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APPLICATIONS TO AERONAUTICS OF THE THEORY OF TRANSFORMATIONS OF NONLINEAR SYSTEMS

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ABSTRACT

We discuss the development of a theory, its application to the control design of nonlinear systems, and results concerning the use of this design technique for automatic flight control of aircraft. The theory examines the transformation of nonlinear systems to linear systems. We show how to apply this in practice, in particular, the tracking of linear models by nonlinear plants. Results of manned simulation are also presented.

INTRODUCTION

Suppose we model a physical plant by a nonlinear system

$$\dot{x}(t) = f[x(t)] + \sum_{i=1}^{m} u_i(t) g_i[x(t)] \quad , \tag{1}$$

where $f, g_1, \ldots, g_m$ are C^∞ vector fields on $\mathbb{R}^n$ and $f(0) = 0$. If we are to have the output of this plant follow a particular path, then we have a difficult problem to consider. However, if there are new state space coordinates and new controls under which equation (1) becomes a linear system, then our task appears to be much easier because of the known results for controller design on linear systems.

We feel that the following problems are thus of interest:

(a) Find necessary and sufficient conditions for the system (1) to be transformable to a controllable linear systems.

(b) Show how to use these transformations so that the controller design for nonlinear systems can be reduced to that of linear systems.

(c) Apply the above theory to the field of aeronautics.

In the next three sections of this paper we discuss the solutions of these problems.

TRANSFORMATION THEORY

The classification of those nonlinear systems that can be transformed to linear systems is actually a subproblem of a much deeper result, the construction of canonical forms for nonlinear systems. We are presently developing a theory for such canonical forms, and in the case that a nonlinear system is transformable as in this paper, the canonical form is actually the Brunovsky (ref. 1) form for a linear system.

Here we concentrate on the transformation theory developed in references 2, 3 and 4. Other significant research in this area is due to Krener (ref. 5), Brockett (ref. 6), Jakubcyzk and Respondek (ref. 7), and Hermann (The Theory of Equivalence of Pfaffian Systems and Input Systems under Feedback). We also refer to the early work of the first author in references 8 and 9.

If we are to map our nonlinear system (1) to a controllable linear system, we may as well assume that this linear system is in Brunovksy (ref. 1) canonical form with Kronecker indices $\kappa_1, \kappa_2, \ldots, \kappa_m$ satisfying $\sum_{i=1}^{m} \kappa_i = n$ and $\kappa_1 \geq \kappa_2 \geq \ldots \geq \kappa_m$. Hence this system is

$$\dot{y} = Ay + Bw \quad , \tag{2}$$

where A is $n \times n$, B is $n \times m$, $w = (w_1, w_2, \ldots, w_m)$ are the new controls, A is equal to

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$$
\begin{array}{c}
\\ \\ \kappa_1\Big\{ \\ \\ \\ \hline \\ \\ \kappa_2\Big\{ \\ \\ \\ \hline \\ \hline \\ \\ \kappa_m\Big\{ \\ \\ \\
\end{array}
\left[\begin{array}{c|c|c|c}
0\;1\;0\;\cdots\;0 & 0\;0\;\cdots\;0 & & 0\;0\;\cdots\;0\\
0\;0\;1\;0\;\cdots\;0 & 0\;0\;\cdots\;0 & & \vdots\\
\vdots\quad\ddots & \vdots & \cdots & \vdots\\
\qquad 1 & & & \\
0\;0\;\cdots\;0 & 0\;0\;\cdots\;0 & & 0\;0\;\cdots\;0\\
\hline
0\;0\;\cdots\;0 & 0\;1\;0\;\cdots\;0 & & 0\;0\;\cdots\;0\\
\vdots & 0\;0\;1\;0\;\cdots\;0 & & \vdots\\
\vdots & \vdots\quad\ddots & \cdots & \vdots\\
 & \qquad 1 & & \\
0\;0\;\cdots\;0 & 0\;0\;\cdots\;0 & & 0\;0\;\cdots\;0\\
\hline
\vdots & \vdots & \ddots & \vdots\\
\hline
0\;0\;\cdots\;0 & 0\;0\;\cdots\;0 & & 0\;1\;0\;\cdots\;0\\
\vdots & \vdots & & 0\;0\;1\;0\;\cdots\;0\\
\vdots & \vdots & \cdots & \ddots\\
 & & & \qquad 1\\
0\;0\;\cdots\;0 & 0\;0\;\cdots\;0 & & 0\;0\;\cdots\;0
\end{array}\right],
$$

