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FROM: Norma Brennan, Director, Editorial Department

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CORRELATION OF PRESTON-TUBE DATA WITH LAMINAR SKIN FRICTION

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and

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ABSTRACT

Preston-tube data have been obtained on a sharp ten-degree cone in the NASA Ames Eleven-Foot Transonic Wind Tunnel. Data were obtained over a Mach number range of 0.30 to 0.95 and unit Reynolds numbers of 9.84, 13.1, and 16.4 million per meter. The portions of these data, that were obtained within laminar boundary layers, have been correlated with the corresponding values of theoretical skin friction. The rms scatter of skin-friction coefficient about the correlation is of the order of one percent, which is comparable to the reported accuracy for calibrations of Preston-tubes in incompressible pipe-flows. In contrast to previous works on Preston-tube/skin-friction correlations, which are based on the physical height of the probe's face, this very satisfactory correlation for compressible boundary-layer flows is achieved by accounting for the effects of a variable "effective" height of the probe. The coefficients, which appear in the correlation, are dependent on the particular tunnel environment. The general procedure can be used to define correlations for other wind tunnels.

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NOMENCLATURE

C_f	=	skin-friction coefficient
$\bar{C}_f$	=	nondimensional difference between theoretical and correlated skin-friction coefficient $[(C_{f,t} - C_{f,c})/C_{f,t}]$
$(C_p)_c$	=	pressure coefficient on surface of cone, $(P_w - P_\infty)/q_\infty$
$(C_p)_{rms}$	=	root-mean square pressure coefficient based on microphone data over a frequency range of 0.2 to 30 kHz
d	=	external diameter of a round Pitot probe
D	=	internal diameter of a pipe
h	=	external height of face of a flattened Pitot probe
K_{eff}	=	nondimensional effective height of Preston tube $(2Y_{eff}/h)$
L	=	axial length of cone, 113 cm
M	=	Mach number
P_e	=	static pressure at outer edge of boundary layer
P_p	=	Preston-tube pressure
P_w	=	static pressure at wall
ΔP_p	=	difference in pressure between a Preston tube and the local static pressure, $P_p - P_w$
q_∞	=	freestream dynamic pressure, $\rho_\infty U^2_\infty/2$
R_d	=	Reynolds number based on U_e and diameter of a circular Preston tube, $U_e d/\nu_e$
Re_T	=	Reynolds number based on freestream properties and X_T
T	=	temperature
u	=	velocity parallel to boundary
u^+	=	nondimensional velocity used in the law-of-the-wall, u/U_τ
U_e	=	velocity at outer edge of boundary layer
U_m	=	mean or average velocity in a pipe flow
U_p	=	velocity calculated from Preston-tube data
U_τ	=	classical wall-shear-stress velocity, $(\tau_w/\rho_w)^{\frac{1}{2}}$
U_∞	=	freestream velocity
w	=	external width of face of a flattened Pitot probe in a direction parallel to the wall but normal to the undisturbed streamlines
x	=	distance along axis of cone
x^*	=	dimensionless pressure difference for incompressible, isothermal flow, $\log_{10}[\Delta P_p\, d^2/4\, \rho\nu^2]$
X	=	distance along surface of cone measured from apex
X_ℓ	=	station at which Preston-tube measurements began
X_t	=	distance along surface of cone from apex to onset of boundary-layer transition

X_T	=	distance along surface of cone from apex to end of boundary-layer transition
X^*	=	dimensionless pressure difference for compressible, nonadiabatic flow, $\log_{10}(U_p Y_{eff}/\nu_w)^2$
X_A^*	=	Allen's correlation parameter, $\log_{10}(U_p d/\nu')$
y	=	distance measured normal to the wall
y_c	=	distance of geometric center of Preston-tube from the wall
y^+	=	nondimensional distance from the wall as used in the law-of-the-wall, $U_\tau y/\nu$
y^*	=	dimensionless shear stress for incompressible, isothermal flow, $\log_{10}[\tau_w d^2/4\rho\nu^2$ or $0.25(U_\tau d/\nu)^2]$
Y_{eff}	=	effective height of face of Preston tube = height above the wall of an undisturbed streamline which has a total pressure equal to the measured Pitot pressure
Y^*	=	dimensionless shear stress for compressible, nonadiabatic flow, $\log_{10}(\tau_w Y_{eff}^2/\rho_w\nu_w^2)$
Y_A^*	=	Allen's correlation parameter, $\log_{10}[(2\tau_w/\rho')^{\frac{1}{2}}d/\nu']$
α	=	angle-of-attack, defined to be positive for nose up
β	=	yaw angle, defined to be positive when nose is to portside
δ	=	cone semi-vertex angle
θ_L	=	momentum thickness of laminar boundary layer
μ	=	molecular viscosity
ν	=	kinematic viscosity
ρ	=	density of fluid
τ_w	=	shear stress at wall

Subscripts

e	=	evaluated at conditions corresponding to outer edge of boundary layer
w	=	evaluated at wall conditions

Superscript

'	=	evaluated at the reference temperature of Sommer and Short, Eq. (11)

I. INTRODUCTION

Since the first transonic wind tunnel became operational at NASA Langley in the late 1940's, there has been a need for a procedure to calibrate the effects of wall-generated noise on tunnel flow quality. As noted by Dougherty and Steinle[1], the primary indicators of flow quality in a wind tunnel are variations of: (1) Mach number, (2) flow angularity within the empty test-section, and (3) the Reynolds number at which transition from a laminar to a turbulent boundary layer occurs on models. Variations in Mach number and flow angularity can be calibrated with conventional Pitot-static probes and yawmeters, e.g., see Reed, et al.[2] And in the case of low-speed wind tunnels, the Reynolds number at which the drag coefficient of a sphere equals 0.30 can be used to define a turbulence factor, as described by Pope and Harper.[3] An "effective" unit Reynolds number for a given tunnel can then be defined by

$$(Re_m)_{eff} = (TF)\ (Re_m) \quad .$$

However, when Mach number exceeds about 0.35, compressibility effects cause the classical turbulence factor to become increasingly erroneous and therefore not useful. Recently, Miller and Bailey[4] have reviewed the status of knowledge concerning the drag of a sphere at transonic speeds. Even today, the precise variation of sphere drag with Mach number and Reynolds number is not well defined. Thus, the classical turbulence-sphere method is not applicable to the calibration of transonic wind tunnels.

During the mid-sixties, engineers at Arnold Engineering Development Center (AEDC) designed and experimented with a sharp ten-degree cone which had a traversing Pitot probe resting on the surface to directly detect boundary-layer transition.* This geometry has the advantage that no shock is generated along the surface at transonic speeds, and thereby avoids shock/boundary-layer interactions such as occur on airfoil and wings. (Paragraph continues on the next page)

*This, of course, is not a new measurement technique. In fact, the first Wright Brother's Lecture by Jones[5] in 1937 describes the utility of this technique for flight tests.

The design eventually evolved to what is now called the AEDC Transition Cone. A schematic of this cone and some of the associated instrumentation are shown in Fig. 1. Since the cone was designed to calibrate the effects of tunnel noise on boundary-layer transition, it also has two miniature microphones imbedded in the surface at 45.7 cm (18 in.) and 66 cm (26 in.) aft of the nose for noise measurements. Additional description of this cone can be found in the papers by Dougherty and Steinle[1] and Dougherty and Fisher.[6]

The need for such a calibration device was indicated by discrepancies between numerous transonic wind-tunnel tests of models at ostensibly identical flow conditions. A particularly well-documented study of differences in static aerodynamic data has been obtained with the same model of a Lockheed C-5A transport aircraft in three major transonic wind tunnels; the results have been reported by Treon, et al.[7] The differences between the three different sets of tunnel data were reduced by accounting for "relative" Reynolds number effects between facilities. The AEDC Transition Cone was used to define the differences in "relative" Reynolds number.

As observed by Dougherty and Steinle[1]: "These results substantiated the need for developing a method for predicting these corrections to Reynolds number to improve extrapolation of wind-tunnel test results to full-scale conditions, i.e., a "turbulence factor" for transonic tunnels." This illustration of improvement in agreement of test results between transonic facilities demonstrated that the use of transition Reynolds number on a standard model could be of practical use.

With the establishment of the fact that freestream disturbances can significantly affect transonic wind-tunnel data, an extensive test program was begun during 1971 in which the AEDC Cone was tested in twenty-three tunnels and finally was flight-tested on the nose of a McDonnell-Douglas F-15 aircraft. A summary of the resulting noise and transition data has been reported by Dougherty and Fisher.[6] In this concluding report, Dougherty and Fisher found for the range of

$(C_p)_{rms}$ observed that the data for transition Reynolds number, based on the product of local unit Reynolds number and distance from nose to end-of-transition (X_T), appear to correlate with $(C_p)_{rms}^{-.25}$ within an error band of $\pm$ 20%.

Clearly, correlations of this type represent a significant step forward in the development of a procedure to calibrate flow quality in transonic wind tunnel. However, this correlation exhibits a singularity when $(C_p)_{rms} = 0$. This relation, with the value of the proportionality constant suggested by Whitfield and Dougherty[8], is compared in Fig. 2 with some transition data obtained with the AEDC Cone in seven different tunnels (Dougherty and Steinle[1] and Mabey[9]) and a flight test at $M_\infty = 0.80$. Two straight lines are also shown in Fig. 2 which suggest the relation between Re_T and noise is dependent on spectral distribution of intensity (i.e., edgetones versus organ-pipe type noise) rather than simply total intensity.

