theorem stated in paragraph V, where the conditions in equation (34) for ASIL are established, is given.

From equation (33), the following expression for $\dot{V}$ is obtained:

$$\dot{V} = \sum_{\substack{i=1 \\ (i\neq j)}}^{n} \sum_{j=1}^{n} \left[q_{ii} w_{ii} e_i^2 + (q_{ii} w_{ij} + q_{jj} w_{ji}) e_i e_j + q_{jj} w_{jj} e_j^2 \right] \tag{A1}$$

where q_{ij} and w_{ij} are the elements of the matrices Q and W, respectively, and $i \neq j$ is to be taken in the crossterms of (A1).

For $\dot{V}$ to be negative definite, at a first glance it seems that a good choice will be to take:

$$\begin{aligned} q_{ii} w_{ii} &\leq -C < 0 \\ q_{jj} w_{jj} &\leq -C < 0 \end{aligned} \tag{A2}$$

and try to get the rest of the right part of equation (A1) to form a square. The constant C in equation (A2) is an arbitrary, positive constant. We shall examine, in the sequel, three different cases (I - III).

Case I: We can choose to satisfy the following conditions:

$$\sqrt{q_{ii} q_{jj} w_{ii} w_{jj}} = \frac{1}{2}(q_{ii} w_{ij} + q_{jj} w_{ji}) \tag{A3a}$$

together with:

$$\begin{aligned} q_{ii} w_{ii} &= -C < 0 \\ q_{jj} w_{jj} &= -C < 0 \end{aligned} \tag{A3b}$$

for:

$i = 1,2,\ldots n$

$j = 1,2,\ldots n$ ($i \neq j$ in the crossterms)

$\forall \underline{e}(t)$ and $\forall t$.

In this case, equation (A1) becomes:

$$\dot{V}_I = \sum_{\substack{i=1 \\ (i\neq j)}}^{n} \sum_{j=1}^{n} (-C\, e_i^2 + 2C\, e_i e_j - C\, e_j^2)$$

$$= -C \sum_{i=1}^{n} \sum_{j=1}^{n} (e_i - e_j)^2 \leq 0 \qquad \text{(A4)}$$

Since $\dot{V}_I$ is, in this case, a negative, semi-definite function ($\dot{V}_I \leq 0$), the Lyapunov stability conditions for ASIL are not met and, therefore, conditions in equation (A3) are not satisfactory. Despite this fact, it is indicated to use conditions in equation (A3) as an initial, starting condition, in order to obtain a better feeling for the choice of the gains m_{ij} and n_{ij}.

Case II: Here, one may choose the conditions

$$\sqrt{q_{ii}q_{jj}w_{ii}w_{jj}} > \frac{1}{2}(q_{ii}q_{ij} + q_{jj}w_{ji}) \qquad \text{(A5a)}$$

or:

$$\sqrt{q_{ii}q_{jj}w_{ii}w_{jj}} = \frac{1}{2}(q_{ii}w_{ij} + q_{jj}w_{ji}) + \gamma^2 \qquad \text{(A5b)}$$

for:

$i = 1,2,\ldots n$

$j = 1,2,\ldots n$ ($i \neq j$ in the crossterms)

$\forall\, \underline{e}(t)$ and $\forall t$

together with conditions in equation (A3b), whereas γ is an arbitrary constant. Substituting, in equation (A1), one gets:

$$\dot{V}_{II} = \sum_{\substack{i=1 \\ (i\neq j)}}^{n} \sum_{j=1}^{n} \left[-C\, e_i^2 - 2(\gamma^2 - C) e_i e_j - C\, e_j^2\right] \qquad \text{(A6)}$$

If the following choice is made:

$$\gamma^2 = C \qquad \text{(A7)}$$

so that the following equality holds:

$$q_{ii}w_{ij} + q_{jj}w_{ji} = 0 \qquad \text{(A8)}$$

one obtains for $\dot{V}_{II}$ the following expression:

$$\dot{V}_{II} = - C \sum_{i=1}^{n} \sum_{j=1}^{n} (e_i^2 + e_j^2) < 0 \qquad \text{(A9)}$$

This time, $\dot{V}_{II}$ is an absolute negative definite function and, therefore, conditions in equations (A3b) and (A8) will ensure asymptotic stability in the large.

