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Design of Analytical Failure-Detection Systems Using Secondary Observers

M. Sidar

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National Aeronautics and
Space Administration

limits of applicability of this code was evaluated. Comparisons of
computed results were made with available experimental data. Results
indicate that the code is robust, accurate (when significant viscous
effects are not present), and efficient. TWING generally produces
solutions an order of magnitude faster than other conservative full

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Design of Analytical Failure-Detection Systems Using Secondary Observers

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DESIGN OF ANALYTICAL FAILURE-DETECTION SYSTEMS USING SECONDARY OBSERVERS

M. Sidar*

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SUMMARY

The problem of designing analytical failure-detection systems (FDS) for sensors and actuators, using observers, is addressed. These failure-detection systems can be applied to linear, constant, and possibly time-varying multi-input, multi-output systems with measurement noise. The use of observers in FDS is related to the examination of the n-dimensional observer error vector which carries the necessary information on possible failures. The problem is that in practical systems, in which only some of the components of the state vector are measured, one has access only to the m-dimensional observer-output error vector, with $m \leq n$. In order to cope with these cases, a secondary observer is synthesized to reconstruct the entire observer-error vector from the observer output error vector. This approach leads toward the design of highly sensitive and reliable FDS, with the possibility of obtaining a unique fingerprint for every possible failure (abrupt or soft). The use of the secondary observers allows us also to solve the measurement noise problem in a very efficient way. Further, in order to keep the observer's (or Kalman filter) false-alarm rate (FAR) under a certain specified value, it is necessary to have an acceptable matching between the observer (or Kalman filter) models and the system parameters. Only properly designed adaptive observers are able to detect abrupt changes in the system (actuator, sensor failures, etc.) with adequate reliability and FAR. A previously developed adaptive observer algorithm is used here to maintain the desired system-observer model matching, despite initial mismatching or system parameter variations. Conditions for convergence for the adaptive process are obtained, leading to a simple adaptive law (algorithm) with the possibility of an a priori choice of fixed adaptive gains. Simulation results show good tracking performance with small observer output errors, while accurate and fast parameter identification, in both deterministic and stochastic cases, is obtained.

I. INTRODUCTION

The use of the analytical redundancy approach for sensor and actuator failure detection in complex, dynamic control systems is by now widely accepted as a feasible concept for redundancy management (refs. 1-3). Besides an appreciable saving in cost, volume, and weight, the analytical failure-detection systems have to provide at least the same high performances as the classical voting systems, which are based on simple threshold examinations and on some crude decision logic. In aeronautical designs, and in particular for flight-control purposes, values of mission abort probability (MAP) of 10^{-4} to 10^{-5} per flight hour, associated with typical false-alarm rates (FAR) of 10^{-3} to 10^{-4}, are rather commonly imposed by operational requirements (ref. 1).

To compete successfully with the triple and quadruple redundant systems based exclusively on voting schemes, the analytical-redundant failure-detection systems have to exhibit certain basic features. For example:

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1. Simplicity and fault-tolerant properties in both the software conception and the hardware implementation.

2. High reliability and high probability of failure detection.

3. Low false-alarm rates, despite external disturbances such as wind gusts, abrupt maneuvering (in flight-control systems), instrumentation noise, and, in some cases, process noise.

4. Ability to determine, as precisely and as rapidly as possible, the failure source, the extent of the failure, and in some cases, the time of failure.

5. In addition to abrupt failure detection (mainly for sensor and actuator failures), the analytical-redundancy schemes have to handle the problem of soft-failures detection, such as the detection of biases or scale factor changes in the instrumentation, some degradations in actuator performances, etc.

Two principal analytical concepts are used in guidance and flight control for analytical failure-detection purposes:

1. Kalman filters (ref. 3-13) where the innovation sequence $\underline{\nu}(t)$ is tested for unbiasedness and whiteness (orthogonality condition test).

2. Linear observers (refs. 3-15) (full- and reduced-order) in which the error between the measured output and the reconstructed one, for example, the so-called residual errors $\underline{e}(t)$, are tested for failure assessment. The gains of those observers are determined such that $\underline{e}(t)$ will reveal the occurrence of a specific failure.

It is useful at this point to remark that results reported or published so far are based on the assumption that the dynamic system has fixed and known parameters.

Another important problem is that in practical cases the dimension $(1 \times m)$ of the observer-output error vector $\underline{\epsilon}(t)$ is lower than the dimension of the observer error vector $\underline{e}(t)$, of dimension $(1 \times n)$, where $m \leq n$. In this case, much of the information about failures is contained in those components of $\underline{e}(t)$, which are not accessible for measurement. Thus, failure events (hard and soft failures) are not easily detected and, certainly, are not detected in a unique way. By analyzing the $\underline{\epsilon}(t)$ vector only, one may obtain a failure-detection system (FDS) with a low failure-detection sensitivity and a nonunique fingerprint for a specific system failure. This crucial problem of designing observers for FDS's with a unique fingerprint for a specific failure has been addressed in the past by various contributors (refs. 5, 9, and 14).

In reference 5 an attempt is made, by using a certain transformation of the observer output residual vector $\underline{\epsilon}(t)$, to obtain a fingerprint related to a specific failure in the system actuators or sensors. This approach does not assure uniqueness and leads to a low-sensitivity failure detection with the probability of a high false-alarm rate (FAR). Besides, the design procedure is cumbersome, and the algorithm includes some difficult numerical procedures. In reference 9, the approach to solving the failure-detection problem is similar to that of reference 5, using a somewhat different algorithm in order to obtain the transformation matrix and taking into account the possibility of stochastic random noise in the output measurement. Reference 14 describes a possible approach to the design for instrument failure detection only, for uncertain linear systems. But in using this approach, one encounters the same difficulties as in the other approaches, because only the limited amount of information contained in $\underline{\epsilon}(t)$ is examined.

The approach taken in the work described herein is more comprehensive. It uses a secondary observer to reconstruct the vector $\hat{\underline{e}}(t)$, from the measured error vector $\underline{\epsilon}(t)$, obtained from the primary observer. The purpose of the secondary observer is twofold:

1. To produce the n-dimensional error vector $\hat{\underline{e}}(t)$ needed for a correct, unique, and high-sensitivity failure assessment.

2. To reduce the susceptibility of the FDS to measurement noise, by using steady-state Kalman (filter) optimal gains.

By this approach one obtains a sensitivity-enhanced FDS with unique failure-fingerprints, and the effect of the measurement noise is reduced. It is appropriate here to point out that in order to be of practical value in applications and to provide reliable systems, the major problem of failure-detection and analytical redundancy theory is to achieve the conflicting objectives of low noise sensitivity, low false-alarm rate (considering noise) and high failure-detection sensitivity.

As stated before, a common assumption used in the references mentioned above, is that the dynamic system has constant parameters. Moreover, in some FDS's one has to use decision algorithms, especially for the detection of soft-failures and for the assessment of the extent of failures. Most of the decision algorithms — such as sequential likelihood ratio test (SLRT) for mean values and functional compatibility (refs. 8, 11, 12, and 13); generalized likelihood ratio (GLR) approach (refs. 4 to 7); and recursive GLR (refs. 7, 8, 12) — assume also (with the exception of ref. 7) that the dynamic system is known and constant.

As will be shown later in this report, it is absolutely necessary when using either observers or Kalman filters, that those devices be "matched" to the dynamic system in order to obtain low observer-output errors and, therefore, low false-alarm rates. A good matching will also provide adequate properties to the decision algorithms in order to assess the time, the place, and the extent of the failure without errors (see ref. 15).

At this point it is worth noting that when the plant parameter variations are themselves the results of some kind of failures, the adaptive matching of the observer to the plant may unintentionally "cover up" those failures. For this reason, it is expected that a complete FDS would include also some on-line parameter-identification procedure to support the failure-detection algorithm. However, it seems possible to relax this need, if the adaptive observers are time-varying and if observer parameters are updated deterministically in open loop, by having parameters stored as a function of flight condition or by changing the parameters according to air data computer outputs.

A complete parameter-adaptive and tracking observer for linear, multi-input, multi-output FDS's, incorporating primary and secondary observers, is designed, presented, and analyzed for convergence and stability in this report.

A short overview of observers (Kalman filters) for failure-detection purposes is presented in section 2 in the interest of completeness.

The secondary observer concept for the deterministic case is introduced in section 3. Different schemes for FDS's, based on mixed primary and secondary observers, are introduced and discussed.

The effect of the measurement noise is discussed in section 4, where the design of FDS's in a stochastic environment is presented.

In sections 5 and 6, an algorithm for adaptive and tracking observer design is presented, together with the appropriate conditions for convergence and stability. Simulation results for a deterministic and stochastic multi-input, multi-output, linear, constant, and time-varying system, are presented and discussed in section 7. Concluding remarks and some suggestions for further study and research are presented in section 8.

An alternative scheme for implementing observers is given in appendix A; use of linear quadratic theory to obtain an asymptotically-stable-in-the-large solution for the FDS adaptive observer is described in appendix B; and the proof for conditions necessary for convergence and stability is given in appendix C.

2. FAILURE-DETECTION SYSTEMS BASED ON OBSERVERS

As pointed out in the Introduction, various analytical redundant schemes for FDS's are based on the utilization of observers of full or reduced order (refs. 1-5, 13-15). Besides the possibility of enhancing the detectability of certain specific failures in a unique way, the analytical redundancy FDS's based on the use of observers lead also to important hardware savings (see, for example, fig. 3 in ref. 15). In the interest of completeness, this section presents a short discussion of some of the basic notions related to the observer theory. First, we shall assume the following mathematical model for the linear dynamic system under consideration:

$$\begin{aligned} \dot{\underline{x}}(t) &= A\,\underline{x}(t) + B\,\underline{u}(t) \\ \underline{y}(t) &= C\,\underline{x}(t) \end{aligned} \tag{1}$$

where $\underline{x}(t)$ is the $(n \times 1)$ state vector, and $\underline{y}(t)$ is the $(m \times 1)$ measurement vector, with $m \leq n$. The system is assumed both completely controllable and observable. The well-known observer model ("matched" case) (ref. 4) is described by

$$\dot{\hat{\underline{x}}}(t) = A\,\hat{\underline{x}}(t) + K[\underline{y}(t) - C\,\hat{\underline{x}}(t)] + B.\underline{u}(t) \tag{2}$$

where $\hat{\underline{x}}(t)$ is the $(n \times 1)$ estimated (or reconstructed) state vector, and K is a fixed-gain matrix $(n \times m)$, with constant entries. This model does not take into consideration various external perturbations and noises that affect the observer output and that can cause high FAR's. The observer error, $\underline{e}(t)$ (residual), is defined by

$$\underline{e}(t) \stackrel{\Delta}{=} \underline{x}(t) - \hat{\underline{x}}(t) \tag{3}$$

and the observer output error (output residual) is defined as

$$\underline{\epsilon}(t) \stackrel{\Delta}{=} \underline{y}(t) - \hat{\underline{y}}(t) \tag{4}$$

The output residual vector $\underline{\epsilon}(t)$ is the quantity that one has access to and therefore it can be used for failure detection and assessment. A block diagram of a failure-detection scheme with an observer is presented in figure 1.

From equations (1) to (3), the following differential equation is obtained:

$$\dot{\underline{e}}(t) = (A - KC)\ \underline{e}(t) \tag{5}$$

One method of choosing the gain matrix K is to place the eigenvalues of the matrix (A - KC) so that all of them have negative real parts (refs. 9 and 10). Under these conditions, the observer will be stable and, as $t \to \infty$, $\underline{e}(t)$ and $\underline{\epsilon}(t)$ will go to zero. Therefore, after a short initial transient, the estimated state $\hat{\underline{x}}(t)$ will follow $\underline{x}(t)$ such that $\hat{\underline{x}}(t) \cong \underline{x}(t)$, $\forall t \in [t_o, \infty]$, although the only measurable vector is $\underline{y}(t)$.

A second approach for choosing K is to enhance the observer's probability of failure detection. After the transient has died out, and if a hard failure of one of the actuators or sensors occurs at $t = T_f$, then a jump in $\underline{e}(t)$ will be observed at T_f, and the vector $\underline{e}(t) \neq 0$, for all $t > T_f$ (see fig. 2). Indeed, one has to remember that by using one (primary) observer, the only access we have for error measurements and analysis is to the $(1 \times m)$ vector $\underline{\epsilon}(t)$. The information about failures, included in $\underline{\epsilon}(t)$, is only partial, and if the output's vector dimension m is much lower than the system's order n, the failure detection may be insensitive, nonunique, and have a high FAR.