and B is given as

$$
\begin{array}{c}
\\ \\ \kappa_1\Big\{ \\ \\ \\ \hline \\ \\ \kappa_2\Big\{ \\ \\ \\ \hline \\ \hline \\ \\ \kappa_m\Big\{ \\ \\ \\
\end{array}
\left[\begin{array}{c}
0\;0\;\cdots\;0\\
0\;0\;\cdots\;0\\
\vdots\\
\vdots\\
1\;0\;\cdots\;0\\
\hline
0\;0\;\cdots\;0\\
0\;0\;\cdots\;0\\
\vdots\\
\vdots\\
0\;1\;0\;\cdots\;0\\
\hline
\vdots\\
\hline
0\;0\;\cdots\;0\\
0\;0\qquad 0\\
\vdots\\
\vdots\\
0\;0\;\cdots\;1
\end{array}\right].
$$

The transformation results we present are actually local (in some open neighborhood of the origin in $(x_1, x_2, \ldots, x_n)$ space), and global theorems are found in reference 3. We simplify notation by saying $\mathbb{R}^n$ when we actually mean an open neighborhood of $(0, 0, \ldots, 0)$ in $\mathbb{R}^n$. However, $\mathbb{R}^m$ means $(u_1, u_2, \ldots, u_m)$ space (or $(w_1, w_2, \ldots, w_m)$ space) and this is not local.

We discuss the allowable transformations mapping system (1) to system (2). We want a C^∞ map $Y = (y_1, y_2, \ldots, y_n, w_1, w_2, \ldots, w_m)$ mapping $\mathbb{R}^n \times \mathbb{R}^m[(x_1, x_2, \ldots, x_n, u_1, u_2, \ldots, u_m)$ space] to $\mathbb{R}^n \times \mathbb{R}^m$ $[(y_1, y_2, \ldots, y_n, w_1, w_2, \ldots, w_m)$ space] that satisfies the following conditions:

1. Y maps the origin to the origin,
2. $y_1, y_2, \ldots, y_n$ are functions of $x_1, x_2, \ldots, x_n$ only and have a nonsingular Jacobian matrix,
3. $w_1, w_2, \ldots, w_m$ are functions of $x_1, x_2, \ldots, x_n, u_1, u_2, \ldots, u_m$ and for fixed $x_1, x_2, \ldots, x_n$, the $m \times m$ Jacobian matrix of $w_1, w_2, \ldots, w_m$ with respect to $u_1, u_2, \ldots, u_m$ is nonsingular,
4. Y maps system (1) to system (2),
5. Y is a one-to-one map of $\mathbb{R}^n \times \mathbb{R}^m$ onto $\mathbb{R}^n \times \mathbb{R}^m$.

Next we introduce some basic definitions from differential geometry.

