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NOSS Algorithm Specifications for Ocean Current Mapping

Volume I

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National Aeronautics and
Space Administration

Goddard Space Flight Center
Wallops Flight Center
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Volume I

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1. OVERVIEW

1.1 INTRODUCTION

This chapter gives an overview of the algorithms for detecting ocean currents and estimating geostrophic velocities with satellite altimeter data. Chapters 2 through 6 describe the theory of operation and the specifications for each algorithm.

1.2 ALGORITHM DESCRIPTIONS

The current-detection and velocity-estimation algorithms process single tracks of residual satellite altimeter data and yield the following outputs:

- Detected locations of specified ocean-current signatures along the satellite subtrack
- Estimated amplitudes of the detected signatures
- Estimated rms errors for the locations and amplitudes of detected signatures
- Estimated cross-track component of the boundary-current geostrophic velocity and an rms error bound for the estimate
- Expected number of false alarms.

The residual altimeter data are inputs to the algorithms and are computed from raw altimeter data in three steps by

- Applying corrections for known error sources

- Interpolating the data through intervals in which the data are in serious error (e.g., outliers)

- Subtracting an estimated gravimetric geoid profile along the satellite subtrack.

The resulting residual data are noisy measurements of the dynamic sea-surface height. The characteristics of the noise in these depend on the noise in the raw altimeter data and on the accuracies of the error corrections and the geoid profiles. The detection algorithm exploits both the statistical properties of the noise in the residual data and the known average properties of ocean-current signatures in the altimeter data. For specified models of the noise and oceanographic signature, the algorithm maximizes the probability of detection at a specified probability of false alarm and minimizes the rms errors in the estimated current signature parameters.

As depicted in Fig. 1.2-1, the detection algorithm consists of four subalgorithms that perform separate functions.

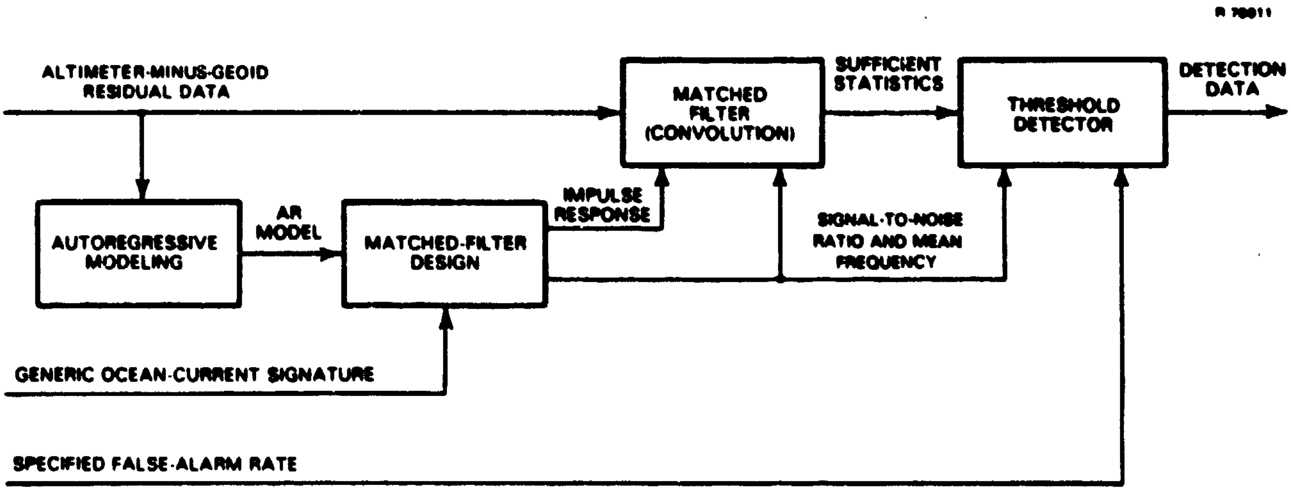

Figure 1.2-1 Structure of Data-Adaptive Current Detection Algorithm

- AUTOREGRESSIVE MODELING - The residual data are analyzed to determine a stochastic autoregressive (AR) model for the process that generated the data. The order of the AR model is selected to minimize the Akaike information criterion.

- MATCHED-FILTER DESIGN - The AR model, together with a user-specified generic ocean-current signature, are used to compute the impulse response of the optimal matched filter for detecting and locating the generic signature in the noisy residual data.

- MATCHED FILTER - The impulse response of the matched filter is convolved with the residual altimeter data to compute a sequence of sufficient statistics for the threshold detector.

- THRESHOLD DETECTOR - The detector compares the sufficient statistics with a threshold value that is chosen to yield a specified false-alarm rate. A detection occurs when the statistic exceeds the threshold. The estimated location of the detected signature is given by the location of the local maximum of the statistic.

1.3 GENERIC OCEAN-CURRENT SIGNATURES

This section describes two parametric families of ocean-current signatures. The first family is used for designing matched filters to detect warm-core and cold-core current rings. The second family is intended for detecting boundary currents, such as the Gulf Stream, and for estimating geostrophic current velocities.

Ring-Current Signatures - A family of generic altimetric signatures is described for modeling the dynamic sea-surface features caused by cold-core and warm-core current rings. The sea-surface height $H(x)$ at radial position x with respect to the ring's center is modeled as

$$H(x) = - D \exp(-9.21 (x/W)^2) \qquad (1.3\text{-}1)$$

D = signature depth

W = signature width

D is positive for cold rings and negative for warm rings. This parametric model has a simple mathematical form and appears to be in reasonable agreement with available data on ring signatures (e.g., Refs. 1-3).

The width W is defined as the diameter at which the signature is 10 percent of its central value:

$$H(W/2) = H(0)/10 \qquad (1.3\text{-}2)$$

Equation 1.3-1 has the form of a Gaussian probability density. Therefore, these are referred to as Gaussian ring signatures. An example of a Gaussian ring signature is shown in Fig. 1.3-1, where the central depth is 0.5 meter and the width is 150 km.

The tangential current-velocity distribution implied by a ring signature may be computed by setting the radial slope of the sea surface equal to the sum of the horizontal Coriolis acceleration and the centrifugal acceleration divided by the acceleration of gravity:

