General Disclaimer

One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.

- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.

- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.

- This document is paginated as submitted by the original source.

- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

Produced by the NASA Center for Aerospace Information (CASI)

ALGORITHMS FOR AUTONOMOUS
STAR IDENTIFICATION

OCTOBER 1980

NASA

National Aeronautics and
Space Administration

Goddard Space Flight Center
Greenbelt, Maryland 20771

ALGORITHMS FOR AUTONOMOUS STAR IDENTIFICATION

OCTOBER 1980

FOREWORD

The Systems Technology Laboratory (STL) is a computational research facility located at the Goddard Space Flight Center of the National Aeronautics and Space Administration (NASA/GSFC). The STL was established in 1978 to conduct research in the area of flight dynamics systems development. The laboratory consists of a VAX-11/780 and a PDP-11/70 computer system, along with an image-processing device and some microprocessors. The operation of the Laboratory is managed by NASA/GSFC (Systems Development and Analysis Branch) and is supported by SYSTEX, Inc., Computer Sciences Corporation, and General Software Corporation.

The main goal of the STL is to investigate all aspects of systems development of flight dynamics systems (software, firmware, and hardware), with the intent of achieving system reliability while reducing total system costs. The flight dynamics systems include the following: (1) attitude determination and control, (2) orbit determination and control, (3) mission analysis, (4) software engineering, and (5) systems engineering. The activities, findings, and recommendations of the STL are recorded in the Systems Technology Laboratory Series, a continuing series of reports that includes this document. A version of this document was also issued as Computer Sciences Corporation document CSC/TM-80/6303.

The primary contributors to this document include

    Pascal Gambardella   (Computer Sciences Corporation)

Other contributors include

    Raymond Byrne   (Computer Sciences Corporation)
    Malcom Shuster   (Computer Sciences Corporation)
    Eugene Lefferts   (Goddard Space Flight Center)
    Roger Werking   (Goddard Space Flight Center)

Single copies of this document can be obtained by writing to

    Keiji Tasaki
    Code 582.1
    NASA/GSFC
    Greenbelt, Maryland 20771

ABSTRACT

Algorithms for onboard autonomous star identification are presented. The algorithms are applicable to two types of spacecraft missions, those flown with nearly inertially fixed attitude (SMM type); and those flown with smoothly time-varying attitude (Landsat-D type).

ii A

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

SECTION 1 - INTRODUCTION

One of the key objectives of NASA's Multimission Modular spacecraft (MMS) program is the autonomous onboard determination of spacecraft attitude (Reference 1). The processing of star camera data and the subsequent identification of stars are essential elements of any autonomous attitude determination system. An example of an autonomous attitude determination algorithm is shown in Figure 1-1. This memorandum provides a detailed description of a semi-autonomous star identification algorithm suitable for implementation by a microprocessor onboard the spacecraft.

The algorithm is applicable to two types of spacecraft missions, those flown with nearly inertially fixed attitude (SMM type) (Reference 2); and those flown with smoothly time-varying attitude (Landsat-D type) (Reference 3).

The onboard star identification algorithm is divided into three separate modules (Figure 1-2) which are characterized by the functions they perform. The three functions are snapshot processing (Figure 1-3), generation of a subcatalog from an onboard star catalog, and the performance of pattern matching between observed and catalog stars. A snapshot is formed of all stars present in a star camera's field of view (FOV). Using onboard-propagated spacecraft attitudes the coordinates of the stars in the snapshot are rotated into an inertial frame to facilitate identification with reference stars stored in an onboard catalog. Direct or pattern matching techniques are necessary to ensure accurate star identification. Star observations are processed one at a time through the entire system, i.e., from raw data through the completion of star identification. This processing minimizes the microprocessor core requirements.
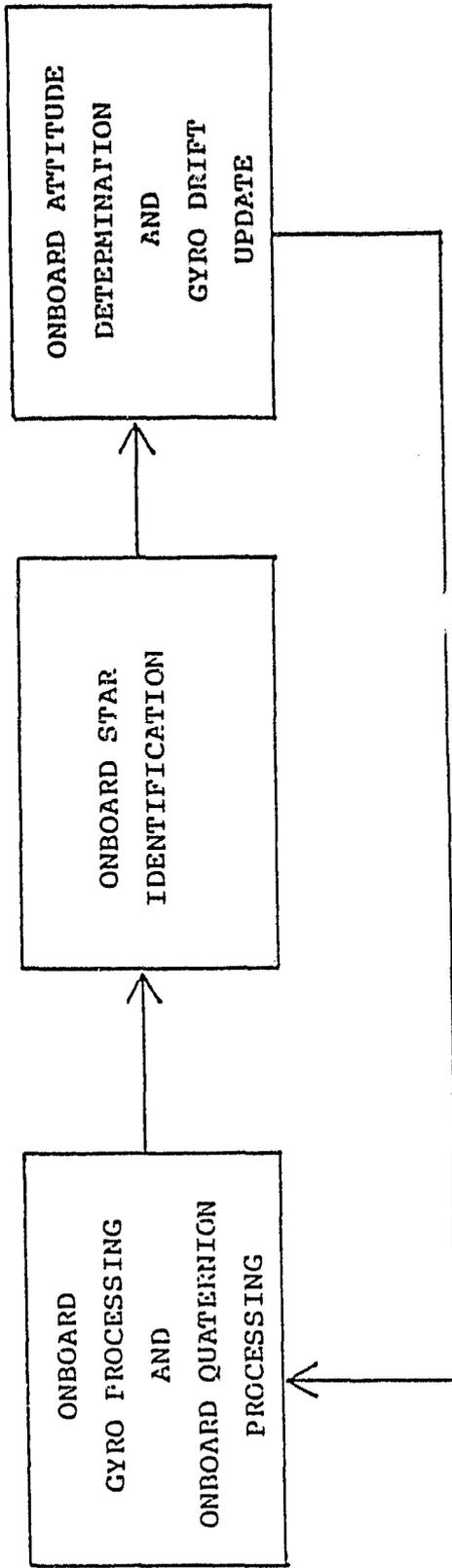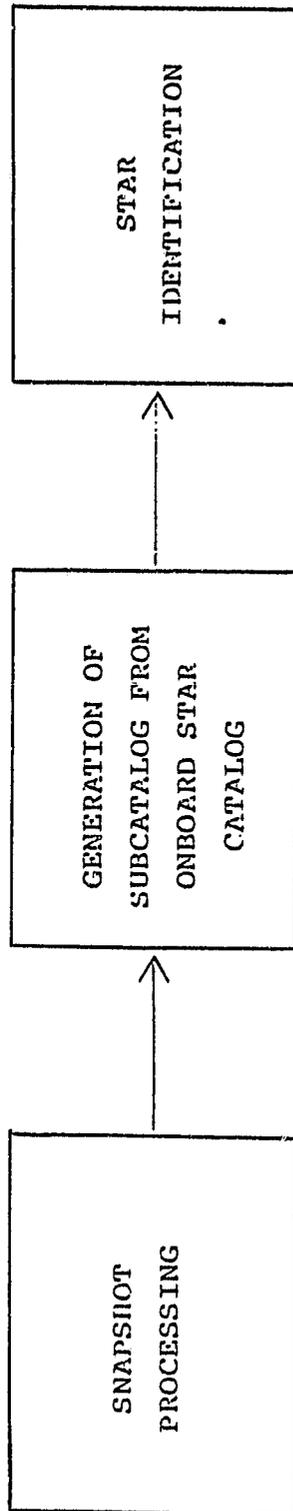
FIGURE 1-1. AUTONOMOUS ATTITUDE DETERMINATION

