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March, 1982 SSL#5-82

(Under NASA Grant #NAG1-126)

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Introduction

The aerospace community is anticipating the construction of some very large structural assemblies in space in the future. Examples of these may include panels of solar cells hundreds of meters long, or in the case of a satellite solar power system, several kilometers long, and microwave antennas hundreds of meters in diameter. For reasons of economy, the mass of these structures will have to be held to a minimum, and thus they will have little inherent structural rigidity. It is expected that active control will be required to hold the figure of these assemblies and damp structural vibrations in addition to the usual requirements for attitude control and station keeping.

In order that the control system can adequately damp the many vibrational modes of such a large structure, and control its figure to a tolerance which will be very demanding in the case of an antenna, many sensors and actuators will be required—probably hundreds of them in some cases. The system designer will likely have considerable freedom of choice as to the number of these components to specify and where to place them on the structure. For example, rate gyro sensors and control moment gyros could be located almost anywhere on a truss-like structure. With so many components to place and so many possible locations to choose from, the designer will need help in resolving these questions of component number and placement.

This report presents a methodology intended to serve this purpose. This approach is intended to be useful in the early stages of system design—before a control system has been designed in detail. The usual control system design problem is to decide how the actuator commands are to be related to the sensor outputs. But this process presumes a set of sensors and actuators to be given. We address here a step which must precede this process—which is to decide, at least tentatively, how many sensors and actuators to incorporate in the system and where to locate them. After a proposed control system has been designed, it must, of course, be evaluated in careful detail to see if it will meet the mission requirements. That evaluation may shed additional light on the adequacy of the set of components incorporated in the design.

One factor which must be accounted for, both in the early assessment of component number and location and in the later evaluation of a specific system configuration, is the likelihood of some failures among the sensors and actuators. With the large number of components involved and the long interval desired between visits for maintenance and resupply, it would be totally unrealistic to design the control system under the assumption that all components will function properly over that interval. For example, if the interval between maintenance visits is three years and the control system utilizes a total of 400 sensors and actuators—each with an exponential distribution of time to failure with a mean time to failure of 100,000 hours (optimistic by today's standards)—the probability that none of these components will fail in this interval is about 2×10^{-46}, which indicates with essential certainty that one or more failures will occur. In fact, the expected number of component failures in the interval under these conditions is 92!

This paper utilizes a method developed in a previous report[1] to compute the Degree of Controllability and Degree of Observability of a system for a given set (number and location) of actuators and sensors. These measures of controllability and observability are quite different from the usual indications of linear system controllability and observability which are just yes-no indicators; these measures are quantitative indicators of how well the system can be controlled and observed with given sets of actuators and sensors.

The issue of component unreliability is introduced by computing an average expected Degree of Controllability and Observability for the system over its operating lifetime accounting for the likelihood of various combinations of component failures. These measures are independent of how failure detection and identification might be implemented in the system, or how the control system might be reconfigured following a failure. They reflect instead the basic capability of the actuator set to control the system state, and the capability of the sensor set to observe it, in the context of the failures which will probably occur.

One can then optimize actuator and sensor performance for a given number of components by computing these average measures for every allowable set of locations. In most cases, the optimal component configuration when unreliability is considered will be the same as that for 100% operation, but an example is provided in which this is not the case. One can also vary the number of components in the system to strike a balance between the marginal cost of adding actuators and sensors and the resulting improvement in controllability and observability measures. This will provide the designer with a meaningful basis for choice of number and location of control system sensors and actuators.

REVIEW OF CONTROLLABILITY AND OBSERVABILITY

The Degree of Controllability developed in [1] is based upon minimizing the amount of control energy

$$E = \frac{1}{2}\int_{0}^{T} \underline{u}^{T}R\underline{u}dt \qquad (1)$$

that is used in bringing a linear system from some initial perturbed state $\underline{x}(0)$ to the origin in a given time T. The result of this minimization is an ellipsoidal surface in state space which bounds the initial states which can be returned to the origin with constrained time and control energy. The interior of the space bounded by this surface

$$E_s = \frac{1}{2} \underline{x}_o^T V_o^{-1} \underline{x}_o \tag{2}$$

is denoted the "recovery region," and V_o is found by solving the differential equation

$$\begin{aligned} \dot{V} &= AV + VA^T - BR^{-1} B^T \\ V(T) &= 0 \end{aligned} \tag{3}$$

for V at t = 0.