Although use of the cone to define a "transonic turbulence factor" was successful in the C-5A studies in terms of correlating data taken in different tunnels, additional research is needed to establish the limitations and relevance of this technique to basic fluid-mechanic data such as skin friction. The objective of this work is to infer skin friction along the AEDC Cone, on the same relative basis, in both wind tunnel and flight tests and to compare the results with measured noise levels and transition Reynolds numbers. The results are expected to provide new information on how to define an "effective" freestream unit Reynolds number for transonic wind tunnels.

The basic approach, which was selected to achieve this objective, is to interpret the surface Pitot-probe data as Preston-tube data, i.e., total pressures near the wall which can be related to skin friction. In the process of conducting this investigation, a new procedure was developed to correlate the Preston-tube data, within laminar boundary layers on the cone, with the corresponding theoretical values of skin friction.

II. PRESTON-TUBE/SKIN-FRICTION CORRELATIONS

According to Preston[10], the British engineers Stephens and Haslem[11] suggested in 1938 that it should be possible to use the data from a Pitot tube traversed along a surface to infer skin friction. Apparently, this idea was not pursued until Preston's work during the early 1950's. He developed a correlation between skin friction and the total pressure as measured with circular Pitot tubes resting on the inside wall of a pipe. In order to develop this correlation, Preston assumed the classical law-of-the-wall is valid across the face of the probe and chose the characteristic length to be the height of the geometric center of the probe above the wall, i.e., d/2. This leads to the following relation between Preston-tube pressure and skin friction.

$$\frac{\tau_w d^2}{4\rho\nu^2} = F\left\{\frac{\Delta P_p d^2}{4\rho\nu^2}\right\} \qquad (1)$$

Using Eq. (1) as a guide, Preston obtained measurements inside a pipe flow with circular Pitot tubes having four different external diameters but a nearly constant ratio of internal to external diameter of 0.6. Pipe Reynolds number was varied over the range $10^4 < Re_D < 10^5$. Skin friction was determined via measurements of pressure drop over a known length of constant diameter pipe, viz., $\tau_w = (P_1 - P_2)\, D/4L$. An empirical fit of the data led to the following correlation.

$$y^* = -1.396 + \frac{7}{8}\, x^* \qquad \text{[Preston, 1954]} \tag{2}$$

Where $y^* \equiv \log_{10} (\tau_w d^2/4\rho\nu^2)$ and $x^* \equiv \log_{10} (\Delta P_p d^2/4\rho\nu^2)$.

In 1964, Patel [12] published the results of an extensive set of tests with fourteen different circular Pitot-probes and three different pipe diameters. He obtained a more accurate calibration for Preston tubes and established limits on the pressure-gradient conditions within which his calibration can be used with prescribed accuracy. Patel obtained empirical equations for $y^* = f(x^*)$ over three regions of y^*: (1) $3.5 < y^* < 5.3$, (2) $1.5 < y^* < 3.5$, and (3) $y^* < 1.5$. These three regions correspond, respectively, to the fully-turbulent, the buffer or transition zone, and the viscous-sublayer regions of the classical law-of-the-wall. The normal Reynolds number range of Preston-tube measurements in incompressible flow corresponds to the buffer zone, and for this region Patel obtained

$$y^* = 0.8287 - 0.1381\, x^* + 0.1437\, (x^*)^2 - 0.0060\, (x^*)^3, \tag{3}$$

where $1.5 < y^* < 3.5$ or $5.6 < U_\tau d/2\nu < 55$. Patel reported this correlates his data to within $\pm$ 1.5% of τ_w.

In the viscous-sublayer region, Patel found his data was correlated by

$$y^* = 0.5x^* + 0.037, \tag{4}$$

when $y^* < 1.5$ or $U_\tau d/2\nu < 5.6$. In this near-wall region, the classical law-of-the-wall exhibits the linear relation

$$u^+ \equiv u/U_\tau = U_\tau\, y/\nu \equiv y^+. \tag{5}$$

In order to relate Eqs. (4) and (5), Patel introduced K_{eff} and defined the "effective" center of a round Pitot tube to be at

$$Y_{eff} \equiv K_{eff}\, d/2\ . \tag{6}$$

By definition of the effective center, the velocity inferred from a Preston tube measurement, U_p, is the true velocity in the undisturbed boundary layer at Y_{eff}.

$$\therefore\ \Delta P_p = \frac{1}{2}\, \rho U_p{}^2 = \frac{1}{2}\, \rho(u^2)_{y = Y_{eff}} \tag{7}$$

If this is substituted in to Eq. (5) and the definitions of x^* and y^* are employed, the results are

$$y^* = 0.5x^* - 0.5 \log_{10} (0.5 K_{eff}^2) . \qquad (8)$$

Now equating Eqs. (4) and (8) and solving for K_{eff}, a value of 1.3 is obtained.

The traversing Pitot probes, used during wind-tunnel tests with the AEDC Transition Cone, are of the flattened or oval-shaped type. Since Patel's results are for circular Preston tubes, they cannot be applied directly to the AEDC Cone tests. In addition, these tests were conducted at transonic speeds, and compressibility effects are expected. With regard to flattened Preston tubes, Quarmby and Das[13] conducted an experimental study and calibration of six oval-shaped Preston tubes. When $x^* > 4.6$, they found these probes gave exactly the same calibration relation between y^* and x^* as was obtained by Patel (Eq. 3) when the external height of the probe face is used in place of d. At lower values of x^*, the negative displacement of the effective center caused by wall proximity was larger (≈ 5%) for the flattened probes with aspect ratios between 1.5 and 1.9.† The following calibration equation correlated the measurements of Quarmby and Das to within 1.5% of τ_w.

$$y^* = 0.5152 + 0.1693\, x^* + 0.0651\, (x^*)^2$$
$$\text{for } 3.38 < x^* < 6 \qquad (9)$$

Since these results for oval-shaped Preston tubes agree so closely with Patel's results, and Patel's value for $K_{eff} = 1.3$ appeared to be appropriate in the viscous-sublayer of a turbulent wall-flow, it was initially decided to use this same value in an attempt to correlate the traversing Pitot-probe data obtained within the laminar boundary layer on the AEDC Cone. This appeared to be

†This, is consistent with the idea that flow about the face becomes more two-dimensional as aspect ratio increases, and more of the flow passes up and over the face rather than around the sides.

reasonable in light of the fact that the x*'s for the cone data were greater than 5.3. Although this is equivalent to assuming K_{eff} is independent of Mach number, Reynolds number, velocity gradient across the face, and aspect ratio, this assumption was attractive because it greatly simplified the analytical work.

III. DEVELOPMENT OF A CORRELATION FOR TRANSONIC WIND TUNNEL DATA

Now turning our attention to compressibility and Mach number effects, Allen[14] has performed a comprehensive analysis of Preston tubes in supersonic boundary layers. He developed a correlation using three independent sets of simultaneous measurements of Preston-tube pressures and skin friction via a floating-element force balance. These data were obtained within flat-plate, turbulent boundary layers and with freestream Mach numbers in the range: $1.6 \leq M_\infty \leq 4.6$. Allen selected the same basic dimensionless parameters as Patel; except, he chose to evaluate the fluid properties ρ and ν at a reference temperature developed by Sommer and Short[15], and the velocity U_J was calculated from P_p and the wall pressure P_w (= P_e) using standard compressible flow relations.†

$$X_A^* \equiv \log_{10}\left[\frac{\rho'}{\rho_e}\frac{\mu_e}{\mu'}R_d\frac{U_p}{U_e}\right] = \log_{10}(U_p\, d/\nu') \qquad (10a)$$

$$Y_A^* \equiv \log_{10}\left[\frac{\mu_e}{\mu'}R_d(\rho' C_f/\rho_e)^{\frac{1}{2}}\right] = \log_{10}\left[(2\tau_w/\rho')^{\frac{1}{2}}d/\nu'\right] \qquad (10b)$$

The primes denote properties evaluated at the Sommer and Short reference temperature, viz.,

$$T'/T_e = 0.55 + 0.035\, M_e^2 + 0.45\, T_w/T_e \quad . \qquad (11)$$

The correlation derived by Allen is

$$Y_A^* = -0.4723 + 0.7814\, X_A^* + 0.01239(X_A^*)^2 \quad . \qquad (12)$$

†The details can be found in the report by Allen[16].

Allen found that the majority of the skin-friction-coefficient data were within +15% to -12% of Eq. (12). This rather large scatter, compared to the incompressible pipe-flow calibrations of Patel and Quarmby and Das, is at least partly associated with the much greater sensitivity and vulnerability of floating-element balances to extranneous errors.++

Obviously, the parameters used by Allen are logical candidates in any attempt to correlate the transonic cone data. However, the basic purpose of a reference temperature is to permit use of skin-friction formulas for incompressible flow to estimate compressible skin friction by evaluating fluid properties at the reference temperature. Thus, the resulting reference properties represent an "average" value across a boundary layer. Whereas, small Preston tubes encounter only the flow near the wall. Therefore, it appeared to us that properties based simply on the wall temperature would be more apropos. The utility of evaluating properties at both of these temperatures was investigated, and the results are reported following a summary of the wind tunnel data.

A. Transonic Wind Tunnel Data

Although the AEDC Cone has been tested in twenty-three different tunnels, only the analyses of subsonic data from the NASA Ames 11-Ft Transonic Wind Tunnel (TWT) is reported herein. Table 1 lists nineteen subsonic flow conditions at which the cone was tested. Pitot-probe surveys were taken along the surface of the cone between axial stations 10 cm (4 in.) and 89 cm (35 in.) downstream of the nose tip. The face of the oval-shaped Pitot-probe, used in these tests, had an external height of 0.025 cm (0.0097 in.), a width of 0.046 cm (0.0180 in.) and an aspect ratio of 1.86. The pattern of typical pressure surveys at high and low Reynolds numbers are shown, respectively, in Figs. 3 and 4.

