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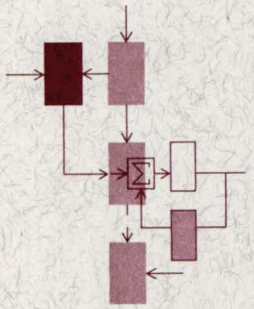

REAL TIME ESTIMATION AND PREDICTION OF SHIP MOTIONS USING KALMAN FILTERING TECHNIQUES

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      using computerized simulations. The equations of motion as derived from
      hydrodynamics, their form and the physical mechanisms involved and the
      general form of the approximation are discussed. The modeling of the sea
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REAL TIME ESTIMATION AND PREDICTION OF SHIP MOTIONS USING KALMAN FILTER TECHNIQUES

ABSTRACT

A study of the real time estimation and prediction of ship motions, velocities and accelerations is presented. The ship motion estimations are of particular interest for operations in rough seas such as aircraft or helicopter landing, transfer of equipment or cargo at sea and off-shore installations.

In the present study the estimation and prediction of heave, pitch, roll, sway, yaw motions of a DD-963 destroyer is considered, using Kalman filter techniques, for application in VTOL landing.

The governing equations are obtained from hydrodynamic considerations in the form of linear differential equations with frequency dependent coefficients. In addition non-minimum phase characteristics are obtained due to the spatial integration of the water wave forces.

The resulting transfer matrix function is irrational and non-minimum phase. The conditions for a finite-dimensional approximation are considered and the impact of the various parameters is assessed.

A detailed numerical application for a DD-963 destroyer is presented and simulation results of the estimations obtained from Kalman filters are discussed. The effect of

the various modeling parameters on the rms error is assessed and simplifying conslusions are drawn.

The models developed are used to predict the motions a few seconds ahead. An upper bound for prediction time of about five seconds is established, with the exception of roll which can be predicted up to ten seconds ahead. The effect of noise and modeling errors on the rms prediction error is investigated in detail.

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INTRODUCTION

The present study started as part of the effort directed toward designing an efficient scheme for landing VTOL aircraft on destroyers in rough seas. A first study [14] showed a significant effect of the ship model used on the thrust level required for safe landing.

In a landing scheme therefore it would be desirable to have accurate ship models capable of providing a good real time estimation of the motions, velocities and accelerations of the landing area, resulting in safer operations and with reduced thrust requirements.

The modeling is quite complex and a substantial effort is required to reduce the governing equations to a finite dimensional system of reasonable order.

The study contains a first chapter on the equations of motion as derived from hydrodynamics, their form and the physical mechanisms involved and the general form of the approximation.

The second chapter describes the modeling of the sea, which proved to be a crucial part of the overall problem.

The third chapter describes the derivation of the state-space equations for the DD-963 destroyer.

In the fourth chapter the Kalman filter studies are presented and the influence of the various parameters is assessed.

In the fifth chapter the feasibility of predicting the ship motions a few seconds ahead in time is studied within the present formulation.

Finally the appendices provide the characteristics of the destroyer, hydrodynamic information and some computer programs used.

OVERVIEW

The real time estimation of the rigid body motions, velocities and accelerations of a vessel in rough seas requires accurate modeling of the wave exciting forces and the hydrodynamic coefficients of the ship.

The wave forces are obtained after an integration over the ship hull of the pressure forces, so that their evaluation requires a seakeeping program, while their magnitude and phase represent clearly an infinitely dimensional system with non-minimum phase characteristics.

The complexity of the resulting equations is due primarily to the wave formation as the vessel moves, which is a mechanism of energy dissipation and additionally it introduces memory effects.

The wave spectrum contains a rather narrow band of frequencies so that an efficient approximation of the ship characteristics can be achieved within this frequency band.

A DD-963 destroyer was used as the basis for the present study. First the geometric and mass properties of the vessel were analysed by the M.I.T. Ocean Engineering Department Seakeeping program and its hydrodynamic forces and coefficients were obtained.

Subsequently a finite dimensional approximation was fitted in this data within the wave frequency range. Two groups of ship motions were distinguished, the heave-pitch and the roll-sway-yaw sets of motion, which up to the first order are uncoupled to each other.

The parameters of the approximations are four:

The speed of the vessel, the direction of the waves, the significant wave height and the modal frequency of the wave spectrum.

These models were used to estimate the ship motions, velocities, accelerations using noisy measurements of the motions. The Kalman filter designed for this purpose gives very good results when a relatively accurate estimate of the modal frequency of the spectrum is available. The modal frequency was found to be the most significant parameter in the overall scheme since it influences the estimation error significantly and is the most difficult to estimate.

The ship speed and the wave heading are important parameters also, but can be estimated easily and accurately.

The double peak spectrum, i.e. seas containing swell also, require separate treatment, because the low frequency peak is hard to estimate, while its influence is quite important.

The predictability of ship motions has been investigated within the frame of the present study. First perfect state information is assumed and by propagating the prediction error covariance from zero initial value it has been established that within 25% rms error over rms motion, the prediction time is about five seconds for all motions with the exception of roll which can be predicted up to ten seconds ahead. Simulations confirmed these results.

The effect of noise and modeling errors is to reduce the prediction time. Omission of the non-minimum phase zeros has a particularly pronounced effect.

In summary, the approximations described in the sequel provide a good model of the quite complex ship equations of motion within the wave frequency range. The derived models can be used for a real time estimation and prediction of the ship motions and other responses using Kalman filter techniques.

Computer programs have been prepared that provide the required model matrices once the parameter of the problem has been specified.

Chapter 1: EQUATIONS OF MOTION

Definitions

The rigid body motions of a ship in six degrees of freedom are shown in Figure 1.1: We define the x_1z_1 plane to coincide with the symmetry plane of the ship, with the z_1 axis pointing vertically upwards when the vessel is at rest, and the y_1 axis so as to obtain an orthogonal right-hand system,while the origin need not coincide with the center of gravity. The $x_oy_oz_o$ system is an inertial system with x_oy_o fixed on the undisturbed sea surface, while the x y z system is moving with the steady speed of the vessel (i.e. it follows the vessel but it does not participate in its unsteady motion). Then the linear motions along the x_1, y_1, z_1 axes are surge, sway and heave respectively. In order to define the angular motions, we normally require Euler angles, in the present case, though, we consider small motions so that the tensor of angular displacements can be replaced by a vector of small angular displacements, which are roll, pitch, yaw around the x_1, y_1, z_1 axes respectively.

The characteristics of a ship are its slender form, i.e. $L/B>>1$, $L/T>>1$, where L is the length, B the beam and T the draft. Also, the ship is symmetric about the xz plane and near symmetric about the yz plane. For this reason

$$I_{yz} = I_{zy} = 0$$

$$I_{xy} = I_{yx} = 0$$

The value of I_{xz} is typically small compared with I_{xx}, I_{yy}. The justification of using the linearity assumption is as follows: The excitation consists of wave induced forces, which include fluid inertia forces and hydrostatic forces. It is well established that the wave height to wave length ratio is small, since at a typical upper value of 1/7 the wave breakes and loses all its energy [15] (Figure 1.2). As a result, the major part of the wave force is a linear function of the wave elevation and can be obtained by a first order perturbation expansion of the nonlinear fluid equation, using the wave height to length ratio as the perturbation parameter [15].

The wave spectrum, as will be shown later, has a frequency range between typically 0.2 and 2 rad/sec. Given the large mass of the vessel, the resulting motions, within this frequency range, are of the order of a few feet, or a few degrees, so that the equations of motion can be linearized.

The only motion that requires attention is roll, because due to the slender form of the ship, the rolling motion may become large, in which case nonlinear damping becomes important.

Simple Derivation

We derive the equation of motion for a simple two dimensional object to demonstrate the overall procedure.

Let us assume that we wish to derive the motion of a two dimensional cylinder subject to wave excitation, allowed to move in heave only (Figure 1.3).

The incoming wave of amplitude a_o and frequency ω_o will cause a force on the cylinder, and, therefore, heave motion. Due to the linearity of the problem, the following decomposition can be used, which simplifies the problem considerably.

(a) Consider the sea calm and the ship forced to move sinusoidally with unit heave amplitude, and frequency ω_o, and find the resulting force.

(b) Consider the ship motionless and find the force on the cylinder due to the incoming waves and the diffraction effects (diffraction problem).

(c) In order to find the heave amplitude, within linear theory, we equate the force found in (a) times the (yet unknown) heave amplitude, with the force found in (b). (Figure 1.4)

The force in (b) can be decomposed further for modeling purposes, again due to linearity: One part is due to the undisturbed incoming waves and the other part due to the diffracted waves. The first is called the Froude-Krylov force and the second the diffraction force. The total force is called the excitation force [15].

The force in (a) due to linearity can be also decomposed: The first part is simply the hydrostatic force. The second part is the dissipative force, caused by the fact that the refraction waves carry energy from the ship to infinity. For this reason, we define a damping coefficient B so that the dissipative force will be $-B\dot{x}$ where $\dot{x}$ is the heave velocity. The third part is

is an inertia force, caused by the fact that the heaving ship causes the fluid particles to move in an unsteady motion so that we define and "added" mass A and the inertia force becomes $-A\ddot{x}$ with $\ddot{x}$ the heave acceleration. If we denote the undisturbed incoming wave elevation amidships as $\eta(t)$:

$$\eta(t) = a_o e^{i\omega_o t} \qquad (1)$$

Where the real part of all complex quantities is meant, here and in the sequel. Then the excitation force will be

$$F = F_o e^{i\omega_o t} a_o \qquad (2)$$

Where F_o is complex (to take into account the phase difference with respect to the wave elevation), and the equation of motion becomes:

$$M\ddot{x} = F - A\ddot{x} - B\dot{x} - Cx \qquad (3)$$

Where M the mass of the cylinder; the motion is also sinusoidal so with x_o complex:

$$x(t) = x_o e^{i\omega_o t} \qquad (4)$$

A very important remark is that F_o,A,B depend on the frequency of the incoming wave ω_o . This can be easily understood by the fact that at various frequencies the heaving cylinder will produce waves with different wavelength. We rewrite, therefore, equation (3) as:

$$-Mx_o\omega_o^2 e^{i\omega_o t} = \{F_o(\omega_o)a_o + [A(\omega_o)\omega_o^2 - i\omega_o B(\omega_o) - C]\, x_o\} e^{i\omega_o t} \qquad (3a)$$

By dropping $e^{i\omega_o t}$, we can rewrite equation (3a) as:

$$\{-[M + A(\omega_o)]\omega_o^2 + i\omega_o B(\omega_o) + C\}\, x_o = F_o(\omega_o)\, a_o \qquad (3b)$$

The motion of a cylinder in water, therefore, results in an increase in the mass and damping term. Equation (3b) is used because of its similarity to a second order system, it is strictly valid, though, only for a monochromatic wave.

Ultimately, we wish to obtain the response in a random sea, so equation (3b) must be extended for a random sea. This can be done by obtaining the inverse Fourier transform of (3a), i.e.

$$\int_{-\infty}^{\infty} K_a(t-\tau)\,\ddot{x}(\tau)\,d\tau + \int_{-\infty}^{\infty} K_u(t-\tau)\,\dot{x}(\tau)\,d\tau + C\,x(t) = \int_{-\infty}^{\infty} K_f(t-\tau)\,\eta(t)\,d\tau \tag{5}$$

Where K_a, K_u, K_f the inverse Fourier transform of $-\omega^2[M + A(\omega)]$, $i\omega B(\omega)$ and $F_o(\omega)$ respectively. The random undisturbed wave elevation is denoted by $\eta(t)$. Equation (5) is not popular with hydrodynamicists, because the effort required to evaluate the kernels K_a, K_u, K_f is by far greater than that required to find the added mass, damping and excitation force. For this reason, equation (5) is rewritten in a hybrid form as follows:

$$-[M + A(\omega)]\,\ddot{X}(t) + B(\omega)\,\dot{x}(t) + C\,x(t) = F(\omega)\,\eta(t) \tag{6}$$

This is an integro-differential equation (or differential equation with frequency dependent coefficients), whose meaning is in the sense of equation (5).

Strip Theory

The evaluation of $A(\omega)$, $B(\omega)$, $F(\omega)$ is not an easy task for complex geometries, such as the hull of a ship. The hydrodynamic particulars can be found in a later section, but we can give a simple description here of a technique used to simplify the derivations: [15], [17]

The ship can be divided in many transverse strips as shown in Figure (1.5). Due to its elongated form and for high frequencies, each strip has small interactions with the other strips, except near the ends. Usually these end effects are small, so that instead of solving the overall three dimensional problem, we can solve many two dimensional problems (one for each strip) and sum up all the partial results. For the case of heave, for example, if $A(\omega,x)$ $B(\omega,x)$ are the added mass and damping in heave of a strip at location x, then

$$A(\omega) = \int_{-L/2}^{L/2} A(\omega,x)\, dx \tag{7}$$

$$B(\omega) - \int_{-L/2}^{L/2} B(\omega,x)\, dx \tag{8}$$

The strip theory has larger errors at smaller frequencies. It so happens, though, that at small frequencies the hydrostatic forces are predominant, so that the motion error is quite small. Comparison with experiments has shown that for slender ship

configurations, the strip theory provides very good predictions [15],[17].

Relation Between Added Mass And Damping

The added mass and damping coefficients are not independent of each other, because their frequency dependence is caused by the same refraction waves. If we define

$$T(\omega) = \omega^2 [A(\omega) - \frac{B(\omega)}{\omega}] \qquad (9)$$

Then $T(\omega)$ is an analytic function [16]. As a result, $A(\omega)$, $B(\omega)$, which are real, are related by the Kramers-Kronig relations, in order to describe a causal system. This fact will be used later to obtain a single approximation for $T(\omega)$ instead of two separate approximations for $A(\omega)$, $B(\omega)$.

Speed Effects

As it can be seen in Figure 1.6 when the ship is heaving with a small angle θ and at the same time is moving forward with speed U, then a heave velocity results, which is $\dot{x} = U\theta$. The effect of the forward speed, therefore, is to couple the various motions by speed dependent coefficients. As it can be found in Appendix 1, there are simplified expressions for the added mass, damping and exciting force with a parametric dependence on the speed U. Then

expressions greatly facilitate the evaluation of the ship motions.

Frequency Of Encounter

An additional effect of the ship speed is the change in the frequency of encounter. If the incident wave has a frequency ω and a wave number k, then the frequency of encounter ω_e is

$$\omega_e = \omega + k\, U \cos \phi \tag{10}$$

Where ϕ is the angle between the x axis of the ship and the direction of wave propagation (Figure 1.7). In deep water, the dispersion relation for water waves is

$$\omega^2 = kg \tag{11}$$

so that we can rewrite (10) as

$$\omega_e = \omega + \frac{\omega^2}{g}\, U \cos \phi \tag{12}$$

A very important consideration in the difference between frequency of encounter and wave frequency is the following: The ship motions due to linearity will be of frequency ω_e so that the refraction waves are of frequency ω_e and the added mass and damping can be written as $A(\omega_e)$, $B(\omega_e)$.

The amplitude of the exciting force though, consists of the Froude Krylov part which depends on ω and the diffraction and speed dependent parts which depend on ω_e. The time dependence is again $\omega_e \cdot t$, i.e.

$$F(t) = a_o F(\omega, \omega_e, \nu)\, e^{i\omega_o t} \qquad (13)$$

with a_o the incident wave amplitude.

This is a very crucial observation and can cause significant errors if not taken into account.

Equation of Motion

Following the notation of Appendix 1, we write the equations of motion. It should be noted that, due to the slenderness of the ship, the surge motion is left out as a second order motion. This is in agreement with experiments [17]. Within linear theory and using the ship symmetry, the heave and pitch motions are not coupled with the group of sway, roll, yaw motions. This is not to imply that the motions are independent, because they are excited by the same wave, so there is a definite relation both in amplitude and phase.

(1) Heave - Pitch Motions

$$\left\{\begin{bmatrix} M & 0 \\ 0 & I_y \end{bmatrix} + \begin{bmatrix} A_{33} & A_{35} \\ A_{53} & A_{55} \end{bmatrix}\right\} \ddot{\underline{x}}_v + \begin{bmatrix} B_{33} & B_{35} \\ B_{53} & B_{55} \end{bmatrix} \dot{\underline{x}}_v +$$

$$+ \begin{bmatrix} C_{33} & C_{35} \\ C_{53} & C_{55} \end{bmatrix} \underline{x}_v = \begin{bmatrix} F_3 \\ F_5 \end{bmatrix} \eta \qquad (14)$$

(2) Sway - Roll - Yaw Motion

$$\left\{\begin{bmatrix} M & M_{24} & M_{26} \\ M_{42} & I_x & I_{xz} \\ M_{62} & I_{xz} & I_z \end{bmatrix} + \begin{bmatrix} A_{22} & A_{24} & A_{26} \\ A_{42} & A_{44} & A_{46} \\ A_{62} & A_{64} & A_{66} \end{bmatrix}\right\} \ddot{\underline{x}}_u +$$

$$\begin{bmatrix} B_{22} & B_{24} & B_{26} \\ B_{42} & B_{44} & B_{46} \\ B_{62} & B_{64} & B_{66} \end{bmatrix} \dot{\underline{x}}_u + \begin{bmatrix} 0 & 0 & 0 \\ 0 & C_{44} & 0 \\ 0 & 0 & 0 \end{bmatrix} \underline{x}_u = \qquad (15)$$

$$= \begin{bmatrix} F_2 \\ F_4 \\ F_6 \end{bmatrix} \eta$$

Where A_{ij}, B_{ij}, C_{ij} the added mass, damping, hydrostatic coefficient matrices; F_j the exciting forces; η the wave elevation;

$$\underline{x}_v = \{x_3, x_5\}^T \tag{16}$$

$$\underline{x}_u = \{x_2, x_4, x_6\}^T \tag{17}$$

The frequency and velocity dependence is not written explictly, but is understood, as described in the previous sections.

Heave - Pitch Approximation

We start with the heave and pitch motions approximation. As it is obvious from equation (14), it involves two stages:

(a) Approximation of the exciting force

(b) Approximation of the added mass and damping coefficients

Data are provided by the hydrodynamic theory for both components and within the wave frequency range.

A. Exciting Force Approximation

Figure 1.8 shows the exciting heave force on a box-like ship [25]. This information is important to demonstrate several zeros of the amplitude of the heaving force. Figure 1.9 shows the amplitude and phase of the exciting force on a destroyer, where, again, the same zeros appear, accompanied in the phase plot by jumps in the phase.

The transfer function between the wave elevation and the heave force cannot be represented as a ratio of polynomials of finite degree as evidenced by Figure 1.9. Similar plots can be obtained for the pitch moment. Within the wave frequency range, though, only the first zero is important, while the remaining peaks are of minor significance. This is not true for other types of vehicles such as the semi-submersible, but for ships it is valid for both heave force and pitch moment, so it will be used to simplify considerably the modeling procedure.

As it was mentioned before, the exciting force changes with frequency ω_e, but its amplitude is determined on the basis of the frequency ω. The following variables must be included in an

appropriate modeling of the exciting forces

(1) frequency ω

(2) speed V

(3) wave angle ϕ

$$F_3\ (t,\ \alpha_o,\ \phi,\ U) = F_3\ (\omega,\ \phi)\alpha_o\ e^{i\omega_e t} \tag{18}$$

$$F_5\ (t,\ \alpha_o,\ \phi,\ U) = \{F_5\ (\omega,\ \phi)\ + \frac{U}{i\omega}\ f_3(\omega_e,\phi)\}\ e^{i\omega_e t} \tag{19}$$

where a_o the wave amplitude, f_3 the heave diffraction force. Equations (18), (19) show that the heave force does not depend on the ship speed, whereas the pitch moment does, in a linear fashion.

In order to approximate $F_3(\omega,\phi)$, $F_5(\omega,\phi)$, $f_3(\omega_e,\phi)$ we use the plots in Figure 1.9 as well as Figure 1.10, which show the approximate influence of the wave angle on the excitation force.

In order to model the DD963 destroyer, the M.I.T. five degrees of freedom seakeeping program [27] was used to derive hydrodynamic results. The following model was derived to model shape of the heave force at V = 0 and ϕ = 0 (no speed, head seas)

$$F_3(s) = \frac{\alpha_1\ \eta}{\left[1 + 2J\frac{s}{\omega_a} + \frac{s^2}{\omega_a^2}\right]^2} \tag{20}$$

Where J = 0.707, α_1 a constant to be determined from hydrodynamic data, η the wave elevation and ω_a the corner frequency. Remembering the analysis above concerning the dependence of the force on ω and Figure 1.10, we can derive

$$\omega_a = \sqrt{\frac{2\pi g}{L\cos\phi + B}} + \frac{2\pi}{L\cos\phi + B} U\cos\phi \qquad (21)$$

where L is the ship length, B the beam.

Before we establish a relation similar to (20) above, we have to discuss Figure 11, where it is shown that <u>for long waves, the heave force and the pitching movement are 90° out of phase</u>. This means that the transfer function between heave and pitch is a non-minimum phase one, because the amplitude is constant, while the phase is 90°. We choose to attribute the non-minimum phase to pitch. Also, the pitch angle tends to the wave slope for large wavelengths, so the pitching moment can be written as

$$F_5 = \alpha_2 \frac{1 - s/\omega_o}{1 + s/\omega_o} \frac{s^2\cos\phi}{\left[1 + 2J\frac{s}{\omega_a} + \frac{s^2}{\omega_a^2}\right]^2} \eta \qquad (22)$$

Where α_2 a constant to be determined, ω_a is the same (for simplicity) as in equation (21) and ω_o is an artificial frequency to model the non-minimum phase. It will be chosen to be equal to the wave spectrum modal frequency, so we defer the discussion until the corresponding section.

B. <u>Added Mass and Damping</u>

By using equation (9), we can rewrite the equations of motion as

$$\begin{bmatrix} T_{33}s^2 + Ms^2 + C_{33} & T_{35} + UsT_{33} + C_{35} \\ T_{53}s^2 - UsT_{33} + C_{35} & Is^2 + T_{55}s^2 - U^2T_{33} + C_{55} \end{bmatrix} \begin{bmatrix} x_3 \\ x_5 \end{bmatrix} = \begin{bmatrix} F_3 \\ F_5 \end{bmatrix} \eta \qquad (23)$$

Here we construct a simplified model where

$$Tij = Aij - \frac{Bij}{i\omega} \tag{24}$$

with Aij, Bij to be evaluated from the hydrodynamic data. Then equation (23) can be written, after we define

$$\begin{aligned} y_1 &= x_3 \qquad y_2 = \dot{x}_3 \\ y_3 &= x_5 \qquad y_4 = \dot{x}_5 \\ \underline{y} &= \{y_1, y_2, y_3, y_4\}^T \end{aligned} \tag{25}$$

in the form:

$$\begin{bmatrix} \dot{y}_2 \\ \dot{y}_4 \end{bmatrix} = \begin{bmatrix} A_{33} & A_{35} \\ A_{53} & A_{55} \end{bmatrix}^{-1} \left\{ - \begin{bmatrix} C_{33} & B_{33} & C'_{35} & B_{35} \\ C'_{53} & B_{53} & C'_{55} & B_{55} \end{bmatrix} \underline{y} + \begin{bmatrix} F_3 \\ F_5 \end{bmatrix} \eta \right\} \tag{23a}$$

where

$$\begin{aligned} C'_{35} &= C_{35} + U\, A_{33} \\ C'_{53} &= C_{35} - U\, A_{33} \\ C'_{55} &= C_{55} - U^2 \times A_{33} \end{aligned} \tag{26}$$

Sway-Roll-Yaw Approximation

Next we approximate the sway-roll-yaw group of motions, which is uncoupled to first order from the heave-pitch group of motions. Again a two stage approximation is required, i.e.:

(a) Approximation of the exciting force
(b) Approximation of the added mass and damping coefficients

Data are obtained by using the sea-keeping program.

A. Exciting force approximation

The same infinite-dimensional form is obtained for the exciting force as seen in Figure 1.12 (17abc) for all three motions, as in the case of pitch and heave. Again, within the wave frequency range, a finite dimensional approximation can be achieved, and of reasonably small order.

The important fact that the exciting force depends on the wave frequency rather than the frequency of encounter, is used, while the following three quantities define the exciting force amplitude and phase

(1) Wave frequency
(2) Speed V
(3) Wave angle ϕ

In Appendix 1 the strip theory approximation of the sway, roll,yaw forces can be found. Using the M.I.T. five degrees of freedom seakeeping program the following finite dimensional approximation was found in case of $V = 0$, $\phi = 90^0$ for sway, roll and yaw

$$F_2(S) = \frac{A_2 \quad S^2}{\left(\frac{S}{\omega_2}\right)^2 + 2J_2 \frac{S}{\omega_2} + 1} \tag{27}$$

$$F_4(S) = \frac{A_4 \quad S^2}{\left(\frac{S}{\omega_4}\right)^2 + 2J_4 \frac{S}{\omega_4} + 1} \tag{28}$$

$$F_6(S) = \frac{A_6 \quad S^2}{\left(\frac{S}{\omega_6}\right)^2 + 2J_6 \frac{S}{\omega_6} + 1} \tag{29}$$

where $\omega_2 = 0.65$ $J_2 = .5$

$\omega_4 = 0.85$ $J_4 = .3$

$\omega_6 = 0.85$ $J_6 = .3$ (30)

and A_2, A_4, A_6 are obtained from hydrodynamic data.

We redefine the value of ω_2, ω_4, ω_6 such that it will be valid for angles ϕ other than 90^0, and speed other than 0:

$$\omega'_j = (\omega_j + \omega_j^2 \frac{V}{g} \cos\phi) \sin\phi \tag{31}$$

where $j = 2,4,6$ and ω_j is given above.

It should be noted that the sway, roll and yaw forces are proportional to the wave slope, i.e. 90^0 out of phase with respect to the wave amplitude. This means that they belong to the same group with pitch, and the same non-minimum phase transfer function:

$$\frac{s - \omega_0}{s + \omega_0}$$

must be used for all three of them, when the total system (all 5 degrees of motion) is considered.

B. Added Mass and Damping

The amplitude of the transfer function between the wave elevation and the rolling motion has a very narrow peak so that the coefficients can be approximated as constant [17]. Using Appendix 1:

$$A_{44} = A^0_{44} \qquad B_{44} = B^0_{44}$$

$$A_{42} = A^0_{24} \qquad B_{42} = B^0_{24}$$

$$A_{46} = A^0_{46} + \frac{V}{\omega^2} B^0_{24}$$

$$B_{46} = B^0_{46} - V A^0_{24} \tag{32}$$

using the value of ω at the roll peak. It should be noted that roll involves a significant nonlinear (viscous) damping, which is approximated by introducing an additional "equivalent" damping coefficient B^*_{44} [3]

Similarly, we calculate the sway, yaw coefficients at the same frequency:

$$A_{22} = A^0_{22} \qquad B_{22} = B^0_{22}$$

$$A_{26} = A^0_{26} + \frac{V}{\omega^2} B^0_{22}$$

$$B_{26} = B^0_{26} - V A^0_{22}$$

$$A_{62} = A^0_{26} - \frac{V}{\omega^2} B^0_{22}$$

$$B_{62} = B^0_{26} + V A^0_{22}$$

$$A_{66} = A^0_{66} + \frac{V^2}{\omega^2} A^0_{22}$$

$$B_{66} = B^0_{66} + \frac{V}{\omega^2} B^0_{22} \tag{33}$$

$$\left[S^2 \{A_{ij}+M_{ij}\} + S\{B_{ij}\} + \{C_{ij}\} \right] \begin{Bmatrix} X_2 \\ X_4 \\ X_6 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_4 \\ F_6 \end{Bmatrix} \qquad \begin{matrix} i = 2,4,6 \\ j = 2,4,6 \end{matrix} \tag{34}$$

where $C_{ij} = 0$ except for C_{44}, which is the roll hydrostatic constant, i.e. $C_{44} = \Delta \cdot (GM)$ with Δ the ship displacement and (GM) the metacentric height.

Due to the special form of the matrix C, a zero-pole cancellation results from a direct state space representation of the equation above. After some manipulations, the following representation can be obtained which avoids zero-pole cancellation problems:

$$\dot{\underline{X}} = T\,\underline{X} + U\,\underline{F}$$

$$\text{where } \underline{X} = \{X_2 \;,\; \dot{X}_4 \;,\; X_4 \;,\; X_6\}^T$$

$$\underline{F} = \{\int F_2,\; F_2 \;,\; F_4 \;,\; \int F_6,\; F_6\} \tag{35}$$

where $\int F$ indicates the time integral of F and $T = \{t_{ij}\}$ and $U = \{u_{ij}\}$, with:

$$t_{11} = \frac{r_{12}}{r_{22}} P_{21} - P_{11} \;, t_{12} = \frac{r_{12}}{r_{22}} \;,\; t_{13} = \frac{r_{12}}{r_{22}} P - P_{12} \;,\; t_{14} = \frac{r_{12}}{r_{22}} P_{23} - P_{13}$$

$$t_{41} = \frac{r_{32}}{r_{22}} P_{21} - P_{31} \;, t_{42} = \frac{r_{32}}{r_{22}} \;,\; t_{43} = \frac{r_{32}}{r_{22}} P_{22} - P_{32} \;,\; t_{44} = \frac{r_{32}}{r_{22}} P_{23} - P_{33}$$

$$t_{21} = -P_{21} t_{11} - P_{23} t_{41} \qquad t_{22} = -P_{21} t_{12} - P_{23} t_{42} - P_{22}$$

$$t_{23} = -P_{21} t_{13} - P_{23} t_{43} - r_{22} C_{44}$$

$$t_{24} = -P_{21} t_{14} - P_{23} t_{44}$$

$$t_{3i} = 0 \quad \text{except for} \quad t_{32} = 1 \tag{36}$$

$$U_{ij} = 0 \quad \text{except for} \quad U_{11} = r_{11} - \frac{r_{12}\, r_{21}}{r_{22}}$$

$$U_{14} = r_{13} - \frac{r_{12}\, r_{23}}{r_{22}} \;, \quad U_{41} = r_{31} - \frac{r_{32}\, r_{21}}{r_{22}}$$

$$U_{44} = r_{33} - \frac{r_{32}\, r_{23}}{r_{22}} \;, \qquad U_{21} = -P_{21} U_{11} - P_{23} U_{41}$$

$$U_{22} = r_{21} \;,\; U_{23} = r_{22} \;,\; U_{24} = -P_{21} U_{14} - P_{23} U_{44}$$

$$U_{25} = r_{23} \tag{37}$$

$$\text{where} \quad R = \{r_{ij}\} = [A+M]^{-1} \;, \quad P = \{P_{ij}\} = R\,B \tag{38}$$

Figures 1.13 through 1.15 show the actual force and moments versus frequency and the achieved finite dimensional approximation for zero speed and beam seas (ϕ=90°). It should be noted again that the approximation is not as good outside the wave frequency range.

Figures 1.16 through 1.18 show the same quantities for speed U= 15.5 ft/sec and 45° angle of incidence. From these figures it can be seen that above 1 rad/sec the approximation is poor, nonetheless no significant wave energy is contained in that range, so the approximation is acceptable.

Figures 1.19 through 1.21 show the overall transfer function between the corresponding motion and the sea elevation.