$$\frac{dH(x)}{dx} = \frac{f\, v(x) + v^2(x)/x}{g} \qquad (1.3\text{-}3)$$

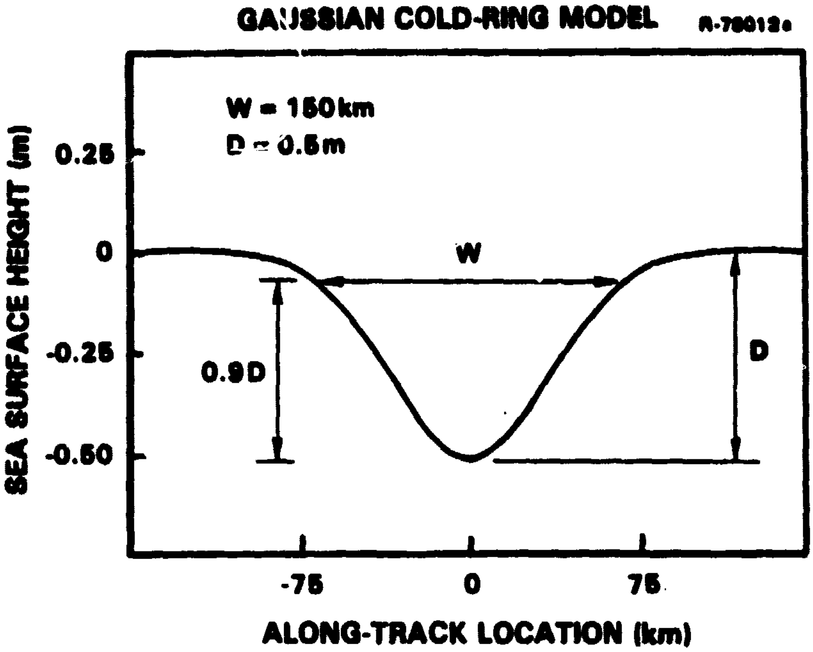

Figure 1.3-1 Gaussian Cold-Ring Signature, Depth = 0.5 m, Width = 150 km

$f = 2\Omega \sin\phi$ = Coriolis parameter

Ω = earth's rotational velocity

ϕ = latitude

$v(x)$ = tangential current velocity

g = acceleration of gravity.

The geostrophic velocity component is

$$V_g(x) = \frac{g}{f} \frac{dH(x)}{dx} \tag{1.3-4}$$

Solving Eq. 1.3-3 for the total current velocity v(x) yields

$$v(x) = \frac{x\ f}{2}\left[\sqrt{1 + \frac{4V_g(x)}{x\ f}} - 1\right] \tag{1.3-5}$$

For a Gaussian 0.5-m 150-km ring signature at 45-degrees latitude, Eqs. 1.3-4 and 1.3-5 yield the velocity distributions shown in Fig. 1.3-2. The geostrophic approximation is seen to over-estimate the maximum velocity by approximately 0.1 m/s (14%).

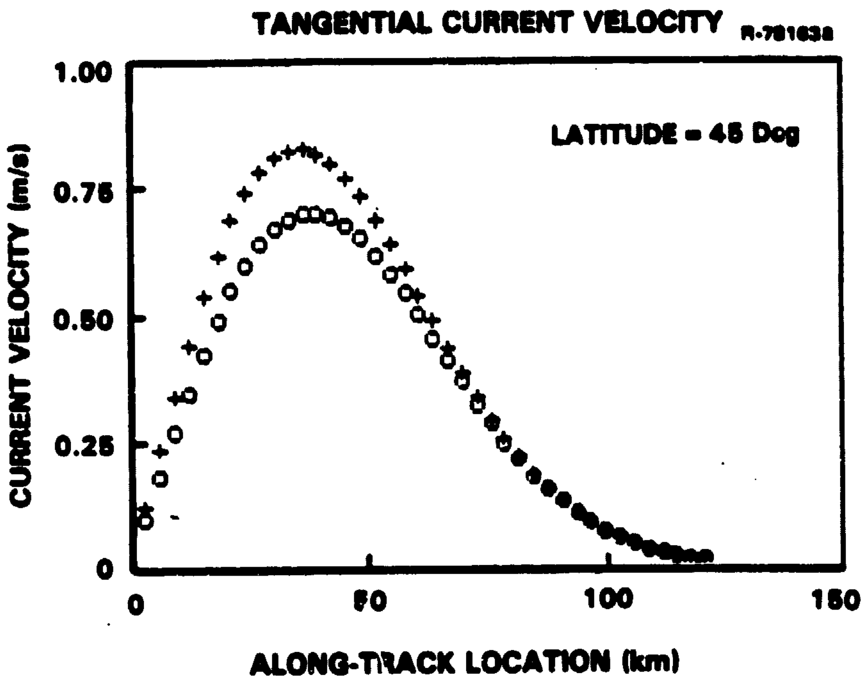

Figure 1.3-2 Tangential Current-Velocity Distributions in Gaussian Cold Ring. Crosses = geostrophic approximation; circles = geostrophic approximation with centrifugal correction.

Boundary-Current Signatures - A family of generic altimetric signatures for boundary currents is defined with the aid of Fig. 1.3-3, which depicts a satellite subtrack crossing a current at angle θ. At position x along the subtrack, the dynamic height H(x) is modeled with the hyperbolic tangent function.

$$H(x) = -(A/2)\tanh(3\,x\,\sin\theta/W_c) \tag{1.3-6}$$

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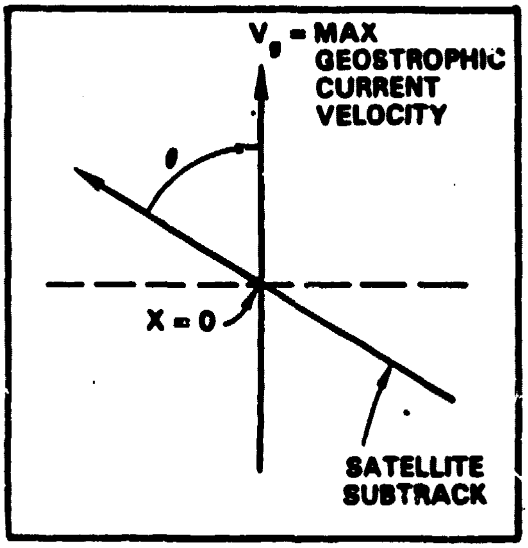

Figure 1.3-3 Geometry of Geostrophic Current and Satellite Subtrack

A = amplitude of dynamic height change

θ = track angle with respect to current velocity

W_c = width of current (90% height change)

The along-track slope of the signature is

$$\frac{dH(x)}{dx} = - \frac{3\ A\ \sin\theta}{2\ W_c} \operatorname{sech}^2\left[\frac{3\ x\ \sin\theta}{W_c}\right] \qquad (1.3\text{-}7)$$

For the coordinate system in Fig. 1.3-3, this slope is related to the cross-track component $V_c(x)$ of the geostrophic velocity as follows

$$V_c(x) = - \frac{g}{f} \frac{dH(x)}{dx} \qquad (1.3\text{-}8)$$

For tracks that cut across the current, the geostrophic velocity profile $V_g(x)$ along the subtrack is proportional to the cross-track velocity $V_c(x)$

$$V_g(x) = V_c(x)/\sin\theta \qquad (1.3\text{-}9)$$

For the signature slope given by Eq. 1.3-7, the geostrophic velocity profile is therefore

$$V_g(x) = \frac{3}{2}\frac{A}{f}\frac{g}{W_c}\ \mathrm{sech}^2\left[\frac{3\ x\ \sin\theta}{W_c}\right] \qquad (1.3\text{-}10)$$

The signature amplitude parameter A is proportional to the maximum geostrophic velocity $V_g(0)$

$$A = \frac{2\ f\ W_c}{3\ g}\ V_g(0) \qquad (1.3\text{-}11)$$

For tracks that intersect the current, the signature width W_s is proportional to the current width W_c

$$W_s = W_c/\sin\theta \qquad (1.3\text{-}12)$$

Typical model parameters for the Gulf Stream in the western North Atlantic are an amplitude of A = 1 m, a maximum geostrophic velocity of $V_g(0)$ = 2 m/s, and a latitude of ϕ = 45 deg. From Eq. 1.3-11 the current's width is W_c = 71 km. For a nominal track crossing angle of 60 deg, the along-track width of the signature is W_s = 82 km. Figure 1.3-4 shows the dynamic sea-surface height signature for these parameter values, while Fig. 1.3-5 depicts the geostrophic velocity profile implied by the height signature.