To better illustrate the second approach, let us examine the case of an actuator failure (ith actuator), and the possibility of enhancing the detection of this event. From equations (1) to (3), one obtains the following result:

$$\dot{\underline{e}}(t) = (A - KC)\ \underline{e} + \underline{b}_i u_i \tag{6}$$

where $\underline{b}_i$ is the ith column of the time-invariant matrix B, and u_i is the ith control of the system. The solution of equation (6) is given by:

$$\underline{e}(t) = \exp\left[(A - KC)(t - T_o)\right] \cdot \underline{e}(T_o) + \left\{\int_{T_o}^{t} \exp\left[(A - KC)(\tau - T_o)\right] u_i(\tau)\, d\tau\right\} \underline{b}_i \tag{7}$$

The first term is negligible (in both the deterministic and the stochastic cases), since we assume that the failure occurs at some time T_f during the system's operation, after the initial transient has died-out ($T_o \ll T_f$). Let us assume that the effects of measurement noise and other perturbations on $\underline{e}(t)$ are small. Therefore, the term containing the abrupt failure information is the second one. Choosing, for C = I (this being a very special and simple case with n = m)

$$(A - KC) \triangleq -I \cdot \frac{1}{T} \tag{8}$$

where I is the $(n \times n)$ identity matrix and T is a convenient, arbitrarily chosen time-constant (ref. 15), one gets:

$$\underline{e}^T(t) \cong \underline{b}_i^r \int_{T_f}^{t} \exp - \left[\frac{(\tau - T_f)}{T}\right] \cdot u_i(\tau)\, d\tau \qquad \forall t > T_f \tag{9}$$

Therefore, the error vector $\underline{e}(t)$ will point in a specific direction in the E^n space, for example, in the direction defined by $\underline{b}_i$, associated with the failure of the ith actuator. Since the only access one has to the system is by measuring the vector $\underline{\epsilon}(t)$, the measured residual will point in the direction of $C\underline{b}_i$. Since, in general, the matrix C is an $(m \times n)$ matrix, it may very well happen that the vector $C\underline{b}_i$ will have only a few components or, perhaps, even none if $\underline{b}_i$ is the null space of C. If, for instance, $m = 2$ and $n = 6$, one measures only two components of $\underline{e}(t)$ and not necessarily the most sensitive ones (see the example and simulation results in section 7).

By a similar treatment, one is able to show how sensor failures can be detected, but in this case $\underline{e}(t)$ lies in a two-dimensional plane. In such a case, it is possible to arrive at a feasible scheme, so that the detection of the failed sensor will be simple and unique. As will be shown later, by processing the information with a secondary observer in an optimal way, a failure direction may be determined, even in the presence of measurement noise.

The following is an alternative way to look at observers as failure-sensitive devices. Suppose we look again at the observer's equation (2); one can rewrite that differential equation in the following form:

$$\dot{\hat{\underline{x}}}(t) = Q\, \hat{\underline{x}}(t) + K\, \underline{y}(t) + B\, \underline{u}(t) \tag{10}$$

where

$$Q \stackrel{\Delta}{=} A - KC \tag{11}$$

Then it is possible to write the solution of equation (10) as a linear combination of three vector functions:

$$\hat{\underline{x}}(t) = W(t)\hat{\underline{x}}(0) + \underline{\phi}(t) + \underline{\rho}(t) \tag{12}$$

where the functions $W(t)$, $\underline{\phi}(t)$, and $\underline{\rho}(t)$ are the solutions of the following differential equations with appropriate initial conditions (see appendix A):

$$\dot{W}(t) = QW(t) \qquad w(0) = I \tag{13a}$$

$$\dot{\underline{\phi}}(t) = Q\underline{\phi}(t) + K\underline{y}(t) \qquad \underline{\phi}(0) = 0 \tag{13b}$$

$$\dot{\underline{\rho}}(t) = Q\underline{\rho}(t) + B\underline{u}(t) \qquad \underline{\rho}(0) = 0 \tag{13c}$$

The matrix differential equation (13a) determines the transient of the observer and, therefore, is of no practical importance for failure detection, since we are assuming that the transient is very short and that the failures may occur in the system after this transient died out. By looking now at figure 3 it is easy to see that sensor failures will affect only the vector $\underline{\phi}(t)$, and that actuator and system failures will affect both vectors $\underline{\phi}(t)$ and $\underline{\rho}(t)$. Moreover, figure 3 shows that measurement noise is affecting only the vector $\underline{\phi}(t)$; this fact will be taken into consideration later. Implementing an observer in the FDS in the form suggested by equations (12) and (13) (as shown in fig. 3), makes it possible to make an immediate distinction between sensor and actuator failures simply by examining the vectors $\underline{\phi}(t)$ and $\underline{\rho}(t)$.

Let us examine again the case in which $m = n$. In this case it is possible not only to locate arbitrarily the eigenvalues of the matrix Q but also to determine the entries of Q in any arbitrary way. If one chooses, for instance, $Q = I$, one obtains because of the initial conditions of equations (13),

$$\left.\begin{aligned} \dot{w}_{ii}(t) &= w_{ii}(t) \\ w_{ij}(t) &= 0 \end{aligned}\right\} \quad \begin{aligned} i &= 1,2,\ldots, n \\ &\forall t \end{aligned} \tag{14}$$

$$\dot{\underline{\phi}}(t) = \underline{\phi}(t) + \sum_{i=1}^{n} \underline{k}_i \, y_i(t) \tag{15}$$

$\underline{k}_i$ being the ith column of the matrix K. Similarly, one obtains for $\underline{\rho}(t)$ the following differential equation:

$$\dot{\underline{\rho}}(t) = \underline{\rho}(t) + \sum_{i=1}^{n} \underline{b}_j \cdot u_j(t) \tag{16}$$

where $\underline{b}_j$ is the jth column of the matrix B. In order to implement the observer in this configuration, we need to solve (in this case) only $(1 + 2n)$ first-order differential equations, a relatively easy task. The benefit of such an observer implementation is obvious: by measuring each of the components of $\underline{\phi}(t)$ and $\underline{\rho}(t)$, one can assess immediately when and where the failure occurred, as well as the extent of the failure. In this manner, for $m = n$, the fingerprint of every possible failure is unique. In some applications it may be worthwhile to use additional sensors (if possible), in order to arrive at the situation where $m = n$.

The problem becomes more complicated for the output measurement case when $m < n$. In this case, although we still can place the poles of the Q matrix arbitrarily, one cannot, in general, obtain $Q = I$. Therefore, the number of integrations will increase, to $n(2 + n)$.

For this reason, when $m < n$, a practical way to solve the FDS problem is to introduce the concept of the secondary observer, as explained in the next section.

3. DESIGN OF FAILURE-DETECTION SYSTEMS WITH PRIMARY AND SECONDARY OBSERVERS

As explained in section 2, the determination of a failure can be made in a reliable and unique way only by examining the $(1 \times n)$ observer error vector $\underline{e}(t)$. Since, when $m < n$, one has access only to the $(1 \times m)$ observer output error $\underline{\epsilon}(t)$, we are proposing here to implement a novel concept and to use a secondary observer in order to reconstruct the $(1 \times n)$ error vector $\hat{\underline{e}}(t)$. The proposed implementation is shown, schematically, in figure 4.

We propose for the second observer's differential equation the following structure:

$$\dot{\hat{\underline{e}}}(t) = T\,\hat{\underline{e}}(t) + L[\underline{\epsilon} - C\,\hat{\underline{e}}] \tag{17}$$

where T is a fixed $(n \times n)$ matrix (to be determined later) and L is a $(n \times m)$ gain matrix (arbitrarily chosen, for the time being).

Let us now define the second observer's error vector $\delta(t)$, as following:

$$\underline{\delta}(t) \triangleq \underline{e}(t) - \hat{\underline{e}}(t) \tag{18}$$

Since the differential equation for $\underline{e}(t)$, as given in equation (5), was

$$\dot{\underline{e}}(t) = (A - KC)\,\underline{e}(t) \tag{19}$$

one can obtain, from equations (17)-(19), the following result:

$$\dot{\underline{\delta}}(t) = (A - KC - LC)\,\underline{e}(t) - (T - LC)\,\hat{\underline{e}}(t) \tag{20}$$

By choosing

$$T \triangleq A - KC \tag{21}$$

one gets

$$\dot{\underline{\delta}}(t) = (A - KC - LC)\,\underline{\delta}(t) \tag{22}$$

If the eigenvalues of the $(n \times n)$ matrix $(A - KC - LC)$ are adequately located in the left half-plane, the solution of equation (22) will be asymptotically stable and will vanish as t goes to infinity:

$$\lim_{t \to \infty} \underline{\delta}(t) = 0 \tag{23}$$

The output of the second observer $\hat{\underline{e}}(t)$, which is the reconstructed error vector of the first observer, will follow $\underline{e}(t)$ after a short, initial transient.

From equations (17) and (21) one finally obtains the second observer's differential equation:

$$\dot{\hat{\underline{e}}}(t) = (A - KC)\,\hat{\underline{e}}(t) + L[\underline{\epsilon}(t) - C\hat{\underline{e}}(t)] \tag{24}$$

The input of the second observer is the first observer's output error vector $\underline{\epsilon}(t)$. To illustrate in a better way how the second observer reconstructs the whole error vector of the first observer, the evolution of the time functions $e_i(t)$ and $\hat{e}_i(t)$ for $i = 1, 2, 3$, for a third-order system (see details in sec. 7) with two outputs, is plotted in figure 5.

Figure 6 shows in a more detailed form and with different scale factors, the evolution of every component of the vector $\underline{e}(t)$ versus the corresponding component of vector $\hat{\underline{e}}(t)$. Also, note that the most sensitive component of $\underline{e}(t)$ is $e_3(t)$

and that it could not be observed without the second observer. From figures 5 and 6, it is easy to see effective tracking and the zeroing of $\underline{\hat{e}}(t)$ following $\underline{e}(t)$, after a short transient. In our case, this transient is of little interest, because our aim is to discover the changes in $\underline{\hat{e}}(t)$ owing to possible system failures.

Equations (19) and (24) suggest that the entire system, including the use of the error vector of the first observer and the output vector of the second observer as state variables, can be represented in the following augmented form (which will be of use later on):

$$\dot{\underline{\gamma}}(t) = \begin{bmatrix} A - KC & 0 \\ LC & A - KC - LC \end{bmatrix} \cdot \underline{\gamma}(t) \tag{25}$$

where $\underline{\gamma}^T \triangleq [\underline{e}, \underline{\hat{e}}]$. Note that this representation is valid only for the nonfailure case and only when the observers are matched to the system dynamics.

Let us now examine in the sequel the modeling problem of two of the most important kind of failures: sensor failures and actuator failures.

Modeling Sensor Failures

Suppose, in this case, that one of the system's measurement sensors fails (completely) at some time T_f and that no more than one failure will occur at the same time. Under the condition of sensor failure, the dynamic system equations will be

$$\begin{cases} \dot{\underline{x}}(t) = A\,\underline{x}(t) + B\,\underline{u}(t) \\ \underline{y}(t) = C^f\,\underline{x}(t) \end{cases} \tag{26}$$

where C^f is the measurement matrix, considering the failure of one of the sensors. We will also define a new matrix ΔC,

$$\Delta C \triangleq C - C^f \tag{27}$$

where C is the nominal (no-failure) measurement matrix and ΔC is an $(m \times n)$ matrix with all entries zero, besides one specific entry Δc_{ij}, modeling for the ijth sensor failure. Actually, since we are measuring the outputs by using only distinct measurements, the matrix ΔC will have all zero entries, besides one unity entry at the ith sensor which failed. Taking into account equation (27), the first observer differential equation will be:

$$\begin{aligned} \dot{\underline{\hat{x}}}(t) &= A\,\underline{\hat{x}}(t) + K[\underline{y} - C\underline{\hat{x}}] + B\,\underline{u}(t) \\ &= A\underline{\hat{x}}(t) + K[C^f\,\underline{x} - C\underline{\hat{x}}] + B\underline{u}(t) \end{aligned} \tag{28}$$

Making use of equation (3), one obtains

$$\dot{\underline{e}}(t) = (A - KC)\,\underline{e}(t) + K.\Delta C.\underline{x}(t) \tag{29}$$

or, in terms of the estimated state $\underline{\hat{x}}(t)$,

$$\underline{\dot{e}}(t) = (A - KC^f)\ \underline{e}(t) + K.\Delta C.\underline{\hat{x}}(t) \tag{30}$$

The first observer output error will be given by

$$\underline{\epsilon}(t) = \underline{y} - C\underline{\hat{x}} = C\underline{e} - \Delta C.\underline{x} \tag{31}$$

The second observer equation (24) will have, for the sensor-failure case, the following form:

$$\underline{\dot{\hat{e}}}(t) = (A - KC - LC)\ \underline{\hat{e}}(t) + L.C^f\ \underline{e}(t) - L.\Delta C.\underline{\hat{x}}(t) \tag{32}$$

From equations (27) and (30) one can easily obtain the differential equation for the first-observer error vector $\underline{e}(t)$, for $t \geq T_f$, given that a sensor failure occurred at T_f:

$$\underline{\dot{e}}(t) = (A - KC^f)\ \underline{e}(t) + \underline{k}_i.\hat{x}_i(t) \tag{33}$$

The vector $\underline{k}_i$ is the ith column of the fixed gain matrix K, and $\hat{x}_i(t)$ is the scalar, ith component of the reconstructed (estimated) state vector $\underline{\hat{x}}(t)$. From equation (33), it is also clear that the vector $\underline{k}_i.\hat{x}_i(t)$, referred to below as the ith sensor-failure fingerprint vector, is acting as a driving (input) function for equation (33). From equations (32) and (33), one can also conclude that after a short transient starting at T_f, the vector $\underline{\hat{e}}(t)$ will also be pointing in a fixed direction in the E^n space. Following the discussion of actuator-failure modeling, we shall return to consider this problem in more detail.

Modeling Actuator Failures

In equation (6), we already obtained the differential equation for $\underline{e}(t)$, given that the ith actuator failed at T_f:

$$\underline{\dot{e}}(t) = (A - KC)\ \underline{e}(t) + \underline{b}_i\ u_i(t) \tag{6}$$

where the vector $\underline{b}_i$ is the ith column of the matrix B, and the scalar $u_i(t)$ is the ith component of the control vector. This equation is, formally, similar to equation (33) and, therefore, we can conclude that the modeling of sensor and actuator failures, being formally similar, will make possible a unique treatment in the sequel. At this point it should be mentioned that using the same approach, one could easily obtain similar models for sensor bias errors, scale factor failures, etc.