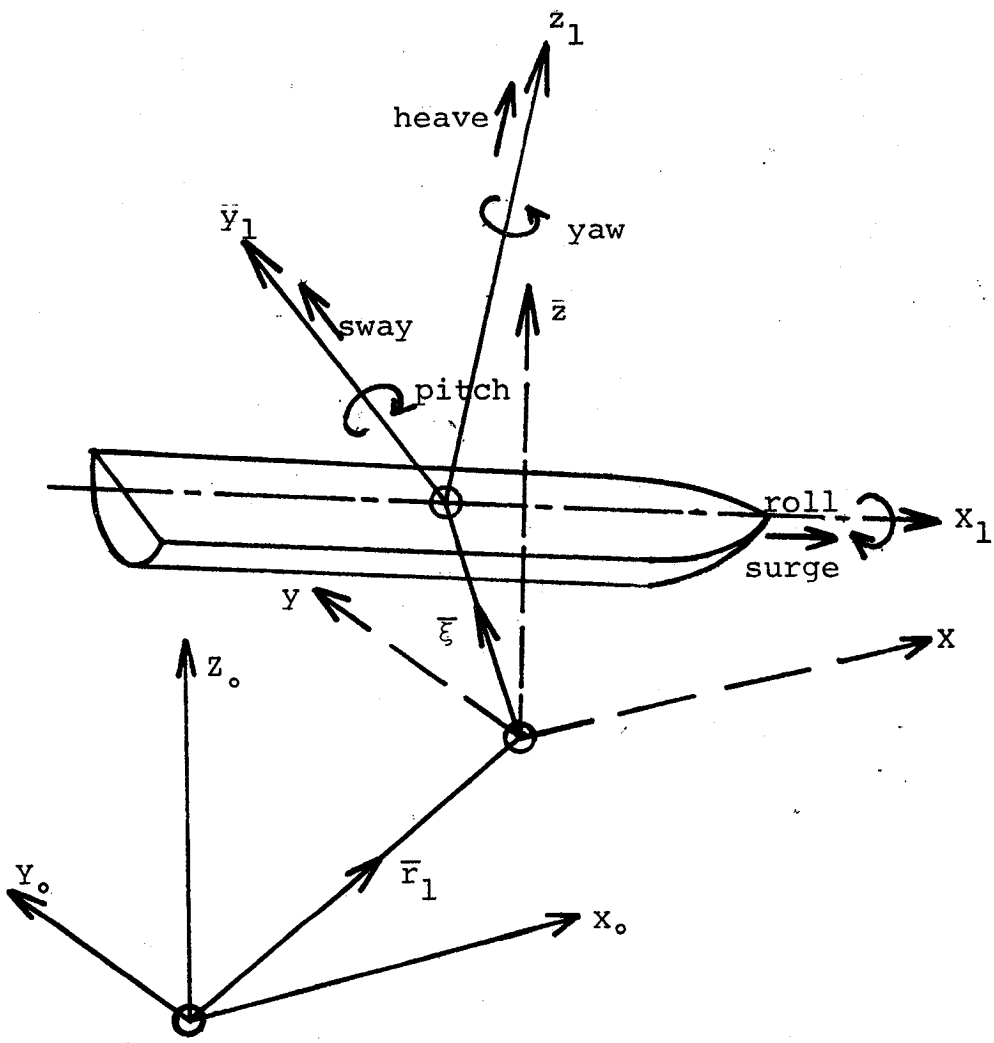

Figure 1.1

Ship Reference Systems

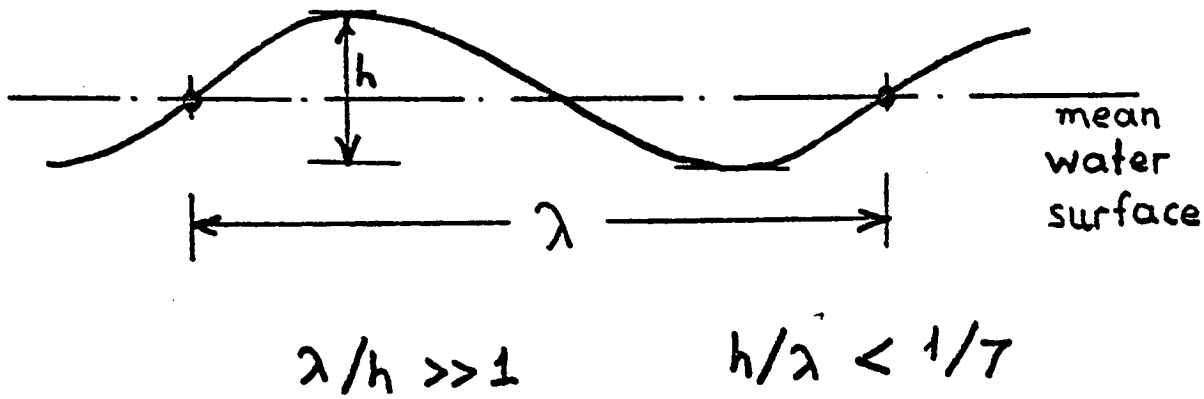

Figure 1.2

Wave Profile

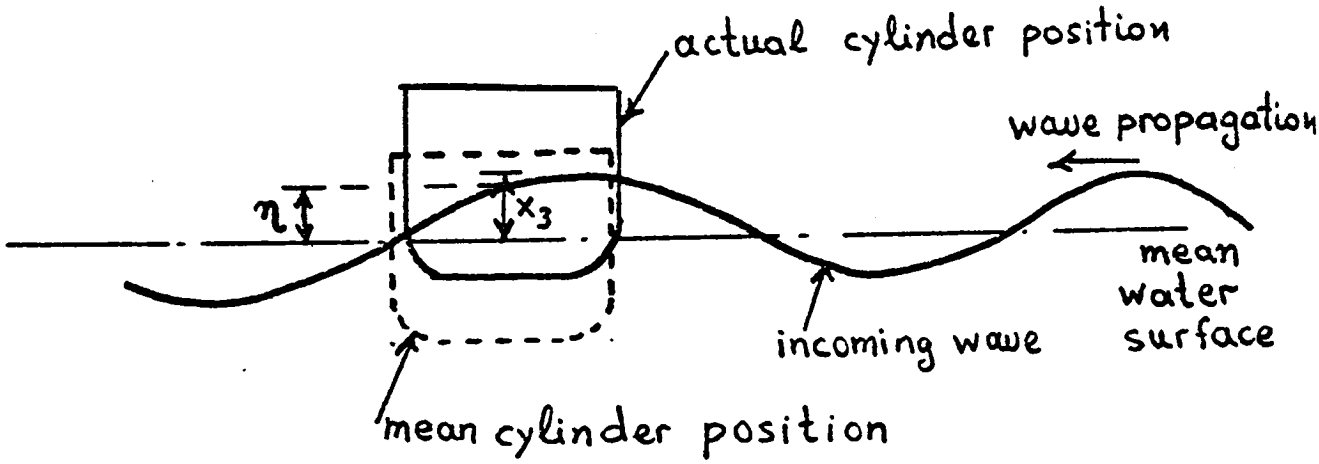

Figure 1.3

Hydrodynamic Problem

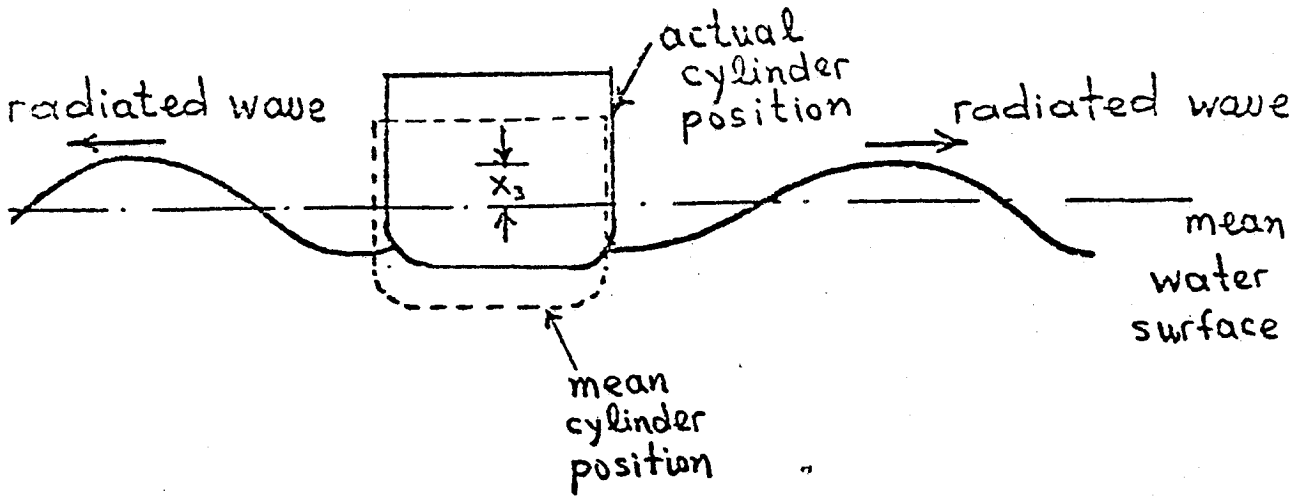

Figure 1.4a
Radiation Problem

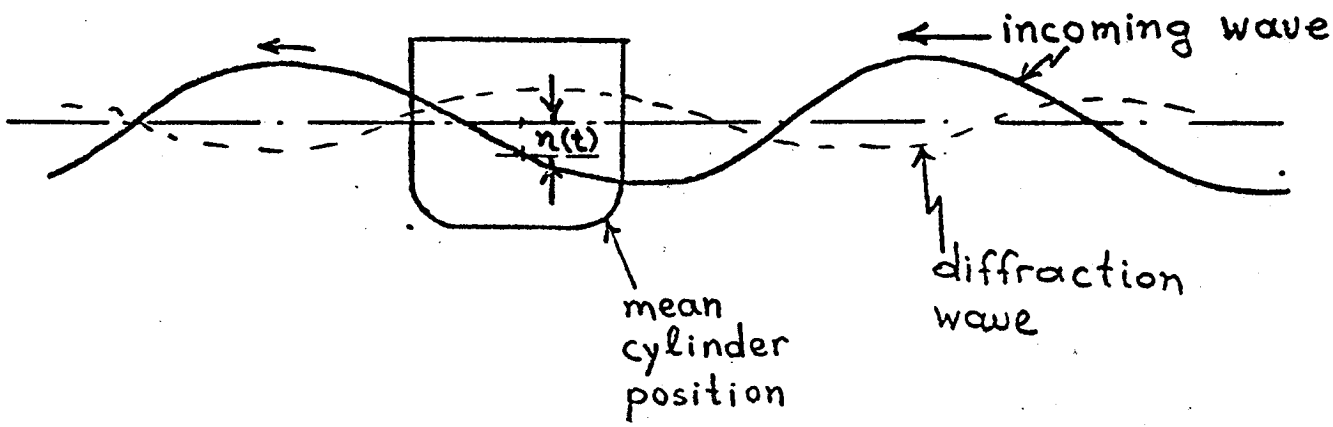

Figure 1.4b
Diffraction Problem

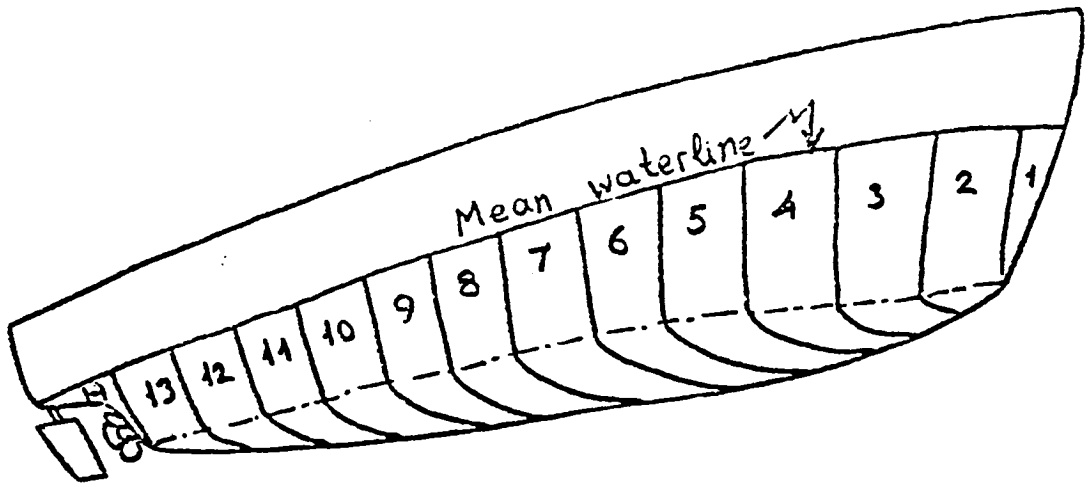

Figure 1.5
Strip Subdivision of Ship Hull

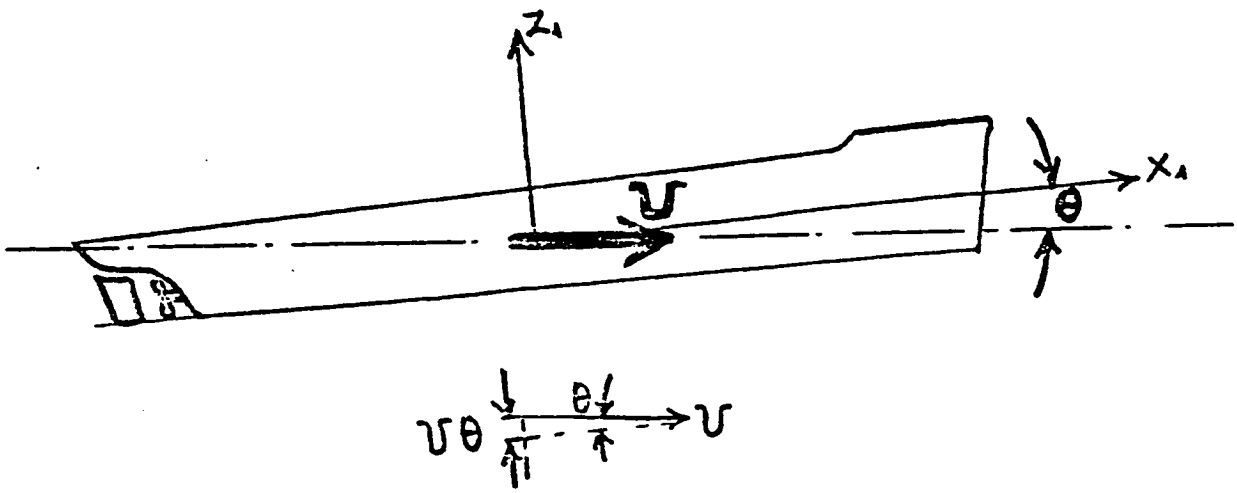

Figure 1.6
Effects of Ship Speed

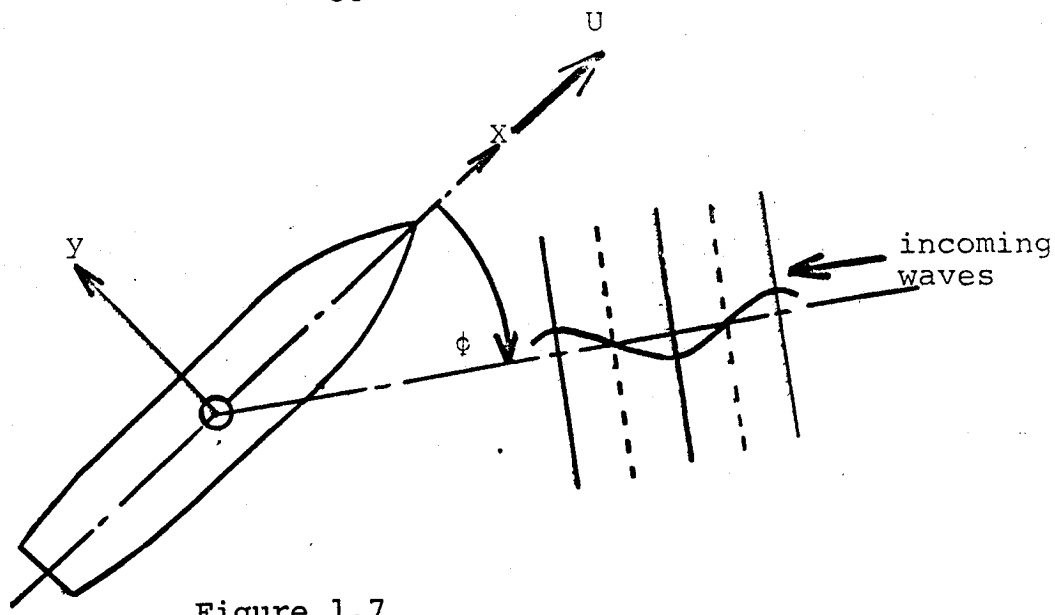

Figure 1.7
Angle of Incoming Waves

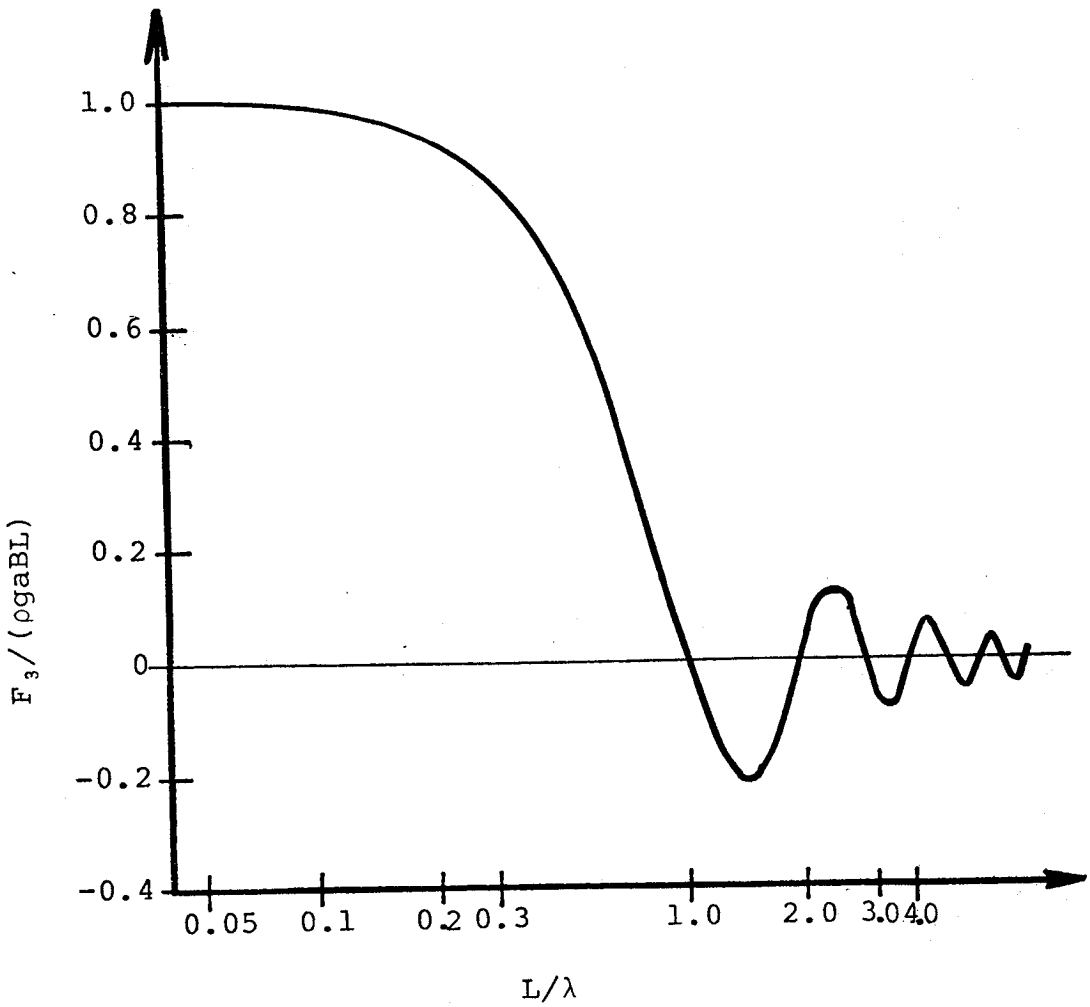

Figure 1.8
Heave Exciting Force on a Rectangular Barge
of Length L, Beam B

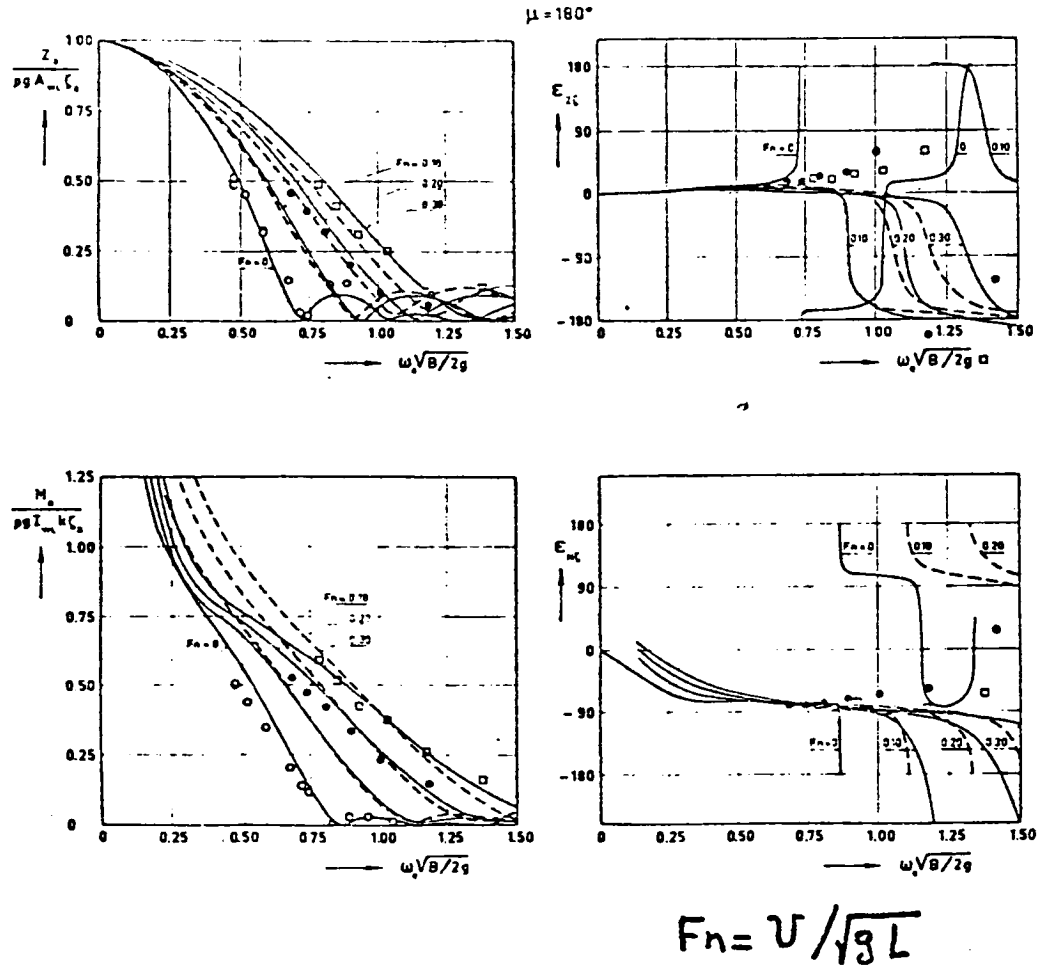

Figure 1.9

Effect of Ship Speed on Heave Force and Pitch Moment at Head Seas

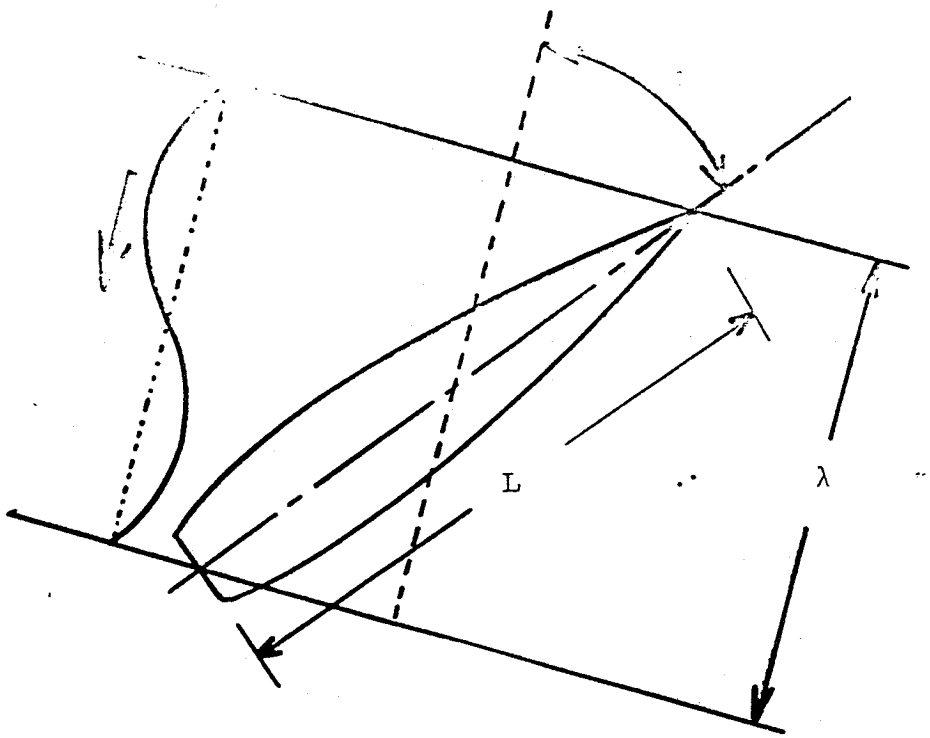

Figure 1.10

The Equivalent Wavelength for Heave and Pitch is $\lambda/\cos\phi$

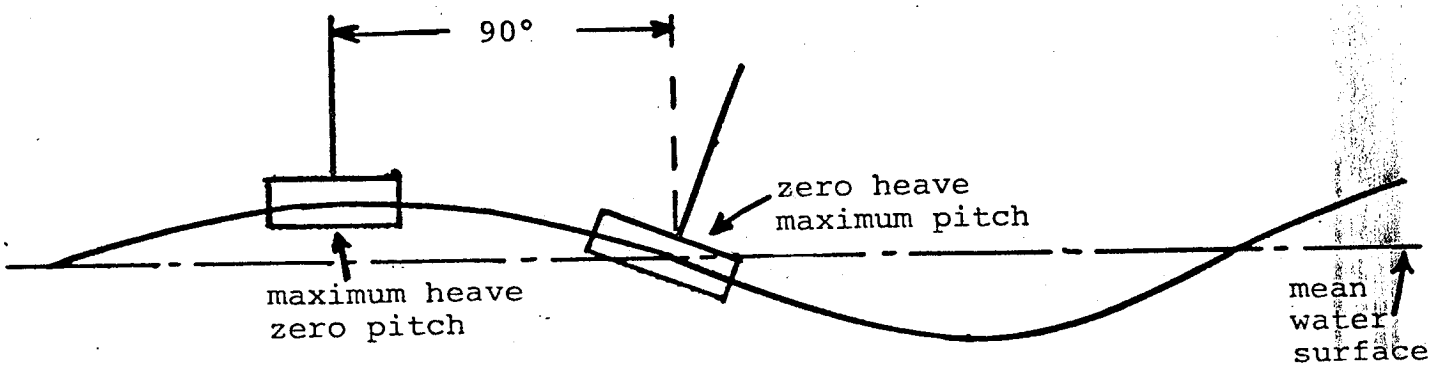

Figure 1.11

Phase Difference Between Heave and Pitch

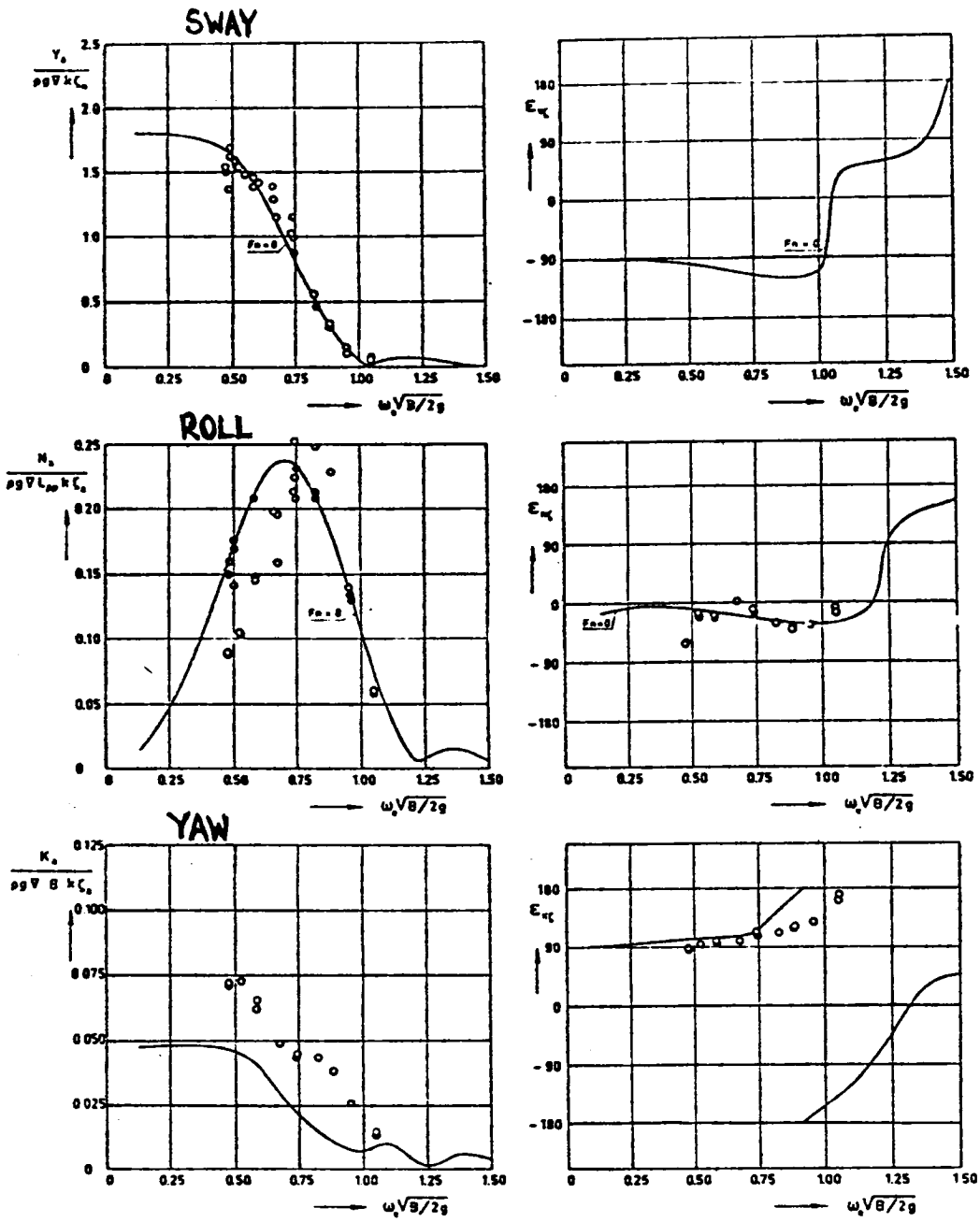

Figure 1.12

Wave Force and Moments in Sway, Roll, and Yaw at 60^0 Angle and Zero Forward Speed

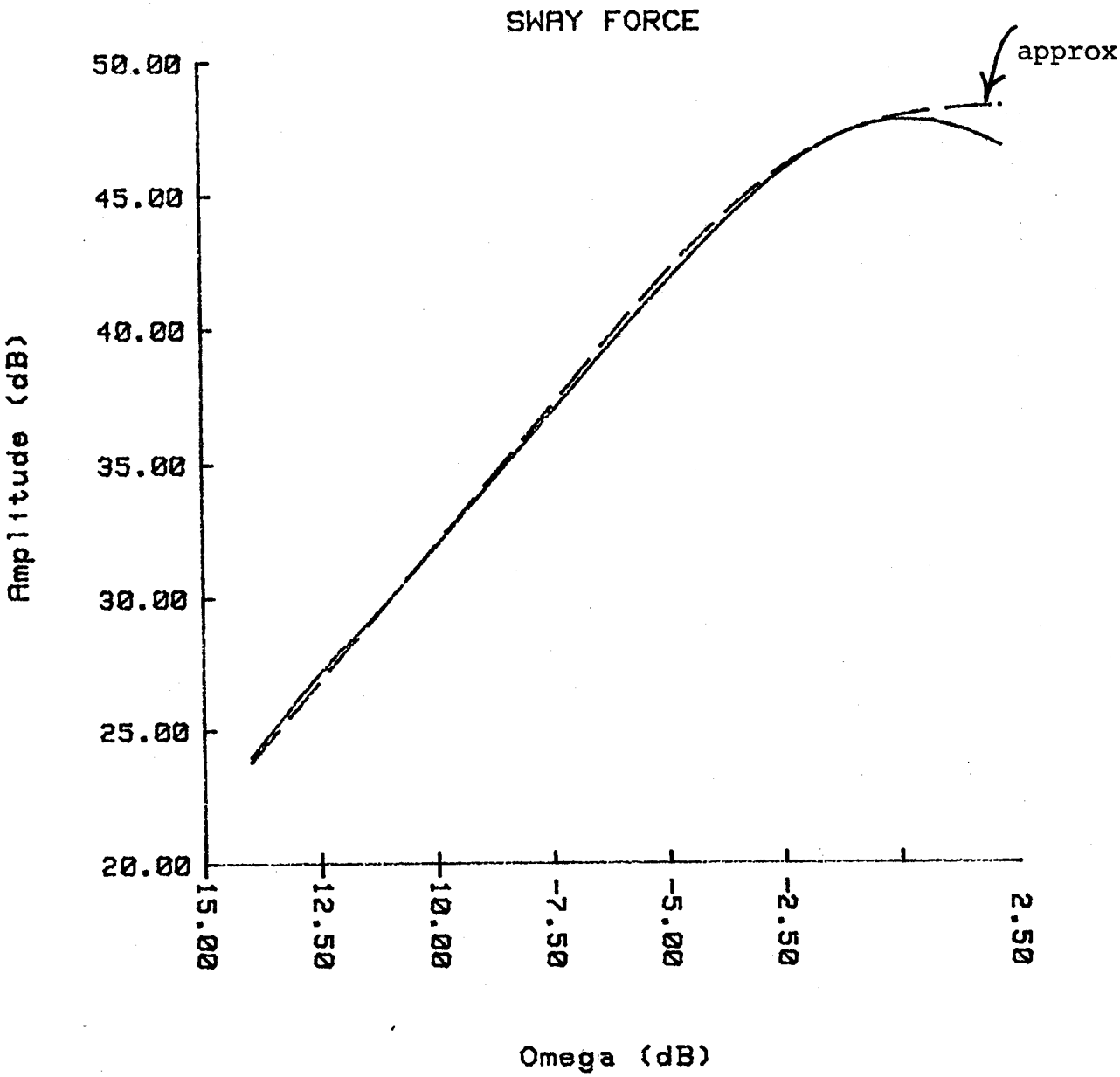

Figure 1.13

Sway Force Versus Frequency for the DD963 Destroyer and Its Finite Dimensional Approximation (dotted line) for Speed U = 15.5 ft/sec and 45^0 Angle of Incidence

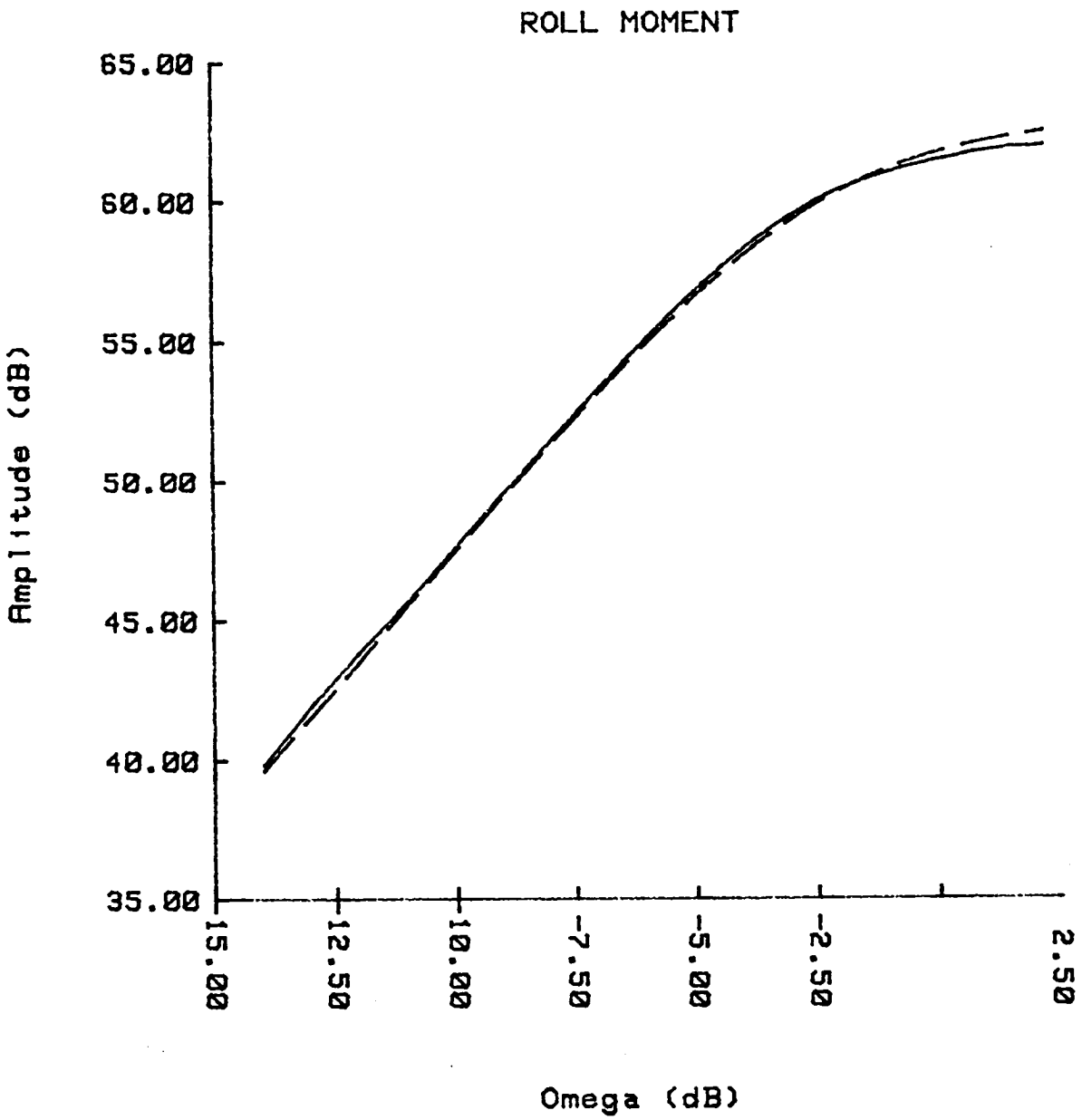

Figure 1.14

Roll Moment for DD963. Same conditions as in 1.13.

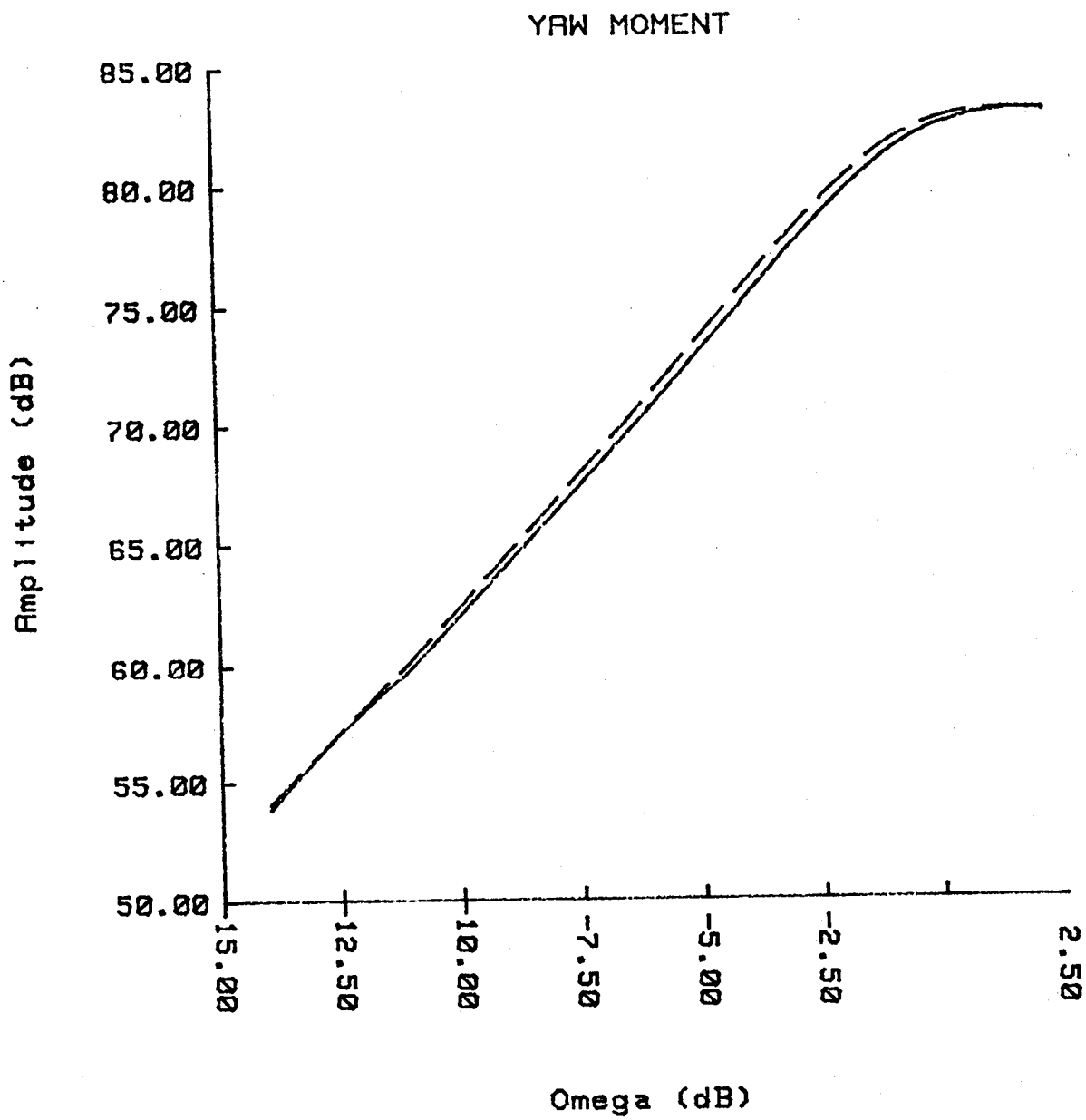

Figure 1.15

Yaw Moment for DD963. Same conditions as in 1.13.