The Degree of Controllability is then defined as a linear measure of the weighted volume of the recovery region in scaled state space:

$$DC = \left[V_S + \frac{V_S}{V_E} (V_E - V_S)\right]^{\frac{1}{n}} \tag{4}$$

$$V_E = \prod_i \sqrt{\nu_i}$$

$$V_S = (\nu_{i_{min}})^{\frac{n}{2}}$$

where n is the dimension of the state space and ν_i are the eigenvalues of DV_oD. The scaling matrices D and R are defined as

$D = \text{diag} \left(\frac{1}{x_{i_{min}}}\right)$ x_i = minimum initial value of x_i to be driven to zero

$R = \text{diag} \left(\frac{r_i}{m}\right)$ r_i - reflects relative costs of different actuators

m = number of actuators

The Degree of Observability is based upon the use of the Kalman Filter to derive the maximum amount of information about the system state in time T starting with zero information. Since the Degree of Observability is to be a property of the system and not the environment in which it operates, the state driving noise is excluded and the information matrix at time T is found by solving

$$\dot{J} = -JA - A^T J + C^T N^{-1} C \qquad J(0) = 0 \tag{5}$$

where C is the measurement matrix and N is the sensor noise intensity matrix.

The amount of information gained about the system states in time T is reflected by the size of J(T). One measurement which indicates matrix size is the volume contained within the surface

$$\underline{v}^T J_T^{-1} \underline{v} = 1 \tag{6}$$

But the variables are scaled first to reflect the relative importance of errors in the different state variables:

$$\underline{w} = F\underline{v} \tag{7}$$

where $F = \text{diag}\,(e_{i_{max}})$ $\qquad e_{i_{max}}$ = maximum tolerable error in estimate of x_i

The Degree of Observability is defined with respect to this volume in the space of equally important errors ($\underline{w}$) just as the Degree of Controllability was defined for the volume of the recovery region in the space of equally important control characteristics:

$$DO = [V_S + \frac{V_S}{V_E} (V_E - V_S)]^{\frac{1}{n}} \tag{8}$$

$$V_E = \Pi \sqrt{\nu_i}$$

$$V_S = (\nu_{i_{min}})^{\frac{n}{2}}$$

where ν_i are the eigenvalues of FJ_TF. Reference [1] gives analytic solutions for V_o and J_T in the case of LSS dynamics.

MEASURES WHICH REFLECT COMPONENT FAILURES

Because of the realistic possibility of components failing during the operating lifetime of the system, one would like the Degree of Controllability (and Observability) to be averaged in some way over the set of component failure combinations which the system may experience. To this end, let f be an indicator of the state of failures of the components, and let the vector $\underline{\ell}$ represent their locations. Then for a given set of operating actuators, one can compute the Degree of Controllability, DC $(\underline{\ell},f)$, using the method previously described.

The component locations indicated by $\underline{\ell}$ are deterministic; they will subsequently be adjusted to optimize the Degree of Controllability. But f is a random variable with a time-dependent probability distribution. Thus DC $(\underline{\ell},f)$ is also a random variable with a time-dependent probability distribution defined by the distribution of f. To define a meaningful deterministic performance measure, one would logically use the expected value of DC($\underline{\ell}$,f) with the expectation taken over the distribution of f, the failure state for the system components. This yields a performance measure which depends on time, t. It represents a measure of the expected performance of the system at time t in view of the probabilities of the various failure states at that time.

But this control system is required to operate over a certain period T_m which might represent the time between maintenance visits. Rather than optimize the degree of controllability at any one time, such as the end of that period, it would seem more meaningful to optimize the average degree of controllability over the whole period. In this average, the performance resulting from failure states which are likely over longer periods would be weighted more heavily than those likely to exist over shorter periods. And a probability-weighted measure of performance over the whole operating period is obtained rather than just a measure of performance at one time.

The average of the expected Degree of Controllability over the mission period T_m is taken as the final measure:

$$DC_{AVE}(\underline{\ell}) = \frac{1}{T_m}\int_0^{T_m} \overline{DC}(\underline{\ell},f)\, dt \qquad (9)$$

But the expected DC is simply a weighted sum over the different failure states,

$$\overline{DC}(\underline{\ell},f) = \sum_i DC(\underline{\ell},f_i)\ P_i(t) \qquad (10)$$

where $P_i(t)$ is the probability of failure state f_i at time t. The final measure can be expressed as

$$DC_{AVE}(\underline{\ell}) = \sum_i DC(\underline{\ell},f_i)\ \frac{1}{T_m}\int_0^{T_m} P_i(t)\ dt \qquad (11)$$

and depends on T_m and the component failure statistics as well as the locations. The modified Degree of Observability is computed in the same way.