We shall now discuss in more detail the unified approach of the failure-fingerprint problem. From equations (6) and (33) we can note, in a general way, that after a failure the vector $\underline{e}(t)$ will be the solution of the following type of differential equation:

$$\underline{\dot{e}}(t) = Q.\underline{e}(t) + \underline{\beta}_i.f_i(t) \tag{34}$$

with $Q \triangleq A - KC$ and where $\underline{\beta}_i \triangleq \underline{k}_i$ and $f_i(t) \triangleq x_i(t)$ for a sensor failure, and where $\underline{\beta}_i \triangleq \underline{b}_i$ and $f_i(t) \triangleq u_i(t)$ for an actuator failure. From equation (34), it is clear that for every possible failure, a certain direction for the vector $\underline{e}(t)$ in E^n [and, therefore, for the vector $\hat{\underline{e}}(t)$] can be chosen, such that every failure will now have its own distinct fingerprint. To implement such a failure fingerprint one has to fulfill the following condition:

$$\text{rank } [\underline{\beta}_i, Q\underline{\beta}_i, \ldots\ldots Q^{n-1}\underline{\beta}_i] = 1 \tag{35}$$

Therefore, it is possible to dedicate a pair of primary and secondary observers for every type of failure detection. The matrices K and L make possible both the fulfillment of the condition expressed by equation (35) and the arbitrary location of the observers' poles.

This design approach, although demanding an additional computational effort, offers a general and practical solution to the failure-detection problem via the second-observer concept, making use of the entire (reconstructed) error vector $\hat{\underline{e}}(t)$. In the sequel we shall call this design method the "failure-dedicated, multiple-observer-pairs" approach.

In some cases, in order to reduce the computational effort whenever needed, one can use a single pair of primary and secondary observers for the failure-detection system. In such a case we cannot allocate a priori, for $\underline{e}(t)$ and $\hat{\underline{e}}(t)$, a desired direction in E^n associated with a specific failure. Instead, the vector $\hat{\underline{e}}(t)$ will provide a definite and unique fingerprint associated with every failure, although this time, unspecified in advance. Nonetheless, by simulating the various possible failures, one can obtain, in advance, the various failure fingerprints and thereby easily determine from $\hat{\underline{e}}(t)$ when the failure occurred and what failed in the system. The simulation results discussed in section 7 show some of the fingerprints obtained in those cases for various sensor and actuator failures.

An intermediate way to solve the FDS problem is to use a limited number of dedicated pairs of primary and secondary observers, optimized for some important systems failures to be determined in a unique manner, and still maintaining the possibility of determining the occurrence of various failures by examining the fingerprint of $\hat{\underline{e}}(t)$.

4. FDS WITH PRIMARY AND SECONDARY OBSERVERS IN A STOCHASTIC ENVIRONMENT

One of the important questions that must be asked when analyzing and designing FDS's based on observers is the following: What is the extent of the effect of measurement noise on the FDS false-alarm rate? This question was addressed by others (e.g., in refs. 5, 9, 14, and 15), but the problem was never solved in a satisfactory manner. With the use of a single observer, it is possible to reduce the influence of the measurement noise by choosing the gain matrix K such that the observer will be less susceptible to noise. But this can be done only at the expense of the observer's sensitivity with respect to the failure-detection task, such that from the overall FAR point of view the benefits of this approach are very questionable. The use of primary and secondary observer pairs allows the noise-reduction problem to be solved without sacrificing the sensitivity of the FDS. The solution to the measurement-noise question is as follows. First, one chooses the gain matrix K of the first observer such that the desired fingerprint with respect to some specific failure is obtained, the direction of $\underline{e}(t)$ in E^n being specified. Then, by choosing the gain matrix L of the second observer as the optimal steady-state (Kalman) filter gain, one obtains the smoothed vector $\hat{\underline{e}}(t)$.

When measurement noise $\underline{n}(t)$ is present, the system dynamics is given by

$$\begin{aligned} \dot{\underline{x}}(t) &= A\,\underline{x}(t) + B\,\underline{u}(t) \\ \underline{y}(t) &= C\,\underline{x}(t) + \underline{n}(t) \end{aligned} \tag{36}$$

It is easy to show that the first-observer error differential equation will be

$$\dot{\underline{e}}(t) = (A - KC)\,\underline{e}(t) - K\,\underline{n}(t) \tag{37}$$

and that the second-observer's output $\hat{\underline{e}}(t)$ will satisfy the following differential equation:

$$\dot{\hat{\underline{e}}}(t) = (A - KC - LC)\,\hat{\underline{e}}(t) + L\,\underline{\epsilon}(t) \tag{38}$$

The primary and secondary observer pair is described by the augmented system dynamics [eq. (39)]:

$$\dot{\underline{\gamma}}(t) = \begin{bmatrix} A - KC & 0 \\ LC & A - KC - LC \end{bmatrix} \underline{\gamma}(t) - K'\,\underline{n} \tag{39}$$

where $\underline{\gamma}^T(t) = [\underline{e}^T(t), \hat{\underline{e}}^T(t)]$ and the $(2n \times m)$ matrix K' is defined as

$$K' \triangleq \begin{bmatrix} K \\ \hline 0 \end{bmatrix} \tag{40}$$

Note the formal similarity between equations (37) and (39), which helps to explain the procedure described above.

As explained before, the suitable choice of the gain matrices K and L in equation (39) allows one to design an FDS that is both sensitive in terms of event (failure) detection and minimally susceptible to measurement noise.

5. ADAPTIVE, PARAMETER TRACKING, PRIMARY AND SECONDARY OBSERVERS FOR A FAILURE-DETECTION SYSTEM

In sections 2-4 we tacitly assumed that the parameters of the dynamical system are constant and known and that the primary and the secondary observers are "matched" to the dynamic (real) system. Unfortunately, in practical applications the system parameters are not exactly known and may even vary with time. Such is the case, in flight-control and guidance systems. This problem was solved, and presented for an FDS that included a single observer, in reference 15. The same reference also includes a short review of the state of the art of adaptive observers, reviewing in particular references 16-24. In reference 25, a method of analysis is presented and an attempt is made to develop a unified method for analysis of adaptive processes.

In this section, an analysis is carried out to show the influence of non-matching conditions on the FAR of failure-detection systems. This condition occurs when the observer (or KF parameters) does not match or track the dynamic system parameters, which are time-varying. As stated above, this is crucial in aeronautical engineering applications of FDS and analytical redundancy concepts, where plant parameter variations are caused by dynamic pressure variations in different flight conditions. The effects on the FAR of mismatching the actual plant and the analytic observer (or FK), including primary and secondary observers, will be discussed subsequently.

First, the mismatched primary observer case will be treated, and we shall assume that the analytical implementation of the primary observer is according to the following observer model:

$$\dot{\hat{\underline{x}}}(t) = (A + \Delta A)\ \hat{\underline{x}}(t) + K[\underline{y}(t) - C\ \hat{\underline{x}}(t)] + (B + \Delta B)\ \underline{u}(t) \tag{41}$$

Accordingly, the primary observer residual error will be the solution of the following linear differential equation:

$$\dot{\underline{e}}(t) = (A - KC)\ \underline{e}(t) - \Delta A.\hat{\underline{x}}(t) - \Delta B.\underline{u}(t) \tag{42}$$

where ΔA and ΔB represent the difference between the parameters of the real plant and those of the primary observer. It is easy to see that the last two terms in equation (42) will cause a high residual $\underline{e}(t)$, even after the initial transient has died out. The large value of $\underline{e}(t)$ is directly responsible for an unacceptably high FAR. Acceptable values of FAR will be obtained only for observers that are matched to the plant dynamics. In order to see the effects of ΔA and ΔB on $\underline{e}(t)$ and $\hat{\underline{e}}(t)$, the effect of three parameter changes in the plant dynamics on $\underline{e}(t)$ is shown in figure 7. From figure 7 it is clear that the errors are very large, leading to a prohibitive FAR. (For more details see the simulation results in sec. 7.) Using design methods based on the "robust observer" approach will not be of much use, because that approach will lead to observers that are insensitive to failures. Therefore, it is easy to see the need for adaptive observers that can track the plant parameter variations in FDS applications.

The same "mismatching" problem can also cause serious problems in the FDS, including the Kalman filters used to reduce measurement noise influence on the FAR. In this case, a notable change in the basic characteristics of the innovation sequence will be caused by mismatching conditions. Let us define the dynamic system (plant) equation by

$$\dot{\underline{x}}(t) = A\underline{x}(t) + B\underline{u}(t) + \Gamma.\underline{w}(t) \tag{43}$$

where $\underline{w}(t)$ is the $(q \times 1)$ noise input vector, assumed to be white and Gaussian. The measurement vector $\underline{y}(t)$, $(n \times 1)$, is contaminated by white noise $\underline{n}(t)$, with $E[n] = 0$ and $E[\underline{n}(t)\underline{n}^T(s)] = Q_1\delta(t - s)$:

$$\underline{y}(t) = C\ .\ \underline{x}(t) + \underline{n}(t) \tag{44}$$

Assume, for simplification, only plant-parameter variations causing the following mismatching conditions:

$$\begin{aligned} \tilde{A} &\triangleq A + \Delta A \\ \tilde{K} &\triangleq K + \Delta K \end{aligned} \tag{45}$$

where A and K are the matrices used in the Kalman filter implementation. The equation of the Kalman filter is given by

$$\dot{\hat{\underline{x}}}(t) = \tilde{A}.\hat{\underline{x}}(t) + \tilde{K}[\underline{y}(t) - C\,\hat{\underline{x}}(t)] \tag{46}$$

Define $\hat{\underline{x}}(t)$ as the best estimate for the ideal matching conditions and $\Delta\underline{x}(t)$ as the change in the estimate owing to mismatching:

$$\tilde{\underline{x}}(t) \triangleq \hat{\underline{x}}(t) + \Delta\underline{x}(t) \tag{47}$$

Denote also $\tilde{\underline{\nu}}(t)$ as the innovation vector for the mismatched system and $\tilde{\underline{\nu}}(t)$ as the innovation of the ideal-matched KF-system. Based on linearity property, one can write

$$\tilde{\underline{\nu}}(t) \triangleq \underline{\nu}(t) + \Delta\underline{\nu}(t) \tag{48}$$

From equations (44), (47), and (48) one obtains

$$\tilde{\underline{\nu}}(t) = \underline{\nu}(t) - C.\Delta\underline{x}(t) \tag{49}$$

where $\Delta\underline{x}(t)$ is the solution of the following differential equation:

$$\Delta\dot{\underline{x}}(t) = (A - KC)\,\Delta\underline{x}(t) + \Delta K.\tilde{\underline{\nu}}(t) + \Delta A.\tilde{\underline{x}}(t) \tag{50}$$

It is clear from equations (49) and (50), and also shown explicitly in figure 8, that the stochastic process $\tilde{\underline{\nu}}(t)$, which is the actual innovation vector, will be a colored noise process, with $E[\tilde{\underline{\nu}}(t)] \neq 0$. Therefore, no adequate test can be made on $\tilde{\underline{\nu}}(t)$ in order to detect a failure in a reliable way, for example, with a very low, admissible FAR.

In conclusion, to obtain an adequate FAR in a failure-detection system, it is absolutely necessary to have good matching between the observer's (or KF) model parameters and the parameters of the dynamic, real plant. In what follows in this section we introduce an algorithm for an adaptive pair, primary-secondary observer design, the adaptation law providing also for parameter identification and tracking.

The approach presented here is basically similar to the method presented in reference 15 and is based on a simple, yet effective, adaptive law (algorithm) for linear, possibly time-varying, multi-input, multi-output systems. The adaptive law makes use of a-priori-determined adaptive gains and does not require solution of additional differential equations. Therefore, the computational effort required is suitable for the practical needs and objectives of real-time, on-line, simple adaptive observers for failure-detection systems.

As shown previously in equations (41) and (42), the model of the mismatched primary observer leads to an augmented observer output residual $\underline{\epsilon}(t)$, which is given by $\underline{\epsilon} = C\,\underline{e}(t)$, where $\underline{e}(t)$ will be the solution of the differential equation

$$\dot{\underline{e}}(t) = (A - KC)\,\underline{e}(t) - \Delta A.\hat{\underline{x}}(t) - \Delta B.\underline{u}(t) \tag{51}$$

To compensate for ΔA and ΔB, in both the primary and the secondary observers, it is proposed here to change the entries of the observer matrices A_o and B_o according to the following adaptation laws (algorithm):

$$\Delta A_o = M\, \underline{\hat{e}}(t) . \underline{\hat{x}}^T(t) \tag{52a}$$

$$\Delta B_o = N\, \underline{\hat{e}}(t) . \underline{u}^T(t) \tag{52b}$$

or, in the discrete case, according to

$$\Delta A_o(k) = M\, \underline{\hat{e}}(k) . \underline{\hat{x}}^T(k) \tag{53a}$$

$$\Delta B_o(k) = N\, \underline{\hat{e}}(k) . \underline{u}^T(k) \tag{53b}$$

with $k = 1,2,...$

The algorithm (52) is based on measurable values, such as the observer output (the estimated state) $\underline{\hat{x}}(t)$, the plant (and the observer) input $\underline{u}(t)$, and $\underline{\hat{e}}(t)$, the second observer output vector. The matrices $M(n \times n)$ and $N(n \times n)$ are to be chosen in such a way that convergence and good tracking are provided. As shown in the next two sections, the adaptive algorithm introduced here makes possible to

(1) maintain a low value of the first observer output residual error, in spite of plant parameter variations;

(2) quickly adapt both primary and secondary observer parameters to those of the dynamic plant; and

(3) to track the varying dynamic plant parameters by the primary-secondary adaptive observer parameters.