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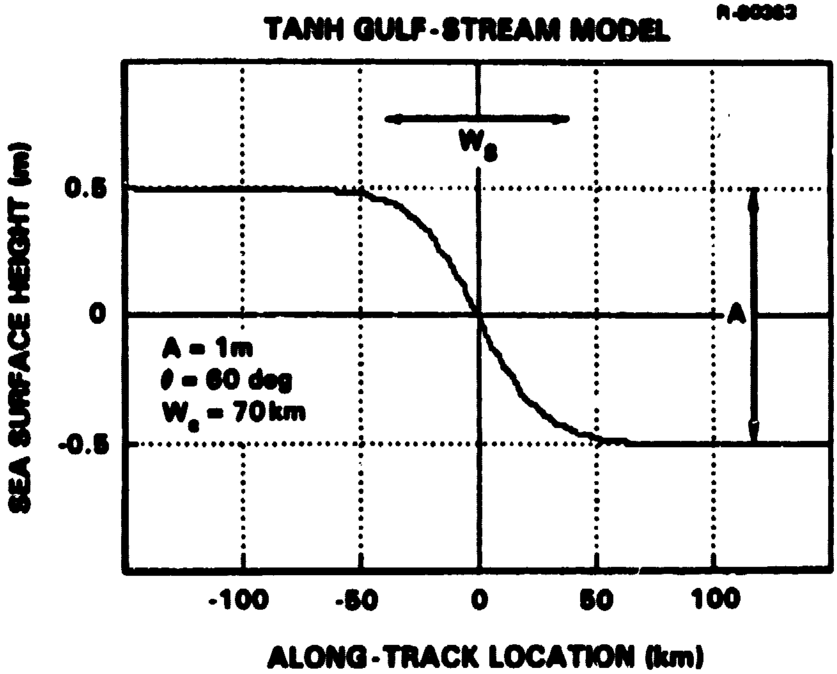

Figure 1.3-4 Dynamic Sea-Surface Height Signature

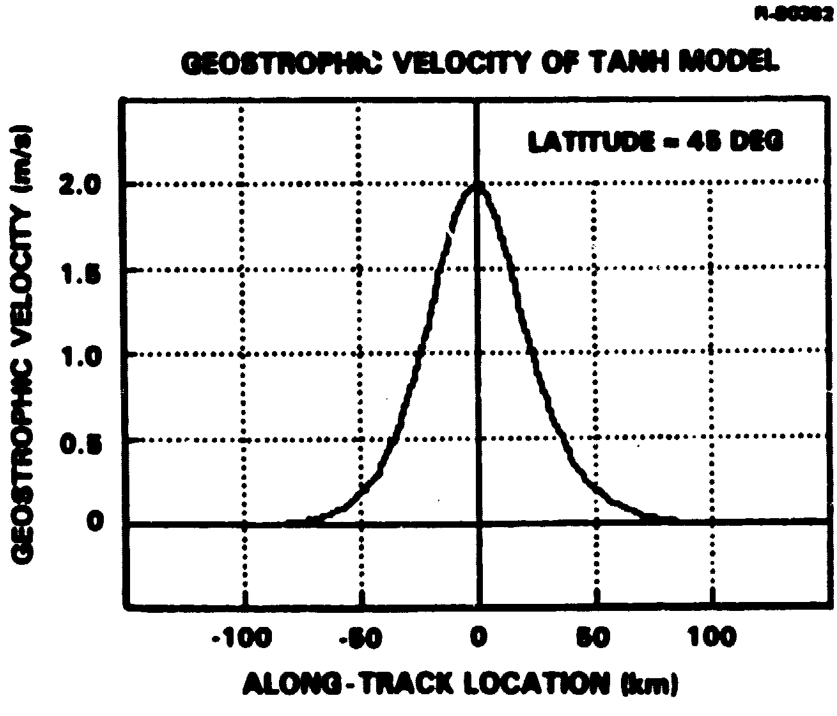

Figure 1.3-5 Geostrophic Velocity Profile

2. AUTOREGRESSIVE MODELING ALGORITHM

2.1 THEORY

An autoregressive (AR) model of order p for a time series $\{x(t);\ t = 1,2,\ldots,n\}$ is a difference equation driven by white noise $w_p(t)$

$$x(t) = C_1\, x(t-1) + C_2\, x(t-2) + \ldots + C_p\, x(t-p) + w_p(t) \qquad (2.1\text{-}1)$$

$$t = p+1,\ p+2,\ \ldots,\ n$$

To identify the best AR model for the underlying random process that generated the x(t) data, a family of AR models is considered. Each member of the family corresponds to a different model order

$$p = 0,1,2,\ldots,pmax$$

When modeling residual altimeter data, sampled at a 1-Hz rate, a reasonable choice for the maximum order is

$$pmax = \text{integer}(n/20)$$

For each order $(p = 0,1,2,\ldots,pmax)$ in turn, the AR coefficients $C_1, C_2, \ldots, C_p$ $(C_o = 1)$ are selected to minimize the sample noise variance

$$\sigma^2(p) = \frac{1}{n-pmax} \sum_{t=pmax+1}^{n} w_p^2(t) \qquad (2.1\text{-}2)$$

This variance is then used to compute the Akaike information criterion (Refs. 5-7)

$$AIC(p) = n \ln(\sigma^2) + 2p \qquad (2.1\text{-}3)$$

The particular order p that minimizes AIC is identified; the corresponding AR parameters

$$C_1, C_2, \ldots, C_p, \sigma^2$$

then define the particular AR model (for the x(t) process) that is best supported by the available data.

There are many known algorithms for computing the AR coefficients to minimize the sample variance in Eq. 2.1-2. An effective algorithm for the present application is based on an augmented version of the COVAR algorithm (Ref. 4), which is described in the following section.

2.2 AUGMENTED COVAR ALGORITHM

This section describes an augmented version of the COVAR algorithm (Ref. 4), which solves the following problem by Cholesky factorization.

GIVEN: 1. Data sequence $\{x(0), x(1), \ldots, x(N-1)\}$

2. Integer M

FIND: 1. Coefficients $\{a_1, a_2, \ldots, a_M\}$ that minimize the sum-squared AR residuals

$$\alpha = \sum_{n=M}^{N-1} [x(n) + \sum_{k=1}^{M} a_k \, x(n-k)]^2 \qquad (2.2\text{-}1)$$

2. Minimized value of α

SOLUTION: 1. Define

$$c_{ik} = \sum_{n=M}^{N-1} x(n-i)\, x(n-k); \quad k = 1,2,\ldots,M \qquad (2.2\text{-}2)$$
$$i = 1,2,\ldots,M$$

2. Solve the following M equations for a_i

$$\sum_{i=1}^{M} a_i c_{ik} = -\, c_{ok}; \; k = 1,2,\ldots,M \qquad (2.2\text{-}3)$$

In the following description of the augmented COVAR algorithm, asterisks are used to indicate additions to the original algorithm in Ref. 4.

