

General Disclaimer

One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.
- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.
- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.
- This document is paginated as submitted by the original source.
- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

(NASA-TM-84797) MATHEMATICAL SPECIFICATIONS
OF THE ONBOARD NAVIGATION PACKAGE (ONPAC)
SIMULATOR (REVISION 1) (NASA) 63 p
HC A04/MF A01

N82-29033

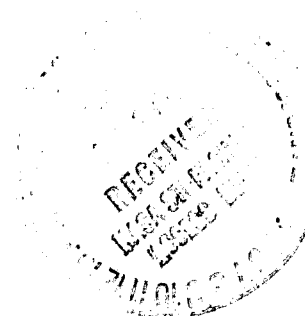
CSSL 09B

Unclas

G3/61 28387

MATHEMATICAL SPECIFICATIONS OF THE ONBOARD NAVIGATION PACKAGE (ONPAC) SIMULATOR (REVISION 1)

FEBRUARY 1981



National Aeronautics and
Space Administration

Goddard Space Flight Center
Greenbelt Maryland 20771

**MATHEMATICAL SPECIFICATIONS OF
THE ONBOARD NAVIGATION
PACKAGE (ONPAC) SIMULATOR
(REVISION 1)**

FEBRUARY 1981



National Aeronautics and
Space Administration

Goddard Space Flight Center
Greenbelt, Maryland 20771

FOREWORD

The Systems Technology Laboratory (STL) is a computational research facility located at the Goddard Space Flight Center of the National Aeronautics and Space Administration (NASA/GSFC). The STL was established in 1978 to conduct research in the area of flight dynamics systems development. The laboratory consists of a VAX-11/780 and a PDP-11/70 computer system, along with an image-processing device and some microprocessors. The operation of the Laboratory is managed by NASA/GSFC (Systems Development and Analysis Branch) and is supported by SYSTEX, Inc., Computer Sciences Corporation, and General Software Corporation.

The main goal of the STL is to investigate all aspects of systems development of flight dynamics systems (software, firmware, and hardware), with the intent of achieving system reliability while reducing total system costs. The flight dynamics systems include the following: (1) attitude determination and control, (2) orbit determination and control, (3) mission analysis, (4) software engineering, and (5) systems engineering. The activities, findings, and recommendations of the STL are recorded in the Systems Technology Laboratory Series, a continuing series of reports that includes this document. A version of this document was also issued as Computer Sciences Corporation document CSC/SD-81/6028.

The primary contributor to this document was

Joan B. Dunham (Computer Sciences Corporation)

Other contributors include

Anne C. Long (Computer Sciences Corporation)
William Wooden (Goddard Space Flight Center)

Single copies of this document can be obtained by writing to

Keiji Tasaki
Code 582.1
NASA/GSFC
Greenbelt, Maryland 20771

ABSTRACT

This document, which is an update to Computer Sciences Corporation document number CSC/SD-78/6002, describes the mathematical theory of the computational algorithms employed in the Onboard Navigation Package (ONPAC) System. This system, which simulates an onboard navigation processor, was developed to aid in the design and evaluation of onboard navigation software. The mathematical formulations presented include the factorized UDU^T form of the extended Kalman filter, the equations of motion of the user satellite, the user clock equations, the observation equations and their partial derivatives, the coordinate transformations, and the matrix decomposition algorithms.

Use of the ONPAC system, with sample input and output, is described in a companion document, the ONPAC User's Guide (Reference 1).

TABLE OF CONTENTS

<u>Section 1 - Introduction</u>	1-1
<u>Section 2 - Estimation Equations</u>	2-1
2.1 Extended Kalman Filter	2-1
2.2 UDU ^T Filter Derivation	2-3
2.3 ONPAC Filter	2-9
<u>Section 3 - State Parameter Propagation Equations</u>	3-1
3.1 State Vector	3-1
3.2 Force Model	3-2
3.3 Clock Modeling	3-6
3.3.1 Correction of the User Clock	3-7
3.4 Modified Euler Integrator	3-8
3.5 Coordinate Transformations	3-9
3.5.1 Earth-Centered Inertial (ECI) to Earth-Centered Earth-Fixed (ECEF) Coordinate Transformation	3-9
3.5.2 ECI to UVW Coordinate Transformation	3-10
3.5.3 Keplerian Orbital Elements	3-11
3.6 State Transition Matrix	3-13
3.7 Fading Memory	3-15
<u>Section 4 - Observation Model and Partial Derivatives</u>	4-1
4.1 Pseudorange Measurement	4-1
4.2 Delta Pseudorange Measurement	4-2
<u>Section 5 - Derivation of the State Process Noise Covariance Matrix</u>	5-1
<u>Appendix A - Matrix Decomposition</u>	
<u>Appendix B - Data Simulation</u>	
<u>Glossary</u>	
<u>References</u>	

SECTION 1 - INTRODUCTION

The Onboard Navigation Package (ONPAC) Simulator simulates an onboard navigation processor assembly using a modified version of the design proposed in Reference 2. The pseudorange and delta pseudorange observations for the estimation of user spacecraft position and time will be measured by the onboard receiver/processor assembly from information broadcast by the NAVSTAR/Global Positioning System (GPS). The pseudorange observation is modeled as the line of sight distance from the GPS to the user, and the delta pseudorange is modeled as the change in the pseudorange over a period of time. The system is currently being used for analysis and evaluation of the algorithms presented in this document for premission studies.

The ONPAC system is being developed on a Digital Equipment Corporation (DEC) PDP 11/70 computer which is similar to the DEC LSI 11 which the navigation processor assembly will use. One proposed use of the processor assembly is to be a part of an experimental package to be placed on Landsat-D for onboard orbit determination using Phase I GPS. A sample case and sample output for Landsat-D are presented in the appendixes of the user's guide (Reference 1).

1.1 OVERVIEW OF ONPAC CAPABILITIES

The ONPAC system processes GPS pseudorange and delta pseudorange observations sequentially to estimate and apply corrections to a host vehicle state, which includes the satellite position and velocity, two terms to describe corrections to the host vehicle clock, and a satellite drag coefficient. The integrator used to predict the position and velocity from observation to observation is an Euler integrator; the force model used in this integrator may be varied by the user. The force model options are detailed in Section 3.2. The covariance matrix is propagated with a state transition matrix which is a Taylor series expansion of the analytical state transition matrix to Δt , Δt^2 , or Δt^3

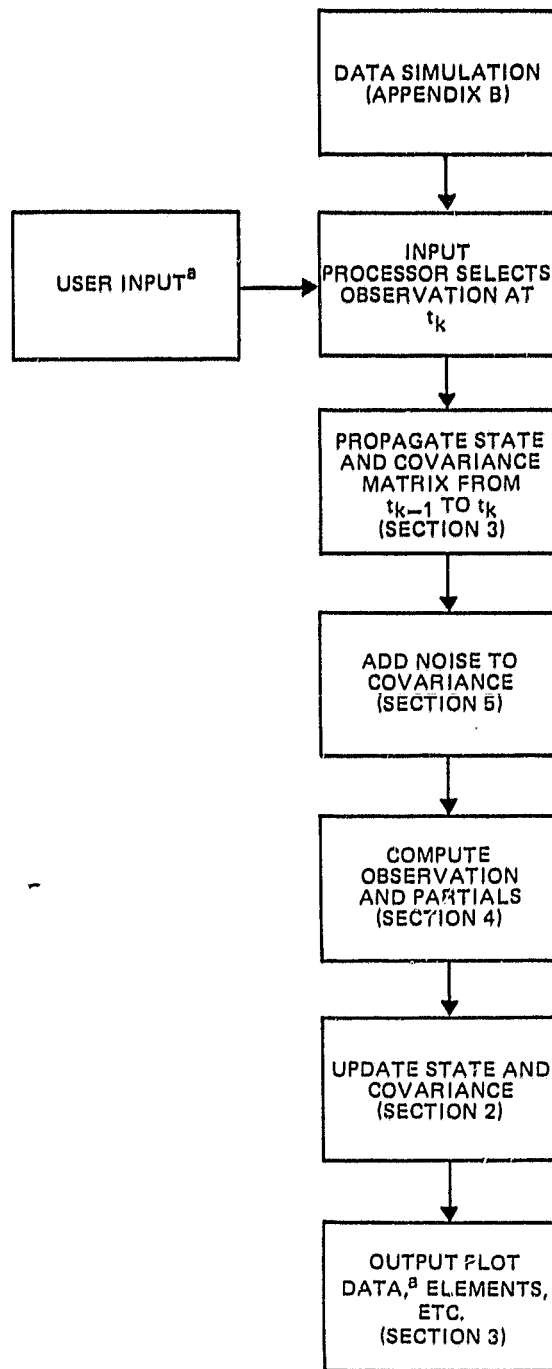
as specified by the user. A state process noise covariance matrix is also computed to allow for errors in the knowledge of the state and is added to the covariance matrix. The variance of the state noise in position, in the drag coefficient, and in the clock terms may be specified by the user.

The ONPAC system design is modular so that algorithms can be replaced or added with minimal changes to the existing simulator. Procedures for making such changes are discussed in the user's guide (Reference 1).

1.2 SUMMARY

This document describes the basic mathematical algorithms used in ONPAC. Derivations for most of the algorithms are available in standard texts and sources are given in the references. Section 2 describes the extended Kalman filter and gives a brief derivation of the UDU^T filter. The step-by-step application of the UDU^T filter in ONPAC is presented, with the points identified at which editing and smoothing are performed. The state vector and the state propagation equations are given in Section 3. The mathematical models in ONPAC use four different coordinate systems. Transformations from the inertial to the other three systems are also given in Section 3. The pseudorange and delta pseudorange observation models and partial derivatives are described in Section 4. The state process noise covariance matrix used in ONPAC is given in Section 5. Appendix A describes the matrix decomposition algorithms. The data simulation which is performed using the Goddard Trajectory Determination System (GTDS) is described in Appendix B. The relationship of these elements of the ONPAC program is shown in Figure 1-1.

