STUDY OF THE GLOBAL POSITIONING SYSTEM FOR MARITIME

CONCEPTS/APPLICATIONS: STUDY OF THE FEASIBILITY OF

REPLACING MARITIME SHIPBORNE NAVIGATION SYSTEMS WITH

NAVSTAR

FINAL TECHNICAL REPORT

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I. Introduction

The NAVSTAR Global Positioning System (GPS) is a satellite navigation system currently under development by the Department of Defense. A joint-Service program office was established in 1974 at the Air Force Space and Missile Systems Organization in Los Angeles, California. Tests conducted between 1974 and the present have verified the system concept and the predicted system performance. GPS will consist of twenty four satellites, inclined 60 degrees to the equator, in twelve hour orbits. The GPS signal is a spread spectrum signal with two pseudo-random noise codes in quadature. A 50 bit per second data bit stream is modulated on both codes. The data stream contains system time, satellite clock characteristics data, satellite ephemerides, plus other status indicator data.

The National Aeronautics and Space Administration (NASA), Goddard Space Flight Center, has developed a concept utilizing a geostationary reference satellite (REFSAT) that broadcasts every four seconds updated GPS satellite coordinates. This procedure reduces the complexity of the GPS receiver. For an overview of the NASA concept, reference (1) is an excellent paper. This paper should be reviewed to understand the REFSAT and GPS receiver interface.

NASA initial direction was to:

1. Quantify the economic and performance payoffs associated with replacing maritime shipborne navigation systems with NAVSTAR, and

2. Evaluate the use of NAVSTAR for measurements of ocean currents in the broad ocean areas of the world.

During this contract period, a single channel GPS receiver was developed by Systematics General Corporation. This unit includes the RF/IF hardware, L-band synchronizer and frequency reference, code tracking loop, carrier tracking loop and digital processor. The software control and signal processing software were developed by Howard University, Washington, D.C. There was a requirement to develop a dynamic software program to position a ship at sea. Colorado State University (CSU) was redirected to develop this software and integrate the software into the REFSAT receiver.

2. Initial Research

Cargo and transport ship velocities in the range of 10 to 20 knots can have their speed through the water reduced by 10-20% by ocean currents. The general trend in ocean currents is quite well understood and has been the subject of numerous analytical efforts (2) and also many empirical studies with resulting sea charts published by the United States Navy. The Pilot Charts include recommended sea-lanes and make the distinction between full and low-powered ships(3).

These currents are time-varying. In particular, both the Gulf Stream and the Japanese Current, two of the predominant sources of general oceanic circulation for the Northern Hemisphere, can have marked variations, particularly along the edges of the mainstream of the current itself. As a result, it is worthwhile to seek minimum-fuel ocean passages which rely on an understanding of the fine grain, time-varying structure of those ocean currents.

Receivers using the GPS signal have the capability to accurately measure ocean currents. This information could be relayed via satellite to a central data reduction center where the real-time current data could be integrated into the computer data banks. The updated data file could then be used to project the minimum sailing time route back to the ship.

If the use of GPS would allow an improvement of half a knot in forward speed, there is a rather remarkable cost savings in dollars-per-year for the tanker fleet. From Table 1, one can see that a tanker in the 100,000-ton class can realize savings of as much as $90,000 per year, if the knowledge of ocean currents allows a speed improvement of .46 knots. Figure 1 is an extract from the pilot chart of the North Atlantic for July 1978. It shows a path which is prescribed for low-powered ships to gulf ports; the path passes through regions of extremely confused currents.

TANKER PERFORMANCE CHART (4)

Tanker Displacement	Avg. Speed Knots	TTU/Year*	$/TTU	3% Speed Improvements	ΔTTU** Δ3% Speed	Savings/ Year
19,000 (FOMALHAUT)	14.5	873	$1,276	.435 kts.	2.86%	31,858
52,000 (VIRGO)	16.25	2882	$ 892	.487 kts.	2.80%	71,980
94,090 (DRACO)	15.3	4810	$ 666	.459 kts.	2.81%	90,017