Case III: In this case, we obtain a set of conditions for ASIL that are easier to fulfill, and, in the same time, we can fix an a priori, upper bound for $\dot{V}$, increasing the convergence rate of the adaptive algorithm (up to a certain limit, because of the stochastic measurement noise susceptibility problem). Let us choose:

$$\begin{aligned} q_{ii}w_{ii} &\leq - C < 0 \\ q_{jj}w_{jj} &\leq - C < 0 \end{aligned} \qquad \text{(A10)}$$

Instead of equation (A10), one writes:

$$\begin{aligned} q_{ii}w_{ii} &= - C - C_1 \\ q_{jj}w_{jj} &= - C - C_2 \\ i &= 1,2,\ldots n \quad ; \quad j = 1,2,\ldots n \end{aligned} \qquad \text{(A11)}$$

where: $C > 0$, $C_1 > 0$, $C_2 > 0$, are arbitrarily chosen constants. Making use of equation (A11), one obtains:

$$\dot{V}_{III} = \sum_{\substack{i=1 \\ (i\neq j)}}^{n} \sum_{j=1}^{n} \left[-(C + C_1)e_i^2 + (q_{ii}w_{ij} + q_{jj}w_{ji})e_i e_j - (C + C_2)e_j^2 \right] \qquad \text{(A12)}$$

or:

$$\dot{V}_{III} = - C \sum_{i=1}^{n} \sum_{j=1}^{n} (e_i^2 + e_j^2) - \sum_{\substack{i=1 \\ (i\neq j)}}^{n} \sum_{j=1}^{n} \left[C_1 e_i^2 - (q_{ii}w_{ij} + q_{jj}w_{ji})e_i e_j + C_2 e_j^2 \right] \qquad \text{(A13)}$$

Choosing the following condition:

$$\frac{(q_{ii}w_{ij} + q_{jj}w_{ji})}{2} = \sqrt{C_1 C_2} \qquad \text{(A14)}$$

one obtains for $\dot{V}_{III}$, the following value:

$$\dot{V}_{III} = - C\sum_{i=1}^{n}\sum_{j=1}^{n}(e_i^2 + e_j^2)$$
$$- \sum_{\substack{i=1 \\ (i\neq j)}}^{n}\sum_{j=i}^{n}\left[\sqrt{C_1 e_i} - \sqrt{C_2 e_j}\right]^2 \qquad \text{(A15)}$$

By comparing equation (A15) with equation (A9), we can easily see that:

$$\dot{V}_{III} \leq - C\sum_{i=1}^{n}\sum_{j=1}^{n}\left[e_i^2 + e_j^2\right] \qquad \text{(A16a)}$$

and, therefore:

$$\dot{V}_{III} \leq \dot{V}_{II} < 0 \qquad \text{(A16b)}$$

for $\forall \underline{e}(t)$ and $\forall t$.

From equation (A11) one has:

$$\sqrt{q_{ii}w_{ii}q_{jj}w_{jj}} = \sqrt{(C + C_1)(C + C_2)} > \sqrt{C_1 C_2} \qquad \text{(A17)}$$

and, therefore, from equation (A14):

$$\sqrt{q_{ii}w_{ii}q_{jj}w_{jj}} > \frac{\left(q_{ii}w_{ij} + q_{jj}w_{ji}\right)}{2} \qquad \text{(A18)}$$

for: $i = 1,2,\ldots n$; $j = 1,2,\ldots n$ $(i \neq j)$. Summing up, the conditions for ASIL, formerly established, can be enunciated by the following:

Theorem

For the time-varying system in equation (27):

$$W(\underline{e},t) = [A - KC - \|\hat{\underline{x}}\|^2 MC - \|\underline{u}\|^2 NC]$$

to be asymptotically stable in the large, about the singular stable point $\underline{e} = 0$, the following conditions are to be satisfied:

$$V > 0 \quad , \quad \forall \underline{e} \neq 0 \ , \ \forall t \qquad \text{(A19a)}$$

$$q_{ii}w_{ii} \leq - C < 0 \qquad i = 1,2,\ldots n \qquad \text{(A19b)}$$

$$q_{jj}w_{jj} \leq - C < 0 \qquad j = 1,2,\ldots n \qquad \text{(A19c)}$$

$$\sqrt{q_{ii}w_{ii}q_{jj}w_{jj}} \geq \frac{\left(q_{ii}w_{ij} + q_{jj}w_{ji}\right)}{2} \tag{A19d}$$