FIGURE 1-2. ONBOARD STAR IDENTIFICATION
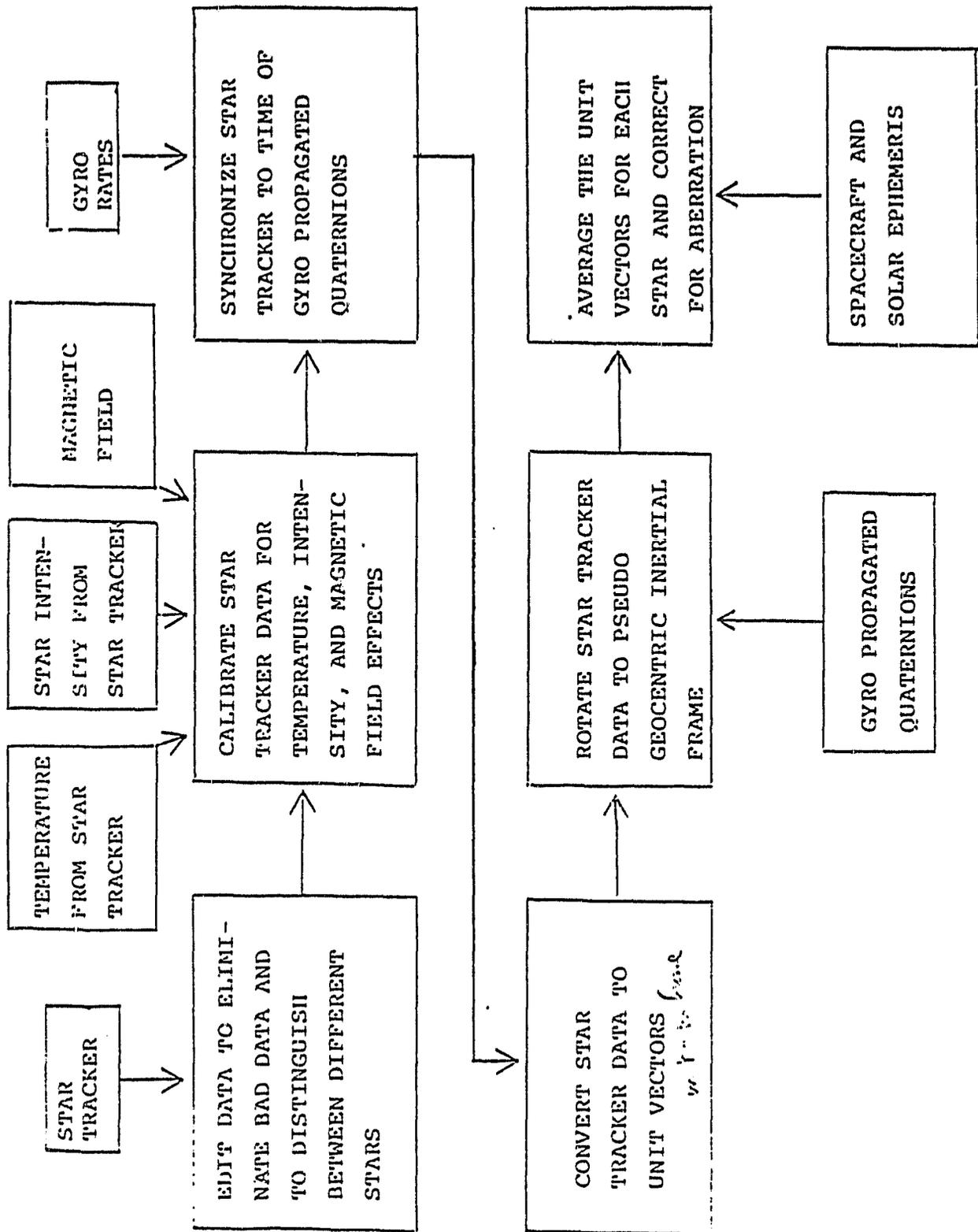
Figure 1.3. Snapshot Processing

The identified star observations and their corresponding catalog stars and other relevant data are stored in an array for access by the attitude determination system.

The onboard star identification algorithm is designed to satisfy the following general requirements (Reference 5) necessary for optimal onboard attitude determination using an onboard microprocessor. The algorithm accepts star observations directly from the star trackers, i.e., it does not assume any pre-processing. It provides for quality checking of the star observations as well as for checking of the sensor operational flags. It accepts star tracker alignments, scale factors, and a priori biases and calibration coefficients from an onboard data base. It accepts spacecraft ephemeris[1] and a star catalog from an onboard data base. It processes star tracker data and performs star identification activities in realtime. It provides star observations and corresponding identified catalog stars to an onboard attitude determination system. The onboard attitude determination system provides for onboard estimation of sensor parameters, such as gyro drift biases, whose short term instability is likely to degrade the attitude accuracy. The spacecraft can operate autonomously for a period of at least 3 days without any ground intervention. The exact period of autonomy is limited by the size of the onboard catalog.

---

1 The Navstar Global Positioning System (GPS) is a possible source of the spacecraft ephemeris (Reference 6).

SECTION 2 - SNAPSHOT PROCESSING

2.1 OVERVIEW

In this section, the operation of the Fixed Head Star Tracker (FHST) and algorithms for onboard snapshot processing are discussed. Star observations are processed one at a time through the entire system, i.e., from raw data through the completion of star identification.

2.2 COORDINATE SYSTEMS

The coordinate systems referred to in this memorandum are described in the following subsections.

2.2.1 GEOCENTRIC INERTIAL COORDINATES

The X_I, Y_I, and Z_I coordinate axes refer to the geocentric inertial coordinates (GCI), where X_I axis points to the vernal equinox; the Z_I axis points to the North Celestial Pole; and the Y_I axis points along the direction of $\hat{Z}_I \times \hat{X}_I$.