INPUTS: N, $\{x(t);\ t=0,1,\ldots,N-1\}$, M

N = number of time-series data

$x(t)$ = datum at time t

M = maximum autoregressive order

OUTPUTS: M, $\{C(k);\ k = 0,1,2,\ldots,M\}$, $\{MS(k);\ k = 0,1,2,\ldots,M\}$

M = maximum AR order

$C(k)$ = k^{th} AR coefficient

* $MS(k)$ = mean-square value of AR residuals for model of order k

ALGORITHM:

1. Compute c_{00}, c_{10}, and c_{11} using Eq. 2.2-2

2. Initialize the following parameters

$$a_{00} = 1 \qquad (2.2\text{-}4)$$

$$\alpha_0 = c_{00} \qquad (2.2\text{-}5)$$

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$$* \quad MS(0) = \alpha_0/(N-M) \tag{2.2-6}$$

$$k_1 = -c_{10}/c_{11} \tag{2.2-7}$$

$$a_{10} = 1 \tag{2.2-8}$$

$$a_{11} = k_1 \tag{2.2-9}$$

$$b_{01} = 1 \tag{2.2-10}$$

$$\beta_0 = c_{11} \tag{2.2-11}$$

$$\alpha_1 = \alpha_0 - k_1^2\beta_0 \tag{2.2-12}$$

$$* \quad MS(1) = \alpha_1/(N-M) \tag{2.2-13}$$

3. Recursively stepping m (for m = 1,2,...,M-1), compute:

$c_{m+1,0}$ using Eq. 2.2-2

$$c_{m+1,k} = c_{m,k-1} + x(M-m-1)\, x(M-k) - x(N-m-1)\, x(N-k); \quad k = 1,2,\ldots,m+1 \tag{2.2-14}$$

$$\gamma_{mn} = \frac{1}{\beta_n} \sum_{j=1}^{n+1} c_{m+1,j}\, b_{nj}; \quad n = 0,1,\ldots,m-1 \tag{2.2-15}$$

$$b_{mj} = - \sum_{i=j-1}^{m-1} \gamma_{mi}\, b_{ij}; \quad j = 1,2,\ldots,m \tag{2.2-16}$$

$$b_{m,m+1} = 1 \tag{2.2-17}$$

$$\beta_m = \sum_{j=1}^{m+1} c_{m+1,j}\, b_{mj} \tag{2.2-18}$$

$$k_{m+1} = -\frac{1}{\beta_m} \sum_{i=0}^{m} c_{m+1,i}\, a_{mi} \tag{2.2-19}$$

$$a_{m+1,0} = 1 \tag{2.2-20}$$

$$a_{m+1,i} = a_{mi} + k_{m+1}\, b_{mi}; \quad i = 1,2,\ldots,m \tag{2.2-21}$$

$$a_{m+1,m+1} = k_{m+1} \tag{2.2-22}$$

$$\alpha_{m+1} = \alpha_m - k_{m+1}^2\, \beta_m \tag{2.2-23}$$

* $MS(m+1) = \alpha_{m+1}/(N-M)$ (2.2-24)

STEP m OF RECURSION IS COMPLETED

4. Termination (end of step M-1)

$$a_k = a_{Mk};\ k = 1,2,\ldots,M \qquad (2.2\text{-}25)$$

$$\alpha = \alpha_m \qquad (2.2\text{-}26)$$

* $C(0) = 1$ (2.2-27)

* $C(k) = -a_k;\ k = 1,2,\ldots,M$ (2.2-28)

The augmented COVAR algorithm is called as a subroutine in the AR modeling algorithm (ACOVAR) used for ocean-current detection, which is specified below.

2.3 SPECIFICATION FOR THE AUTOREGRESSIVE MODELING ALGORITHM

For ocean-current detection, the ACOVAR algorithm is used for autoregressive modeling. ACOVAR uses the augmented COVAR algorithm (Section 2.2) as a subroutine. Formal specifications for ACOVAR are given in the following.

NAME: ACOVAR

PURPOSE: Compute the parameters of an optimal autoregressive (AR) model for one track of residual altimeter data.

INPUTS: N, $\{D(k);\ k = 1,2,\ldots,N\}$

N = number of residual altimeter data

$D(k)$ = k^{th} sample in the time series of residual altimeter data along one satellite subtrack

OUTPUTS: P, $\{C(k);\ k = 0,1,\ldots,P\}$, VAR

P = order of selected AR model

$C(k)$ = k^{th} AR-model coefficient

VAR = mean-square value of AR residuals

ALGORITHM:

1. Use the augmented COVAR algorithm to compute the AR coefficients C(k) and the mean-square residuals MS(k) (for k = 0,1,2,...,M) by using the following assignments

$$x(k) = D(k+1);\ k = 0,1,\ldots,N-1 \qquad (2.3\text{-}1)$$

$$M = \text{INTEGER }(N/20) \qquad (2.3\text{-}2)$$

Set PMAX = M. (2.3-3)

2. Compute the Akaike information criterion {AIC(k); k = 0,1,...,PMAX} for each order of AR model

$$\begin{aligned} AIC(k) &= N \ln(MS(k)) + 2k \\ k &= 0,1,\ldots,PMAX \end{aligned} \qquad (2.3\text{-}4)$$

3. Determine the smallest J such that

$$\begin{aligned} &0 \leq J \leq PMAX \\ &AIC(J) \leq AIC(I);\ I = 0,1,\ldots,PMAX \end{aligned} \qquad (2.3\text{-}5)$$

4. If (J = PMAX) or (J = 0) then

   P = J (2.3-6)

   VAR = MS(J) (2.3-7)

   GOTO STEP 5

   If (J < PMAX) then

   GOTO STEP 1 BUT USE M=J (2.3-8)

5. OUTPUT P, {C(k); k = 0,1,...,P}, VAR

6. END

3. MATCHED-FILTER DESIGN ALGORITHM

3.1 THEORY

The theory of optimal matched filters for detecting deterministic signatures in additive colored noise is discussed in several text books (e.g., Refs. 8, 9, 11). The key results of that theory for ocean-current detection are summarized in the following.

The problem of detecting ocean-current signatures in residual altimeter data is formalized as follows.

GIVEN: $D(t)$ = time series of residual altimeter data

$m(t)$ = ocean-current signature time series

$N(t)$ = stationary Gaussian noise model for residual altimeter data that are free of $m(t)$

T = specified time (location) in the data $D(t)$

A_s = unknown signature amplitude scale factor

H_T = hypothesis that $D(t) = N(t) + A_s m(t-T)$ with $A_s \neq 0$

H_0 = hypothesis that $D(t) = N(t)$

FIND: An optimal decision rule for correctly choosing between hypotheses H_0 and H_T; and an optimal estimate of the amplitude A_s when H_T is chosen.

OPTIMALITY: Maximize the probability of correct detection for a specified probability of false alarm.

SOLUTION: Compute the likelihood ratio

$$LR = \frac{\text{Likelihood of } D(t) \text{ under } H_T}{\text{Likelihood of } D(t) \text{ under } H_0}$$

Select H_T when LR > threshold value.

Select H_0 when LR $\leq$ threshold value.

As depicted in Fig. 3.1-1, the optimal decision rule can be efficiently implemented by processing the residual altimeter data D(t) with one matched filter and a threshold detector to test H_T against H_0 for all possible values of T. Once a detection is made (i.e., H_T is selected), the best estimates of the location T and the amplitude scale factor A_s are easily computed from the matched-filter output.