$i = 1,2,\ldots n$

$j = 1,2,\ldots n \quad (i \neq j \text{ in the crossterms})$

If the conditions of the theorem are satisfied, it is guaranteed that the time derivative of the Lyapunov function will be negative definite everywhere in the n-dimensional vector space E^n spanned by $\underline{e}$, i.e.,

$$\dot{V}_{III} < 0 \tag{A20}$$

for: $\forall \underline{e}(t)$ and $\forall t$, the function $V(e)$ being therefore an admissible Lyapunov function for the system in equation (27).

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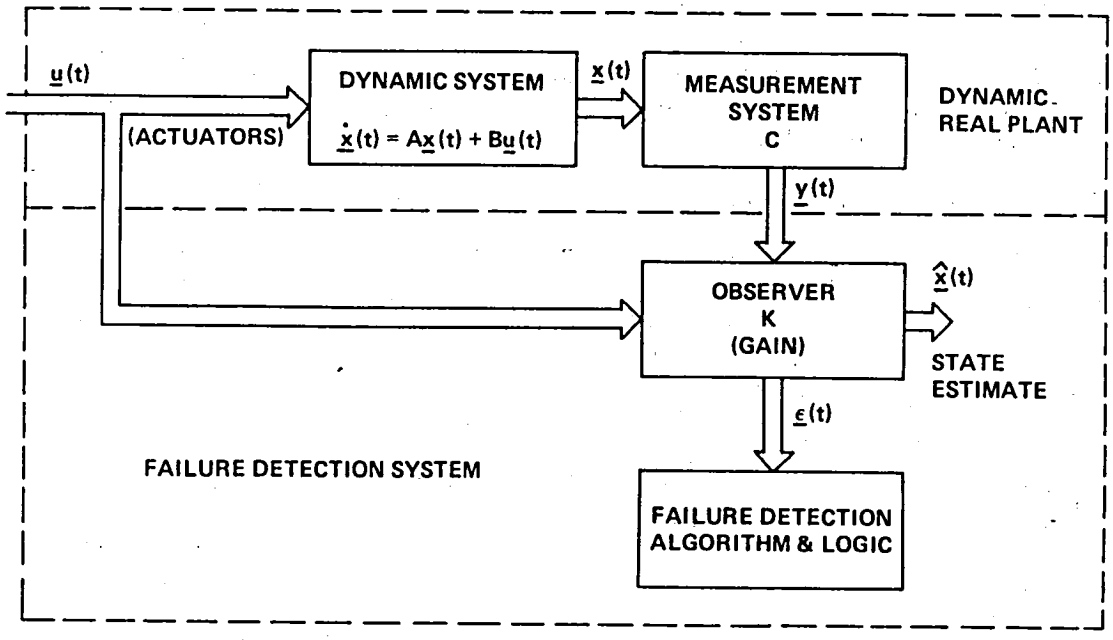


Figure 1.- Schematic block diagram of failure detection system, including an observer.