If f and g are C^∞ vector fields on $\mathbb{R}^n$, the Lie bracket of f and g is

$$[f,g] = \frac{\partial g}{\partial x} f - \frac{\partial f}{\partial x} g$$

where $\frac{\partial g}{\partial x}$ and $\frac{\partial f}{\partial x}$ are Jacobian matrices. We let

$$(ad^0 f,g) = g$$
$$(ad^1 f,g) = [f,g]$$
$$(ad^2 f,g) = [f,(f,g)]$$
$$(ad^k f,g) = [f(ad^{k-1} f,g)] \quad .$$

A collection of C^∞ vector fields $h_1, h_2, \ldots, h_r$ is <u>involutive</u> if there exists C^∞ functions γ_{ijk} such that

$$[h_i,h_j](x) = \sum_{k=1}^{r} \gamma_{ijk}(x)h_k(x) \quad , \quad 1 \le i,\ j \le r,\ i \ne j \quad .$$

Let $\langle \cdot,\cdot \rangle$ denote the duality between one forms and vector fields. If $\omega = \omega_1\, dx_1 + \omega_2 dx_2 + \ldots + \omega_n dx_n$ is a differentiable one form and f a vector field on $\mathbb{R}^n$, then

$$\langle \omega, f \rangle = \omega_1 f_1 + \omega_2 f_2 + \ldots + \omega_n f_n$$

To state the main result from reference 4 giving necessary and sufficient conditions for transforming system (1) to system (2) we need the following sets:

$$C = \Big\{g_1, [f,g_1], \ldots, (ad^{\kappa_1 - 1} f, g_1), g_2, [f,g_2], \ldots, (ad^{\kappa_2 - 1} f, g_2), \ldots,$$
$$g_m, [f,g_m], \ldots, (ad^{\kappa_m - 1} f, g_m)\Big\}$$

$$C_j = \Big\{g_1, [f,g_1], \ldots, (ad^{\kappa_j - 2} f, g_1), g_2, [f,g_2], \ldots, (ad^{\kappa_j - 2} f, g_2), \ldots,$$
$$g_m, [f,g_m], \ldots, (ad^{\kappa_j - 2} f, g_m)\Big\} \quad \text{for } j=1,2,\ldots,m \quad .$$

<u>Theorem 2.1</u> There exists a transformation $Y = (y_1, y_2, \ldots, y_n, w_1, w_2, \ldots, w_m)$ satisfying conditions i) through v) above if and only if on $\mathbb{R}^n$

1) the set C spans an n dimensional space,
2) each set C_j is involutive for $j = 1,2,\ldots,m$, and,
3) the span of C_j equals the span of $C_j \cap C$ for $j = 1,2,\ldots,m$.

Let $\sigma_1 = \kappa_1$, $\sigma_2 = \kappa_1 + \kappa_2$, $\ldots$, $\sigma_m = \kappa_1 + \kappa_2 + \ldots + \kappa_m = n$. Then the transformation is constructed in reference 4 by solving the partial differential equations

$$\langle dy_1, (ad^j f, g_i) \rangle = 0, j=0,1,\ldots,\kappa_1 - 2 \text{ and } i=1,2,\ldots,m,$$
$$\langle dy_{\sigma_1+1}, (ad^j f, g_i) \rangle = 0, j=0,1,\ldots,\kappa_2 - 2 \text{ and } i=1,2,\ldots,m,$$

$$\vdots$$

$$\langle dy_{\sigma_{m-1}+1}, (ad^j f, g_i)\rangle = 0, j=0,1,\ldots,\kappa_m-2 \text{ and } i=1,2,\ldots,m, \tag{3}$$

$$\langle dy_{\sigma_1}, f\rangle + \sum_{i=1}^{m} u_i \langle dy_{\sigma_1}, g_i\rangle = w_1$$

$$\langle dy_{\sigma_2}, f\rangle + \sum_{i=1}^{m} u_i \langle dy_{\sigma_2}, g_i\rangle = w_2$$

$$\vdots$$

$$\langle dy_n, f\rangle + \sum_{i=1}^{m} u_i \langle dy_n, g_i\rangle = w_m \quad ,$$

where the matrix

$$\begin{bmatrix} \langle dy_1, (ad^{\kappa_1-1} f, g_1)\rangle & \cdots & \langle dy_1, (ad^{\kappa_1-1} f, g_m)\rangle \\ \langle dy_{\sigma_1+1}, (ad^{\kappa_2-1} f, g_1)\rangle & \cdots & \langle dy_{\sigma_1+1}, (ad^{\kappa_2-1} f, g_m)\rangle \\ \vdots & & \vdots \\ \langle dy_{\sigma_{m-1}+1}, (ad^{\kappa_m-1} f, g_1)\rangle & \cdots & \langle dy_{\sigma_{m-1}+1}, (ad^{\kappa_m-1} f, g_m)\rangle \end{bmatrix} \tag{4}$$

is nonsingular.