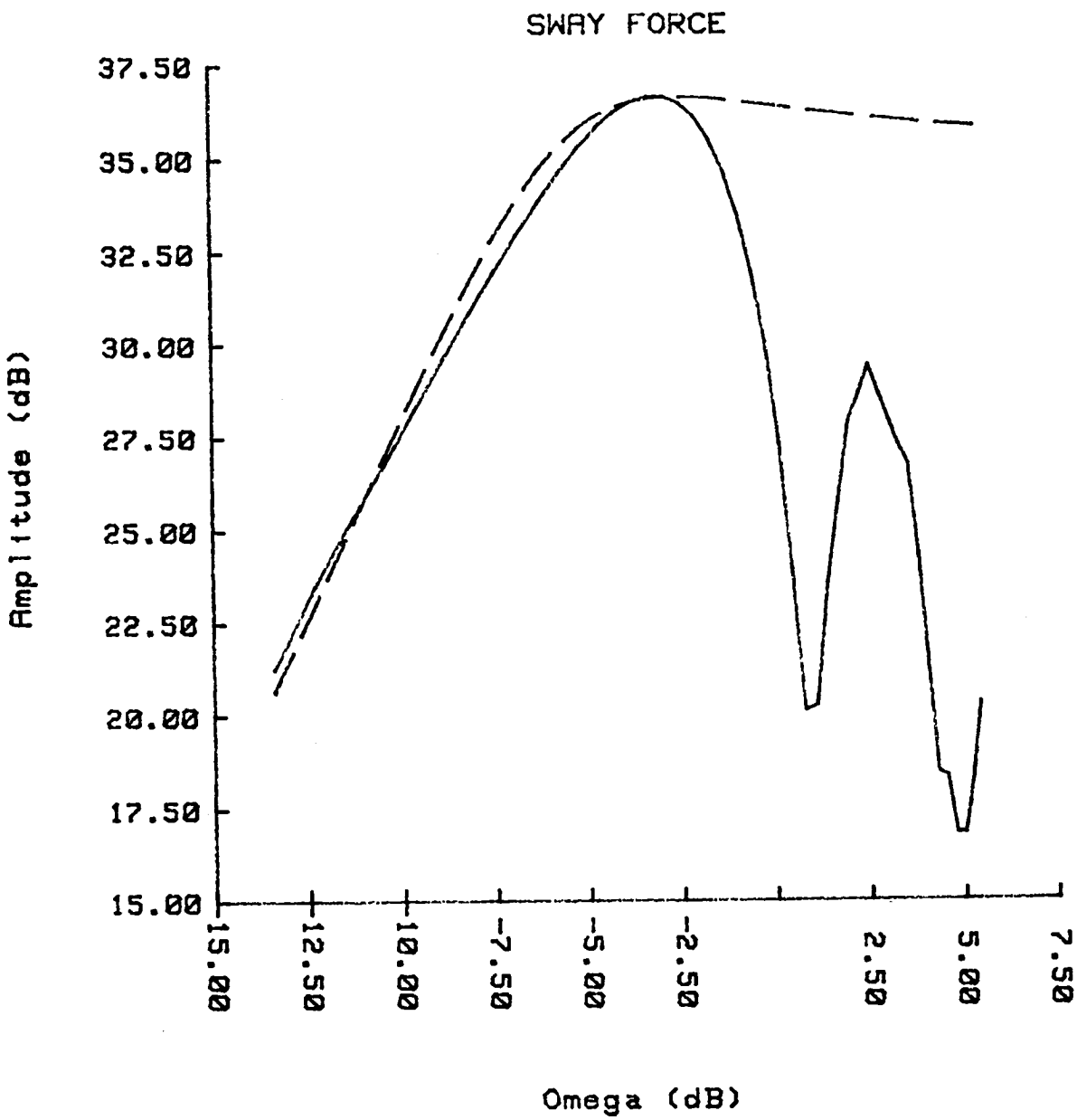

Figure 1.16

Sway Force Versus Frequency for the DD963 Destroyer and Its Finite Dimensional Approximation (dotted line), for Speed U = 15.5 ft/sec and 45^0 Angle of Incidence

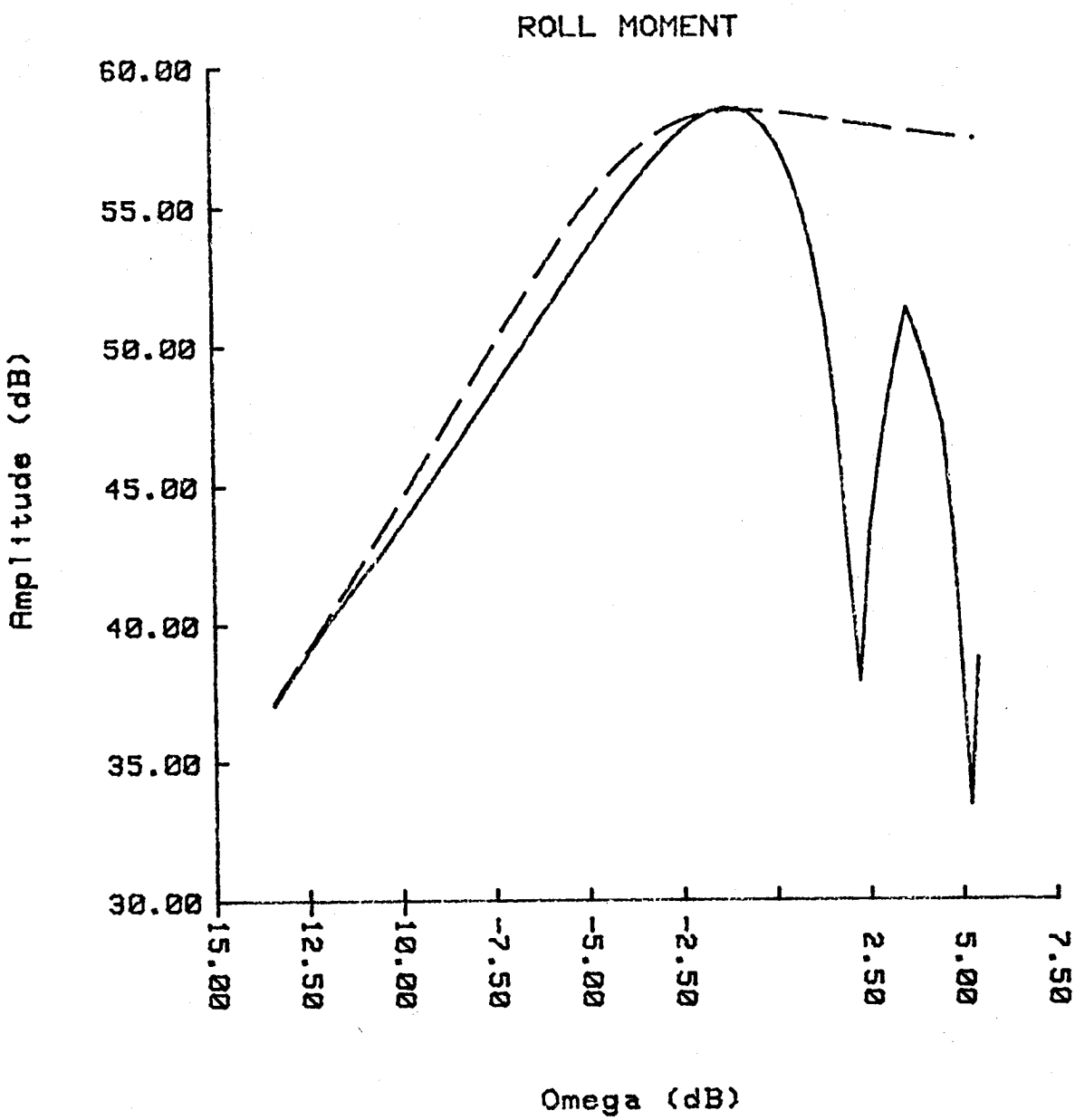

Figure 1.17

Roll Moment for DD963. Same conditions as in 1.16.

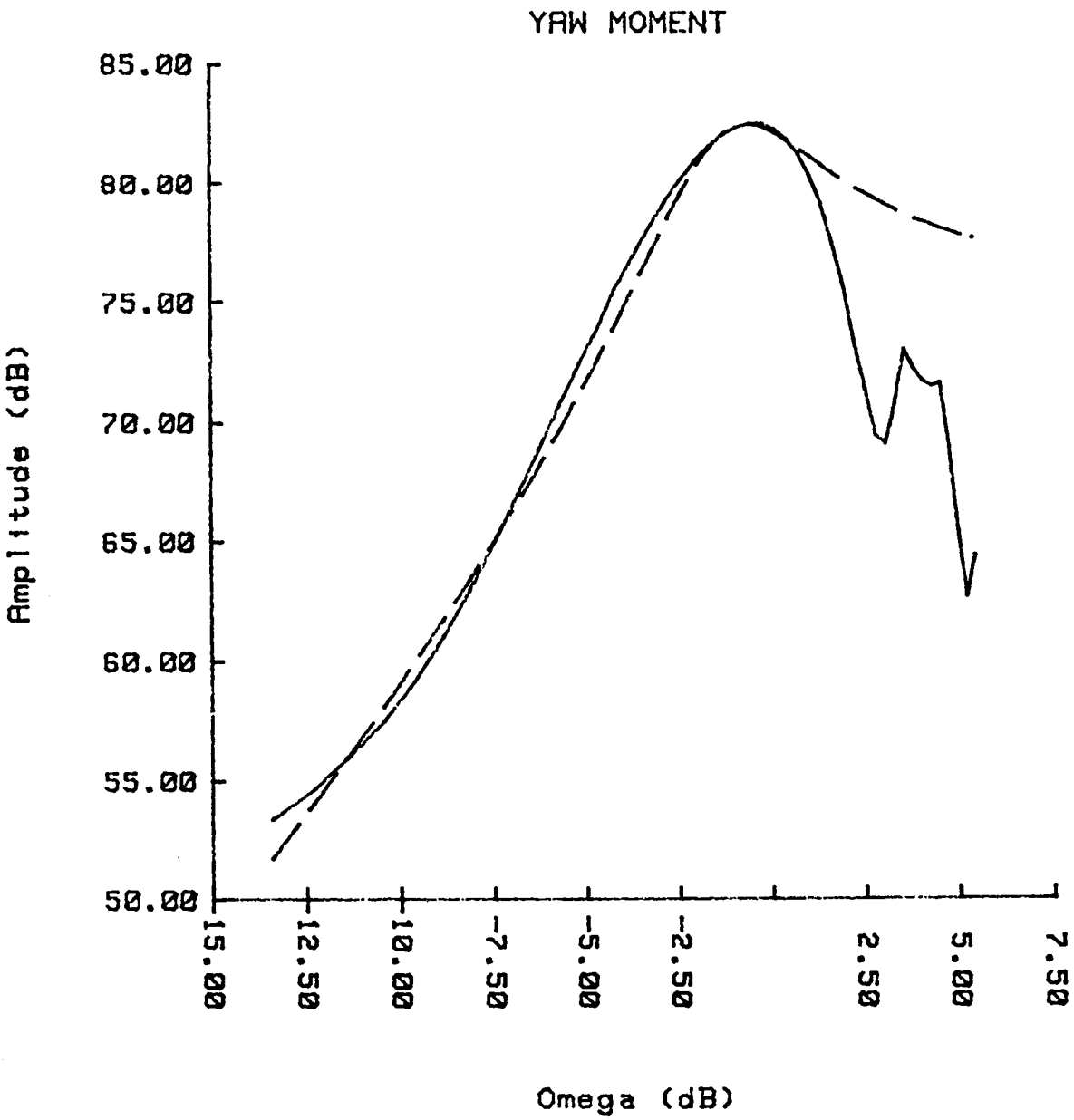

Figure 1.18

Yaw Moment for DD963. Same conditions as in 1.16.

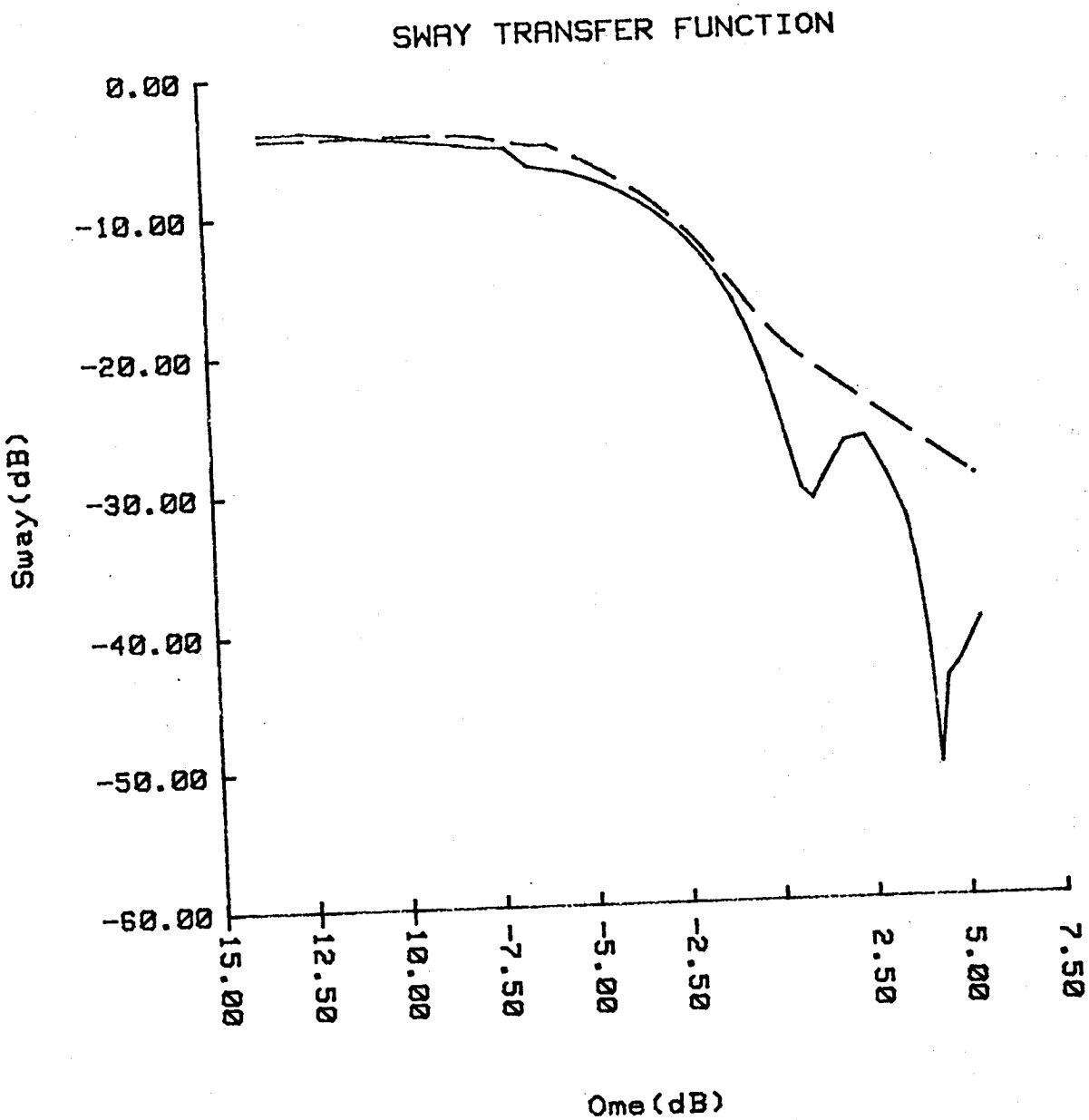

Figure 1.19

Sway Transfer Function for the DD963 Destroyer and Its Finite Dimensional Approximation (dotted line), for Speed U = 15.5 ft/sec and 45^0 Angle of Incidence

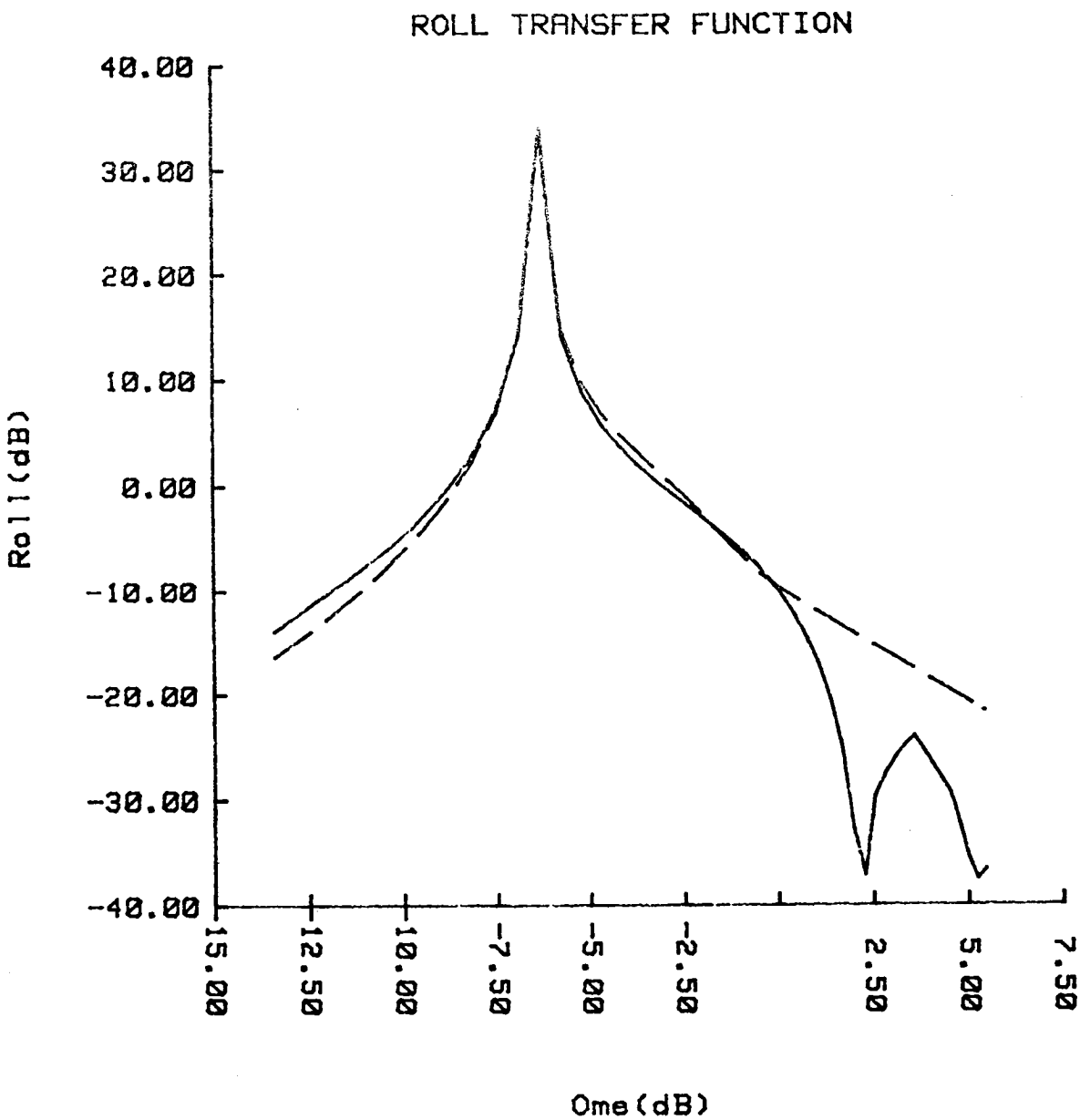

Figure 1.20

Roll Transfer Function for DD963. Same conditions as in 1.19.

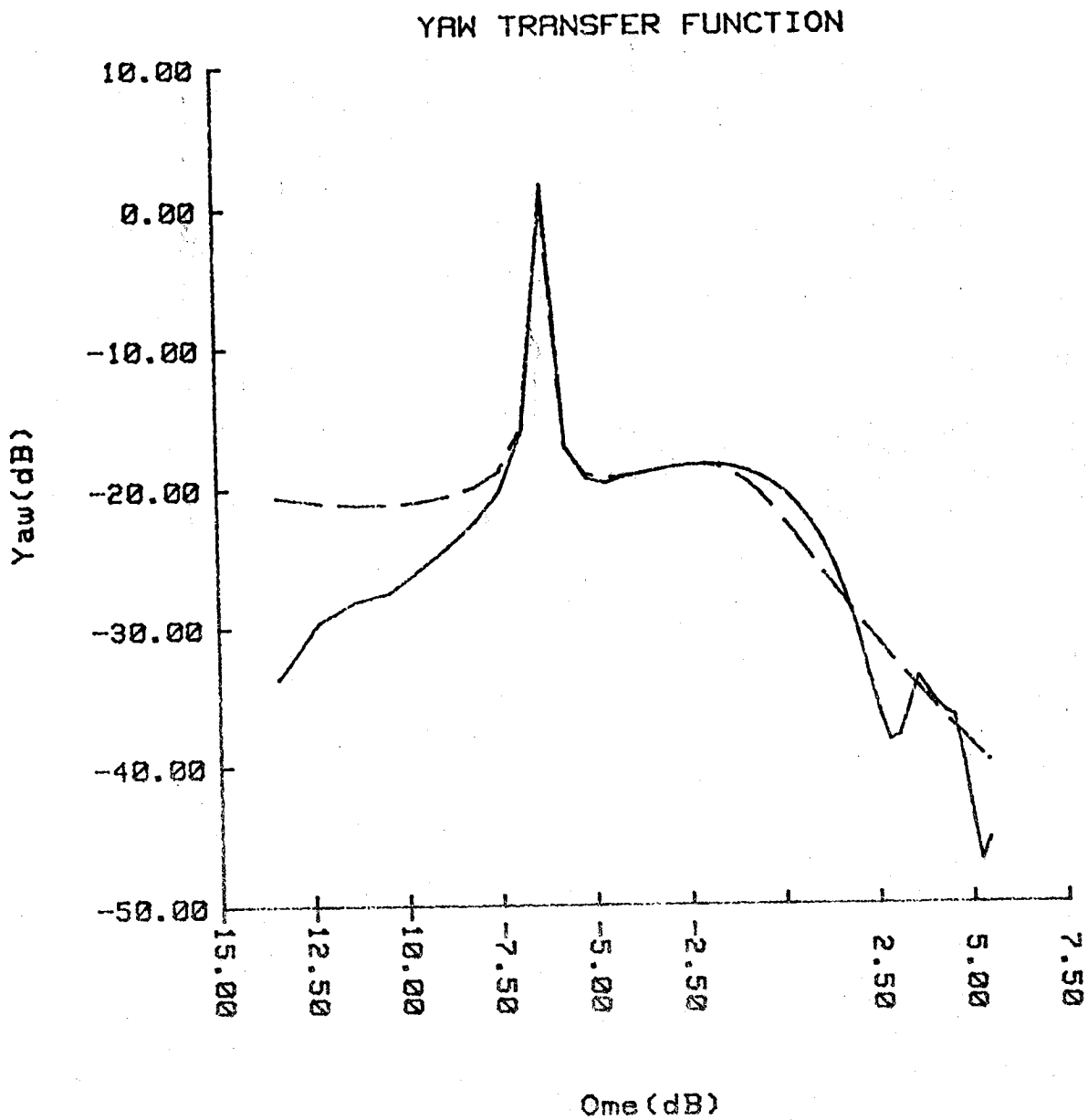

Figure 1.21

Yaw Transfer Function for DD963. Same conditions as in 1.19.

Chapter 2: SEA MODELING

The sea waves are generated by the wind, except for very few cases. The process of wave generation is of importance in modeling, so we will outline, briefly, a simple theoretical model:

When the wind starts blowing over a calm water surface, it contains gust components of high frequency, which cause wavelets on the surface. This is due to the inherent instability of the wave air interface. As soon as the surface becomes rough, a significant drag force develops between air and water, which becomes zero only if the average wind speed (which causes the major part of the drag) equals the phase wave velocity. As a result, the steady-state condition of the sea develops slowly by creating waves whose phase velocity is close to the wind speed. Since the process starts with high frequencies, we conclude that a young storm will contain a peak at high frequency. We usually distinguish between a developing storm and a fully developed storm.

As soon as the wind stops blowing, then the water viscosity dissipates the high frequency waves so that the so called swell (decaying seas) forms, which consists of long waves (low frequency content), which travel away from the storm that originates them. For this reason, swell can be found together with another local storm (Figure 2.1).

A storm usually contains one peak (except if swell is present when it contains two peaks) and the peak frequency ω_m is called the modal frequency (Figure 2.2). Also, the intensity of the storm is required, which can be described in a number of waves: Beaufort Scale, Sea State, Wind Average Velocity, Significant Wave Height. The best is the significant wave height H defined as the statistical average of the 1/3 highest waveheight. For a narrow band spectrum of area M_o

$$H \simeq 4 \sqrt{M_o} \tag{39}$$

From our discussion on sea storm generation, we conclude that it is important to model a storm by both H (intensity) and ω_m (duration of storm). For this reason, the Bretschneider Spectrum will be used defined as:

$$S(\omega) = \frac{1.25}{4} H^2 \frac{\omega_m^4}{\omega^5} \exp \{ - 1.25 (\frac{\omega_m}{\omega})^4 \} \tag{40}$$

The spectrum was developed by Bretschneider for the North Atlantic, for unidirectional seas, with unlimited fetch, infinite depth and no swell. It was developed to satisfy asymptotic theoretical predictions and to fit North Atlantic data. It was found to fit reasonably well in any sea location. Also, by combining two such spectra, we can model the swell as well. Its main limitations are unidirectionality and unlimited fetch.

It was felt, however, that it could provide an adequate description for the present application for open sea.

As it has already been mentioned, the forward speed of the vessel causes a shift in the wave frequency to the frequency of encounter. The spectrum, now, can be defined for ship coordinates as follows

$$S(\omega_e) = \left[\frac{S(\omega)}{d\omega_e/d\omega} \right] \quad \omega = f(\omega_e) \tag{41}$$

where

$$\omega = f(\omega_e) = \frac{-1 + \sqrt{1 + 4\omega_e \frac{U\cos\phi}{g}}}{2 \frac{U}{g} \cos\phi} \tag{42}$$

A rational approximation was found to (29) subject to (30) in the following form

$$S_a(\omega_e) = \frac{1.25}{4\omega_m^5} H^2 B(\alpha) \frac{(\omega_e/\omega_o)^4}{\left[1 + \left(\frac{\omega_e}{\omega_o} \right)^4 \right]^3} \tag{43}$$

where $S_a(\omega_e)$ the approximate spectrum

$$\alpha = \frac{U}{g} \omega_m \cos\phi \tag{44}$$

$B(\alpha)$, $\gamma(\alpha)$ functions given in table 2.1

Now a transfer function can be deriven from (43) such as to provide an output with the spectrum in (40) when driven by white noise. It is easy to see that

$$H_a(s) = \sqrt{S_o} \; \frac{(s/\omega_o)^2}{\left[1 + 2J\,\frac{s}{\omega_o} + \left(\frac{s}{\omega_o}\right)^2\right]^3} \tag{45}$$

where

$$S_o = \frac{1.25}{4\omega_m} \; H^2 \quad B(\alpha) \tag{46}$$

$$\omega_o = \gamma(\alpha) \; \omega_m \tag{47}$$

$$J = 0.707$$

A plot of the spectrum for various wind speeds is given in Figure 2.3, while the spectrum and its rational approximation is plotted in Figure 2.4 for fully developed seas and $H^{1/3} = 3$ m.

Important Remark

It is customary to define the power spectral density as the Fourier transform of the autocorrelation $R(\tau)$

$$S(\omega) = \int_{-\infty}^{\infty} R(\tau) e^{-i\omega\tau} \, d\tau \tag{48}$$

In wave theory, the spectrum is defined one sided (for positive frequencies only) as follows

$$S_\beta(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} R(\tau) e^{-i\omega\tau} \, d\tau \qquad \omega > 0 \tag{49}$$

For this reason, the relation between the spectrum $S(\omega)$ as required for the present application and the Bretschneider spectrum $S_\beta(\omega)$ is:

$$S(\omega) = \begin{cases} \pi\ S_\beta(\omega) & \omega > 0 \\ \pi\ S_\beta(-\omega) & \omega < 0 \end{cases} \tag{50}$$

Therefore, the intensity of the white noise required for driving the transfer function (33) is π (or equivalently we can multiply the transfer function by $\sqrt{\pi}$).

TABLE 2.1

Sea Spectrum Coefficients

α	γ (α)	β (α)
.00	.9538	1.8861
.10	1.0902	1.6110
.20	1.1809	1.3827
.30	1.2717	1.2116
.40	1.3626	1.0785
.50	1.4539	.9718
.60	1.5448	.8845
.70	1.6360	.8116
.80	1.7272	.7498
.90	1.8182	.6968
1.00	1.9095	.6509
1.10	2.0008	.6106
1.20	2.0918	.5750
1.30	2.1833	.5434
1.40	2.2744	.5150
1.50	2.3657	.4895
1.60	2.4567	.4664
1.70	2.5481	.4454
1.80	2.6395	.4262
1.90	2.7306	.4085
2.00	2.8218	.3923

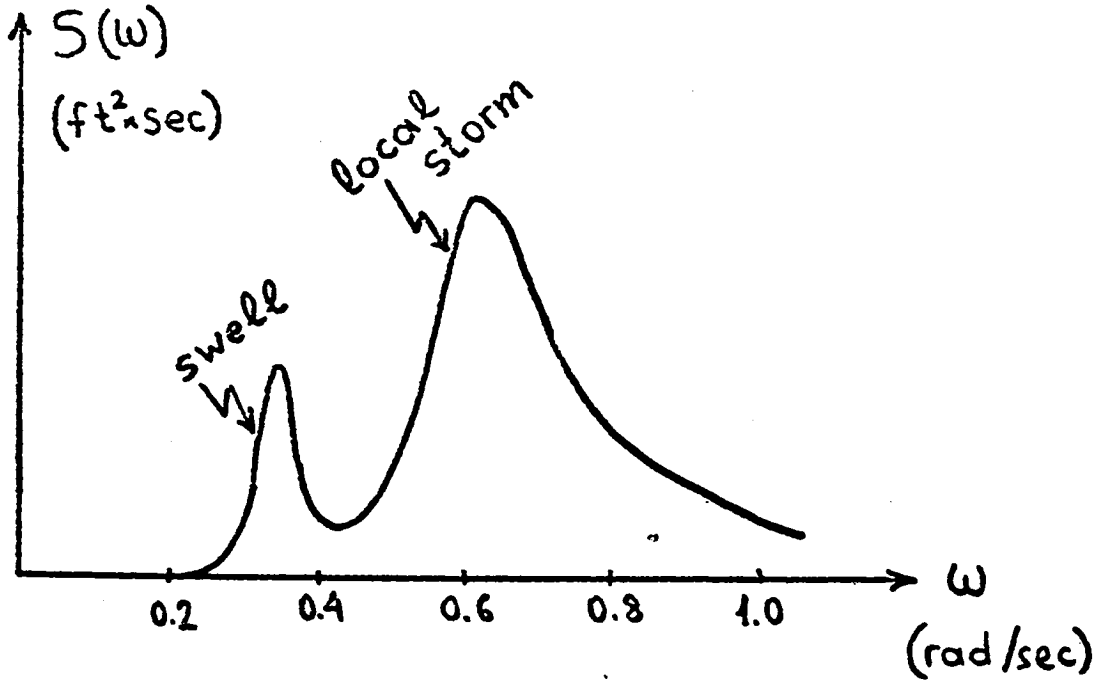

Figure 2.1

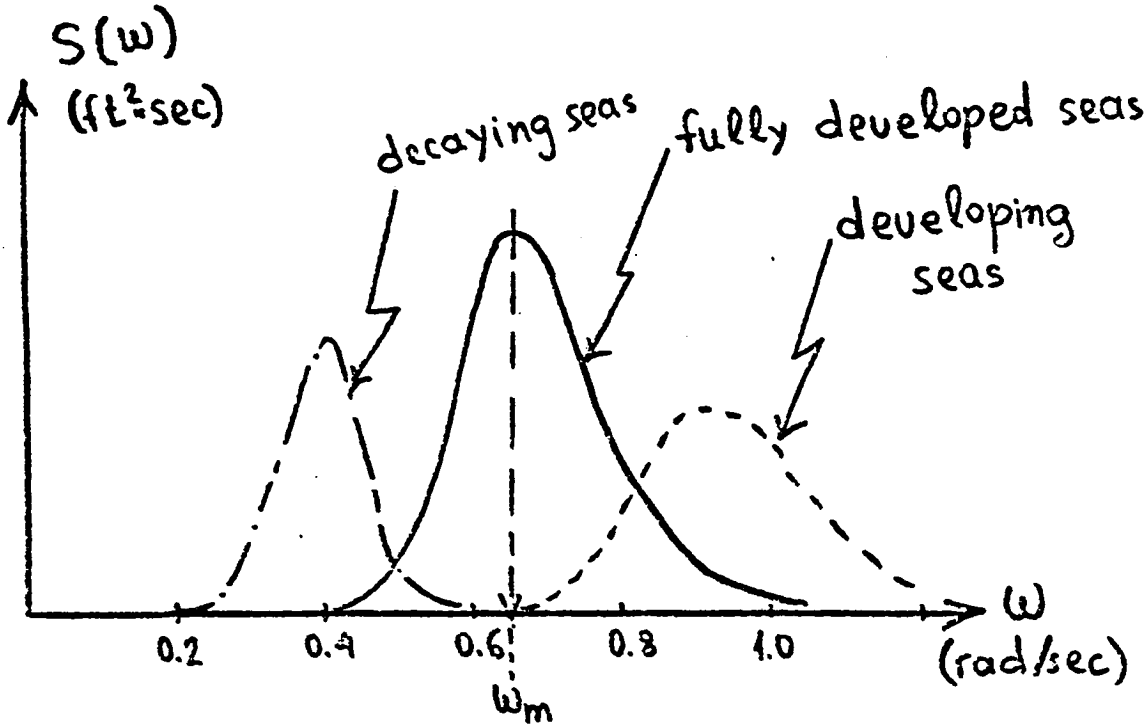

Figure 2.2

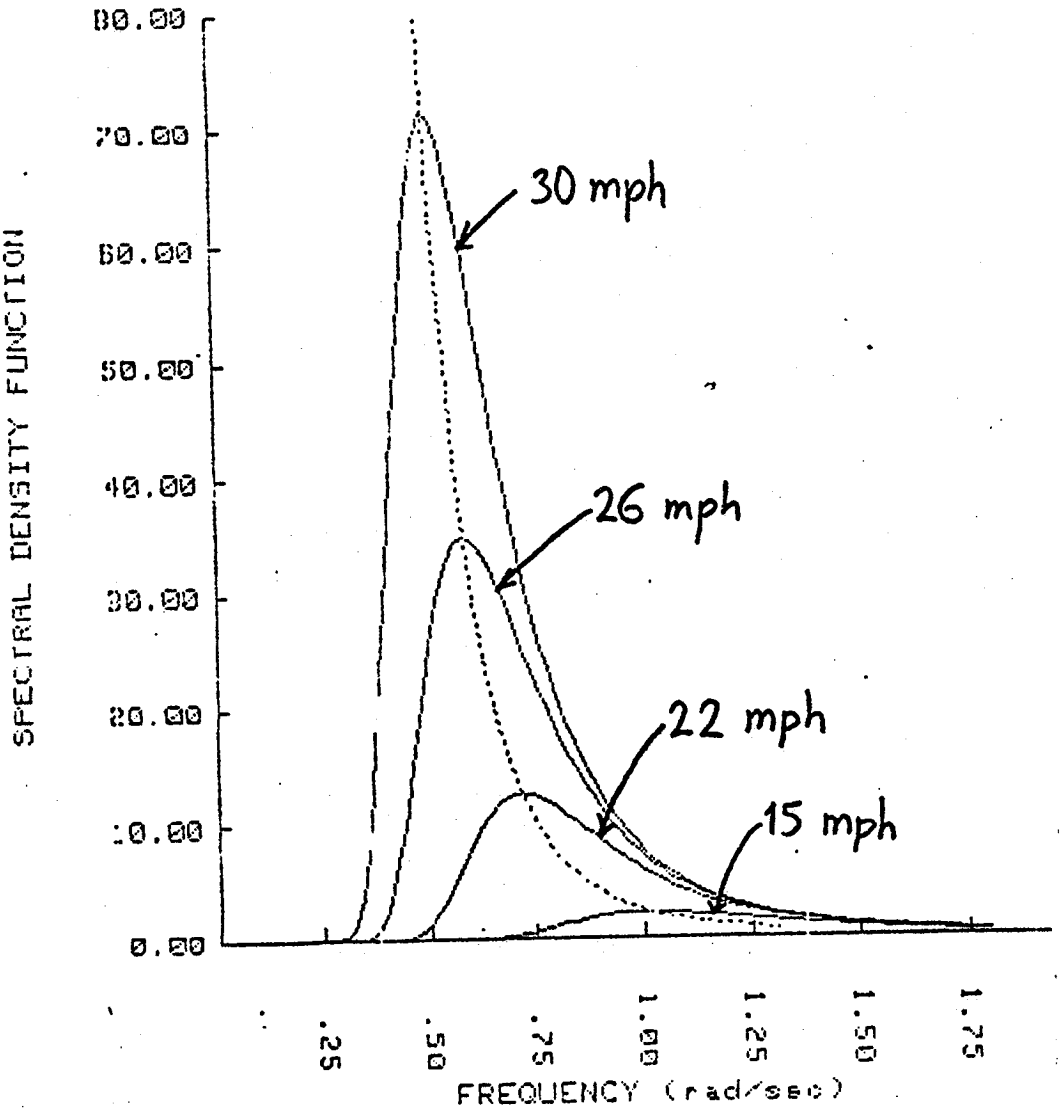

Figure 2.3 Bretschneider spectrum at various wind speeds (fully developed seas).