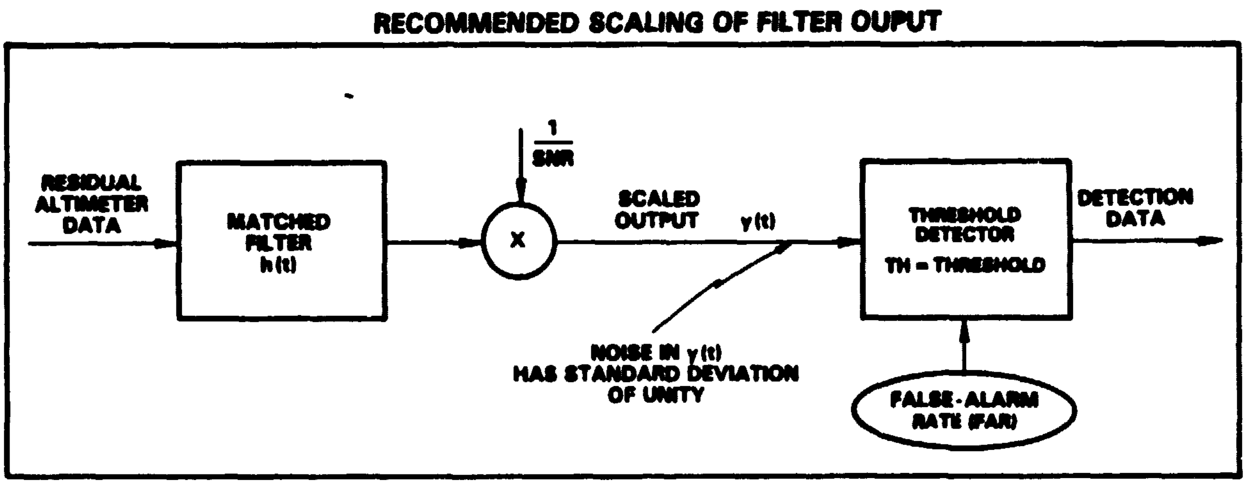

Figure 3.1-1 Matched-Filter Detector

The optimal matched filter for long data sets is a convolution operator having the frequency response H(F) and the impulse response h(t) (e.g., Ref. 11, pp. 325-329)

$$h(t) = \int_{-\frac{1}{2}}^{\frac{1}{2}} H(F)\, e^{i2\pi Ft}\, dF \qquad (3.1\text{-}1)$$

$$H(F) = \frac{M(-F)}{S_{NN}(F)} \qquad (3.1\text{-}2)$$

F = frequency (cycles/sample)

M(F) = Fourier transform of the ocean-current signature m(t)

$S_{NN}(F)$ = power spectrum of the residual altimetry N(t)

The Fourier transform of m(t) is defined as

$$M(F) = \sum_{t=-\infty}^{\infty} m(t)\, e^{-i2\pi Ft} \qquad (3.1\text{-}3)$$

$$m(t) = \int_{-\frac{1}{2}}^{\frac{1}{2}} M(F)\, e^{i2\pi Ft}\, dF \qquad (3.1\text{-}4)$$

When the residual altimeter noise model N(t) is autoregressive (AR), the optimal matched filter can be implemented as a finite-impulse-response (FIR) filter. This means that the matched-filter impulse response h(t) has finite support, i.e., h(t) = 0 for $t < t_{min}$ and $t > t_{max}$, for finite t_{min} and t_{max}.

The AR model for the noise N(t) is a difference equation of order p driven by white noise W(t)

$$N(t) = C_1\, N(t-1) + C_2\, N(t-2) + \ldots + C_p\, N(t-p) + W(t) \qquad (3.1\text{-}5)$$

$$t = \ldots -1, 0, 1 \ldots$$

$$\text{Mean}(W(t)) = 0;\ \text{Variance}(W(t)) = \sigma^2$$

From linear-system theory (Refs. 10 and 11), the power spectrum $S_{NN}(F)$ of N(t) is

$$S_{NN}(F) = \frac{\sigma^2}{G(F)\ G(-F)} \tag{3.1-6}$$

$$G(F) = 1 - \sum_{k=1}^{p} C_k\ e^{-i2\pi Fk} \tag{3.1-7}$$

From Eqs. 3.1-2 and 3.1-6, the matched-filter frequency response may be expressed as

$$H(F) = \frac{M(-F)}{S_{NN}(F)} = \sigma^{-2} G(F)\ G(-F)\ M(-F) \tag{3.1-8}$$

The Fourier transform of Eq. 3.1-8 yields the following expression for the impulse response h(t) of the matched filter

$$h(t) = \sigma^{-2} g(t) * g(-t) * m(-t) \tag{3.1-9}$$

$$g(t) = \int_{-\frac{1}{2}}^{\frac{1}{2}} G(F)\ e^{i2\pi Ft}\ dF \tag{3.1-10}$$

$$g(t) = 0;\ t < 0 \tag{3.1-11}$$

$$g(0) = 1 \tag{3.1-12}$$

$$g(1) = -C_1 \tag{3.1-13}$$

$$\vdots$$

$$g(p) = -C_p \tag{3.1-14}$$

$$g(t) = 0;\ t > p \tag{3.1-15}$$

Since the convolutions in Eq. 3.1-9 contain only a finite number of non-zero terms when m(t) has finite support, the impulse response h(t) also has finite support.

The rms signal-to-noise ratio achieved by the matched filter is defined as

$$SNR = \frac{\text{Peak Filter Output Due to Signature } m(t)}{\text{Rms Filter Output Due to Noise } N(t)} \qquad (3.1\text{-}16)$$

The SNR of an optimal matched filter can be computed with the formula

$$SNR = \sqrt{\sum_{j=-\infty}^{\infty} h(j)\ m(-j)} \qquad (3.1\text{-}17)$$

where only a finite number of terms contribute. The rms value of the noise in the filter output is numerically equal to SNR when the filter is optimized

$$\text{Rms Noise in Filter Output} = SNR \qquad (3.1\text{-}18)$$

The peak filter output value due to the signature m(t) is also expressible in terms of SNR:

$$\text{Peak Filter Output Due to Signature } m(t) = SNR^2 \qquad (3.1\text{-}19)$$

Since the SNR equals the rms value of the modeled noise in the filtered output, it is convenient to scale the filter output by the factor 1/SNR as shown in Fig. 3.1-1. This yields a test statistic Y(t) that contains a random component having a standard deviation of unity.

The mean output frequency F_m of the filter is a number that measures the average rate at which the filter output changes sign

$$F_m = \begin{array}{l}\text{Half the Average Rate of Zero Crossings} \\ \text{of Noise in Filter Output}\end{array} \qquad (3.1\text{-}20)$$

By using results in Ref. 10, p. 199, it may be shown for the Gaussian noise in the filter output that

$$F_m = \frac{1}{2\pi} \cos^{-1}[x/SNR^2] \text{ (cycles/sample)} \qquad (3.1\text{-}21)$$

$$x = \sum_{k=-\infty}^{\infty} h(k)\, m(1-k)$$

Equation 3.1-21 is derived in the Appendix of Ref. 12.