It can be shown that matrix (4) being invertible means we can solve for $u_1, u_2, \ldots, u_m$ in terms of $w_1, w_2, \ldots, w_m$ in the last m equations in equation (3).

Equation (3) can be formally solved by considering a sequence of ordinary differential equations as in reference 4, but we shall not mention details here.

If a nonlinear system is transformable to a linear system, we study the process of using the transformation to construct a controller for the nonlinear system.

TRANSFORMATIONS IN CONTROLLER DESIGN

Let $Y = (y_1, y_2, \ldots, y_n, w_1, w_2, \ldots, w_m)$ be the transformation from system (1) to system (2) as before. The structure of the control system using transformation theory is illustrated in figure 1. The design scheme is implemented on the "linear part" of the diagram, and this system is in Brunovsky form.

We ask that the output of the nonlinear system follow a particular path which corresponds to a trajectory for the output of the linear model. If we know how to design for the linear system, then we actually have a tracking of a linear model by a nonlinear plant.

Linear design is used to generate an open loop command w_c, for the system (2), and we find the corresponding y coordinates y_c by plugging w_c into equation (2). The transformation Y maps the measured x space to y space and y is compared to y_c and the difference is an error e_y. The regulator yields a control δw which sends e_y to zero, and variations in plant dynamics and disturbances are compensated for in this way.

The controls w_c and δw are added and transformed through the inverse map R (actually $w_c + \delta w$ is substituted into the last m equations in equation (3) and $u = (u_1, u_2, \ldots, u_m)$ is generated) to obtain a control which is applied to the plant. Thus we have an exact model follower, and the difficult problem of finding an open loop control and the regulator control are constrained to the linear system.

The remainder of this paper contains the application of the transformation theory to aeronautics.

AUTOMATIC FLIGHT CONTROLLER DESIGN

The aircraft will be represented by a rigid body moving in 3-dimensional space in response to gravity, aerodynamics and propulsion.

The state

$$x = \begin{pmatrix} r \\ v \\ C \\ \omega \end{pmatrix} \epsilon\, X \subset \mathbb{R}^3 \times \mathbb{R}^3 \times SO(3) \times \mathbb{R}^3 \tag{5}$$

where r and v are inertial coordinates of body center of mass position and velocity, respectively; C is the direction cosine matrix of body fixed axes relative to the runway fixed axes (inertial), and ω is the angular velocity.

The controls,

$$u = \begin{pmatrix} u^M \\ u^P \end{pmatrix} \epsilon\, U \subset \mathbb{R}^3 \times \mathbb{R} \tag{6}$$

where u^M is the 3-axis moment control such as ailerons, elevator and rudder in a conventional aircraft or roll cyclic, pitch cyclic and tail rotor collective in a helicopter; and u^P controls power - throttle in a conventional aircraft, and the main rotor collective in a helicopter. The state equation consists of the translational and rotational kinematic and dynamic equations:

$$\begin{aligned} \dot{r} &= v \\ \dot{v} &= f^F(x,u) \\ \dot{C} &= S(\omega)C \\ \dot{\omega} &= f^M(x,u) \end{aligned} \tag{7}$$

where f^F and f^M are the total force and moment generation processes and $x \epsilon X$. We wish to transform equation (7) into a linear system.