++Allen[17] has discussed the various error sources in floating-element force balances, and he has recently suggested an improved design for this type of instrument, Allen.[18]

Table 1. Wind Tunnel Cases Used To Develop Laminar Correlations

RUN NO.	M_∞	$Re_m \times 10^{-6}$	q_∞(kPa)	α^o	β^o
15.231	0.95	13.1	33.1	-0.05	0.02
19.289	0.8	13.1	29.5	-0.00	-0.02
21.318	0.7	13.1	26.2	-0.01	-0.03
23.346	0.6	13.1	22.8	-0.00	-0.03
25.376	0.5	13.1	19.3	-0.01	-0.03
29.440	0.3	13.1	11.0	-0.01	-0.03
40.547	0.6	16.4	28.1	0.02	0.02
41.548	0.7	16.4	32.6	0.02	0.02
42.549	0.8	16.4	36.4	0.01	0.02
43.550	0.9	16.4	40.3	0.01	0.02
44.551	0.95	16.4	41.8	0.01	0.02
56.631	0.9	9.8	23.6	0.06	0.01
57.632	0.8	9.8	21.7	0.07	0.01
58.633	0.7	9.8	19.5	0.07	0.01
59.634	0.6	9.8	17.1	0.08	0.01
60.635	0.5	9.8	14.5	0.07	0.01
61.636	0.4	9.8	11.8	0.07	0.01
70.726	0.7	13.1	25.8	0.04	0.02
72.748	0.8	13.1	29.0	0.03	0.02

B. Computation of Boundary Layer and Data Analysis

The distribution of static pressure along the surface of the sharp cone at subsonic speeds is assumed to be defined by the inviscid theory of Wu and Lock.[19] Predictions for pressure coefficient along the surface of a 10-degree cone are shown in Fig. 5 as a function of freestream Mach number. This information and the known tunnel freestream conditions are used to calculate flow conditions along the outer edge of the boundary layer. The conical laminar boundary layer is then calculated using a computer program developed at Stanford University by Crawford and Kays[20] which they have labelled STAN-5. The resulting distributions of laminar skin friction and boundary-layer properties are then matched with the corresponding values of surface-Pitot measurements.

It was arbitrarily decided to only use Preston-tube data at 1.27 cm (0.5 in.) intervals beginning with the most forward station at which data were obtained. In each individual case, the final point was selected to be upstream of X_t, the station at which transition begins. This resulted in a total of 136 data points along the cone for the various M_∞ and Re_m listed in Table 1. The locations of the various data points are tabulated in Table 2. The following quadratic equation was used to correlate the Preston-tube measurements with the corresponding values of theoretical, laminar skin friction.

$$Y^* = A(X^*)^2 + BX^* + CT^* + E \tag{13}$$

where

$$Y^* \equiv \log_{10} (\tau_w Y_{eff}^2/\rho_w \nu_w{}^2) = \log_{10} (U_\tau Y_{eff}/\nu_w)^2 \tag{14a}$$

$$X^* \equiv \log_{10} (U_p Y_{eff}/\nu_w)^2 \tag{14b}$$

$$T^* \equiv \log_{10} (T'/T_e) \tag{14c}$$

$$Y_{eff} = K_{eff}\, h/2 = 0.65\, h \tag{14d}$$

The reference temperature was introduced to account for small departures of the fluid properties ρ and ν from adiabatic-wall values. The coefficients A, B, C, and E were determined by a least-squares fit of the data. This resulted in the following semi-empirical correlation.

Table 2. Wind Tunnel Data Used in Development Of Laminar Correlation

RUN NO.	X_t(cm)	X(cm)	P_p(kPa)	K_{eff}	$(C_p)_{rms}$
15.231		10.2	70.0	1.015	1.397
		11.4	68.8	1.032	
		12.7	68.0	1.053	
		14.0	67.1	1.069	
		15.2	66.4	1.087	
		16.5	65.9	1.108	
		17.8	65.5	1.129	
		19.1	65.1	1.147	
		20.3	64.7	1.160	
		21.6	64.2	1.170	
	24.1				
19.289		14.0	80.7	1.138	1.706
		15.2	80.1	1.162	
		16.5	79.7	1.184	
		17.8	79.2	1.208	
		19.1	78.8	1.228	
		20.3	78.5	1.247	
	22.9				
21.318		11.4	91.2	1.100	1.807
		12.7	90.2	1.105	
		14.0	89.3	1.111	
		15.2	88.8	1.135	
		16.5	88.3	1.153	
		17.8	87.9	1.171	
		19.1	87.6	1.191	
		20.3	87.2	1.208	
	21.6				
23.346		10.2	104.4	1.077	1.395
		11.4	103.6	1.101	
		12.7	102.7	1.105	
		14.0	102.0	1.118	
		15.2	101.6	1.163	
		16.5	101.2	1.159	
		17.8	101.0	1.185	
		19.1	100.7	1.208	
		20.3	100.4	1.223	
	21.6				
25.376		12.7	120.9	1.106	0.793
		14.0	120.6	1.139	
		15.2	120.2	1.158	
		16.5	119.8	1.177	
		17.8	119.6	1.197	
		19.1	119.3	1.217	
		20.3	119.1	1.236	
		21.6	118.9	1.257	
		22.9	118.8	1.279	
	24.1				

Table 2. (Cont'd)

29.440		16.5	179.4	1.054	0.400
		17.8	179.1	1.053	
		19.1	178.9	1.056	
		20.3	178.8	1.069	
		21.6	178.6	1.065	
	24.1				
40.547		11.4	126.8	0.965	1.583
		12.7	126.0	0.979	
		14.0	125.4	0.999	
		15.2	124.7	1.013	
		16.5	124.2	1.027	
		17.8	123.5	1.032	
		19.1	123.1	1.049	
	20.3				
41.548		11.4	112.4	0.955	1.970
		12.7	111.4	0.971	
		14.0	110.7	0.993	
		15.2	110.0	1.005	
		16.5	109.1	1.011	
	17.8				
42.549		11.4	101.3	0.973	1.793
		12.7	100.3	0.996	
		14.0	99.2	1.009	
		15.2	98.3	1.023	
		16.5	97.5	1.029	
	18.4				
43.550		12.7	90.2	0.911	1.512
		14.0	89.0	0.923	
		15.2	88.1	0.946	
		16.5	87.1	0.959	
	18.4				
44.551		12.7	85.1	0.868	1.391
		14.0	83.8	0.880	
		15.2	82.9	0.898	
		16.5	82.0	0.911	
	18.4				
56.631		21.6	48.0	1.149	1.488
		22.9	47.9	1.176	
		24.1	47.8	1.203	
		25.4	47.8	1.229	
		26.7	47.7	1.250	
		27.9	47.7	1.283	
		29.2	47.7	1.314	
	30.5				

Table 2. (Cont'd)

57.632		20.3	53.7	1.021	1.745
		21.6	53.5	1.035	
		22.9	53.5	1.057	
		24.1	53.3	1.069	
		25.4	53.2	1.081	
		26.7	53.0	1.088	
	29.2				
58.633		14.0	64.7	1.126	1.866
		15.2	64.4	1.152	
		16.5	64.2	1.183	
		17.8	64.0	1.207	
		19.1	63.8	1.230	
		20.3	63.6	1.245	
		21.6	63.5	1.269	
		22.9	63.3	1.293	
		24.1	63.1	1.303	
		25.4	63.0	1.319	
		26.7	62.8	1.333	
	27.9				
59.634		16.5	74.5	1.209	1.468
		17.8	74.4	1.238	
		19.1	74.2	1.258	
		20.3	74.1	1.280	
		21.6	73.9	1.360	
		22.9	73.8	1.324	
		24.1	73.6	1.342	
		25.4	[illegible]	1.358	
		26.7	73.4	1.381	
		27.9	73.2	1.382	
	29.2				
60.635		16.5	88.5	1.219	0.862
		17.8	88.3	1.237	
		19.1	88.2	1.272	
		20.3	88.2	1.308	
		21.6	88.0	1.324	
		22.9	88.0	1.356	
		[illegible]	87.8	1.371	
		25.4	87.7	1.385	
		26.7	87.6	1.392	
		27.9	87.5	1.416	
		29.2	87.4	1.422	
	29.8				

Table 2. (Cont'd)

61.636		16.5	110.1	1.226	0.534
		17.8	110.0	1.261	
		19.1	109.9	1.282	
		20.3	109.8	1.314	
		21.6	109.7	1.339	
		22.9	109.7	1.369	
		24.1	109.6	1.393	
		25.4	109.4	1.402	
		26.7	109.4	1.422	
		27.9	109.2	1.426	
	29.2				
70.726		16.5	85.6	1.082	1.918
		17.8	85.2	1.100	
		19.1	85.0	1.120	
		20.3	84.7	1.139	
		21.6	84.5	1.157	
	22.9				
72.748		17.8	75.3	1.067	1.788
		19.1	75.0	1.088	
		20.3	74.8	1.111	
		21.6	74.6	1.132	
	22.9				

$$Y^* = 0.273(X^*)^2 - 2.618X^* + 1.645T^* + 8.92. \quad (15)$$

A plot of this equation is presented in Fig. 6 along with the individual data points. The corresponding differences in skin friction coefficients are shown in Fig. 7. The rms error in $C_{f,c}$ is 5.85%. When the correlation parameters of Allen are used to fit the same data, the rms error in $C_{f,c}$ is 8.6%. Thus, the parameters defined in Equation (14) appear to be superior for correlating this particular data.

