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CONTROLLABILITY OF INHERENTLY DAMPED LARGE FLEXIBLE SPACE STRUCTURES

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CONTROLLABILITY OF INHERENTLY DAMPED LARGE FLEXIBLE SPACE STRUCTURES*

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Abstract

Graph theoretic techniques are used to study controllability of linear systems which could represent large flexible orbiting space systems with inherent damping. The controllability of the pair of matrices representing the system state and control influence matrices is assured when all states in the model are reachable in a digraph sense from at least one input and also when the term rank of a Boolean matrix whose non trivial components are based on the state and control influence matrices has a term rank of the order of the state vector. It is seen that the damping matrix does not influence the required number of actuators but gives flexibility to the possible locations of the actuators for which the system is controllable, and that the stiffness matrix term rank deficiency dictates the number as well as the location of the required actuators. Specific examples include a model of a shallow spherical orbiting shell where both orientation and shape control are required, and also a smaller dimensional numerical example (unrelated to the shell) which readily demonstrates the effect of damping.

Nomenclature

A	nxn system matrix
$\bar{A}$	2nx2n system matrix
B, B_2	nxm control influence matrices
$\bar{B}$	2nxm control influence matrix
D	nxn damping matrix
D'	nxn modified damping matrix
A_B, B_B, D'_B	Boolean equivalents of A, B and D' matrices
K	nxn stiffness matrix
M	nxn mass matrix
R	reachability matrix
R'	nxn submatrix of matrix R
S	diagonal matrix (rxr)
U	mx1 input vector
V_1	nxn unitary orthogonal matrix
V_2	mxm unitary orthogonal matrix
X	nx1 vector
Σ	nxm matrix
μ	small parameter
σ_i	i^{th} singular value of matrix A

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I. Introduction

Any linear, time invariant dynamical system can be, in general represented by:

$$\dot{X} = AX + BU \tag{1}$$

where

X is an nx1 state vector of the system
A is an nxn system state matrix
B is an nxm control influence matrix
U is an mx1 input of the system.

The system described by equation (1) is said to be controllable if, with finite U and in finite time, the system (1) can be transferred from any state to any other state. This concept was first introduced by Richard E. Kalman.[1] The verification of controllability is essential for control system design as no control law should be designed for a system which is not controllable. The controllability concept is even more important for large space structural systems whose dimensionality is very large. If the design of the control system is undertaken without first verifying controllability, a considerable amount of effort may be wasted, through the failure to arrive at any satisfactory control law for an uncontrollable system.

In the following sections, the concept of controllability, as a property of the A and B matrices, is reviewed.

The system, (1) is controllable if and only if[1,2]:

$$\text{rank}\ (B, AB, A^2B, \ldots\ldots, A^{n-1}B) = n \tag{2}$$

$$\text{or} \quad \text{rank}\ (B, A-\lambda_i I) = n, \quad i = 1,2,\ldots n \tag{3}$$

where λ_i are the eigenvalues of the matrix, A.

The determination of the rank of the matrix in equation (2) poses the problem of selecting n independent columns out of nm columns and this could be done by such numerical techniques as the singular value decomposition of a matrix.[3]

The singular value decomposition is a numerical algorithm used to find the numerical rank of a rectangular matrix, A, (nxm, say n>m) through the evaluation of two orthogonal (unitary) matrices, V_1 and V_2 such that

$$A = V_1\ \Sigma\ V_2^T. \tag{4}$$

where

$$\Sigma = \begin{bmatrix} S & 0 \\ 0 & 0 \end{bmatrix} = V_1^T A V_2 \tag{5}$$

and $S = \text{diag}\ (\sigma_1, \sigma_2, \ldots, \sigma_r)$

with $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r > 0$, being singular values of A. The $\sigma_1, \sigma_2, \ldots, \sigma_r$ are the non zero eigenvalues of the matrix AA^T (or A^TA) and r is the rank of the matrix A. The technique of finding the singular values of A by evaluating the eigenvalues of AA^T or A^TA is, in general, less accurate than using singular value decomposition techniques and may result in erroneous conclusions, as is demonstrated by the following example.[3]

Let

$$A = \begin{bmatrix} 1 & 1 \\ u & 0 \\ 0 & u \end{bmatrix} \tag{6}$$

Then [if $1+u^2=1$, but $1+u \neq 1$, for the computer accuracy involved]

$$A^TA = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \tag{7}$$

The eigenvalues of this approximation to A^TA are $\sqrt{2}$ and 0, respectively, whereas the precise eigenvalues of A^TA are found to be $\sqrt{2+u^2}$ and u, respectively.