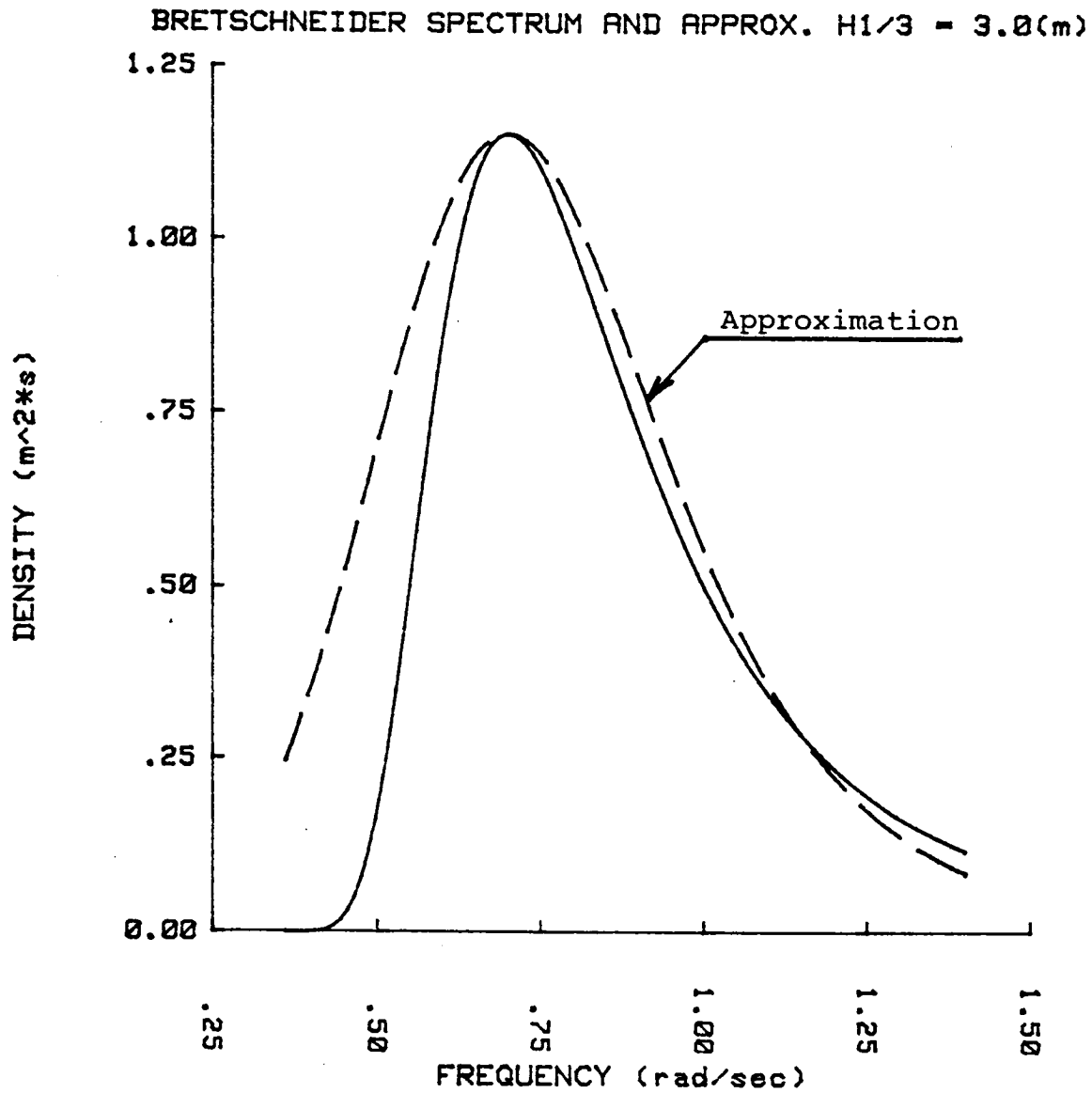

Figure 2.4 Bretschneider Spectrum and Approximation

Chapter 3: DERIVATION OF THE STATE - SPACE EQUATION

We proceed to derive a state-space form of the equations of motion. Starting with the sea, we can construct the following representation (three cascaded second order systems)

$$\dot{\underline{x}}_s = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\omega_o^2 & -2J\omega_o & 0 & \omega_o^2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\omega_o^2 & -2J\omega_o & 0 & \omega_o^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\omega_o^2 & -2J\omega_o \end{bmatrix} \underline{x}_s + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \omega \qquad (51)$$

$$\eta = \begin{bmatrix} \sqrt{S_o} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \underline{x}_s$$

or

$$\dot{\underline{x}}_s = A_s \underline{x}_s + B_s \omega \qquad (52)$$

$$\eta = C_s \underline{x}_s$$

Heave-Pitch Model

The following model is derived for the force (some algebra was involved to reduce the dimension of the state).

$$\dot{\underline{x}}_f = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\omega_o & -2J\omega_o & \omega_o^2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\omega_o^2 & -2J\omega_o & 0 \\ \theta_1 & \theta_2 & \theta_3 & \theta_4 & 1 \end{bmatrix} \underline{x}_f + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \eta$$

$$\begin{bmatrix} F_3 \\ F_5 \end{bmatrix} = \begin{bmatrix} \alpha_1 \omega_o^2 & \dot{0} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha_2 \cos\phi \end{bmatrix} \underline{x}_f \qquad (53)$$

or written in short

$$\dot{\underline{x}}_f = A_f \underline{x}_f + B_f \eta$$

$$\begin{bmatrix} F_3 \\ F_5 \end{bmatrix} = C_f \underline{x}_f \qquad (54)$$

The inertia model can be written as follows:

$$\dot{\underline{x}}_m = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \beta_1 & \beta_2 & \beta_3 & \beta_4 \\ 0 & 0 & 0 & 1 \\ \beta_5 & \beta_6 & \beta_7 & \beta_8 \end{bmatrix} \underline{x}_m + \begin{bmatrix} 0 & 0 \\ D_1 & D_2 \\ 0 & 0 \\ D_3 & D_4 \end{bmatrix} \begin{bmatrix} F_3 \\ F_5 \end{bmatrix}$$

$$\begin{bmatrix} x_3 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \underline{x}_m \qquad (55)$$

where

$$\begin{bmatrix} D_1 & D_2 \\ D_3 & D_4 \end{bmatrix} = \begin{bmatrix} A_{33} & A_{35} \\ A_{53} & A_{55} \end{bmatrix}^{-1} \tag{56}$$

$$\begin{bmatrix} \beta^1 & \beta^2 & \beta^3 & \beta^4 \\ \beta^5 & \beta^6 & \beta^7 & \beta^8 \end{bmatrix} = - \begin{bmatrix} D_1 & D_2 \\ D_3 & D_4 \end{bmatrix} \begin{bmatrix} C_{33} & B_{33} & C_{35} & B_{35} \\ C_{53} & B_{53} & C_{55} & B_{55} \end{bmatrix} \tag{57}$$

and we can write in short

$$\begin{aligned} \dot{\underline{x}}_m &= A_m \underline{x}_m + B_m \begin{bmatrix} F_3 \\ F_5 \end{bmatrix} \\ \begin{bmatrix} x_3 \\ x_5 \end{bmatrix} &= C_m \underline{Xm} \end{aligned} \tag{58}$$

Now the total model can be constructed as follows:

$$\begin{aligned} \dot{\underline{x}}_t &= A_t \underline{x}_t + B_t W \\ \begin{bmatrix} x_3 \\ x_5 \end{bmatrix} &= C_t \underline{x}_t \end{aligned} \tag{59}$$

where

$$\begin{aligned} A_t &= \begin{bmatrix} A_s & 0 & 0 \\ 0 & A_m & B_m C_f \\ B_f C_s & 0 & A_f \end{bmatrix} \\ B_t &= \begin{bmatrix} B_s & 0 & 0 \end{bmatrix}^T \\ C_t &= \begin{bmatrix} 0 & C_m & 0 \end{bmatrix} \end{aligned} \tag{60abc}$$

Data were obtained for the DD-963 destroyer from the M.I.T. Seakeeping Program and are given in Appendix 2. Appendix 4 lists a computer program that produces the A_t, B_t, C_t matrices once the ship speed, wave heading, significant wave height and modal frequency were specified. The output can be used directly as input to the LIDS control and filter design package.

Table 3.1 provides the numerical values of matrix A for speed 20 knots, angle 0° (head seas), significant wave height 10 ft. and modal frequency 0.72 rad/sec (sea state 5).

Sway-Roll-Yaw Model

The sway, roll, yaw exciting forces and moments are essentially driven by the slope of the sea elevation, which for regular waves equals the wavenumber times the amplitude, or for deep water we can write

$$\text{Slope} = \frac{\omega^2}{g} a$$

i.e. in the time domain:

$$\text{Slope} = -\frac{1}{g}\frac{d^2}{dt^2}\eta \tag{61}$$

As outlined in Chapter 1, the fact that only roll has a spring constant causes zero pole cancellation problems, which can be avoided by introducing the matrices T and U described by equations (36) through (38). The original equation is in the form

$$(M+A)\,\underline{\ddot{x}}_1 + B\underline{\dot{x}}_1 + C\underline{x}_1 = \underline{F}_1 \tag{62}$$

where $\underline{x}_1$ is of dimension 3:

$$\underline{x}_1 = \begin{bmatrix} \text{sway} \\ \text{roll} \\ \text{yaw} \end{bmatrix} = \begin{bmatrix} x_2 \\ x_4 \\ x_6 \end{bmatrix}, \quad \underline{F}_1 = \begin{bmatrix} F_2 \\ F_4 \\ F_6 \end{bmatrix} \tag{63}$$

M is the mass matrix, A and B the added mass and damping matrices respectively, C is the hydrostatic matrix and F the vector of exciting force and moments. Then

$$S^2\underline{x}_1 = -[M+A]^{-1}\, s\underline{x}_1 - [M+A]^{-1}C\underline{x}_1 + [M+A]^{-1}\underline{F}_1 \tag{64}$$

By letting

$$\underline{x}^T = [x_2 \ , \ \dot{x}_4 \ , \ x_4 \ , \ x_6]$$

$$\underline{F}^T = [\int F_2, \ F_2, \ F_4, \ \int F_4, \ F_6] \qquad (65ab)$$

and using the T and U matrices of equations (36),(38) we obtain a state space description of (64) without zero-pole cancellation, in the form

$$\dot{\underline{x}} = T \ \underline{x} + U \ F \qquad (66)$$

The state space representation of equations (27) through (29) is in the form

$$\dot{\underline{x}}_j = \begin{bmatrix} 0 & 1 \\ -\omega_j^2 & -2\zeta_j\omega_j \end{bmatrix} \underline{x}_j + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \dot{\eta} \qquad (67)$$

where j = 2,4,6, while

$$\begin{bmatrix} \int F_j \\ F_j \end{bmatrix} = \begin{bmatrix} A_j\omega_j^2 & 0 \\ 0 & A_j\omega_j^2 \end{bmatrix} \underline{x}_j \qquad (68)$$

For this reason we build a force matrix A_F:

$$A_F = \begin{bmatrix} A_2 & \emptyset & \emptyset \\ \emptyset & A_4 & \emptyset \\ \emptyset & \emptyset & A_6 \end{bmatrix} \qquad (69)$$

and a matrix B_F driving the force dynamics with $\dot{\eta}$, i.e. using the sea model, which is exactly the same as in the case of heave, pitch (6 states), so

$$B_F(i,j) = 0$$

except

$$B_F(2,2) = -A_2\omega_2^2$$

$$B_F(4,2) = -A_4\omega_4^2 \qquad (70abcd)$$

$$B_F(6,2) = -A_6\omega_6^2$$

Then using the same sea model described in equations (52ab) we obtain the overall model as

$$\underset{(16x16)}{\dot{\underline{x}}_t} = \underset{(16x16)}{A_t}\ \underline{x}_t + \underset{(16x1)}{B_t W} \qquad (71ab)$$

$$\begin{bmatrix} x_2 \\ x_4 \\ x_6 \end{bmatrix} = \underset{(3x16)}{C_t}\ \underline{x}_t$$

where

$$A_t = \begin{bmatrix} A_s & \emptyset & \emptyset \\ B_F & A_F & \emptyset \\ \emptyset & U & T \end{bmatrix}$$

$$B_t^T = [\, B_s^T \quad \emptyset] \qquad (72abc)$$

$$C_t = \left[\begin{array}{c:cccc} \emptyset & 1 & 0 & 0 & 0 \\ & 0 & 0 & 1 & 0 \\ & 0 & 0 & 0 & 1 \end{array}\right]$$

Data for all quantities involved are given in Appendix 2, while Appendix 4 lists the computer programs that can produce the matrices A_t, B_t, C_t once the ship speed, wave heading, significant wave height and modal frequency are specified.

Table 3.1 provides the A_t matrix for speed 15.5 knots, heading 45°, significant wave height 10 ft. and modal frequency 0.72 rad/sec (sea state 5).

TABLE 3.1

Matrix A for heave and pitch (15 x 15)

0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-1.491E+00	-1.726E+00	0.	1.491E+00	0.	0.	0.	2.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-1.491E+00	-1.726E+00	0.	1.491E+00	0.	0.	0.	0.	0.	2.	0.	0.	0.
0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	-1.491E+00	-1.726E+00	0.	1.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	-1.213E+00	-5.350E-01	-9.452E+00	-2.190E+01	7.850E-01	0.	0.	0.	-5.027E-01
0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	8.818E-04	3.106E-04	-1.163E+00	-4.347E-01	-1.529E-03	0.	0.	0.	1.605E-02
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-6.635E-01	-1.152E+00	6.635E-01	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-6.635E-01	-1.152E+00	0.
6.595E+02	0.	0.	0.	0.	0.	0.	0.	0.	0.	9.728E-01	1.249E+00	-9.728E-01	4.402E-01	-1.058E+00
0.	0.	0.	0.	0.	0.	0.	0.							

Matrix A for sway, roll, yaw (16 x 16)

0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-1.136E+00	-1.507E+00	0.	1.136E+00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-1.136E+00	-1.507E+00	0.	1.136E+00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	-1.136E+00	-1.507E+00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.
0.	5.722E+01	0.	0.	0.	0.	-2.610E-01	-5.282E-01	0.	1.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	-4.575E-01	-6.696E-01	0.	0.	0.	0.	0.	0.
0.	6.859E+02	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-8.111E-01	-4.458E-01	0.	0.	0.	0.
0.	6.481E+03	0.	0.	0.	0.	0.	0.	0.	0.	-4.469E-06	0.	-8.343E-03	2.748E-01	1.097E-01	8.041E+00
0.	0.	0.	0.	0.	0.	2.423E-03	0.	0.	3.142E-06	-1.999E-08	-1.804E-07	4.343E-05	-1.961E-02	-2.344E-01	1.286E-02
0.	0.	0.	0.	0.	0.	3.218E-06	1.375E-06	0.	0.	0.	0.	0.	1.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-4.611E-04	-2.319E-02	4.892E-04	-3.242E-02
0.	0.	0.	0.	0.	0.	-4.057E-06	0.	0.	0.	1.299E-07	0.				

Chapter 4: KALMAN FILTER AND SIMULATION

The heave-pitch approximation resulted in a 15 state system and the sway-roll-yaw approximation in a 16 state system. Given that 6 states describe the sea, the total system required for 5 degree of freedom motion studies would contain 25 states. If the sea spectrum contains two peaks then a 31 state model is required.

The heave-pitch group is not coupled with the sway-roll-yaw group so that the study of each group can be independent. This is not to indicate that in a total design the two groups must remain independent, since they are excited by the same sea.

Heave-Pitch Motions

It is assumed that the heave and pitch motions are measured. The gyroscopes can provide accurate measurements of angles, up to about 1/10 degree. The noise therefore is due to structural vibrations, which in the longitudinal direction can be significant due to the beam-like response of the vessel. As a result the measurement noise was estimated based on data from ship vibrations. The same applies to the heave measurement noise.

A Kalman filter was designed for speed V = 21 ft/sec and waves coming at 0° (head seas) with significant wave height H = 10 ft. and modal frequency ω_m = 0.73 rad/sec (sea state 5). The measurement noise intensity matrix was selected from ship vibration

data to be

$$V = \begin{bmatrix} 0.75 & 0 \\ 0 & 0.0003 \end{bmatrix}$$

The model poles are shown in table 4.1, while the filter poles are within a radius of 1.3 rad/sec as seen in table 4.2. Typical simulation results are shown in Figures 4.1 and 4.2. In these figures exact knowledge is assumed for the significant wave height and modal frequency. The accuracy of the filter is very good both for heave and pitch.

Subsequently, the same filter was used combined with a ship and sea model different than the nominal one, to investigate the sensitivity to the following parameters:

The influence of the significant wave height is very small when the modal frequency is accurately known. On the contrary, the influence of the modal frequency is quite critical; particularly for pitch (see Figures 4.6 and 4.7). The same conclusion is reached when a double peak sea spectrum is used [22],[23].

The effect of the forward speed and wave direction was found to be unimportant particularly for heave, while for small changes in wave angle ($\pm$ 15°) the pitch prediction error was not affected significantly [22].

Sway-Roll-Yaw Motions

As in the case of heave and pitch, the measurement noise consists primarily of structural vibrations rather than instrument noise. For roll such vibrations are quite small for a destroyer vessel and similarly for sway and yaw the vibrations are smaller than in the case of heave and pitch.

The noise intensity used was nonetheless similar to the heave, pitch noise, so as to bound the filter eigenvalues below 2 rad/sec, which is the typical wave bandwidth.

A specific example has been worked out for a forward ship speed of 15.5 ft/sec and waves at 45° and sea state 5 (significant wave height of 10 ft. and modal freqeuncy of 0.72 rad/sec). The measurement noise intensity matrix was

$$\text{diag } \{0.1 \text{ ft}^2 \ , \ 2*10^{-4}(\text{rad})^2 \ , \ 2*10^{-4}(\text{rad})^2\}$$

The simulation shows very good estimation as seen in figures 4.3, 4.4, and 4.5. Yaw is very small and the measurement noise is large relative to the yaw motion, nonetheless the yaw estimation based primarily on the roll, sway measurements is very good.

Table 4.2 presents the results of a sensitivity study of the influence of the various parameters involved. The most critical parameter is again the modal frequency. The ship speed and the wave direction are not critical for the estimation error. This is a very important conclusion as far as the wave direction is concerned, because in reality, seas are directional and very difficult to measure, or even model appropriately.

The influence of systematic measurement errors was studied by using a calibration factor. This factor is defined to be the ratio of the measurement fed to the filter over the actual measurement, thus introducing a systematic error. If C is the calibration factor, then the systematic error as a percentage of the actual measurement is $100 \cdot (1-C)$. In the case of a 10% error, the most significant change was found in the case of the roll motion. In the case of a calibration factor 0 (indicating a disconnected measurement) significant errors resulted, especially for roll in the case of disconnected roll measurements (Table 4.3).

Table 4.4 presents the poles of the model used, while Table 4.5 shows the poles of the Kalman Filter derived for the nominal condition as described above. Figures 4.8 through 4.10 are simulation results and show the significant effect of the modal frequency on the estimation error. Finally, Figure 4.11 shows the simulation of the sway motion estimation when the roll measurement is disconnected.

Ship Speed: U = 21 ft/sec	Heading angle: 0°
SEA: H=10 ft, $w_m = 0.72 \frac{rad}{sec}$	H=10 ft, $w_m = 0.52 \frac{rad}{sec}$
$P_{1,2} = -0.199 \pm 1.111i$	same
$P_{3,4} = -0.286 \pm 1.016i$	same
$P_5 = -1.058$	$P_5 = -0.696$
$P_{6,7} = -0.576 \pm 0.576i$	same
$P_{8,9} = -0.576 \pm 0.576i$	same
$P_{10,11} = -0.863 \pm 0.863i$	$P_{10,11} = -0.571 \pm 0.571i$
$P_{12,13} = -0.863 \pm 0.863i$	$P_{12,13} = -0.571 \pm 0.571i$
$P_{14,15} = -0.863 \pm 0.863i$	$P_{14,15} = -0.571 \pm 0.571i$

TABLE 4.1: Poles of the heave, pitch model

Ship Speed: U = 21 ft/sec, Heading angle: 0°	
SEA: H = 10 ft	W_m = 0.72 rad/sec

$$P_{1,2} = -1.289 \pm 0.540\,i$$

$$P_{3,4} = -1.134 \pm 1.033\,i$$

$$P_5 = -1.340$$

$$P_{6,7} = -0.903 \pm 0.7280\,i$$

$$P_{8,9} = -0.777 \pm 1.376\,i$$

$$P_{10,11} = -0.273 \pm 1.572\,i$$

$$P_{12,13} = -0.248 \pm 0.918\,i$$

$$P_{14,15} = -0.0936 \pm 0.0940\,i$$

TABLE 4.2 Poles of the heave, pitch Kalman Filter

TABLE 4.3 Sensitivity of the RMS error of sway, roll, yaw motion, to changes in the parameters of the ship-sea model. C (sway) indicates a calibration coefficient in the sway measurement (and similarly for the other motions), to handle systematic errors in the measurements.

Parameter Changed	Error Sway (ft)	Error Roll (deg)	Error Yaw (deg)
Basic Case	0.241	0.56	0.0776
U=20 ft/sec	0.245	0.568	0.0963
ω_m=0.52 rad/sec	0.314	0.91	0.0858
ϕ=60°	0.296	0.624	0.112
C(sway)=0.9	0.255	0.586	0.081
C(roll)=0.9	0.247	0.708	0.0808
C(yaw)=0.9	0.242	0.56	0.0777
C(sway)=0.0	0.518	1.21	0.1408
C(roll)=0.0	0.376	4.08	0.158
C(yaw)=0.0	0.242	0.563	0.0785
RMS values of the motions (nominal case)	0.60	4.56	0.227
Measurement noise intensity	$(.316 \text{ ft})^2$	$(.81°)^2$	$(.81°)^2$

Ship speed : U=15.5ft/s Wave heading angle : ϕ=45°	
SEA : H=10ft ω_m=0.72rad/s	SEA : H=12ft ω_m=0.4807rad/s
Ship model poles :	
$p_{1,2} = -0.754 \pm 0.754\,i$	$p_{1;2} = -0.470 \pm 0.470\,i$
$p_{3,4} = -0.754 \pm 0.754\,i$	$p_{3,4} = -0.470 \pm 0.470\,i$
$p_{5,6} = -0.754 \pm 0.754\,i$	$p_{5,6} = -0.470 \pm 0.470\,i$
$p_{7,8} = -0.223 \pm 0.873\,i$	
$p_{9,10} = -0.335 \pm 0.588\,i$	
$p_{11,12} = -0.260 \pm 0.440\,i$	SAME
$p_{13,14} = -0.00983 \pm 0.484\,i$	
$p_{15,16} = -0.0204 \pm 0.0597\,i$	

TABLE 4.4 : Poles of the sway, roll, yaw model

SEA H=10ft ω_m=0.72rad/s			
Kalman filter poles :			
$p_{1,2}$	= -1.067	±	1.086 i
$p_{3,4}$	= -0.457	±	1.312 i
$p_{5,6}$	= -1.279	±	0.477 i
$p_{7,8}$	= -0.210	±	0.934 i
$p_{9,10}$	= -0.365	±	0.523 i
$p_{11,12}$	= -0.087	±	0.446 i
$p_{13,14}$	= -0.159	±	0.165 i
$p_{15,16}$	= -0.0203	±	0.0595 i

TABLE 4.5: Poles of the sway, roll, yaw Kalman filter

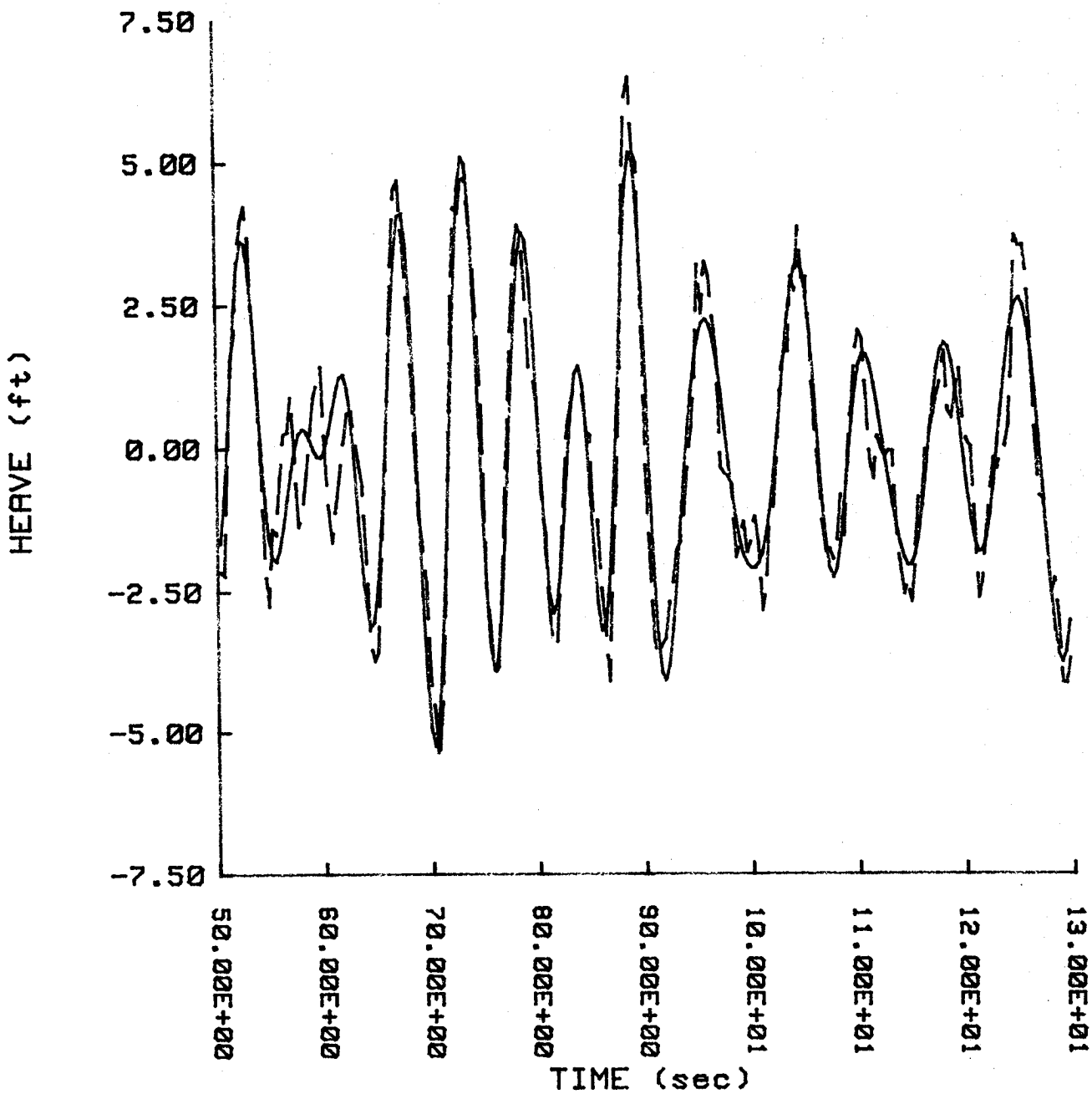

Figure 4.1

Results of Heave Simulation and Its Kalam Filter Estimate (dotted line), Using Accurate Model at U=21 ft/sec and 0^0 Angle of Incidence and in sea state 5.

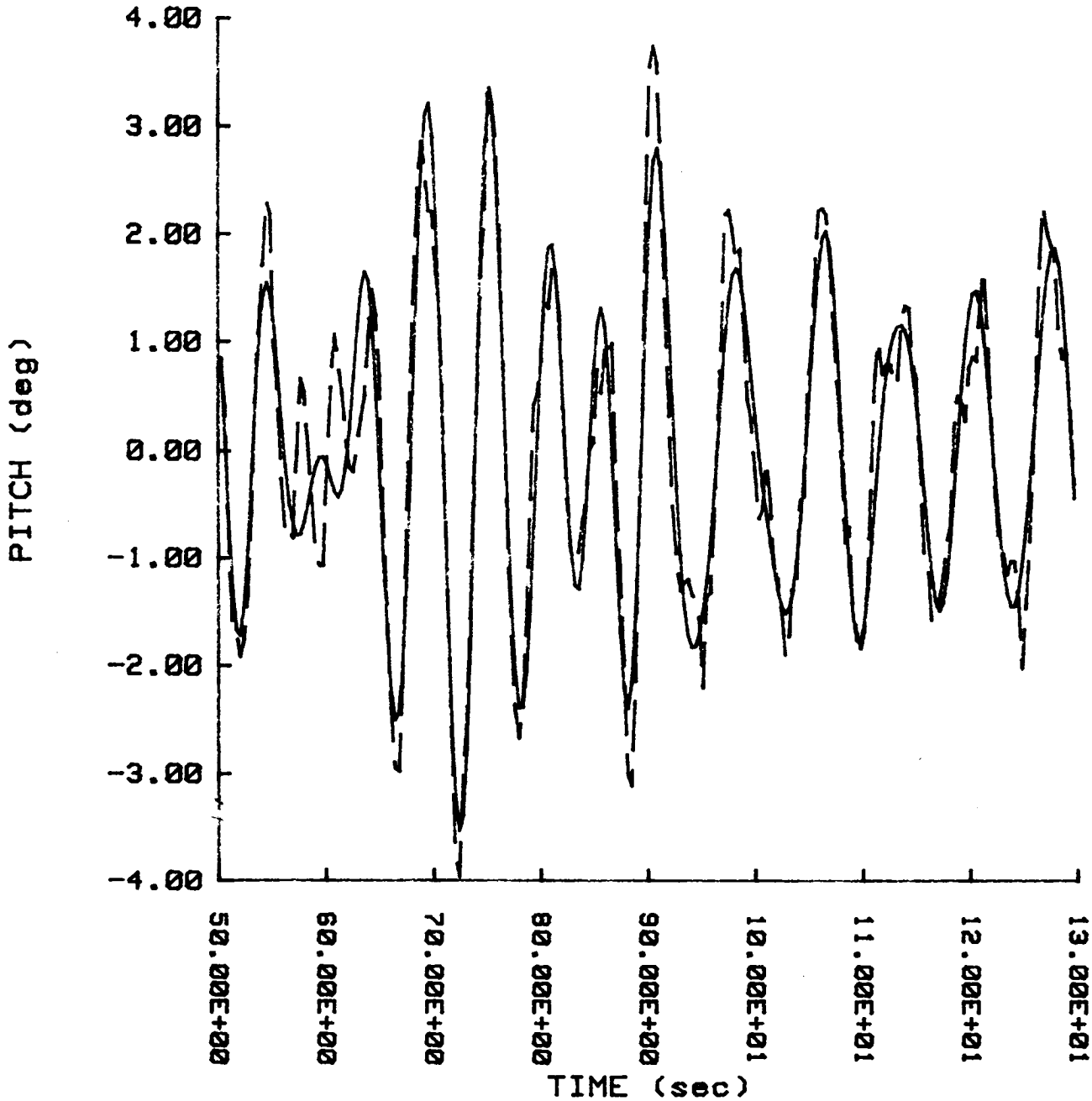

Figure 4.2

Results of Pitch Simulation and Its Kalman Filter Estimate. Same conditions as in 4.1.

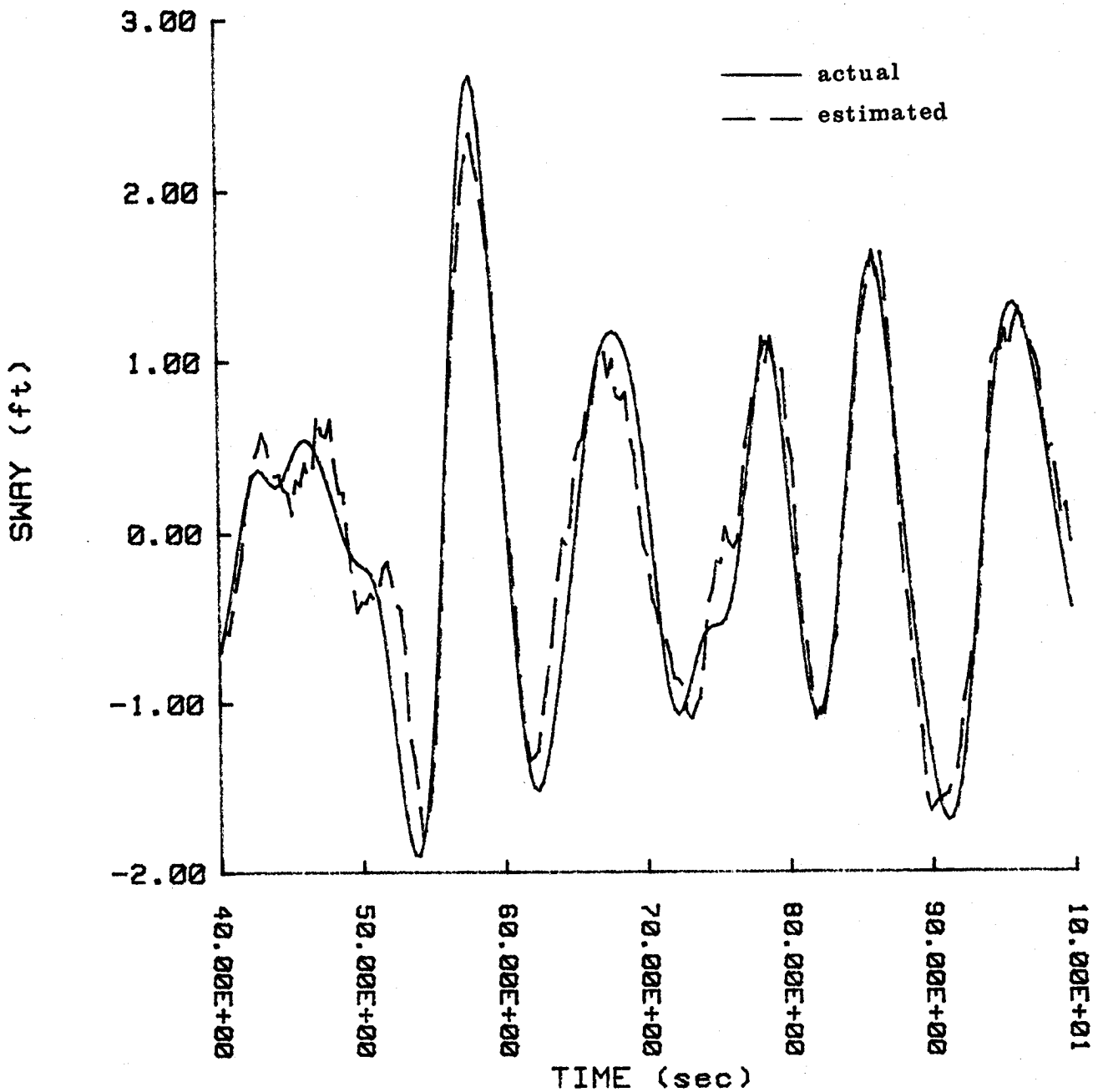

Figure 4.3

Results of Sway Simulation and Its Kalman Filter Estimate (dotted line), Using Accurate Model at U=15.5 ft/sec and 45^0 Angle of Incidence, and in sea state 5.

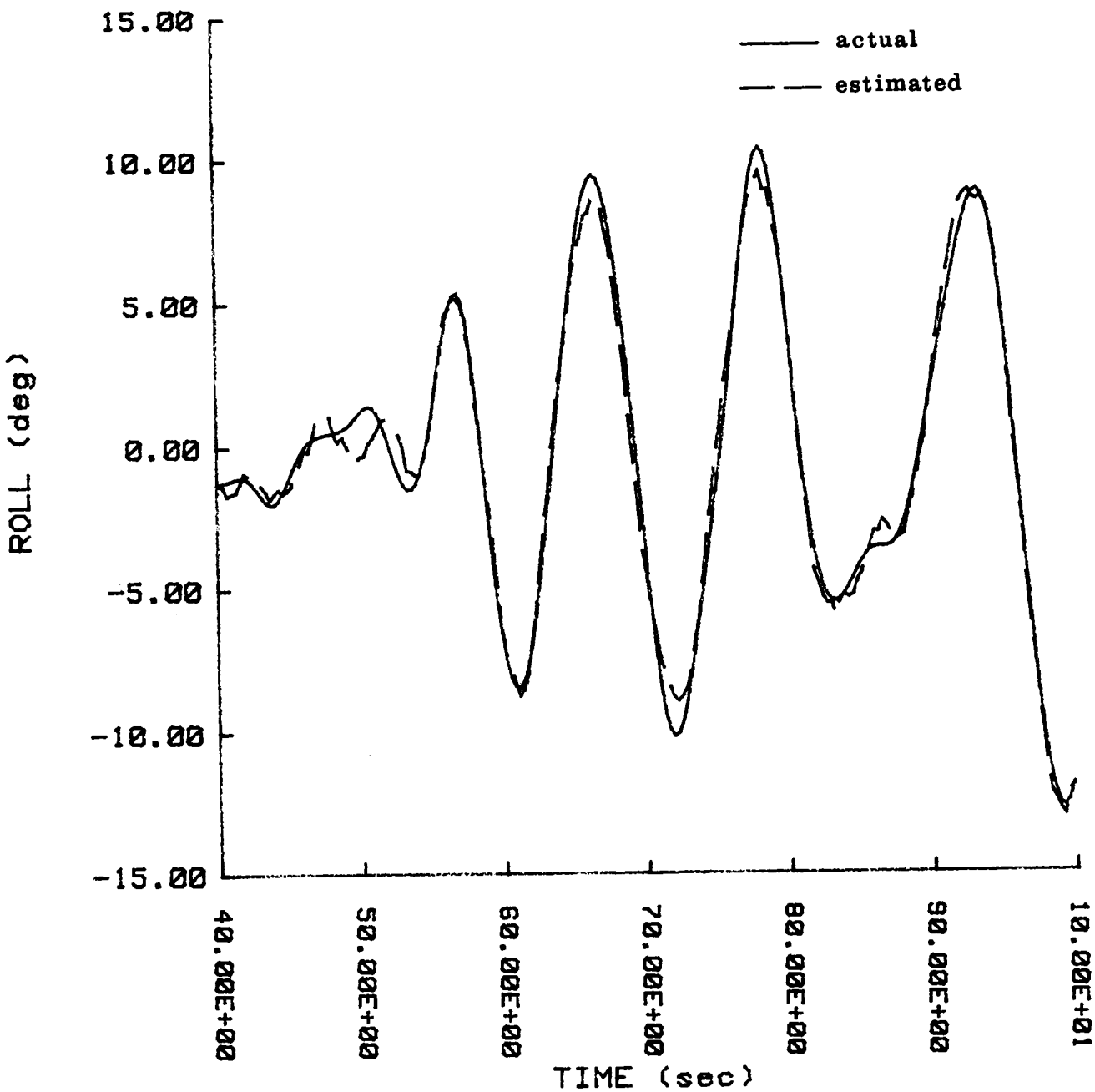

Figure 4.4

Results of Roll Simulation and Its Kalman Filter Estimate. Same conditions as in 4.3.