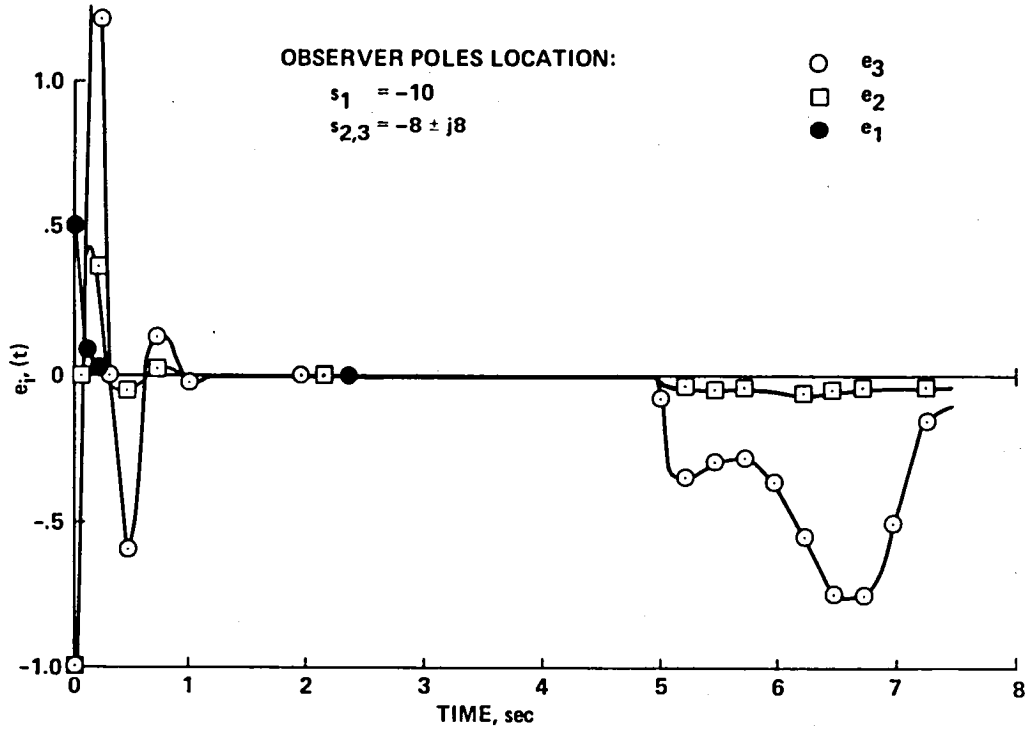
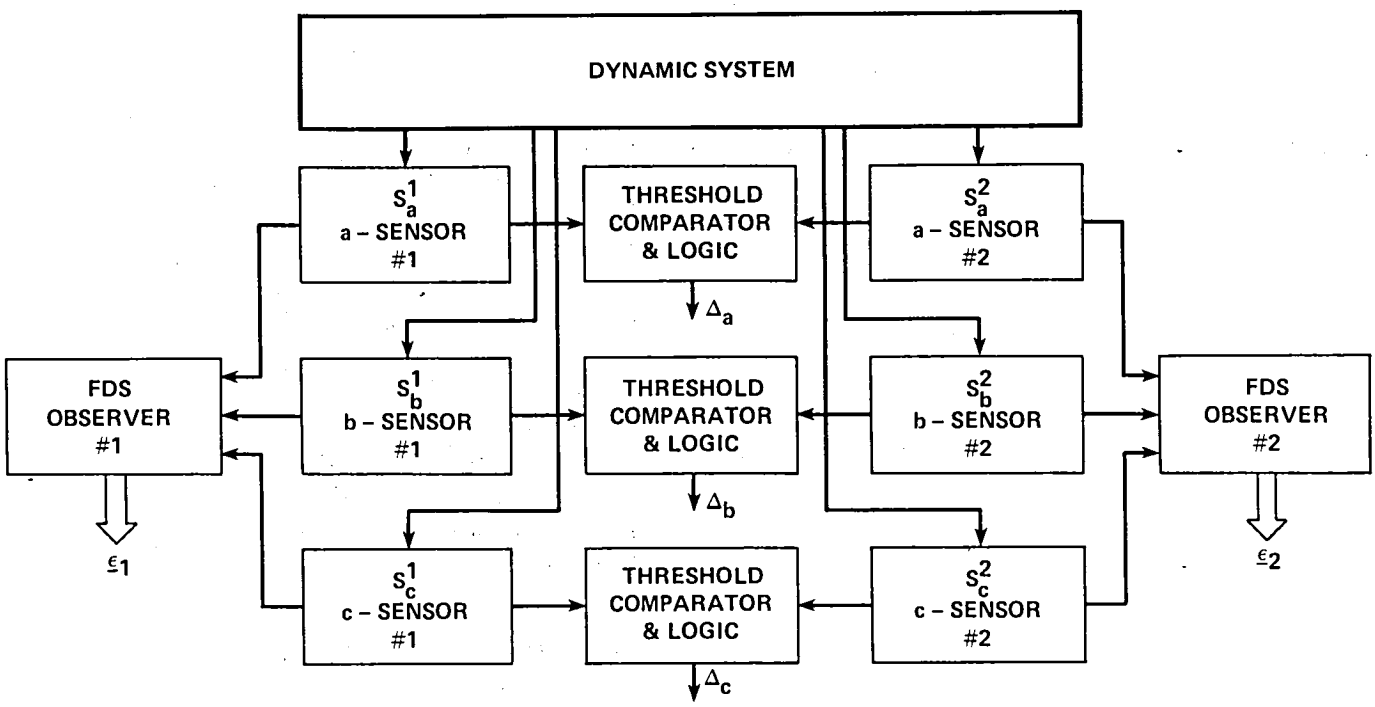


Figure 2. - Observer errors (residuals) for a third-order system (ex. 2) with actuator failure at t = 5 sec.