ORIGINAL PAGE IS
OF POOR QUALITY



^aDESCRIBED IN THE USER'S GUIDE (REFERENCE 1).

Figure 1-1. The ONPAC System

SECTION 2 - ESTIMATION EQUATIONS

Matrix factorization techniques have been demonstrated to improve the stability and accuracy of Kalman filters (Reference 3). The factoring of the covariance matrix, P , into square root

$$P = SS^T$$

or upper triangular, U , and diagonal, D , components

$$P = UDU^T$$

reduces the occurrence of numerical problems by keeping the covariance matrix positive definite. The filter in ONPAC is an application of the U-D formulation of an extended Kalman filter (EKF).

2.1 EXTENDED KALMAN FILTER

A derivation of the EKF, also known as the extended sequential filter, can be found in many sources (Reference 4 and 5). The EKF equations are presented in this section as a basis for the UDU^T filter discussion which follows.

Given a state vector and covariance matrix at time t_{k-1} , the prediction and update equations at time t_k are as follows:

1. Given $\hat{\underline{X}}(t_{k-1})$, the estimated n-dimensional state vector at t_{k-1} , and $\hat{P}(t_{k-1})$, the covariance at t_{k-1} , integrate the state equations of motion $\underline{F}(\underline{X}, t)$,

$$\dot{\underline{X}} = \underline{F}(\underline{X}, t) \quad (2-1)$$

from t_{k-1} to t_k , with the initial conditions $\hat{\underline{X}}(t_{k-1})$. This gives the predicted state $\underline{X}(t_k)$. The carets over \underline{X} and P indicate the updated estimates after processing an observation.

Compute the state transition matrix $\Phi(t_k, t_{k-1})$, by integrating the differential equations

$$\dot{\Phi}(t, t_{k-1}) = A(t) \Phi(t, t_{k-1}) \quad (2-2)$$

where $A(t)$ is the matrix of partial derivatives

$$A(t) = \left[\frac{\partial F(\underline{X}, t)}{\partial \underline{X}} \right] \quad (2-3)$$

evaluated at $\underline{X} = \underline{X}(t_{k-1})$. The initial conditions for the integration are $\Phi(t_{k-1}, t_{k-1}) = I$, the identity matrix.

Propagate the covariance matrix using the state transition matrix to obtain the predicted covariance matrix at time t_k (without process noise),

$$P(t_k) = \Phi(t_k, t_{k-1}) \hat{P}(t_{k-1}) \Phi^T(t_k, t_{k-1}) \quad (2-4)$$

2. Obtain the observation at time t_k , $Y(t_k)$. Compute the predicted observation, $G(\underline{X}(t_k), t_k)$, where G is a nonlinear function of the state parameters and time.

Compute the $(1 \times n)$ matrix of observation partial derivatives

$$H(t_k) = \left[\frac{\partial G(\underline{X}, t_k)}{\partial \underline{X}} \right]_{\underline{X}=\underline{X}(t_k)} \quad (2-5)$$

The observation residual, or observed minus computed observation (O - C) is given by

$$g(t_k) = Y(t_k) - G(\underline{X}(t_k), t_k) \quad (2-6)$$

3. The Kalman gain, $\underline{K}(t_k)$, is

$$\underline{K}(t_k) = P^T(t_k) H^T(t_k) \left[H(t_k) P(t_k) H^T(t_k) + \sigma_m^2 \right]^{-1} \quad (2-7)$$

where σ_m^2 is the measurement variance.

The quantity within the brackets is a scalar and $\underline{K}(t_k)$ is a vector with n components.

The Kalman gain is used to compute the updated state vector

$$\hat{\underline{X}}(t_k) = \underline{X}(t_k) + \underline{K}(t_k) g(t_k) \quad (2-8)$$

and the updated covariance matrix

$$\hat{P}(t_k) = [I - \underline{K}(t_k) H(t_k)] P(t_k) \quad (2-9)$$

The updated state vector and covariance matrix at time t_k are then the input for propagation to time t_{k+1} in the next step.

2.2 UDU^T FILTER DERIVATION

The covariance matrix is a positive definite square matrix which may be factored into a triangular matrix and its transpose. It may also be factored into a triangular matrix with unity on the diagonal, its transpose, and a diagonal matrix.

The EKF equations can then be formulated using the component matrices instead of the covariance matrix. The covariance matrix is computed as shown below.

$$P = UDU^T \quad (2-10)$$

where U is an upper triangular matrix and D is a diagonal matrix.

The initial conditions for the t_k th step of the EKF filter are

$$\hat{\underline{x}}(t_{k-1}), \hat{U}(t_{k-1}), \hat{D}(t_{k-1})$$

Propagating the covariance matrix (without process noise) to time t_k yields,

$$\begin{aligned} P(t_k) &= U(t_k) D(t_k) U^T(t_k) \\ &= \Phi(t_k, t_{k-1}) \hat{U}(t_{k-1}) \hat{D}(t_{k-1}) \hat{U}^T(t_{k-1}) \Phi^T(t_k, t_{k-1}) \\ &= \Phi(t_k, t_{k-1}) \hat{U}(t_{k-1}) \hat{D}(t_{k-1}) [\Phi(t_k, t_{k-1}) \hat{U}(t_{k-1})]^T \end{aligned} \quad (2-11)$$

The covariance update equation is

$$\hat{P}(t_k) = [I - K(t_k) H(t_k)] P(t_k) \quad (2-12)$$

For the following equations, all quantities are evaluated at t_k . Replacing K in Equation (2-12) with Equation (2-7) gives

$$\hat{P} = P - P^T H^T \left(H P H^T + \sigma_m^2 \right)^{-1} H P \quad (2-13)$$

Substituting Equation (2-10) into Equation (2-13) gives

$$\begin{aligned}\hat{P} &= \hat{U}\hat{D}\hat{U}^T \\ &= UDU^T - UDU^T H^T \left(HUDU^T H^T + \sigma_m^2 \right)^{-1} HUDU^T \\ &= U \left[D - DU^T H^T \left(HUDU^T H^T + \sigma_m^2 \right)^{-1} HUD \right] U^T\end{aligned}\quad (2-14)$$

For a single observation, the term

$$HUDU^T H^T + \sigma_m^2$$

is the scalar

$$\alpha_n = \sigma_m^2 + \sum_{i=1}^n d_i \left(h_i + \sum_{j=1}^{i-1} h_j u_{j,i} \right)^2 \quad (2-15)$$

where n is the dimension of the state vector, $d_i = d_{i,i}$, the diagonal terms of D , $h_i = h_{1,i}$, the elements of the H matrix and $u_{i,j}$ are the elements of the U matrix.

Substituting this relationship into Equation (2-14) gives

$$\hat{U}\hat{D}\hat{U}^T = U \left[D - \frac{(DU^T H^T)(DU^T H^T)^T}{\alpha_n} \right] U^T \quad (2-16)$$

Equation (2-16) is then premultiplied by U^{-1} and postmultiplied by U^{-T} to give

$$U^{-1}\hat{P}U^{-T} = U^{-1}\hat{U}\hat{D}\hat{U}^T U^{-T} \quad (2-17)$$

which is, substituting in (2-16)

$$U^{-1}\hat{P}U^{-T} = D - \frac{(DU^T H^T)(DU^T H^T)^T}{\alpha_n} \quad (2-18)$$

Because both \hat{P} and U (and U^{-1}) are positive definite matrices, $U^{-1}\hat{P}U^{-T}$ is also positive definite (Reference 3). Therefore, the matrix (2-18) is a positive definite matrix and can itself be decomposed into upper triangular and diagonal component matrices.

Let

$$M = D - \frac{(DU^T H^T)(DU^T H^T)^T}{\alpha_n} \quad (2-19)$$

and

$$M = BWB^T \quad (2-20)$$

where B is an upper triangular matrix with unity on the diagonal and W is a diagonal matrix.

Then,

$$\hat{U}\hat{D}\hat{U}^T = UBWB^T U^T$$

and the following can be identified:

$$\begin{aligned} \hat{U} &= UB \\ \hat{D} &= W \end{aligned} \quad (2-21)$$

ORIGINAL PAGE IS
OF POOR QUALITY

The matrices B and W can be found by applying the factorization equations given in Appendix A.