Although these results are good compared to the correlation of Allen[14], they are rather large compared to the very small scatter ($\approx$1%) of the correlations for incompressible flow of Patel (Eq. 3) and Quarmby and Das (Eq. 9). Although greater scatter may be expected for compressible flows, somewhat less scatter is expected for a correlation of the subsonic cone data because errors associated with floating-element balances are not present as they are in the data considered by Allen. Thus, the question arises: how can the data be better correlated? This led to a reexamination of the data and the development of an improved correlation when K_{eff} is treated as a variable.

IV. DEVELOPMENT OF AN IMPROVED CORRELATION FOR TRANSONIC FLOW

Reexamination of the papers by Patel[12], McMillan[21], and Quarmby and Das[13,22] led us to conclude that the effective center of a Pitot probe in an incompressible viscous flow is a function of the following variables.

$$K_{eff} = K_{eff}\,(U_\tau h/\nu,\ y_c/h,\ w/h) \quad (16)$$

In the case of a Preston tube, $y_c/h = 0.5$, and aspect ratio (w/h) is a constant for a given probe. When these restrictions apply, Eq. (16) reduces to $K_{eff}(U_\tau h/\nu)$. Since, in general, wall shear stress is a function of Reynolds number, pressure gradient, Mach number and heat transfer, we can expect K_{eff} for a given Preston tube to also be a function of these variables. If this conclusion is true, it is necessary to interpolate K_{eff} from the STAN-5 boundary-layer profiles. This has been done by finding the position within the theoretical laminar profiles at which the total pressure is equal to the measured Pitot pressures. Table 2 provides a summary of the results for each wind-tunnel flow condition. In addition to K_{eff} and P_p, Tables 2 also includes noise measurements of $(C_p)_{rms}$ which were obtained with a 0.635cm (0.25 in) microphone mounted flush with the surface of the cone at a distance of 45.7cm (18 in) aft of the nose and 135 degrees around from the Preston tube. As discussed in the introduction, Dougherty and Fisher[6] have correlated boundary-layer transition with this type of noise data.

The method used to define K_{eff} has the effect of adjusting the height above the wall at which P_p is measured. This procedure is expected to lead to an improved correlation between P_p and C_f because the Preston-tube pressure is forced to be consistent with the theoretical boundary-layer profile and skin friction. However, a high or low value of P_p and K_{eff} for a given value of C_f and h leads to a numerically different relationship between X* and Y*. Higher values of P_p produce a more nonlinear correlation. This naturally leads to the question of accuracy of the measured pressures, and how can erroneous data for a given wind-tunnel condition be identified? This can be qualitatively assessed by comparing the corresponding values of K_{eff} with the distribution of K_{eff} for the majority of the data.** For this purpose, K_{eff} has been plotted as a function of $U_\tau h/\nu_w$, M_∞ and Re_m and is shown in Fig. 8. It is relevant to here note that the pressure gradients are negligible over the range $0.09 < X/L < 0.26$ for which Preston-tube data are available, see Fig. 5 and Table 2. Thus, the systematic variations in K_{eff} are apparently caused by changes in flow about the face of the probe with changes in: (1) Reynolds number, (2) Mach number, (3) and

**Here we assume the bulk of the data provides a valid reference.

tunnel freestream disturbance levels. These variations in effective probe height must be properly accounted for if a single correlation equation is to be uniformly valid with respect to Mach number.

Only two subsonic wind-tunnel conditions were repeated, viz., $M_\infty = 0.7$ and 0.8 at a Reynolds number of 13×10^6. Comparisons of K_{eff} for each of these cases indicate a difference of 0.075 for $M_\infty = 0.7$ and 0.15 for $M_\infty = 0.80$. These differences translate, respectively, to differences in measured pressure of 1.1 kPa (0.16 psi) and 2.3 kPa (0.33 psi). Since the full-scale range of the pressure transducer used in the probe is 34.5 kPad (5 psid), the corresponding percent errors in pressure are 3.2% and 6.6%, respectively. These values are a measure of the repeatability and precision of the Preston-tube data.

Since the distribution of K_{eff} for a given M_∞ is expected to be continuous, the discontinuities between the data for unit Reynolds numbers of 9.8 and 13 are also a measure of precision. The 11-Ft TWT was shut down between the runs for different unit Reynolds number, and individual Mach number cases were run in the order listed in Table 1. However, there were two exceptions to this order. The tunnel was started for run numbers 44-47 and was shut down afterwards.* The second exception occurred for run numbers 70 ($M_\infty = 0.7$ and $Re_m = 13 \times 10^6$) and 72 ($M_\infty = 0.8$ and $Re_m = 13 \times 10^6$) which were performed at a higher unit Reynolds number immediately after the preceeding runs (56-61) for a lower Reynolds number. Thus, for these two runs, it is suspected that the pressure transducer was being influenced by unsteady temperatures and may not have achieved an equilibrium temperature. This phenomenon may have also contributed to errors in pressure measurement for other cases. For example, the K_{eff} for run number 57 ($M_\infty = 0.95$ and $Re_m = 16.4 \times 10^6$) appear to be low.

*Only data from run number 44 is being used in this work.

Analyses have shown that departures of K_{eff} from the pattern defined by the majority of the data lead to greater scatter in $\overline{C}_f$. Thus, the data for run number 57 was deleted and the remaining 132 values of K_{eff}, shown in Fig. 8, were used to define a correlation between Preston-tube pressures and theoretical, laminar skin friction. The equation obtained from a least-squares fit of a quadratic to the data is

$$Y^* = 0.0227(X^*)^2 + 0.2663X^* - 0.1558T^* + 0.6130, \quad \text{for } 5.4 < X^* < 6.3 \text{ and } M_\infty < 1.0 \quad . \tag{17}$$

A graph of this equation and the corresponding data are shown in Fig. 9. The associated scatter in $\overline{C}_f$ is shown in Fig. 10. The rms error in $C_{f,c}$ is now 1.04%. This amount of scatter is comparable to the pipe-flow calibrations of Patel[12] and Quarmby and Das[13,22]. However, it is here emphasized that the numerical values of K_{eff} and the coefficients in Eq. (17) are valid only for the Ames 11-Ft TWT and the particular probe used during these tests. The numbers are expected to be different for different wind-tunnel environments and for probes with significantly different aspect ratio and/or face geometry. In particular, the coefficients in Eq. (17) are believed to contain information on the freestream disturbance levels which are peculiar to the 11-Ft TWT.

Thus, Eq. (17) is not considered to be a universal correlation applicable to all wind tunnels, Preston tubes and models with arbitrary pressure gradients. Rather the described procedure for developing a correlation is applicable to the data obtained with the AEDC Cone in twenty two other wind tunnels (see Dougherty and Fisher[6]). The utility of a correlation like Eq. (17) is that it can be used with a similar correlation, based on flight data, to define an "effective" unit Reynolds number for a transonic tunnel. In analogy to the classical definition of an effective freestream unit Reynolds number based on equal values of the drag of a sphere, correlations based on wind tunnel and flight data can be used to attain the same objective by equating values of skin friction. This can be accomplished by inputting all the information to the flight-data correlation required to calculate a value for C_f which can be assumed to be valid for no freestream disturbances. Then when this value of C_f and all the other specified variables, except Re_m, are substituted into the wind-tunnel correlation, Eq. (17), an "effective" freestream unit Reynolds number can be calculated. The two correlations are expected to be different because it is known that freestream vorticity biases the pressure measured with a Pitot probe, e.g., Becker and Brown.[23] This procedure is currently being developed.

V. SUMMARY AND CONCLUDING REMARKS

Preston-tube data from transonic wind-tunnel tests of the AEDC Transition Cone have been correlated with theoretical, laminar skin friction. A single correlation equation with constant coefficients is rendered uniformly valid with respect to Mach number by introducing the concept of a variable "effective" probe height. A precise correlation for compressible boundary-layer flows (i.e., rms error in $C_{f,c}$ of the order of 1%) requires proper modeling of the variation of effective height of the probe with wall shear stress, Mach number and Reynolds number. The need for this information is a significant limitation on the usefulness of Preston tubes to measure skin friction in arbitrary compressible boundary layers. The distributions of effective height indicate the accuracy of Preston-tube calibrations is very sensitive to pressure measurement errors in compressible boundary layers. The described procedure for developing a correlation is applicable to data obtained with the AEDC Cone in other wind tunnels.

This procedure is also being applied to flight data obtained with the AEDC Transition Cone mounted on the nose of a McDonnell-Douglas F-15 plane. The resulting correlation will then be compared with the wind-tunnel correlation presented herein. The two correlations are expected to be different because it is known that vorticity alters the reading of a Pitot probe. Since these two correlations will be based on a flow model which ignores the effects of noise and freestream vorticity, any significant differences between the two may be attributable to freestream disturbance levels in the wind-tunnel. Such a comparison may lead to a new procedure for defining an "effective" unit Reynolds number for transonic wind tunnels.