		POSSIBLE SENSOR FAILURES						
		NO	S_a^1	S_a^2	S_b^1	S_b^2	S_c^1	S_c^2
OBSERVERS AND COMPARATORS OUTPUT ERROR	ϵ_1	0	1	0	1	0	1	0
	ϵ_2	0	0	1	0	1	0	1
	Δ_a	0	1	1	0	0	0	0
	Δ_b	0	0	0	1	1	0	0
	Δ_c	0	0	0	0	0	1	1

Figure 3.- Hybrid, voting system with observers for analytic redundancy management.

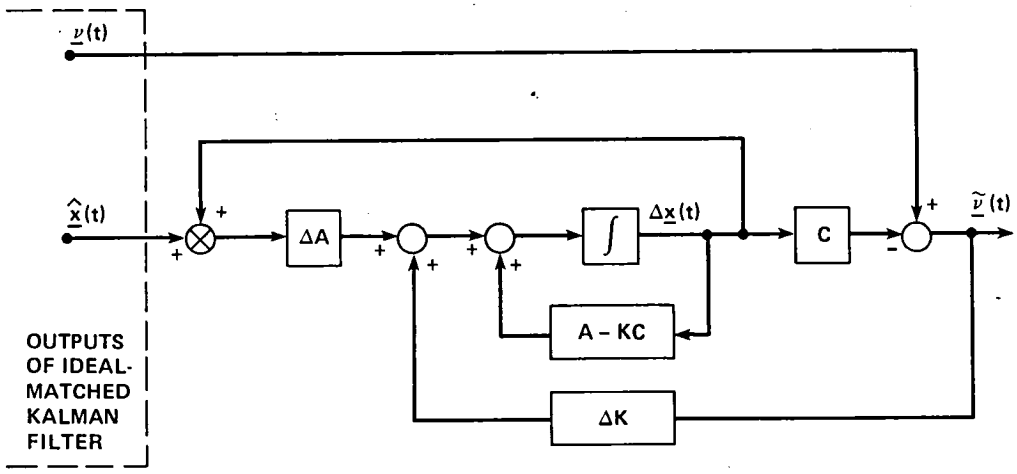


Figure 4.- The modeling of the plant-KF "mismatching" effect on the innovation stochastic process.