In figure 9, a block diagram of the primary-secondary adaptive observer is presented; it points out the simplicity of the adaptive law and the fact that this algorithm only makes use of accessible measurable functions.

Although one has to show that the algorithm proposed in equations (52) provides for stability and convergence for the entire primary-secondary observer, it is worthwhile to do, at the beginning, a simple and approximate analysis for the first observer only, assuming that the matrix A_o of the second observer is also adequately tracking the real dynamic system A matrix.

Substituting equation (52) into equation (51), one gets

$$\begin{aligned} \underline{\dot{e}}(t) &= (A - KC)\, \underline{e}(t) - \|\underline{\hat{x}}\|^2\, M\, \underline{\hat{e}}(t) - \|\underline{u}\|^2\, N\, \underline{\hat{e}}(t) \\ &\cong (A - KC)\, \underline{e}(t) - \|\underline{\hat{x}}\|^2\, M\, \underline{e}(t) - \|\underline{u}\|^2\, N\, \underline{e}(t) \end{aligned} \tag{54}$$

Equation (54) can be put in the more compact form:

$$\underline{\dot{e}}(t) = [(A - KC) - \|\underline{\hat{x}}\|^2\, M - \|\underline{u}\|^2\, N]\, \underline{e}(t) \tag{55}$$

To obtain an asymptotically-stable-in-the-large (ASIL) solution for the time-varying, nonlinear, differential equation, several approaches can be taken. The first is a heuristic one; although the matrix included in the square bracket is time-varying because of the time-dependent positive scalars $\|\underline{x}\|^2$ and $\|\underline{u}\|^2$, it is conjectured here that by an appropriate choice of M and N, based on a priori knowledge of $\underline{x}(t)$ and $\underline{u}(t)$, the adaptive algorithm [eq. (52)] can be made asymptotically convergent. Loosely speaking, the M and N matrices allow us to locate the eigenvalues of this square matrix so that all of them will have negative real values, providing us with the result: $\underline{e}(t) \to 0$ as $t \to \infty$. A second way to obtain an ASIL solution for equation (55) is to make use of a version of Perron's theorem (refs. 24, 27, and 28) and to determine, accordingly, the entries of the gain matrices M and N. A more appropriate way to obtain a convergent adaptive law is to determine the gain matrices M and N by making use of Lyapunov's second method (refs. 15 and 29) and this approach will be presented in the next section. A fourth method to show that the algorithm (52) leads to an ASIL solution for the FDS adaptive observer, provided the parameter changes are small, is to use the linear quadratic regulator theory results. This new and interesting approach is presented in appendix B.

Given parameter changes in the real, dynamic system, one can now write the exact primary-secondary observers equations, based on equations (24) and (51), as follows:

$$\dot{\underline{\gamma}}(t) = \overline{Q}\,\underline{\gamma}(t) + \overline{\Delta A}.\hat{\underline{x}} + \overline{\Delta B}.\underline{u} \tag{56}$$

where $\underline{\gamma}^T(t) \triangleq [\underline{e}(t), \hat{\underline{e}}(t)]^T$

$$\overline{Q} \triangleq \begin{bmatrix} A - KC & 0 \\ LC & A - KC - LC \end{bmatrix} \tag{57}$$

$$\overline{\Delta A} \triangleq \begin{bmatrix} \Delta A \\ \hline 0 \end{bmatrix} \tag{58a}$$

$$\overline{\Delta B} = \begin{bmatrix} \Delta B \\ \hline 0 \end{bmatrix} \tag{58b}$$

The adaptive algorithm (52) can also be presented in the following alternative form:

$$\Delta A_o = \overline{M}\,\underline{\gamma}(t)\,\hat{\underline{x}}^T(t) \tag{59a}$$

$$\Delta B_o = \overline{N}\,\gamma(t)\,\underline{u}^T(t) \tag{59b}$$

where

$$\overline{M} \triangleq \left[\begin{array}{c|c} 0 & M \\ \hline 0 & 0 \end{array}\right] \tag{60a}$$

$$\overline{N} \triangleq \left[\begin{array}{c|c} 0 & N \\ \hline 0 & 0 \end{array}\right] \tag{60b}$$

From equation (56) one obtains

$$\begin{aligned} \dot{\underline{\gamma}}(t) &= \overline{Q}.\underline{\gamma}(t) + \overline{M}\,\|\hat{\underline{x}}\|^2\,\underline{\gamma}(t) + \overline{N}\,\|\underline{u}\|^2\,\underline{\gamma}(t) \\ &= [\overline{Q} + \overline{M}\,\|\hat{\underline{x}}\|^2 + \overline{N}\,\|\underline{u}\|^2].\underline{\gamma}(t) \end{aligned} \tag{61}$$

By taking into account equations (59) and (60) and by substituting them into equation (61), one gets

$$\dot{\underline{\gamma}}(t) = \begin{bmatrix} A - KC & \|\hat{\underline{x}}\|^2 M + \|\underline{u}\|^2 N \\ \\ LC & A - KC - LC \end{bmatrix} \underline{\gamma}(t) \tag{62}$$

Equation (62) shows that the correct adaptive algorithm for the first observer is indeed the algorithm given in equation (52), provided that we prove convergency and stability; on the other hand, one has to adapt the A matrix in the second observer simultaneously with the first-observer adaptation. Therefore, in the next section a proof of stability and convergency for the system described by equation (62) will be presented.

If measurement noise is to be taken into account, the gain matrices K and the adaptive gain matrices M and N will have to meet some requirements in addition to those imposed by the appropriate convergency conditions. In this case, a trade-off is to be made in the choice of M and N, between fast parameter-tracking requirements and minimal noise susceptibility. Finally, the gain matrices M and N and, in particular, the gain matrix K, have to be chosen such that the observer sensitivity in terms of failure detection will be maximal.

To summarize, besides the necessary convergency conditions, the gains K, L, M, and N are to be judiciously determined by taking into account such considerations as (1) the minimum parameter alignment time (rate of convergence), (2) fast-tracking capabilities, (3) minimum noise susceptibility for minimal FAR, and (4) maximum sensitivity for high-probability failures detections.

6. CONDITIONS FOR CONVERGENCE AND STABILITY

In the previous section, a procedure for choosing M and N matrices based on a heuristic approach was discussed briefly. Here, a procedure for determining the matrices M and N, based on Lyapunov's theorem for asymptotic stability, will be developed. It will be shown that for a system described by a differential equation such as equation (62) and that has a general form, such as that of equation (63),

$$\dot{\underline{\gamma}}(t) = W(\underline{\gamma},t) \cdot \underline{\gamma}(t) \tag{63}$$

where

$$W(\underline{\gamma},t) \triangleq \begin{bmatrix} A - KC & \|\hat{\underline{x}}\|^2 M + \|\underline{u}\|^2 N \\ \\ LC & A - KC - LC \end{bmatrix} \tag{64}$$

the solution is uniformly asymptotically stable in the large, about the zero solution $\underline{\gamma}(t) = 0$, which is the equilibrium point, if the entries of the matrix $W(\underline{\gamma},t)$ satisfy certain requirements, provided by some inequality conditions.

Let us consider the following positive-definite scalar quadratic function $V(\underline{\gamma})$ as a candidate for a Lyapunov function:

$$V(\underline{\gamma}) = \underline{\gamma}^T Q \underline{\gamma} \tag{65}$$

with Q being an arbitrary, constant, diagonal, positive-definite matrix, such that

$$V(\underline{\gamma}) \begin{cases} = 0 & \text{if } \underline{\gamma} = 0 \\ > 0 & \forall \underline{\gamma} \neq 0 \quad , \forall t \end{cases} \tag{66}$$

In addition, equation (65) provides us with

$$\lim_{\|\underline{\gamma}\| \to \infty} V(\underline{\gamma}) = \infty \tag{67}$$

To obtain ASIL conditions for the system in equation (63), in addition to the conditions in equations (66) and (64), it is necessary that $\dot{V} \triangleq dV/dt$ meet the following condition:

$$\dot{V}(\underline{\gamma}) < 0 \quad , \quad \forall t \quad , \forall \underline{\gamma} \neq 0 \tag{68}$$

We will now proceed to obtain the necessary conditions to be fulfilled by $W(\underline{\gamma},t)$ in order to satisfy conditions in equations (66) to (68). If those conditions are satisfied, then $V(\underline{\gamma})$ from equation (65) will be an adequate Lyapunov function for the system in equation (63), and the ASIL property will be obtained.

From equations (63) and (65), we get the following expression for $\dot{V}$:

$$\dot{V} = \underline{\gamma}^T [Q\, W(\underline{\gamma},t) + W^T(\underline{\gamma},t)\, Q]\, \underline{\gamma} \tag{69}$$

To satisfy the condition in equation (68), the matrix $P \triangleq [Q\,W + W^T Q]$ has to be negative-definite (ref. 29). The symmetric matrix P is a function of the gains $\{K, L, M, N\}$ and depends also on the matrix Q and the functions $\underline{u}(t)$ and $\underline{\gamma}(t)$. We shall proceed further to seek the necessary conditions for the elements p_{ij} of P such that $\dot{V} < 0$. By expanding the quadratic form given in equation (69) the following expression for $\dot{V}$ is found.

$$\dot{V} = \sum_{\substack{i=1 \\ (i\neq j)}}^{n} \sum_{j=1}^{n} [q_{ii}\, w_{ii}\, \gamma_i^2 + (q_{ii}\, w_{ij} + q_{jj}\, w_{ji})\, \gamma_i\, \gamma_j + q_{jj}\, w_{jj}\, \gamma_j^2] \tag{70}$$

where q_{ij} and w_{ij} are the elements of the matrices Q and W, respectively, and we take $i \neq j$ in the cross-terms of the expression (70).

In order to obtain appropriate conditions for convergence and ASIL stability of the adaptation algorithm from equation (52), it is necessary that the conditions established in the following theorem hold.

<u>Theorem</u>: In order for the time-varying system described by equations (63) and (64) to have an ASIL solution (asymptotically stable in the large), about the singular stable point $\underline{\gamma} = 0$, the following conditions must be satisfied:

$$V > 0 \; , \quad \forall t \; , \; \forall \underline{\gamma} \neq 0 \tag{71a}$$

$$q_{ii} w_{ii} \leq - D < 0 \quad i = 1,2,\ldots, n \tag{71b}$$

$$q_{jj} w_{jj} \leq - D < 0 \quad j = 1,2,\ldots, n \tag{71c}$$

$$\sqrt{q_{ii} w_{ii} q_{jj} w_{jj}} \geq \frac{(q_{ii} w_{ij} + q_{jj} w_{ji})}{2} \tag{71d}$$

$i = 1,2,\ldots, n$

$j = 1,2,\ldots, n$ ($1 \neq j$ in the cross-term)

If the conditions of the theorem are satisfied, it is guaranteed that the time derivative of the Lyapunov function will be negative-definite everywhere in the 2n-dimensional vector space E^{2n} spanned by $\underline{\gamma}$, that is,

$$\dot{V} < 0 \; , \quad \forall t \; , \; \forall \underline{\gamma} \neq 0 \tag{72}$$

the function $V(\underline{\gamma})$ being, therefore, an admissible Lyapunov function for the system in equation (63).

The conditions established in equation (71) are not difficult to meet, since the values of D, q_{ij}, and those of the gains m_{ij} and n_{ij} (contained in w_{ij}) can be arbitrarily chosen. The proof of the theorem is given in appendix C, where it is also shown that if the conditions given in equations (71b), (71c), and (71d) are satisfied, the value of the function $\dot{V}$ will be

$$\dot{V} \leq - D \sum_{i=1}^{n} \sum_{j=1}^{n} (\gamma_i^2 + \gamma_j^2) < 0 \tag{73}$$

From equation (73), it is easy to see that by an appropriate choice of the matrix Q and of the constant D, it is possible to modify and accelerate the convergence rate of the adaptation process. But as pointed out before, a trade is to be made between high convergence rate and susceptibility to possible existing measurement noise.

It should be noted that conditions similar to those in equation (71) can be obtained by applying Sylvester's theorem for negative definiteness directly to the system matrix P. This alternative approach is not explicitly shown in this paper, since the establishment of the ASIL conditions following this approach is associated with a lengthy and tedious algebraic manipulation. Also, as we already mentioned above, we obtained similar conditions by applying linear quadratic regulator theory, as explained in appendix B.

7. SIMULATION RESULTS

To illustrate the utilization of the proposed approach for FDS's — namely, (1) the use of primary and secondary observers for failure detection and (2) the use of the adaptive observer algorithm introduced in this paper — the results obtained for a linear third-order system with two outputs are shown later. The nominal system equations are

$$\begin{cases} \dot{x}_1 = -a_n x_1 + x_2 \\ \dot{x}_2 = x_3 - c_n x_1 \\ \dot{x}_3 = b_n u \end{cases}$$

with $a_n > 0$, $b_n > 0$, $C_n > 0$ and with two output measurements

$$\underline{y} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \underline{x}$$

The input $u(t)$ was the sum of sinusoids

$$u(t) = \sin \pi t + \frac{1}{3} \cos t$$

In this case $u(t)$ was of a persistently exciting type.

The discretization time chosen was $\Delta T = 0.05$ sec (a fairly high value). The simulation results are divided into three groups:

1. In the first group of results (figs. 10-12), the fingerprints obtained for various sensor and actuator failures are shown. The various components of the vector $\hat{\underline{e}}(t)$ are also shown with appropriate scale factors.