* A TTU is 10^6 ton miles
** 10,000 mile round voyage

Table 1. Cost Savings Due to Speed Improvements

Note that the dashed lines on Figure 1 reflect current uncertainty. The pilot charts reflect many examples of such unpredictable currents. An important case is that in the Gulf of Alaska, in which the area has extensive dashed currents for the month of July, 1978. This is the area currently being traversed by the Alaskan tanker fleet. Shown on Table 1 are the overall savings in dollars-per-year versus speed improvements for three classes of ship.
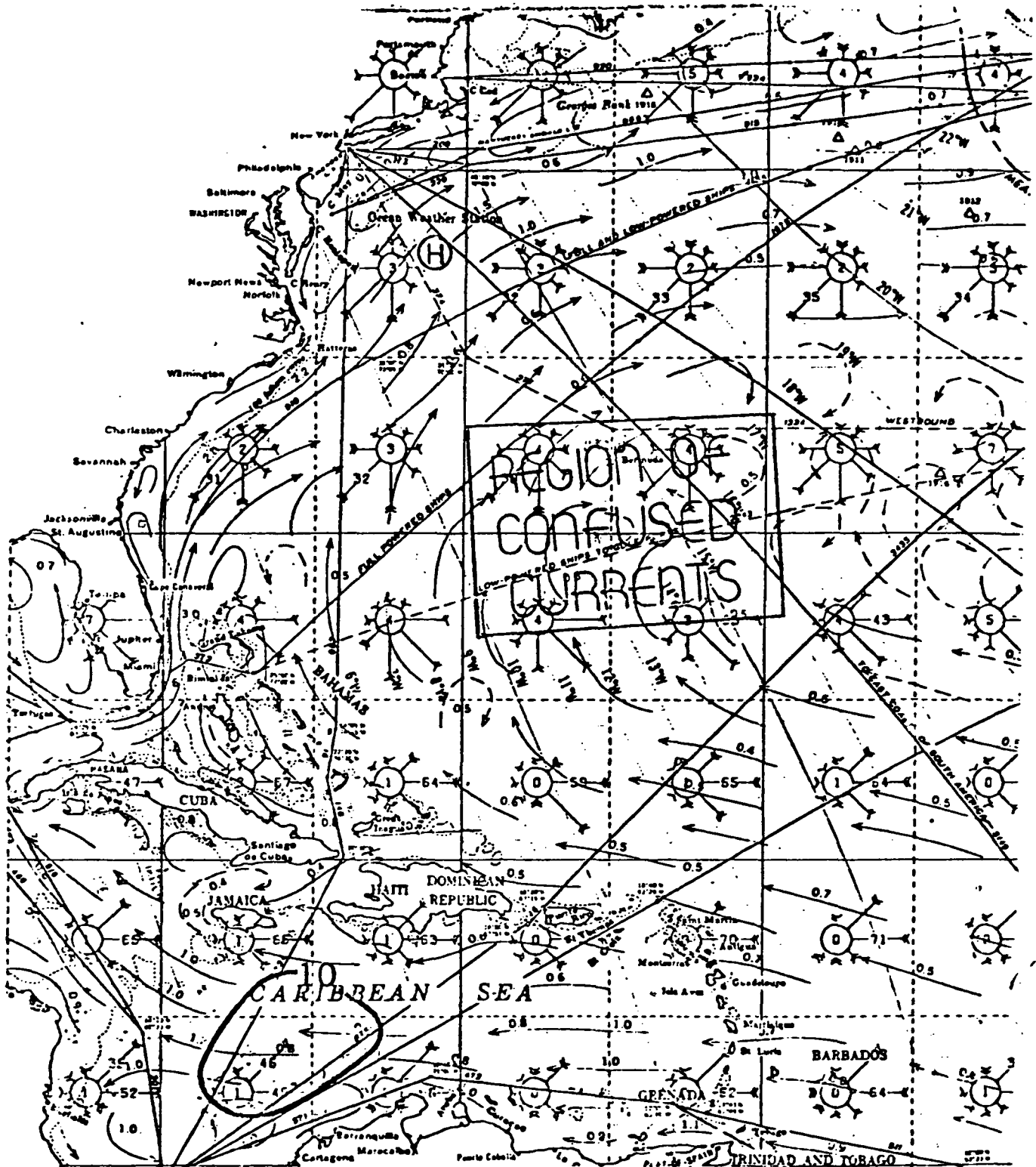
Figure 1. Ocean Currents from North Atlantic Pilot Chart

The initial research was directed toward constructing an algorithm that would give the minimum sailing time route between two points on the earth. To specify a position on the earth's surface we need two angles, θ and ϕ. Let (θ_0,ϕ_0) be the position of the point of origin and (θ_1,ϕ_1) the destination. Using θ as a parameter, a function $\Phi(\theta)$ is determined which provides the minimum sailing time.

First an integral is developed which provides the sailing time in terms of ocean currents, ship speed and $\Phi(\theta)$. To find the function that minimizes this integral we apply calculus of variations methods to a second order differential equation - Euler's equation with boundary conditions. The Euler's equation is quite long and nonlinear. Solving the equation numerically, the interval from θ_0 to θ_1 is divided into n subintervals. The first and second derivatives of $\Phi(\theta)$ at each end point of the subinterval are approximated by using Taylor's theorem. In place of the Euler's equation we now have a system of n-1 nonlinear algebraic equations in the unknowns $\Phi(\theta_0+i\Delta\theta)(i=1,\ldots,n-1)$ plus the boundary conditions.

Let $\underline{F}$ be this system of equations and $\underline{X}$ the n-1 vector of values of $\Phi(\theta_0+i\Delta\theta)$. Next the system $\underline{F}(\underline{X})=\underline{0}$ is solved. This system can be rewritten

$$\underline{F}(\underline{X}) + \underline{X} = \underline{G}(\underline{X}) = \underline{X}.$$

This system is solved by iterating on an initial guess $\underline{X}_0$ using the equation

$$\underline{X}_{k+1} = \underline{G}(\underline{X}_k) \qquad \text{where } k = 0,1,3\ldots$$

CSU was in the process of implementing the system on the computer when we were redirected to develop a navigation filtering algorithm to support the REFSAT approach to low cost GPS terminals.

Some analytical approaches to solving this optimization problem are described in Appendix A.

3. Development of Navigational Filtering Algorithms.

CSU was directed to support the REFSAT development by developing navigation filtering algorithms. This research includes coordination with Howard University and Systematics General. The hardware developed by Systematics General Corporation was to be integrated with the digital processing and operational software developed by Howard University. Development problems delayed the integration. CSU was requested to have the software available for integration in the summer of 1981. Unfortunately, funding to the hardware and software activities was not available. Funds set aside for the integration of the navigation software have been returned by CSU to NASA.

The navigation software was to be resident in the REFSAT receiver processor. As the digital processing and operational software developed, it appeared that insufficient space would be available, both from a throughput and a memory point of view. It was therefore concluded that the software should not be developed independent of the operating REFSAT receiver. Information on the Navigation Function Driver (NAVFUNC) has been received from Howard University. It is possible that some of the data calculated in the subroutines could be used to reduce the computation time of the navigation filter. If the NAVFUNC data is not retrievable, then a method similar to the one used by A. J. Van Dierendonck (5) must be used to obtain the NAVSTAR satellite coordinates.

Time to calcualte the navigation solution is very limited. It is estimated that it will take from 5-10 seconds to calculate the necessary variables needed for the navigation algorithm. The solution of the ship's position will take from 15-30 seconds. For example, to solve for the linear system represented by

$$H\overline{x} = \overline{y} \text{ (rank 5)}$$

takes from 1.7 to 2.1 seconds to solve. A listing of the program to solve this system is listed in Appendix B. The dead reckoning portion will take another 5-10 seconds, for a total computational time of slightly less than one minute. Times are based on an 8080/Z80 based microprocessor ($\sim$ 2MHZ clock).

The accuracy of the position and velocity of the ship will be impacted by the ship's motion: headway, surge, leeway, sway, heave, roll, pitch and yaw. Of these motions, roll will have the greatest adverse impact. Large ships (over 150 meters in length) can have roll periods up to 13 seconds. By taking a sampling period much greater ($\sim$ 5τ) than the period of roll the short term vessel's motion could be treated as increased noise.

The initial navigation algorithm will be based on the work of Noe, Myers and Wu (6). The navigation solution will be strongly dependent upon system noise. Since the REFSAT receiver performance has not been fully defined, the navigation accuracy can not yet be determined.