Acknowledgements

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FIGURE CAPTION SHEET

NOTE: CS = Cone Station = Distance in inches aft of the nose

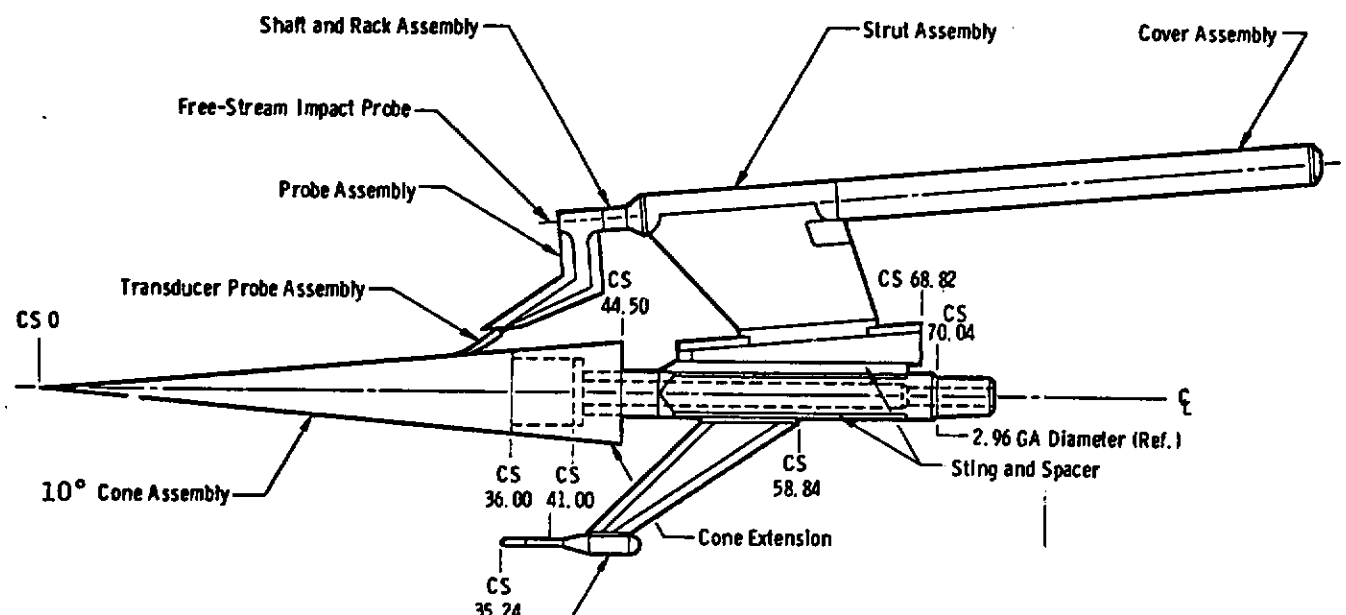

Figure 1. AEDC Boundary Layer Transition Cone

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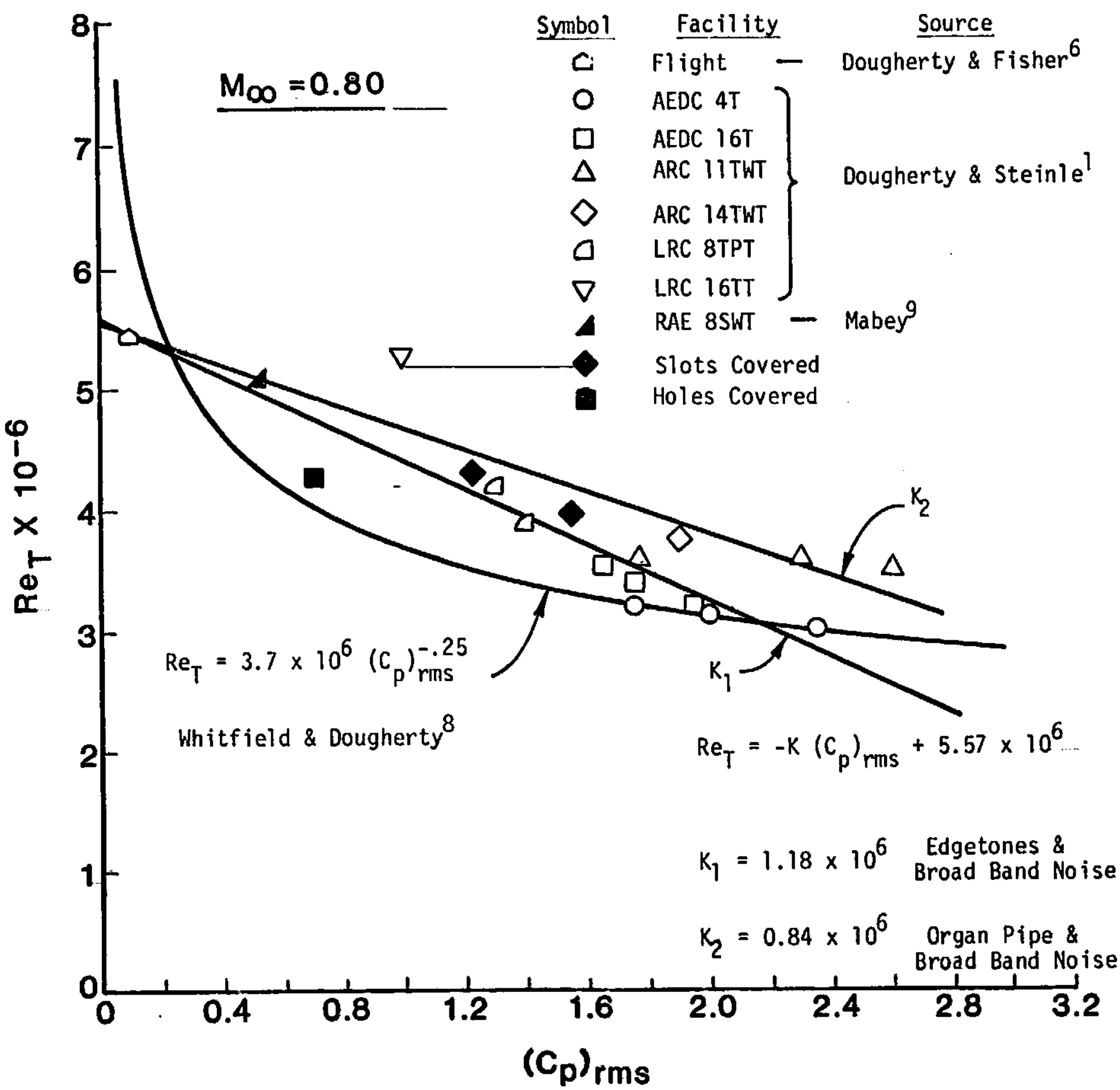