The use of the singular value decomposition technique, itself, may result in numerical problems when the magnitude of the largest singular value of the matrix A, is an order of magnitude (or more) different from that of the smallest singular value. [A complete discussion of the singular value decomposition technique is given in Ref. 3. It should be noted that with the use of this algorithm, it is unnecessary to directly evaluate the singular values of the A matrix. The rank determination is accomplished based on the unitary, orthogonal properties of the matrices V_1 and V_2 within the algorithm.]

The application of the controllability condition in equation (3) requires the determination of the eigenvalues of the matrix A and the evaluation of the rank of a matrix of order $[nx(n+m)]$ for each of the n eigenvalues. This scheme is more attractive than that of condition (2) as the problem of rank evaluation of an $nxmn$ matrix is reduced to the rank evaluations of n matrices each of dimension $[nx(n+m)]$. The eigenvalues are, in general, needed for the structural dynamic analysis of the system and may, thus, be already available for this phase of the control system design.

II. Controllability of Large Space Structures

The dynamical equations of a large space structure system are, in general, described by a set of linear second order coupled differential equations as:

$$M\ddot{X} + D\dot{X} + KX = B_p U \tag{8}$$

where

- X is the nx1 vector of the generalized coordinates
- M is the mass matrix (nxn)
- D is the damping matrix (nxn) (can include viscous damping as well as gyroscopic effects)
- K is the stiffness matrix
- B_p is the control influence matrix
- U is the mx1 vector of inputs.

The dynamical system of equations (8) can be rewritten as a set of first order differential equations (in standard state space form as):

$$\begin{bmatrix} \dot{X} \\ \ddot{X} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix} \begin{bmatrix} X \\ \dot{X} \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}B_p \end{bmatrix} U \tag{9}$$

Equation (9) can be considered as

$$\dot{\bar{X}} = \bar{A}\bar{X} + \bar{B}U \tag{10}$$

where $\bar{X} = [X, \dot{X}]^T$

$$\bar{A} = \begin{bmatrix} 0 & I \\ A & D' \end{bmatrix} \quad \bar{B} = \begin{bmatrix} 0 \\ B \end{bmatrix} \tag{11}$$

and $A = -M^{-1}K$, $D' = -M^{-1}D$, $B = M^{-1}B_p$.

The controllability condition (2) for this system can be written as:

$$\text{rank } [\bar{B}, \bar{A}\bar{B}, \ldots, \bar{A}^{2n-1}\bar{B}] = 2n \tag{12}$$

If we assume D=0, which is true for many idealized free vibrating structures, the controllability matrix[4]

$$C = [\bar{B}, \bar{A}\bar{B}, \ldots, \bar{A}^{2n-1}\bar{B}], \tag{13}$$

becomes

$$C = \left[\begin{array}{c} 0 \\ \hline B \end{array} \middle| \begin{array}{c} B \\ \hline 0 \end{array} \middle| \begin{array}{c} 0 \\ \hline AB \end{array} \middle| \begin{array}{c} AB \\ \hline 0 \end{array} \middle| \cdots \middle| \begin{array}{c} A^{n-1}B \\ \hline 0 \end{array} \right] \tag{14}$$

It can be very easily seen that C has a rank 2n if and only if

$$\text{rank } [B, AB, \ldots, A^{n-1}B] = n \tag{15}$$

which leads to the following theorem[4]:

__Theorem:__ The pair $\begin{bmatrix} 0 & I \\ A & 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ B \end{bmatrix}$ is controllable if and only if the pair [A,B] is controllable.