2.2.2 RIGHT ASCENSION - DECLINATION SYSTEM

The right ascension, α, is measured eastward in the plane of the celestial equator from the vernal equinox direction. The declination, δ, is measured northward from the celestial equator to the line of sight. A unit vector in the geocentric inertial frame is expressed in terms of α and δ as follows,

$$\left(\hat{r}\right)_{GCI} = \begin{pmatrix} \cos\delta & \cos\alpha \\ \cos\delta & \sin\alpha \\ \sin\delta \end{pmatrix} \qquad (2-1)$$

2.2.3 SPACECRAFT BODY FIXED REFERENCE FRAMES

The spacecraft reference frame is fixed with respect to the structure of the vehicle. Sensor coordinate reference frames are defined by constant transformations from the spacecraft reference frame. The matrix $_T M_{SC}$ represents

the transformation from the spacecraft reference frame to
each of the star tracker reference frames, i = 1,2.

2.3 FHST OPERATION

The FHST detects stars and provides star position and star
magnitude data. It searches for and acquires stars of
visual magnitude between +5.7 and 2 (class GOV) in an
8-degree square FOV (Reference 4). It tracks the acquired
star with a small tracking pattern, following the star
throughout the FOV. It provides accurate two-axis position
signals indicating the position of the star in the FOV. Upon
command, the search mode is restarted and the next star
encountered in the FOV is acquired. An independent bright
object sensor (BOS) and a shutter mounted in a separate
shutter housing assembly, with Sun shade attached, protect
the image dissector tube from excess energy.

The FHST operates as follows: When power is supplied to
the system and no other mode is commanded, the FHST auto-
matically starts searching the total field of view (TFOV).
The search scan pattern is a left-to-right, right-to-left
horizontal (H) direction sweep with a staircase signal
applied to the vertical (V) direction (raster pattern).
Four commandable threshold levels are provided for setting
the minimum sensitivity from +6.0 to +3.0 visual magnitudes.
The search scan continues until the first star that exceeds
the commanded threshold level is encountered. When this
occurs, the star is acquired and the track scan begins.
(The time required to search the TFOV does not exceed
10 seconds.)

The track scan pattern centers on the star image, and two-
axis position signals are provided at the system output.
If the star moves in the FOV because of small vehicle
attitude changes, the track scan pattern follows and remains
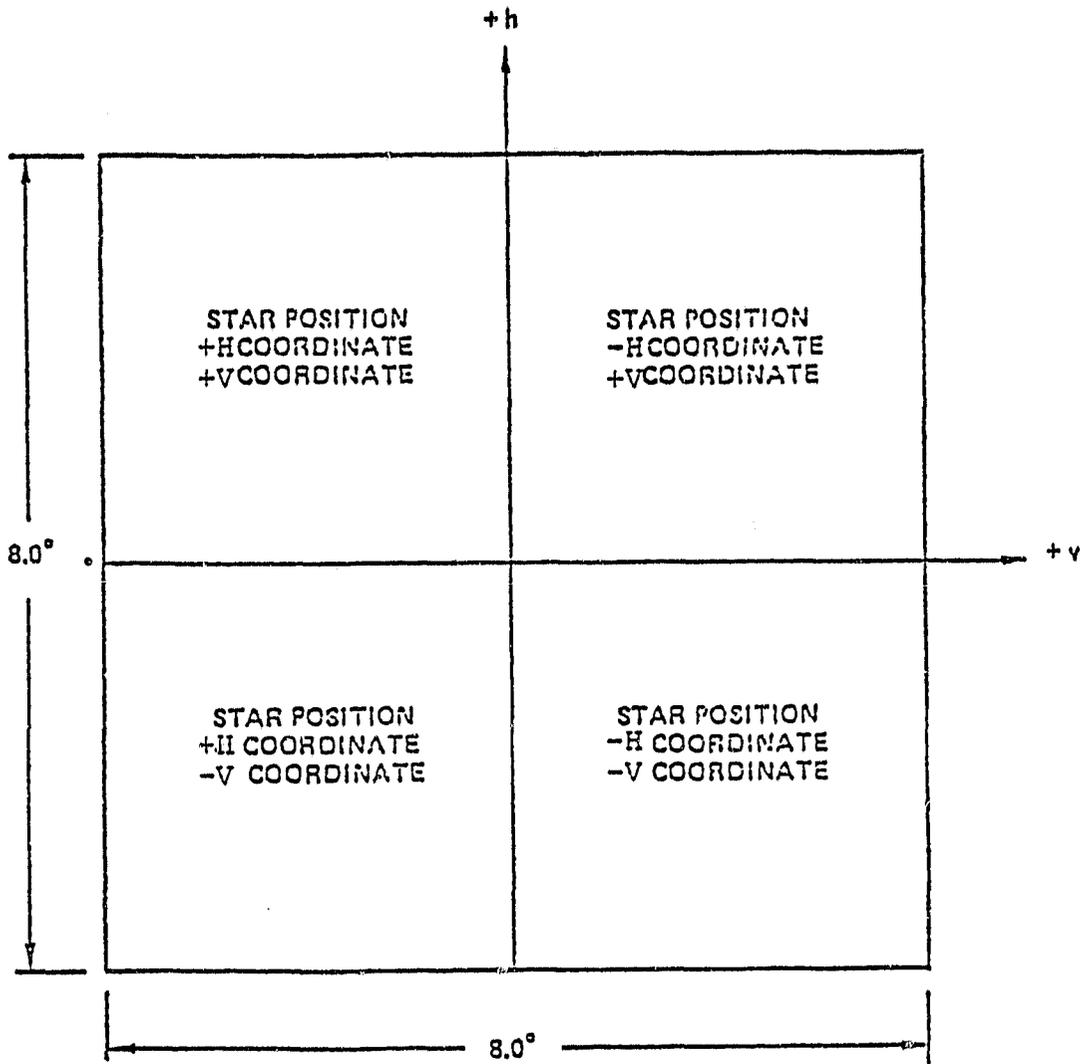
Total field of view (TFOV) as viewed from inside
the FHST looking out at the star field with the interface
connectors facing down.

Figure 2-1.   FHST Coordinate System

centered on the star image.[1] The star position is identified
by a positive or negative H coordinate and a positive or
negative V coordinate based on the coordinate system shown
in Figure 2-1. Positive position signals in the H and V
directions represent positive rotations about the h and v
axes. Tracking of the same star continues until (1) it leaves
the TFOV, (2) the amplitude of the signal falls below the
commanded threshold, or (3) a BREAK TRACK command (Reference
4) is received from the user.

An estimate of the average number of stars in the 8 degree
square FOV is provided in Table 2-1 for the SMM star trackers.
This information is useful in estimating the size of the
onboard catalog.