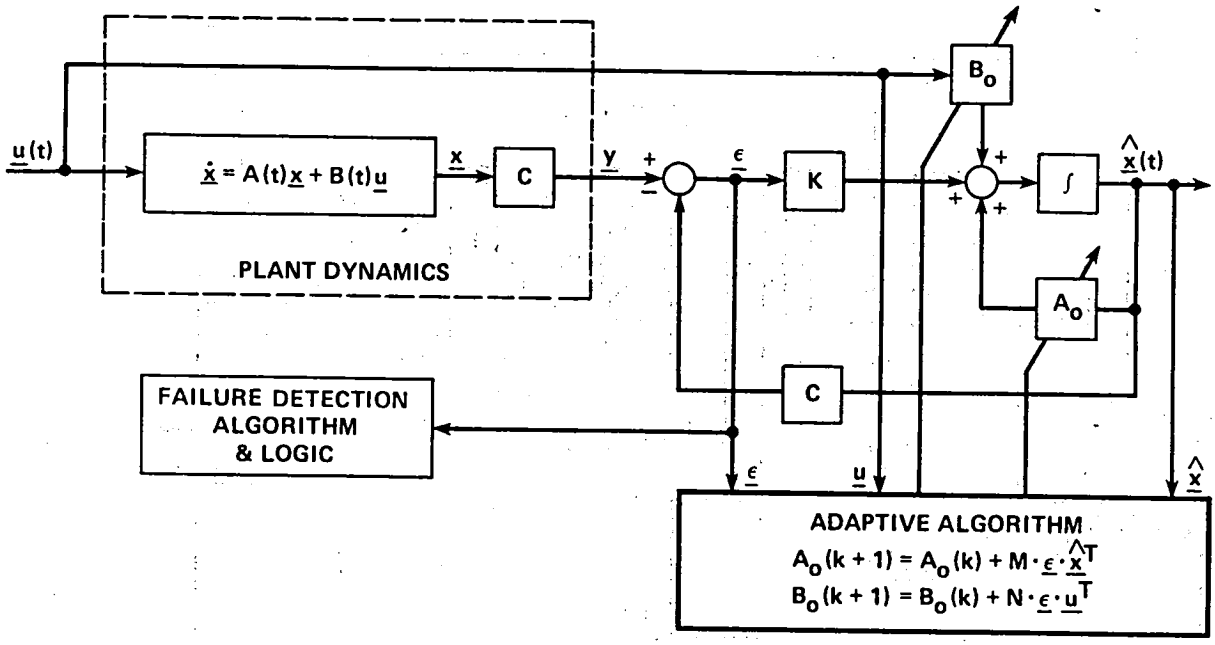


Figure 5.- Failure detection system with adaptive observer.

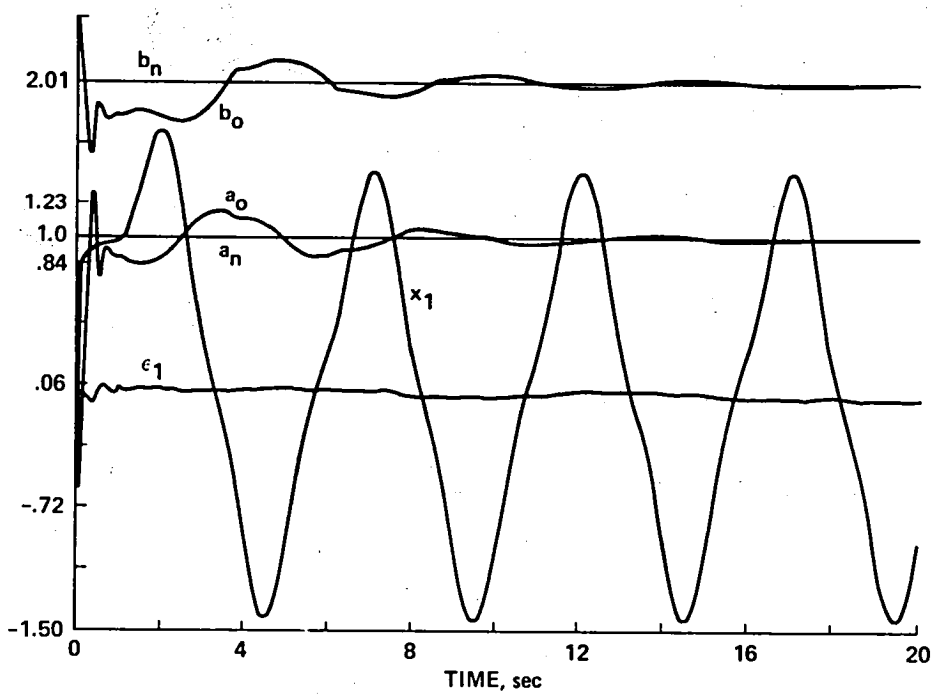


Figure 6.- First example: observer output error and parameter adaptation process.