This theorem reduces the determination of the controllability of a $2n^{th}$ order system to the determination of the controllability of an equivalent n^{th} order system. In general for large space structure applications, n itself may still be sufficiently large and, thus, numerical techniques would be required in order to determine controllability. This theorem is based on the inherent assumption that D=0 and no insight can be drawn when D is not equal to zero. The effect of the matrix, D, on controllability is studied in this paper using the graph theoretic definition of controllability.

III. Graph Theoretic Definition of Controllability[3]

Given the general, linear, time invariant dynamical system described by equation (1), repeated here as:

$$\dot{X} = AX + BU \tag{16}$$

the pair $[A,B]$ is controllable if and only if:

(1) the term rank of $\begin{bmatrix} A_B, & B_B \\ 0 & 0 \end{bmatrix} = n$

where A_B, B_B are the Boolean equivalents of the matrices A and B, respectively; and
(2) all states in the system are reachable from at least one input in the digraph sense.

The two terms, term rank and reachability, are explained here.

Term rank is the maximum rank a matrix can achieve due to the locations of the non zero, non fixed elements of the matrix rather than due to the numerical values of the elements. A complete discussion of this concept is provided in the Appendix.

Reachability. If one draws a digraph for the extended square matrix $\begin{matrix} A & B \\ 0 & 0 \end{matrix}$ and finds the input-state reachability matrix as explained in the Appendix, there must be at least one non zero entry for every row of the submatrix (R') in the reachability matrix, R, formed from the row and column indices 1 to n, and n+1 to n+m, respectively, as shown in equation (17) where n is the number of states in the system and m is the number of actuators.

$$R = \begin{bmatrix} \begin{matrix} 1 & & & n \\ x & \cdots & & x \\ x & \cdots & & x \\ \vdots & & & \vdots \\ x & \cdots & & x \end{matrix} & \begin{matrix} n+1 & & n+m \\ & R' & \\ & & \end{matrix} \\ \hline 0 & 0 \end{bmatrix} \begin{matrix} 1 \\ \\ n \\ n+1 \\ \\ n+m \end{matrix} \tag{17}$$

IV. Controllability of Systems with Inherent Damping

The dynamics of large space systems with inherent damping can be written as (repeating equation (7) with the notation defined in equation (10))

$$\begin{bmatrix} \dot{X} \\ \ddot{X} \end{bmatrix} = \underbrace{\left[\begin{array}{c|c} 0 & I \\ \hline A & D' \end{array}\right]}_{\bar{A}} \begin{bmatrix} X \\ \dot{X} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ B \end{bmatrix}}_{\bar{B}} U \tag{18}$$

The digraph for the system matrices $\bar{A}$, $\bar{B}$ can be drawn in general as shown in Fig. 1.

The elements of $\bar{A}(i,i+n)$, $i=1,2,\ldots,n$ - i.e. those elements of $\bar{A}$ appearing in the identity matrix - are represented in the digraph by the (solid) lines joining the nodes $(i+n)$ to i where $i=1,2,\ldots,n$. The elements of $\bar{A}(i+n,j)$, $i=1,2,\ldots,n$, $j=1,2,\ldots,n$ i.e. those elements of $\bar{A}$ appearing in A are represented in the digraph by the (dashed) lines joining the j^{th} node to the $(i+n)$th node. The elements of $\bar{A}(i+n,j+n)$, $i=1,2,\ldots,n$, $j=1,2,\ldots,n$ i.e. those elements of $\bar{A}$ appearing in D' are represented in the digraph by the (dotted) lines connecting the $(j+n)$th node to the $(i+n)$th node.

The elements of B [the lower half of $\bar{B} = \bar{B}(i+n,j)$], $B_B(i,j)$, $i=1,2,\ldots n$, $j=1,2,\ldots,m$, are represented in the digraph by the (double solid) lines joining the j^{th} actuator [the $(2n+j)$th node, here] to the $(i+n)$th node.