2.4 ALGORITHMS

The output of the star tracker corresponds to many sequential
sightings of each different star. For a given star, one
sighting is called a track point and all the sightings are
referred to as a track group (Figure 2-2). Each track point
is edited and calibrated to form observed star unit vectors
in the spacecraft reference frame. Quaternions are
formed from an initial attitude estimate and gyro data.
These quaternions are used to transform the observed star
unit vectors in a given track group into a common reference
frame, the pseudogeocentric inertial frame (PGCI). It
differs from a true inertial frame because of gyro-related
and propagation errors. The track points for a given track
group will not coincide in the PGCI frame due to those
errors. An average PGCI unit vector for the track group
is then constructed by summing the individual track point
unit vectors and normalizing the result. In the PGCI frame,
the individual track point unit vectors in a given track
group are referred to as a clump (Figure 2-3).

[1]According to camera specifications, the camera can follow a
 star moving in the FOV at a rate of 0.3 degree/second or less.

Table 2-1. FHST Threshold Settings

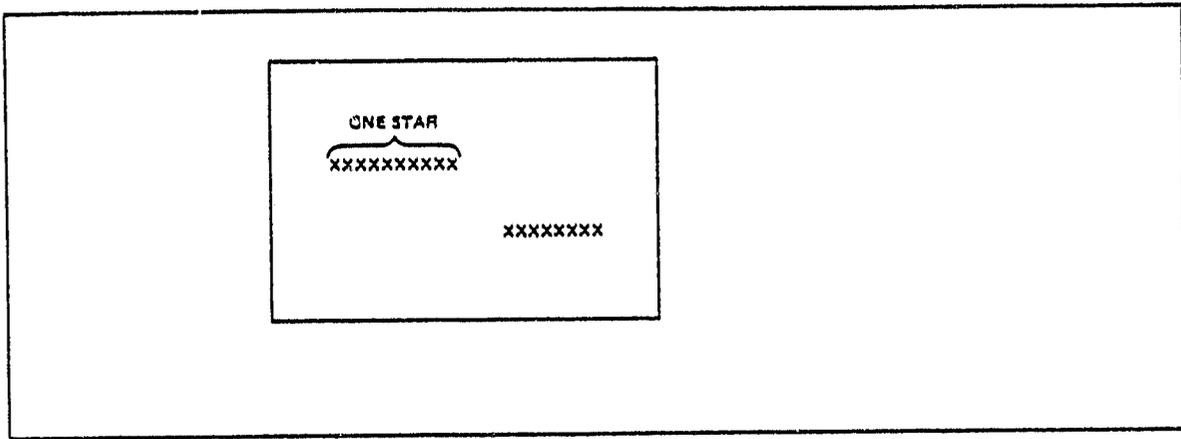| Setting	FHST 1 (S. No. 002)		FHST 2 (S. No. 001)	
	Actual Threshold	Average No. of Stars in FHST TFOV	Actual Threshold	Average No. of Stars in FHST TFOV
3	3.981	0.85	3.81	0.70
4	4.664	1.80	4.50	1.50
5	5.731	5.80	5.54	4.70
6	6.552	14.28	6.44	12.63

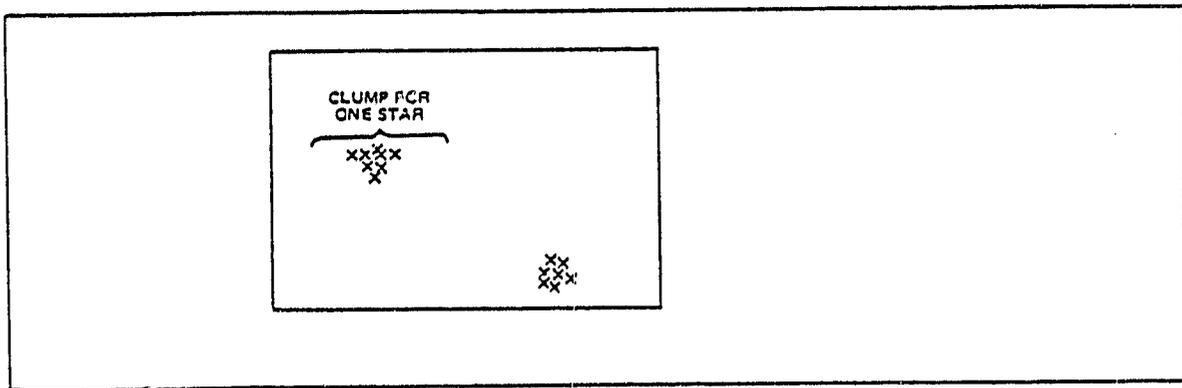Figure 2-2. Observations in Star Tracker FOV

Figure 2-3. Transformation of Star Tracker
             Observations to PGCI Frame

The steps involved in processing data from a single star
tracker are shown in Figure (1-3). Star position da.a,
temperature, and star intensity are output from the star
tracker. The magnetic field is either internally computed
using spacecraft ephemeris data or is provided by an onboard
magnetometer. Gyro rates and gyro propagated quaternions
are provided by an external gyro processing subsystem. The
Solar ephemeris is stored or computed onboard.

2.4.1 DATA EDIT

Tests are provided to ensure the star tracker output data
is constrained by minimum and maximum tolerances, the track
groups for different stars are distinguishable, and bad data
is eliminated. Data is collected during a time τ specified
by the user. Star tracker position output H and V, tempera-
ture output T and intensity output I are tested as follows
to verify they fall within minimum and maximum tolerances,

$$H_{MIN} \leq H_O \leq H_{MAX}$$
$$V_{MIN} \leq V_O \leq V_{MAX}$$
$$T_{MIN} \leq T \leq T_{MAX} \qquad (2\text{-}2)$$
$$I_{MIN} \leq I \leq I_{MAX}$$

where H_{MAX}, H_{MIN}, V_{MIN}, V_{MAX}, T_{MIN}, T_{MAX}, I_{MIN}, and I_{MAX}
are user defined tolerances. If H_i, V_i, $i \geq 1$, represent
a sequence of star camera track points, then track points
satisfying

$$\left| H_{i+1} - H_i \right| < K_1$$
$$\left| V_{i+1} - V_i \right| < K_2 \qquad (2\text{-}2)$$

are considered to belong to the same star, where K_1 and K_2
are user specified constants. The number of track points

collected for each star satisfies minimum and maximum limits.
Track points are edited from the beginning and end of each
track group to eliminate transient effects. Track groups
are discarded if they overlap in time a gyro gap of duration
t_g seconds or greater. Track points are discarded if they
do not have at least one corresponding intensity between
the first and last valid track point.