The general terms for components of the matrix M are

$$m_{i,i} = d_i - \frac{d_i^2 \left(h_i + \sum_{\ell=1}^{i-1} u_{\ell,i} h_i \right)^2}{\alpha_n} \quad (2-22)$$

and for $i \neq j$

$$m_{i,j} = - \frac{d_i d_j \left(h_i + \sum_{\ell=1}^{i-1} u_{\ell,i} h_i \right) \left(h_j + \sum_{q=1}^{j-1} u_{q,j} h_q \right)}{\alpha_n} \quad (2-23)$$

After decomposition and some manipulation, the general term for W is

$$w_i = \frac{d_i \alpha_{i-1}}{\alpha_i} \quad (2-24)$$

where $w_i = w_{i,i}$

$$\alpha_{i-1} = \sigma_{in}^2 + \sum_{j=1}^{i-1} d_j \left(h_j + \sum_{k=1}^{j-1} h_k u_{k,j} \right)^2 \quad (2-25)$$

and

$$\alpha_i = \alpha_{i-1} + d_i \left(h_i + \sum_{k=1}^{i-1} h_k u_{k,i} \right)^2 \quad (2-26)$$

The general off-diagonal term of the B matrix is

$$b_{i-j,i} = \frac{-d_{i-j} \left(h_{i-j} + \sum_{k=1}^{i-j-1} h_k u_{k,i-j} \right) \left(h_i + \sum_{k=1}^{i-1} h_k u_{k,i} \right)}{\alpha_{i-1}} \quad (2-27)$$

Then, the update equations for the U and D matrices are

$$\hat{d}_i = \frac{d_i \alpha_{i-1}}{\alpha_i} \quad (2-28)$$

and

$$\hat{u}_{i,j} = u_{i,j} - \frac{d_i S_i S_j}{\alpha_{j-1}} - \sum_{k=i+1}^j \frac{u_{i,k} d_k S_k S_j}{\alpha_{j-1}} \quad (2-29)$$

where S is defined as

$$S_j = h_j + \sum_{\ell=1}^{j-1} h_\ell u_{\ell,j} \quad (2-30)$$

The Kalman gain is given by

$$K = U D U^T H^T \left(H U D U^T H^T + \sigma_m^2 \right)^{-1} \\ = \frac{U D U^T H^T}{\alpha_n} \quad (2-31)$$

The components of the gain vector are

$$K = \begin{bmatrix} \frac{1}{\alpha_n} \left(d_1 S_1 + \sum_{j=2}^n u_{1,j} d_j S_j \right) \\ \frac{1}{\alpha_n} \left(d_2 S_2 + \sum_{j=3}^n u_{2,j} d_j S_j \right) \\ \vdots \\ \frac{1}{\alpha_n} \left(d_i S_i + \sum_{j=i+1}^n u_{i,j} d_j S_j \right) \\ \frac{1}{\alpha_n} d_n S_n \end{bmatrix} \quad (2-32)$$

2.3 ONPAC FILTER

The filtering algorithm used to process each observation in ONPAC is discussed below.

1. Enter with a state vector, $\hat{\underline{X}}(t_{k-1})$, and covariance matrix components, $\hat{U}(t_{k-1})$ and $\hat{D}(t_{k-1})$, from the previous step. If this is the initial step in the filter, $(\hat{\underline{X}}(t_{k-1}), \hat{U}(t_{k-1}), \hat{D}(t_{k-1}))$ are initialized. Sample initial conditions are given in the user's guide (Reference 1).
2. Retrieve t_k , (the time of the next observation), and $Y(t_k)$, (the observation). Correct the time t_k with the previous estimate of the user clock error (Section 3.3). Propagate the state to t_k using the Euler integrator (Section 3). If $\Delta t = t_k - t_{k-1}$ is larger than the maximum allowable stepsize, the integration from t_{k-1} to t_k is done in substeps no larger than Δt_{\max} .

3. Compute $\Phi(t_k, t_{k-1})$. (the state transition matrix) and $Q(t_k)$ (the noise matrix).
4. Propagate the covariance matrix

$$P(t_k) = \Phi(t_k, t_{k-1}) \hat{U}(t_{k-1}) \hat{D}(t_{k-1}) \left[\Phi(t_k, t_{k-1}) \hat{U}(t_{k-1}) \right]^T + Q(t_k) \quad (2-33)$$

5. Decompose $P(t_k)$ into U and D components using the method given in Appendix A

$$P(t_k) = U(t_k) D(t_k) U^T(t_k) \quad (2-34)$$

6. Fade the filter memory by multiplying the D component of the factored covariance matrix

$$D(t_k) = \rho D(t_{k-1})$$

where

$$\rho = \frac{1}{8} \sum_{i=1}^4 \beta_R(i) + \beta_{\Delta R}(i) \quad (\beta_{\min} < \rho < \beta_{\max}) \quad (2-35)$$

where β_R and $\beta_{\Delta R}$ are vectors of smoothed residuals from each of the GPS satellites in the current constellation, and β_{\min} and β_{\max} are the minimum and maximum values allowed for ρ . If there are fewer than four GPS satellites in view, this step is not performed.

7. Compute the observation

$$\begin{aligned} G(\underline{X}_k, t_k) &= R(t_k) && \text{for pseudorange} \\ &= \Delta R(t_k) && \text{for delta pseudorange} \end{aligned} \quad (2-36)$$

ORIGINAL PAGE IS
OF POOR QUALITY

and the matrix of observation partial derivatives

$$\begin{aligned}
 H &= \left[\frac{\partial R}{\partial \underline{X}} \right] && \text{for range} \\
 &= \left[\frac{\partial \Delta R}{\partial \underline{X}} \right] && \text{for delta range}
 \end{aligned}
 \tag{2-37}$$

These equations will be given in Section 4.

8. Compute the observation residual

$$g(t_k) = Y(t_k) - G(X_k, t_k) \tag{2-38}$$

9. Compute the α 's and S's

$$\alpha_i = \sigma_m^2 + \sum_{j=1}^i d_j \left(h_j + \sum_{\ell=1}^{j-1} h_\ell u_{\ell, j} \right)^2 \tag{2-39}$$

$$S_j = h_j + \sum_{\ell=1}^{j-1} h_\ell u_{\ell, j} \tag{2-40}$$

where $\sigma_m^2 = \sigma_R^2$ for a pseudorange observation
 $= \sigma_{\Delta R}^2$ for a delta pseudorange observation

10. Test the residual for acceptability by computing the square-to-variance ratio

$$\bar{\rho}_k = \frac{[y(t_k)]^2}{\alpha_n} \tag{2-41}$$

11. Test $\bar{\rho}_k$ against ρ_{\max} , a tunable parameter. If $\bar{\rho}_k > \rho_{\max}$, the observation is rejected. In that case, set

$$\hat{X}(t_{k-1}) = X(t_k)$$

$$\hat{U}(t_{k-1}) = U(t_k)$$

$$\hat{D}(t_{k-1}) = D(t_k)$$

$$t_{k-1} = t_k$$

$$t_k = t_{k+1}$$

and go back to step 1.

If $\bar{\rho}_k < \rho_{\max}$ and the observation is a range, compute the smoothed residual

$$\beta_R(i) = \beta_R(i) + p(\bar{\rho}_k - \beta_R(i)) \quad (2-42a)$$

If the observation is a delta range, the smoothed residual is

$$\beta_{\Delta R}(i) = \beta_{\Delta R}(i) + p(\bar{\rho}_k - \beta_{\Delta R}(i)) \quad (2-42b)$$

where $(\beta_R(i), \beta_{\Delta R}(i))$ is the smoothed square-to-variance ratio from the previous observation of the i th GPS and p is a tunable parameter. After $(\beta_R(i), \beta_{\Delta R}(i))$ is computed, it is checked against a tunable parameter ρ_2 and, if larger than ρ_2 , set equal to ρ_2 .

12. Update the U and D components of the covariance matrix and compute the Kalman gain, K, using Equations (2-28), (2-29), (2-30), and (2-32).

13. Update the state vector

$$\hat{\underline{X}}(t_k) = \underline{X}(t_k) + \underline{K}(t_k) g(t_k) \quad (2-43)$$

If there are more observations, go to step 1.

SECTION 3 - STATE PARAMETER PROPAGATION EQUATIONS

The ONPAC simulator has nine state parameters, the position and velocity components, two clock terms, and a drag coefficient. For satellites such as Landsat, which are not highly drag perturbed, this can be reduced to eight by leaving out the drag coefficient.

With the intention of reducing the computation time and storage needed, the Taylor series expansion in the algorithms for the state propagation were truncated to the minimum number of terms necessary for achieving the desired accuracy. Observations will be made very frequently at the rate of a pseudorange and delta pseudorange pair in every 6.6 seconds when possible. This will reduce the impact of the neglected terms in the propagation algorithms on the filter accuracy.

3.1 STATE VECTOR

The state vector, \underline{X} , which is used in ONPAC is given by

$$\underline{X} = \begin{bmatrix} x \\ y \\ z \\ b \\ \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{b} \\ d \end{bmatrix} \quad (3-1)$$

where $(x, y, z, \dot{x}, \dot{y}, \dot{z})$ are the Cartesian position and velocity components in Earth-centered Earth-fixed (ECEF) rotating coordinates, (b, \dot{b}) are the clock bias and bias rate expressed in kilometers (km) and kilometers per second (km/sec) (i. e., the bias and bias rates are multiplied by the speed of light yielding km and km/sec. This provides immediate comparison with the errors in pseudorange and delta pseudorange) and d is the drag coefficient

$$d = \frac{C_D A_{cx}}{2m} \quad (3-2)$$

where A_{cx} = the cross-sectional area of the satellite
 m = the mass of the satellite
 C_D = a constant coefficient

3.2 FORCE MODEL

The ONPAC force model is variable, to be set by the user at run time. The equations of motion include the force terms due to the central body attraction and the coriolis and centripetal terms. To this may be added the force terms due to J_2 , or to a 2×2 , 3×3 , 4×4 , or 5×5 geopotential model and atmospheric drag. The gravity potential is a Pines model (Reference 6).

The gravity potential function uses a four parameter model. The potential function is

$$V = \frac{\mu}{r} \left\{ 1 - \sum_{n=1}^{n_{\max}} \left(\frac{R_e}{r} \right)^n \left[J_n A_{n,0}(u) - \sum_{m=1}^n A_{n,m}(u) \left(C_{n,m} r_m(s,t) + S_{n,m} i_m(s,t) \right) \right] \right\} \quad (3-3)$$

where

R_e = mean radius of the Earth

μ = GM, the gravitational parameter (the gravitational constant times the mass of the Earth)

J_n = zonal harmonic coefficients

$C_{n,m}$; $S_{n,m}$ = tesseral harmonic coefficients (the values used are from GTDS)

$r = (x^2 + y^2 + z^2)^{1/2}$ = magnitude of the position vector

$s = x/r$

$t = y/r$

$u = z/r$

$A_{n,m}(u) = d^m P_n(u)/d u^m$ (where $P_n(u)$ = the Legendre polynomial of the first kind of degree n)

n_{max} = a user specified input which = 0, 1, 2, 3, 4, or 5

= 0 to produce a 2-body only geopotential

= 1 to produce a J_2 geopotential

= 2, 3, 4, or 5 giving a 2×2 , 3×3 , 4×4 , or 5×5 geopotential field, respectively

The functions $r_m(s, t)$ and $i_m(s, t)$, the real and complex portions of the potential expansion, are defined by

$$(s + jt)^m = r_m(s, t) + j i_m(s, t)$$

where $j = \sqrt{-1}$.