4. Future Efforts

The incorporation of the navigation filter into the REFSAT receiver should take approximately one month. If the receiver is not operational or persons familiar with the receiver are not available for consultation then the period of effort would extend several months. CSU has the personnel capable of making the receiver operational and would be pleased to continue this effort if and when funding becomes available.

5. References

(1) J. W. Sennott, A. K. Chouldhury, and R. E. Taylor, NASA Technical Memorandum 79655, The REFSAT Approach to Low-Cost GPS Terminals, April 1979.

(2) Stern, M. E., Ocean Circulation Physics, Academic Press, N.Y., 1975.

Von Arx, W., An Introduction to Physical Oceanography, Addison-Wesley, Reading, Mass., 1962.

(3) Pilot Chart of the North Atlantic.

(4) Gannett, E., Tanker Performance and Cost, Cornell Maritime Press, Cambridge, Maryland, 1969.

(5) A. J. Van Dierendonck, S. S. Russell, E. R. Kopitzke and M. Birnbaum, The GPS Message, Journal of the Institute of Navigation, Vol. 25, No. 2, Summer 1978.

(6) P. S. Noe, K. A. Meyers, and J. K. Su, "A Navigation Algorithm for the Low-Cost GPS Receiver," Journal of the Institute of Navigation, Summer 1978.

Appendix A

The trajectory optimization problem discussed in Section 2 is described in more detail in this appendix. A simplified analytical formulation is presented based on a kinematical analysis and a closed form solution to this simplified problem is presented. Next, various complications are added and the form of the optimal controller for each variation is presented. Finally, a dynamical formulation is presented and its solution is discussed.

The problem of determining the optimal trajectory for a ship may be described in simplified terms as follows. Consider the situation represented on Figure A.1. The problem is to determine the steering angle, $\theta(t)$, that results in a transfer of the ship from its initial position (L_o, α_o) to a final position (L_f, α_f) in minimum time. Without loss of generality we may assume that the final latitude, L_f, is zero. Also, without loss of generality, we may assume that the motion takes place on a unit sphere and that the ship's speed relative to the water is unity.

Model No. 1. No Ocean Currents

For this initial problem we assume that there are no ocean currents. This provides a starting point for which we can obtain a closed form analytical solution. We may then add the ocean currents to the model and attempt to build a solution in steps.
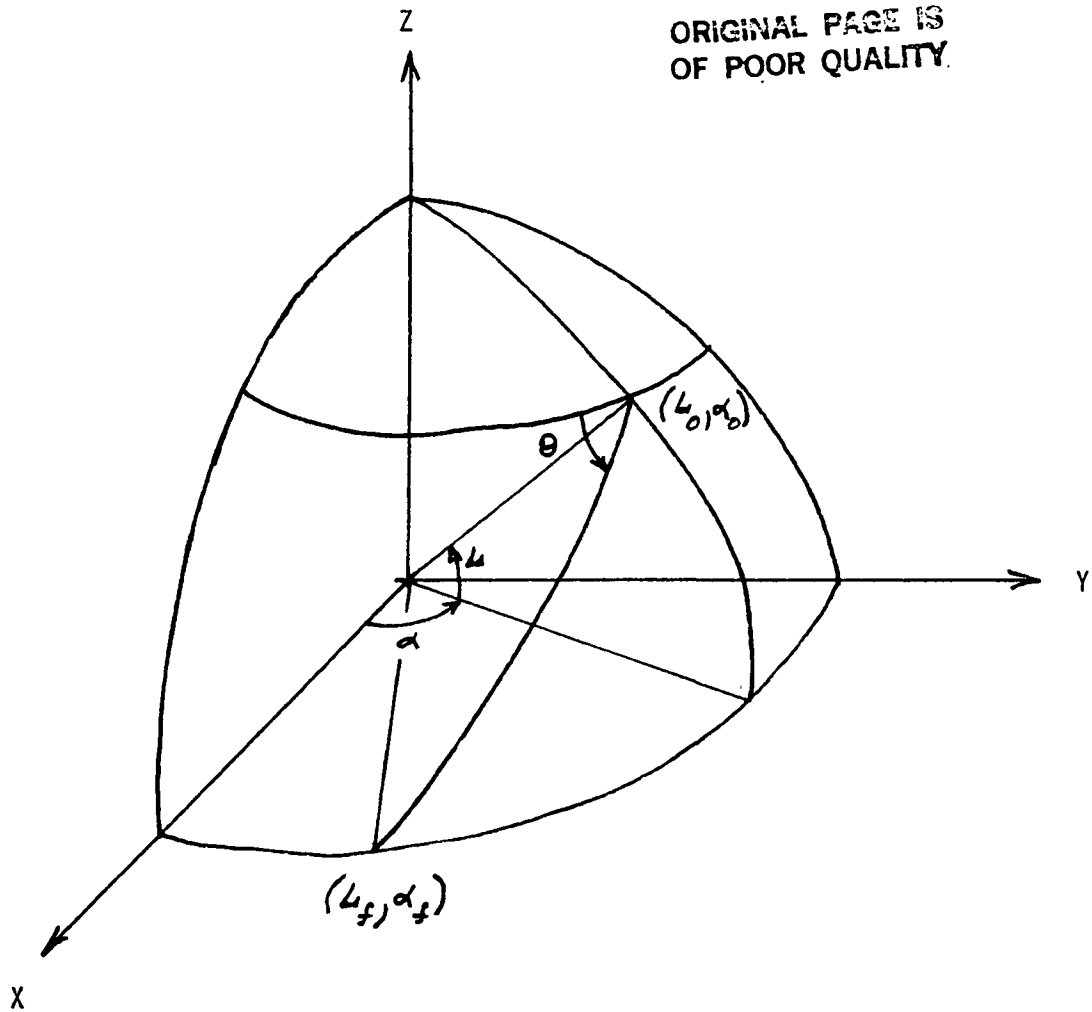
The kinematic equations may be written as