EFFECT OF NOISE ON BOUNDARY LAYER TRANSITION

Figure 2

Fig. 3 PATTERN OF TYPICAL PRESTON TUBE DATA FOR HIGH UNIT REYNOLDS NUMBER

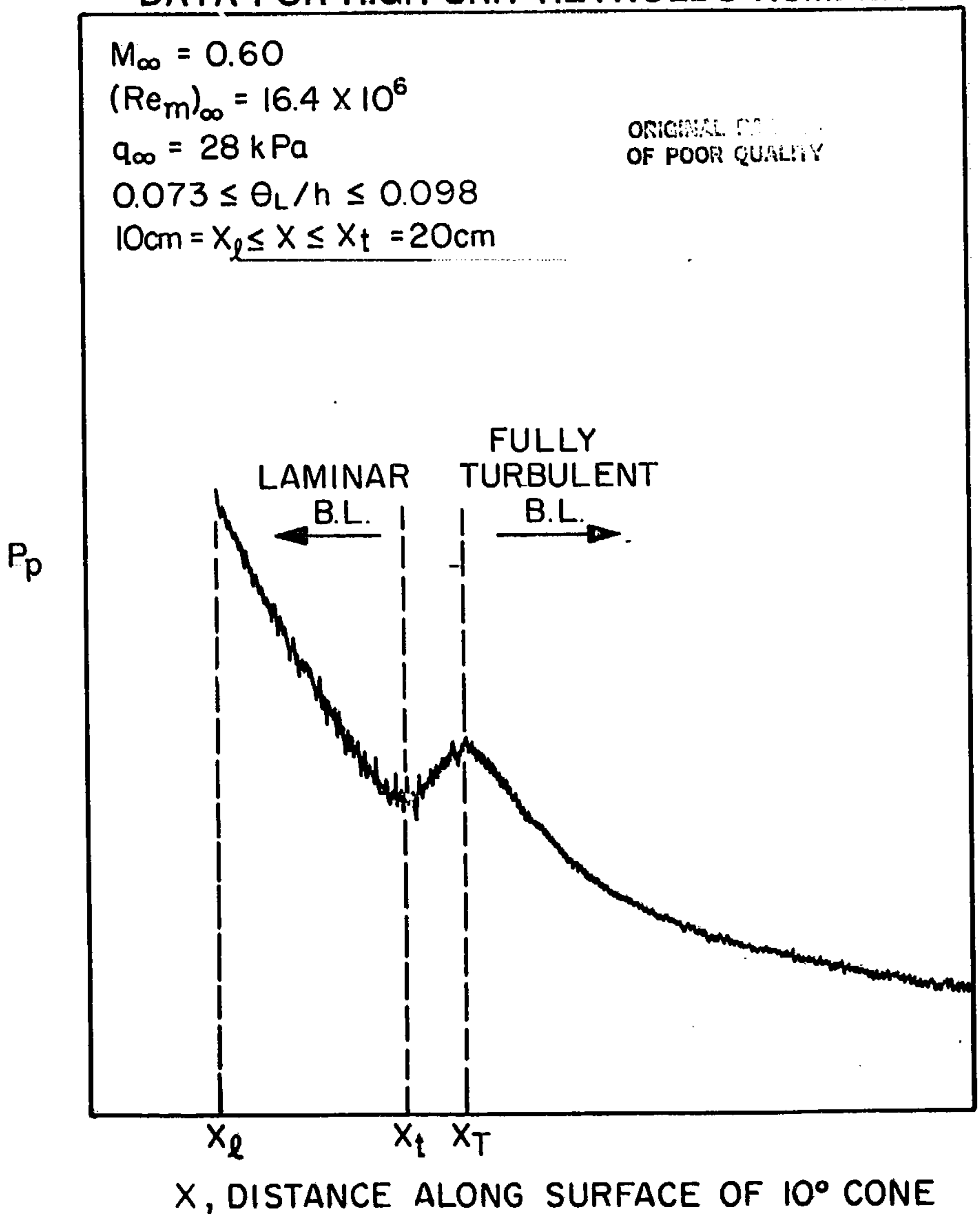
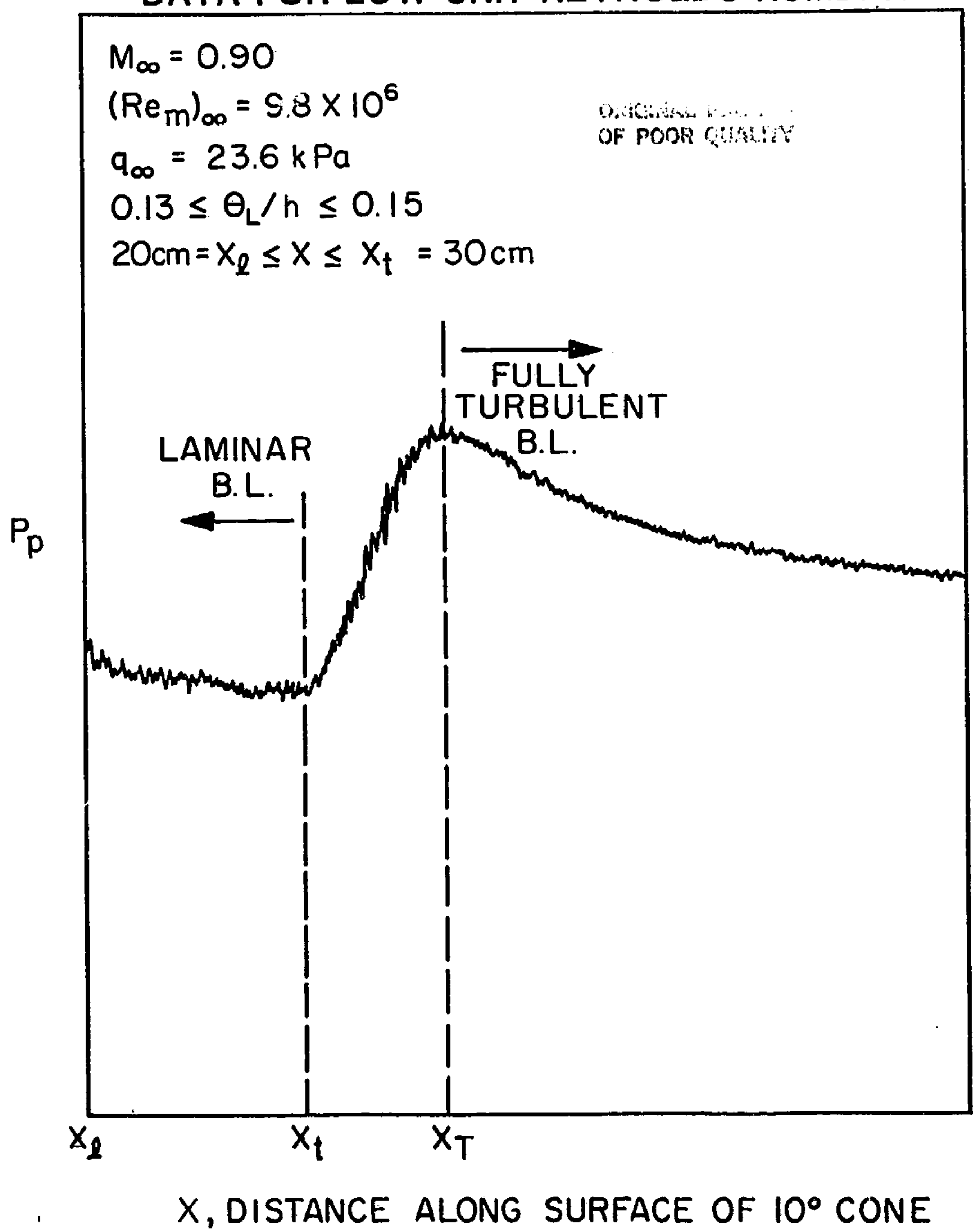