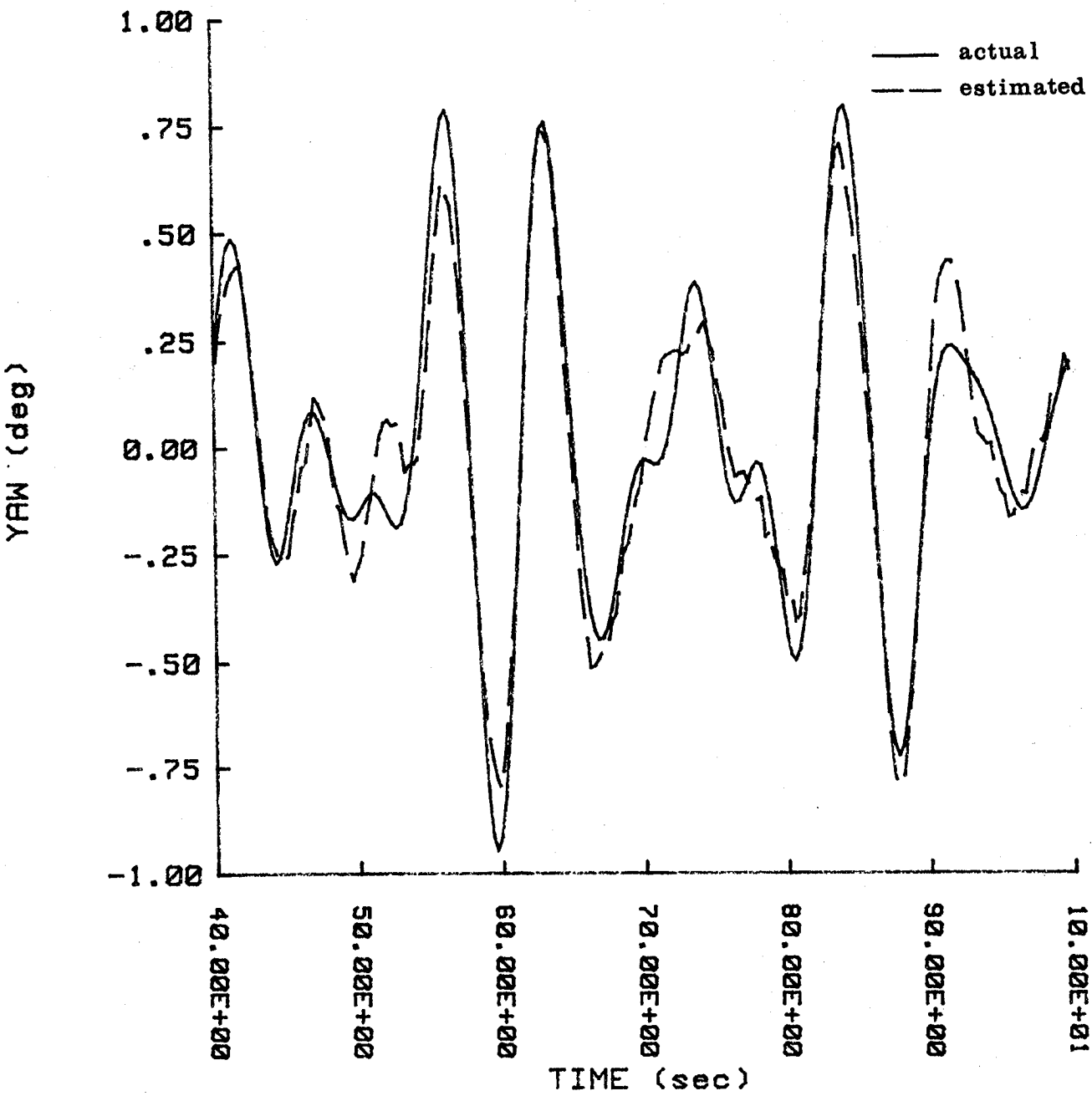

Figure 4.5

Results of Yaw Simulation and Its Kalman Filter Estimate.
Same condition as in 4.3.

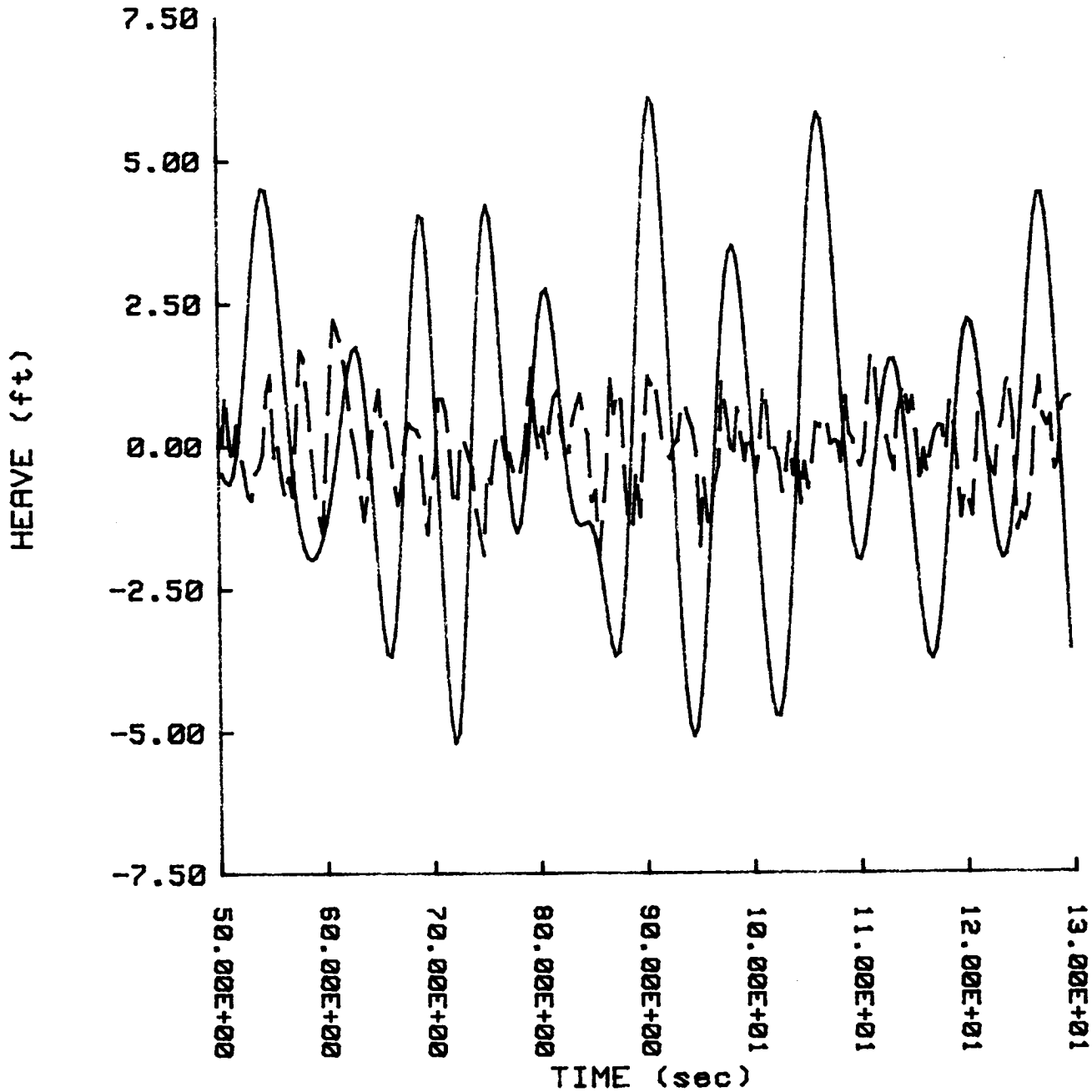

Figure 4.6

Results of Heave Simulation and Its Kalman Filter Estimate (dotted line). The actual wave spectrum model frequency is 0.52 rad/sec, while the value used in the Kalman Filter is 0.72 rad/sec. All other parameters as in 4.1.

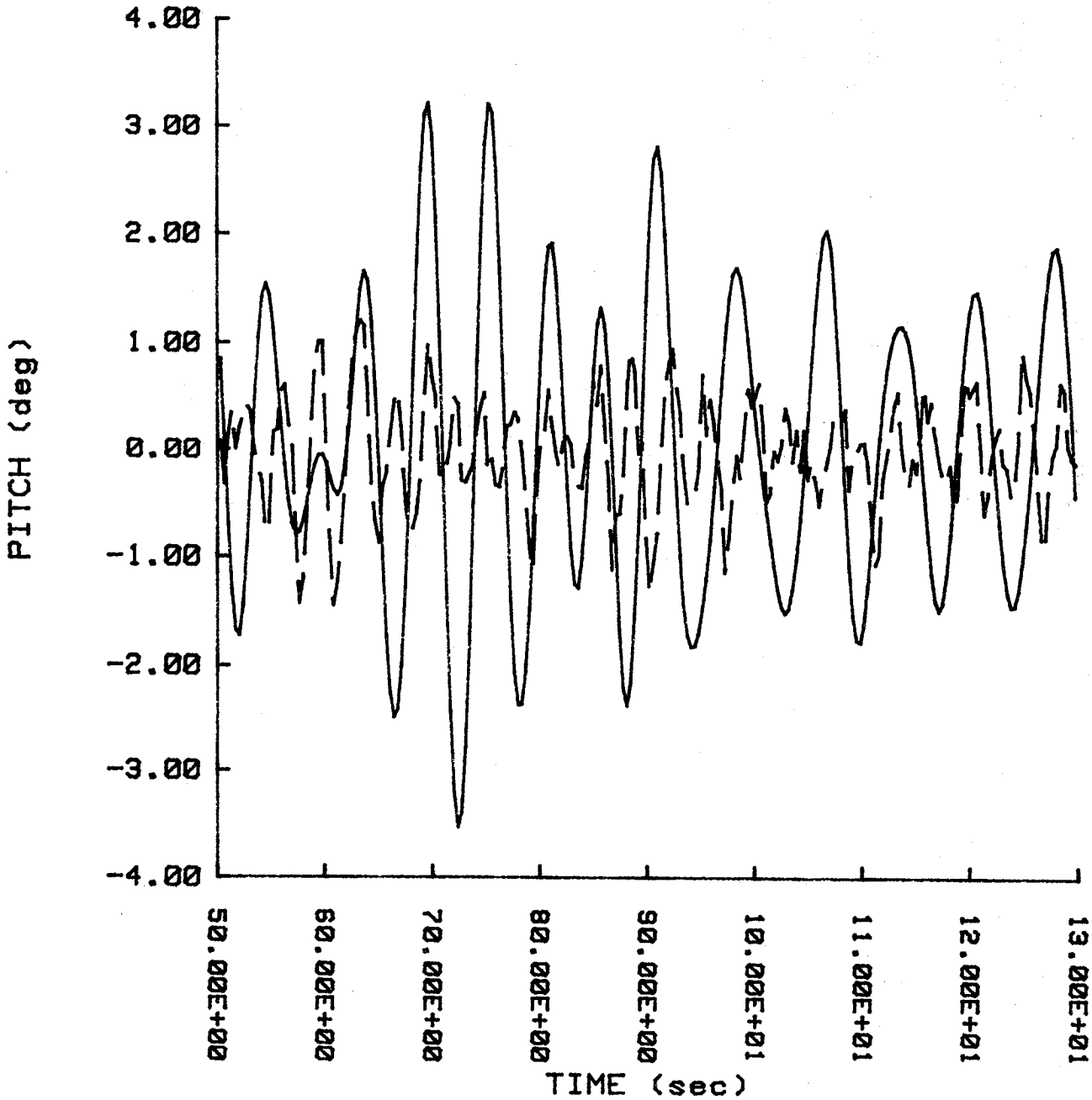

Figure 4.7

Results of Pitch Simulation and Its Kalman Filter Estimate. Same condition as in 4.6.

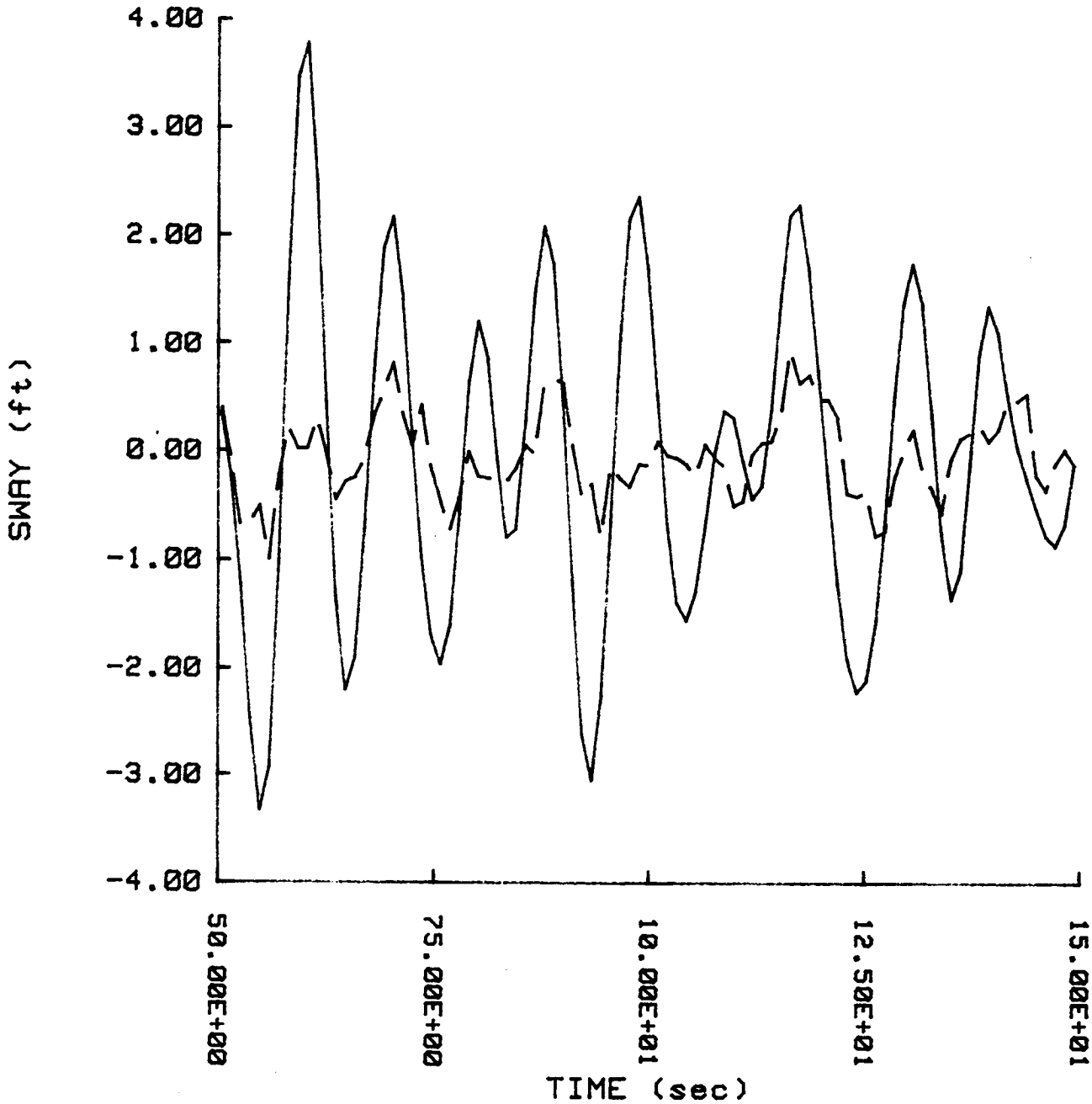

Figure 4.8

Results of Sway Simulation and Its Kalman Filter Estimate (dotted line). Actual W_m=0.52 rad/sec, while in Kalman Filter W_m=0.72 rad/sec. All other parameters as in 4.3.

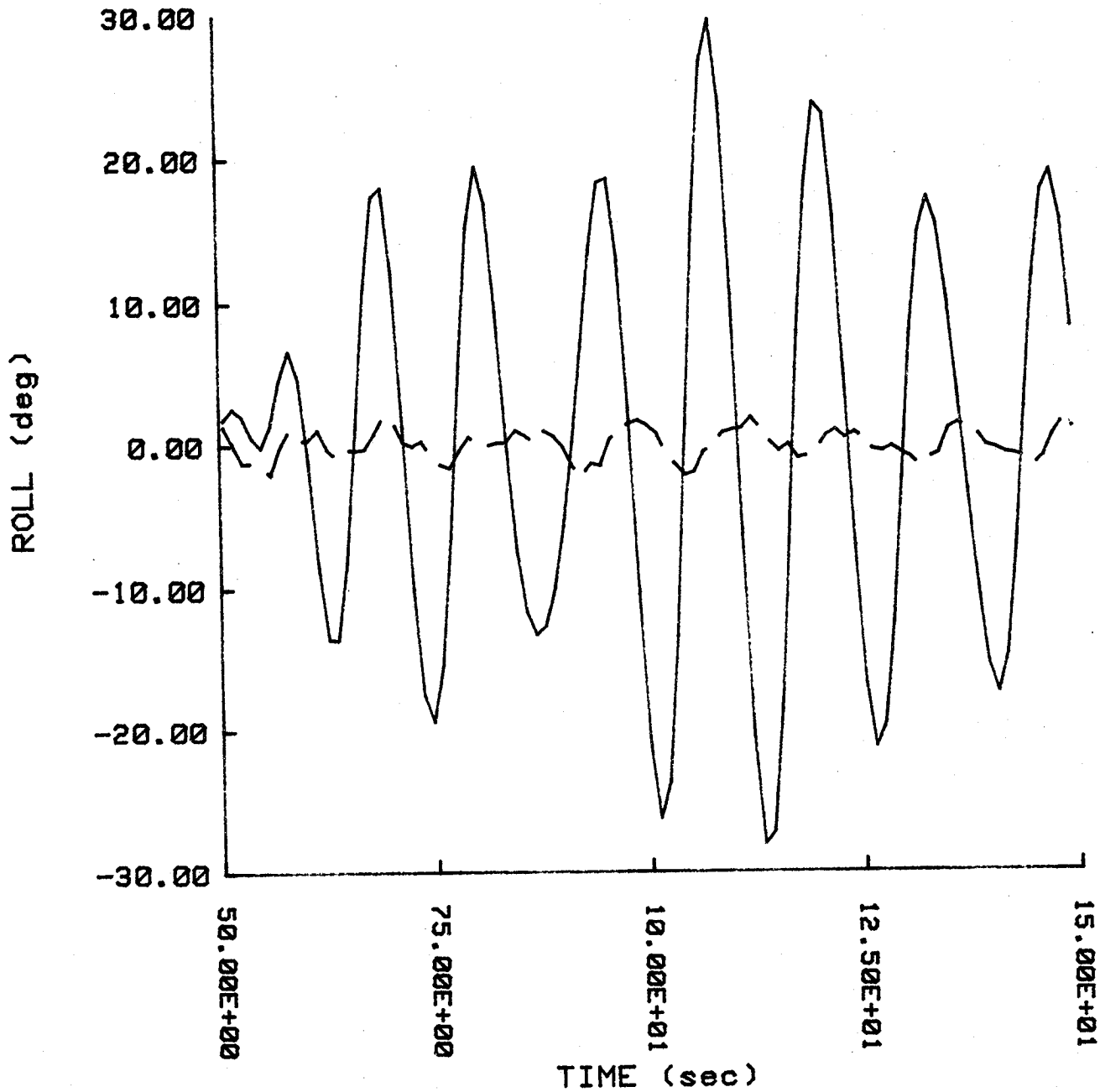

Figure 4.9

Results of Roll Simulation and Its Kalman Filter Estimate. Same conditions as in 4.8.

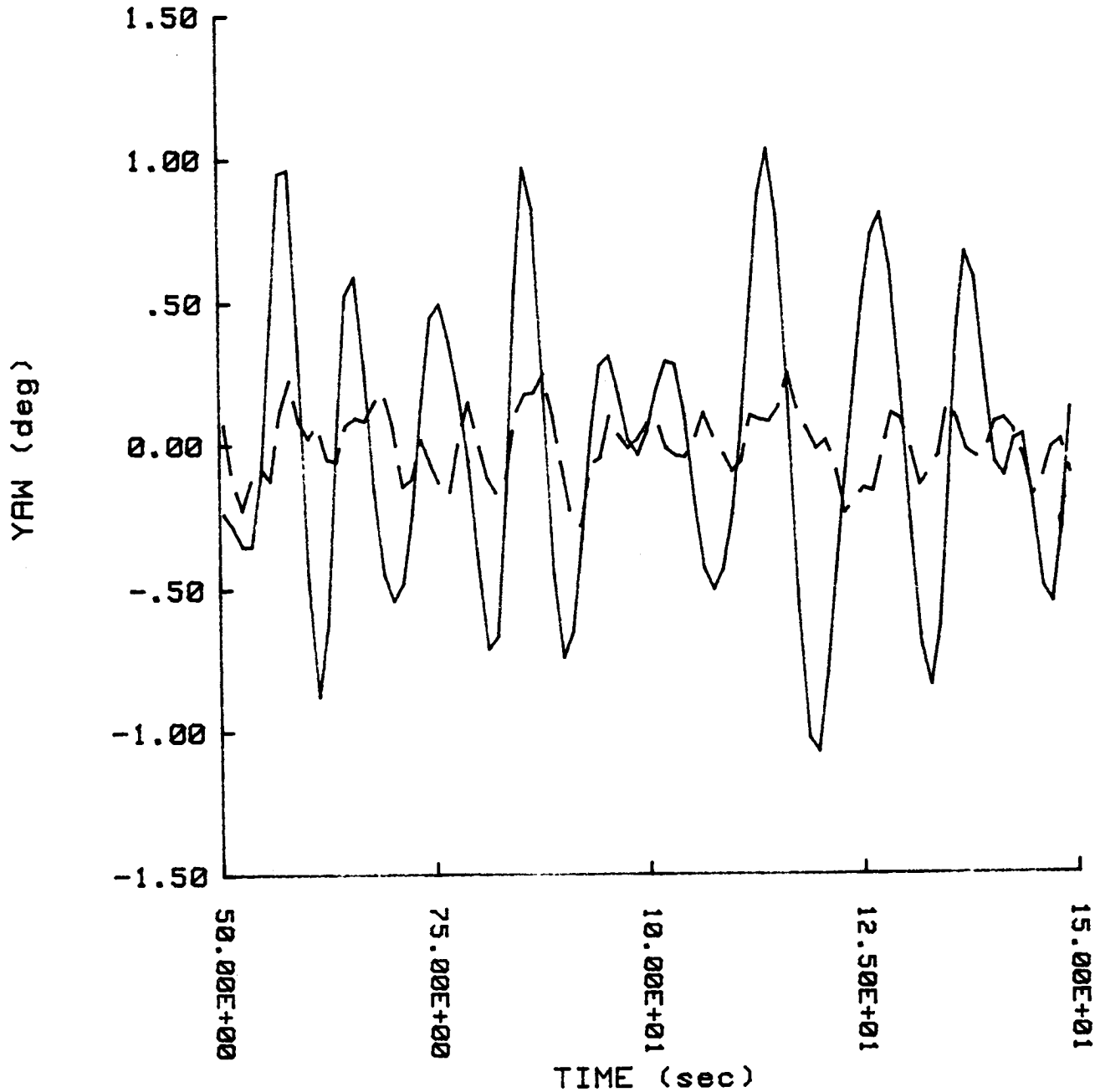

Figure 4.10

Results of Yaw Simulation and Its Kalman Filter Estimate.
Same conditions as in 4.8.

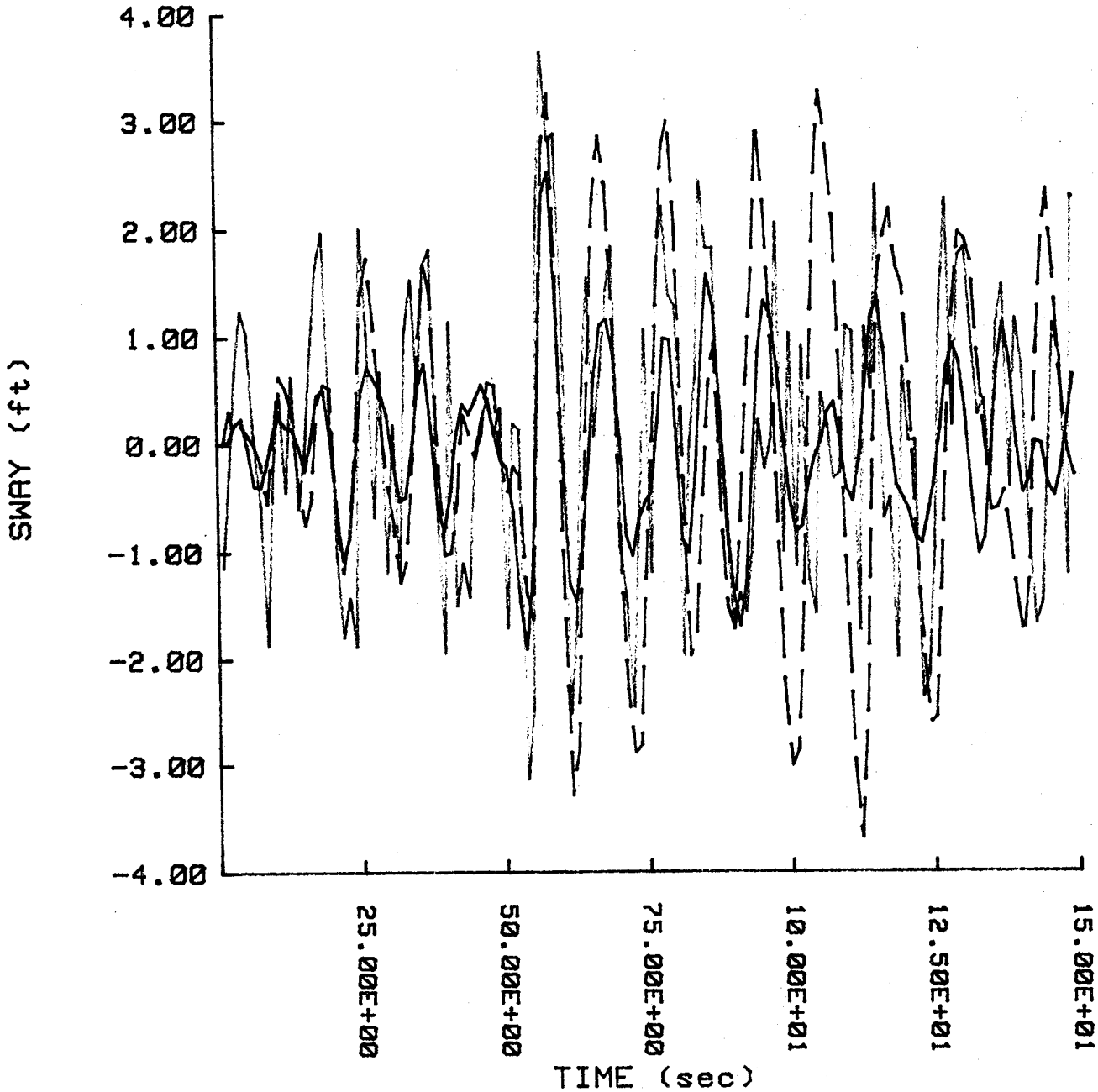

Figure 4.11

Results of Yaw Simulation and Its Kalman Filter Estimate (dotted line), using noisy measurements (light line) when the roll measurement is disconnected. Same other conditions as in 4.3.

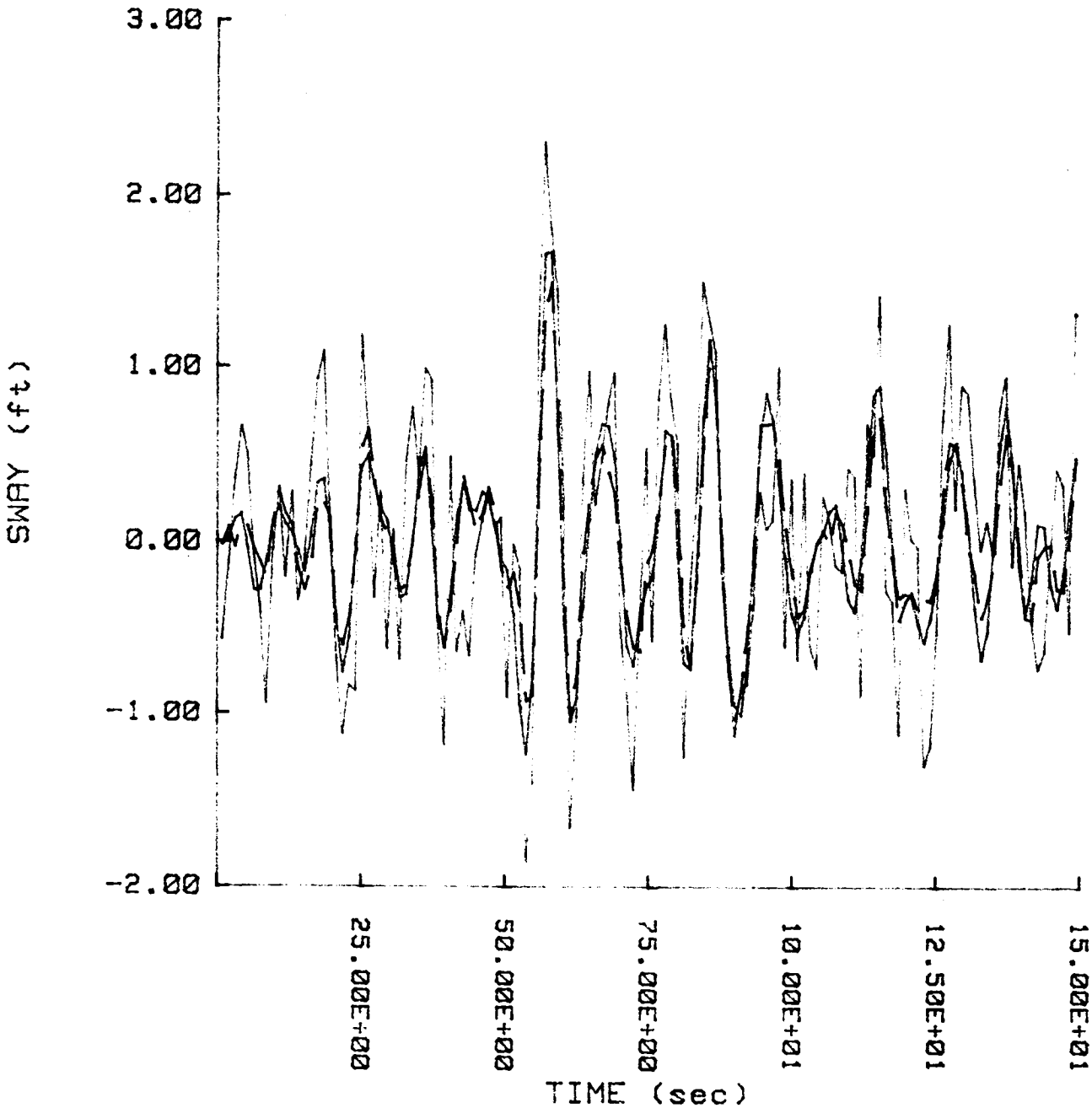

Figure 4.12

Results of Sway Simulation and Its Kalman Filter Estimate (dotted line), using measurements (light line) when the actual angle of incedence is 60^0 and the value used in the Kalman Filter is 45^0. Same other conditions as in 4.3.

Chapter 5: SHIP MOTION PREDICTION

It is of interest to use the models developed in the previous sections to forecast the behavior of the vessel within a few seconds. The feasibility to predict the motions could assist significantly the pilot in committing the aircraft to landing under favorable conditions.

An automatic landing does not require within LOG theory such information since the predictable part of the motions is included in the state and therefore used directly. Nonethless the prediction is of primary importance for pilot landing or semi-automatic landing.

Similarly for offshore operations a display of a prediction of the most critical vessel motions could reduce the operation risk significantly. The operator could choose a time window of minimal motion or acceleration and then transfer cargo or personnel.

The subject has been considered in the literature [6], [11], [24] using both frequency and time domain techniques.

Theoretical Background

The first to treat the subject of developing a predicting filter was Wiener [29]. If a random process has power spectrum $S(\omega)$ a spectral factorization is first required, i.e.

$$A(\omega) = \psi(\omega)\ \psi^*(\omega) \tag{73}$$

where * denotes complex conjugation and $\psi(s)$ is an analytic function of s with the exception of a finite number of poles in the left half plane. Then the transfer function of the optimal predictor $K(\omega)$, in the sense of minimizing the expected value of the error, is

given by the expression:

$$K(\omega) = \left(\frac{1}{\psi(\omega)}\right) \int_0^\infty Y(t+a)e^{-i\omega t} \, dt \tag{74}$$

where $Y(t)$ is the inverse Fourier transform of $\psi(\omega)$ and a is the prediction time. The importance of this result is to provide a number of intuitive results, such as the fact that a narrow band process is predictable while at the extreme a wide band process is unpredictable. The disadvantage of this approach is that it may require differentiators in its implementation, depending on the form of the spectrum [29].

The alternative is to use state space models where no such problems appear. In fact the predictive filter has a very simple form. If the system has a state space description

$$\begin{aligned} \dot{\underline{x}} &= A\underline{x} + \underline{W}_1 \\ \underline{Y} &= C\underline{x} + \underline{W}_2 \end{aligned} \tag{75ab}$$

where $\underline{W}_1$ and $\underline{W}_2$ are white noise signals, it is not hard to see that if the state $\underline{x}$ is perfectly known at t then the predictable part of $\underline{x}(t+\tau)$, denoted by $\underline{x}_p$, is

$$\underline{x}_p(t+\tau) = e^{A\tau} \, \underline{x}(t) \tag{76}$$

If the state is not available the Kalman filter estimate is used instead [5],[8]:

$$\underline{x}_p(t+\tau) = e^{A\tau} \, \hat{\underline{x}}(t) \tag{77}$$

i.e. the propagation of the equation

$$\dot{\underline{x}}_p(t+\tau) = A\,\underline{x}_p(t+\tau)$$
$$\underline{x}_p(t) = \hat{\underline{x}}(t) \qquad (78ab)$$

For a stable matrix A, $\underline{x}_p(t+\tau) \to 0$ as $\tau \to \infty$, reflecting the fact that the influence of the driving white noise completely alters the state of the system, once the homogeneous solution has died out. The covariance of the error

$$\underline{e} = \underline{x}_p - \underline{x}$$

denoted as P_p is governed by the equation

$$\dot{P}_p(\tau) = A\,P_p(\tau) + P_p(\tau)\,A^T + V_1 \qquad (79)$$

When the state is perfectly known the initial condition is:

$$P_p(o) = 0 \qquad (80a)$$

While in the case of using the Kalman filter estimate

$$P_p(o) = P(t) \qquad (80b)$$

where P(t) is the error covariance of the Kalman filter at the "present" time t.

The two models developed for the vertical and horizonatl motions have been used to study the predictability of the ship motions. Figures 5.1 through 5.5 show simulation results assuming perfect state knowledge. Similarly perfect state knowledge has been assumed and the covariance has been propagated using equations (79), (80). Figure 5.6 is a plot of the heave, pitch motions rms error versus prediction time, while Figure 5.7 depicts the sway, roll,

yaw rms erros versus prediction time. The error has been non-dimensionalized with respect to the corresponding rms motions.

As expected, the error tends to 100% for large prediction times. Roll is a narrow band process and as expected it is the most predictable motion, up to ten seconds ahead. The remaining four motions are predictable up to five seconds ahead.

These results hold in the ideal case. The actual performance will be lower due to the presence of noise in the measurements, fewer measurements than states and modeling errors.

To assess the effect of measurement noise some simulations were made, whose results are shown in Figures 5.8 through 5.10. A rather extreme case was considered: In the case of the vertical motions only two measurements were available, heave and pitch, and in the case of the horizontal motions, only the sway, roll, and yaw motions were available. As seen in the figures, the same noise used to derive the Kalman filter gains was used, which is quite significant. As expected, the performance deteriorated although roll is still predictable up to eight seconds. The other four motions are predictable up to about two seconds.