Figure A.1. Ship's Trajectory

$$\frac{dL}{dt} = \sin \theta \qquad \text{(A.1)}$$

$$\frac{d\alpha}{dt} = \frac{\cos \theta}{\cos L} \qquad \text{(A.2)}$$

The payoff function may be defined as

$$\phi = -t_f \qquad \text{(A.3)}$$

where t_f is the final time. We wish to determine $\theta(t)$ such that ϕ is maximized as this is equivalent to minimizing the time required for the transfer to take place. The terminal constraints are

$$\psi_1 = L(t_f) = 0 \qquad \text{(A.4)}$$

$$\psi_2 = \alpha(t_f) - \alpha_f = 0 \qquad \text{(A.5)}$$

The Hamiltonian function is

$$H = \lambda_L \sin \theta + \lambda_\alpha \frac{\cos \theta}{\cos L} \qquad \text{(A.6)}$$

A necessary condition for optimality is that the control, $\theta(t)$, maximizes the Hamiltonian. Therefore the optimal control is determined unambiguously by the equations

$$\sin \theta = \lambda_L \ / \ \sqrt{\lambda_L^2 + \lambda_\alpha^2 \ / \ \cos^2 L} \qquad \text{(A.7)}$$

$$\cos \theta = (\lambda_\alpha / \cos L) \ / \ \sqrt{\lambda_L^2 + \lambda_\alpha^2 \ / \ \cos^2 L} \qquad \text{(A.8)}$$

Equations (A.7) and (A.8) specify the optimal control in terms of the adjoint variables λ_L and λ_α. These, in turn, are determined from the following equations.

$$\frac{d\lambda_L}{dt} = - \frac{\partial H}{\partial L}$$

$$= - \lambda_\alpha \cos \theta \sin L / \cos^2 L \tag{A.9}$$

$$\frac{d\lambda_\alpha}{dt} = - \frac{\partial H}{\partial L}$$

$$= 0 \tag{A.10}$$

The adjoint variables are determined at the final time, t_f, from the transversality condition

$$\lambda^T(t_f) = \left(\frac{\partial \phi}{\partial \chi} - \nu^T \frac{\partial \psi}{\partial \chi}\right)\bigg|_{t_f} \tag{A.11}$$

where $\chi^T = (L, \alpha)$. From equation (A.3),

$$\frac{\partial \phi}{\partial \chi}\bigg|_{t_f} = (0 \quad 0) \tag{A.12}$$

and from equations (A.4) and (A.5)

$$\frac{\partial \psi}{\partial \chi}\bigg|_{t_f} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{A.13}$$

Therefore

$$(\lambda_L, \lambda_\alpha)\Big|_{t_f} = (-\nu_1 \quad -\nu_2) \tag{A.14}$$

The components of ν^T represent the sensitivities in the payoff function, ϕ, to changes in the components of the state vector at the final time. Consequently

$$\nu_1 = -\sin \theta_f$$

and

$$\nu_2 = -\cos \theta_f$$

and therefore

$$\lambda_L(t_f) = \sin \theta_f \tag{A.15}$$

and

$$\lambda_\alpha(t_f) = \cos \theta_f \tag{A.16}$$

An additional necessary condition is that

$$\frac{d}{dt} [\phi - \nu^T \psi]\Big|_{t_f} = 0 \tag{A.17}$$

Expanding,

$$[(\frac{\partial \phi}{\partial \chi} - \nu^T \frac{\partial \psi}{\partial \chi}) \frac{d\chi}{dt} + (\frac{\partial \phi}{\partial t} - \nu^T \frac{\partial \psi}{\partial t})]\Big|_{t_f} = 0 \tag{A.18}$$

Recalling equations (A.1), (A.2), (A.6), and (A.11); and using the fact that $\frac{\partial\phi}{\partial t} = -1$ and $\frac{\partial\psi}{\partial t} = 0$, this reduces to

$$H\Big|_{t_f} = 1 \qquad\qquad (A.19)$$

Also, since the Hamiltonian is not an explicit function of time it must be constant along an optimal trajectory. Therefore, necessarily

$$H \equiv 1$$

along the optimal path. This provides us with the following equation for λ_L:

$$\lambda_L{}^2 = 1 - \cos^2\theta_f / \cos^2 L \qquad\qquad (A.20)$$

The above system of equations may be solved in closed form; the solutions are

$$L(t) = -\sin^{-1}[\sin\theta_f \sin(t_f - t)] \qquad\qquad (A.21)$$

$$\alpha(t) = \alpha_f - \tan^{-1}[\cos\theta_f \tan(t_f - t)] \qquad\qquad (A.22)$$

$$\lambda_L(t) = \cos(t_f - t) / \sqrt{\cot^2\theta_f + \cos^2(t_f - t)} \qquad\qquad (A.23)$$

$$\lambda_\alpha(t) = \cos\theta_f \qquad\qquad (A.24)$$

and

$$\theta(t) = \tan^{-1}[\tan\theta_f \cos(t_f - t)] \qquad\qquad (A.25)$$

These solutions may be verified by substituting them back into the differential equations. For any given value for α_f, the above equations define a two-parameter family of trajectories on the sphere. The two parameters are θ_f and t_f.

Given any arbitrary state (L,α) at some time $t < t_f$ one may solve for θ_f and t_f in terms of L, α, and t. The results are

$$\theta_f = \tan^{-1}\left[\frac{-\tan L}{\sin (\alpha_f - \alpha)}\right] \tag{A.26}$$

and

$$t_f = t + \cos^{-1}\left[\cos L \cos (\alpha_f - \alpha)\right]. \tag{A.27}$$

These relations for θ_f and t_f may be substituted into equations (A.23), (A.24) and (A.25) to obtain the optimal values for the adjoint variables and the control. The results are

$$\lambda_L^*(L,\alpha,t) = -\sin L / \sqrt{\sin^2 L + \tan^2 (\alpha_f - \alpha)} \tag{A.28}$$

$$\lambda_\alpha^*(L,\alpha,t) = \cos L \tan (\alpha_f - \alpha) / \sqrt{\sin^2 L + \tan^2 (\alpha_f - \alpha)} \tag{A.29}$$

and

$$\theta^*(L,\alpha,t) = \tan^{-1}\left[\frac{-\sin L}{\tan (\alpha_f - \alpha)}\right] \tag{A.30}$$

The optimal value for the payoff function is

$$\phi^*(L,\alpha,t) = -t_f^*(L,\alpha,t) - \cos^{-1}\left[\cos L \cos (\alpha_f - \alpha)\right] \tag{A.31}$$

Model No. 2. Constant Current (East or West)

We next consider a situation in which there is a constant ocean current in the easterly or westerly direction. In this case the kinematical equations are

$$\frac{dL}{dt} = \sin \theta \tag{A.32}$$

$$\frac{d\alpha}{dt} = \frac{\cos \theta \pm k}{\cos L} \tag{A.33}$$

The Hamiltonian is

$$H = \lambda_L \cos \theta + \lambda_\alpha \left(\frac{\cos \theta \pm k}{\cos L}\right) \tag{A.34}$$

The Hamiltonian is again maximized by the control

$$\sin \theta = \frac{\lambda_L}{\sqrt{\lambda_L^2 + \lambda_\alpha^2 / \cos^2 L}} \quad , \quad \cos \theta = \frac{\lambda_\alpha \cos L}{\sqrt{\lambda_L^2 + \lambda_\alpha^2 / \cos^2 L}} \tag{A.35}$$

That is, the optimal control is determined by the same equations as for the first problem.