To illustrate the calculation of the average probabilities for the failure states, take the usual assumptions of independence of component failures and the exponential distribution of time to failure for each component. Then for the j^{th} component, the probability that it is working at time t is

$$P(j^{th} \text{ component working at } t) = e^{-\lambda_j t} \qquad (12)$$

where λ_j is the failure rate for this component, the reciprocal of its mean time to failure. Let the i^{th} failure state be characterized by two sets of indices, J_W and J_F, with all components having indices j in the set J_W working and all components having indices j in the set J_F failed. Note that the index of each component in the system must be contained in one or the other of J_W or J_F, but not both. Then the probability of this failure state at t is

$$P_i(t) = \left[\prod_{j \varepsilon J_W} e^{-\lambda_j t}\right] \left[\prod_{j \varepsilon J_F} (1 - e^{-\lambda_j t})\right] \qquad (13)$$

With the definition

$$\lambda_W = \sum_{j \varepsilon J_W} \lambda_j \qquad (14)$$

this can be written as

$$P_i(t) = e^{-\lambda_W t} \prod_{j \varepsilon J_F} (1 - e^{-\lambda_j t}) \qquad (15)$$

The average, over the mission period T_M, of this probability—as is required for the calculation of the Degree of Controllability or the Degree of Observability given in Eq. (11)—can be expressed as

$$\frac{1}{T_M} \int_0^{T_M} P_i(t)\, dt = \sum_{k=0}^{N_F} (-1)^k \, \text{sum}(k) \qquad (16)$$

where

$$\text{Sum}(k) = \sum_{\ell=1}^{\binom{NF}{k}} \frac{1}{(\lambda_W + \sum_k^{\ell} \lambda_j) T_M} \left[1 - e^{-(\lambda_W + \sum_k^{\ell} \lambda_j) T_M}\right] \qquad (17)$$

$$\binom{NF}{k} = \frac{N_F!}{k!\ (N_F-k)!}$$

N_F = the number of elements in J_F (the number of failed components)

$\sum_k^{\ell} \lambda_j$ = for each ℓ, the sum of a different combination of k λ_j with $j \varepsilon J_F$

The first term in the sum of Eq. (16) requires interpretation in the case of no working components. In the usual case with some components working, λ_W given by Eq. (14) is greater than zero and

$$\text{Sum}(0) = \frac{1}{\lambda_W T_M} \left(1 - e^{-\lambda_W T_M}\right) \qquad \lambda_W > 0 \tag{18}$$

If there are no working components, define $\lambda_W = 0$, and Sum(0) = 1.

These expressions can be simplified in the special case of all component failure rates equal. Call the number of working components N_W and the number of failed components N_F as before. Then if all $\lambda_j = \lambda$,

$$\frac{1}{T_M} \int_0^{T_M} P_i(t)dt = \sum_{k=0}^{N_F} \frac{(-1)^k \binom{NF}{k}}{(N_W + k)\lambda T_M} \left[1 - e^{-(N_W+k)\lambda T_M}\right] \tag{19}$$

As before, if $N_W = 0$, the term corresponding to k = 0 is 1.

OPTIMUM COMPONENT PLACEMENT

Having a computable measure of how well the structure can be controlled (observed) with any given set of actuators (sensors), with the expected effect of component failures throughout the mission reflected in the measure, one can then seek to optimize the choice of component locations, for a given number, so as to maximize the performance measure. This task may be computationally burdensome when dealing with a large number of components but it is conceptually straightforward.

A constraint which will likely apply in most applications is that component placement will be restricted to a discrete set of permissible locations. Structural considerations, for example, may require that control moment gyros be mounted only at the joints of a truss structure. If this is true of all the components, then the placement optimization problem is in the nature of an integer programming problem. Many algorithms have been described in the literature for solving integer programming problems; nothing has been added to that art in this work. The examples which follow are intended only to illustrate the nature of this step. They were restricted to a small number of components and optimization was accomplished by global search—by testing all admissible combinations of component locations.

CHOICE OF COMPONENT NUMBER

Having the optimum set of component locations and the corresponding maximum Degree of Controllability (Observability) for a given number of components, one can compute this maximum performance measure for several choices of component number. The choice of how many actuators and sensors to use in the system cannot be resolved as an optimization problem unless additional factors are incorporated in the criterion. The Degree of Controllability or Observability will always improve with additional components if the best locations are used in each case.

However, it should be informative to observe the trend of the performance measure with number of components. Some locations are more advantageous than others—such as the placement of torque actuators near the nodes of important modes. With the realistic restriction that only one component can be placed at any one of the allowable locations, one should expect to see diminishing returns in performance with increasing number as the more favorable locations are occupied. This information should be helpful to the designer in making the trade-off between improved performance and increased cost, power required, etc.