2. In the second group of results (figs. 13-16), the adaptation process of the primary and secondary observer's pair toward the nominal, constant, plant parameters is shown, with and without measurement noise. Also, the three components of the $\hat{\underline{e}}(t)$ vector used for failure detection and assessment, are shown.

3. In the third group of results (figs. 17-19), the adaptation of the primary and secondary observer pair toward the nominal, time-varying, plant parameters is shown, with and without measurement noise. Also, the three components of the $\hat{\underline{e}}(t)$ vector are drawn. In all three groups, the gain K of the first observer was chosen on the basis of a desired fingerprint with respect to a failure, and the gain L of the second observer had to minimize the effect of the measurement noise on $\hat{\underline{e}}(t)$.

In figure 10, the second observer output, for example, the components of the vector $\hat{\underline{e}}(t)$, owing to an abrupt failure of sensor No. 2 at $T_f = 3$ sec, is shown. The first observer gain matrix K was chosen according to the condition established in equation (35) such that the vector $\hat{\underline{e}}(t)$ will point in a predetermined and fixed direction in E^n for a failure in sensor No. 2.

The first-observer eigenvalues, under those conditions, were $s_1 = -16$ and $s_{2,3} = -8 \pm j8$. The optimal gain of the second observer was determined as

$$L = \begin{bmatrix} 10 & 4 \\ 10 & 20 \\ 10 & 50 \end{bmatrix}$$

After an initial, short transient of the observers, and under conditions of no-failure, the three components of $\hat{\underline{e}}(t)$ became very small (an order of magnitude of 10^{-6}). As a result of the failure of sensor No. 2 at $T_f = 3$ sec, one could observe a short and high transient in all three components of $\hat{\underline{e}}(t)$, in particular in $\hat{e}_3(t)$, such that a well-defined alarm signal is provided instantly, pointing out that a failure occurred. The direction of $\hat{\underline{e}}(t)$, with its definite fingerprint, determines that the failure that occurred was in sensor No. 2 Note the difference in scale factors between the plot of $\hat{e}_3(t)$ and that of $\hat{e}_1(t)$ and $\hat{e}_2(t)$.

In figure 11, the effect of the failure of sensor No. 1 on $\hat{\underline{e}}(t)$ is shown. Here also the most sensitive component of $\hat{\underline{e}}(t)$ was $\hat{e}_3(t)$ (note the different scale factors). The fingerprint of this failure is distinct and different from the fingerprint of the previous failure.

Figure 12 shows the effect of an actuator failure on $\hat{\underline{e}}(t)$ and the specific fingerprint obtained in this case. It is noted that the sensitivity of the FDS with respect to the actuator failure is quite low, because criterion (35) was implemented in our example only with respect to a failure of sensor No. 2.

In figure 13, the simultaneous adaptation process of three primary-secondary observer parameters, a_o, b_o, and c_o, is shown. These three parameters converge, respectively, toward the nominal system parameter values: $a_n = 1.0$, $b_n = 1.5$, and $c_n = 3.0$. The starting values of the observers parameters were $a_o(o) = 1.5$, $b_o(o) = 2.0$, and $c_o(o) = 2.5$. Together with the initial parameter mismatching, the following mismatching conditions in the initial conditions values were also used:

$$x_1(o) = 1.0 \qquad x_1(o) = 0.0$$

$$x_2(o) = 0.0 \qquad x_2(o) = 1.0$$

$$x_3(o) = 0.0 \qquad x_3(o) = 1.0$$

After 6 sec (120 steps), the norm of the parameter error vector dropped to less than 5%. The norm of the second-observer output error vector dropped to less than 10^{-2} after 5 sec. The normalized values of m_{ij} and n_{ij} were unity, except the values of m_{3j} and n_{3j} (j_3= 1, 2, 3), which were taken as 0.1 In figure 14, the various components of $\hat{\underline{e}}(t)$ are shown, and it is easy to see that after the adaptation phase, $\hat{\underline{e}}(t)$ becomes very small again, being valid for failure detection.

In figure 15, the simultaneous adaptation process of three primary-secondary observer parameters, a_o, b_o, and c_o, while the first output measurement is contaminated with white noise, is shown. The noise-to-signal ratio was chosen to be intentionally high — about 5% relative to the maximum value of x_1. The adaptation process has essentially the same profile as before, for the same values of mismatching in the states and in the parameters. The identification accuracy, although slightly reduced in this case because of the high measurement noise, is still remarkably good. As shown in figure 16, the components of $\hat{\underline{e}}(t)$, although noisy, are still very low, giving valid information sources for failure alarm and assessment.

The effects of parameter changes in the nominal plant on the primary-secondary adaptive observers adaptation process is shown in figure 17. The nominal system parameters were widely varied, as shown in the following:

$$a_n = \begin{cases} 1.0 & \text{for} \quad 0 \le t \le 1 \text{ sec} \\ 1.0 + 0.2(t - 1) & \text{for} \quad 1 < t \le 14 \text{ sec} \\ 3.6 & \text{for} \quad t > 14 \text{ sec} \end{cases}$$

$$b_n = \begin{cases} 2.0 & \text{for} \quad 0 \le t \le 3 \text{ sec} \\ 2.0 + 0.08(t - 3) & \text{for} \quad 3 < t \le 12 \text{ sec} \\ 2.72 & \text{for} \quad t > 12 \text{ sec} \end{cases}$$

$$c_n = \begin{cases} 3.0 & \text{for} \quad 0 \le t \le 5 \text{ sec} \\ 3.0 - 0.01(t - 4) & \text{for} \quad 4 < t \le 18 \text{ sec} \\ 1.6 & \text{for} \quad t > 18 \text{ sec} \end{cases}$$

While in steady state, the accuracy of the parameter identification was of the order of 95%. The second-observer output, while in the simultaneous tracking phase of a_o, b_o, and c_o, following $a_n(t)$, $b_n(t)$ and $c_n(t)$, was less than unity.

The effects of the output measurement noise of sensor No. 1 on the adaptation process and on the parameter-identification accuracy are shown in figure 18. The adaptation process was only slightly modified by the measurement noise, the noise-to-signal ratio being deliberately chosen to be high — about 5%. The identification accuracy was also only slightly reduced, and in the steady-state phase the three parameters could be identified with high accuracy, as shown in figure 18. As shown in figures 18 and 19, the effect of the measurement noise on the adaptation process and on parameter-identification and failure-detection capabilities was rather minor, mostly because the gains of the second observer were optimal gains.

8. COMMENTS AND CONCLUSIONS

The problem of designing analytical failure-detection systems, using pairs of primary and secondary observers for linear, constant, and, possibly, time-varying, multi-input, multi-output systems, with measurement noise, was described. The use of a secondary observer permits the reconstruction of the entire error vector $\hat{\underline{e}}(t)$, which is the major source of information for failure assessment. The $\hat{\underline{e}}(t)$ vector has a unique fingerprint associated with certain classes of failures. Moreover, by applying criterion (35) the specific fingerprint can be determined a priori, by choosing the K matrix, thereby enhancing the failure-detection sensitivity and detectability.

It was also shown that in order to use primary-secondary observers (or Kalman filters or both) for the purpose of detecting the failures in linear systems, it is necessary to adapt the observers (or the Kalman filter) to the parameters of the dynamic system. If this is not done, prohibitive false-alarm rates will result.

An on-line algorithm for tracking-adaptive primary and secondary observers for multi-input, multi-output linear systems was introduced, and conditions for convergence and asymptotic stability were developed. Those conditions are established a priori, such that the use of the algorithm is simple and effective. In the example shown, in both deterministic and stochastic cases, the adaptive law exhibited satisfactory accuracy and tracking capabilities by maintaining a low observer output error and, simultaneously, by identifying the system parameters in an accurate manner.

The effect of the output measurement noise was minor because of the use of the optimal gain matrix L in the second observer.

Although the results obtained here are encouraging for the detection of sudden or abrupt actuator and sensor failures, the detection of soft failures remains an important topic for further research. In particular, it is essential to minimize the failure-detection time and to do so with a minimal false-alarm rate. To resolve this problem, one has to implement, in addition to the primary-secondary observers, an algorithm based on statistical decision theory such as the generalized likelihood ratio (GLR) or, eventually, the sequential-likelihood-ratio-test (SLRT) approach (see refs. 2 and 9).

Another topic for additional research is the development of a synthesis technique for the optimal choice of the matrices L, M, and N in order to maintain low false-alarm rates associated with high failure-detection sensitivity in stochastic environments such as those that exist in turbulence or when maneuvering.

Another topic for further research is the reorganization of the adaptive observer (or KF) and of the whole FDS after a major failure has occurred. This must be done, no matter what approach and algorithm are used in the analytical failure-detection system.

Acknowledgement

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APPENDIX A

ALTERNATIVE IMPLEMENTATION OF OBSERVERS

Given the observer differential equation (10),

$$\dot{\hat{\underline{x}}}(t) = Q.\hat{\underline{x}}(t) + K\,\underline{y}(t) + B\,\underline{u}(t) \tag{A1}$$

we are looking for a solution for $\hat{\underline{x}}(t)$ having the following form:

$$\hat{\underline{x}}(t) = W(t).\hat{\underline{x}}(0) + \underline{\phi}(t) + \underline{\rho}(t) \tag{A2}$$

where the observer's output is the sum of three different time functions. Taking now the derivative of $\hat{\underline{x}}(t)$ from equation (A2), one obtains

$$\dot{\hat{\underline{x}}}(t) = \dot{W}(t).\underline{x}(0) + \dot{\underline{\phi}}(t) + \dot{\underline{\rho}}(t) \tag{A3}$$

By substituting equation (A2) into (A1), one obtains

$$\dot{\hat{\underline{x}}}(t) = Q\,W(t)\;\hat{\underline{x}}(0) + Q\,\underline{\phi}(t) + Q\,\underline{\rho}(t) + K\,\underline{y}(t) + B\,\underline{u}(t) \tag{A4}$$

By comparing terms between equations (A3) and (A4), one finally gets the following differential equations satisfying $W(t)$, $\underline{\phi}(t)$, and $\underline{\delta}(t)$ in equation (A2), with the appropriate initial conditions:

$$\dot{W}(t) = Q\,W(t) \qquad W(0) = I \tag{A5}$$

$$\dot{\underline{\phi}}(t) = Q\,\underline{\phi}(t) + K\,\underline{y}(t) \qquad \underline{\phi}(0) = 0 \tag{A6}$$

$$\dot{\underline{\rho}}(t) = Q\,\underline{\rho}(t) + B\,\underline{u}(t) \qquad \underline{\rho}(0) = 0 \tag{A7}$$

Since the term $W(t).\hat{\underline{x}}(0)$ represents the effect of the initial transient, the failure-event information is contained in the second and the third terms only. In particular, sensor failures affect only the function $\underline{\phi}(t)$, whereas actuator failures are affecting both $\underline{\phi}(t)$ and $\underline{\rho}(t)$. Therefore, the observer form in equation (A2) will provide the information necessary for failure detection and assessment, instead of the error-vector examination approach, usually used in this context.

Eventually, the output observer form equation (A2) may be useful for the system matrix A identification purposes.

APPENDIX B

ADAPTIVE OBSERVER ALGORITHM VIA LINEAR QUADRATIC REGULATOR THEORY

In this appendix we demonstrate, by invoking well-known results from linear quadratic regulator theory (LQR) that the adaptive observer algorithm suggested in equation (52) will indeed insure convergence and ASIL conditions for the primary-secondary observer pair. In what follows, without any loss of generality, the existence of ASIL conditions will be demonstrated for a single adaptive observer.

From equation (51), if the system matrices A and B were slightly changed, one has

$$\dot{\underline{e}}(t) = (A - KC)\,\underline{e}(t) - \Delta A_o \cdot \hat{x}(t) - \Delta B_o \cdot \underline{u}(t) \tag{B1}$$

Let us write equation (B1) in the following more general form:

$$\dot{\underline{e}} = \bar{A}\,\underline{e} + \underline{q}(t) \tag{B2}$$

where

$$\bar{A} \overset{\Delta}{=} A - KC \tag{B3}$$

$$\underline{q}(t) \overset{\Delta}{=} -\Delta A_o \cdot \underline{x}(t) - \Delta B_o \cdot \underline{u}(t) \tag{B4}$$

Now, one can ask for the optimal control law $\underline{q}^*(t)$, such that the following functional

$$\mathcal{J} = \frac{1}{2}\int_0^{t_f} [\underline{e}^T P\,\underline{e} + \underline{q}^T R\,\underline{q}]\, dt \tag{B5}$$

will be minimized. The meaning of the minimization process is obvious: one tries to minimize and zeroing the error $\underline{e}(t)$, while the adaptation process is carried out with a finite, optimal control $\underline{q}^*(t)$.

From linear optimal control theory, the following necessary conditions for optimality are obtained:

$$\underline{q}^*(t) = -R^{-1}\, S(t)\, \underline{e}(t) \tag{B6}$$

where the matrix $S(t)$ is the solution of the well-known nonlinear Riccati equation. From equations (B4) and B6) one has

$$-R^{-1}\, S(t)\, \underline{e}(t) = -\Delta A_o(t)\, \hat{\underline{x}}(t) - \Delta B_o(t)\, \underline{u}(t) \tag{B7}$$

The question now is under what conditions does equation (B7) hold. Or, in other words, what does the formal structure of $\Delta A_o(t)$ and $\Delta B_o(t)$ have to be in order to satisfy equation (B7)? A simple inspection of the terms of equation (B7) reveals that it is sufficient to choose the algorithm,

$$\Delta A_o(t) = M C \underline{e}.\hat{\underline{x}}^T \tag{B8a}$$

$$\Delta B_o(t) = N C \underline{e}.\underline{u}^T \tag{B8b}$$

in order to satisfy, at least formally, equation (B7).