2.4.2 CALIBRATION OF FHST

This subsection describes the FHST calibration procedure.
The form of the calibration equations (Reference 4) is as
follows:

$$f_1(H,V,X) = C_1 + C_2 V + C_3 H + C_4 X + C_5 V^2 + C_6 VH + C_7 VX + C_8 H^2$$

$$+ C_9 HX + C_{10} X^2 + C_{11} V^3 + C_{12} V^2 H + C_{13} V^2 X$$

$$+ C_{14} VH^2 + C_{15} VHX + C_{16} VX^2 + C_{17} H^3 + C_{18} H^2 X + C_{19} HX^2$$

where H = horizontal axis output in counts

V = vertical axis output in counts

X = physical parameter as defined below

$f_1(H, V, X)$ = H value corrected for X

C = calibration coefficients corresponding to
H value corrections (TBD)

The expression $f_2(H, V, X)$ for the V value corrected for X
is of the same form as $f_1(H, V, X)$ except for different
calibration coefficients C.

Five separate applications of the above equation are neces-
sary for each axis. The first application is a flat-field
temperature calibration; X is the temperature in volts.
The second application is for intensity, with X being the
star intensity in volts. The third application has X equal
to the magnetic field along the star tracker h axis in gauss;
and the fifth application has X equal to the magnetic field
along the star tracker v axis. The values of C_i for all

of these calibrations are TBD. For particular attitude
accuracy requirements, a study should be initiated to
determine how these equations can be simplified to minimize
the number of calibration coefficients. For example, the
magnetic field contribution on the SMM mission is less than
1 arc second.

2.4.3 SYNCHRONIZATION OF FHST DATA TO GYRO PROPAGATED QUATERNIONS

It is necessary to synchronize the calibrated FHST position
output H and V to the gyro propagated quaternions; the
quaternions are used to rotate the star data to the PGCI
frame. The quaternion $Q(t_i)$ represents a rotation from the
PGCI frame to the spacecraft reference frame at time t_i.
The gyro processing system supplies, both the gyro rates
$W(t_i)$ in the spacecraft body frame, and the quaternions.
The quaternions and gyro rates are available at a lower
frequency than the star data. An algorithm to synchronize
the FHST data to the quaternion is described in this section.

The star unit vector in the tracker frame is defined by

$$\hat{u} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} = \begin{pmatrix} \cos\theta_V \cos\phi_H \\ -\sin\theta_V \cos\phi_H \\ \sin\phi_H \end{pmatrix} \tag{2-5}$$

where
$$\theta_V = V\beta_1$$
$$\phi_H = H\beta_2$$

and β_1 and β_2 are scale factors. Suppose star tracker
position output occurs at time t' where $t_i \leq t' \leq t_{i-1}$. The
average gyro rate is computed by

$$W_{AVG} = \tfrac{1}{2}(W_i + W_{i-1})$$

and converted to the star tracker reference frame,

$$\left[W_{AVG}\right]_T = T^M{}_{SC}\, W_{AVG}$$

Synchronized star tracker output can be calculated from

$$\begin{pmatrix} 1 \\ -\theta'_V \\ \phi'_H \end{pmatrix} = \begin{pmatrix} 1 \\ -\theta_V \\ \phi_H \end{pmatrix} + \left[W_{AVG} \right]_T \Delta t \qquad (2\text{-}6)$$

where $\Delta t = -t' + t_{i-1}$, if $t' < \tfrac{1}{2} (t_i - t_{i-1})$,

synchronizing θ'_V and ϕ'_H to $Q(t_{i-1})$

and $\Delta t = -t' + t_i$, if $t' \geq \tfrac{1}{2} (t_i - t_{i-1})$

synchronizing θ'_V and ϕ'_H to $Q(t_i)$

The synchronized star unit vector $\hat{u}'$ is then obtained by
inserting θ'_V and ϕ'_H into equation (2-6).

2.4.4 ROTATION OF STAR TRACKER DATA TO PGCI REFERENCE FRAME

The average star vector is transformed to the PGCI reference
frame is given by

$$\vec{u} = \frac{1}{n} \sum_{k=1}^{n} A_Q^T (t_k)_T M_{SC}^T \hat{u}_T^{(k)} \qquad (2\text{-}7)$$

where

$\hat{u}_T^{(k)}$ = synchronized track point unit vectors in the
tracker reference frame, $1 \leq k \leq n$

t_k = the time of the kth track point in the track
group, $1 \leq k \leq n$

$A_Q(t_k)$ = the matrix corresponding to the quaternion
$Q(t_k)$.

It is necessary to correct the observed average star
unit vectors for aberration effects. This correction
can be of the order of 20 arc-seconds. The complexity of
this calculation is considerably reduced if it is applied
to the observed star unit vectors rather than to the catalog
unit vectors. In the latter case, the catalog stars in

2-10

right ascension-declination reference frame would have to
be converted to star unit vectors, corrected for aberration
and then reconverted back to right ascension-declination
reference frame (see Section 3). In addition, the correction
would have to be applied to all the catalog stars which
serve as identification candidates for the observed star.
The vector $\vec{u}$ is unitized to $\hat{u}$ and corrected for aberration
as follows:

$$\hat{u}_c = (\hat{u} - T\hat{v})/ \| \hat{u} - T\hat{v}\| \tag{2-8}$$

with
$$\vec{v} = \vec{v}_E + \vec{v}_{SC} \tag{2-9}$$

and
$$T = \frac{\| \vec{v} \|}{c} \tag{2-10}$$

where $\vec{v}_E$ is velocity vector of the Earth around the Sun in
the GCI reference frame, $\vec{v}_{SC}$ is the velocity of the spacecraft
relative to the Earth and c is the speed of light. The
velocity $\vec{v}_E$ is determined from the Solar ephemeris which
must be stored or computed onboard.

The vector $\hat{u}_c$ is converted to right ascension and declination
using the equations

$$\alpha = \text{arc tan } (u_y/u_x)$$
$$\delta = \text{arc sin } (u_z) \tag{2-11}$$

SECTION 3 - ONBOARD STAR CATALOG

The contents, order, size, and accessing of the onboard
star catalog are discussed. The star catalog contains the
right ascenstion α, declination δ, and magnitude of each
star. The star catalog is ordered by right ascension into
η (nominally 360) subcatalogs with stars in each subcatalog
ordered by declination. The size of the onboard star cata-
log is limited by the available memory in the onboard micro-
processor. The period of spacecraft autonomy depends on
the size of the onboard catalog. An estimate of the average
number of new stars per day for FHST1 (see Table 2-1) for
Landsat-D and SMM in nominal mission mode is given in
Table 3-1. Suppose FHST1 and FHST2 view different regions
of the celestial sphere in one week, then an estimate of
the total number of stars viewed by both trackers in one
day and one week based on this assumption appears in
Table 3-2.

The storage requirements for each star in the catalog is
12 bytes. Storage estimates for the Landsat-D and SMM
missions appear in Table 3-3. To minimize star tracker
processing in the attitude determination system, it may be
advantageous to additionally store the star catalog unit
vectors in the GCI reference frame. This additional storage
would double the star catalog storage.