In general, f^M is invertible with respect to the (vector) pair $(\dot{\omega}, u^M)$, and, for the specific class of helicopter maneuvers being considered (i.e., no 360° rolls), f^F is invertible with respect to the (scalar) pair $(\dot{v}_3, u^P)$. Thus, a function $h: X \times R^4 \rightarrow U$ can be constructed such that if

$$\begin{pmatrix} u^M \\ u^P \end{pmatrix} = h(r,v,C,\omega,\dot{\omega}_0,\dot{v}_{30}) \tag{8}$$

then

$$\begin{aligned} \dot{\omega} &= \dot{\omega}_0 \\ \dot{v}_3 &= \dot{v}_{30} \end{aligned} \tag{9}$$

for all admissible maneuvers. That is, angular and vertical accelerations can be chosen as the new set of independent controls in which case the state equation may be written as follows

$$\begin{aligned} \dot{r} &= v \\ \dot{v} &= f^0(r,v,C,\dot{v}_{30}) + \epsilon f^1(r,v,C,\dot{v}_{30},\omega,\dot{\omega}_0) \\ \dot{C} &= s(\omega)C \\ \dot{\omega} &= \dot{\omega}_0 \end{aligned} \tag{10}$$

where $\epsilon = 1$, and $f^1(r,v,C,0,0,0) = 0$ for all admissible maneuvers.

The function f^0 is invertible with respect to $((\dot{v}_1,\dot{v}_2,E_3(\psi)),C)$ where $E_3(\psi)$ is an elementary rotation about the z-axis and represents the heading of the helicopter. Thus, a function $h^f: \mathbb{R}^8 \times SO(2) \rightarrow SO(3)$ can be constructed such that if

$$C_0 = h^f(r,v,\dot{v}_0,E_3(\psi_0)) \tag{11}$$

then

$$\dot{v} = \dot{v}_0 \tag{12}$$

Equations (8) and (11) are the trim equations of the process equation (10) (with $\epsilon = 0$). That is, for a given path $(r(t), E_3(\psi(t)))$, $t \geq 0$ with $\dot{v}_3(t) = 0$, the corresponding state and control may be constructed as follows

$$
\begin{aligned}
r_0 &= r(t) \\
v_0 &= \dot{r}(t) \\
C_0 &= h^f(r_0, v_0, \dot{v}(t), E_3, (\psi(t))) \\
\omega_0 &= q(\dot{C}C^t) \\
\dot{\omega}_0 &= (\omega_0)^{\boldsymbol{\cdot}} \\
u_0 &= h(r_0, v_0, C_0, \omega_0, \dot{\omega}_0, 0)
\end{aligned}
\tag{13}
$$

where the function q extracts ω from $\dot{C}C^t = S(\omega)$. The required time derivatives in equation (13) can be computed provided that the path (r, E_3) is generated by the system diagrammed in figure 2 where ● represents a scalar integrator and y_0^5 the control (w in the previous section is y^5 here).

We construct an approximation to the linearizing transformation as follows: Y_1, R_1, Q are constructed so that

$$
\begin{aligned}
y &= Y(x) \approx Y_0(x_0) + Y_1\delta x = y_0 + Y_1\delta x \\
u &= R(x, y^5) \approx u_0 + R_1\delta y^5 + Q\delta x
\end{aligned}
\tag{14}
$$

Here Y_0 is the transformation when $\varepsilon = 0$, and δx (and δy^5) is the perturbation about the nominal x_0 (and y_0^5) given in equation (13) (and figure 2).

From equation (10) with $\varepsilon = 0$, it follows that $(C = (I + S(\varepsilon)C_0)$

$$
\begin{aligned}
(\delta r)^{\boldsymbol{\cdot}} &= \delta v \\
(\delta v)^{\boldsymbol{\cdot}} &= \overset{0}{\frac{\partial f}{\partial r}}\,\delta r + \overset{0}{\frac{\partial f}{\partial v}}\,\delta v + \overset{0}{\frac{\partial f}{\partial C}}\,\varepsilon + \overset{0}{\frac{\partial f}{\partial \dot{v}_3}}\,\dot{v}_3 \\
(\varepsilon)^{\boldsymbol{\cdot}} &= \delta\omega \\
(\delta\omega)^{\boldsymbol{\cdot}} &= \delta\dot{\omega}_0 \quad ,
\end{aligned}
\tag{15}
$$

where ε is attitude perturbation.