Making use of equation (B8) in (B7) and after eliminating $\underline{e}(t)$, one obtains

$$R^{-1} S(t) = M C \, \|\hat{\underline{x}}\|^2 + N C \, \|\underline{u}\|^2 \tag{B9}$$

Since the matrices R, C, and S are known, the values of the entries of the M and N matrices can be chosen (at least in principle) so that equation (B9) holds. We are not suggesting that M and N be chosen by using this procedure, because the purpose of this appendix is only to show that the adaptive algorithm introduced in equation (52) satisfies also, in some sense, an optimality criterion and therefore provides adequate stability conditions established on LQR theory grounds.

APPENDIX C

PROOF OF STABILITY THEOREM

In this appendix, a proof of the theorem stated in section 6, where the conditions in equation (74) for ASIL are established, is given.

From equation (70), the following expression for $\dot{V}$ is obtained:

$$\dot{V} = \sum_{\substack{i-1 \\ (i \neq j)}}^{n} \sum_{j=1}^{n} [q_{ii} w_{ii} \gamma_i^2 + (q_{ii} w_{ij} + q_{jj} w_{ji}) \gamma_i \gamma_j + q_{jj} w_{jj} \gamma_j^2] \tag{C1}$$

where q_{ij} and w_{ij} are the elements of the matrices Q and W, respectively, and $i \neq j$ is to be taken in the cross-terms of (C1).

For $\dot{V}$ to be negative-definite, at a first glance it seems to be a good choice to take

$$\begin{aligned} q_{ii} w_{ii} &\leq - D < 0 \\ q_{jj} w_{jj} &\leq - D < 0 \end{aligned} \tag{C2}$$

and to try to get the rest of the right-hand side of equation (C1) to form a square. The constant D in equation (C2) is an arbitrary, positive constant. We shall examine, in the sequel, three different cases.

<u>Case I</u>: We can choose to satisfy the following conditions:

$$\sqrt{q_{ii} q_{jj} w_{ii} w_{jj}} = \frac{1}{2}(q_{ii} w_{ij} + q_{jj} w_{ji}) \tag{C3a}$$

together with

$$\begin{aligned} q_{ii} w_{ii} &= -D < 0 \\ q_{jj} w_{jj} &= -D < 0 \end{aligned} \tag{C3b}$$

for

$i = 1,2,\ldots, n$

$j = 1,2,\ldots, n$ ($1 \neq j$ in the cross-terms)

$\forall \underline{\gamma}(t)$ and $\forall t$

In this case, equation (C1) becomes

$$\dot{V}_I = \sum_{\substack{i=1 \\ (i \neq 1)}}^{n} \sum_{j=1}^{n} (-D\gamma_i^2 + 2D\gamma_i\gamma_j - D\gamma_j^2)$$
$$= -D \sum_{i=1}^{n} \sum_{j=1}^{n} (\gamma_i - \gamma_j)^2 \leq 0 \qquad \text{(C4)}$$

Since $\dot{V}_I$ is in this case a negative, semi-definite function ($\dot{V}_I \leq 0$), the Lyapunov stability conditions for ASIL are not met and, therefore, conditions in equation (C3) are not satisfactory. Despite this fact, it is indicated that the conditions in equation (C3) be used as an initial, starting condition, in order to obtain a better feeling for the choice of the gains m_{ij} and n_{ij}.

<u>Case II</u>: Here, one may choose the conditions

$$\sqrt{q_{ii}\, q_{jj}\, w_{ii}\, w_{jj}} > \frac{1}{2}(q_{ii}\, q_{ij} + q_{jj}\, w_{ji}) \qquad \text{(C5a)}$$

or

$$\sqrt{q_{ii}\, q_{jj}\, w_{ii}\, w_{jj}} = \frac{1}{2}(q_{ii}\, w_{ij} + q_{jj}\, w_{ji}) + \theta^2 \qquad \text{(C5b)}$$

for

$i = 1,2,\ldots, n$

$j = 1,2,\ldots, n$ ($i \neq j$ in the cross-terms)

$\forall \underline{\gamma}(t)$ and t

together with the conditions in equation (C3b), whereas θ is an arbitrary constant. Substituting, in equation (C1), one gets

$$\dot{V}_{II} = \sum_{\substack{i=1 \\ (i \neq j)}}^{n} \sum_{j=1}^{n} [-D\gamma_i^2 - 2(\theta^2 - D)\gamma_i\gamma_j - D\gamma_j^2] \qquad \text{(C6)}$$

If the following choice is made,

$$\theta^2 = D \qquad \text{(C7)}$$

so that the following equality holds,

$$q_{ii}\, w_{ij} + q_{jj}\, w_{ji} = 0 \qquad \text{(C8)}$$

one obtains for $\dot{V}_{II}$ the following expression:

$$\dot{V}_{II} = -D \sum_{i=1}^{n} \sum_{j=1}^{n} (\gamma_i^2 + \gamma_j^2) < 0 \tag{C9}$$

This time, $\dot{V}_{II}$ is an absolute, negative-definite function and, therefore, the conditions in equations (C3b) and (C8) will ensure asymptotic stability in the large.

Case III: In this case, we obtain a set of conditions for ASIL that are easier to fulfill, and, at the same time, we can fix an a priori, upper bound for $\dot{V}$, increasing the convergence rate of the adaptive algorithm (up to a certain limit, because of the stochastic measurement noise susceptibility problem). Let us choose

$$\begin{aligned} q_{ii} w_{ii} &\leq -D < 0 \\ q_{jj} w_{jj} &\leq -D < 0 \end{aligned} \tag{C10}$$

Instead of equation (C10), one writes

$$\begin{aligned} q_{ii} w_{ii} &= -D - D_1 \\ q_{jj} w_{jj} &= -D - D_2 \\ i = 1,2,\ldots, n \quad &; \quad j = 1,2,\ldots, n \end{aligned} \tag{C11}$$

where $D > 0$, $D_1 > 0$, $D_2 > 0$ are arbitrarily chosen constants. Making use of equation (C11), one obtains

$$\begin{aligned} \dot{V}_{III} = \sum_{\substack{i=1 \\ (i \neq j)}}^{n} \sum_{j-1}^{n} \Big[&-(D + D_1)\gamma_i^2 + (q_{ii} w_{ij} + q_{jj} w_{ji}) \gamma_i \gamma_J \\ &- (D + D_2) \gamma_j^2 \Big] \end{aligned} \tag{C12}$$

or

$$\begin{aligned} \dot{V}_{III} = &-D \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\gamma_i^2 + \gamma_j^2\right) \\ &- \sum_{\substack{i=1 \\ (i \neq j)}}^{n} \sum_{j=1}^{n} \left[D_1 \gamma_i^2 - (q_{ii} w_{ij} + q_{jj} w_{ji}) \gamma_i \gamma_j + D_2 \gamma_j^2\right] \end{aligned} \tag{C13}$$

Choosing the condition

$$\frac{(q_{ii} w_{ij} + q_{jj} w_{ji})}{2} = \sqrt{D_1 D_2} \tag{C14}$$

one obtains for $\dot{V}_{III}$, the following value:

$$\dot{V}_{III} = -D \sum_{i=1}^{n} \sum_{j=1}^{n} (\gamma_i^2 + \gamma_j^2) - \sum_{\substack{i=1 \\ (i \neq j)}}^{n} \sum_{j=i}^{n} \left[\sqrt{D_1}\,\gamma_i - \sqrt{D_2}\,\gamma_j\right]^2 \tag{C15}$$

By comparing equation (C15) with equation (C9), we can easily see that

$$\dot{V}_{III} \leq -D \sum_{i=1}^{n} \sum_{j=1}^{n} \left[\gamma_i^2 + \gamma_j^2\right] \tag{C16a}$$

and, therefore,

$$\dot{V}_{III} \leq \dot{V}_{II} < 0 \tag{C16b}$$

for $\forall \underline{\gamma}(t)$ and $\forall t$.

From equation (A11) one has

$$\sqrt{q_{ii}\, w_{ii}\, q_{jj}\, w_{jj}} = \sqrt{(D + D_1)(D + D_2)} > \sqrt{D_1 D_2} \tag{C17}$$

and, therefore, from equation (C14),

$$\sqrt{q_{ii}\, w_{ii}\, q_{jj}\, w_{jj}} > \frac{(q_{ii}\, w_{ij} + q_{jj}\, w_{ji})}{2} \tag{C18}$$

for $i = 1,2,\ldots, n$; $j = 1,2,\ldots, n$ $(i \neq j)$. Summing up, the conditions for ASIL, formerly established, can be enunciated by the following theorem.

Theorem: For the time-varying system described by equations (63) and (64) to be asymptotically stable in the large, about the singular stable point $\underline{\gamma} = 0$, the following conditions are to be satisfied:

$$V > 0 \quad , \forall \underline{\gamma} \neq 0 \quad , \forall t \tag{C19a}$$

$$q_{ii}\, w_{ii} \leq -D < 0 \qquad i = 1,2,\ldots, n \tag{C19b}$$

$$q_{jj}\, w_{jj} \leq -D < 0 \qquad j = 1,2,\ldots, n \tag{C19c}$$

$$\sqrt{q_{ii}\, w_{ii}\, q_{jj}\, w_{jj}} \geq \frac{(q_{ii}\, w_{ij} + q_{jj}\, w_{ji})}{2} \tag{C19d}$$

$i = 1,2,\ldots, n$

$j = 1,2,\ldots, n$ ($1 \neq j$ in the cross-terms)

If the conditions of the theorem are satisfied, it is guaranteed that the time derivative of the Lyapunov function will be negative-definite everywhere in the 2n-dimensional vector space E^{2n} spanned by $\underline{\gamma}$, that is,

$$\dot{V}_{III} < 0 \tag{C20}$$

for $\forall \underline{\gamma}(t)$ and $\forall t$, the function $V(\gamma)$ being therefore an admissible Lyapunov function for the system in equation (62).

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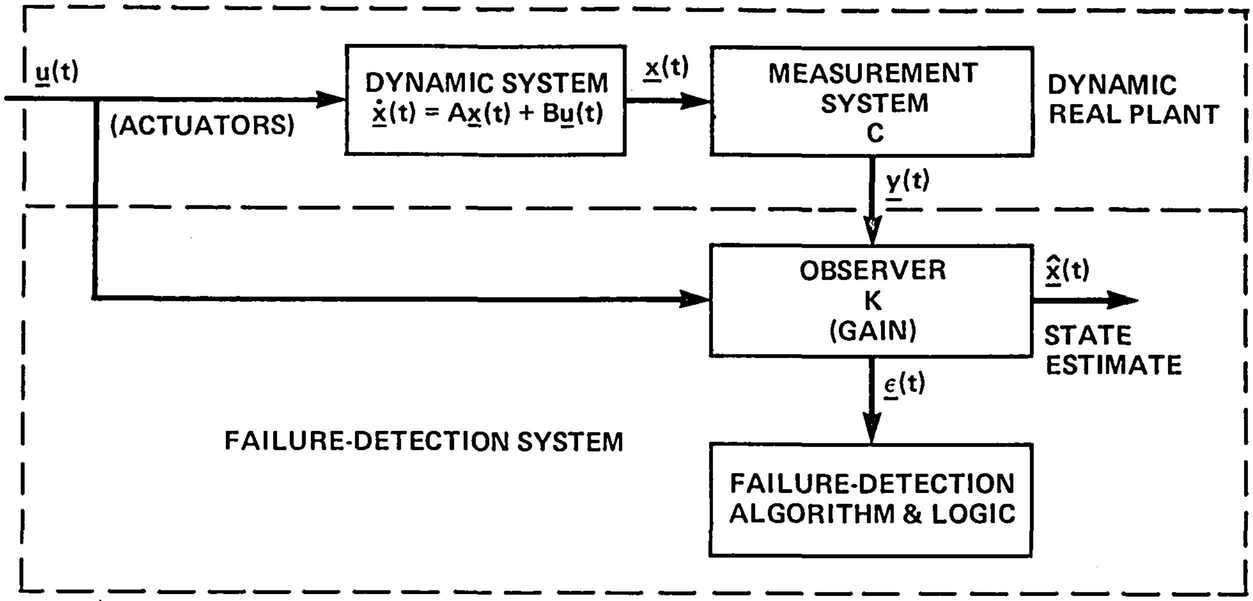

Figure 1.-Schematic block diagram of failure-detection system, including an observer.