In the case of modeling errors, let the correct model be

$$\dot{\underline{x}} = A\underline{x} + \underline{W} \tag{81}$$

while the prediction model is

$$\dot{\underline{x}}_p = A^* \underline{x}_p \tag{82}$$

with $A^* = A + \delta A$. Then the error $\underline{e} = \underline{x}_p - \underline{x}$ is governed by the equation

$$\dot{\underline{e}} = A^*\underline{e} + \delta A\, \underline{x} - \underline{W} \tag{83}$$

or by letting $\underline{x}_1 = \begin{bmatrix} \underline{e} \\ \underline{x} \end{bmatrix}^T$

$$\dot{\underline{x}}_1 = \begin{bmatrix} A^* & \delta A \\ 0 & A \end{bmatrix} \underline{x}_1 + \begin{bmatrix} -\underline{W} \\ \underline{W} \end{bmatrix} \qquad (84)$$

so by denoting:

$$P = E\{\underline{e}\ \underline{e}^T\}, \quad V = E\{\underline{x}\ e^T\}, \quad V = E\{\underline{x}\ \underline{x}^T\} \qquad (85)$$

the following equations are obtained

$$\dot{P} = A^* P + P A^{*T} + \delta A \cdot V + V^T \cdot \delta A^T + V_1$$

$$\dot{V} = A V + V A^{*T} + U \cdot \delta A^T - V_1$$

$$\dot{U} = A \cdot U + U A^T + V_1 \qquad (86abc)$$

U, the covariance of the vessel model, can be assumed to be in steady state so the first two equations can be used to propagate V and P. Figures 5.12, 5.13, and 5.14 depict the error covariance of the vessel motions when the model used is different than the actual one. Since one of the most critical parameters is the modal frequency, its effect has been studied: The nominal value is 0.52 rad/sec, while the value used in the prediction filter is 0.72 rad/sec. The covariance at the initial time is assumed to be zero (perfect state knowledge).

As can be seen from these figures, the effect of modeling error is important in the case of the modal frequency, providing a reduction of about 30-50% in the prediction time within prescribed confidence limits.

The sinusoidal behavior of the covariance propagation at about twice the motion natural frequency (as seen in the case of roll, for example) can be explained by the form of the covariance equation

$$\dot{P} = A\,P + P\,A^T - V \tag{87}$$

For example the unforced equation

$$\begin{aligned} \dot{P} &= A\,P + P\,A^T \\ P(0) &= Q \end{aligned} \tag{88ab}$$

has the solution

$$P(t) = e^{At} Q e^{A^T t} \tag{89}$$

which is composed of exponentials in the form

$$e^{(\lambda_i + \lambda_j)t}$$

where λ_i are the eigenvalues of A. As a result $2\lambda_i$ will appear and in the case of roll it is obvious that twice the roll natural frequency dominates the response.

Reduced Number of States

Another aspect of interest is omitting states. In such a case we denote by $\underline{x}$ the nominal state of dimension n and by $\underline{x}^*$ the implemented state of dimension m, $n \geq m$. We assume that $\underline{x}^*$ is obtained by simply omitting some states, so that

$$\underline{x}^* = W\underline{x} \tag{90}$$

where

$$W = [I_m \vdots \emptyset] \tag{91}$$

with I_m the unit (mxm) matrix. The nominal system equation is

$$\begin{aligned} \dot{\underline{x}} &= A\,\underline{x} + B\underline{W}_1 \\ \underline{z} &= C\underline{x} + \underline{W}_2 \end{aligned} \tag{92ab}$$

while the prediction filter is

$$\begin{aligned} \dot{\underline{x}}_p &= A^*\,\underline{x}_p \\ \underline{z}_p &= C^*\,\underline{x}_p \end{aligned} \tag{93ab}$$

where $\underline{x}_p$ has dimension m, A^* is the mxm reduced system matrix and C^* the reduced observation matrix. Then we define:

$$\delta A = W^T A^* W - A \tag{94}$$

and we obtain the covariance equations [8]:

$$\begin{aligned} \dot{P} &= W^T \cdot A^* \cdot W \cdot P + P \cdot W^T \cdot A^{*T} \cdot W + \delta A \cdot V + V^T \cdot \delta A^T + V_1 \\ \dot{V} &= A \cdot V + V \cdot W^T \cdot A^{*T} \cdot W + U \cdot \delta A^T - V_1 \\ \dot{U} &= AU + UA^T + V_1 \end{aligned} \tag{95abc}$$

where P, V, and U are the same quantities defined before, except that the error is defined as:

$$e = W^T \underline{x}_p - \underline{x} \tag{96}$$

Note that we may eliminate any row from the original system, so its is convenient to set all the rows to be eliminated at the end by performing a row permutation. The matrix that interchanges rows has the form

$$S = \begin{bmatrix} & \cdots & & \\ 0 & 0 \ \cdots & 0 & 1 \ 0 \ \cdots \ 0 \\ & \cdots & & \end{bmatrix} \tag{97}$$

where each row has only one nonzero entry equal to one, and no row is the same as any other one.

The inverse of S is S^T thus minimizing the computational effort. Once the appropriate permutations have been established, it is easy to construct S and equation

$$\begin{aligned} \dot{\underline{x}} &= A\underline{x} + B\underline{W}_1 \\ \underline{y} &= C\underline{x} \end{aligned} \tag{98ab}$$

becomes

$$\begin{aligned} \dot{\underline{z}} &= A_p \underline{z} + B_p \underline{W}_1 \\ \underline{y} &= C_p \underline{z} \end{aligned} \tag{99ab}$$

where

$$\begin{aligned} \underline{z} &= S\underline{x} \\ A_p &= S\ A\ S^T \\ B_p &= S\ B \\ C_p &= C_p\ S^T \end{aligned} \tag{100abcd}$$

Then we proceed to determine the error covariance as explained above using A_p, B_p, and C_p as the system matrices.

The inclusion of non-minimum phase zeros was considered to be an important part of the overall modeling. This was confirmed by studying the effect of omitting these zeros on the prediction error covariance. This is seen in Figure 5.15, where the heave and pitch rms error, non-dimensionalized over the corresponsing rms motion, is plotted versus prediction time. As expected, pitch error increases substantially, since pitch lags heave at low frequencies by 90°. Because heave and pitch are coupled, the error in heave is also affected, resulting in poor prediction of both heave and pitch.

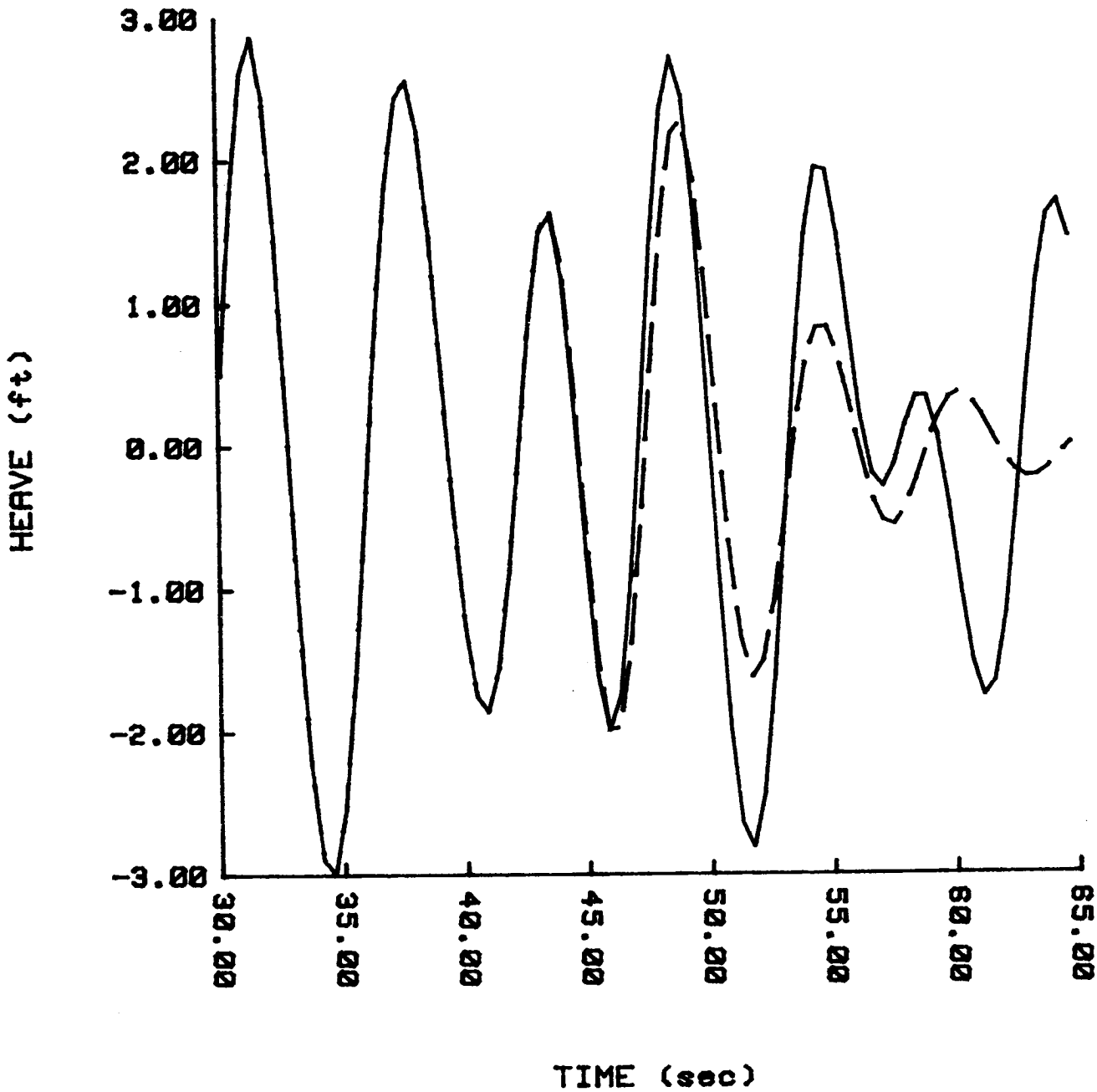

Figure 5.1

Heave Simulation Results and Its Prediction (dotted line starting at t=40 sec) for U=21 ft/sec and $\phi=0^0$, and in sea state 5. Perfect state knowledge is assumed.

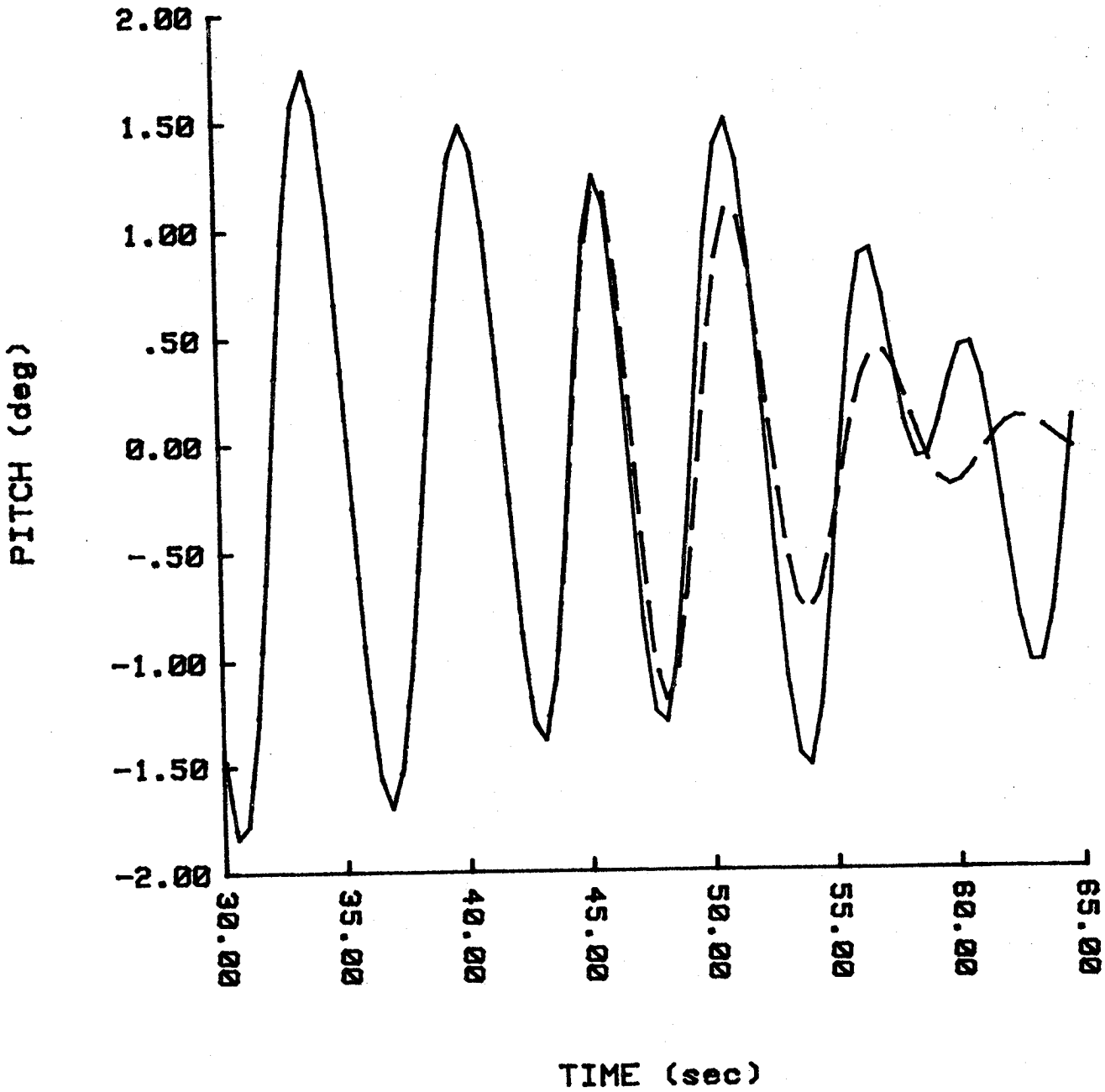

Figure 5.2

Pitch Simulation Results and Its Prediction. Same conditions as in 5.1.

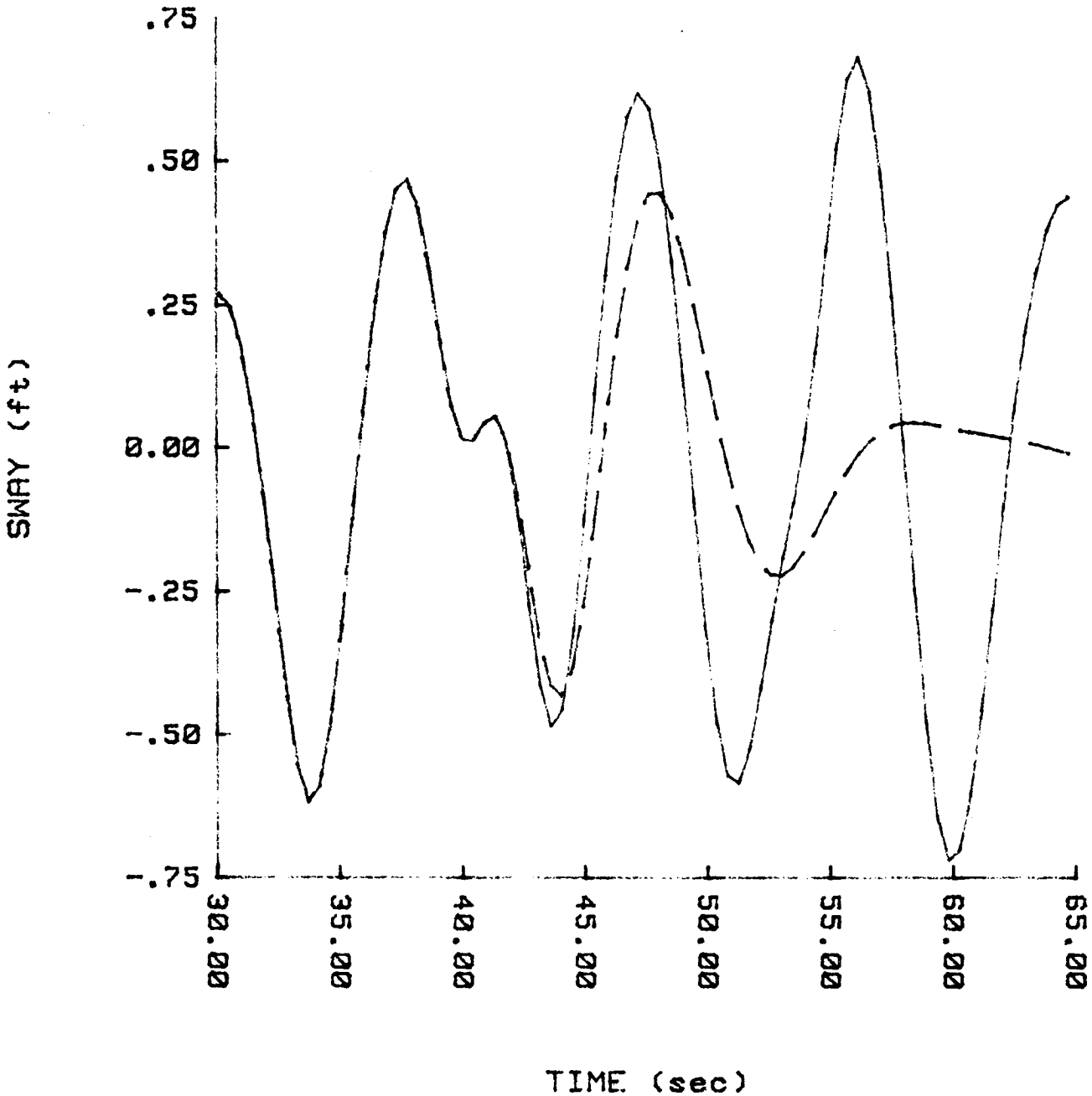

Figure 5.3

Sway Simulation Results and Its Prediction (dotted line starting at t=40 sec) for U=15.5 ft/sec and $\phi=45^0$, and in sea state 5. Perfect state knowledge is assumed.

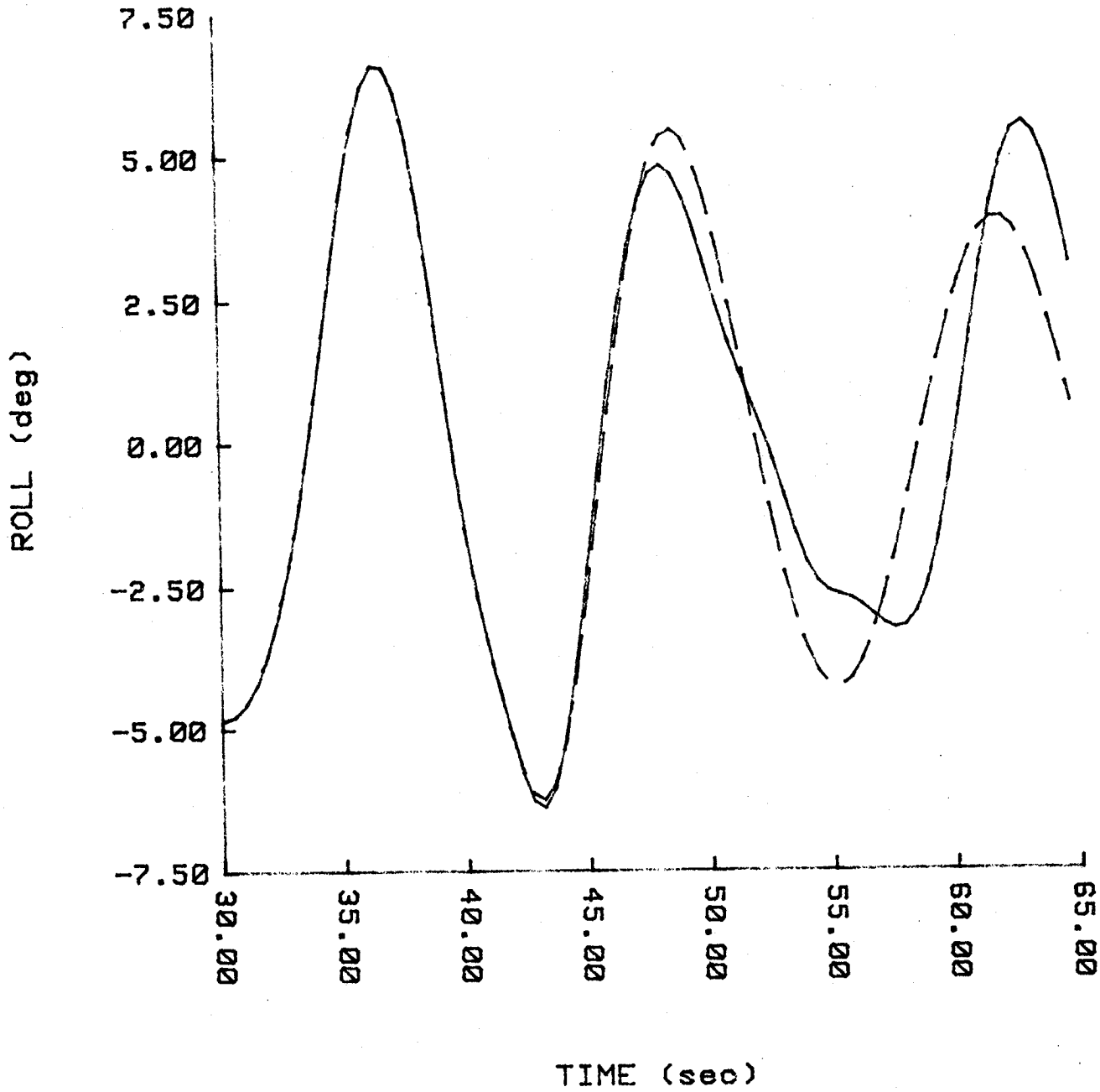

Figure 5.4

Roll Simulation Results and Its Prediction. Same conditions as in 5.3.

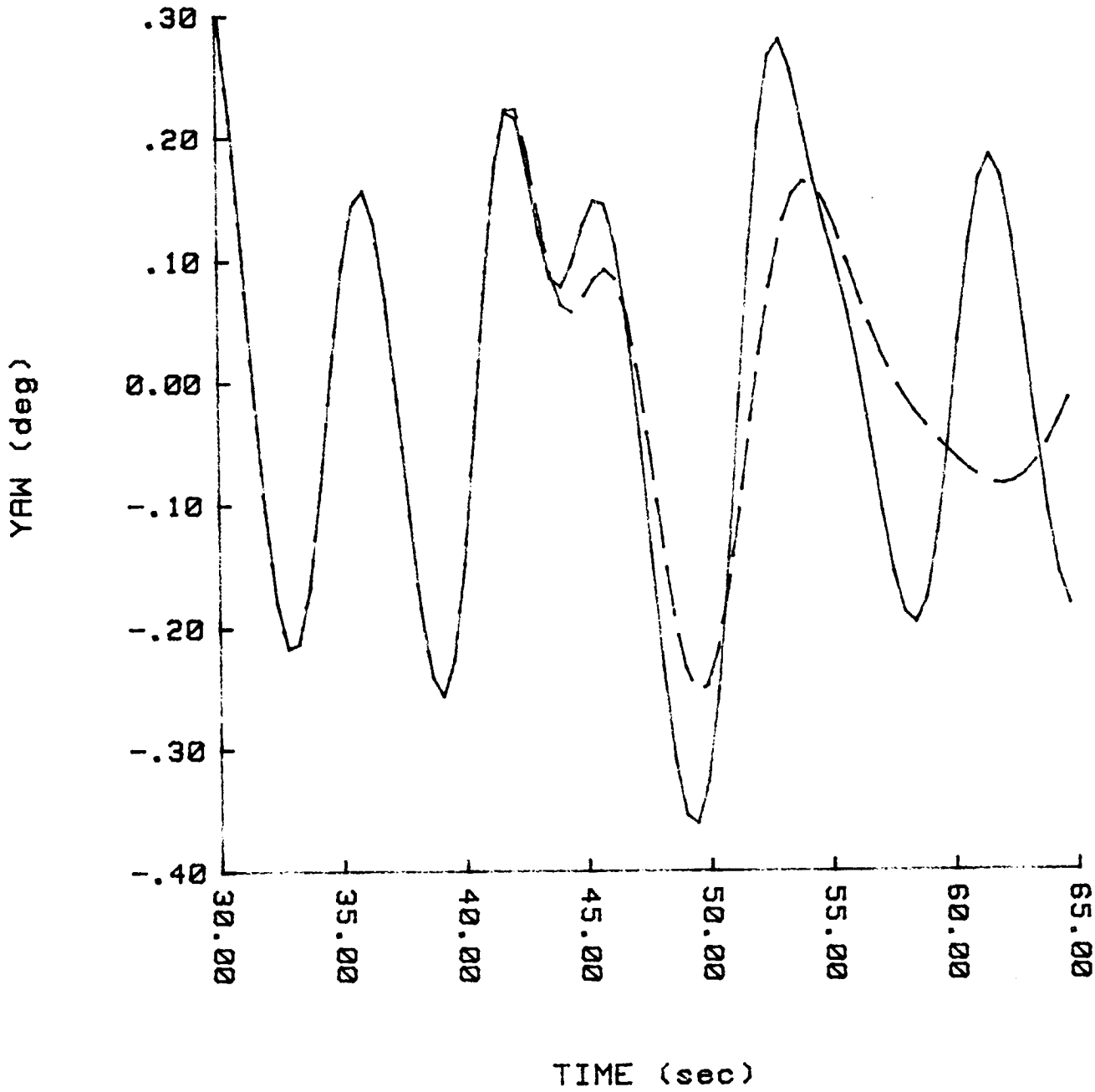

Figure 5.5

Yaw Simulation Results and Its Prediction. Same conditions as in 5.3.

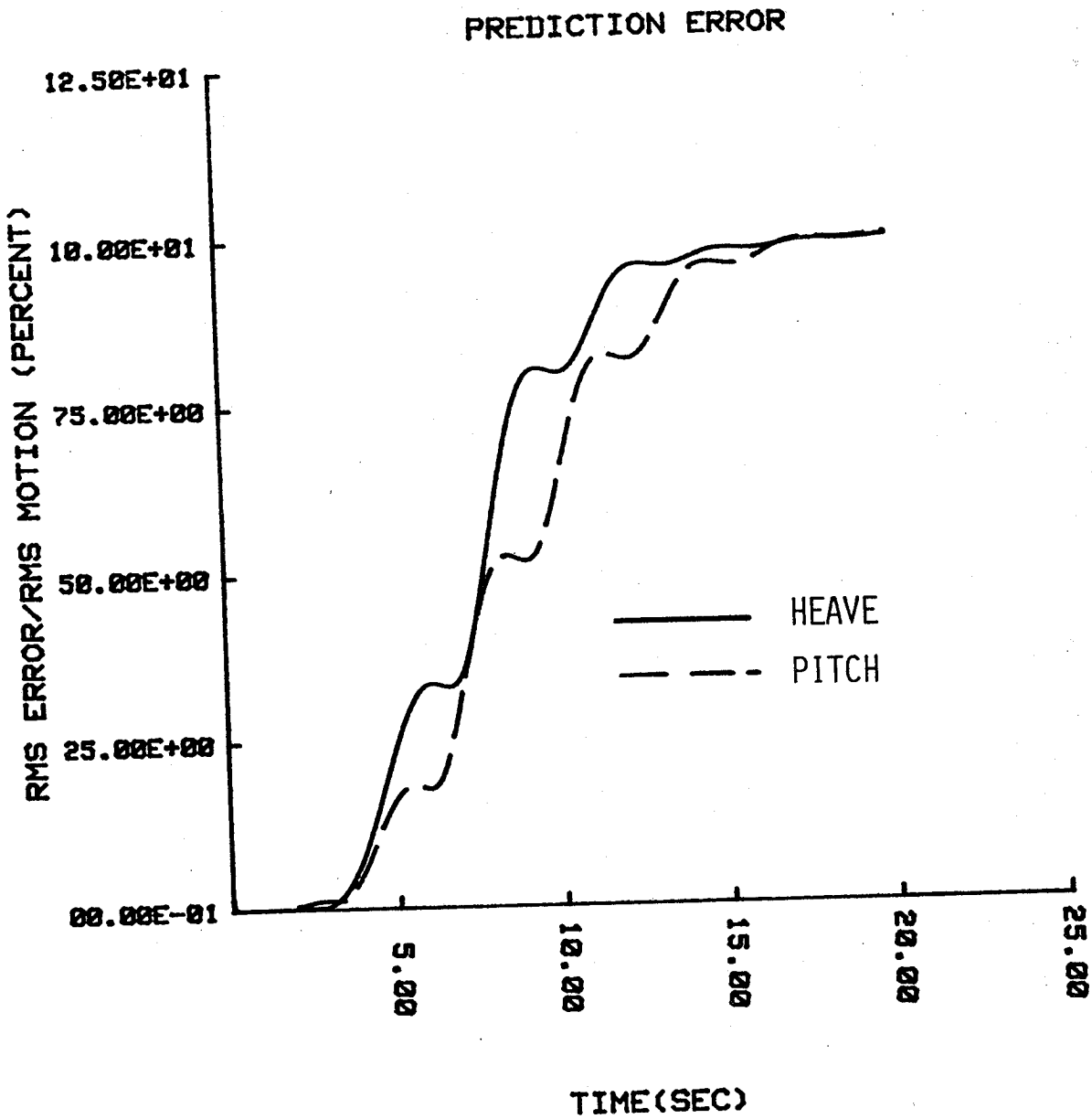

Figure 5.6

RMS Prediction Error Over RMS Motion Versus Prediction Time for Heave and Pitch, U=21 ft/sec, W_m=0.72 rad/sec, sea state 5.

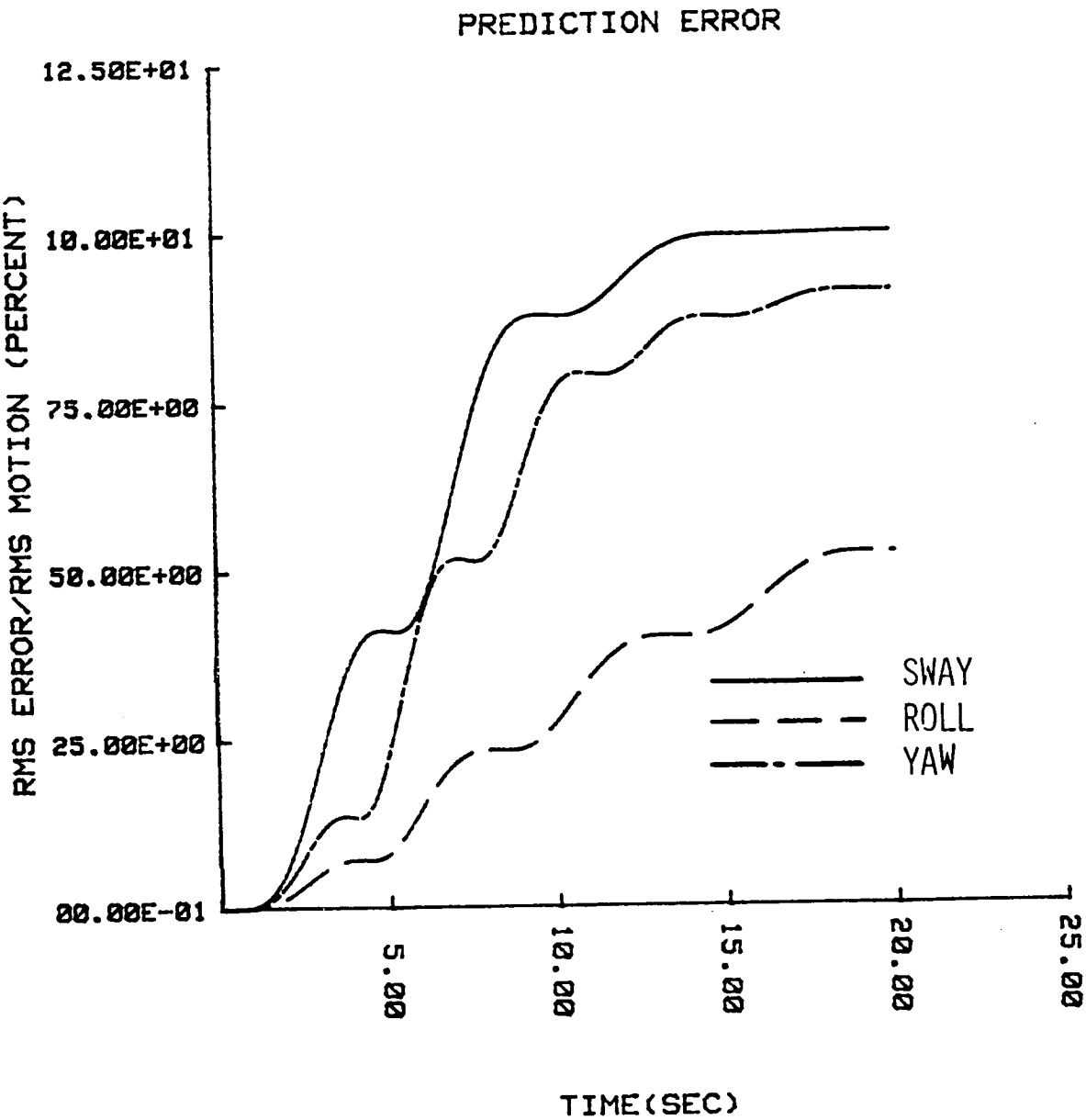

Figure 5.7

RMS Prediction Error Over RMS Motion Versus Prediction Time for Sway, Roll, Yaw. U=15.1 ft/sec, $\phi=45^0$, sea state 5.

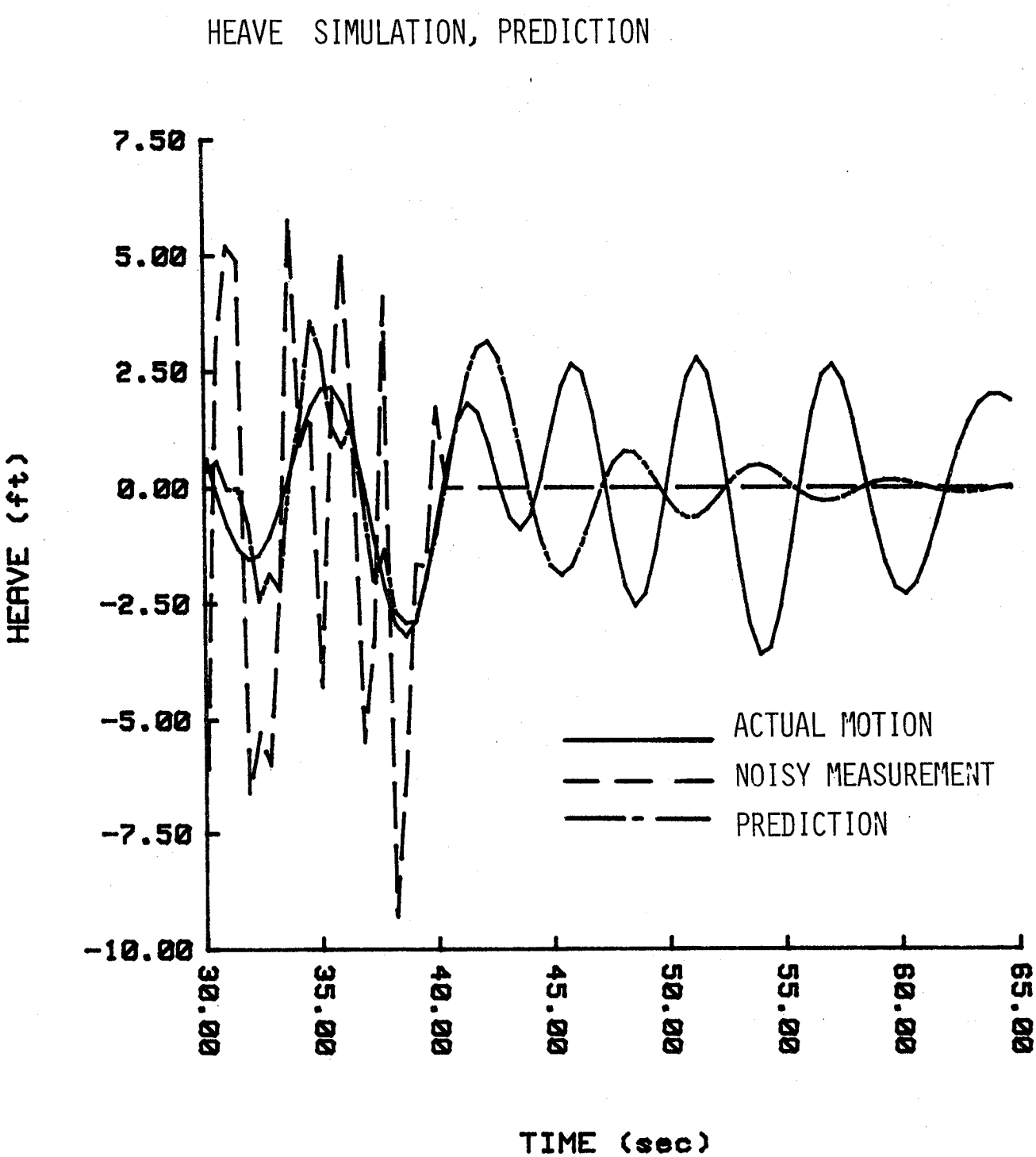

Figure 5.8

Heave Simulation Results, Its Kalman Filter Estimate (up to 40 sec) and Its Prediction Using the Kalman Filter Estimate (after t=40 sec). Same conditions as in 5.1.