APPLICATION TO BEAM

To illustrate the methodology defined above, actuator placement and number were considered for the case of a free-free beam. The beam was modeled as in Reference [1] with the states representing the modal amplitudes and rates of the first three flex modes; force actuators were used for control (control period was 10 sec). In all trials the amplitude rate states were scaled by the factor $1/\omega_i$ relative to the amplitude states where ω_i is the corresponding modal frequency. The actuators were assumed equally efficient ($R_o = I$), but the elements were scaled by (1/no. of actuators) to reflect saturation of the controllers. This scaling was chosen to produce a result which is proportional to the number of actuators of equal effectiveness. For example, two actuators at the same location have a degree of controllability twice that of a single actuator at that position.

The effect of actuator location on the Degree of Controllability of a three-mode representation of a uniform free-free beam is shown in Fig. 1. This figure is a plot of DC as a function of the location of a single force actuator along the length of the beam. As an aid to interpretation of these results, the mode shapes for the three simulated modes are given in Fig. 2. No failures are considered. As one would expect, DC is zero at each node of the three modes because, with just one actuator, one mode is uncontrollable in those cases—and the uncontrollability of any mode is reflected in a zero DC. The Degree of Controllability rises to intermediate peaks between the nodal points and has its maximum at the ends of the beam where the modal deflections of all three modes are greatest. For this system, then, it is clear that the end of the beam is the optimum location of one actuator no matter how many are used.

Optimum actuator locations for this system were found for 1, 2, 3 and 4 actuators with and without component failures considered. One might expect it will usually be true that the best places to locate control system components are the same with and without consideration of possible component failures. But as this example illustrates, this is not always so. Permissible actuator locations were restricted to the 11 discrete locations indicated in Fig. 2. Only half the beam was searched for favorable locations because of the symmetry of all the modes. The component mean time to failure was taken equal to the mission time, so the probability that any one actuator fails before the end of the mission is 0.63, and the average probability of any one actuator failure over the mission period is 0.37. All calculations were performed with a computer program given in [1].

The detailed results are given in Table I. For a single actuator, the optimal location with and without consideration of failures is in position 1 at the end of the beam as was anticipated above. For the case of two actuators, positions #1 and #11 (end and center) are best for no failures and positions #1 and #5 are optimal when failures are considered.

TABLE I. OPTIMAL DC's AND LOCATIONS FOR VARYING ACTUATOR NUMBER

No. Actuators	†Location		Degree of Controllability	
	No Fail	Fail	No Fail	Fail
1	1	1	.1609	.1017
2	11,1	5,1	.2791	.1657
3	5,11,1	10,5,1	.3856	.2305
4	6,5,11,1	6,11,5,1	.4879	.2980

†Location number refers to test position from the end of the beam (actuators were restricted to one side of beam only)

The reason for this difference can be seen by examining Fig. 1 which illustrates degree of controllability vs actuator position for a single actuator along the three-mode beam. The DC at the center of the beam (#11) is zero because that is the location of a node of the second mode. However, the center is also an antinode of the first and third modes (see Fig. 2), so that as long as some control is maintained over the second mode by another actuator, the center is an excellent location for a secondary actuator. Thus 11 and 1 are optimal locations for two actuators and 5, 11, 1 are optimal for three. But once the possibility of an actuator failure is introduced, the penalty of losing an actuator at 1 or 5 and being left with only the one at 11, which leaves the second mode uncontrollable, weighs heavily into the average DC shifting the optimal location from 11 away from the center.

A more extensive parametric study on actuator location was conducted using a two-mode simulation. In this example, the effect of state weighting on optimum actuator locations was explored: The degree of controllability vs actuator position for one variable and one fixed actuator is shown in Figs. 3 and 4 for two cases. The fixed actuator is at the end of the beam since that is always an optimal position for one actuator. In the failure case, the mission period was chosen to be the mean time to failure for a single actuator.

In Fig. 3, the amplitudes of both the first and second modes were weighted equally, and the optimal actuator locations for both cases were #1 and #7. Note that position 7 is near the antinode of mode 2 (see Fig. 2). If mode 2 is made less important to control than mode 1, by decreasing $x_{3_{min}}$ in the definition of the scaling matrix D, then the desirability of having the second actuator at the antinode of mode 2 is diminished. Figure 4 shows the results when the minimum desired controllable excursion of mode 2 is 2/3 that of mode 1. In this case, if no possibility of component failures is considered, the optimum actuator locations are still #1 and #7, but with failures considered the optimum locations have switched to #1 and #2—away from the antinode of mode 2. With the second actuator at position #2, the loss of DC due to the possible failure of the actuator at the end is less severe. If the importance of controlling mode 2 is decreased further, eventually the optimum locations switch to #1 and #2 in the no-failure case as well.

Some detailed results of this study are given in Table II. Decreasing the importance of controlling mode 2 results in increased values of the third and fourth diagonal elements of D according to the structure of D given below Eq. (4). In each case, scaling of the rate variables by $1/\omega_i$ relative to the corresponding amplitude variable was retained.