For each observed star, the onboard catalog is accessed
and a subcatalog is generated for use on star identification.
Suppose α_0 and δ_0 are the right ascension and declination,
respectively, of the observed star. One or two of the 360
subcatalogs are accessed to cover the interval

$$\alpha_0 + \Delta_1 \geq \alpha \geq \alpha_0 - \Delta_1 \qquad (3-1)$$

Threshold Setting	Average Number of New Stars Per Day in FHST 1	
	Landsat-D	SMM
3	2.39	0.11
4	5.06	0.22
5	16.3	0.73
6	40.15	1.79

Table 3-1. Average Number of New Stars Per Day in
One Star Tracker for Landsat-D and SMM

Threshold Setting	Average Number of Stars Viewed by Both Star Trackers			
	Landsat-D		SMM	
	one day	one week	one day	one week
3	38	67	2	3
4	81	142	4	7
5	261	457	12	21
6	643	1125	28.6	50

Table 3-2. Average Number of Stars Viewed by
Both Trackers for Landsat-D and SMM

Period	Storage (K bytes) (FHST Threshold Setting = 6)	
	Landsat-D	SMM
one day	8	0.4
one week	15	0.8
entire catalog	64	64

Table 3-3. Storage Requirement Estimates of the
Onboard Star Catalog for Landsat-D and SMM

where Δ_1 is a mission dependent tolerance (nominally 0.1 degree). Only one subcatalog need be accessed if the subcatalogs overlap in right ascension by Δ_1. The equation

$$T(\alpha, \delta) = (\alpha - \alpha_0)^2 + (\delta - \delta_0)^2 \qquad (3-2)$$

is applied to all stars in the subcatalog which satisfy

$$\delta_0 + \Delta_2 \geq \delta \geq \delta_0 - \Delta_2 \qquad (3-3)$$

If $|\alpha - \alpha_0| > \pi$ or $|\delta - \delta_0| > \pi$ in Equation (3-2), replace them with $2\pi - |\delta - \delta_0|$ or $2\pi - |\alpha - \alpha_0|$, respectively. Catalog stars which satisfy

$$T(\alpha, \rho) \leq \varepsilon \qquad (3-4)$$

where ε is a constant tolerance are considered candidates for the observed star (see Section 4).

SECTION 4 - STAR IDENTIFICATION

Stars are identified primarily by a direct matching technique.
For star identification to be considered successful, at least
n (nominally 3) stars must be matched to catalog stars within
a time t_I. A pairwise matching technique is employed when
successful star identification is not obtained by exclusive
use of the direct matching technique. The output of the
star identification algorithm is the observed and catalog
star positions in a format consistent with the input to the
attitude determination algorithms. The time associated with
the midpoint of the track group for each observed star is
also output. To achieve attitude accuracy requirements it is
necessary to compute an attitude using stars from each
tracker. A star tracker observability error analysis for
SMM is presented in the Appendix. The star identification
algorithm presented here can accept star observations from
both star trackers. The consequence of using stars from
both star trackers is an enlarging of the accessed star
catalog (see Section 3).

For successful star identification by direct match there
must be only one star in an observation window centered on
the observed star. For an attitude error of 30 arc-seconds,
the observation window size is typically 0.1 degree square
as on SMM. Assuming a uniform distribution of stars in the
celestial sphere of density ρ , the probability of two or
more stars in an observation window of area A, is

$$P(\rho , A) = \left[1 - (1 + \rho A) \exp (-\rho A) \right] / \left(1 - e^{-\rho A} \right) \quad = .003$$

This probability is 0.011 using a consecutive estimate of the
density, $\rho = 0.6$ (degree)$^{-2}$, and a window size of 0.1 degree
square. Although this probability is small, it's large
enough to include pairwise matching in the algorithm.

4-1

Each star catalog candidate for an observed star is first
subject to an intensity test. The stellar magnitude of the
observed star is given by

$$M_o = -a \log_{10}I + b$$

where I is the tracker output intensity in volts and a and b
(a > o, b > o) are constants which depend on the particular
star tracker used. Each star catalog candidate must satisfy

$$M_c < F(M_o)$$

where M_c is the catalog magnitude and F (TBD) is a function
of Mo. The function F is designed to compensate for the
star tracker decreasing intensity sensitivity (Reference 7)
as the magnitude increases.

After the intensity test is performed (see Figure 4-1)·, each
observed star could have no catalog star candidate, one cata-
log star candidate or more than one catalog star candidate.
If the observed star has no candidates, it is rejected. If
there are k or more successive stars with no candidate, the
observation window is enlarged. If the observed star has
one candidate, it is placed in a storage area of tentatively
identified stars. When there are n tentatively identified
stars within a time t_I, the n stars are considered identified
and placed in a storage area of identified stars. If the
observed star has more than one candidate, then the pairwise
matching technique is invoked. Suppose the observation with
unit vector $\hat{O}$ has s candidates with unit vectors $\hat{C}_i$ (1<i <s).
Define the function H by

$$H(\hat{C}_i) = \arcsin \| \hat{O} \times \hat{O}_I \| - \arcsin \| \hat{C}_i \times \hat{C}_I \|$$