3.2 SPECIFICATIONS FOR THE MATCHED-FILTER DESIGN ALGORITHM

For a specified AR noise model and a specified ocean-current signature, the DESIGNMF algorithm is used to design the optimal matched filter. Formal specifications for this algorithm are given in the following.

NAME: DESIGNMF

PURPOSE: Compute the impulse response of the matched filter, the signal-to-noise ratio, and the mean output frequency.

INPUTS: P, {C(k); k = 0,1,...,P}, VAR, NM NH, {M(k); k = -NM,-NM+1,...,NM}, SNR, FM

P = order of the AR model for the residual altimeter data

C(k) = k^{th} coefficient of the AR model

VAR = mean-square value of the AR residuals

NM = half-width of the time series containing the oceanographic signature to be detected (support of signature contains 2NM + 1 points)

M(k) = k^{th} sample in the oceanographic signature time series

NH = half-width of the matched-filter impulse response (support of impulse response contains 2NH + 1 points)

SNR = rms signal-to-noise ratio

FM = mean output frequency (cycles/sample)

OUTPUTS: NH, $\{H(k);\ k = -NH, -NH+1, \ldots, NH\}$

NH = half-width of the matched-filter impulse response

$H(k)$ = k^{th} sample in the matched-filter impulse response

ALGORITHM:

1. Compute A(k), the convolution of G(k) and G(-k):

$$G(k) = 0,\ k < 0 \qquad (3.2\text{-}1)$$

$$G(Q) = 1;\ k = 0 \qquad (3.2\text{-}2)$$

$$G(k) = -C(k);\ k = 1,2,\ldots,P \qquad (3.2\text{-}3)$$

$$G(k) = 0;\ k > P \qquad (3.2\text{-}4)$$

$$A(k) = \sum_{j=0}^{P} G(j)\ G(j-k);\ k = -P,-P+1,\ldots,P \qquad (3.2\text{-}5)$$

2. Compute B(k), the convolution of A(k) and M'(-k):

$$M'(k) = 0;\ k < -NM \qquad (3.2\text{-}6)$$

$$M'(k) = M(k);\ k = -NM,-NM+1,\ldots,NM \qquad (3.2\text{-}7)$$

$$M'(k) = 0;\ k > NM \qquad (3.2\text{-}8)$$

$$B(k) = \sum_{j=-P}^{P} A(j)\ M'(j-k);\ k = -NH,-NH+1,\ldots,NH \qquad (3.2\text{-}9)$$

3. Compute H(k), the impulse response of the matched filter, by scaling B(k):

$$H(k) = VAR^{-1}B(k);\ k = -NH,-NH+1,\ldots,NH \qquad (3.2\text{-}10)$$

4. Compute SNR, the rms signal-to-noise ratio:

$$SNR = \left(\sum_{k=-NM}^{NM} H(k)\ M(-k) \right)^{\frac{1}{2}} \tag{3.2-11}$$

5. Compute FM, the mean output frequency:

$$FM = (2PI)^{-1} \cos^{-1}(x/SNR^2) \text{ (cycles/sample)} \tag{3.2-12}$$

$$x = \sum_{k=-NM}^{NM} H(k)\ M(1-k) \tag{3.2-13}$$

$$PI = 3.14159...$$

6. Output: NH, {H(k); k = -NH,-NH+1,...,NH}

7. End

4. MATCHED-FILTER CONVOLUTION ALGORITHM

4.1 THEORY

The optimal matched filter, depicted in Fig. 3.1-1, processes the residual altimeter data D(t) to produce a test statistic Y(t) for the threshold detector. The scaled output of the matched filter is computed through the following convolution of the data D(t) with the filter's impulse response h(t)

$$Y(t) = \frac{1}{SNR} \sum_{k=-\infty}^{\infty} h(k)\, D(t-k) \qquad (4.1\text{-}1)$$

Since both h(t) and D(t) are finite sequences, the convolution in Eq. 4.1-1 has only a finite number of terms.

4.2 SPECIFICATIONS FOR THE MATCHED-FILTER CONVOLUTION ALGORITHM

The matched filter is implemented as a convolution in algorithm CONVOLVE. The formal specifications for this algorithm are given in the following.

NAME: CONVOLVE

PURPOSE: Convolve the residual altimeter data with the matched-filter impulse response and scale the output.

INPUTS: NH, {H(k); k = -NH,-NH+1,...,NH}, N,
{D(k); k = 1,2,...,N}, SNR

NH = half-width of the matched-filter impulse response

$H(k)$ = k^{th} sample of the matched-filter impulse response

N = number of data in the residual altimetry time series

$D(k)$ = k^{th} sample in the residual altimetry time series

SNR = rms signal-to-noise ratio

OUTPUTS: N, $\{Y(k);\ k = 1,2,\ldots,N\}$

N = number of samples in the scaled output of the matched filter

$Y(k)$ = k^{th} sample in the scaled output of the matched filter

ALGORITHM:

1. Compute $Y(k)$, the scaled convolution of $H(k)$ and $D'(k)$:

$$D'(k) = 0;\ k < 1 \tag{4.2-1}$$

$$D'(k) = D(k);\ k = 1,2,\ldots,N \tag{4.2-2}$$

$$D'(k) = 0;\ k > N \tag{4.2-3}$$

$$Y(k) = \mathrm{SNR}^{-1} \sum_{j=-NH}^{NH} H(j)\ D'(k-j); \tag{4.2-4}$$

$$k = NH+1, NH+2, \ldots, N-NH$$

$$Y(k) = 0;\ k = 1,2,\ldots,NH \tag{4.2-5}$$

$$Y(k) = 0;\ k = N-NH+1, N-NH+2, \ldots, N \tag{4.2-6}$$

2. Output: N, $\{Y(k);\ k = 1,2,\ldots,N\}$

3. End

5. THRESHOLD DETECTION ALGORITHM

5.1 THEORY

The threshold detector compares the sequence of scaled test statistics Y(t) from the matched filter against a detection threshold. When Y(t) exceeds the threshold, an alarm is said to occur. These alarms are classified into three categories:

- Correct Detection caused by occurrences of modeled current signatures in the residual altimeter data
- False Alarms caused by random excursions of the modeled noise in the residual altimeter data
- Unmodeled Detections caused by unmodeled current signatures or unmodeled noise in the residual altimeter data.

The statistics of correct detections and false alarms are computed for the specific ocean-current signature and the specific noise model for which the filter was designed. On the basis of these statistics, the expected average performance of the detector is predicted, and the detection threshold is adjusted for a desired tradeoff between detection probabilities and false-alarm rates.

Formulas are listed below for computing the following performance statistics of the detector: the probability of false alarms; the average false-alarm rate, the maximum-likelihood estimates of signature location and amplitude (and their rms accuracies), and the probability of detecting a signature with a prescribed amplitude.