TABLE II. DC AND OPTIMAL LOCATIONS FOR VARYING STATE WEIGHTING FACTORS

				Optimal Location		Degree of Controllability	
D(1,1)	D(2,2)	D(3,3)	D(4,4)	No-fail	Fail	Pos.#1,7	Pos.#1,2
1	.0882	1	.0316	1,7		.5546	.5456
					1,7	.3447	.3418
1	.0882	1.2	.0379	1,7		.6583	.6502
					1,2	.4064	.4073
1	.0882	1.6	.0506	1,7		.8534	.8529
					1,2	.5184	.5343
1	.0882	2.0	.0632	1,2		1.022	1.040
					1,2	.6142	.6515

It can be seen from this table that for mode 1 weighting relative to mode 2 in the range 1.2 to 1.6, the optimum actuator locations are different when failures are acknowledged than without consideration of failures.

Finally, the effect of the number of actuators on the Degree of Controllability of the three-mode representation of the beam is shown in Fig. 5 both with and without failures considered. This is a plot of DC data appearing in Table I; each value is the Degree of Controllability resulting from optimal placement of the corresponding number of actuators. Both curves are seen to be essentially linear over the range of actuator number shown. The reason for this is clear when one notes the DC as a function of the location of a single actuator shown in Fig. 1; after locating the first actuator in position #1, there are several possible positions for the next few actuators which have almost equal effectiveness. If the curve were to be extended to larger numbers of actuators, it would show the expected diminishing returns as the more favorable positions become occupied.

CONCLUSIONS

A methodology has been presented which is intended to assist the designer of a control system for a large space structure to decide how many sensors and actuators should be incorporated in the system and where they should be placed on the structure. This approach is intended to be especially useful in the early stages of the evolution of the system, before a complete control system concept has been defined.

This methodology uses quantitative measures of the controllability and observability of the system for given sets of actuators and sensors which were developed in a previous report. In this work, the effect of possible component failures during the mission period was incorporated in the measures. The question of actuator and sensor placement is then resolved by finding the locations which maximize these performance measures. The number of components to use cannot be determined by optimizing these measures because the controllability and observability always improves with increased number of components if they are optimally located. However, the improvement in these measures with component number can be determined, and this information can be used along with data on cost, power required, etc. to decide how many components to use.

These procedures were illustrated for the case of control of a uniform free-free beam. Optimal actuator locations were found and the variation of maximum Degree of Controllability with number of actuators was determined for up to 4 actuators. Cases were shown in which the recognition of possible actuator failures resulted in significantly different optimum actuator locations than without consideration of failures. The results are intuitively clear when dealing with a simple beam, but it is hoped that this methodology will be useful in more realistically complicated design situations by providing a rational quantitative basis for addressing the questions of control system actuator and sensor number and placement.

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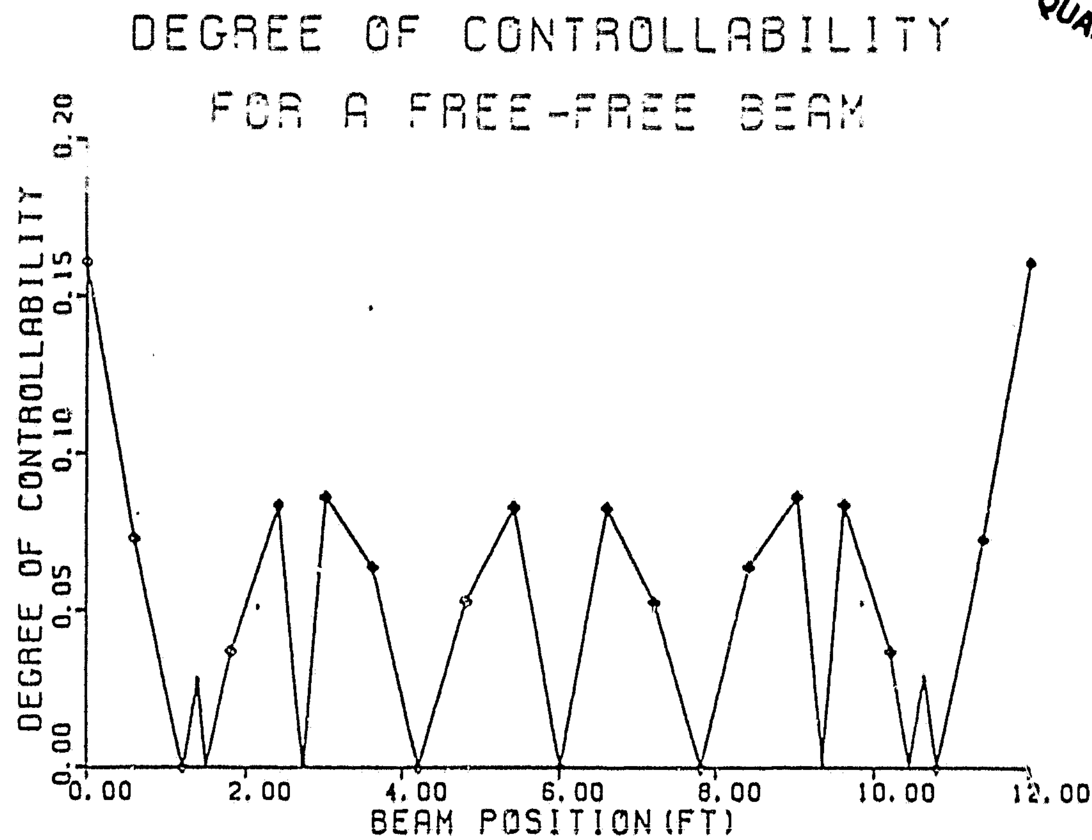