The gravitational acceleration is the gradient of the potential function with respect to the four parameters (r , s , t , u)

$$F = \frac{\partial V}{\partial r} \Delta r + \frac{\partial V}{\partial s} \Delta s + \frac{\partial V}{\partial t} \Delta t + \frac{\partial V}{\partial u} \Delta u \quad (3-4)$$

ORIGINAL PAGE IS
OF POOR QUALITY

The acceleration in Cartesian coordinates due to the gravitational potential is

$$a_x = a_1 - sa_4$$

$$a_y = a_2 - ta_4$$

$$a_z = a_3 - ua_4$$

$$a_1 = \frac{1}{r} \frac{\partial V}{\partial s}$$

(3-5)

$$a_2 = \frac{1}{r} \frac{\partial V}{\partial t}$$

$$a_3 = \frac{1}{r} \frac{\partial V}{\partial u}$$

$$a_4 = \frac{\partial V}{\partial r} - sa_1 - ta_2 - ua_3$$

The equations of motion for the spacecraft position vector are

$$\frac{dx}{dt} = \dot{x}$$

$$\frac{dy}{dt} = \dot{y}$$

(3-6)

$$\frac{dz}{dt} = \dot{z}$$

The equations of motion for the spacecraft velocity vector are

$$\frac{d\dot{x}}{dt} = a_x + 2\Omega\dot{y} + \Omega^2 x - d\eta v\dot{x}$$

$$\frac{d\dot{y}}{dt} = a_y - 2\Omega\dot{x} + \Omega^2 y - d\eta v\dot{y}$$

(3-7)

$$\frac{dz}{dt} = a_z - d\eta v^2$$

where d = the drag scale factor

$$= \frac{C_D \times \text{satellite area in km}^2}{2 \times \text{satellite mass in kg}}$$

C_D = coefficient of drag

$$v = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2} = \text{magnitude of velocity vector}$$

Ω = Earth's rotation rate

η = the atmospheric density, computed as follows:

The height of the satellite, h , above the reference ellipsoid is found from

$$h = r - R_e \left[1 - \left(\frac{\epsilon z}{r} \right)^2 \right]^{1/2} \quad (3-8)$$

where ϵ = Earth's ellipticity

R_e = Semi-major axis of the reference ellipsoid.

The height is scaled with

$$h_s = \frac{r_0 h}{r_0 + h} \quad (3-9)$$

where r_0 is a constant.

The scaled height h_s is compared against a series of heights

$$\Delta h = h_s - h_t(k), \text{ for } k = 1, 6 \quad (3-10)$$

where h_t is a vector of threshold heights.

When the first value of k is found for which

$$\Delta h > 0$$

the atmospheric density is computed as

$$\eta = W_a(k) [W_b(k)\Delta h + 1] W_c(k) \quad (3-11)$$

where W_a , W_b , and W_c are vectors of constants.

If

$$h_s > h_t(\theta)$$

then

$$\eta = 0.$$

The equation to model the behavior of the drag coefficient is

$$\dot{d} = \frac{1}{\tau_d} d \quad (3-12)$$

where τ_d is an input parameter, the time constant of the drag.

3.3 CLOCK MODELING

The clock equations are as follows:

$$\begin{aligned} \frac{db}{dt} &= b \\ \frac{db}{dt} &= -\tau_f b \end{aligned} \quad (3-13)$$

where τ_f is the time constant of the clock.

For a step of Δt from t_{k-1} to t_k ,

$$b(t_k) = b(t_{k-1}) + \Delta t \dot{b}(t_{k-1})$$

$$\dot{b}(t_k) = \dot{b}(t_{k-1})$$
(3-14)

3.3.1 Correction of the User Clock

The independent variable, t , is the user clock. The corrections to the user clock, (b, \dot{b}) , which are estimated in the filter, must be applied to this independent variable.

When an observation is received, the time at which it is received must be corrected with the predicted offset and rate. The clock rate equation (Equation (3-9)) computes the correction in kilometers. To convert it to seconds, it needs to be divided by the speed of light in kilometers per second (c).

The true time, at the k th observation, is approximately

$$t_{\text{true}} \approx t_{\text{obs}} - \frac{\hat{b}_k}{c}$$

$$\approx t_{\text{obs}} - \left(\frac{\hat{b}_{k-1}}{c} + (t_{\text{true}} - t_{k-1}) \frac{\hat{\dot{b}}_{k-1}}{c} \right)$$

or

$$t_{\text{true}} \approx \frac{t_{\text{obs}} - \frac{\hat{b}_{k-1}}{c} + t_{k-1} \frac{\hat{\dot{b}}_{k-1}}{c}}{1 + \frac{\hat{\dot{b}}_{k-1}}{c}}$$
(3-15)

The observed time is corrected to be t_k by

$$t_k = \frac{t_{\text{obs}} - \frac{\hat{b}_{k-1}}{c} + t_{k-1} \frac{\hat{b}_{k-1}}{c}}{1 + \frac{\hat{b}_{k-1}}{c}} \quad (3-16)$$

3.4 MODIFIED EULER INTEGRATOR

The state equations for the spacecraft position and velocity are integrated using a modified Euler integrator. The procedure is described below.

Using the state vector and acceleration vector from the previous step, at t_{k-1} , compute the position and velocity at t_{k-1} .

$$\dot{\underline{r}}_k^- = \dot{\underline{r}}_{k-1} + \Delta t \ddot{\underline{r}}_{k-1} \quad (3-17)$$

$$\underline{r}_k^- = \underline{r}_{k-1} + \Delta t \dot{\underline{r}}_{k-1} + \frac{\Delta t^2}{2} \ddot{\underline{r}}_{k-1}$$

where Δt is the integration stepsize.

This new state $(\underline{r}_k^-, \dot{\underline{r}}_k^-)$ is used to compute the acceleration at t_k , $\ddot{\underline{r}}_k$ by calling the derivative evaluation subroutines. Then, the two acceleration values $(\ddot{\underline{r}}_{k-1}, \ddot{\underline{r}}_k)$ are used to perform the integration

$$\dot{\underline{r}}_k = \dot{\underline{r}}_{k-1} + \frac{\Delta t}{2} (\ddot{\underline{r}}_{k-1} + \ddot{\underline{r}}_k) \quad (3-18)$$

$$\underline{r}_k = \underline{r}_{k-1} + \Delta t \dot{\underline{r}}_{k-1} + \frac{\Delta t^2}{3} \ddot{\underline{r}}_{k-1} + \frac{\Delta t^2}{6} \ddot{\underline{r}}_k$$

The stepsize, Δt , is monitored so that

$$\Delta t \leq \Delta t_{\max}$$

where Δt_{\max} is an input to the integration routines.

3.5 COORDINATE TRANSFORMATIONS

The GPS satellites broadcast their information in Earth-centered Earth-fixed (ECEF) Cartesian coordinates, and the ONPAC ephemeris and estimation algorithms are all written using ECEF coordinates. More information on the filter behavior and error sources can be obtained from examining ONPAC results in other coordinate frames (in particular, the UVW coordinates and the Keplerian orbital elements). To convert from ECEF coordinates to UVW coordinates or to Keplerian elements first requires a conversion to an inertial coordinate frame. The one used for this purpose in ONPAC is the Earth-centered inertial (ECI) coordinate system.

3.5.1 Earth-Centered Inertial (ECI) to Earth-Centered Earth-Fixed (ECEF) Coordinate Transformation

In general, the transformation from an ECEF to an ECI coordinate system is obtained by the rotation

$$\underline{r} = \begin{bmatrix} \cos \Omega t & -\sin \Omega t & 0 \\ \sin \Omega t & \cos \Omega t & 0 \\ 0 & 0 & 1 \end{bmatrix} \underline{r}' \quad (3-19)$$

where \underline{r} is the satellite position vector in true of date ECI coordinates, \underline{r}' is the position vector in ECEF coordinates, Ω is the Earth's rotation rate, and t the elapsed time from the epoch of the true of date system. For ONPAC,

it is necessary only to know the instantaneous correction from one system to the other. In this case, the time from epoch is zero and the transformation from ECEF to ECI is

$$\begin{aligned}\underline{r} &= \underline{r}' \\ \underline{\dot{r}} &= \underline{\dot{r}}' + \underline{\Omega} \times \underline{r}\end{aligned}\tag{3-20}$$

where $\underline{\Omega}$ is the vector directed along the Earth's North polar axis whose magnitude is Ω , the rotation rate of the Earth.

3.5.2 ECI to UVW Coordinate Transformation

The unit vectors in the UVW coordinate system are

$$\hat{u} = \frac{\underline{r} \times \underline{\dot{r}}}{|\underline{r} \times \underline{\dot{r}}|}\tag{3-21}$$

$$\hat{v} = \frac{\underline{\dot{r}}}{|\underline{\dot{r}}|}\tag{3-22}$$

$$\hat{w} = \hat{u} \times \hat{v} = \frac{\underline{\dot{r}} \times (\underline{r} \times \underline{\dot{r}})}{|\underline{\dot{r}} \times (\underline{r} \times \underline{\dot{r}})|}\tag{3-23}$$

The unit vector, \hat{u} , is in the cross-track direction (along the angular momentum vector), \hat{v} is along track (along the velocity vector), and \hat{w} is in the radial direction for a circular orbit. For any other orbit, it is in the orbit plane perpendicular to the \hat{v} and \hat{u} vectors. It will be nearly in the direction of the radius vector for an orbit with low eccentricity.