where $\hat{O}_I$ and $\hat{C}_I$ are the observation and catalog unit vectors,
respectively, of a previously identified star. The candidate
$\hat{C}_i$, which minimizes H, is matched with the observation. This
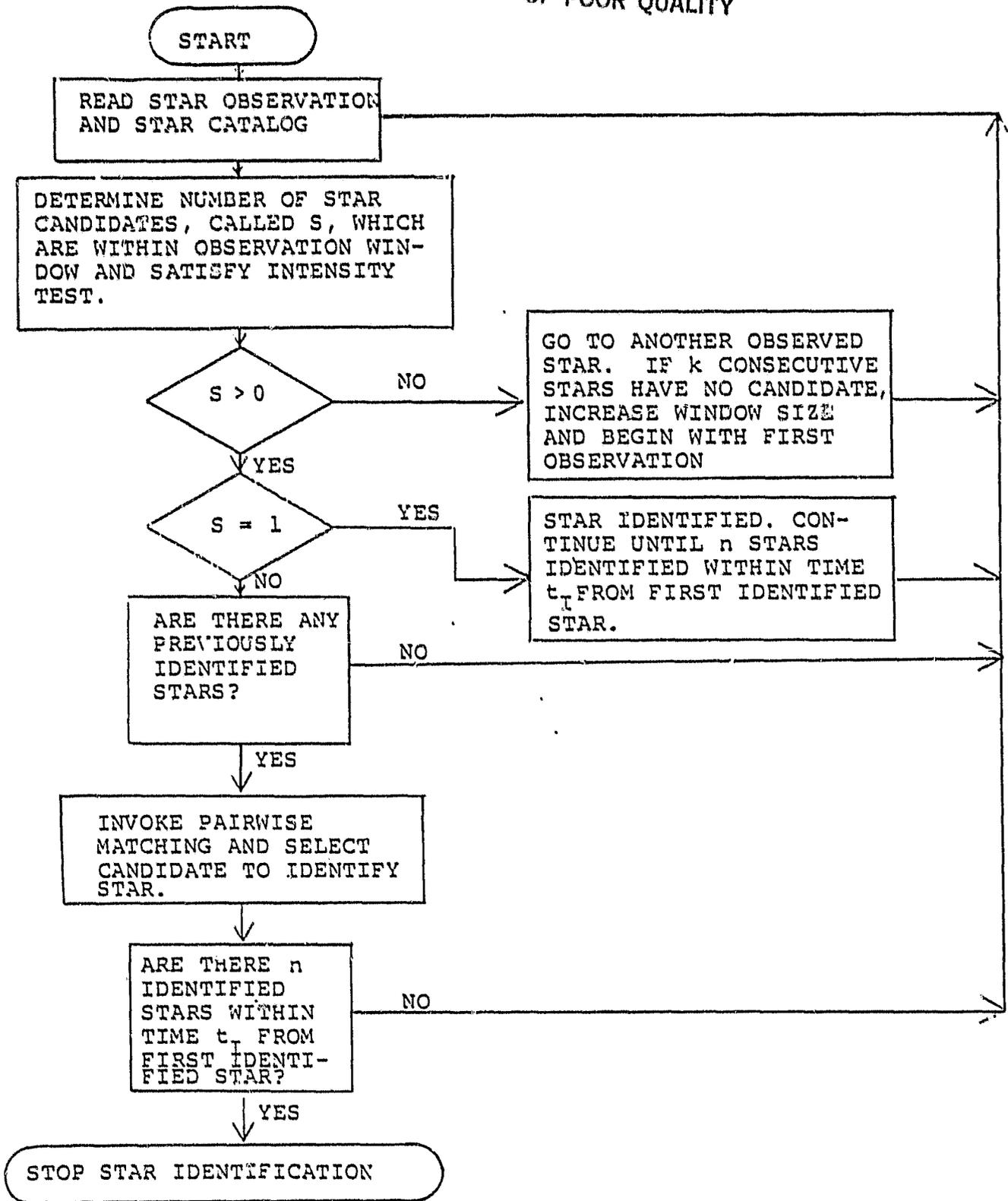algorithm is summarized in Figure 4-1.

Figure 4-1. Star Identification

APPENDIX - STAR TRACKER OBSERVABILITY ERROR ANALYSIS FOR SMM

This appendix computes the attitude error as a function of the angular distance between two star unit vectors.

This computation determines star tracker observability. This is defined as the attitude error corresponding to the relative position of the star tracker boresights.

The result and consequences of the calculation are presented; the derivations are given in Reference 7. Let one star unit vector lie along the x-axis (see Figure A-1) and the other star unit vector lie in the x-y plane at an angle θ from the first star. The attitude error about the x-axis is

$$\Sigma = \left(\frac{\sigma_2^2 + \sigma_1^2 \cos^2 \theta}{\sin \theta} \right)^{\frac{1}{2}} \qquad (A-1)$$

where σ_2 is the star tracker measurement error where $i = 1$ refers to the star along the x-axis and $i = 2$ refers to the other star. As θ approaches zero, Σ diverges, indicating a loss of observability about the x-axis. As θ approaches 90 degrees, $\Sigma = \sigma_2$, indicating complete observability about the x-axis.

A plot of the attitude error Σ as a function of the angle θ between the stars is given in Figure A-2. In this plot σ_1 is set equal to σ_2 to illustrate the observability effect described above.

The star tracker has a 8-degree by 8-degree field of view. If both stars are in one tracker, the range of θ is $0 \leq \theta \leq 8$ degrees. The boresights of the two star trackers on SMM are separated by 73 degrees. If one star is in each tracker, the maximum and minimum θ are 81 degrees and 65 degrees, respectively. The attitude error corresponding to these star configurations is given in Table A-1 with $\sigma = \sigma_1 = \sigma_2$.
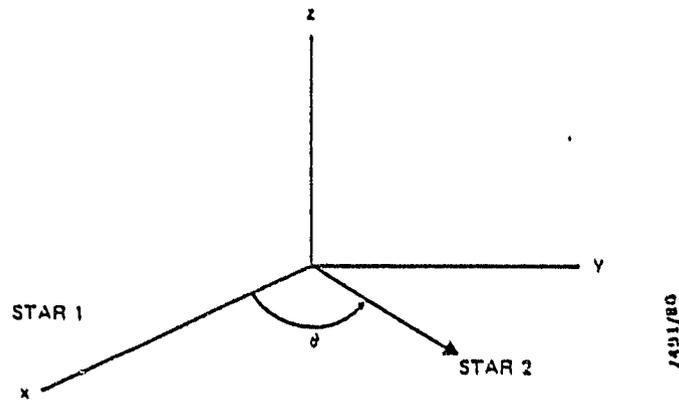
Figure A-1. Star Unit Vectors

$$\Sigma = \frac{(1 + \cos^2 \theta)^{1/2}}{\sin \theta} \, \sigma$$
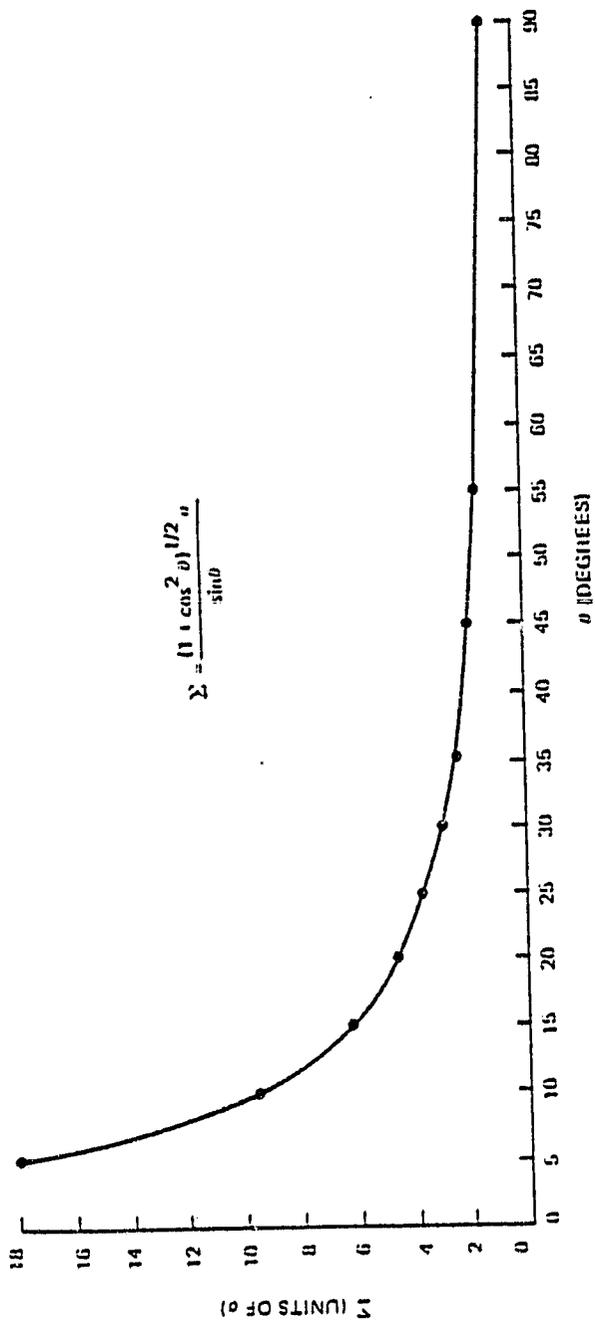
Figure A.2 Plot of the Attitude Error Σ as a Function
of the Angle θ Between the Stars