Figure 1. DC vs. Actuator Position for Three-Mode Representation of the Beam.

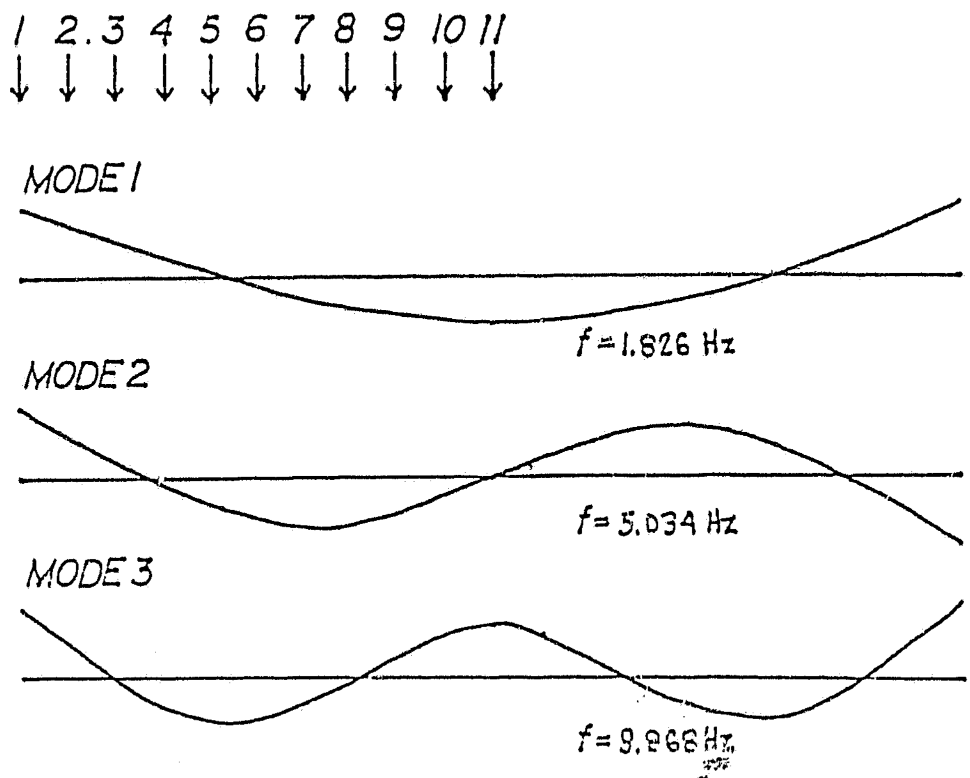

Figure 2. Modeled Mode Shapes and Actuator Test Positions.