Also, we still have that

$$\frac{d\phi}{dt} - \nu^T \frac{d\psi}{dt} \bigg|_{t_f} = 0 \tag{A.36}$$

$$\left(\frac{\partial\phi}{\partial\chi} - \nu^T \frac{\partial\psi}{\partial\chi}\right) \frac{d\chi}{dt} \bigg|_{t_f} + \left(\frac{\partial\phi}{\partial t} - \nu^T \frac{\partial\psi}{\partial t}\right) \bigg|_{t_f} = 0 \tag{A.37}$$

Therefore $\quad \lambda^T F \big|_{t_f} + (\frac{\partial \phi}{\partial t} - \nu^T \frac{\partial \psi}{\partial t}) \big|_{t_f} = \dot{0}$ $\qquad$ (4.38)

But

$$\frac{\partial \phi}{\partial t} = -1 \quad , \quad \frac{\partial \psi}{\partial t} = 0 \qquad (4.39)$$

Also

$$\lambda^T F \big|_{t_f} = H$$

Therefore $\qquad\qquad H\big|_{t_f} = + 1 \qquad\qquad$ (A.40)

Also $H \neq H(t)$; $\therefore \frac{dH}{dt} = 0$ along the optimal trajectory; also, we have that $H \equiv 1$ along the optimal trajectory. Hence

$$\lambda_L \sin\theta + \lambda_\alpha \frac{\cos \theta}{\cos L} \pm \lambda_\alpha \frac{k}{\cos L} \equiv 1 \qquad (A.41)$$

The adjoint equations are

$$\frac{d\lambda_L}{dt} = \frac{-\partial H}{\partial L} = - \lambda_\alpha (\cos \theta \pm k) \frac{\sin L}{\cos^2 L} \qquad (A.42)$$

and

$$\frac{d\lambda_\alpha}{dt} = - \frac{\partial H}{\partial \alpha} = 0 \qquad (A.43)$$

Also, we still have

$$\lambda_\alpha (t_f) = \frac{\partial t_f}{\partial \alpha_f} = \cos \theta_f \qquad (A.44)$$

Also, from $H \equiv 1$,

$$\frac{\lambda_L^2}{\sqrt{\lambda_L^2 + \lambda_\alpha^2 / \cos^2 L}} + \frac{(\lambda_\alpha / \cos L)^2}{\sqrt{\lambda_L^2 + \lambda_\alpha^2 / \cos^2 L}} \pm \frac{\lambda_\alpha k}{\cos L} = 1$$

$$\sqrt{\lambda_L^2 + \frac{\lambda_\alpha^2}{\cos^2 L}} \pm \frac{\lambda_\alpha k}{\cos L} = 1$$

$$\lambda_L^2 + \frac{\lambda_\alpha^2}{\cos^2 L} = (1 \mp \frac{\lambda_\alpha k}{\cos L})^2 = 1 + \frac{\lambda_\alpha^2 k^2}{\cos^2 L} \mp \frac{2\lambda_\alpha k}{\cos L} \qquad (A.45)$$

The solution proceeds along the same lines as for the first problem.

Model Number 3. Constant Current (North and South)

Let V_c = constant in the north or south direction. Then

$$\frac{dL}{dt} = \sin \theta \pm k \qquad\qquad (A.46)$$

$$\frac{d\alpha}{dt} = \frac{\cos \theta}{\cos L} \qquad\qquad (A.47)$$

$$H = \lambda_L (\sin \theta \pm k) + \lambda_\alpha \frac{\cos \theta}{\cos L}$$

$$= \lambda_L \sin \theta + \frac{\lambda_\alpha}{\cos L} \cos \theta \pm k\lambda_L \qquad (A.48)$$

Again the control law will remain the same.

That is

$$\sin \theta = \frac{\lambda_L}{\sqrt{\lambda_L{}^2 + \dfrac{\lambda_\alpha^2}{\cos^2 L}}} \quad , \quad \cos \theta = \frac{\lambda_\alpha / \cos L}{\sqrt{\lambda_L{}^2 + \dfrac{\lambda\alpha^2}{\cos^2 L}}} \qquad (A.49)$$

But the adjoint equations are

$$\frac{d\lambda_L}{dt} = - \frac{\partial H}{\partial L} = - \frac{\lambda_\alpha \cos \theta}{\cos^2 L} \sin L \qquad (A.50)$$

$$\frac{d\lambda_\alpha}{dt} = - \frac{\partial H}{\partial \alpha} = 0 \qquad (A.51)$$

Again we see that the addition of a constant current to the model results in the same form of solution. Clearly the second and third models may be combined.

Model No. 4. Linearly Increasing Currents (East and West)

Now suppose that the ocean current is a linearly increasing function of latitude. That is

$$V_c = \pm kL \qquad (A.52)$$

Then the system equations are

$$\frac{dL}{dt} = \sin \theta \qquad (A.53)$$

$$\frac{d\alpha}{dt} = \frac{\cos \theta \pm kL}{\cos L} \qquad (A.54)$$

The Hamiltonian is

$$H = \lambda_L \sin \theta + \lambda_\alpha (\cos \theta \pm kL)/\cos L \qquad \text{(A.55)}$$

The optimal control law still remains the same as before; the adjoint variables, which actually determine the optimal control as a function of time are given in this case by

$$\frac{d\lambda_L}{dt} = - \lambda_\alpha (\cos \theta \pm kL) \frac{\sin L}{\cos^2 L} \mp \frac{k\lambda_\alpha}{\cos L} \qquad \text{(A.56)}$$

$$\frac{d\lambda_\alpha}{dt} = 0 \qquad \text{(A.57)}$$

The solution technique proceeds in the same manner as before.