From the reachability condition, it is observed that the lines in the digraph due to the D matrix can supplement those lines due to the B_B matrix. For example suppose that due to the structure of D, there is a directed (dotted) line from node $(n+3)$ to node $(n+2)$ in Fig. 1. Suppose that the m^{th} actuator represented by node $(2n+m)$ can directly influence node $(n+3)$. Then it is clear the actuators number 2 and 3 (represented by node 2n+2 and 2n+3, respectively) need not be present in order to influence nodes n+2 and n+3. Thus the damping matrix can allow a greater flexibility in this selection of the actuator locations. Although based on this argument one could construe that the D matrix influences the number of actuators (that could be removed), it should be remembered that the minimum number of actuators required is dictated from the term rank condition which will now be discussed.

For the system represented by equation (18) to be controllable, the term rank (as explained in the Appendix) of the Boolean matrix, m

$$\left[\begin{array}{c|c|c} 0 & I & 0 \\ \hline -A_B & D'_B & B_B \\ \hline 0 & 0 & 0 \end{array}\right] = n \tag{19}$$

must be 2n. Note that the dimensionality of the state vector in equation (18) is 2n. If A has term rank less than n, then the term rank of the $(2n+m)$th order Boolean matrix in equation (19) can only be augmented due to the presence of B_B, since the Det $[A_B] = -$ Det $[A_B]$. Thus, D_B can not be used to augment the term rank of the Boolean equivalent of the state matrix $\bar{A}$ in equation (18).

In summary, the damping matrix, D, has an effect on the location of the actuators, while the matrix A has an impact on the location as well as the number of actuators.

V. Numerical Examples

The use of graph theoretic techniques in the determination of controllability and the amount of information about the location of the actuators and the number of actuators needed is demonstrated using the model of an orbiting shallow spherical shell in orbit with and without the stabilizing dumbbell (Fig 2).[6]

The Boolean Equivalent of the system matrix (A)[7] for a shallow spherical shell is given in Fig. 3. The digraph is given in Fig. 4. From the digraph it can be seen that the nodes may be subgrouped as:

10	11	12	13	14	15	16	17	18
1	2	3	4	5	6	7	8	9

To reach all the 18 states from at least one input, control actuators must directly influence the following nodes: (a) (10 or 11); (b) at least one of the nodes (12-15); (c) (16); (d) (17); and (e) (18). The system matrix, A, has a term rank deficiency of 1 (note the presence of only zeros in the first column, Fig. 3) and, thus, one actuator is required for controllability. This actuator must be placed such that the above mentioned states are directly influenced.

The model of a shallow spherical shell with a stabilizing dumbbell[6,7] is considered as another example for controllability considerations. The Boolean equivalent of the 22nd ordered system matrix A is given in Fig. 5 and the digraph is shown in Fig. 6. From the digraph it can be seen that the total states can be subdivided into two groups

(1)								(2)		
12	16	17	18	19	20	21	22	13	14	15
1	5	6	7	8	9	10	11	2	3	4

The control actuators must directly influence one or more states from group (1) and one or more states from group (2). The system matrix here has full term rank and, thus, one actuator is sufficient to establish controllability.

The two practical examples considered in this section to this point do not specifically illustrate the independence between the number of actuators required and the damping matrix. To illustrate this effect an example of sixth order is created and analyzed.

It is assumed that the system matrix is given by

$$\bar{A} = \left[\begin{array}{ccc|ccc} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \hline 1 & 0 & 0 & 1.0 & 2.0 & 3.0 \\ 1 & 0 & 0 & -10.0 & 5.0 & 6.0 \\ 1 & 0 & 0 & 7.0 & 8.0 & 9.0 \end{array}\right] \quad (20)$$

$$\begin{array}{cc} \quad A \quad & \quad D \quad \end{array}$$

The digraph is drawn as shown in Fig. 7

From the digraph the reachability condition for controllability is satisfied if any one or more of the states: 4,5,6, are directly influenced by the control actuators. The term rank of $\bar{A}_0$ has a deficiency of two and thus two actuators are required for controllability. Even if the damping matrix D=0, the same number (2) of actuators is needed for controllability and, thus, it is shown that damping has no effect on the required number of actuators. But, if the damping matrix D=0, then the location of the actuators must be changed such that the states 4,5&6 can be directly influenced by the control actuators. In order to emphasize the point, when D=0 the dotted lines should be removed from the digraph shown in Fig. 7.