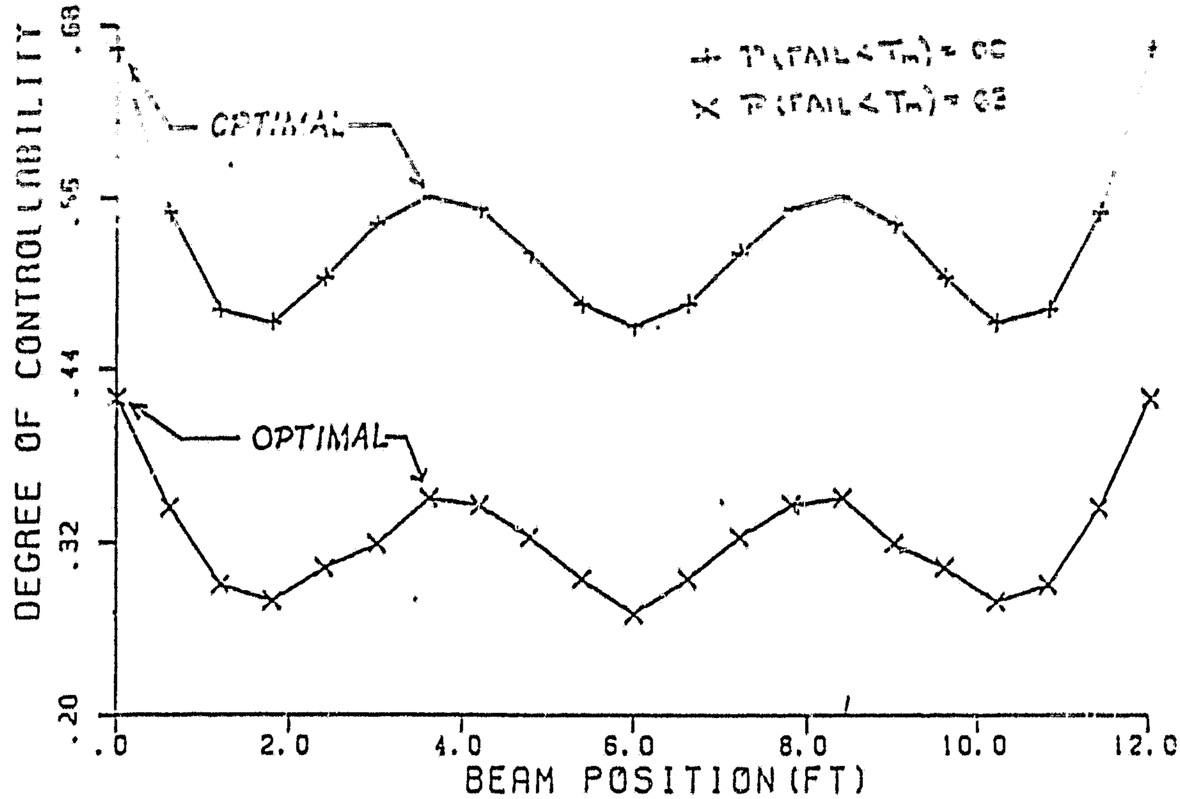

Figure 3. DC vs. Actuator Position for Modal Amplitudes Weighted Equally (one actuator is fixed at the end).

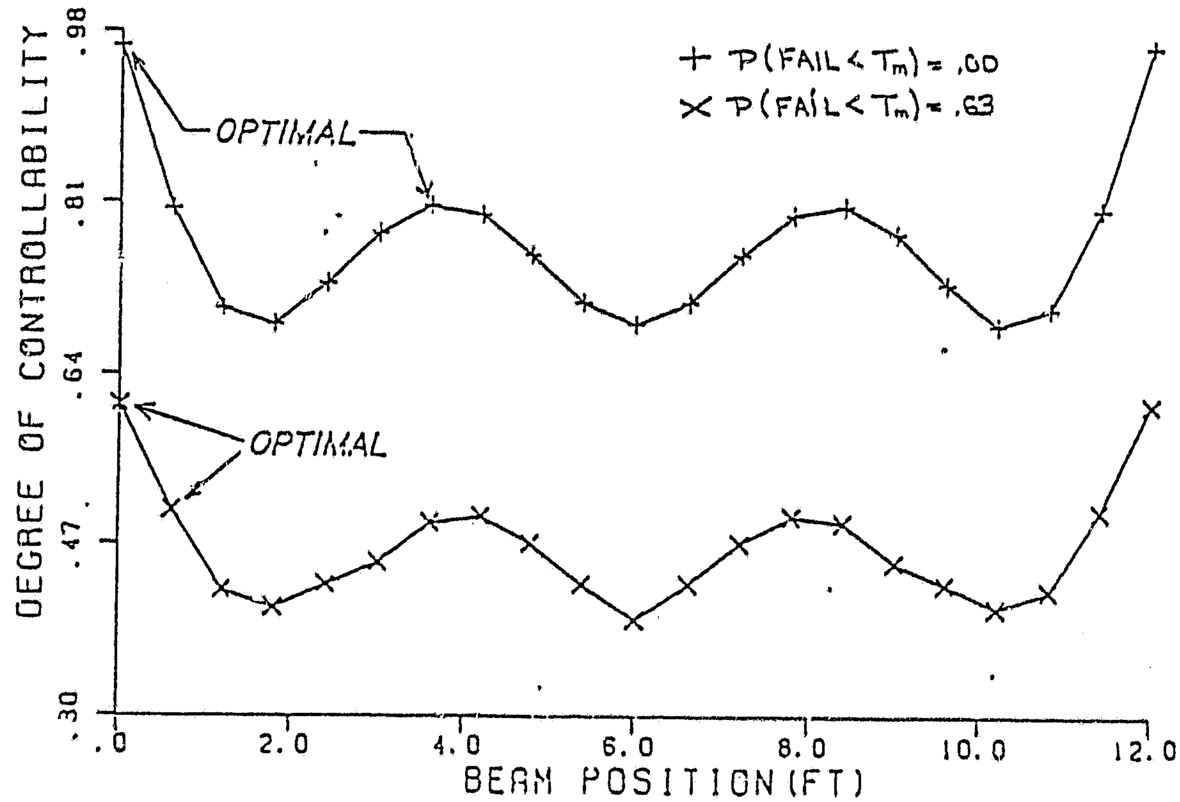

Figure 4. DC vs. Actuator Position for Mode 1 Amplitude Weighted 1.5 Times More Heavily than Mode 2 Amplitude (one actuator is fixed at the end).