The pattern of equation (15) after some rearrangement of coordinates is shown in equation (16).

$$
\begin{bmatrix} r_1 \\ r_2 \\ v_1 \\ v_2 \\ \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ r_3 \\ \omega_1 \\ \omega_2 \\ \omega_3 \\ v_3 \end{bmatrix}^{\boldsymbol{\cdot}}
=
\left[\begin{array}{c|c|cc|cc}
0 & I & \multicolumn{2}{c|}{0} & \multicolumn{2}{c}{0} \\ \hline
0 & C_1 & C_2 & C_3 & 0 & C_4 \\ \hline
0 & 0 & \multicolumn{2}{c|}{0} & \multicolumn{2}{c}{I} \\ \hline
0 & 0 & \multicolumn{2}{c|}{0} & \multicolumn{2}{c}{0}
\end{array}\right]
\begin{bmatrix} r_1 \\ r_2 \\ v_1 \\ v_2 \\ \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ r_1 \\ \omega_1 \\ \omega_2 \\ \omega_3 \\ v_3 \end{bmatrix}
+
\left[\begin{array}{cc}
\multicolumn{2}{c}{0} \\ \hline
0 & C_5 \\ \hline
\multicolumn{2}{c}{0} \\ \hline
\multicolumn{2}{c}{I}
\end{array}\right]
\begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \\ \dot{v}_3 \end{bmatrix}
\tag{16}
$$

In the present case of the helicopter, C_1, C_4, and C_5 are negligible. Their effect will be controlled by the regulator.

The transformations

$$\begin{bmatrix} \delta y_1^1 \\ \delta y_2^1 \\ \delta y_1^2 \\ \delta y_2^2 \\ \delta y_1^3 \\ \delta y_2^3 \\ \delta y_3^3 \\ \delta y_4^3 \\ \delta y_1^4 \\ \delta y_2^4 \\ \delta y_3^4 \\ \delta y_4^4 \end{bmatrix} = \left[\begin{array}{c|c|c|c} I & 0 & 0 & 0 \\ \hline 0 & I & 0 & 0 \\ \hline 0 & \begin{array}{cc} C_2 & C_3 \\ 0 & I \end{array} & 0 \\ \hline 0 & 0 & \begin{array}{cc} C_2 & C_3 \\ 0 & I \end{array} \end{array}\right] \begin{bmatrix} \delta r_1 \\ \delta r_2 \\ \delta r_1 \\ \delta r_2 \\ \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \delta r_3 \\ \delta\omega_1 \\ \delta\omega_2 \\ \delta\omega_3 \\ \delta r_3 \end{bmatrix} \tag{17}$$

$$\begin{bmatrix} \delta\dot{\omega}_1 \\ \delta\dot{\omega}_2 \\ \delta\dot{\omega}_3 \\ \dot{v}_3 \end{bmatrix} = \left[\begin{array}{c|c} C_2^{-1} & -C_2^{-1}C_3 \\ \hline 0 & I \end{array}\right] \begin{bmatrix} \delta y_1^5 \\ \delta y_2^5 \\ \delta y_3^5 \\ \delta y_4^5 \end{bmatrix} \tag{18}$$

take the system in equation (16)(with C_1, C_4, C_5 = 0) into the canonic system:

$$(\delta y)^{\cdot} = \left[\begin{array}{c|c|cc|c} 0 & I & 0 & & 0 \\ \hline 0 & 0 & I & 0 & 0 \\ \hline 0 & 0 & 0 & & I \\ \hline 0 & 0 & 0 & & 0 \end{array}\right] \delta y + \left[\begin{array}{c} 0 \\ \hline 0 \\ \hline 0 \\ \hline I \end{array}\right] \delta y^5 \tag{19}$$

Thus, the linearizing transformation (Y and R in figure 1) is constructed.