VI. Conclusions

The definition of controllability as applied to general linear time invariant dynamics systems and large space systems is reviewed. The special nature of the coupled matrix second order differential equations that are used to describe large space systems is used to arrive at specific controllability conditions. The graph theory approach is employed to define controllability in terms of the term rank and input-state reachability concepts. This approach is used to find the effect of inherent damping present in large space systems on the number of the actuators and their locations. It is observed that the damping does not effect the minimum number of actuators required, but does provide greater flexibility in the possible locations of the actuators. The number of actuators reqired depends on the term rank of the generalized system (stiffness) matrix. The stiffness matrix also influences the location of the actuators.

References

1. Kalman, R.E., "On the General Theory of Control Systems," in Proc. 1st IFAC Congress, vol. 1, London: Butterworth, 1960, pp. 481-491.

2. Paige, C.C., "Properties of Numerical Algorithms Related to Computing Controllability," IEEE Trans. on Automatic Control, vol. AC-26, No. 1, Feb. 1981, pp. 130-138.

3. Klema, V.C., and Laub, A.J., "The Singular Value Decomposition: Its Computation and Some Applications," IEEE Trans on Automatic Control, vol. AC-25, No. 2, April 1980, pp. 164-176.

4. Balas, M.J., "Feedback Control of Flexible Systems," IEEE Trans. on Automatic Control, vol. AC-23, No. 4, Aug. 1978, pp. 673-679.

5. Schizas, C., and Evans, F.J., "Rank Invariant Transformations and Controllability of Large Scale Systems," Electronic Letters, vol. 16, No. 1, 3rd Jan., 1980., pp. 19-20.

6. Kumar, V.K., and Bainum, P.M., "On the Motion of a Flexible Shallow Spherical Shell in Orbit," AIAA 19th Aerospace Sciences Meeting, Jan. 12-15, 1981, St. Louis, Missouri, paper no. AIAA-81-0170; also to appear in AIAA Journal

7. Balnum, P.M., and Reddy, A.S.S.R., "On the Shape and Orientation Control of an Orbiting Shallow Spherical Shell Structure," to be Presented at the Joint IFAC/ESA Symposium on Automatic Control in Space, Noordwijkerhout, the Netherlands, July 5-9, 1982.

Appendix

A. Term Rank of a Matrix

The term rank of a square matrix of dimension $n \times n$ is less than n if and only if the matrix has a zero submatrix "O" of dimension $r \times r'$ with $r + r' > n$. The term rank is different from the numerical rank in the following sense. If a square matrix of order n has two columns or rows that are dependent on each other, then its term rank is not reduced while its numerical rank is reduced by one for each pair of columns of rows that are dependent. For large space systems, the determination of the numerical values of the elements for the system matrices are not exact, and thus the probability that two columns or rows would be identically equal, or that one row is a constant times another is very small. If such a dependency exists that must be detected before subjecting it to the term rank tests for establishing the controllability of large space systems.

B. Input-State Reachability Matrix

The augmented adjacency matrix for the system matrix pair $\{A,B\}$ can be written as

$$C_B = \begin{array}{c} \\ n \\ m \end{array} \begin{array}{c} \leftarrow n \rightarrow \quad\quad \leftarrow m \rightarrow \\ \left[\begin{array}{c|c} A_B & B_B \\ \hline 0_{m \times n} & I_{m \times m} \end{array}\right] \end{array}$$

where A_B and B_B are the adjacency matrices of A and B, respectively.

The states can be reached from any of the inputs and can not be of length more than n. So C_B can be raised to the power n and thus the augmented system reachability matrix is given by

$$R = \left[\begin{array}{c|c} A_B^n & (A_B^{n-1} + A_B^{n-2} + \dots + A_B)B_B \\ & (= R_{IS}) \\ \hline 0 & 0 \end{array}\right] = C_B^n$$

where R_{IS} is the input-state reachability matrix For all the states to be reached from at least one input, every row of R_{IS} must have at least one non zero entry.