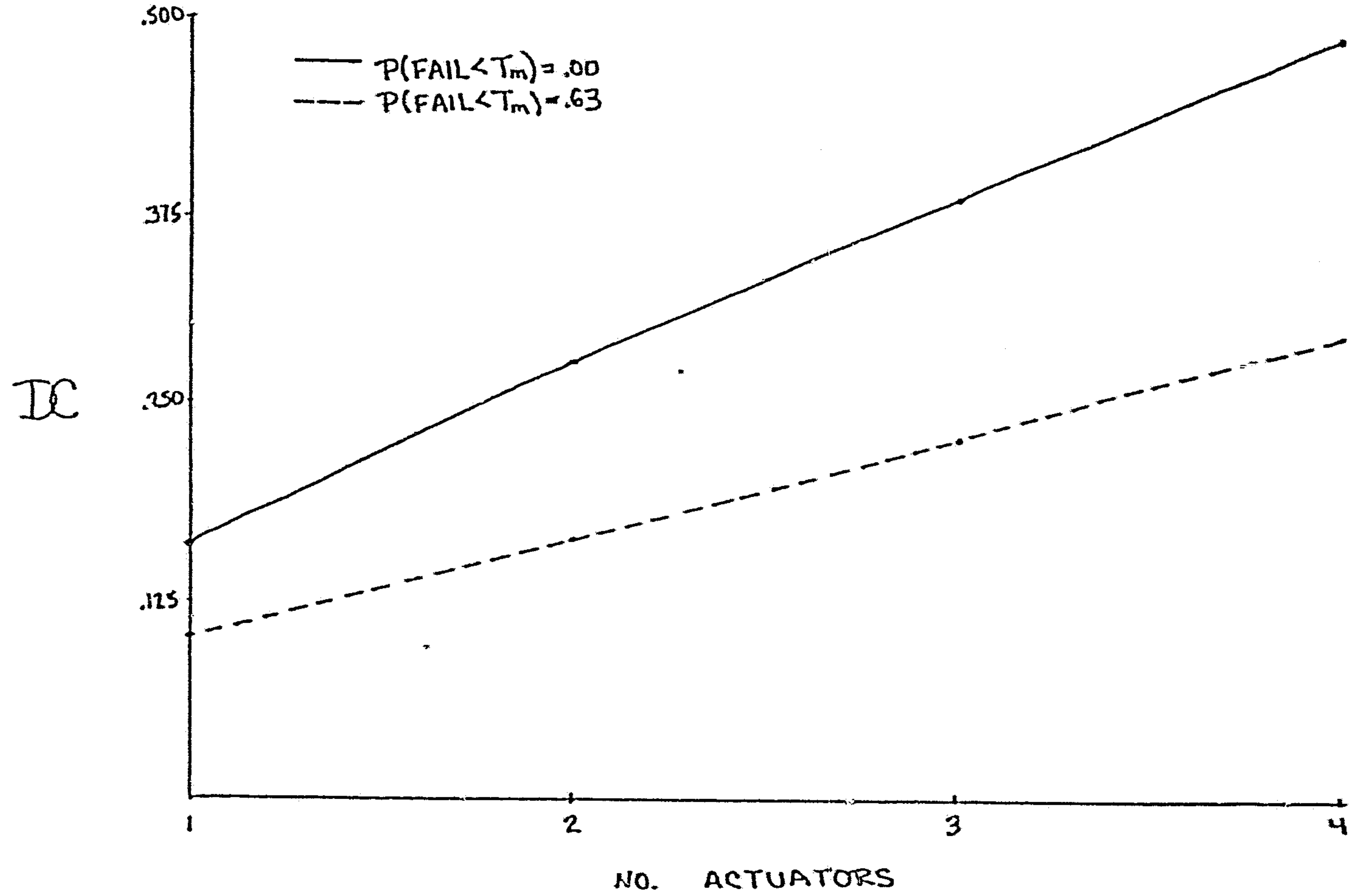

Figure 5. Optimal Degree of Controllability vs. Number of Actuators for Three-mode Simulation.