That the accuracy of the transformation is adequate may be seen from the results of the simulation of the flight experiment to be briefly summarized next.

The test consists in automatically flying a trajectory which exercises the system over a wide range of flight conditions as shown in figures 4 and 5.

Thus, the test takes the helicopter from hover (WP1) to high speed (150 ft/sec) turning acceleration, ascending flight.

Figure 6 shows the resulting tracking errors.

As can be seen, position tracking error e_r is quite small. The acceleration errors $e_{\dot{v}}$, which is due to the neglected terms in the construction of the linearizing transformation is also quite small. In summary, the resulting performance of the system is good.

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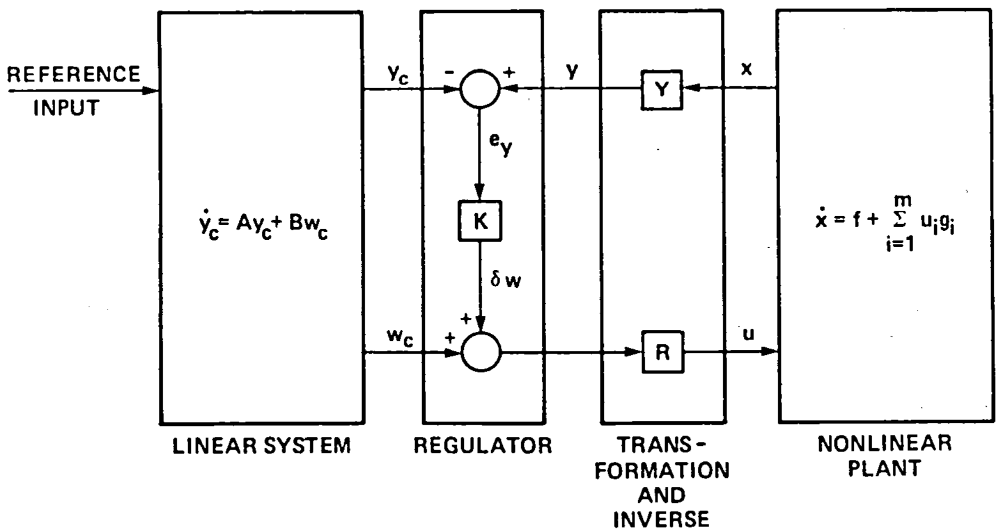