If both stars are in one tracker, the attitude error is
an order of magnitude or larger than the star tracker
measurement error. However, if there is a star in each
tracker, the observability effect simply corresponds to the
addition of from 0.4 arc-second to 4 arc-seconds to the
star tracker measurement error.

Table A-1.   Attitude Error for Stars in the Star
             Trackers

CONFIGURATION	θ (DEGREES)	Σ/σ	Σ WITH σ = 20 ARC-SECONDS
BOTH STARS IN ONE TRACKER	0	DIVERGES	DIVERGES
	8	10.1	202 ARC-SECONDS
A STAR IN EACH TRACKER	65	1.20	24.0 ARC-SECONDS
	81	1.02	20.4 ARC-SECONDS

1451/80

REFERENCES

1. Computer Sciences Corporation, CSC/TM-8C/6099, Ancillary Data and Support Computing Concepts, L. Fallon, G. Nair, L. Jordan, F. Manders, and C. Sturch, May 1980

2. CSC/SD-78/6082, Solar Maximum Mission (SMM) Attitude Systems Functional Specifications and Requirements, R. Byrne, P. Gambardella, J. Garber, G. Nair, D. Sood, and G. Tandon, September 1978

3. CSC/TM-78/6209, Onboard Star Catalog for Landsat-D Mission, P. K. Misra, August 1978

4. Ball Aerospace Systems Division, TM79-04, User's Guide for Standard Star Tracker, R. A. Deters, 1979

5. Computer Sciences Corporation, Internal Memorandum, "Onboard Attitude Determination System," D. Sood, July 1980

6. American Astronautical Society Annual Rocky Mountain Guidance and Control Conference, AAS 79-001, "Space Navigation Using the Navstar Global Positioning System (GPS)", J. E. Farr, February 1979

7. Computer Sciences Corporation, CSC/TM-80/6159, Solar Maximum Mission (SMM) Postlaunch Report, G. Nair. P. Gambardella, G. Neal, Y. Kwon, P. Kammeyer, R. Thompson, and J. Buckley, September 1980

STL BIBLIOGRAPHY

Systems Technology Laboratory, STL-78-001, System Description of the IMP-16C Microprocessor Orbit Determination Program, C. Shenitz, C. Rabbin, and G. Snyder, October 1978

--, STL-78-002, Landsat/NAVPAC System Description and User's Guide, S. R. Waligora, December 1978

--, STL-79-001, Intel 8080 Orbit Propagation Program System Description and User's Guide, C. Rabbin, April 1979

--, STL-79-002, Evaluation of the IMP-16 Microprocessor Orbit Determination System Filter, C. Shenitz, September 1979

--, STL-79-003, Multimission Modular Spacecraft Ground Support Software System (MMS/GSSS) MODCOMP Device and MAX IV Dependency Study, T. Weldon and M. McClellan, December 1979

--, STL-80-001, Orbit Determination Software Development for Microprocessor-Based Systems: Evaluation and Recommendations, C. M. Shenitz, July 1980

--, STL-80-002, The Two-Way TDRSS Observation Model for the LSI-11/23 Microcomputer, C. E. Goorevich, July 1980

--, STL-80-003, Automated Orbit Determination System (AODS) Requirements Definition and Analysis, S. R. Waligora, C. E. Goorevich, J. Teles, and R. S. Pajerski, September 1980

--, STL-80-004, Algorithms for Autonomous Star Identification, P. Gambardella, October 1980

--, STL-80-005, Autonomous Onboard Attitude Determination System Specifications and Requirements, M. D. Shuster, S. N. Ray, and L. Gunshol, December 1980

--, STL-81-001, Systems Technology Laboratory (STL) Library Methods and Procedures, W. J. Decker and P. D. Merwarth, September 1981

--, STL-81-002, Mathematical Specifications of the Onboard Navigation Package (ONPAC) Simulator (Revision 1), J. B. Dunham, February 1981

--, STL-81-003, Systems Technology Laboratory (STL) Compendium of Utilities, W. J. Decker, E. J. Smith, W. A. Taylor, P. D. Merwarth and M. E. Stark, July 1981

--, STL-81-004, <u>Automated Orbit Determination System (AODS) Environment Simulator for Prototype Testing (ADEPT) System Description</u>, S. R. Waligora, J. E. Fry, Jr., B. J. Prusiewicz, and G. N. Klitsch, August 1981

--, STL-81-005, <u>Preliminary Automated Orbit Determination System (AODS)/AODS Environment Simulator for Prototype Testing (ADEPT) User's Guide</u>, S. R. Waligora, Y. Ong, J. E. Fry, Jr. and B. J. Prusiewicz, September 1981

--, STL-82-001, <u>GPSPAC/Landsat-D Interface (GLI) System Description</u>, J. B. Dunham, H. M. Sielski, and W. T. Wallace, April 1982

--, STL-82-002, <u>GPSPAC/Landsat-D Interface (GLI) System User's Guide</u>, H. M. Sielski, J. B. Dunham, and P. D. Merwarth, March 1982

--, STL-82-003, <u>Autonomous Attitude Determination System (AADS), Volume 1: System Description</u>, K. A. Saralkar, Y. G. Frenkel, G. N. Klitsch, K. Liu and E. Lefferts, April 1982

--, STL-82-007, <u>System Description for the Global Positioning Subsystem Experiment Package (GPSPAC) Experiment Data Preprocessor (GEDAP)</u>, P. D. Merwarth and J. F. Cook, June 1982