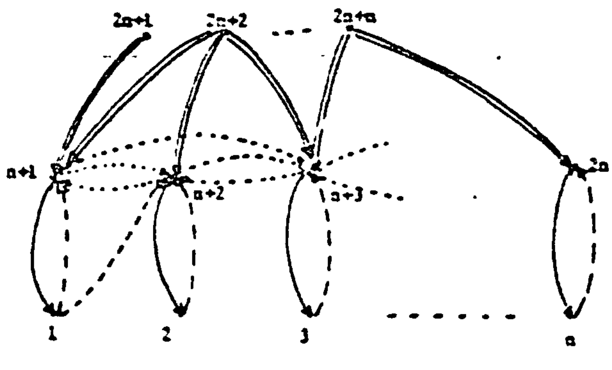

Fig. 1. Digraph of {A,B} matrix pair.

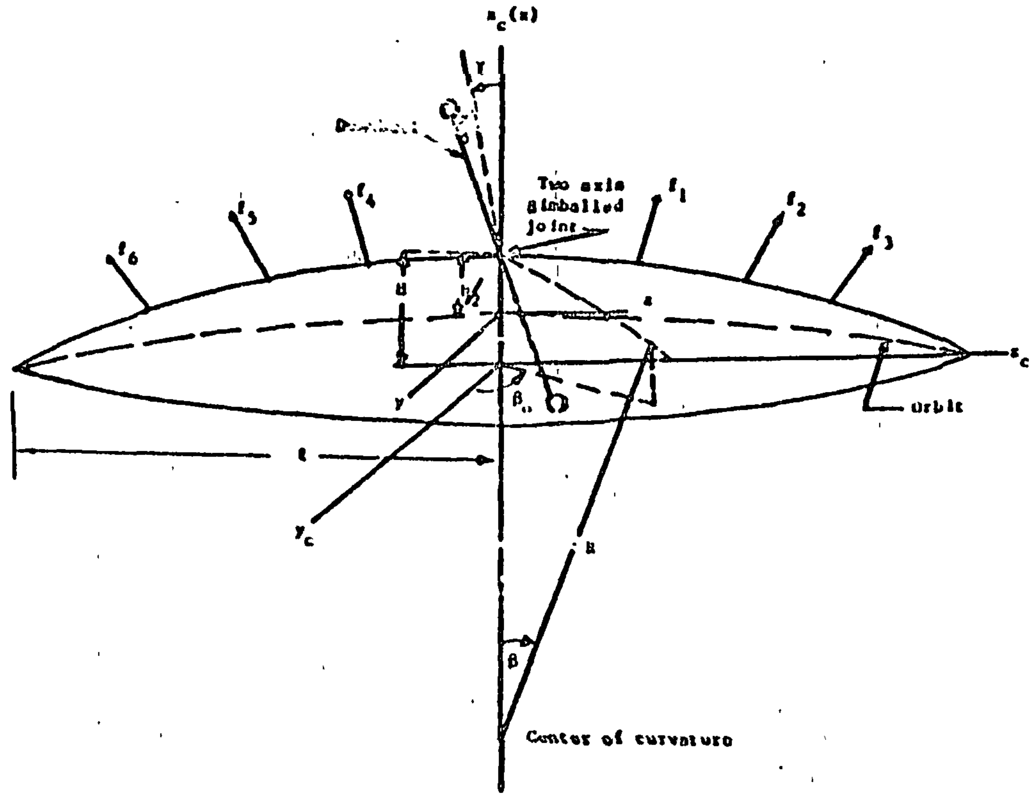

Fig. 2. Shallow spherical shell with dumbbell and actuators.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1										X								
2											X							
3												X						
4													X					
5														X				
6															X			
7																X		
8																	X	
9																		X
10											X							
11		X								X								
12			X										X	X	X			
13				X									X					
14					X								X					
15						X							X					
16							X											
17								X										
18									X									