Probability of False Alarm - Let TH denote the detection threshold. The probability P_f that the noise component alone in Y(t) will exceed TH is given by the standardized normal probability distribution function:

$$P_f = \mathrm{Prob}\{Y(t) > TH;\ \text{noise alone}\} \tag{5.1-1}$$

$$P_f = Q(TH) = \int_{TH}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-x^2/2)\, dx \tag{5.1-2}$$

The detection threshold that yields a prescribed probability of false alarm is

$$TH = Q^{-1}(P_f) \tag{5.1-3}$$

Average False-Alarm Rate - The threshold detector processes data from individual tracks of altimeter data, and it is often reasonable to set the detection threshold so that a specified number of false alarms is expected to occur per unit distance along the track (expressed in the units of alarms per data sample). This false-alarm rate (FAR) is computed as

$$FAR = (F_m) \exp(-TH^2/2) \ \text{(alarms/sample)} \tag{5.1-4}$$

F_m = mean output frequency of modeled noise in filter output

Equation 5.1-4 is derived from results in Ref. 10, p. 492. The expected number of false alarms (EN) along a track of data having N samples of the test statistic Y(t) is

$$EN = N \cdot FAR \tag{5.1-5}$$

Detection Threshold - The detection threshold TH is chosen to yield a specified false-alarm rate FAR

$$TH = \sqrt{-2\ \ln(FAR/F_m)} \qquad (5.1\text{-}6)$$

Equation 5.1-6 is obtained by solving Eq. 5.1-4 for TH.

Detected Signature Location and Amplitude - The best estimate of the location of a detected signature is the value of t for which Y(t) achieves its local maximum value above the threshold TH. Let t_o denote this estimate of signature location. The Cramer-Rao (C-R) lower bound on the rms error of this estimate depends on the filter's maximum scaled output $Y(t_o)$:

$$\text{C-R Lower Bound} = \frac{1}{2\pi\ F_m\ Y(t_o)}\ \text{(samples)} \qquad (5.1\text{-}7)$$

The maximum-likelihood estimate of the signature amplitude scale factor A_s is

$$\hat{A}_s = \frac{Y(t_o)}{SNR} \qquad (5.1\text{-}8)$$

and the rms (one-sigma) error in this estimate is 1/SNR. The best estimate of the detected signature is then $\hat{A}_s \cdot m(t-t_o)$.

Probability of Detection - The probability of detecting the signature $A_s\ m(t)$ is

$$P_d(A_s) = Q(TH - A_s \cdot SNR) \qquad (5.1\text{-}9)$$

5.2 SPECIFICATIONS FOR THE THRESHOLD DETECTION ALGORITHM

The threshold detector is implemented by algorithm DETECTOR. The formal specifications for this algorithm are given in the following.

NAME: DETECTOR

PURPOSE: Compare the scaled output of the matched filter against a detection threshold; locate and count all alarms; compute a signature amplitude scale factor for each detection.

INPUTS: N, {Y(k); k = 1,2,...,N}, NH, SNR, FM, FAR

N = number of samples in scaled output of matched filter

Y(k) = k^{th} sample in scaled output of matched filter

NH = half-width of matched-filter impulse response

SNR = rms signal-to-noise ratio

FM = mean output frequency (cycles/sample)

FAR = average false-alarm rate (alarms/sample) (FAR must be < FM)

OUTPUTS: NALARM, {A(k); k = 1,2,...,NALARM}, {L(k); k = 1,2,...,NALARM}, EN, {CR(k); k = 1,2,...,NALARM}, AE

NALARM = number of alarms

A(k) = signature amplitude scale factor for k^{th} detection

L(k) = location of the k^{th} detection (sample number)

EN = expected number of false alarms

CR(k) = Cramer-Rao lower bound on rms location error of k^{th} detection (samples)

AE = standard deviation of the errors in the estimates of the signature amplitude scale factors

ALGORITHM:

1. Compute detection threshold TH

$$TH = \sqrt{2 \ln(FM/FAR)} \qquad (5.2\text{-}1)$$

2. Count number of alarms (NALARM), record the location L(k) of the k^{th} positive-going threshold crossing, and mark locations in array A(N) of all samples Y(k) that exceed the threshold

```
NALARM = 0                                        (5.2-2)

  FLAG = 0                                        (5.2-3)

 FOR K = NH + 1 to N - NH

IF FLAG = 0 AND Y(K) > TH THEN
                      NALARM = NALARM + 1         (5.2-4)
                        FLAG = 1                  (5.2-5)
                   L(NALARM) = K                  (5.2-6)

IF Y(K) > TH THEN A(K) = 1                        (5.2-7)

IF Y(K) ≤ TH THEN FLAG = 0                        (5.2-8)

NEXT K
```

3. Skip to output if there are no threshold crossings

```
IF NALARM = 0 THEN GOTO STEP 7
```

4. Compute the estimated signature location and signature amplitude scale factor for each alarm.

```
FOR I = 1 TO NALARM

     FOR K = L(I) TO N-NH
          IF A(K) = 0 THEN
                    KE = K (NEGATIVE-GOING
                        THRESHOLD CROSSING)   (5.2-9)
                     K = N-NH (EXIT LOOP)     (5.2-10)

     NEXT K

TMAX = -1E38                                  (5.2-11)

  KB = L(I)                                   (5.2-12)

FOR K = KB TO KE-1

     IF Y(K) > TMAX THEN TMAX = Y(K)          (5.2-13)
                          LL = K              (5.2-14)

NEXT K
```

$$L(I) = LL \text{ (LOCATION OF I-TH DETECTION)} \tag{5.2-15}$$

$$A(I) = Y(LL)/SNR \text{ (SCALE FACTOR FOR I-TH DETECTION)} \tag{5.2-16}$$

$$CR(I) = 1/(2\pi \cdot FM \cdot Y(LL)) \text{ (C-R LOWER BOUND)} \tag{5.2-17}$$

NEXT I

5. Compute EN, the expected number of false alarms caused by modeled noise in the residual altimeter data.

$$EN = (N-2 \cdot NH)FAR \tag{5.2-18}$$

6. Compute AE, the standard deviations of the errors of the estimated signature amplitude scale factors

$$AE = 1/SNR \tag{5.2-19}$$

7. Output: NALARM, {L(k); k = 1,2,...,NALARM}, {A(k); k = 1,2,...,NALARM}, EN

8. End

6. GEOSTROPHIC-VELOCITY ESTIMATION ALGORITHM

6.1 THEORY

Nearly geostrophic boundary currents, such as the Gulf Stream in the western North Atlantic, produce characteristic signatures in altimeter data when the satellite subtracks intersect the current. Figure 1.3-1 depicts a satellite subtrack crossing a current at an angle θ. At position x along the subtrack, the dynamic sea-surface height H(x) is related to the cross-track component of the geostrophic velocity $V_c(x)$ by the equation

$$V_c(x) = -\frac{g}{f}\frac{dH(x)}{dx} \qquad (6.1\text{-}1)$$

$f = 2\Omega\sin\phi$ = Coriolis parameter

Ω = earth's rotational velocity

ϕ = north latitude

g = acceleration of gravity

For the hyperbolic-tangent current signature H(x) described in Section 1.3, the cross-track velocity is

$$V_c(x) = \frac{3}{2}\frac{g}{f}\frac{A}{W_s}\operatorname{sech}^2(3\,x/W_s) \qquad (6.1\text{-}2)$$

A = amplitude of dynamic height change

W_c = width of current (90% height change)

$W_s = W_c/\sin\theta$ = width of signature when $\sin\theta \neq 0$

θ = track angle with respect to current velocity

The maximum cross-track geostrophic velocity is

$$[V_c]_{max} = V_c(0) = \frac{3}{2}\frac{g}{f}\frac{A}{W_s} \qquad (6.1\text{-}3)$$

Equation 6.1-3 indicates that the maximum cross-track velocity can be computed from estimates of the amplitude A and width W_s of the boundary-current signature. The required estimates of A and W_s are computed from a track of residual altimeter data in two steps.