The transformation from ECI to UVW is

$$\underline{r}'' = \begin{bmatrix} \hat{u} \cdot \hat{i} & \hat{u} \cdot \hat{j} & \hat{u} \cdot \hat{k} \\ \hat{v} \cdot \hat{i} & \hat{v} \cdot \hat{j} & \hat{v} \cdot \hat{k} \\ \hat{w} \cdot \hat{i} & \hat{w} \cdot \hat{j} & \hat{w} \cdot \hat{k} \end{bmatrix} \underline{r} \quad (3-24)$$

where $(\hat{i}, \hat{j}, \hat{k})$ are the ECI unit vectors and \underline{r}'' is the position vector in UVW coordinates.

3.5.3 Keplerian Orbital Elements

The orbital elements are determined using ECI satellite position and velocity $(\underline{r}, \dot{\underline{r}})$. The angular momentum is

$$\begin{aligned} \underline{c} &= \underline{r} \times \dot{\underline{r}} \\ &= c_i \hat{i} + c_j \hat{j} + c_k \hat{k} \end{aligned} \quad (3-25)$$

The vector along the nodal line is

$$\begin{aligned} \underline{n} &= \hat{k} \times \underline{c} \\ &= n_i \hat{i} + n_j \hat{j} \\ &= -c_j \hat{i} + c_i \hat{j} \end{aligned} \quad (3-26)$$

The Laplace vector, whose magnitude is the eccentricity, e , and which points along the periapsis line is

$$\begin{aligned} \underline{e} &= \frac{1}{\mu} \left[\left(\dot{r}^2 - \frac{\mu}{r} \right) \underline{r} - (\underline{r} \cdot \underline{r}) \dot{\underline{r}} \right] \\ e &= |\underline{e}| \end{aligned} \quad (3-27)$$

and $\mu = GM$, the gravitational parameter.

The semilatus rectum is

$$p = \frac{c \cdot c}{\mu} \quad (3-28)$$

The semimajor axis is

$$a = \frac{p}{1 - e^2} \quad (3-29)$$

The inclination of the orbit to the X-Y plane, I , is given by

$$\cos I = \frac{c_k}{|c|} \quad (3-30)$$

The inclination is always less than 180 degrees.

The longitude of the ascending node, measured from the X axis, is given by

$$\cos \Omega = \frac{n_i}{|n|} \quad (3-31)$$

If $n_j < 0$, the orbit is retrograde and the longitude = $\Omega + 180^\circ$.

The argument of perigee is given by

$$\cos \omega = \frac{n \cdot e}{|n|e} \quad (3-32)$$

If $e_k < 0$, the argument of perigee = $\omega + 180^\circ$.

The true anomaly is given by

$$\cos f = \frac{\underline{e} \cdot \underline{r}}{|\underline{e}| |\underline{r}|} \quad (3-33)$$

If $\underline{r} \cdot \dot{\underline{r}} < 0$, the true anomaly = $f + 180^\circ$.

The eccentric anomaly E is given by

$$\tan \frac{E}{2} = \sqrt{\frac{1-e}{1+e}} \tan \frac{f}{2} \quad (3-34)$$

3.6 STATE TRANSITION MATRIX

A state transition matrix propagates the estimated corrections to the state vector forward in time and it is computed by integrating the linearized form of the equations of motion. In ONPAC, the covariance matrix is propagated by using an analytic approximation to the state transition matrix. The analytic state transition matrix is expanded in a Taylor series and may be truncated at $O(t)$, $O(t^2)$ or $O(t^3)$ at the user's option. The force model for the state transition matrix includes the central body attraction and the coriolis terms. The user may also include some terms due to drag.

The general differential equation for the state transition matrix Φ is

$$\dot{\Phi}(t, t_{k-1}) = A(t) \Phi(t, t_{k-1}) \quad (3-35)$$

which has the solution

$$\begin{aligned} \Phi(t_k, t_{k-1}) &= \exp \left[\int_{t_{k-1}}^{t_k} A(t) dt \right] \\ &\approx \exp [A(t_{k-1}) \Delta t] \\ &\approx I + A(t_{k-1}) \Delta t + \frac{A^2(t_{k-1}) \Delta t^2}{2!} + \dots \end{aligned} \quad (3-36)$$

where $\Delta t = t_k - t_{k-1}$. The matrix $A(t)$ is composed of the partial derivatives of the equations of motion $\underline{F}(t)$, with regard to the state, \underline{X} .

$$\Phi(t_k, t_{k-1}) = \begin{bmatrix} \phi_{11} & 0_{3 \times 1} & \phi_{12} & 0_{3 \times 1} & \phi_{13} \\ 0_{1 \times 3} & 1 & 0_{1 \times 3} & \Delta t & 0 \\ \phi_{21} & 0_{3 \times 1} & \phi_{22} & 0_{3 \times 1} & \phi_{23} \\ 0_{1 \times 3} & 0 & 0_{1 \times 3} & 1 - \Delta t / \tau_f & 0 \\ 0_{1 \times 3} & 0 & 0_{1 \times 3} & 0 & 1 - \Delta t / \tau_d \end{bmatrix} \quad (3-37)$$

where $\phi_{11} = I$ (and is a 3 by 3 submatrix)

$0_{3 \times 1} = 3$ by 1 zero submatrix

$\phi_{12} = I \Delta t + \phi_a \Delta t^2 / 2$ (and is a 3 by 3 submatrix)

$0_{1 \times 3} = 1$ by 3 zero submatrix

$\phi_{21} = \phi_g \Delta t$ (and is a 3 by 3 submatrix)

$\phi_{22} = I + \phi_a \Delta t$ (and is a 3 by 3 submatrix)

$\Delta t = t_k - t_{k+1}$

$$\phi_g = \frac{\mu}{r^5} \begin{bmatrix} 2x^2 - y^2 - z^2 & 3xy & 3xz \\ 3xy & 2y^2 - x^2 - z^2 & 3yz \\ 3xz & 3yz & 2z^2 - x^2 - y^2 \end{bmatrix} \quad (3-38)$$

$$\phi_a = \frac{-d\eta}{v} \begin{bmatrix} 2\dot{x}^2 + \dot{y}^2 + \dot{z}^2 & \dot{x}\dot{y} & \dot{x}\dot{z} \\ \dot{x}\dot{y} & \dot{x}^2 + 2\dot{y}^2 + \dot{z}^2 & \dot{y}\dot{z} \\ \dot{x}\dot{z} & \dot{y}\dot{z} & \dot{x}^2 + \dot{y}^2 + 2\dot{z}^2 \end{bmatrix} \quad (3-39)$$

$$+ \begin{bmatrix} 0 & 2\Omega & 0 \\ -2\Omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\phi_{23} = \begin{bmatrix} -\eta v \dot{x} \Delta t \\ -\eta v \dot{y} \Delta t \\ -\eta v \dot{z} \Delta t \end{bmatrix} \quad (3-40)$$

$$\phi_{13} = \frac{\Delta t \phi_{23}}{2} \quad (3-41)$$

where Ω = rotation rate of the Earth.

The equations of motion, $F(t)$, are Equations (3-6) and (3-7) defined in Section 3.2.

3.7 FADING MEMORY

The most recently made observations are weighted more heavily than previous ones by fading the filter memory. When the covariance is propagated, as described by Equation (2-4), an additional factor is included

$$P(t_k) = s\phi(t_k, t_{k-1}) \hat{P}(t_{k-1}) \Phi^T(t_k, t_{k-1}) \quad (3-42)$$

where the variable s can be described as

$$s = e^{-\Delta t/\tau} \quad (3-13)$$

with Δt the interval between observations and τ a time constant. Since the covariance is factored into U-D components for gain and update computation, the D component of the propagated covariance is multiplied instead of the entire covariance,

$$D(t_k) = s D(t_k) \quad (3-14)$$

which is equivalent to (3-12) when there is process noise covariance. In ONPAC, the s term is approximated with a combination of smoothed square-to-variance ratios of range and delta range observations from the GPS satellites in the constellations, as described in Section 2.3.

The fading memory can be compared to the process noise covariance matrix given in Section 5. The process noise terms, which are added to the covariance, provide a minimum value for specific terms of the covariance at any given time. The fading memory multiplies the entire covariance and can impact the covariance much more than the process noise. This limits the use of the fading memory to periods of good GPS satellite visibility (four or more in view). When used together, the fading memory will overwhelm the process noise but there is reason for having both available to ONPAC. When the GPS satellite visibility is poor, only the process noise is used. When it is good, both the process noise and the fading memory are used.

SECTION 4 - OBSERVATION MODEL AND PARTIAL DERIVATIVES

The pseudorange observation equation is the equation for the length of the line-of-sight vector from the user to the GPS satellite. The delta pseudorange observation equation is the difference between two range measurements made over a short time span. The partial derivatives of the pseudorange equation are straightforward. Those of the delta pseudorange are the first terms of an expansion in which the assumption is made that a linear approximation will suffice over a short time span.

4.1 PSEUDORANGE OBSERVATION MODEL

The pseudorange observation equation at time t_k is

$$R(t_k) = \left[(s_x - x)^2 + (s_y - y)^2 + (s_z - z)^2 \right]^{1/2} + b \quad (4-1)$$

where (x, y, z) = the user satellite position at t_k

(s_x, s_y, s_z) = the GPS satellite position at t_k

b = the user clock bias at t_k

The partial derivatives of the pseudorange observation with respect to the state parameters are

$$\frac{\partial R}{\partial x} = -\ell_x(t_k)$$

$$\frac{\partial R}{\partial y} = -\ell_y(t_k) \quad (4-2)$$

$$\frac{\partial R}{\partial z} = -\ell_z(t_k)$$

$$\frac{\partial R}{\partial b} = 1$$

$$\frac{\partial R}{\partial x} = \frac{\partial R}{\partial y} = \frac{\partial R}{\partial z} = \frac{\partial R}{\partial b} = \frac{\partial R}{\partial d} = 0 \quad (4-2)$$

(Cont'd)

where the components of the unit vector along the line-of-sight from the user satellite to the GPS satellite are given by

$$\begin{aligned} \ell_x(t_k) &= \frac{s_x(t_k) - x(t_k)}{R(t_k) - b(t_k)} \\ \ell_y(t_k) &= \frac{s_y - y}{R - b} \\ \ell_z(t_k) &= \frac{s_z - z}{R - b} \end{aligned} \quad (4-3)$$

4.2 DELTA PSEUDORANGE OBSERVATION MODEL

The delta pseudorange observation equation at time t_k is

$$\Delta R(t_k) = R(t_k) - R(t_{k-1}) \quad (4-4)$$

This equation assumes that a pseudorange measurement precedes the delta pseudorange measurement at time t_{k-1} .