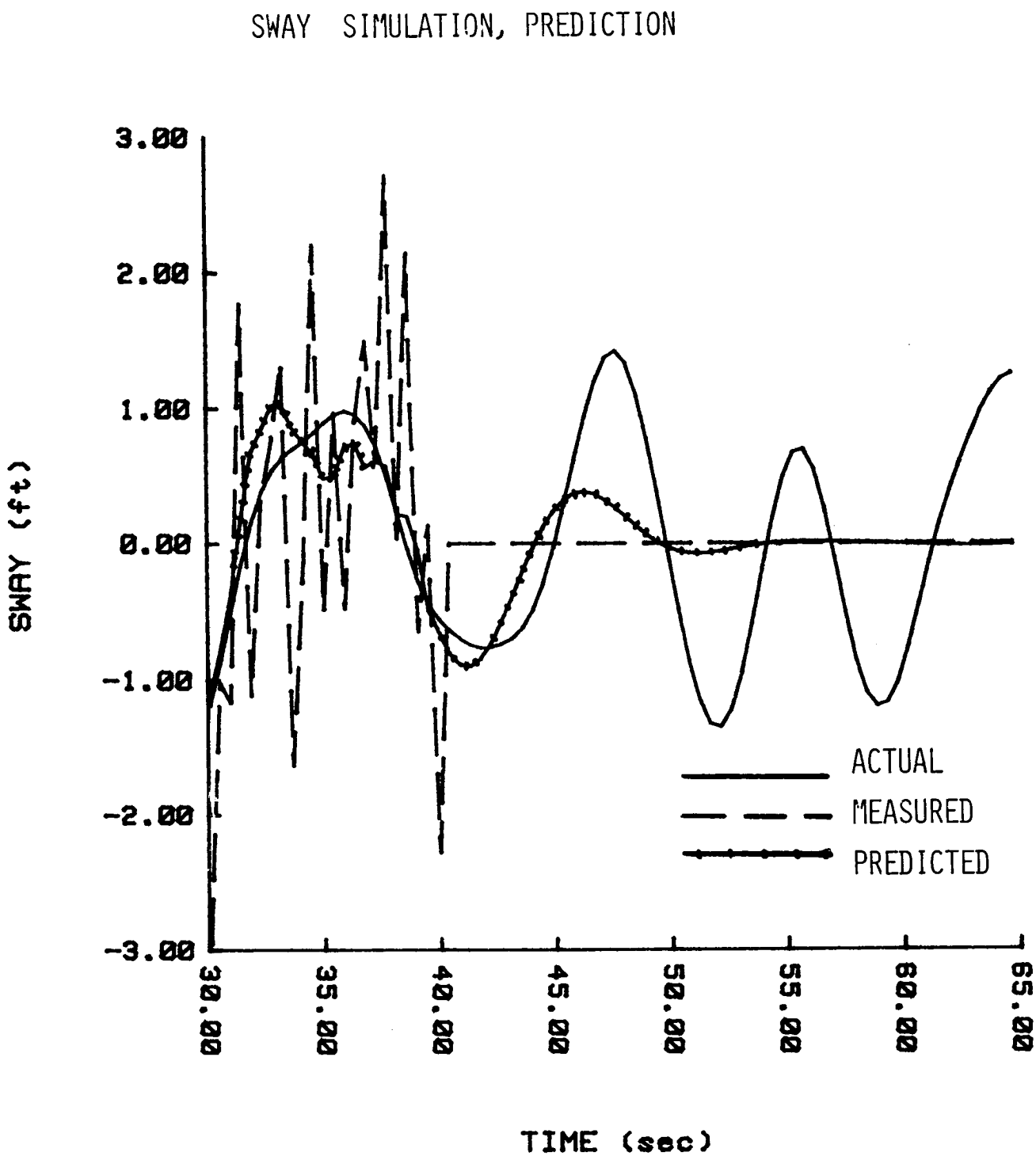

Figure 5.9

Sway Simulation Results, Its Kalman Filter Estimate (up to 40 sec) and Its Prediction Using the Kalman Filter Estimate (after t=40 sec). Sam conditions as in 5.3.

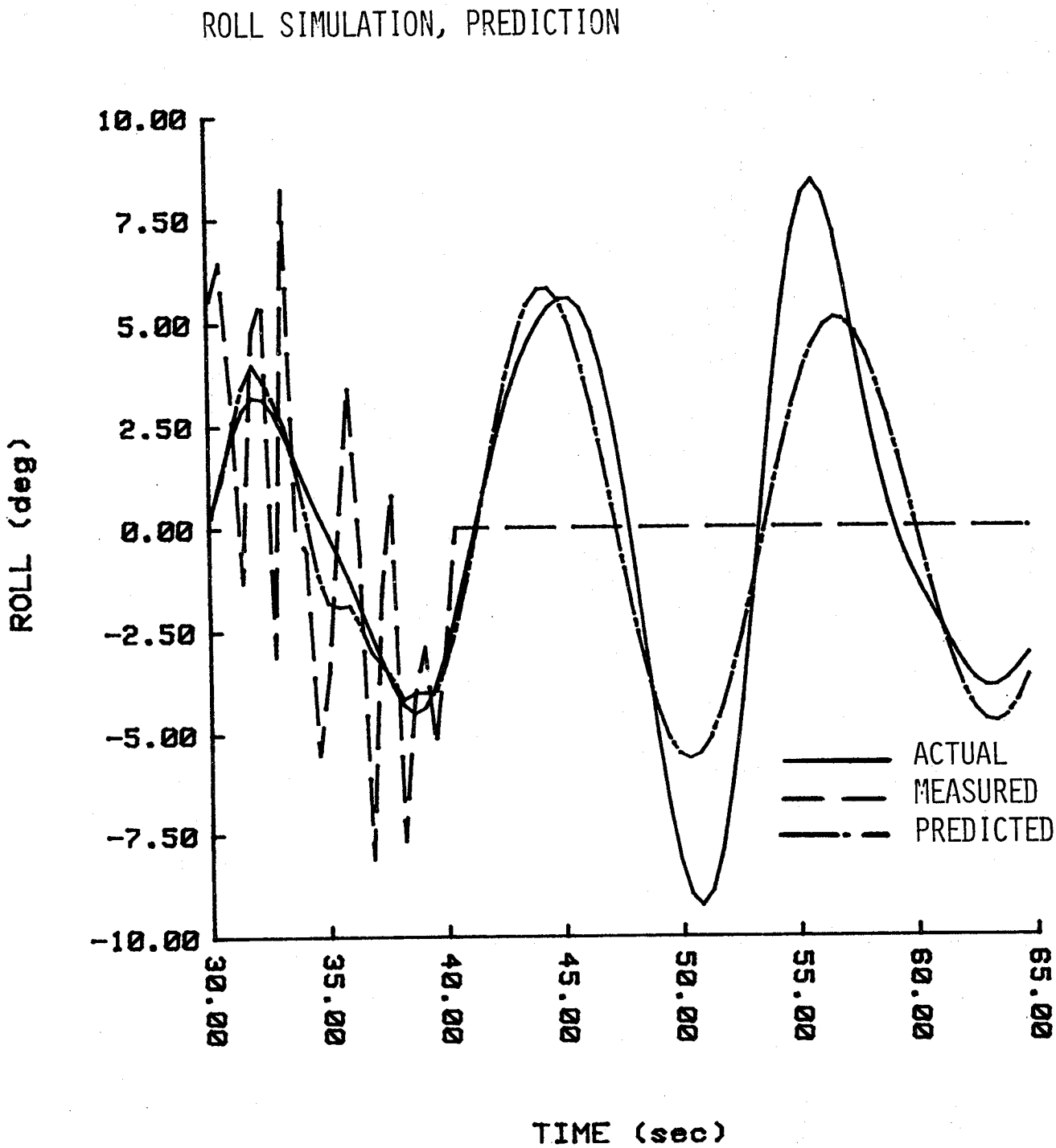

Figure 5.10

Roll Simulation and Prediction. Same conditions as in 5.9.

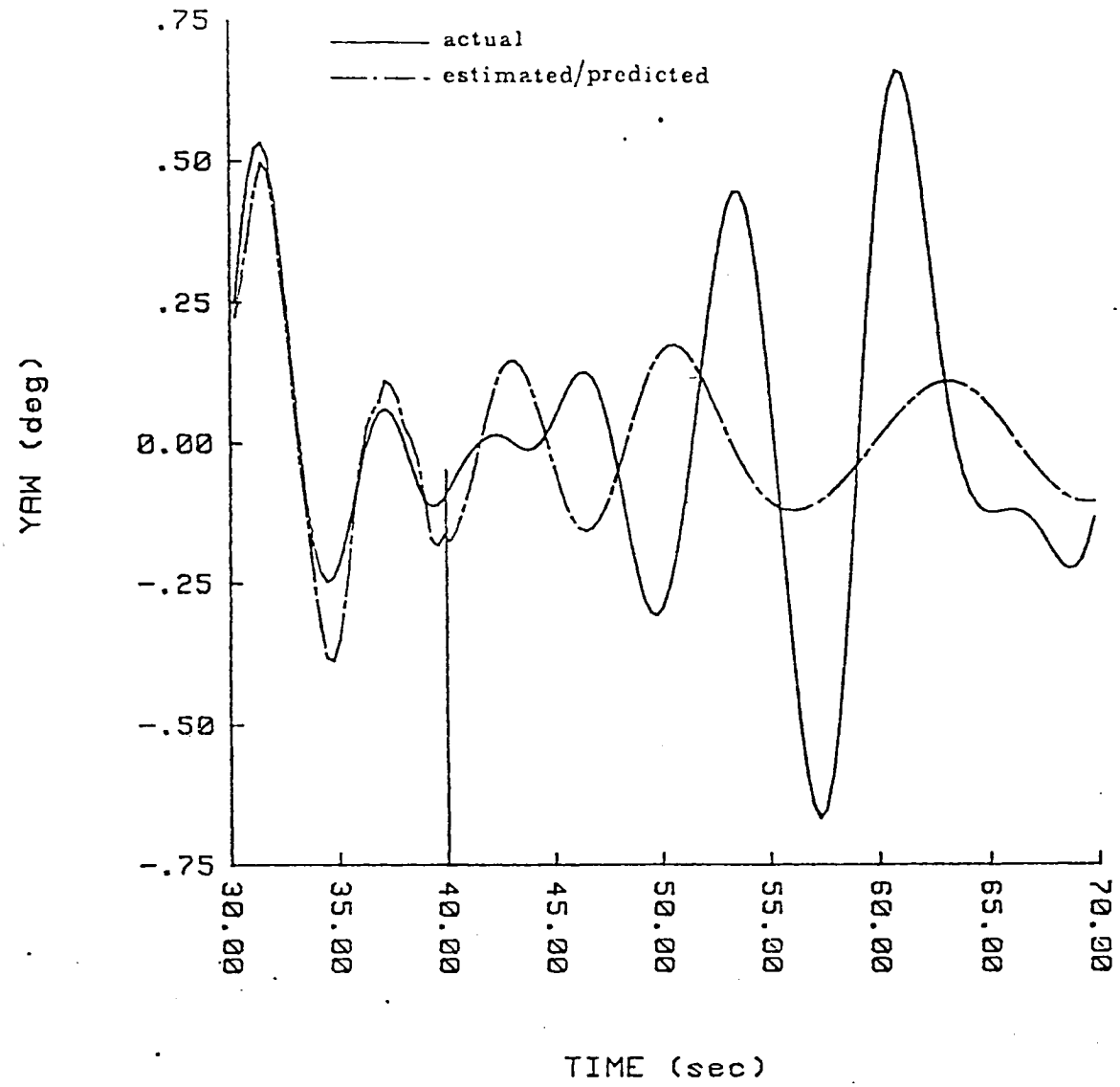

Figure 5.11

Yaw Simulation and Prediction. Same conditions as 5.9.

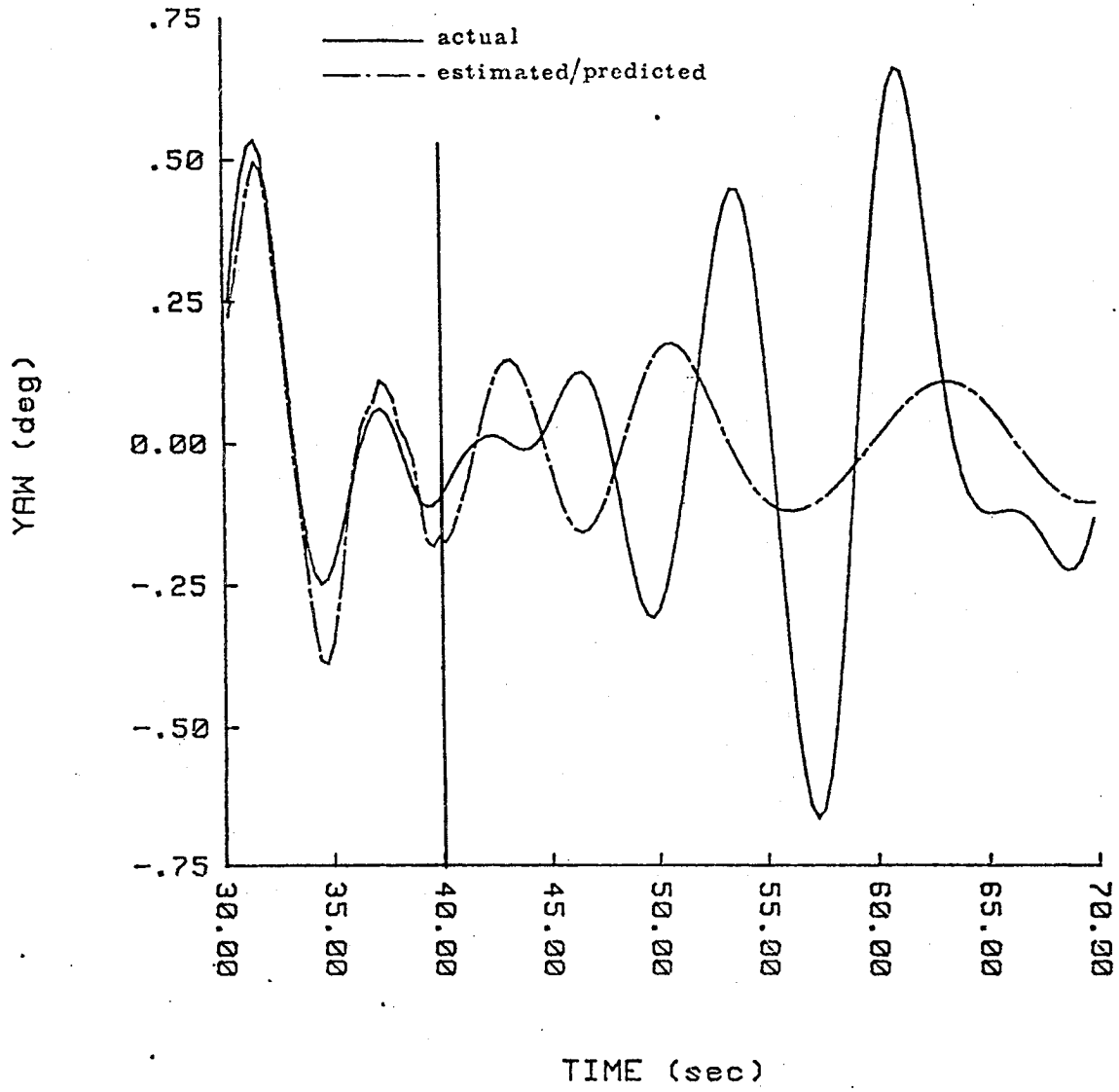

Figure 5.11:

Yaw Simulation and Prediction. Same conditions as 5.9.

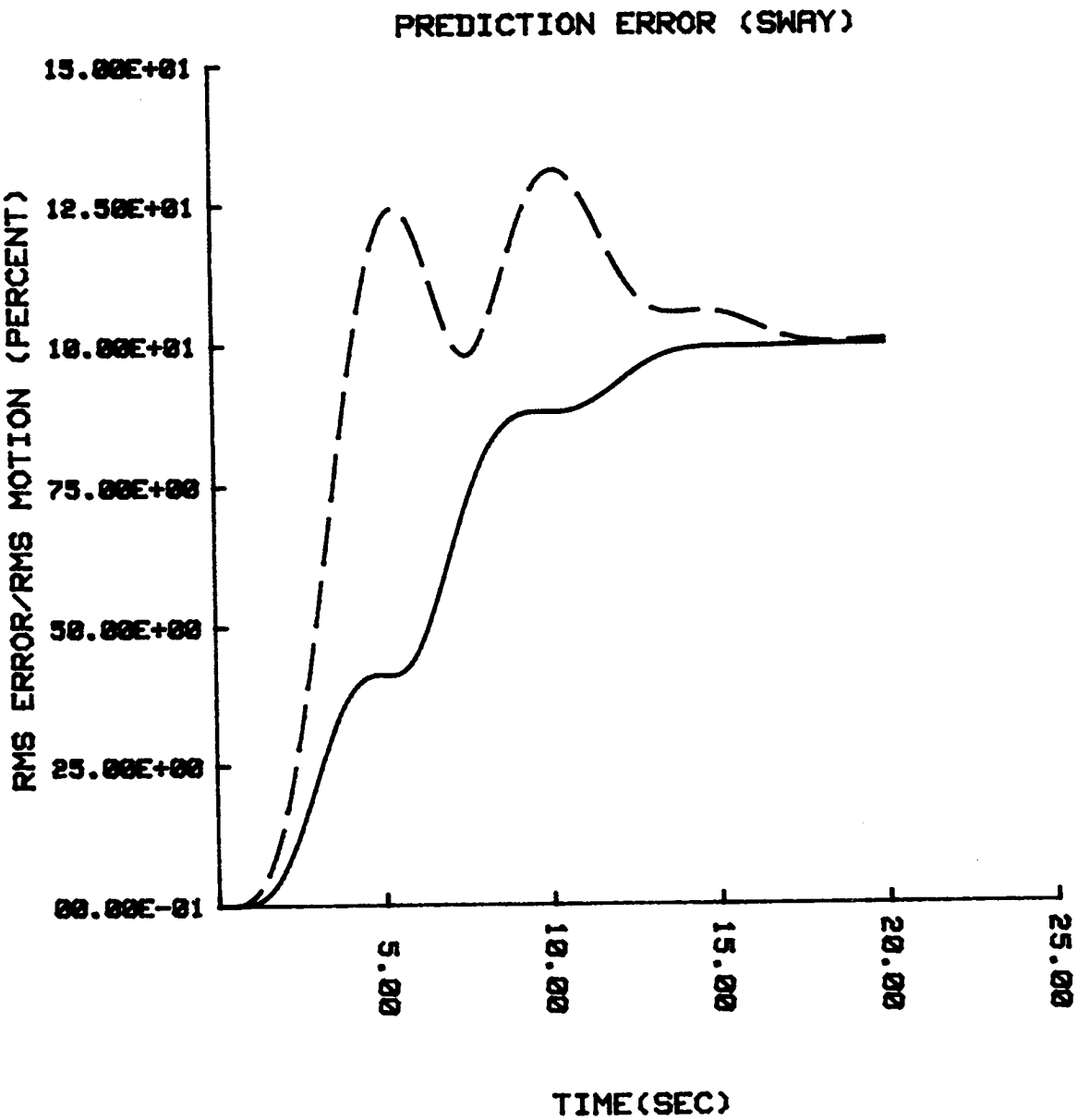

Figure 5.12

RMS Prediction Error Versus RMS Motion Versus Prediction Time for Sway. Actual W_m=0.52 rad/sec, used W_m=0.72 rad/sec. All other conditions as in 5.3.

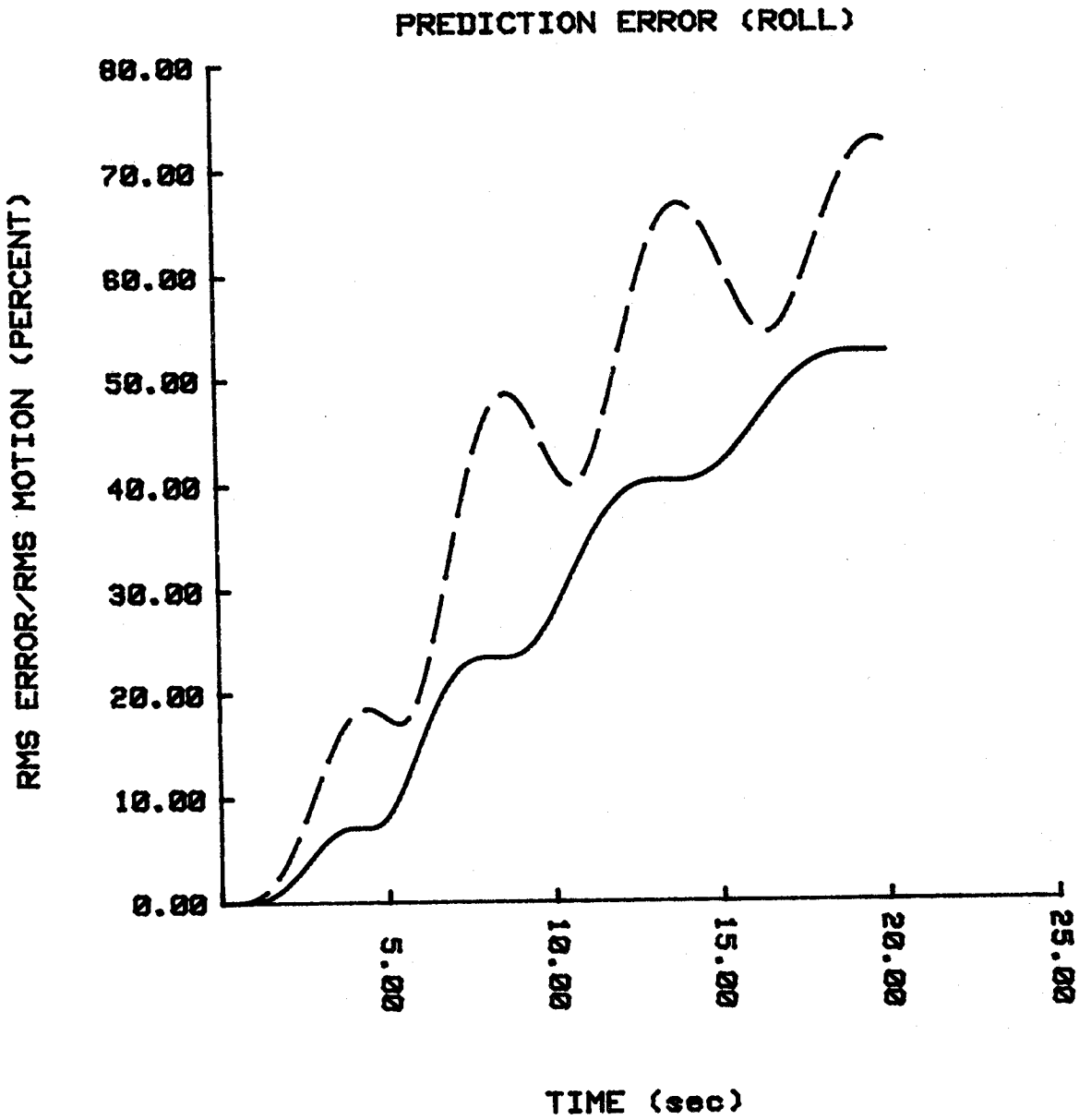

Figure 5.13

RMS Error Versus Prediction Time Roll. Same condition as in 5.12.

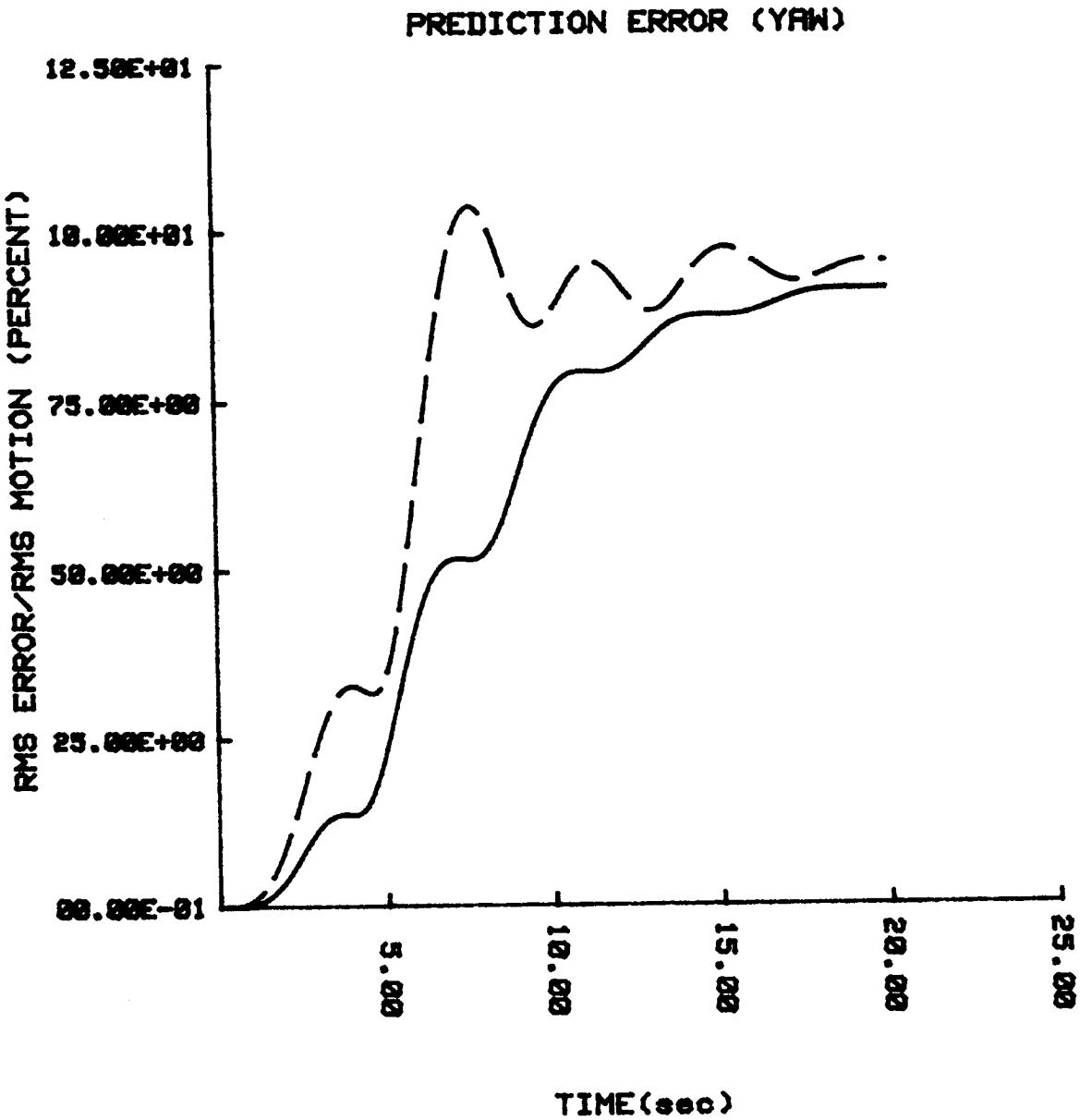

Figure 5.14

RMS Error Versus Prediction Time for Yaw. Same conditions as in 5.12.

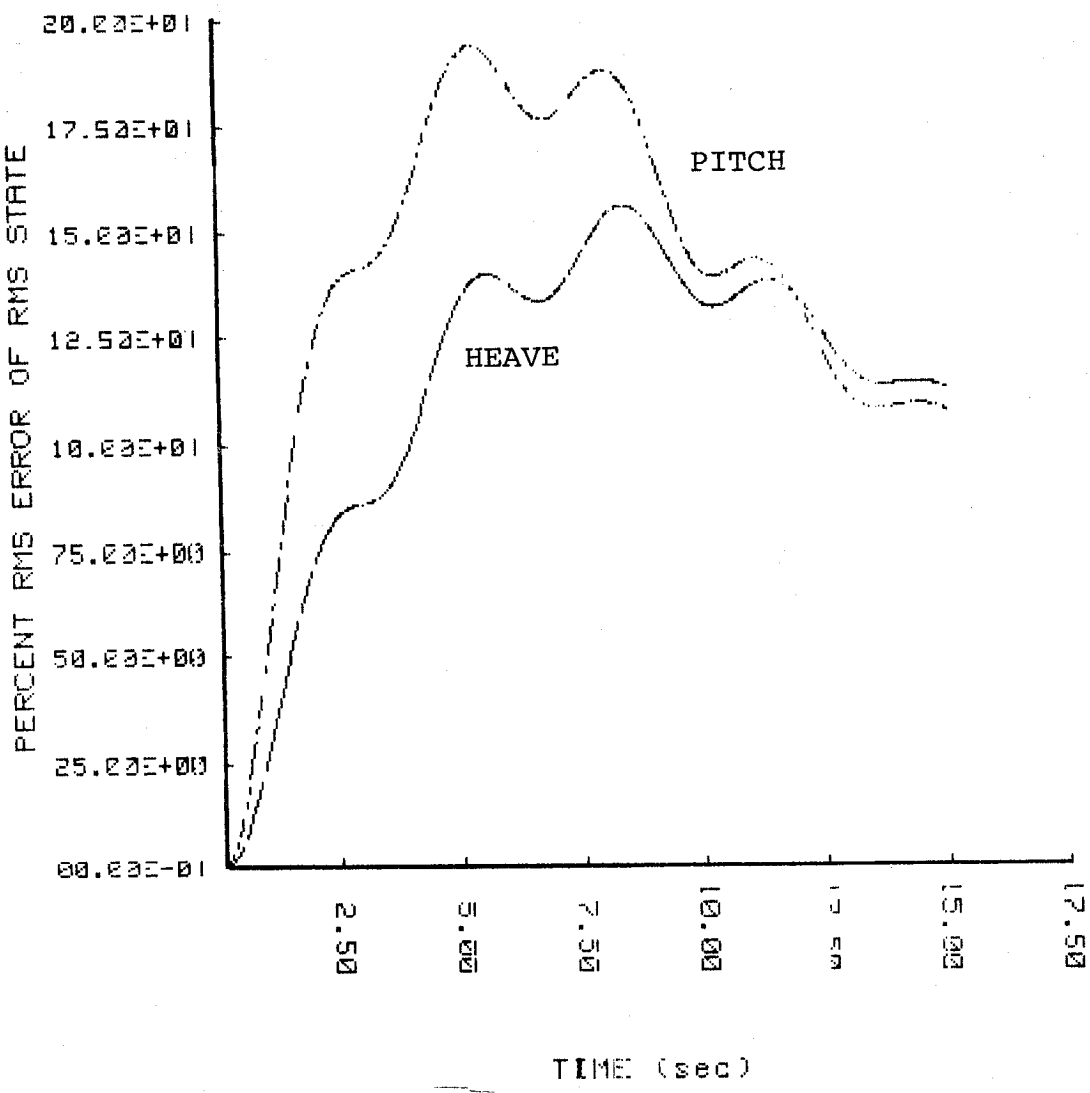

Figure 5.15

RMS Prediction Error Over RMS Motion for Heave and Pitch. U=21 ft/sec, $\phi=0^0$, sea state 5. In the prediction model the nonminimum phase zero have been omitted.

CONCLUSIONS

A satisfactory approximation of the ship motion equations as provided by hydrodynamic theory has been achieved. The approximation is valid within the wave frequency range and for seas described by the Bretschneider spectrum, whose major limitations are

(a) uni-directional seas

(b) unlimited fetch, deep water

The resulting two groups of motions, i.e., heave-pitch and sway-roll-yaw can be approximated separately requiring 15 and 16 states respectively. If both must be used a 25 state system is required.

The model depends parametrically on the ship speed, the wave angle and the significant wave height and modal frequency.

The Kalman filter is designed using as measurement noise intensity, values from ship vibration amplitudes. For sway-roll-yaw the vibration levels are small, nonetheless, to bound the filter eigenvalues below 2.0 rad/sec similar values were used, as for heave-pitch.

It should be remembered that heave and pitch are related by a non-minimum phase transfer function, resulting in reduced filter accuracy. Actually, heave is 90° out of phase for low frequencies with respect to all the other motions.

A sensitivity analysis of the filter performance indicates that the most critical parameter is the spectrum modal frequency. It should be remembered that a sea spectrum may contain more than one peak, in which case it is essential to obtain an accurate estimate of both peak frequenices.

Of particular interest is the fact that the wave direction does not have a significant influence on the estimation error. This means that although our modeling used a uni-directional Bretschneider spectrum, it can be applied in its present form for directional seas.

The models derived herein can be used to predict the ship's motions up to 5 seconds ahead in time for all motions and 10 seconds for roll. When modeling errors and noise are taken into account, a more realistic estimate of 2-3 seconds for all motions and 6-8 seconds for roll is obtained.

Again, the modal frequency of the sea spectrum is the most critical parameter. Also, the nonminimum phase zeros can deteriorate the performance of the predictor significantly if omitted.

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REFERENCES

(1) Athans, M., "The Role and Use of the Stochastic Linear-Quadratic - Gaussian Problem in Control System Design", IEEE Trans. on Automatic Control, Vol. AC-16, Dec. 1971.

(2) Balchen, J.G., Jenssen, N.A., Saelid, S., "Dynamic Positioning Using Kalman Filtering and Optimal Control Theory," Automation in Offshore Oil Field Operations, 1976, pp. 183-188.

(3) Barr, R.A., Ankudinov, V., "Ship Rolling, Its Prediction and Reduction Using Roll Stabilization," Marine Technology, Jan. 1977.

(4) Bodson, M., "Lateral Control System Design for VTOL Landing on a DD963 in High Sea States," S.M. Thesis, Dept. of Electrical Engineering and Computer Science, MIT, Cambridge, Mass. 1982.

(5) Bryson, A., Ho, Y.C., "Applied Optimal Control," Hemisphere Publishing, Co., New York, 1975.

(6) Dalzell, J.F., "A Note on Short-Time Prediction of Ship Motions," Journal of Ship Research, Vol. 9, No. 2, 1965.

(7) Doyle, J.C., Stein, G., "Robustness with Observers," IEEE Trans. on Automatic Control, Vol. AC-24, August 1979.

(8) Gelb, A., editor, "Applied Optimal Estimation," MIT Press, Cambridge, MA. 1974.

(9) Grimble, M.J., Patton, R.J., Wise, D.A., "The Design of a Dynamic Ship Positioning Control System Using Extended Kalman Filtering Technique," Oceans '79, San Diego, CA, Sept. 1979.

(10) Kallstrom, C.G., "Simulation of Ship Steering," Dept. of Automatic Control, Lund Institute of Technology, Lund, Sweden, Coden: LUTFD2/(TFRT-7109)/1-353/1976.

(11) Kaplan, P., Kaff, A.I., "Evaluation and Verification of Computer Calculations of Wave-Induced Ship Structural Loads," Ship Structure Committee, Report No. SSC-229, 1972.

(12) Kaplan, P., "A Study of Prediction Techniques for Aircraft Carrier Motion at Sea," AIAA Sixth Aerospace Science Meeting, Paper No. 68-123, 1968.

(13) Kim, C.H., Chou, F.S., Tien, D., "Motions and Hydrodynamic Loads of a Ship Advancing in Oblique Waves," Trans. SNAME, Vol. 88, 1980.

(14) Kwakernaak, H., Sivan, R., "Linear Optimal Control Systems," Wiley, 1972.

(15) McMuldroch, C.G., "VTOL Controls for Shipboard Landing," Laboratory for Information and Decision Systems, Report LIDS-TH-928, 1979, MIT, Cambridge, MA.

(16) Newman, J.N., "Marine Hydrodynamics," MIT Press, 1978, Cambridge, MA.

(17) Ogilvie, T.F., "Recent Progress Toward the Understanding and Prediction of Ship Motions," Fifth Symposium on Naval Hydrodynamics, Bergen, Norway, 1964.

(18) Salvesen, N., Tuck, E.O., Faltinsen, O., "Ship Motions and Sea Loads," Trans. SNAME, Vol. 78, 1970.

(19) Sidar, M., Doolin, B.F., "On the Feasibility of Real Time Prediction of Aircraft Carrier Motion at Sea," NASA Techn. Memo X-6245, 1975.

(20) Smith, O.J.M., "Feedback Control Systems," McGraw Hill, New York 1958.

(21) St. Denis, M., Pierson, W.J., "On the Motions of Ships in Confused Seas," Trans. SNAME, 1953.

(22) Tick, L.J., Differential Equations with Frequency Dependent Coefficients," J. of Ship Research, Vol. 3, No. 2., 1959.

(23) Triantafyllou, M., Athans, M., "Real Time Estimation of the Motions of a Destroyer Using Kalman Filtering Techniques," Laboratory for Information and Decision Systems Report, MIT, Cambridge, MA 1981.

(24) Triantafyllou, M., Athans, M., "Real Time Estimation of the Heaving and Pitching Motions of a Ship Using a Kalman Filter," Proc. Oceans '81, Boston, MA, Sept. 1981.

(25) Triantafyllou, M., Bodson, M., "Real Time Prediction of Marine Vessel Motion Using Kalman Filtering Techniques," Proc. OTC, Houston, Texas, 1982.

(26) Vugt, J.H., "The Hydrodynamic Forces and Ship Motions in Oblique Waves," Netherlands Ship Research Center TNO Report No. 150S, Dec., 1971.

(27) Yumori, I.R., "Real Time Prediction of Ship Response to Ocean Waves Using Time Series Analysis," Oceans '81, Boston, MA., Sept. 1981.

(28) "5-Degree of Freedom Seakeeping Program Manual," Design Laboratory, Ocean Engineering Dept., MIT, Cambridge, MA, 1979.

(29) "Dynamic Control Systems Software - User's Manual," Laboratory for Information and Decision Systems, MIT, Cambridge, MA. 1980.