Fig 3. Location of non-zero elements in the system matrix of the shallow spherical shell without the dumbbell in orbit

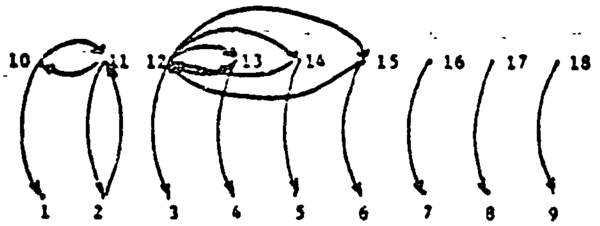

Fig. 4. Digraph of the shallow spherical shell system matrix.

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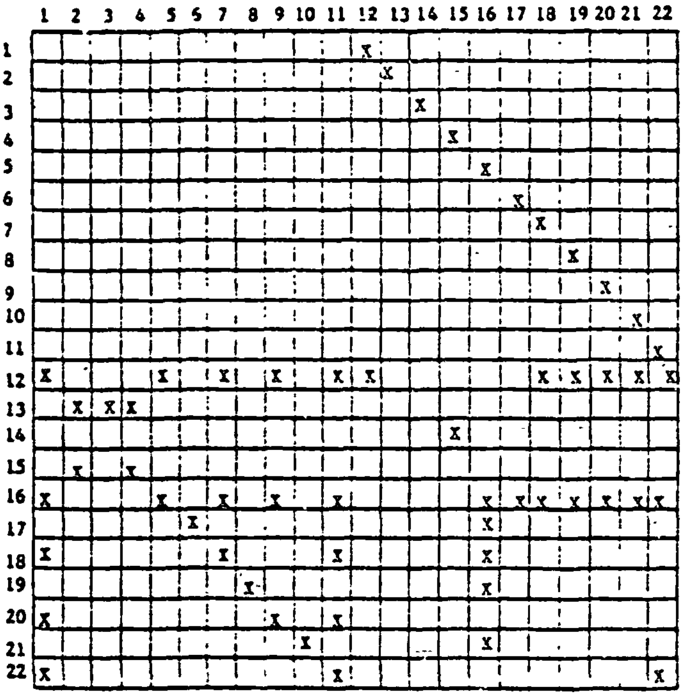

Fig 5. Location of non-zero elements of the system matrix of the shell with the dumbbell in orbit.

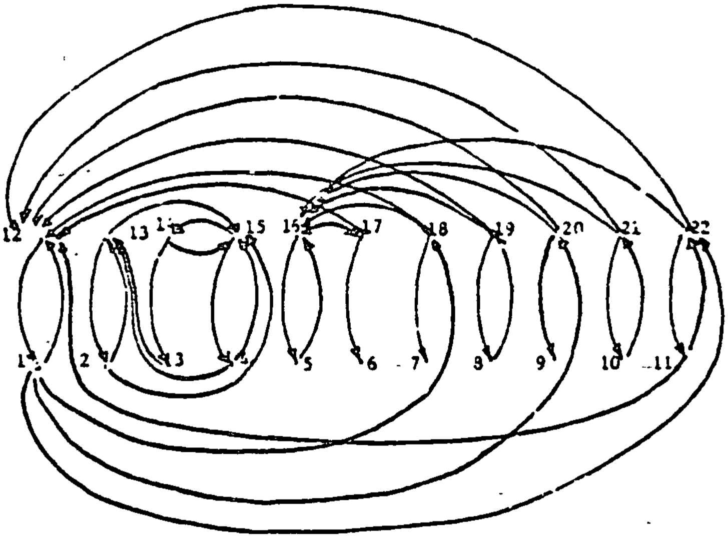

Fig 6. Digraph of the shallow spherical shell system matrix with dumbbell.

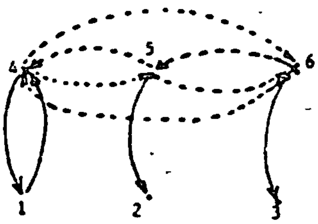

Fig 7. Digraph of system matrix given in equation (20)

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