Fig. 4 PATTERN OF TYPICAL PRESTON TUBE DATA FOR LOW UNIT REYNOLDS NUMBER



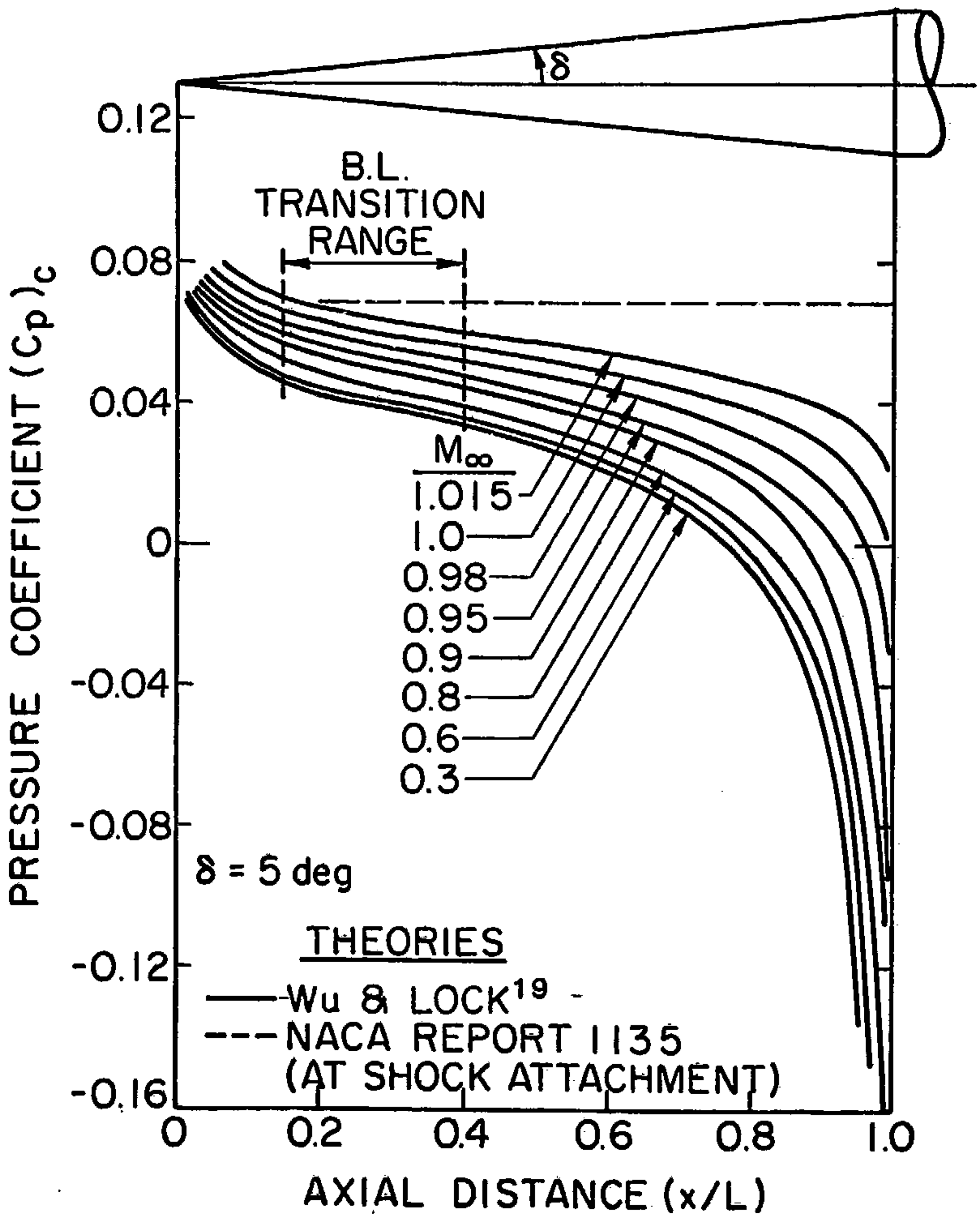


Fig. 5 INVISCID PRESSURE DISTRIBUTION ABOUT A 10° CONE AT TRANSONIC SPEEDS

Fig. 6 PRESTON-TUBE/LAMINAR-SKIN-FRICTION CORRELATION BASED ON A CONSTANT EFFECTIVE PROBE HEIGHT

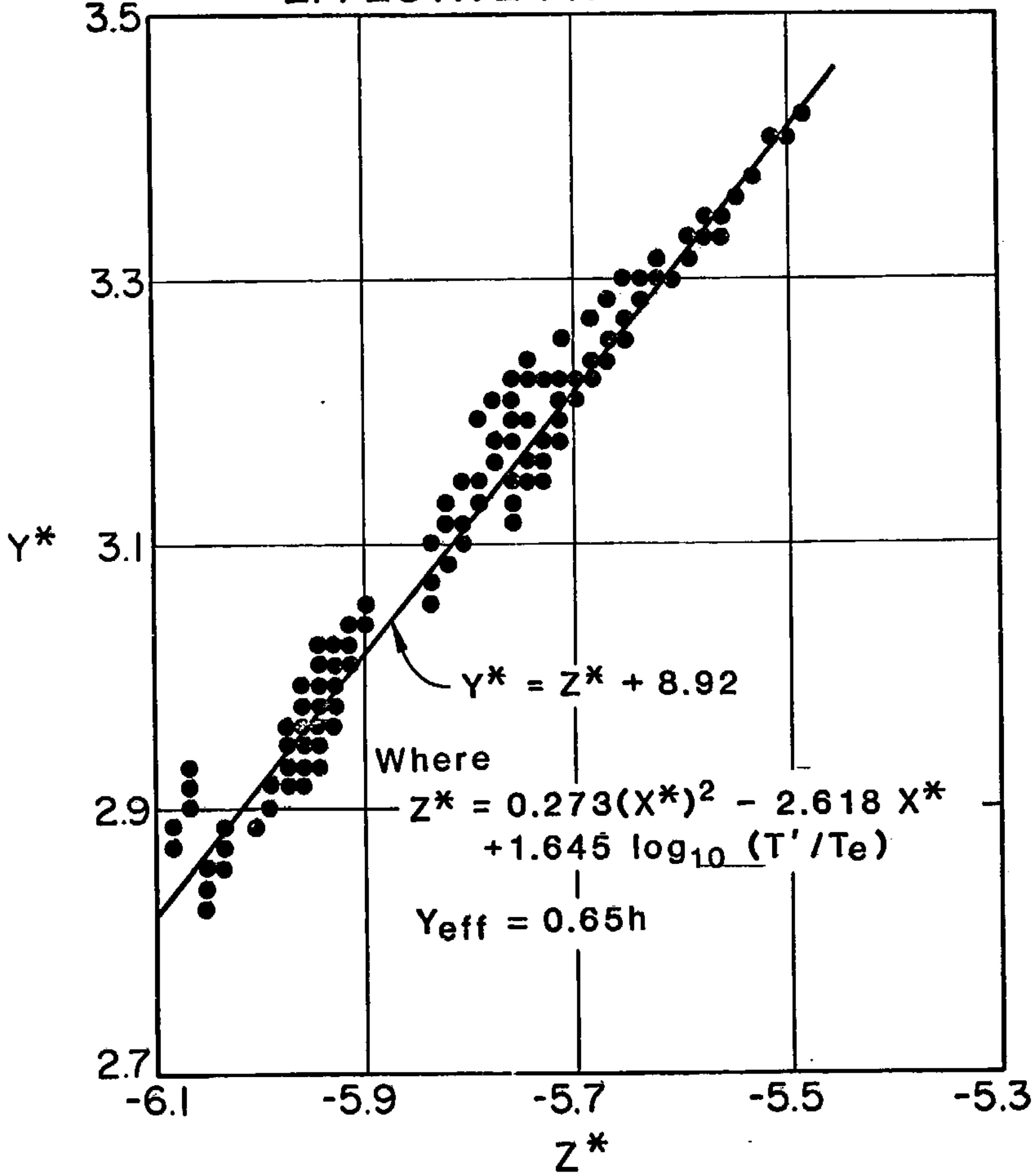
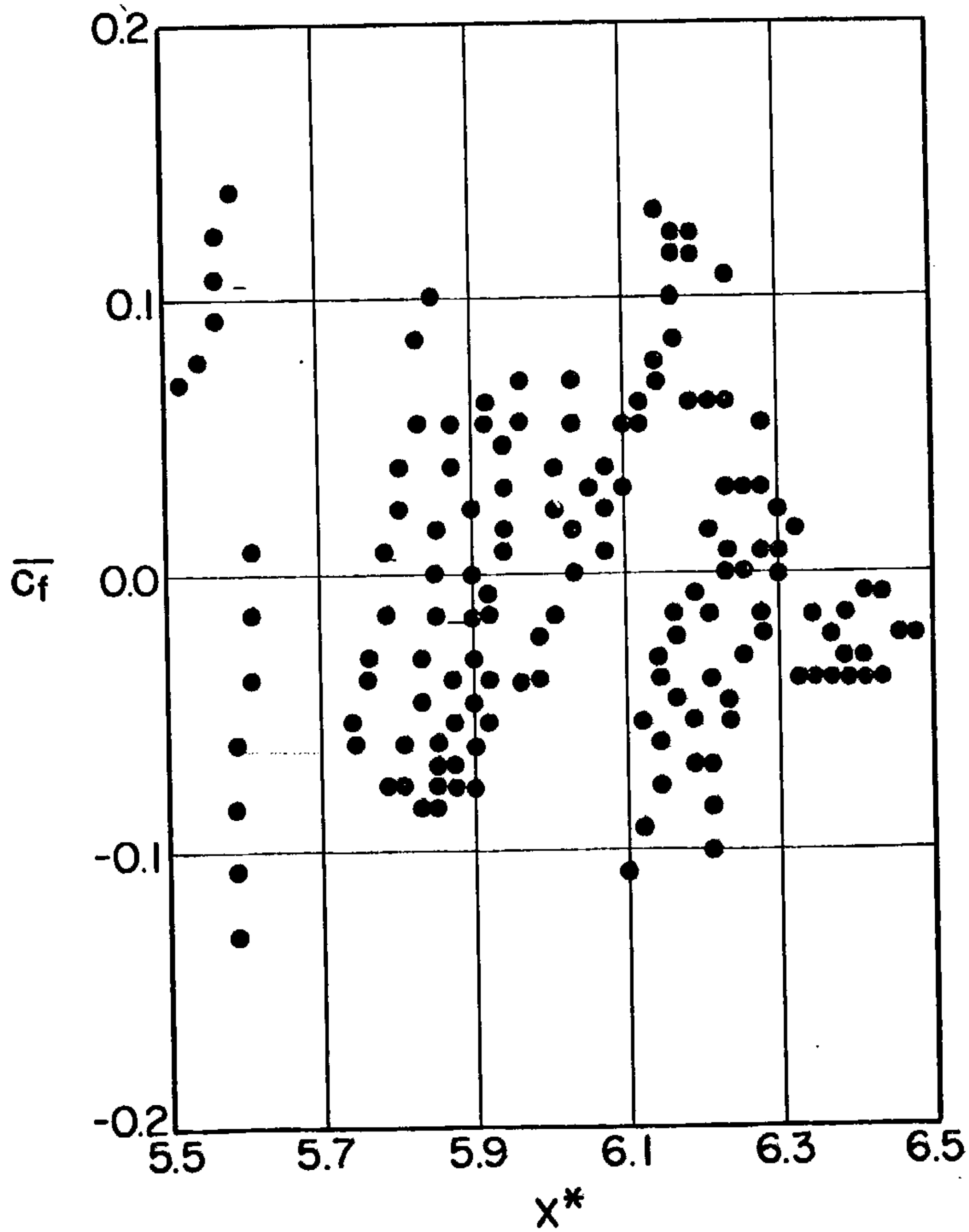


Fig. 7 SCATTER OF LAMINAR SKIN FRICTION COEFFICIENT ABOUT FIRST CORRELATION



RMS ERROR IN $C_{f,c}$ = 5.85%

Figure 8. VARIATION OF EFFECTIVE HEIGHT OF PROBE.