Model No. 5. Linearly Increasing Currents (North or South)

In this case consider

$$V_c = \pm k\alpha \qquad \text{(A.58)}$$

Then the system equations are

$$\frac{dL}{dt} = \sin \theta \pm k\alpha \qquad \text{(A.59)}$$

$$\frac{d\alpha}{dt} = \frac{\cos \theta}{\cos L} \qquad \text{(A.60)}$$

The Hamiltonian is

$$H = \lambda_L (\sin \theta \pm k\alpha) + \lambda_\alpha \cos \theta/\cos L \qquad \text{(A.61)}$$

Again there is no change in the optimal control law. The adjoint equations are

$$\frac{d\lambda_L}{dt} = - \lambda_\alpha \cos \theta \sin L/\cos^2 L \qquad (A.62)$$

and

$$\frac{d\lambda_\alpha}{dt} = \mp k\lambda_L \qquad (A.63)$$

Thus we are no longer have that λ_α is constant along an optimal trajectory.

Model No. 6. Dynamical Model

The problem of determining the optimal steering angle to minimize transfer time is similar to the problem of steering a ship to minimize propulsion losses. This latter problem has been addressed [A.1], [A.2] and is briefly discussed below. Comsider the representation depicted in Figure A.2. The ship's equations of motion may be written as

$$\begin{bmatrix} (m - Y_{\dot{v}}) - Y_{\dot{r}} & - Y_{\dot{p}} & 0 \\ -N_{\dot{v}} & (I_{zz} - N_{\dot{r}}) & - (N_{\dot{p}} + I_{xz})0 \\ -K_{\dot{v}} & -(K_{\dot{r}} + I_{xz}) & (I_{xx} - K_{\dot{p}})0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{p} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} Y_v & Y_r - mU & Y_p & Y_\phi \\ N_v & N_r & N_p & N_\phi \\ K_v & K_r & K_p & K_\phi \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ p \\ \phi \end{bmatrix} + \begin{bmatrix} Y_{\delta r} \\ N_{\delta r} \\ K_{\delta r} \\ 0 \end{bmatrix} \delta r + \begin{bmatrix} Y_D \\ N_D \\ K_D \\ 0 \end{bmatrix} \qquad (A.64)$$

where

ψ = yaw

v = sway

ρ = roll

and Y_D, N_D, K_D represent the external disturbances on the ship due to the seaway. In addition, the ship's lateral motion is defined by

$$\dot{y} = v + U\psi \qquad (A.65)$$
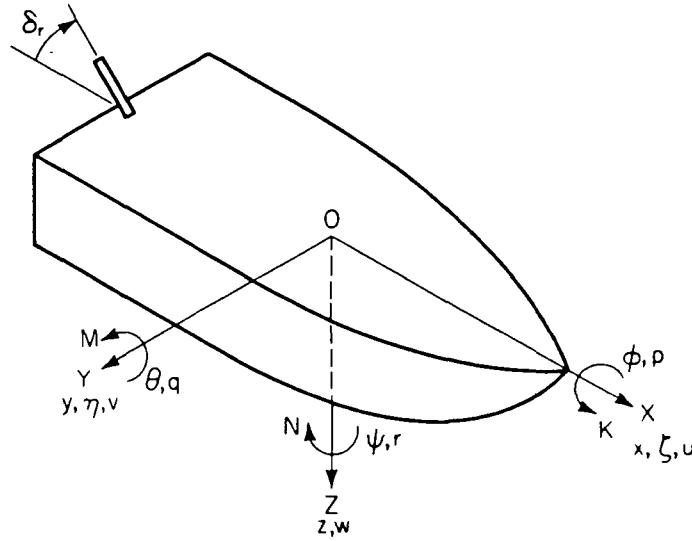
$$\dot{\psi} = r. \qquad (A.66)$$

Fig. A.2   Ship coordinate system-body axes coordinates,
forces, and moments.   Abstracted from [A.1].

Equations (A.64), (A.65), and (A.66) may be written in state space representation as

$$A\dot{X} = BX + Cu + DW ,$$ (A.67)

or, rewriting in standard form

$$\dot{X} = FX + B^{T}u + GW$$ (A.68)

where $F=A^{-1}B$, $B^{T}=A^{-1}C$, and $G=A^{-1}D$.   The state vector is X, the control vector is u, and the disturbance vector is W.

Representative open loop lateral plane system eigenvalues for a highspeed containership at full-load design condition for speeds of 16, 23, and 32 knots are [A.1]:

16 Knots -------0.00

$$-0.0584 \times 10^{-2}$$

$$-0.792 \times 10^{-1}$$

23 Knots -------0.00

$$-0.839 \times 10^{-2}$$

$$-0.114$$

32 Knots -------0.00

$$-0.117 \times 10^{-1}$$

$$-0.158$$

A simulation model for this system has been developed [A.1] and is represented pictorially by the block diagram shown on Figure A.3.