In the first step, the residual altimetry are processed with a bank of five matched filters; each filter is optimized for a different width of signature (W_s = 50, 60, 75, 100, 150 km). The particular filter that produces the largest scaled test statistic $[Y(t)]_{max}$ in response to the boundary current is identified; its value for W_s is the maximum-likelihood estimate $\hat{W}_s$ for the sample of data being processed.

In the second step, the maximum-likelihood estimate $\hat{A}$ of the signature amplitude is computed as

$$\hat{A} = A \cdot \hat{A}_s \qquad (6.1\text{-}4)$$

A = signature amplitude used in designing the matched filter

$\hat{A}_s$ = estimated amplitude scale factor based on matched-filter output

$\hat{A}_s = [Y(t)]_{max}/SNR$

SNR = matched-filter rms signal-to-noise ratio

The estimated maximum cross-track velocity is then computed as

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$$\hat{V}_c(0) = \frac{3\, g\, \hat{A}}{2\, f\, \hat{W}_s} \tag{6.1-5}$$

The standard deviation of the error in estimating $\hat{A}$ is

$$\delta\hat{A} = A\cdot\delta\hat{A}_s \tag{6.1-6}$$

$$\delta\hat{A}_s = 1/SNR = \text{standard deviation of error in } \hat{A}_s$$

The standard deviation of the velocity estimate is estimated as

$$\delta\hat{V}_c(0) = \frac{3\, g\, \delta\hat{A}}{2\, f\, \hat{W}_s} \tag{6.1-7}$$

6.2 SPECIFICATIONS FOR THE THRESHOLD DETECTION ALGORITHM

The geostrophic-velocity estimation algorithm is named GVE. The formal specifications for this algorithm are given in the following.

NAME: GVE

PURPOSE: Estimate the location and magnitude of the maximum cross-track velocity in a geostrophic current.

REQUIRED EXTERNAL
ALGORITHMS: ACOVAR, DESIGNMF, CONVOLVE, and DETECTOR

ACOVAR = autoregressive modeling algorithm

DESIGNMF = matched-filter design algorithm

CONVOLVE = matched-filter implementation algorithm

DETECTOR = threshold detection algorithm

INPUTS: N, $\{D(k); k = 1,2,\ldots,N\}$, $\{LAT(k); k=1,2,\ldots,N\}$

N = number of data in the residual altimetry time

D(k) = k^{th} sample in the residual altimetry time series

LAT(k) = north latitude of k^{th} sample in altimetry time series

OUTPUTS: NL, V, LE, VE

NL = estimated location of maximum geostrophic velocity (data sample number)

V = estimated maximum cross-track geostrophic velocity (meters per second)

LE = Cramer-Rao lower bound on rms error in estimated location NL of the maximum geostrophic velocity (samples)

VE = standard deviation of the error in the velocity estimate V, due to modeled noise (meters per second)

ALGORITHM:

1. Use the algorithms ACOVAR, DESIGNMF, CONVOLVE, and DETECTOR to test if a geostrophic current signature is in the residual altimetry data $\{D(k); k = 1,2,\ldots,N\}$. For this test, input to DESIGNMF the tanh model current signature defined in Section 1.3 with the following parameter values: A = 0.5 m; W_s = 75 km.

   If a geostrophic current is not detected, then set V = 0, NL = 0, LE = 0, VE = 0 and skip to Step 7.

2. Truncate the data set $\{D(k); k = 1,2,\ldots,N\}$ to remove the detected geostrophic current signature. Process the truncated data set with ACOVAR to compute an autoregressive (AR) model.

3. Use the AR model computed in Step 2 with the algorithm DESIGNMF to design a bank of five matched filters, each matched to a tanh current signature having a different width. Suggested widths are W_s = 50, 60, 75, 100, and 150 km. The signature amplitudes are all set to A = 0.5 meter.

4. Implement the five matched filters from Step 3 by using the algorithm CONVOLVE. Filter the residual altimeter data (that part which contains the detected current signature) five times, once with each of the matched filters.

5. Compare the scaled test statistics Y(t) coming from each of the five matched filters in Step 4, and identify that output sequence which achieves the largest value in the vicinity of the geostrophic current.

6. Use the algorithm DETECTOR to compute the geostrophic current location NL, the Cramer-Rao lower bound LE on the rms location error, the signature amplitude scale factor $\hat{A}_s$, and the rms error AE in the $\hat{A}_s$ estimate. Compute the estimated maximum geostrophic velocity V

$$V = \frac{3\, g\, \hat{A}_s\, 0.5}{2\, f\, \hat{W}_s} \; ; \quad \hat{W}_s = \text{signature width corresponding to largest Y(t) in Step 5}$$

$$g = 9.81\ \mathrm{m/s^2}$$

$$f = 7.29 \times 10^{-5} \cdot 2 \cdot \sin(\mathrm{LAT(NL)})$$

Compute the standard deviation VE of the error in the velocity estimate V

$$VE = V \cdot AE/\hat{A}_s$$

7. Output: NL, V, LE, and VE

8. End

7. SUMMARY

This report documents the specifications of NOSS algorithms for ocean current mapping and summarizes the detection theory on which the algorithms are based. The inputs to the algorithms are individual tracks of residual satellite radar altimeter data from which estimated geoid profiles have been subtracted. The algorithms are based on the fact that cold-core and warm-core current rings and boundary currents can be detected by identifying the occurrence of characteristic sea-surface height signatures in the residual altimeter data. In the case of nearly geostrophic boundary currents, the cross-track component of the current velocity can be inferred by estimating the along-track sea-surface slope from the altimeter data and then using the geostrophic equation to compute the velocity.

Optimal matched filters are used to detect, locate, and estimate the amplitudes of generic current signatures in the residual altimeter data. The algorithms automatically analyze each track of residual altimeter data and compute an optimal autoregressive model for the noise signal in the data. Using this noise model, together with a parametric model for the deterministic ocean-current signature that is to be detected, the algorithm designs a statistically optimal matched-filter detector for discriminating between the noise and the signature. The detector is optimal in the sense that the probability of detecting the ocean-current signature is maximized for a specified probability of false alarm (a false alarm occurs when the random noise excursions in the altimeter data masquerade as a current signature and cause a false detection). The algorithm adjusts the sensitivity of the detector to achieve a specified average false-alarm rate (e.g., 1 false alarm per 10,000 km of data along the satellite subtrack).

The algorithm for estimating the geostrophic velocities of boundary currents employs a bank of five matched-filter detectors; each filter is matched to a different width for the current signature. The algorithm determines that signature width which is most probable (given the available altimeter data) and computes a maximum-likelihood estimate of the current signature amplitude. From this information, the algorithm estimates the maximum along-track slope of the sea surface and uses the geostrophic equation to compute the estimated geostrophic velocity. The rms accuracy of the velocity estimate is also computed by using the Cramer-Rao lower bound on the variance of the estimated signature amplitude.

The development of the algorithms is documented in Ref. 12. This reference also summarizes the results of verification tests in which the algorithms are used to detect cold-core current rings and to estimate geostrophic velocities with SEASAT-A altimeter data and Marsh-Chang geoid estimates (Ref. 13).

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