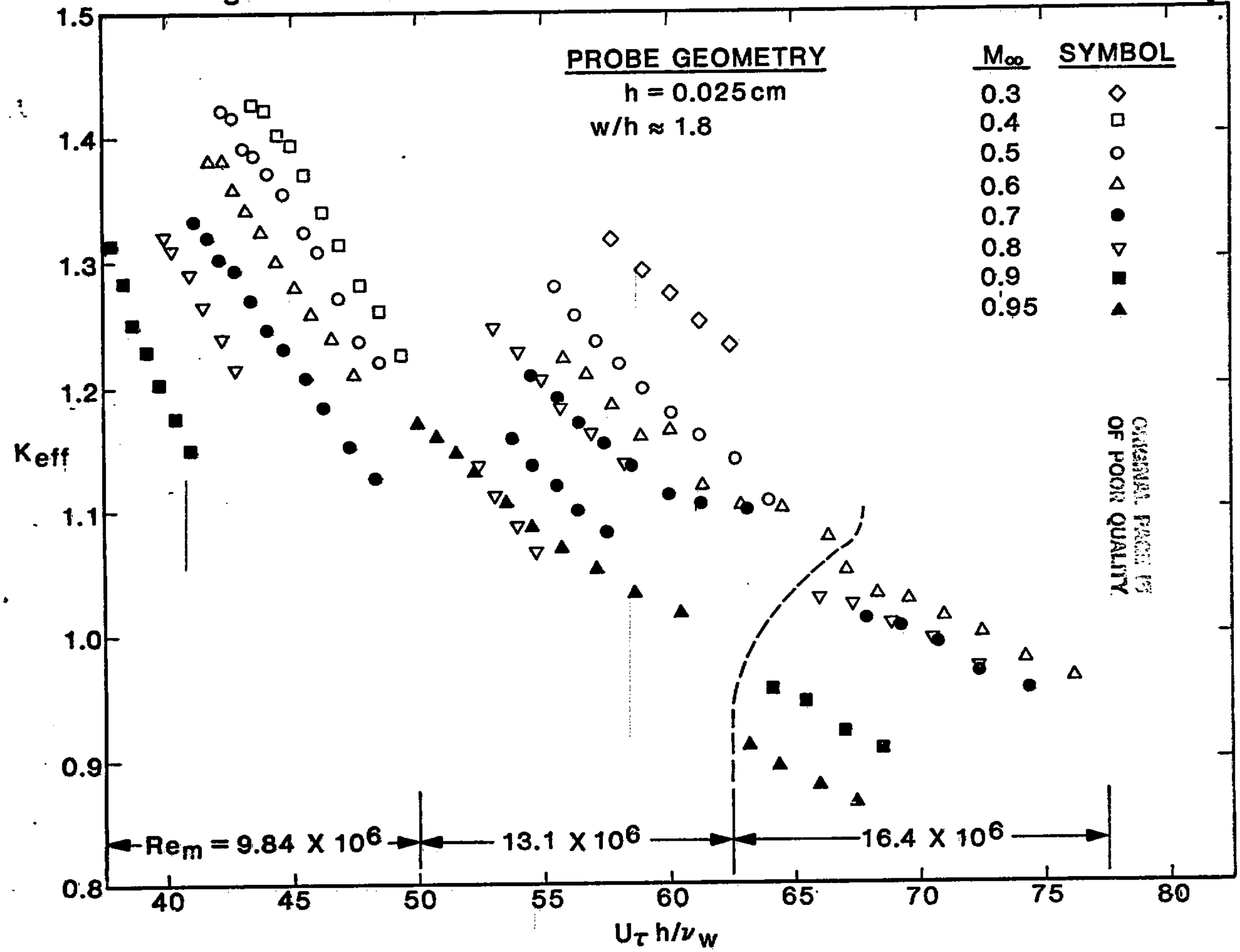


Fig. 9 PRESTON-TUBE/LAMINAR-SKIN-FRICTION CORRELATION BASED ON A VARIABLE EFFECTIVE PROBE HEIGHT

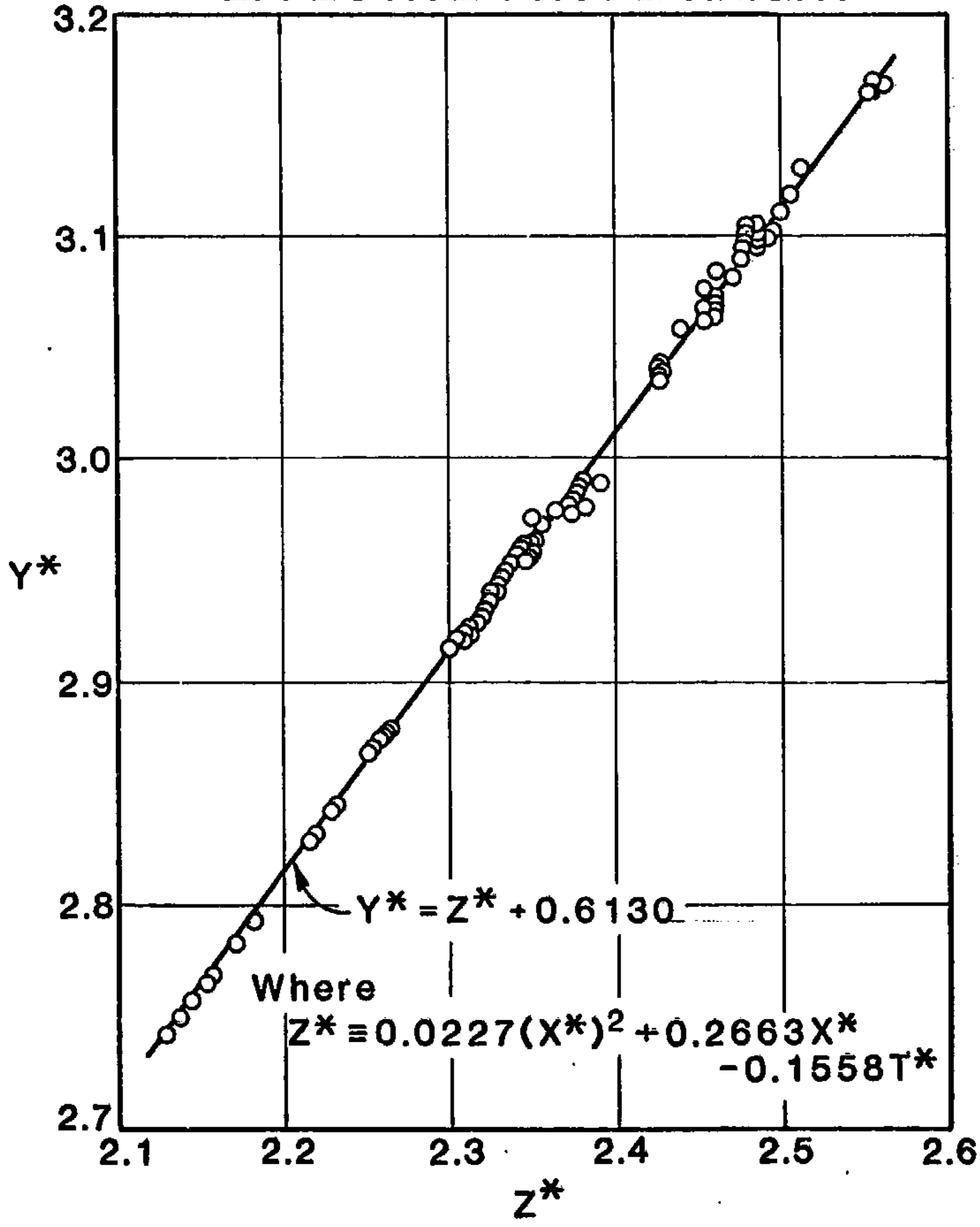
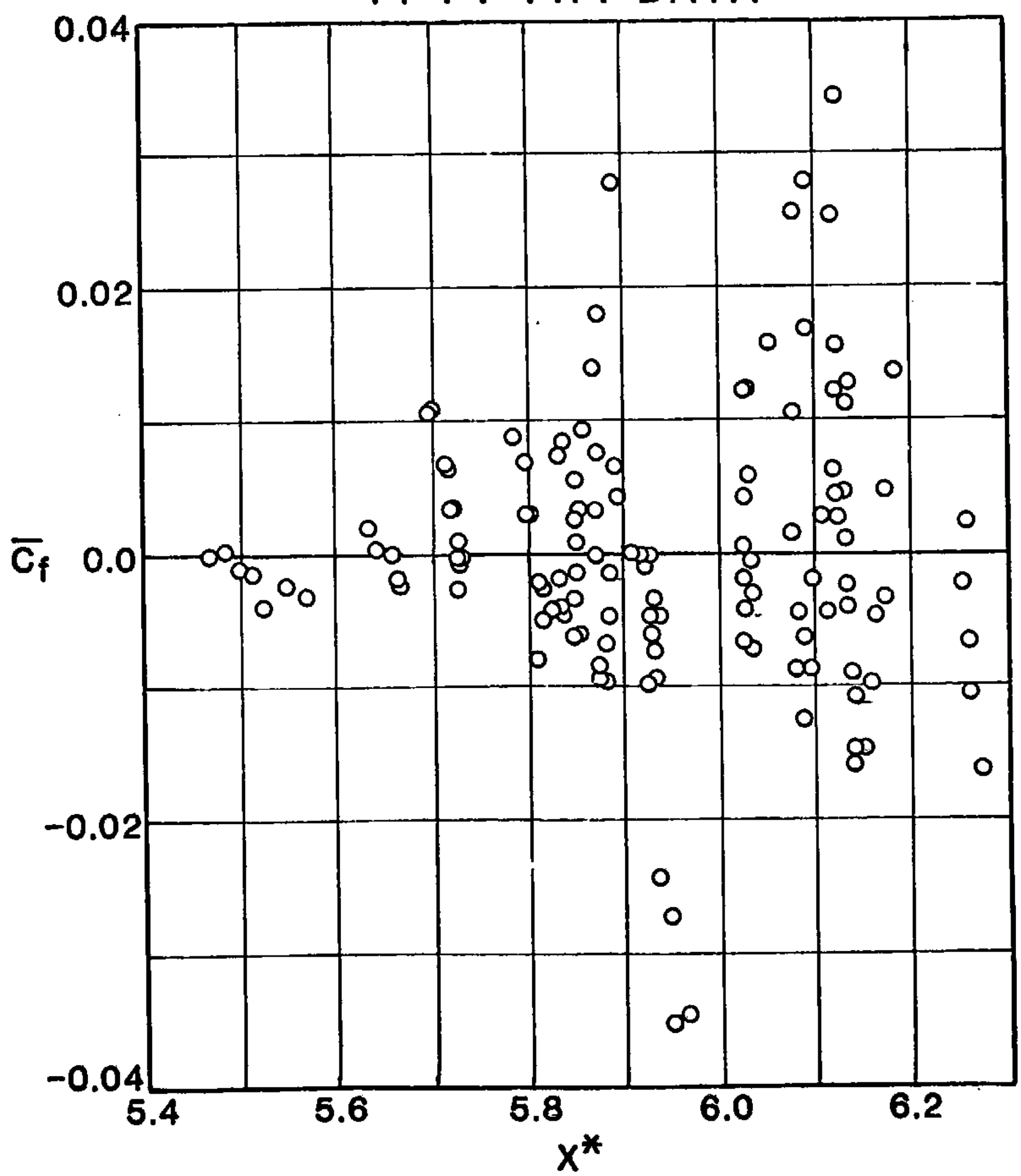


Fig. 10 SCATTER OF LAMINAR SKIN FRICTION ABOUT THE FINAL CORRELATION FOR 11-FT TWT DATA



RMS ERROR IN $C_{f,c}$ = 1.04%