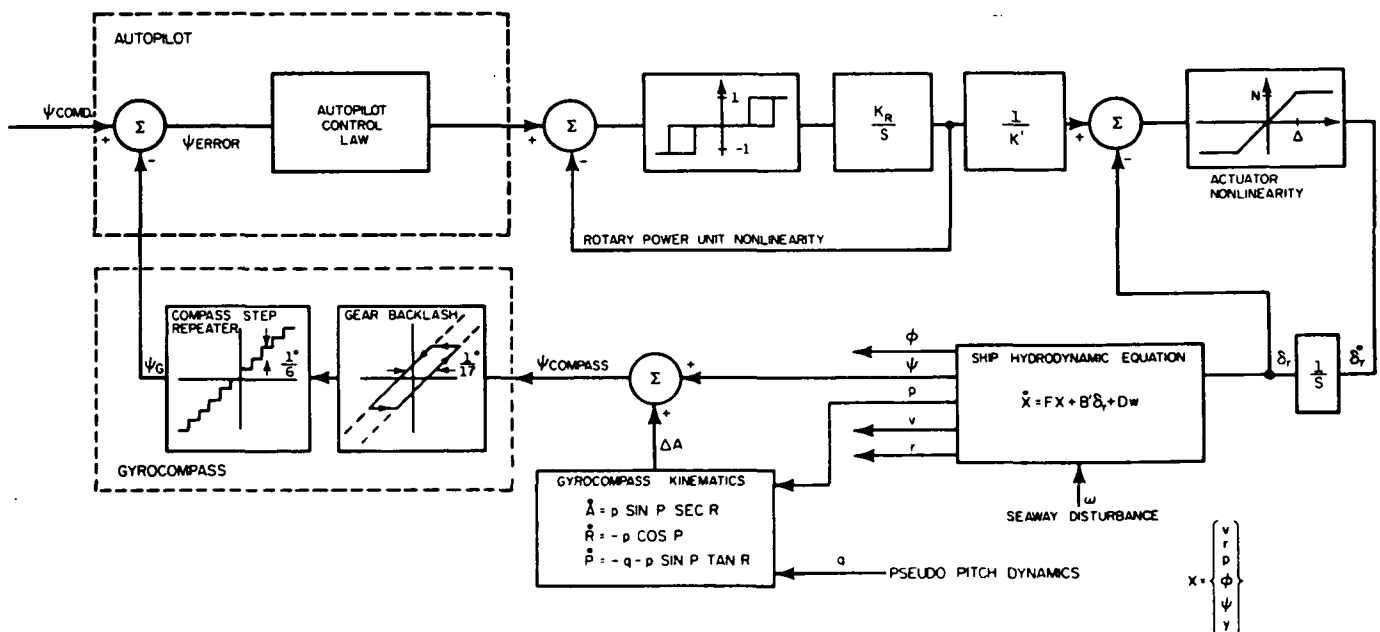

Figure A.3   Nonlinear simulation model.   Abstracted from [A.1].

The problem is to determine the controller that will minimize propulsion losses due to steering. These losses may be shown to result from excess power consumption per unit distance travelled in the X-direction caused by the added resistance due to steering. This is related to the instantaneous surge due to steering, which is

$$\Delta X = (m + (\rho/2) \, LAX'_{vr}) \, vr + (1/2)](\rho/2) \, AX'_{vv}] \, v^2 \qquad (A.69)$$

$$+ (1/2)[(\rho/2) \, AX'_{\delta\delta} \, U^2] \, \delta^2$$

where m denotes mass of ship; ρ, density of sea water; L, ship length between perpendiculars; $A=L^2$; U, ship's water speed; v, sway velocity of ship; r, yaw rate of ship; δ, rudder angle; X'_{vr}, force coefficient due to yaw/sway (+ve); $X'_{\delta\delta}$, force coefficient due to rudder angle (-ve); X'_{vv}, force coefficient due to sway. From this, the mean surge relevant to steering may be written as:

$$\overline{\Delta X} = [m + (\rho/2) \, LAX'_{vr}] \, (v_a \, r_a/2) \cos (\phi_v - \phi_r)$$

$$+ (1/2) \, [(\rho/2) \, AX'_{vv}](v_a^2/2)$$

$$+ (1/2) \, [(\rho/2) \, AX'_{\delta\delta} \, U^2](\delta_a^2/2) \qquad (A.70)$$

Since the ship motions resulting from seaway disturbances are oscillatory, a performance criterion for added resistance due to steering may be formulated as:

$$J = (1/2) \int_{o}^{\infty} [-\lambda'' \, vr + \eta v^2 + \delta^2] \, dt \qquad (A.71)$$

where

$$\lambda'' = 2[m + (\rho/2) \, LAX'_{vr}]/[(\rho/2) \, AX'_{\delta\delta} \, U^2] \qquad (A.72)$$

$$\eta = X'_{vv}/X'_{\delta\delta} \, U^2 \qquad (A.73)$$

both inversely proportional to speed squared.

The simulation program described in Ref. [A.1] has been used for the purpose of evaluating controllers to minimize the cost function, J, defined by equation (A.71).

By combining the two methods of analysis described in this appendix the global optimal controller may be determined.

References

A.1. R. E. Reid and J. W. Moore, "Optimal Steering Control of High Speed Containerships: Part 1 System Dynamics," Proceedings of the 1980 Joint Automatic Control Conference, Vol. 1, August 1980.

APPENDIX B

This appendix includes the listing of the program to solve the linear system $H\bar{x} = \bar{y}$ of rank five. Two routines are used to solve the same linear system.

Subroutine SOLVEO is written in the normal FORTRAN manner using Do-loops, double subscripting, etc. Subroutine SOLVE is written in a more primitive way. SOLVE evaluates all subscripts by single additions, uses no Do-loops, and isolates floating point operations in a way that will facilitate an easy transition to an assembly language version of the code.

Code developed for the navigation filter will incorporate the features of SOLVE. If the time required to obtain a ship's position needs to be reduced then the program could be written with a minimum of effort.

On the MDS-230 system ($\sim$2 MHz) the approximate times required for the two methods are:

SOLVEO   2.1 sec.