The partial derivatives of the delta pseudorange ΔR with respect to the state, \underline{X} , at time t_k are

$$\begin{aligned} \frac{\partial \Delta R(t_k)}{\partial \underline{X}(t_k)} &= \frac{\partial R(t_k)}{\partial \underline{X}(t_k)} - \frac{\partial R(t_{k-1})}{\partial \underline{X}(t_{k-1})} \frac{\partial \underline{X}(t_{k-1})}{\partial \underline{X}(t_k)} \\ &= \frac{\partial R(t_k)}{\partial \underline{X}(t_k)} - \frac{\partial R(t_{k-1})}{\partial \underline{X}(t_{k-1})} \Phi(t_{k-1}, t_k) \end{aligned} \quad (4-5)$$

ORIGINAL MANUSCRIPT
OF POOR QUALITY

The state transition matrix, $\Phi(t_{k-1}, t_k)$, can be approximated by the expansion

$$\Phi(t_{k-1}, t_k) \approx I_{9 \times 9} + A(t_{k-1})_{9 \times 9} (t_{k-1} - t_k) \quad (4-6)$$

For the nine-parameter state, a further approximation for the matrix A can be introduced, where

$$A \approx \begin{bmatrix} 0_{3 \times 3} & I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix} \quad (4-7)$$

Then, the partial derivatives with respect to the state parameters are

$$\frac{\partial \Delta R}{\partial x} = \ell_x(t_{k-1}) - \ell_x(t_k)$$

$$\frac{\partial \Delta R}{\partial y} = \ell_y(t_{k-1}) - \ell_y(t_k)$$

$$\frac{\partial \Delta R}{\partial z} = \ell_z(t_{k-1}) - \ell_z(t_k)$$

$$\frac{\partial \Delta R}{\partial b} = 0 \quad (4-8)$$

$$\frac{\partial \Delta R}{\partial \dot{x}} = -\Delta t \ell_x(t_{k-1})$$

$$\frac{\partial \Delta R}{\partial \dot{y}} = -\Delta t \ell_y(t_{k-1})$$

$$\frac{\partial \Delta R}{\partial \dot{z}} = -\Delta t \ell_z(t_{k-1})$$

$$\frac{\partial \Delta R}{\partial b} = \Delta t$$

(4-8)
(Cont'd)

$$\frac{\partial \Delta R}{\partial d} = 0$$

where $\Delta t = t_k - t_{k-1}$. In the pseudorange measurement, the Δt is small (on the order of 0.6 second). The ONPAC program assumes that a delta pseudorange observation at time t_k is following a range observation at t_{k-1} .

SECTION 5 - DERIVATION OF THE STATE PROCESS NOISE
COVARIANCE MATRIX

The covariance matrix in a Kalman filter or EKF will become saturated, the terms becoming very small, after a large number of observations have been processed. The result is such that the filter will no longer significantly correct the state (i.e., the state corrections become infinitesimal). To prevent this, a noise matrix which compensates for neglected terms in the force model is computed and added to the covariance matrix when it is propagated. Then, Equation (2-4) becomes

$$P(t_k) = \Phi(t_k, t_{k-1}) \hat{P}(t_{k-1}) \Phi^T(t_k, t_{k-1}) + Q(t_k) \quad (5-1)$$

where $Q(t_k)$ is the state process noise covariance matrix.

The noise errors are assumed to be uncorrelated in time (see Reference 4) and the position-velocity and drag terms are uncorrelated with the clock terms.

The vector \underline{X}_e is defined as the error in the state.

$$\underline{X}_e = \begin{bmatrix} \delta \underline{r} \\ \delta b \\ \delta \underline{v} \\ \delta \dot{b} \\ \delta d \end{bmatrix} \quad (5-2)$$

where $\delta \underline{r}$ = position error
 δb = clock bias error
 $\delta \underline{v}$ = velocity error
 $\delta \dot{b}$ = clock drift rate error
 δd = drag scale error

Then, the differential equations to describe this system can be written

$$\dot{\underline{x}}_e = \left[\frac{\partial \mathbb{F}}{\partial \underline{x}} \right] \underline{x}_e + \underline{\omega}(t) \quad (5-3)$$

where $\underline{\omega}(t)$ is a random forcing function to account for the errors in the force models used in the differential equations. This is called the state noise. The function $\underline{\omega}(t)$ is assumed to satisfy the following:

$$\begin{aligned} E[\underline{\omega}(t)] &= 0 \\ E[\underline{\omega}(t) \underline{\omega}^T(s)] &= \underline{W} \delta(t - s) \end{aligned} \quad (5-4)$$

where δ = the Dirac delta function

$$\delta(t - s) = \begin{cases} 1, & t = s \\ 0, & t \neq s \end{cases}$$

E = the expectation operator

\underline{W} = spectral density matrix

This model assumes that the state noise is unbiased and uncorrelated in time.

The covariance matrix for the noise is

$$\begin{aligned} Q(t) &= E \left[\int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^{t_k} \Phi(t_k, u) \underline{\omega}(u) \underline{\omega}^T(s) \Phi^T(t_k, s) du ds \right] \\ &= \int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^{t_k} \Phi(t_k, u) E[\underline{\omega}(u) \underline{\omega}^T(s)] \Phi^T(t_k, s) du ds \end{aligned} \quad (5-5)$$

If the position-velocity and drag terms are uncorrelated with the clock terms, the state noise matrix can be partitioned and the clock terms derived separately. The noise matrix can be divided into the following:

$$Q(t) = \begin{bmatrix} Q_{11} & 0_{3 \times 1} & Q_{12} & 0_{3 \times 1} & Q_{13} \\ 0_{1 \times 3} & q_{bb} & 0_{1 \times 3} & q_{bb} & 0 \\ Q_{12}^T & 0_{3 \times 1} & Q_{22} & 0_{3 \times 1} & Q_{23} \\ 0_{1 \times 3} & q_{bb} & 0_{1 \times 3} & q_{bb} & 0 \\ Q_{13}^T & 0 & Q_{23}^T & 0 & Q_{33} \end{bmatrix} \quad (5-6)$$

The clock process noise covariance matrix terms are

$$q_{bb} = \frac{1}{2} \sigma_0^2 \Delta t + 2 \sigma_{-1}^2 \Delta t^2 + \frac{2}{3} \pi^2 \sigma_{-2}^2 \Delta t^3 + \frac{1}{4} \sigma_D^2 \Delta t^4$$

$$q_{bb} = 2 \sigma_{-1}^2 \Delta t + \pi^2 \sigma_{-2}^2 + \frac{1}{2} \sigma_D^2 \Delta t^2 \quad (5-7)$$

$$q_{bb} = \sigma_{-1}^2 c + 2 \pi^2 \sigma_{-2}^2 \Delta t + \sigma_D^2 \Delta t^2$$

where $c = 4.6 \frac{\Delta t}{\tau_{\min}} \left(1 - \frac{\Delta t}{2 \tau_{\min}} \right)$

σ_0^2 is the clock white noise Allan variance factor

σ_{-1}^2 is the clock flicker noise Allan variance factor

σ_{-2}^2 is the clock random walk Allan variance factor

σ_D^2 is the deterministic clock drift rate variance

and τ_{\min} is the scaled minimum time where flicker noise predominates on the Allan variance curve

$Q'(t)$ is defined as the subset of $Q(t)$ which concerns the position-velocity and drag terms

$$Q'(t) = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{12}^T & Q_{22} & Q_{23} \\ Q_{13}^T & Q_{23}^T & Q_{33} \end{bmatrix} \quad (5-8)$$

Equation (5-4) can be rewritten in the form

$$E[\underline{\omega}(u) \underline{\omega}^T(s)] = \begin{bmatrix} 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 1} \\ 0_{3 \times 3} & \sigma_a^2 I_{3 \times 3} & 0_{3 \times 1} \\ 0_{1 \times 3} & 0_{1 \times 3} & \sigma_d^2 \delta(u - s) \end{bmatrix} \quad (5-9)$$

for the position-velocity and drag noise terms where σ_a^2 and σ_d^2 are the rates of the noise variances for the acceleration and the drag coefficient.

Because Equation (5-5) for computing $Q(t)$ contains both Φ and Φ^T , the state transition matrix, Φ may be approximated with fewer terms than those given in Section 3.