Figure 1. Structure of the Control System

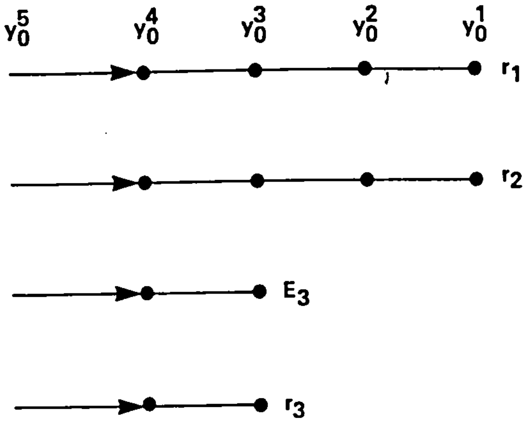

Figure 2. Reference Canonic System

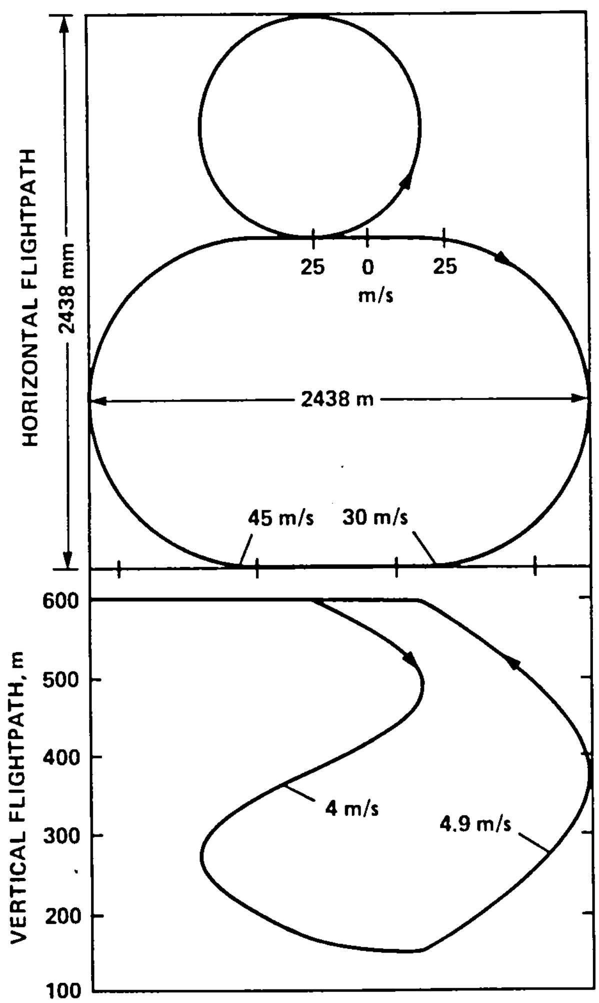

Figure 3. Experimental Flightpath Shown in Horizontal and Vertical Planes

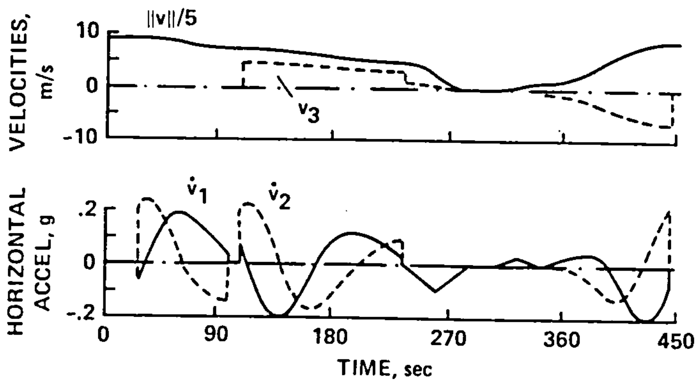

Figure 4. Speed and Acceleration of Experimental Trajectory

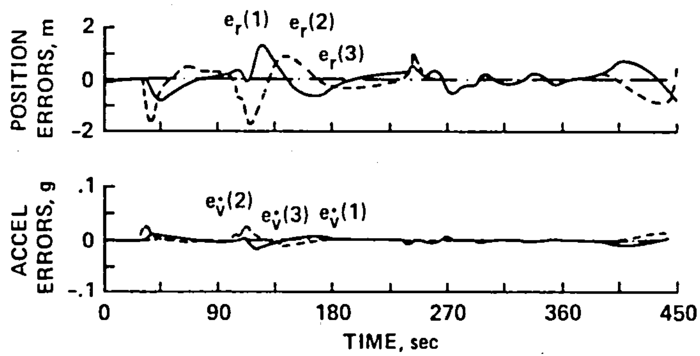

Figure 5. Tracking Errors

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16. Abstract

We discuss the development of a theory, its application to the control design of nonlinear systems, and results concerning the use of this design technique for automatic flight control of aircraft. The theory examines the transformation of nonlinear systems to linear systems. We show how to apply this in practice; in particular, the tracking of linear models by nonlinear plants. Results of manned simulation are also presented.

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