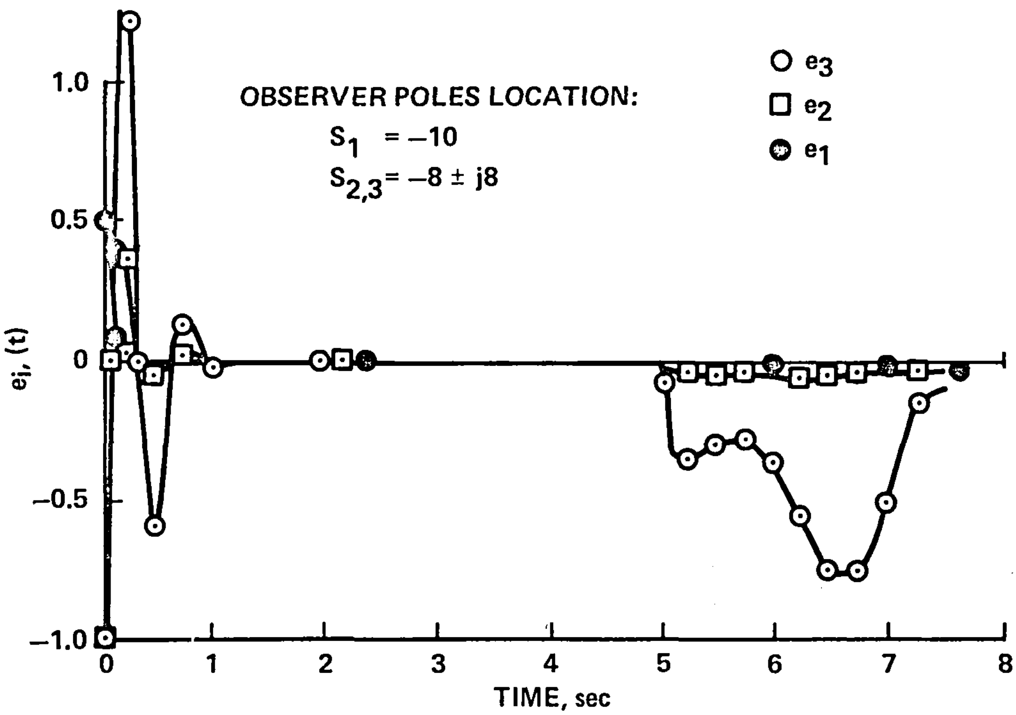

Figure 2.- Observer errors (residuals) for a third-order system, with actuator failure at T_f = 5 sec.

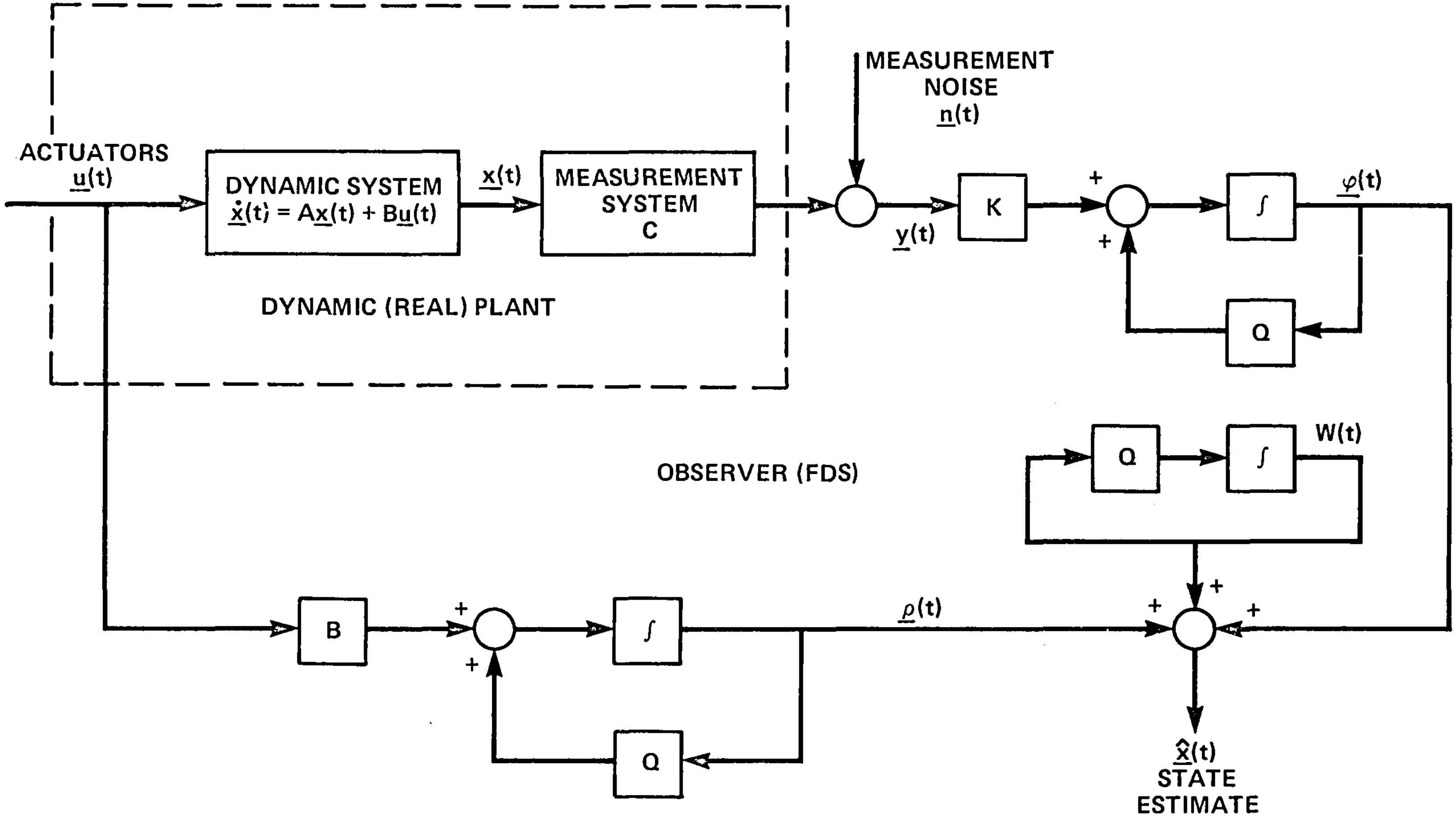

Figure 3.- An alternative observer implementation for FDS.

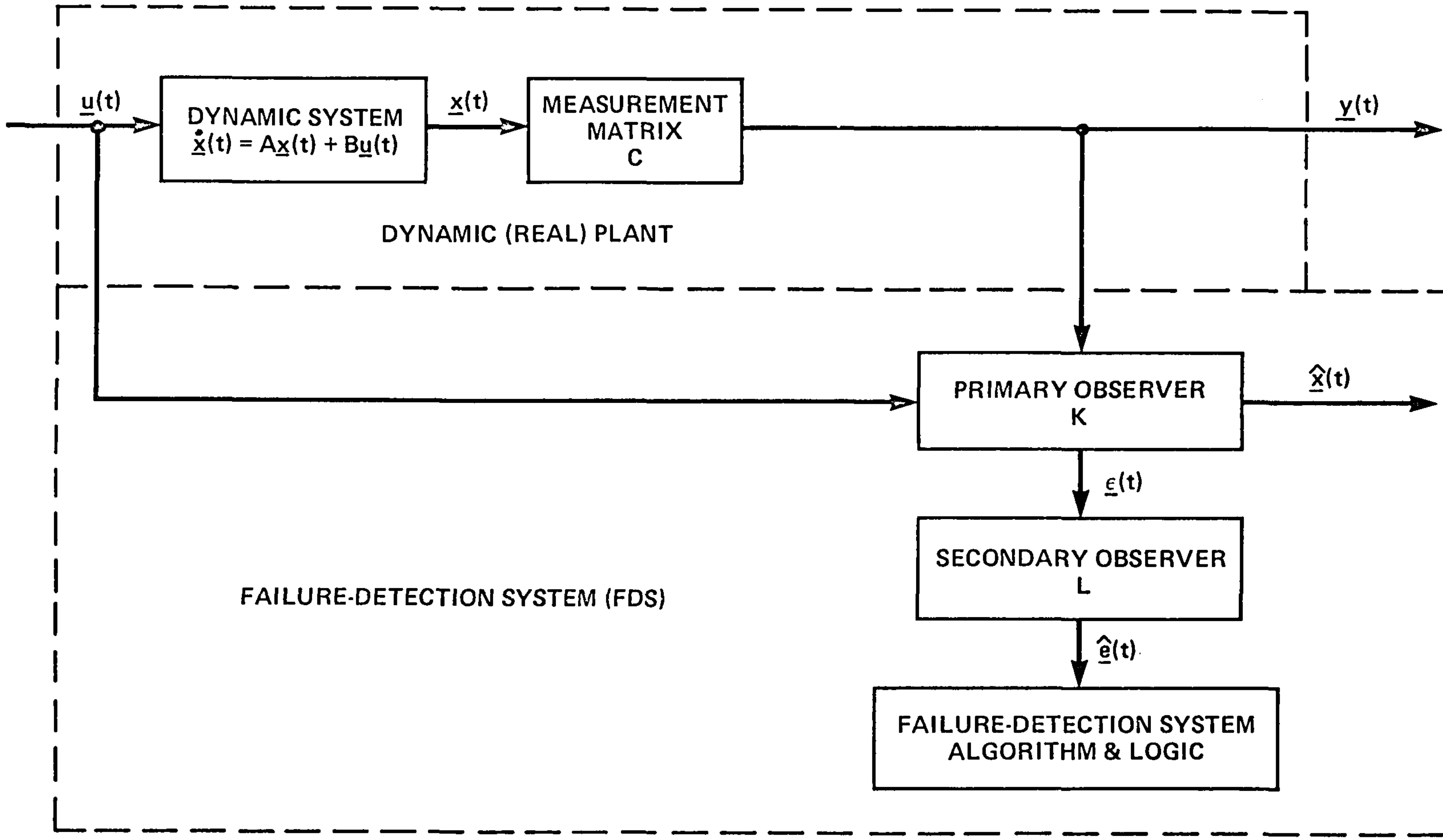

Figure 4.- FDS with primary and secondary observers.

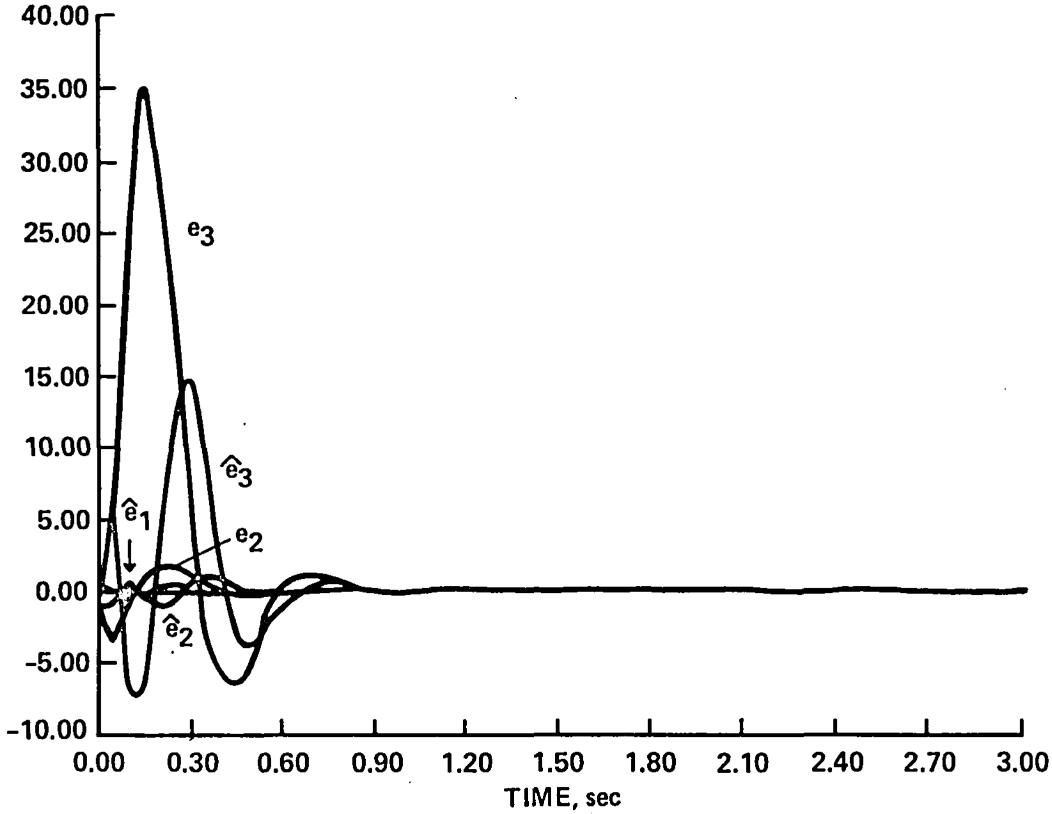

Figure 5.- First observer error functions vs second observer output functions.