The transition matrix is approximated for use in Equation (5-5) by

$$\Phi(t_k, t_{k-1}) \approx \begin{bmatrix} \phi_{11} & \phi_{12} & 0_{3 \times 1} \\ 0_{3 \times 3} & \phi_{22} & \phi_{23} \\ 0_{1 \times 3} & 0_{1 \times 3} & 1 - \Delta t / \tau_d \end{bmatrix} \quad (5-10)$$

ORIGINAL PAGE IS
OF POOR QUALITY

where $\Delta t = t_k - t_{k-1}$

$$\phi_{11} = I_{3 \times 3}$$

$$\phi_{12} = \Delta t I_3$$

$$\phi_{22} = \begin{bmatrix} 1 & 2\Omega \Delta t & 1 \\ -2\Omega \Delta t & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\phi_{23} = \begin{bmatrix} -\eta v \dot{x} \Delta t \\ -\eta v \dot{y} \Delta t \\ -\eta v \dot{z} \Delta t \end{bmatrix}$$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$$

Ω = rotation rate of the Earth

η = the atmospheric density

Then,

$$Q_{11} = \frac{\sigma_a^2 \Delta t^4}{4} I_3 \quad (5-11)$$

$$Q_{12} = \sigma_a^2 \begin{bmatrix} \frac{\Delta t^3}{2} & \frac{-\Omega \Delta t^4}{2} & 0 \\ \frac{\Omega \Delta t^4}{2} & \frac{\Delta t^3}{2} & 0 \\ 0 & 0 & \frac{\Delta t^3}{2} \end{bmatrix} \quad (5-12)$$

$$Q_{13} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (5-13)$$

$$Q_{22} = \sigma_a^2 \begin{bmatrix} \Delta t^2 + \Omega^2 \Delta t^4 & 0 & 0 \\ 0 & \Delta t^2 + \Omega^2 \Delta t^4 & 0 \\ 0 & 0 & \Delta t^2 \end{bmatrix} \quad (5-14)$$

$$Q_{23} = -\eta v \sigma_d^2 \frac{\Delta t^2}{2} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \quad (5-15)$$

$$Q_{33} = \sigma_d^2 \Delta t \quad (5-16)$$

The terms $\sigma_d^2 \Delta t^2$ and $\sigma_d^2 \Delta t^3$ were neglected since they are smaller than the neglected terms in the state transition matrix.

The state noise process covariance matrix is a positive definite matrix. Consideration of the position-velocity components $[Q_{11}, Q_{12}]$ shows that

$$\begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix}$$

is a matrix which may be factored into U - D components. Let Q'' be the subset of Q described above. Then,

$$Q'' = \sigma_a^2 \Delta t^2 \left[(I_{6 \times 6} + q_1) (q_2) (I_{6 \times 6} + q_1)^T \right] \quad (5-17)$$

where

$$q_1 = \begin{bmatrix} 0_{3 \times 3} & q_1^1 \\ 0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix}$$

$$q_1^1 = \begin{bmatrix} \frac{\Delta t}{2(1 + \Omega^2 \Delta t^2)} & -\frac{\Omega \Delta t^2}{2(1 + \Omega^2 \Delta t^2)} & 0 \\ \frac{\Omega \Delta t^2}{2(1 + \Omega^2 \Delta t^2)} & \frac{\Delta t}{2(1 + \Omega^2 \Delta t^2)} & 0 \\ 0 & 0 & \frac{\Delta t}{2} \end{bmatrix}$$

and q_2 is a diagonal 6 by 6 matrix with elements

$$q_2 = [0, 0, 0, 1 + \Omega^2 \Delta t^2, 1 + \Omega^2 \Delta t^2, 1]$$

APPENDIX A - MATRIX DECOMPOSITION

A.1 CHOLESKY DECOMPOSITION

Any positive definite square matrix can be factored into a triangular matrix and its transpose. If A is such a matrix,

$$A = BB^T$$

The matrix B is not unique and may be either an upper or lower triangular matrix. If it is an upper triangular matrix, the decomposition algorithms for an $n \times n$ matrix are as follows:

$$b_{n,n} = \sqrt{a_{n,n}}$$

$$b_{i,n} = \frac{a_{i,n}}{b_{n,n}} \text{ for } i = 1 \text{ to } n - 1$$

$$b_{j,j} = \sqrt{a_{j,j} - \sum_{i=j+1}^n b_{j,i}^2} \text{ for } j = n - 1 \text{ to } 1 \tag{A-1}$$

$$b_{l,j} = \frac{a_{l,j} - \sum_{k=j+1}^n b_{j,k} b_{l,k}}{b_{j,j}} \text{ for } l = j - 1 \text{ to } 1$$

A.2 UDU^T DECOMPOSITION

The matrix A can also be factored into a triangular matrix with unity on the diagonal and a diagonal matrix,

$$A = CEC^T \tag{A-2}$$

which can also be written as

$$A = (CE^{1/2})(CE^{1/2})^T \quad (A-3)$$

implying that

$$B = CE^{1/2} \quad (A-4)$$

When C is an upper triangular matrix, the algorithms for the C and E components are as follows:

$$\begin{aligned} e_{n,n} &= a_{n,n} \\ c_{n-i,n} &= \frac{a_{n-i,n}}{e_{n,n}} \quad \text{for } i = 1 \text{ to } n - 1 \end{aligned} \quad (A-5)$$

Then, for $j = 1$ to $n - 1$ the diagonal terms are

$$e_{n-j,n-j} = a_{n-j,n-j} - \sum_{k=n-j+1}^n c_{n-j,k}^2 e_{k,k} \quad (A-6)$$

and the off-diagonal terms in C are for $m = 1$ to $n - j - 1$

$$c_{n-j-m,n-j} = \left(\frac{a_{n-j-m,n-j} - \sum_{l=n-j+1}^n c_{n-j-m,l} c_{n-j,l} e_{l,l}}{e_{n-j,n-j}} \right) \quad (A-7)$$

The diagonal terms of C are

$$c_{i,i} = 1, \text{ for } i = 1, n$$

APPENDIX B - DATA SIMULATION

Simulated pseudorange and delta pseudorange measurements from GPS broadcasts are provided by using the ANALYSIS program of GTDS. Further information on the ANALYSIS program is available in Reference 7. Data may be simulated for either ONPAC or the FILTER and DC programs in GTDS.

For a data simulation computation, the user satellite orbit is computed using the GTDS EPHEM program, and the true coordinates are read as input to the ANALYSIS program. The GPS satellite coordinates at a given time are determined depending on the configuration chosen. The options available are 3 GPS satellites in 2 orbits (Phase I), 6 satellites spaced in 3 orbits as set by the user (modified Phase I); 12 satellites with unequal separation (Phase IIa), 12 satellites with equal separation (Phase IIb), and 24 satellites with equal separation (Phase III).

The visibility of the GPS satellites is constrained by the observer antenna cone angle and the ionospheric cutoff. The user may cycle through all visible GPS satellites, select a subset of four by choosing those four which minimize the Geometric Dilution of Precision (GDOP), or use all those visible except only one of the two or more that are essentially in the same direction from the user.

The user and GPS satellite positions can have random noise added. The GPS position and velocity errors are simulated by computing errors in HCL coordinates, with the along-track error increasing with time.

The clock errors are a total of the GPS and user satellite clock errors. The GPS clock offset is a constant for each GPS satellite. The constant value is chosen at random for an input standard deviation. The user clock offset may be computed using a quadratic or a Markov process model.

The quadratic model of the user clock offset, T_u^B , at time t is

$$T_u^B(t) = T_u^B + f_u^B (t - t_0) + \frac{1}{2} f_u^D (t - t_0)^2 \quad (B-1)$$

where T_u^B = the user clock bias

f_u^B = the user clock drift which equals the user frequency offset in parts per reference frequency at time t

f_u^D = the user clock drift rate which equals the user discrete change in frequency per reference frequency

t = the observation time (in seconds)

t_0 = the epoch of the clock model

In the Markov process, the drift rate f_u^D is computed as a random number at discrete update times. The time interval between updates is set by input, as are the mean and standard deviation of the random number generator. The drift rate is assumed constant between update times. The clock drift at time t_1 , $f_u^B(t_1)$, is

$$f_u^B(t_1) = f_u^B(t_0) + \sum_{j=0}^1 f_u^D(t_{j+1}) u(t_1 - t_{j+1}) \quad (B-2)$$

where $u(\tau)$ is a step function

$$u(\tau) = \begin{cases} 0, & \tau < 0 \\ 1, & \tau \geq 0 \end{cases}$$

and the t_{j+1} are the times of updates.

The user clock bias is then

$$T_u^B(t_1) = T_u^B(t_0) + f_u^B(t_0)(t_1 - t_0) + \sum_{j=0}^1 f_u^D(t_{j+1})(t_1 - t_{j+1}) u(t_1 - t_{j+1}) \quad (B-3)$$

For the quadratic model, the user offset is computed by evaluating Equation (B-1). In ONPAC, the user clock bias, b , is expressed in kilometers, and is compared to T_u^B by multiplying T_u^B times the speed of light in km/sec.

The pseudorange observation is the true range plus the user and GPS clock offsets plus the range measurement noise, computed as a random number with a user-supplied standard deviation. The delta pseudorange is the difference of two true ranges plus its measurement noise computed from the user-specified standard deviation. The filter programs are also supplied with GPS coordinates which will be obtained from the broadcasts. The coordinates are Cartesian ECEF and contain the simulated GPS ephemeris errors.

The ONPAC data tape also includes a record with the true observation, true user coordinates, and true user clock offset and drift for each observation record. This information is used in analyzing the results.