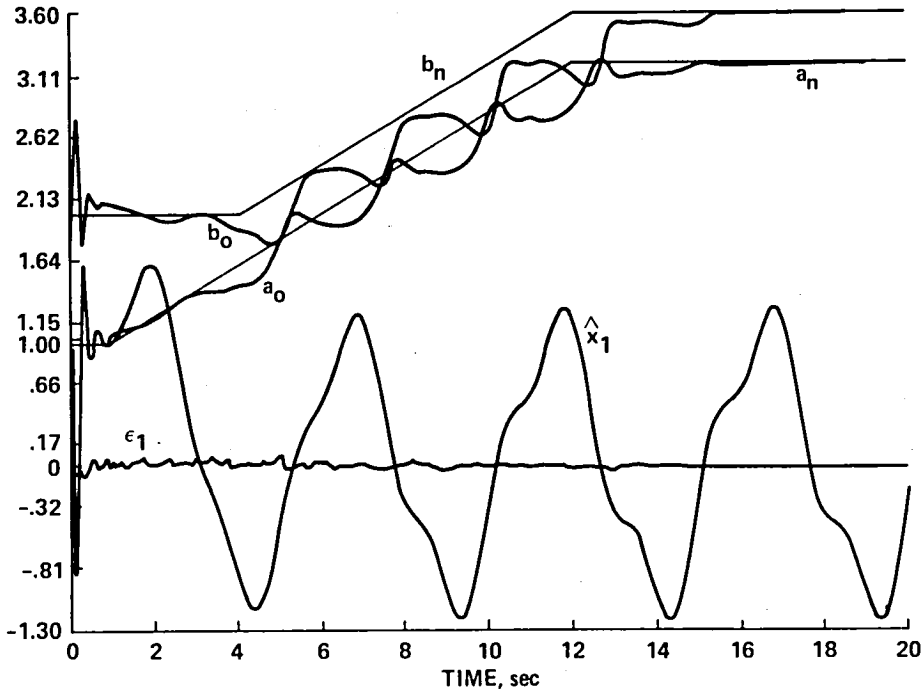


Figure 7.- First example: parameter-tracking adaptation.

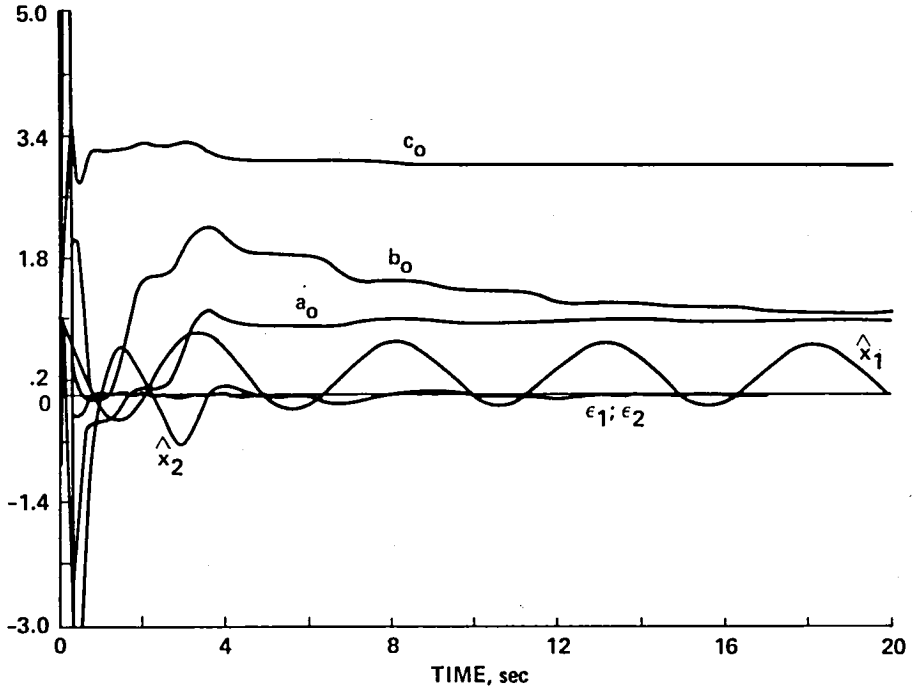


Figure 8.- Second example: observer output error and parameter adaptation process.