SOLVE   1.7 sec.

```
C
C
C
C
C

      PROGRAM NS
      DIMENSION H(24,5),HTH0(5,6),HTH(5,6),Y(24),X0(5),X(5)
      DATA IW/ 6/
C
      OPEN(UNIT=IW,FILE=':F1:OUTPT')
      READ(5,*,END=999,ERR=999) ITER
      WRITE(IW,5) ITER
    5 FORMAT(//' NUMBER OF ITERATIONS ON EACH LOOP = ',I4/)
C
      DO 20   I=1,24
      Y(I)= SIN(FLOAT(I)/(24.*3.1416))
      DO 10   J=1,5
      H(I,J)= FLOAT(I+J) + 1./FLOAT(I+J)
   10 CONTINUE
   20 CONTINUE
C
      DO 40   I=1,24
      WRITE(IW,30) (H(I,J),J=1,5),Y(I)
   30 FORMAT(5(1X,E10.4),4X,E10.4)
   40 CONTINUE
C
      PAUSE ' BEGIN FIRST LOOP  '
      DO 50   I=1,ITER
      CALL SOLVE0(H,HTH0,Y,X0)
   50 CONTINUE
      PAUSE ' END OF FIRST LOOP  '
C
      WRITE(IW,60)
   60 FORMAT(/)
      DO 70   I=1,5
      WRITE(IW,30) (HTH0(I,J),J=1,6)
   70 CONTINUE
      WRITE(IW,80) (X0(I),I=1,5)
   80 FORMAT(//' SOLUTION VECTOR X0'/5(1X,E11.5)/)
C
      PAUSE ' BEGIN SECOND LOOP  '
      DO 90   I=1,ITER
      CALL SOLVE(H,HTH,Y,X)
   90 CONTINUE
      PAUSE ' END OF SECOND LOOP  '
C
      DO 100   I=1,5
      WRITE(IW,30) (HTH(I,J),J=1,6)
  100 CONTINUE
      WRITE(IW,110) (X(I),I=1,5)
  110 FORMAT(//' SOLUTION VECTOR X'/5(1X,E11.5)/)
C
  999 STOP
      END
C
C
C
C
C
```

```
      SUBROUTINE SOLVE0(H,G,Y,X)
      DIMENSION H(24,5),G(5,6),Y(24),X(5)
C
      DO 100   J=1,5
      DO 80    I=1,5
      IF(I.GT.J) GO TO 80
      S= 0.
      DO 50    K=1,24
      S= S + H(K,J)*H(K,I)
   50 CONTINUE
      G(I,J)= S
      G(J,I)= S
   80 CONTINUE
  100 CONTINUE
C
      DO 150   I=1,5
      S= 0.
      DO 120   J=1,24
      S= S + Y(J)*H(J,I)
  120 CONTINUE
      G(I,6)= S
  150 CONTINUE
C
C
      DO 300   J=1,4
      R0= -1./G(J,J)
      JP1= J+1
C
      DO 280   I=JP1,5
      R= R0*G(I,J)
C
      DO 250   K=JP1,6
      G(I,K)= R*G(J,K) + G(I,K)
  250 CONTINUE
  280 CONTINUE
  300 CONTINUE
C
      X(5)= G(5,6)/G(5,5)
      DO 350   II=1,4
      I= 5-II
      IP1= I+1
C
      S= G(I,6)
      DO 320   J=IP1,5
      S= S - G(I,J)*X(J)
  320 CONTINUE
      X(I)= S/G(I,I)
C
  350 CONTINUE
      RETURN
      END
C
C
C
C
```

```
      SUBROUTINE SOLVE(H,G,Y,X)
      DIMENSION H(1),G(1),Y(1),X(1)
      INTEGER  GADR, HADR, YADR, GPTR, GTBASE, HBASE, GTPTR,
     +         HTBASE, HPTR, HTPTR, YBASE, YPTR



      INTEGER  XADR, R0PTR, GJBASE, GIBASE, RPTR,
     +         GIPTR, GJPTR, GBASE, XPTR
      INTEGER*1 I, J, K
C
      HADR= 1
      GADR= 1
      YADR= 1
      XADR= 1
C
C
      GPTR= GADR - 1
      GTBASE= GADR - 6
      HBASE= HADR - 25
C
      J= 6
   10 J= J-1
      IF(J. EQ. 0) GO TO 100
      GTBASE= GTBASE + 1
      GTPTR= GTBASE
      HTBASE= HADR - 25
      HBASE= HBASE + 24
C
      I= 6
   20 I= I-1
      IF(I. EQ. 0) GO TO 10
      GPTR= GPTR + 1
      GTPTR= GTPTR + 5
      HTBASE= HTBASE + 24
      IF(I. LT. J) GO TO 20
C
      HPTR= HBASE
      HTPTR= HTBASE
      S= 0.
      K= 25
   30 K= K-1
      IF(K. EQ. 0) GO TO 50
      HPTR= HPTR + 1
      HTPTR= HTPTR + 1
      S= S + H(HPTR)*H(HTPTR)
      GO TO 30 .
   50 G(GPTR)= S
      IF(I. EQ. J) GO TO 20
      G(GTPTR)= S
      GO TO 20
```

```
C
  100 HPTR= HADR - 1
       YBASE= YADR - 1
C
       J= 6
  110 J= J-1
       IF(J.EQ.0) GO TO 140
       GPTR= GPTR + 1
       YPTR= YBASE
       S= 0
       I= 25
  120 I= I-1
       IF(I.EQ.0) GO TO 130
       HPTR= HPTR + 1
       YPTR= YPTR + 1




       S= S + H(HPTR)*Y(YPTR)
       GO TO 120
  130 G(GPTR)= S
       GO TO 110
C
C
  140 R0PTR= GADR - 6
       GJBASE= GADR - 6
C
       J= 0
  210 J= J+1
       IF(J.EQ.5) GO TO 300
       R0PTR= R0PTR + 6
       R0= -1./G(R0PTR)
       RPTR= R0PTR
C
       GJBASE= GJBASE + 6
       GIBASE= GJBASE
C
       I= J
  220 I= I+1
       IF(I.EQ.6) GO TO 210
       RPTR= RPTR + 1
       R= R0*G(RPTR)
       GIBASE= GIBASE + 1
       GIPTR= GIBASE
       GJPTR= GJBASE
C
       K= J
  230 K= K+1
       IF(K.EQ.7) GO TO 220
       GIPTR= GIPTR + 5
       GJPTR= GJPTR + 5
       G(GIPTR)= R*G(GJPTR) + G(GIPTR)
       GO TO 230
```

```
C
  300 GBASE= GADR + 30
      I= 6
  310 I= I-1
      IF(I.EQ.0) GO TO 340
      GBASE= GBASE - 1
      GPTR= GBASE
      XPTR= XADR + 5
      S= G(GPTR)
C
      J= 6
  320 J= J-1
      IF(J.EQ.I) GO TO 330
      GPTR= GPTR - 5
      XPTR= XPTR - 1
      S= S - G(GPTR)*X(XPTR)
      GO TO 320
C
  330 XPTR= XPTR - 1
      GPTR= GPTR - 5
      X(XPTR)= S/G(GPTR)
      GO TO 310
C
  340 RETURN



      END
```

```
SOLUTION VECTOR X0
-.12363E+00   .29913E+00 -.62771E-01 -.21159E+00   .11195E+00

SOLUTION VECTOR X
-.12363E+00   .29913E+00 -.62771E-01 -.21159E+00   .11195E+00
```