GLOSSARY

$A(t)$	Partial derivatives of the equations of motion defined in Equation (2-3)
A_{cx}	Cross-sectional area of the satellite
$A_{n,m}$	Associated Legendre polynomials
a	Semimajor axis defined in Equation (3-25)
a_1, a_2, a_3, a_4	Acceleration components in Pine's model defined in Equation (3-5)
a_x, a_y, a_z	Acceleration components
B	Upper right triangular matrix in U-D update derivation
b	Clock bias expressed in units of km
\dot{b}	Clock drift rate expressed in units of km/sec
$b_{i,j}$	Element of B defined in Equation (2-27)
C_D	Coefficient of drag
$C_{n,m}$	Tesseral harmonic coefficients
\underline{c}	Angular momentum vector defined in Equation (3-25)
c_x, c_y, c_z	Angular momentum components
D	Diagonal component matrix of covariance matrix
d	Drag coefficient defined in Equation (3-2)
d_i	Diagonal components of D matrix
δd	Drag coefficient error
E	Eccentric anomaly
$E[]$	Expectation operator

e	Eccentricity
\underline{e}	Laplace vector defined in Equation (3-27)
F(X, t)	Equations of motion
f	True anomaly defined in Equation (3-13)
G(X(t), t)	Observation equation
g(t)	Observed minus computed observation defined in Equation (2-6)
H(t)	Matrix of partial derivatives of G(X(t), t)
h	Satellite height defined after Equation (3-6)
h_i	Components of H(t) matrix
h_s	Scaled height
h_t	Vector of threshold heights
I	Identity matrix
I	Satellite orbit inclination defined in Equation (3-30)
$i_m(s, t)$	Imaginary part of the potential expansion defined after Equation (3-3)
J_n	Zonal harmonic coefficients
j	$\sqrt{-1}$
K(t)	Kalman gain vector defined in Equation (2-31)
ℓ	Line-of-sight unit vectors defined in Equation (4-3)
M	Inner matrix in the U-D update derivation Equation (2-19)
m	Satellite mass
$m_{i,j}$	Element of M matrix defined in Equations (2-22) and (2-23)
n	The number of state parameters

$P(t)$	Covariance matrix defined in Equations (2-33) and (2-34)
P_n	Legendre polynomials
p	Semilatus rectum defined in Equation (3-28)
P	Filter tunable parameter used to compute smoothed residual
$Q(t)$	State noise process covariance matrix defined in Equation (5-6)
Q_{11}, Q_{12}, Q_{13} Q_{22}, Q_{23}, Q_{33}	Submatrix components of $Q(t)$
Q'	Position-velocity and drag submatrix of $Q(t)$ defined in Equation (5-8)
Q''	Position-velocity submatrix of $Q(t)$ defined in Equation (5-17)
q_1, q_2	Component matrices in Q'' defined after Equation (5-17)
$R(t)$	Pseudorange defined in Equation (4-1)
$\Delta R(t)$	Delta pseudorange, defined in Equation (4-4)
R_e	Mean Earth radius
\underline{r}	Satellite position vector
$\delta \underline{r}$	Satellite position error
$r_m(s, t)$	Real part of potential expansion defined after Equation (3-3)
r_0	Satellite height scaling factor, defined after Equation (3-6)
S_i	Partial sum in the U-D update defined in Equation (2-30)
$S_{n,m}$	Tesseral harmonic coefficients

s	Coordinate in Pines force model defined in Equation (3-3)
s_x, s_y, s_z	GPS satellite position components
t	Time
t	Coordinate in Pines force model defined in Equation (3-3)
Δt	Filter stepsize
Δt_{\max}	Maximum allowable stepsize
U	Upper right triangular matrix component of the covariance
u	Coordinate in Pines force model defined in Equation (3-3)
\hat{u}	Crosstrack unit vector defined in Equation (3-21)
$u_{i,j}$	Element of U matrix
V	Gravity potential defined in Equation (3-3)
\underline{v}	Satellite velocity vector
$\underline{\delta v}$	Satellite velocity error
\hat{v}	Along track unit vector defined in Equation (3-22)
W	Diagonal matrix in U-D update derivation
w_i	Diagonal element of W matrix, defined in Equation (2-24)
\hat{w}	Unit vector in UVW coordinate system defined in Equation (3-23)
W_a, W_b, W_c	Vectors of constants used in atmospheric density computation, defined after Equation (3-6)
$\underline{X}(t)$	State vector defined in Equation (3-1)
\underline{X}_e	Error in state vector defined in Equation (5-2)

x	Position component of satellite
\dot{x}	Velocity component of satellite
$Y(t)$	Observation
y	Position component of satellite
\dot{y}	Velocity component of satellite
z	Position component of satellite
\dot{z}	Velocity component of satellite
α_n	Scalar divisor for EKF update defined in Equation (2-15)
$\beta_R, \beta_{\Delta R}$	Vectors of smoothed residuals (2-42)
$\beta_{\min}, \beta_{\max}$	Filter tunable parameters
γ	Term in atmospheric density computation defined after Equation (3-6)
ϵ	Earth's ellipticity
η	Atmospheric density defined after Equation (3-6)
μ	Gravitational parameter, GM
ρ	Fading memory factor, defined in Equation (2-35)
$\bar{\rho}_k$	Ration of the square of the residual to α_n defined in Equation (2-40)
ρ_2	Tunable parameter used to limit $\beta_R, \beta_{\Delta R}$
σ_a^2	Acceleration noise variance rate
σ_0^2	Clock white noise Allan variance factor
σ_1^2	Clock flicker noise Allan variance factor
σ_2^2	Clock random walk Allan variance rate factor

σ_D^2	Deterministic clock drift rate variance
σ_d^2	Drag noise variance rate
σ_m^2	Measurement variance
σ_R^2	Pseudorange measurement variance
$\sigma_{\Delta R}^2$	Delta pseudorange measurement variance
τ_d	Time constant of drag defined in Equation (3-12)
τ_{\min}	Scaled minimum time where flicker noise predominates on the Allan variance curve.
$\Phi(t_k, t_{k-1})$	State transition matrix defined in Equation (3-37)
ϕ_{ij}	Submatrix component of Φ defined after Equation (3-37)
ϕ_a	Submatrix component of Φ defined in Equation (3-39)
ϕ_g	Submatrix component of Φ defined in Equation (3-38)
Ω	Rotation rate of Earth in Section 3.5.1
Ω	Longitude of the ascending node in Section 3.5.3
ω	Argument of perigee

REFERENCES

1. Computer Sciences Corporation, CSC/SD-80/6077, Onboard Navigation Package (ONPAC) Simulator User's Guide (Revision 1), J. B. Dunham August 1980
2. Magnavox Corporation, Advanced Products Division, Technical Proposal for the Receiver/Processor Assembly To Be Used in Conjunction With the GPS Navigation Satellites, September 1976
3. G. J. Bierman, Factorization Methods for Discrete Sequential Estimation, Academic Press, 1977
4. A. Gelb, Ed. Applied Optimal Estimation, M. I. T. Press, 1974
5. B. D. Tapley, "Statistical Orbit Determination Theory," pp. 396-425, Recent Advances in Dynamical Astronomy, B. D. Tapley, and V. Szebehely, eds., D. Reidel, 1973
6. Magnavox Corporation, GPS 77-07, (memo to GPSPAC Analysis File) "Selection of a Gravity Potential Algorithm for GPSPAC," R. T. Uyeminami, January 18, 1977
7. Computer Sciences Corporation, (GTDS Task Specification), Goddard Trajectory Task Specification, Global Positioning System (GPS) Data Implementation in the GTDS R and D Analysis Program, P. Desai and H. Stull, September 21, 1976

STL BIBLIOGRAPHY

- Systems Technology Laboratory, STL-78-0001, System Description of the IMP-16C Microprocessor Orbit Determination Program, C. Shenitz, C. Rabbin, and G. Snyder, October 1978
- , STL-78-002, Landsat/NAVPAC System Description and User's Guide, S. R. Waligora, December 1978
- , STL-79-001, Intel 8080 Orbit Propagation Program System Description and User's Guide, C. Rabbin, April 1979
- , STL-79-002, Evaluation of the IMP-16 Microprocessor Orbit Determination System Filter, C. Shenitz, September 1979
- , STL-79-003, Multimission Modular Spacecraft Ground Support Software System (MMS/GSSS) MODCOMP Device and MAX IV Dependency Study, T. Weldon and M. McClellan, December 1979
- , STL-80-001, Orbit Determination Software Development for Microprocessor-Based Systems: Evaluation and Recommendations, C. M. Shenitz, July 1980
- , STL-80-002, The Two-Way TDRSS Observation Model for the LSI-11/23 Microcomputer, C. E. Goorevich, July 1980
- , STL-80-003, Automated Orbit Determination System (AODS) Requirements Definition and Analysis, S. R. Waligora, C. E. Goorevich, J. Teles, and R. S. Pajerski, September 1980
- , STL-80-004, Algorithms for Autonomous Star Identification, P. Gambardella, October 1980
- , STL-80-005, Autonomous Onboard Attitude Determination System Specifications and Requirements, M. D. Shuster, S. N. Ray, and L. Gunshol, December 1980
- , STL-81-001, Systems Technology Laboratory (STL) Library Methods and Procedures, W. J. Decker and P. D. Merwarth, September 1981
- , STL-81-002, Mathematical Specifications of the Onboard Navigation Package (ONPAC) Simulator (Revision 1), J. B. Dunham, February 1981
- , STL-81-003, Systems Technology Laboratory (STL) Compendium of Utilities, W. J. Decker, E. J. Smith, W. A. Taylor, P. D. Merwarth and M. E. Stark, July 1981

--, STL-81-004, Automated Orbit Determination System (AODS) Environment Simulator for Prototype Testing (ADEPT) System Description, S. R. Waligor, J. E. Fry, Jr., B. J. Prusiewicz, and G. N. Klitsch, August 1981

--, STL-81-005, Preliminary Automated Orbit Determination System (AODS)/AODS Environment Simulator for Prototype Testing (ADEPT) User's Guide, S. R. Waligora, Y. Ong, J. E. Fry, Jr. and B. J. Prusiewicz, September 1981

--, STL-82-001, GPSPAC/Landsat-D Interface (GLI) System Description, J. B. Dunham, H. M. Sielski, and W. T. Wallace, April 1982

--, STL-82-002, GPSPAC/Landsat-D Interface (GLI) System User's Guide, H. M. Sielski, J. B. Dunham, and P. D. Merwarth, March 1982

--, STL-82-003, Autonomous Attitude Determination System (AADS), Volume 1: System Description, K. A. Saralker, Y. G. Frenkel, G. N. Klitsch, K. Liu and E. Lefferts, April 1982