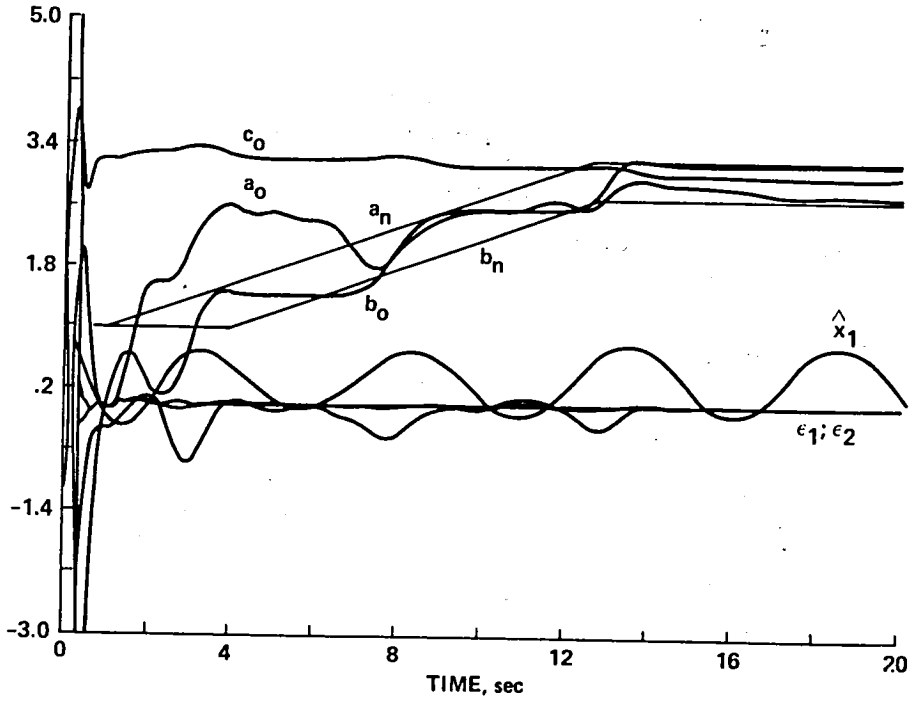


Figure 9.- Second example: adaptive parameter tracking.

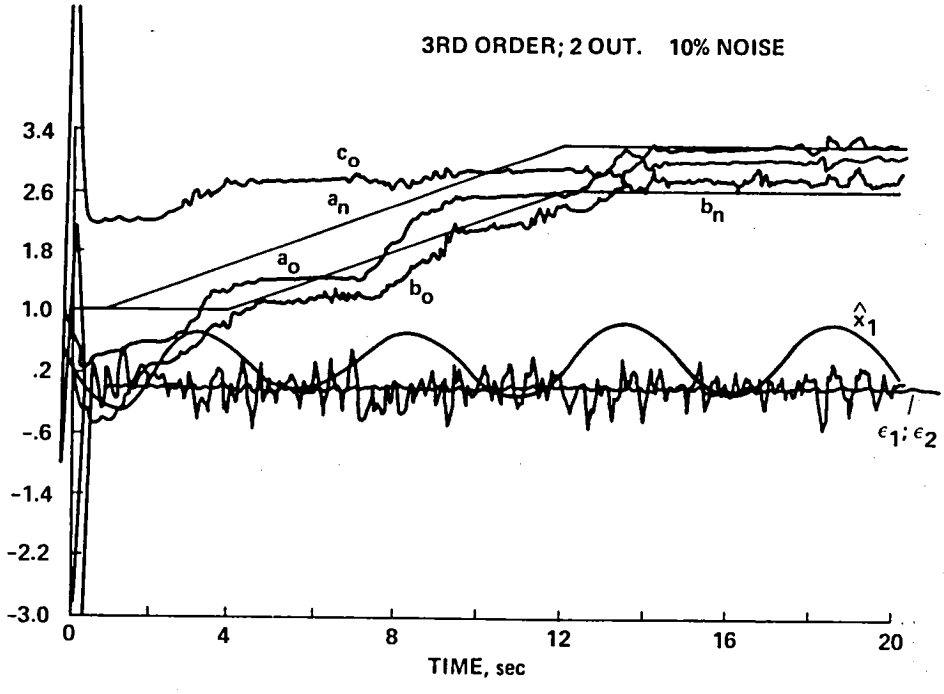


Figure 10.- Second example: adaptive parameter tracking in stochastic environment.

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16. Abstract

The design problem of adaptive observers for failure-detection purposes, applied to linear, constant and variable parameters, multi-input, multi-output systems is considered here. It is shown that, in order to keep the observer's (or Kalman filter) false-alarm rate (FAR) under a certain specified value, it is necessary to have an acceptable proper matching between the observer (or KF) model and the system parameters. An adaptive observer algorithm is introduced here in order to maintain the desired system-observer model matching, despite initial mismatching and/or system parameter variations. Only a properly designed adaptive observer is able to detect abrupt changes in the system (actuator, sensor failures, etc.) with adequate reliability and FAR. Conditions for convergence for the adaptive process are obtained, leading to a simple adaptive law (algorithm) with the possibility of an a priori choice of fixed adaptive gains. Simulation results show good tracking performance with small observer output errors and accurate and fast parameter identification, in both deterministic and stochastic cases.

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