(30) Wiener, N., "Time Series," MIT Press, Cambridge, MA. 1949.

APPENDIX 1

Hydrodynamic Theory

In the text, the simple one dimensional equation of heave motion was derived to demonstrate the principles involved. Here we will proceed to write the overall equations of motion.

We will avoid extensive hydrodynamic theory developments since [15] and [17] provide an in depth coverage. Within linear theory, we intend to write the added mass, damping and exciting force terms.

The equations of motion can be written as

$$\{-\omega^2 \quad [M + A] + i\omega\, B + C\}\, \underline{x} = \underline{F}\eta \tag{1}$$

where

$$M = \{M_{ij}\}\ ,\ A = \{A_{ij}\},\ B = \{B_{ij}\},\ C = \{C_{ij}\} \tag{2}$$

We can find the various matrices from hydrodynamic theory [15], [17] and dynamics. We omit surge as a second order quantity so that $\underline{x}$ is a vector of dimension five:

$$M = \begin{bmatrix} m & 0 & -mz_c & 0 & 0 \\ 0 & m & 0 & 0 & 0 \\ -mz_c & 0 & I_{xx} & 0 & I_{xz} \\ 0 & 0 & 0 & I_{yy} & 0 \\ 0 & 0 & I_{zx} & 0 & I_{zz} \end{bmatrix} \tag{3}$$

with m the mass of the ship, z_c the distance of the center of

gravity, vertically, from the origin.

$$A = \begin{bmatrix} A_{22} & 0 & A_{24} & 0 & A_{26} \\ 0 & A_{33} & 0 & A_{35} & 0 \\ A_{42} & 0 & A_{44} & 0 & A_{46} \\ 0 & A_{53} & 0 & A_{55} & 0 \\ A_{62} & 0 & A_{64} & 0 & A_{66} \end{bmatrix} \tag{4}$$

$$B = \begin{bmatrix} B_{22} & 0 & B_{24} & 0 & B_{26} \\ 0 & B_{33} & 0 & B_{35} & 0 \\ B_{42} & 0 & B_{44} & 0 & B_{46} \\ 0 & B_{53} & 0 & B_{55} & 0 \\ B_{62} & 0 & B_{64} & 0 & B_{66} \end{bmatrix} \tag{5}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \rho g A_{\omega p} & 0 & C_{35} & 0 \\ 0 & 0 & \Delta(GM)_{\tau} & 0 & 0 \\ 0 & C_{53} & 0 & \Delta(GM)_{L} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{6}$$

With $A_{\omega p}$ the waterplane area of the ship, Δ the displacement, $(GM)_{\tau}$ the transverse metacentric height and $(GM)_{L}$ the longitudinal metacentric height.

The following relations hold true within strip theory [17]

$$A_{33} = A^{\circ}_{33} - \frac{U}{\omega^2} b^{A}_{33}$$

$$B_{33} = B^{\circ}_{33} + U\, \alpha^{A}_{33}$$

$$A_{35} = A^{\circ}_{35} - \frac{U}{\omega^2} + \frac{U}{\omega^2}\, x_A\, b^{A}_{33} - \frac{U^2}{\omega^2}\, \alpha^{A}_{33}$$

$$B_{35} = B^{\circ}_{35} + UA^{\circ}_{33} - Ux_A\, \alpha^{A}_{33} - \frac{U^2}{\omega^2}\, b^{A}_{33}$$

$$A_{53} = A^{\circ}_{35} + \frac{U}{\omega^2}\, B^{\circ}_{33} + \frac{U}{\omega^2}\, x_A\, b^{A}_{33}$$

$$B_{53} = B^{\circ}_{35} - UA^{\circ}_{33} - Ux_A\, \alpha^{A}_{33}$$

$$A_{55} = A^{\circ}_{55} + \frac{U^2}{\omega^2}\, A^{\circ}_{33} - \frac{U}{\omega^2} x_A^2\, b^{A}_{33} + \frac{U^2}{\omega^2} x_A\, \alpha^{A}_{33}$$

$$B_{55} = B^{\circ}_{55} + \frac{U^2}{\omega^2}\, B^{\circ}_{33} + Ux_A^2\, \alpha^{A}_{33} + \frac{U^2}{\omega^2} x_A\, b^{A}_{33}$$

The superscript O denotes quantities at zero speed. X_A is the distance to the aftermost cross-section of the ship and a^{A}_{ij}, b^{A}_{ij} are its sectional added mass characteristics. These last two quantities are important only for cruiser stern ships. The A°_{ij}, B°_{ij} can be found using the M.I.T. Seakeeping Program [27]. Similarly, we can find:

$$A_{22} = A^{\circ}_{22} - \frac{U}{\omega^2}\, b^{A}_{22}$$

$$B_{22} = B^{\circ}_{22} + U\, \alpha^{A}_{22}$$

$$A_{24} = A_{42} = \overset{\circ}{A}_{24} - \frac{U}{\omega^2} b^A_{24}$$

$$B_{24} = B_{42} = \overset{\circ}{B}_{24} + U\,\alpha^A_{24}$$

$$A_{26} = \overset{\circ}{A}_{26} + \frac{U}{\omega^2}\overset{\circ}{B}_{22} - \frac{U}{\omega^2} x_A\, b^A_{22} + \frac{U^2}{\omega^2}\alpha^A_{22}$$

$$B_{26} = \overset{\circ}{B}_{26} - U\overset{\circ}{A}_{22} + Ux_A\,\alpha^A_{22} + \frac{U^2}{\omega^2} b^A_{22}$$

$$A_{44} = \overset{\circ}{A}_{44} - \frac{U}{\omega^2} b^A_{24}$$

$$B_{44} = \overset{\circ}{B}_{44} + U\,\alpha^A_{44} + B^*_{44}$$

with B^*_{44} the equivalent nonlinear damping

$$A_{46} = \overset{\circ}{A}_{46} + \frac{U}{\omega^2}\overset{\circ}{B}_{24} - \frac{U}{\omega^2} x_A\, b^A_{24} + \frac{U^2}{\omega^2}\alpha^A_{24}$$

$$B_{46} = \overset{\circ}{B}_{46} - U\overset{\circ}{A}_{24} + Ux_A\,\alpha^A_{24} + \frac{U^2}{\omega^2} b^A_{24}$$

$$A_{62} = \overset{\circ}{A}_{26} - \frac{U}{\omega^2}\overset{\circ}{B}_{22} - \frac{U}{\omega^2} x_A\, b^A_{22}$$

$$B_{62} = \overset{\circ}{B}_{26} + U\overset{\circ}{A}_{22} + Ux_A\,\alpha^A_{22}$$

$$A_{64} = \overset{\circ}{A}_{46} - \frac{U}{\omega^2}\overset{\circ}{B}_{24} - \frac{U}{\omega^2} x_A\, b^A_{24}$$

$$B_{64} = \overset{\circ}{B}_{46} + U\overset{\circ}{A}_{24} + Ux_A\,\alpha^A_{24}$$

$$A_{66} = \overset{\circ}{A}_{66} + \frac{U^2}{\omega^2}\,\overset{\circ}{A}_{22} - \frac{U}{\omega^2}\,x_A^2\,b_{22}^A + \frac{U^2}{\omega^2}\,x_A\,\alpha_{22}^A$$

$$B_{66} = \overset{\circ}{B}_{66} + \frac{U^2}{\omega^2}\,\overset{\circ}{B}_{22} + Ux_A^2\,\alpha_{22}^A + \frac{U^2}{\omega^2}\,x_A\,b_{22}^A$$

$$F_2 = \alpha_o\,\rho \int (f_2 + h_2)\,d\xi + \alpha_o\,p\,\frac{U}{i\omega}\,h_2^A$$

$$F_6 = \alpha_o\,\rho \int \left[\xi\,(f_2 + h_2) + \frac{U}{i\omega}\,h_2\right] d\xi + \alpha_o\,\rho\,\frac{U}{i\omega}\,x_A\,h_2^A$$

$$F_4 = \alpha_o\,\rho \int (f_4 + h_4)\,d\xi + \alpha_o\,\rho\,\frac{U}{i\omega}\,h_4^A$$

$$F_3 = \rho\,\alpha_o \int (f_3 + h_3)\,d\xi + \rho\,\alpha_o\,\frac{U}{i\omega}\,h_3^A$$

$$F_5 = -\rho\,\alpha_o \int \left[\xi(f_3 + h_3) + \frac{U}{i\omega}\,h_3\right] d\xi - \rho\,\alpha_o\,\frac{U}{i\omega}\,x_A h_3^A$$

where f_j is the sectional Froude-Kryloff force and h_j the sectional diffraction force.

VCG = Vertical Center of Gravity = - 4.6 ft.
(from waterplane)

GM = Metacentric Height = 4.16 ft.

XCG = Longitudinal Center of Gravity = 1.07 ft. AFT
(from admidships)

Δ = Displacement = 6,800 ton

C_b = Block Coefficient = 0.461

APPENDIX 2

The M.I.T. Sea-keeping Program [27] was used to derive the hydrodynamic data. The values used to develop the simple models for heave-pitch and sway-roll-yaw (as already mentioned, to first order heave and pitch are uncoupled from sway, roll and yaw).

All units are consistent such that the forces are obtained in tons, the moments in ton·ft., the linear motions in ft. and the angular motions in radians. The programs change the angular motions and express them in degrees only in the final (output) stage.

Heave-Pitch Characteristics

M	= 214	C_{33}	= 587
$A^0 33$	= 281	B^0_{33}	= 260
$A^0 35$	= 15500	B^0_{35}	= 15500
C_{35}	= 260		
I_{55}	= $3.76*10^6$	A_{55}	= $4.20*10^6$
B_{55}	= $3.8*10^6$	C_{55}	= $9.53*10^6$
A1	= 550		
A2	= 120000		

The principal ship characteristics have as follows

L_{BP} = Length between perpendiculars = 529 ft.

B = Beam = 55 ft.

T = Draft = 18 ft.

Sway - Roll - Yaw Characteristics

A_{44} = 22,800

I_{44} = 104,000

B_{44} = 800 + B^{*}_{44}

C_{44} = 28,800

A_{24} = -760 $\qquad$ B_{24} = -50

A_{46} = 181,000 $\qquad$ B_{46} = 5,600

A_{22} = 220 $\qquad$ B_{22} = 10

A_{66} = $4.16*10^{6}$

I_{66} = $3.8 * 10^{6}$

B_{66} = 130,000

A_{26} = 14,500 $\qquad$ B_{26} = 370

A_{2} = 380

A_{4} = 2,400

A_{6} = 23,000

APPENDIX 3

Simulation

The simulation of a continuous system

$$\dot{x} = Ax + BW_1$$

$$y = Cx + W_2$$

where the white noise signals W_1, W_2 have intensity V_1, V_2 respectively, is performed by constructing the equivalent discrete system

$$x(T+\delta t) = A_k\, x(t) + B_k\, W_{k1}$$

$$y(t) = C\, x(t) + W_{k2}$$

where $A_k = e^{A\delta t} = I + A\cdot\delta t + A^2\cdot\delta t^2/2\,! + \ldots$

$$B_k W_{k1} = \int_t^{t+\delta t} e^{A(t-\tau)} BW_1(\tau)\, d$$

An approximation would be

$$x(t+\delta t) = (I+A\cdot\delta t)x(t)+(B\cdot\delta t)W_{k1}$$

$$y(t) = Cx(t) + W_{k2}$$

where W_{k1}, W_{k2} discrete white noise of intensity V_{k1}, V_{k2} respectively, given by:

$$V_{k1} = V_1/\delta t$$

$$V_{k2} = V_2/\delta t$$

If a random number generator is provided with a range between 0 and 1, the following relation provides W_{k1} (and similarly W_{k2})

$$W_{k1} = (RND-0.5) \cdot \sqrt{12 \cdot V_{k1}}$$

where RND is a random number between 0 and 1.

Higher order approximations to A_k may be necessary in some cases. Problems appear in particular with lightly damped systems, i.e., when the matrix A possesses eigenvalues $-a \pm ib$ where $0<a<<b$. Such for example, is the case of roll whose damping is typically around 10% of the critical value.

The approximation:

$$e^{(-a\pm ib)\delta t} \simeq (1-a\cdot\delta t) \pm ib\cdot\delta t$$

is valid provided

$$a\cdot\delta t<<1$$

$$b\cdot\delta t<<1$$

The next order term is:

$$(-a+ib)^2 \frac{\delta t^2}{2}$$

so for accuracy we require,

$$\left|(a^2-b^2)\frac{\delta t^2}{2}\right| << a\delta t$$

$$ab\delta t^2 << b\delta t$$

which results in a single additional condition

$$\delta t << \left| \frac{2a}{a^2 - b^2} \right|$$

It is easy to see that when $a << b$ then

$$\delta t << \min \left[\frac{1}{2a} , \frac{2}{b} \cdot \left(\frac{a}{b} \right) \right]$$

This requirement may be very demanding for simulation by imposing an extremely small time step. If an appropriate step is not chosen then the real part of the (neglected) second order term reduces the first order real part, thus resulting in a reduction of the already small damping. By using a higher order approximation, in the form

$$e^{A \cdot \delta t} \simeq I + A \cdot \delta t + \ldots + \frac{A^m \delta t^m}{m!}$$

such problems are resolved. In the present case, the sway-roll-yaw model requires such treatment for efficient simulation.

Appendix 4

Computer Program Listing

```
      DIMENSION A(15,15),C(2,15),SIGMA(15,15),XI(2,2)                   MOD00010
       FX(X)=5.*(-X**4+1.)*(1.+2.*X*ALFA)-2.*ALFA*X**5                  MOD00020
      WRITE(6,2000)                                                     MOD00030
 2000 FORMAT(3X,'INPUT THE SPEED (FT/SEC) AND WAVE ANGLE (DEGREES)')    MOD00040
      READ(5,*)U,T1                                                     MOD00050
      T=T1*3.14159/180.                                                 MOD00060
      G=32.2                                                            MOD00070
      WRITE(6,2010)                                                     MOD00080
 2010 FORMAT(3X,'INPUT THE SIGN. WAVE HEIGHT (FT) AND MODAL FREQ. (RAD/SMOD00090
     1EC)')                                                             MOD00100
      READ(5,*)ZZ,OMM                                                   MOD00110
      ALFA=U/G*OMM*COS(T)                                               MOD00120
      U1=.2                                                             MOD00130
       U2=5.                                                            MOD00140
      AA=FX(U1)                                                         MOD00150
  903 UM=(U1+U2)*.5                                                     MOD00160
      IF(ABS(U1-U2).LT..0001*UM) GO TO 901                              MOD00170
      CC=FX(UM)                                                         MOD00180
      IF(AA*CC.GT.0.) GO TO 902                                         MOD00190
      U2=UM                                                             MOD00200
      GO TO 903                                                         MOD00210
  902 U1=UM                                                             MOD00220
      AA=CC                                                             MOD00230
      GO TO 903                                                         MOD00240
  901 ZS=UM+UM**2*ALFA                                                  MOD00250
      GAM=ZS*2.**.25                                                    MOD00260
      ZS1=(1.+(ZS/GAM)**4)**3/ZS**4                                     MOD00270
      ZS2=EXP(-1.25/UM**4)/UM**5/(1.+2.*ALFA*UM)                        MOD00280
      BE1=ZS1*ZS2                                                       MOD00290
      A33=495.                                                          MOD00300
      B33=260.                                                          MOD00310
      C33=537.                                                          MOD00320
      A35=15500.                                                        MOD00330
      B35=11700.+280.*U                                                 MOD00340
      C35=260.*U+17250.                                                 MOD00350
      A53=15500.                                                        MOD00360
      B53=11700.-280.*U                                                 MOD00370
      C53=-260*U+17250.                                                 MOD00380
      A55=7.99*10.**6                                                   MOD00390
      B55=3.8*10**6                                                     MOD00400
      C55=-280.*U**2+9.53*10.**6                                        MOD00410
      D=A33*A55-A35*A53                                                 MOD00420
      D1=A55/D                                                          MOD00430
      D2=-A35/D                                                         MOD00440
      D3=-A53/D                                                         MOD00450
      D4=A33/D                                                          MOD00460
      Z1=-(D1*B33+D2*B53)                                               MOD00470
      Z2=-(D1*C33+D2*C53)                                               MOD00480
      Z3=-(D1*B35+D2*B55)                                               MOD00490
      Z4=-(D1*C35+D2*C55)                                               MOD00500
      Z5=-(D3*B33+D4*B53)                                               MOD00510
      Z6=-(D3*C33+D4*C53)                                               MOD00520
      Z7=-(D3*B35+D4*B55)                                               MOD00530
      Z8=-(D3*C35+D4*C55)                                               MOD00540
      RL=529.                                                           MOD00550
```

Program to prepare matrices for the heave-pitch motions.

```
      B=55.                                                                  MOD00560
      FO=SQRT(2.*3.14159*G/(RL*COS(T)+B))+2.*3.14159/(RL*COS(T)+B)*U*COSMOD00570
     1(T)                                                                    MOD00580
      ZT=.7071                                                               MOD00590
       OMNE=OMM+OMM**2*U/32.2*COS(T)                                         MOD00600
      TH1=(2.*ZT*FO**5+FO**4*OMNE)                                           MOD00610
      TH2=(FO**4*(4.*ZT**2-1.)+2.*ZT*FO**3*OMNE)                             MOD00620
      TH3=-(OMNE*FO**4+2.*ZT*FO**5)                                          MOD00630
      TH4=FO**4                                                              MOD00640
      A1=550.                                                                MOD00650
      A2=120000.                                                             MOD00660
       TH5=D1*A1*FO**2                                                       MOD00670
      TH6=D2*A2*COS(T)                                                       MOD00680
      TH7=D3*A1*FO**2                                                        MOD00690
      TH8=D4*A2*COS(T)                                                       MOD00700
      OMN=OMM*GAM                                                            MOD00710
      S0=1.25/4.*ZZ**2/OMM**5*BET                                            MOD00720
      VP1=OMN**2                                                             MOD00730
      VP2=OMN*2.*ZT                                                          MOD00740
      VP3=VP1*SQRT(S0)                                                       MOD00750
      DO 800 I=1,15                                                          MOD00760
      DO 800 J=1,15                                                          MOD00770
  800 A(I,J)=0.                                                              MOD00780
      A(1,2)=1.                                                              MOD00790
      A(3,4)=1.                                                              MOD00800
      A(5,6)=1.                                                              MOD00810
      A(7,8)=1.                                                              MOD00820
      A(9,10)=1.                                                             MOD00830
      A(11,12)=1.                                                            MOD00840
      A(13,14)=1.                                                            MOD00850
      A(15,15)=-OMNE                                                         MOD00860
      A(8,7)=Z2                                                              MOD00870
      A(8,8)=Z1                                                              MOD00880
      A(8,9)=Z4                                                              MOD00890
      A(8,10)=Z3                                                             MOD00900
      A(10,7)=Z6                                                             MOD00910
      A(10,9)=Z5                                                             MOD00920
      A(10,9)=Z8                                                             MOD00930
      A(10,10)=Z7                                                            MOD00940
      A(15,11)=TH1                                                           MOD00950
      A(15,12)=TH2                                                           MOD00960
      A(15,13)=TH3                                                           MOD00970
      A(15,14)=TH4                                                           MOD00980
      A(8,11)=TH5                                                            MOD00990
      A(8,15)=TH6                                                            MOD01000
      A(10,11)=TH7                                                           MOD01010
      A(10,15)=TH8                                                           MOD01020
      A(2,1)=-VP1                                                            MOD01030
      A(2,2)=-VP2                                                            MOD01040
      A(2,4)=VP1                                                             MOD01050
      A(4,3)=-VP1                                                            MOD01060
      A(4,4)=-VP2                                                            MOD01070
      A(4,6)=VP1                                                             MOD01080
      A(6,5)=-VP1                                                            MOD01090
      A(6,6)=-VP2                                                            MOD01100
```

```
      A(12,11)=-FO**2                                                   MOD01110
      A(12,12)=-2*ZT*FO                                                 MOD01120
      A(12,13)=FO**2                                                    MOD01130
      A(14,13)=-FO**2                                                   MOD01140
      A(14,14)=-2*FO*ZT                                                 MOD01150
      A(14,1)=VP3                                                       MOD01160
      WRITE(6,2200)                                                     MOD01170
 2200 FORMAT('** A MATRIX')                                             MOD01180
      DO 890 I=1,15                                                     MOD01190
     -    WRITE(9,2400) (A(I,J),J=1,15)                                 MOD01200
 2400 FORMAT(8E10.4)                                                    MOD01210
  890 CONTINUE                                                          MOD01220
      END                                                               MOD01230
```

```
C
C MODEL FOR THE SWAY ROLL YAW MOTIONS OF A DD-963 DESTROYER
C REVISED VERSION
C LAST CORRECTED SEPTEMBER 11 1981
C
      DOUBLE PRECISION A(16,16),XI(16,16),C(3,16),CV(3,16),CA(3,16)
      DOUBLE PRECISION THETA(3,3)
      DOUBLE PRECISION AH(3,3),BH(3,3)
      DOUBLE PRECISION P(3,3),R(3,3),T(4,4),U(4,6)
      DOUBLE PRECISION ZET(16,16),WR(16),WI(16),APS(16,16)
C
C SPEED, ANGLE AND SEA STATE INPUT
C
      WRITE (6,100)
  100 FORMAT(2X,'INPUT THE SPEED (FT/SEC) AND ANGLE (DEGREES)')
      READ(5,*)V,T1
      T1=T1/57.29578
      WRITE(6,101)
  101 FORMAT(2X,'INPUT THE SIGNIFICANT WAVE HEIGHT (FT) ',
     1'AND MODAL FREQUENCY (RAD/SEC)')
      READ(5,*)ZZ,OMM
C
C SEA MODEL
C
      ALFA=V/32.2*OMM*COS(T1)
      ZETTA=0.707
      DELTA=SQRT(1.-2.*ZETTA**2+SQRT(9.-4.*ZETTA**2+4.*ZETTA**4))/2.
      OMN=ABS(OMM*(1.+ALFA)/DELTA)
      BET=((1.-DELTA**2)**2+4.*ZETTA**2*DELTA**2)**3/DELTA**4
      BET=ABS(BET*EXP(-1.25)/(1.+2.*ALFA))
      SO=0.3125*ZZ**2/OMM*BET
      VP1=OMN**2
      VP2=OMN*ZETTA*2
C
C HYDRODYNAMIC DATA
C   FORCES
C
      A2=310.*SIN(T1)
      A4=2120.*SIN(T1)
      A6=11300.*SIN(T1)
      ZET2=0.72*SIN(T1)
      ZET4=0.7*SIN(T1)
      ZET6=0.35*SIN(T1)
      OM2=(0.6+V*COS(T1)/89.444)*SIN(T1)
      OM4=(0.76+V*COS(T1)/55.748)*SIN(T1)
      OM6=(0.96+V*COS(T1)/34.939)*SIN(T1)
C
C   ADDED MASS - DAMPING
C
      OMES=(0.425+V*COS(T1)/178.27)**2
      AH(1,1)=223.
      AH(1,2)=-759.
      AH(2,1)=-759.
      AH(2,2)=22900.
      BH(1,1)=10.6
```

SRY00010
SRY00020
SRY00030
SRY00040
SRY00050
SRY00060
SRY00070
SRY00080
SRY00090
SRY00100
SRY00110
SRY00120
SRY00130
SRY00140
SRY00150
SRY00160
SRY00170
SRY00180
SRY00190
SRY00200
SRY00210
SRY00220
SRY00230
SRY00240
SRY00250
SRY00260
SRY00270
SRY00280
SRY00290
SRY00300
SRY00310
SRY00320
SRY00330
SRY00340
SRY00350
SRY00360
SRY00370
SRY00380
SRY00390
SRY00400
SRY00410
SRY00420
SRY00430
SRY00440
SRY00450
SRY00460
SRY00470
SRY00480
SRY00490
SRY00500
SRY00510
SRY00520
SRY00530
SRY00540
SRY00550

Program to prepare matrices

for the sway-roll-yaw motions

```
      BH(1,2)=-55.4                                                    SRY00560
      BH(2,1)=-55.4                                                    SRY00570
      BH(2,2)=887.                                                     SRY00580
      AH(1,3)=14600.+V*BH(1,1)/OMES                                    SRY00590
      AH(3,1)=14600.-V*BH(1,1)/OMES                                    SRY00600
      AH(2,3)=182000.+V*BH(1,2)/OMES                                   SRY00610
      AH(3,2)=182000.-V*BH(2,1)/OMES                                   SRY00620
      AH(3,3)=4.18E6+V**2*AH(1,1)/OMES                                 SRY00630
      BH(1,3)=423.-V*AH(1,1)                                           SRY00640
      BH(3,1)=423.+V*AH(1,1)                                           SRY00650
      BH(2,3)=6270.-V*AH(1,2)                                          SRY00660
      BH(3,2)=6270.+V*AH(1,2)                                          SRY00670
      BH(3,3)=144000.+V**2*BH(1,1)/OMES                                SRY00680
C     NONLINEAR ROLL FACTOR                                            SRY00690
      FBV=3.                                                           SRY00700
      BH(2,2)=BH(2,2)*FBV                                              SRY00710
C                                                                      SRY00720
C   MASS AND SPRING CONSTANT                                           SRY00730
C                                                                      SRY00740
      AH(1,1)=AH(1,1)+215.                                             SRY00750
      AH(1,2)=AH(1,2)+988.                                             SRY00760
      AH(1,3)=AH(1,3)-230.                                             SRY00770
      AH(2,1)=AH(2,1)+988.                                             SRY00780
      AH(2,2)=AH(2,2)+104000.                                          SRY00790
      AH(3,1)=AH(3,1)-230.                                             SRY00800
      AH(3,3)=AH(3,3)+3.76E6                                           SRY00810
      C44=28800.                                                       SRY00820
C                                                                      SRY00830
C CONSTRUCTION OF THE MATRICES T AND U                                 SRY00840
C                                                                      SRY00850
      CALL RINV(AH,R)                                                  SRY00860
      CALL RMUL(R,BH,P)                                                SRY00870
      DO 400 I=1,4                                                     SRY00880
      DO 401 J=1,4                                                     SRY00890
  401 T(I,J)=0.                                                        SRY00900
      DO 400 J=1,6                                                     SRY00910
  400 U(I,J)=0.                                                        SRY00920
      T(1,1)=R(1,2)*P(2,1)/R(2,2)-P(1,1)                               SRY00930
      T(1,2)=R(1,2)/R(2,2)                                             SRY00940
      T(1,3)=R(1,2)*P(2,2)/R(2,2)-P(1,2)                               SRY00950
      T(1,4)=R(1,2)*P(2,3)/R(2,2)-P(1,3)                               SRY00960
      U(1,1)=R(1,1)-R(1,2)*R(2,1)/R(2,2)                               SRY00970
      U(1,5)=R(1,3)-R(1,2)*R(2,3)/R(2,2)                               SRY00980
      T(4,1)=R(3,2)*P(2,1)/R(2,2)-P(3,1)                               SRY00990
      T(4,2)=R(3,2)/R(2,2)                                             SRY01000
      T(4,3)=R(3,2)*P(2,2)/R(2,2)-P(3,2)                               SRY01010
      T(4,4)=R(3,2)*P(2,3)/R(2,2)-P(3,3)                               SRY01020
      U(4,1)=R(3,1)-R(3,2)*R(2,1)/R(2,2)                               SRY01030
      U(4,5)=R(3,3)-R(3,2)*R(2,3)/R(2,2)                               SRY01040
      T(3,2)=1                                                         SRY01050
      T(2,1)=-P(2,1)*T(1,1)-P(2,3)*T(4,1)                              SRY01060
      T(2,2)=-P(2,1)*T(1,2)-P(2,3)*T(4,2)-P(2,2)                       SRY01070
      T(2,3)=-P(2,1)*T(1,3)-P(2,3)*T(4,3)-R(2,2)*C44                   SRY01080
      T(2,4)=-P(2,1)*T(1,4)-P(2,3)*T(4,4)                              SRY01090
      U(2,1)=-P(2,1)*U(1,1)-P(2,3)*U(4,1)                              SRY01100
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      U(2,2)=R(2,1)                                                     SRYO1110
      U(2,4)=R(2,2)                                                     SRYO1120
      U(2,5)=-P(2,1)*U(1,5)-P(2,3)*U(4,5)                               SRYO1130
      U(2,6)=R(2,3)                                                     SRYO1140
C                                                                       SRYO1150
C CONSTRUCTION OF THE MODEL MATRICES                                    SRYO1160
C                                                                       SRYO1170
      DO 402 I=1,16                                                     SRYO1180
      DO 402 J=1,16                                                     SRYO1190
      A(I,J)=0.                                                         SRYO1200
  402 XI(I,J)=0.                                                        SRYO1210
      XI(6,6)=SO*3.1415926                                              SRYO1220
      A(1,2)=1.                                                         SRYO1230
      A(2,1)=-VP1                                                       SRYO1240
      A(2,2)=-VP2                                                       SRYO1250
      A(2,4)=VP1                                                        SRYO1260
      A(3,4)=1.                                                         SRYO1270
      A(4,3)=-VP1                                                       SRYO1280
      A(4,4)=-VP2                                                       SRYO1290
      A(4,6)=VP1                                                        SRYO1300
      A(5,6)=1.                                                         SRYO1310
      A(6,5)=-VP1                                                       SRYO1320
      A(6,6)=-VP2                                                       SRYO1330
      A(7,8)=1.                                                         SRYO1340
      A(8,2)=A2*OM2**2                                                  SRYO1350
      A(8,7)=-OM2**2                                                    SRYO1360
      A(8,8)=-2.*ZET2*OM2                                               SRYO1370
      A(9,10)=1.                                                        SRYO1380
      A(10,2)=A4*OM4**2                                                 SRYO1390
      A(10,9)=-OM4**2                                                   SRYO1400
      A(10,10)=-2.*ZET4*OM4                                             SRYO1410
      A(11,12)=1.                                                       SRYO1420
      A(12,2)=A6*OM6**2                                                 SRYO1430
      A(12,11)=-OM6**2                                                  SRYO1440
      A(12,12)=-2.*ZET6*OM6                                             SRYO1450
      DO 404 I=1,4                                                      SRYO1460
      IP=I+12                                                           SRYO1470
      DO 405 J=1,6                                                      SRYO1480
      JP=J+6                                                            SRYO1490
  405 A(IP,JP)=U(I,J)                                                   SRYO1500
      DO 406 J=1,4                                                      SRYO1510
      JP=J+12                                                           SRYO1520
  406 A(IP,JP)=T(I,J)                                                   SRYO1530
  404 CONTINUE                                                          SRYO1540
C     OUTPUT MATRICES                                                   SRYO1550
      DO 403 I=1,3                                                      SRYO1560
      DO 403 J=1,16                                                     SRYO1570
      C(I,J)=0.                                                         SRYO1580
      CV(I,J)=0.                                                        SRYO1590
  403 CA(I,J)=0.                                                        SRYO1600
      C(1,13)=1.                                                        SRYO1610
      C(2,15)=1.                                                        SRYO1620
      C(3,16)=1.                                                        SRYO1630
      CV(2,14)=1.                                                       SRYO1640
      DO 407 I=1,6                                                      SRYO1650
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      IP=I+6                                                            SRY01660
      CV(1,IP)=U(1,I)                                                   SRY01670
      CV(3,IP)=U(4,I)                                                   SRY01680
      CA(1,IP)=-P(1,1)*U(1,I)-P(1,3)*U(4,I)                             SRY01690
      CA(2,IP)=U(2,I)                                                   SRY01700
      CA(3,IP)=-P(3,1)*U(1,I)-P(3,3)*U(4,I)                             SRY01710
  407 CONTINUE                                                          SRY01720
      CA(1,8)=R(1,1)                                                    SRY01730
      CA(1,10)=R(1,2)                                                   SRY01740
      CA(1,12)=R(1,3)                                                   SRY01750
      CA(3,8)=R(3,1)                                                    SRY01760
      CA(3,10)=R(3,2)                                                   SRY01770
      CA(3,12)=R(3,3)                                                   SRY01780
      DO 408 I=1,4                                                      SRY01790
      IP=I+12                                                           SRY01800
      CV(1,IP)=T(1,I)                                                   SRY01810
      CV(3,IP)=T(4,I)                                                   SRY01820
      CA(1,IP)=-P(1,1)*T(1,I)-P(1,3)*T(4,I)                             SRY01830
      CA(2,IP)=T(2,I)                                                   SRY01840
      CA(3,IP)=-P(3,1)*T(1,I)-P(3,3)*T(4,I)                             SRY01850
  408 CONTINUE                                                          SRY01860
      CA(1,14)=CA(1,14)-P(1,2)                                          SRY01870
      CA(1,15)=CA(1,15)-R(1,2)*C44                                      SRY01880
      CA(3,14)=CA(3,14)-P(3,2)                                          SRY01890
      CA(3,15)=CA(3,15)-R(3,2)*C44                                      SRY01900
C                                                                       SRY01910
C OUTPUT                                                                SRY01920
C                                                                       SRY01930
      GO TO 399                                                         SRY01940
      IER1=0                                                            SRY01950
      IER2=0                                                            SRY01960
      CALL TRNATB(16,16,16,16,A,APS)                                    SRY01970
      CALL LYPCND(16,16,16,APS,XI,ZET,WR,WI,IER1,IER2)                  SRY01980
      DO 174 I=1,16                                                     SRY01990
      XI(I,I)=-XI(I,I)                                                  SRY02000
  174 WRITE(9,175)XI(I,I)                                               SRY02010
  175 FORMAT(D11.5)                                                     SRY02020
  399 CONTINUE                                                          SRY02030
      WRITE(9,310)                                                      SRY02040
  310 FORMAT(10X,'---------   MATRIX A   -----------')                  SRY02050
      WRITE(9,320)((A(I,J),J=1,16),I=1,16)                              SRY02060
  320 FORMAT(8E14.4)                                                    SRY02070
      WRITE(9,330)                                                      SRY02080
  330 FORMAT(10X,'---------  MATRIX C  -----------')                   SRY02090
      WRITE(9,320)((C(I,J),J=1,16),I=1,3)                               SRY02100
      WRITE(9,340)                                                      SRY02110
  340 FORMAT(10X,'---------  MATRIX XI  -----------')                  SRY02120
      WRITE(9,320)((XI(I,J),J=1,16),I=1,16)                             SRY02130
      DO 350 I=1,3                                                      SRY02140
      DO 350 J=1,3                                                      SRY02150
  350 THETA(I,J)=0.                                                     SRY02160
      WRITE(6,360)                                                      SRY02170
  360 FORMAT(2X,'INPUT THE SWAY-ROLL-YAW NOISE INTENSITIES (3 NUMBERS)')SRY02180
      READ(5,*)THETA(1,1),THETA(2,2),THETA(3,3)                         SRY02190
      WRITE(9,370)                                                      SRY02200
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  370 FORMAT(10X,'---------  MATRIX THETA -----------')                  SRYO2210
      WRITE(9,320)((THETA(I,J),J=1,3),I=1,3)                            SRYO2220
      END                                                               SRYO2230
      SUBROUTINE RINV(A,B)                                              SRYO2240
C     3X3 MATRIX INVERSION                                              SRYO2250
      DOUBLE PRECISION A(3,3),B(3,3),C(3,3)                             SRYO2260
      C(1,1)=A(2,2)*A(3,3)-A(3,2)*A(2,3)                                SRYO2270
      C(1,2)=A(2,1)*A(3,3)-A(3,1)*A(2,3)                                SRYO2280
      C(1,3)=A(2,1)*A(3,2)-A(3,1)*A(2,2)                                SRYO2290
      C(2,1)=A(1,2)*A(3,3)-A(3,2)*A(1,3)                                SRYO2300
      C(2,2)=A(1,1)*A(3,3)-A(3,1)*A(1,3)                                SRYO2310
      C(2,3)=A(1,1)*A(3,2)-A(3,1)*A(1,2)                                SRYO2320
      C(3,1)=A(1,2)*A(2,3)-A(2,2)*A(1,3)                                SRYO2330
      C(3,2)=A(1,1)*A(2,3)-A(2,1)*A(1,3)                                SRYO2340
      C(3,3)=A(1,1)*A(2,2)-A(2,1)*A(1,2)                                SRYO2350
      DET=A(1,1)*C(1,1)-A(1,2)*C(1,2)+A(1,3)*C(1,3)                     SRYO2360
      B(1,1)=C(1,1)/DET                                                 SRYO2370
      B(1,2)=-C(2,1)/DET                                                SRYO2380
      B(1,3)=C(3,1)/DET                                                 SRYO2390
      B(2,1)=-C(1,2)/DET                                                SRYO2400
      B(2,2)=C(2,2)/DET                                                 SRYO2410
      B(2,3)=-C(3,2)/DET                                                SRYO2420
      B(3,1)=C(1,3)/DET                                                 SRYO2430
      B(3,2)=-C(2,3)/DET                                                SRYO2440
      B(3,3)=C(3,3)/DET                                                 SRYO2450
      RETURN                                                            SRYO2460
      END                                                               SRYO2470
      SUBROUTINE RMUL(A,B,C)                                            SRYO2480
C     3X3 MATRIX MULTIPLICATION                                         SRYO2490
      DOUBLE PRECISION A(3,3),B(3,3),C(3,3)                             SRYO2500
      DO 100 I=1,3                                                      SRYO2510
      DO 100 J=1,3                                                      SRYO2520
      C(I,J)=0                                                          SRYO2530
      DO 100 K=1,3                                                      SRYO2540
  100 C(I,J)=C(I,J)+A(I,K)*B(K,J)                                       SRYO2550
      RETURN                                                            SRYO2560
      END                                                               SRYO2570
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