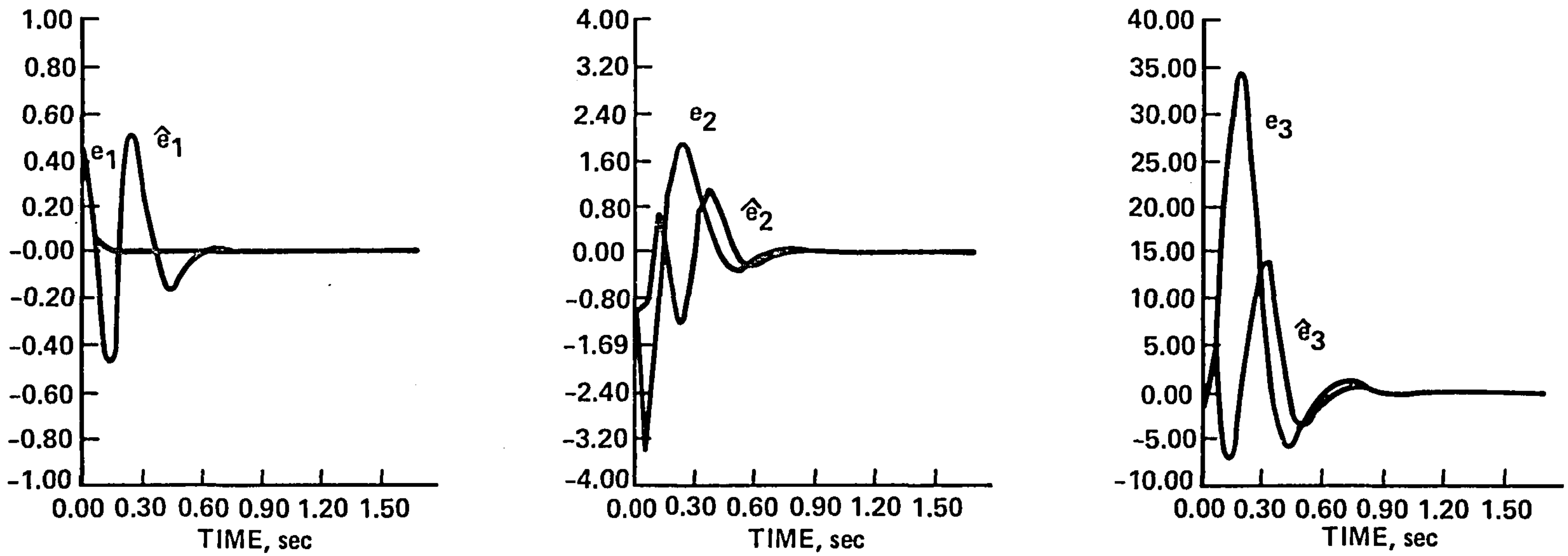

Figure 6.- Detailed plot of the first observer error functions vs second observer output functions.

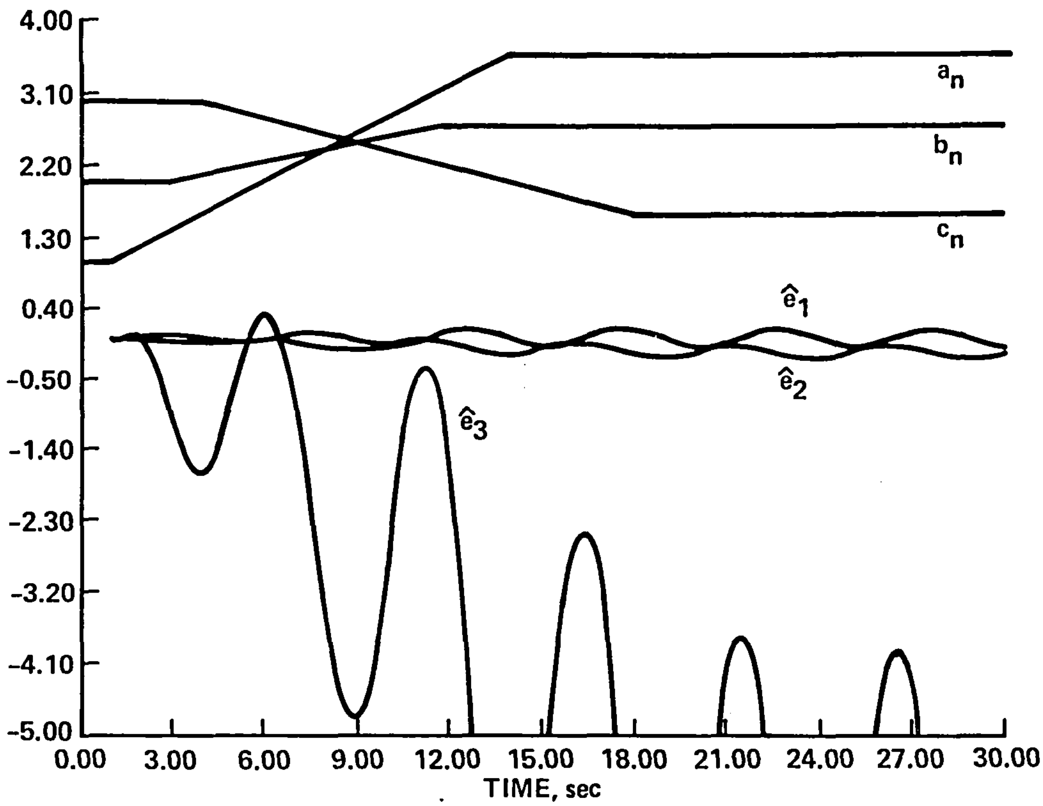

Figure 7.- The impact of plant parameter changes on $\underline{\hat{e}}(t)$.

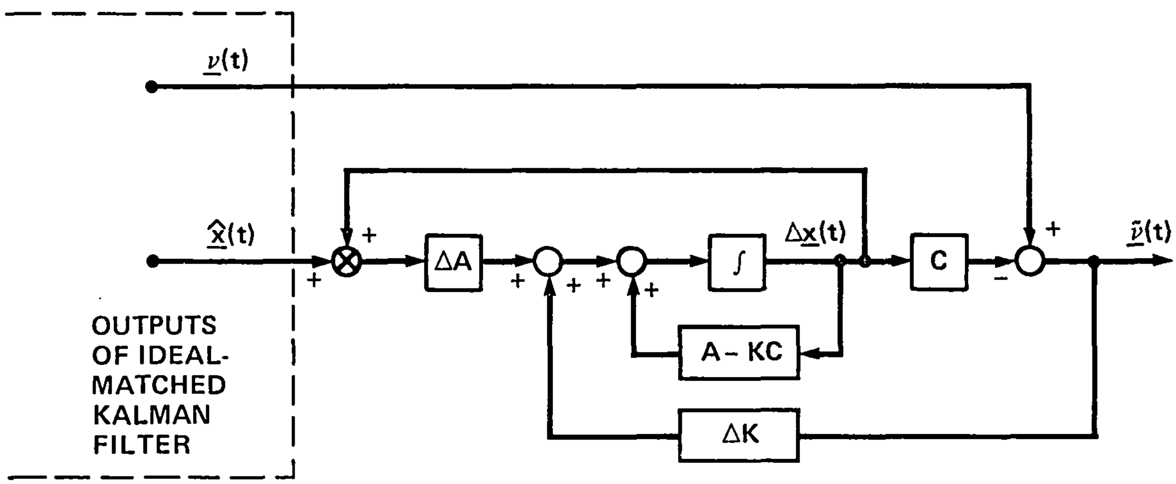

Figure 8.- The modeling of plant - KF "mismatching" effects on the innovation stochastic process.

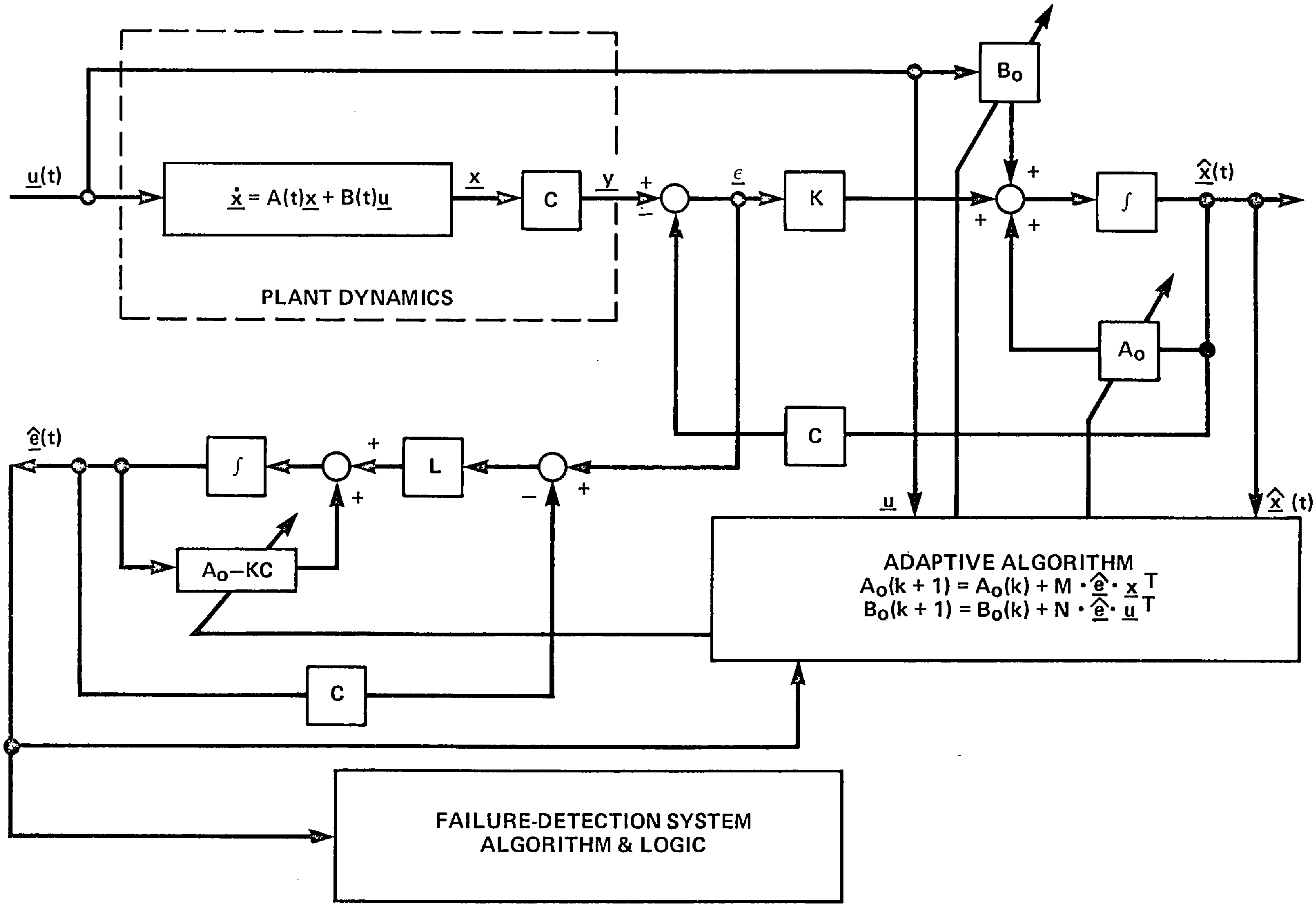

Figure 9.- Failure-detection system with adaptive primary/secondary observers.

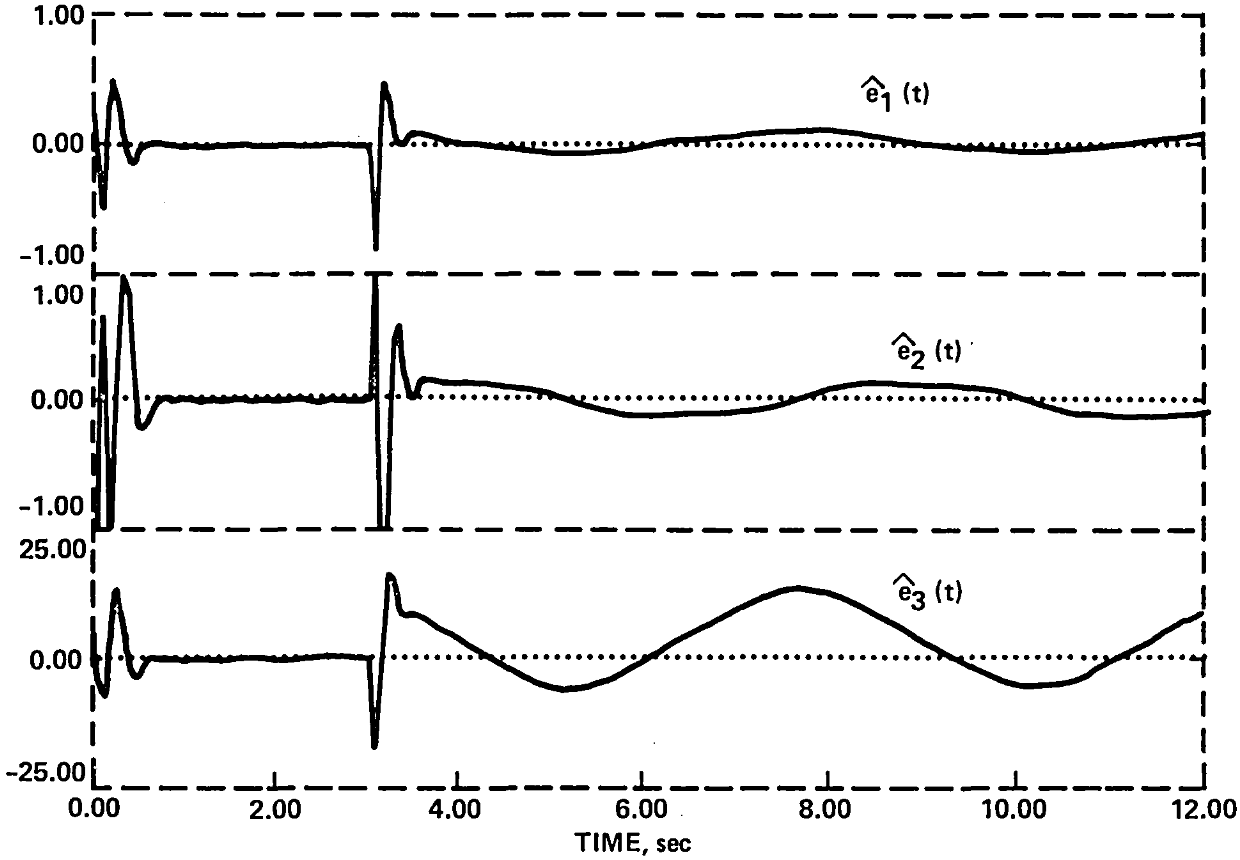

Figure 10.- Second observer output due to sensor No. 2 failure at $T_f = 3$ sec.

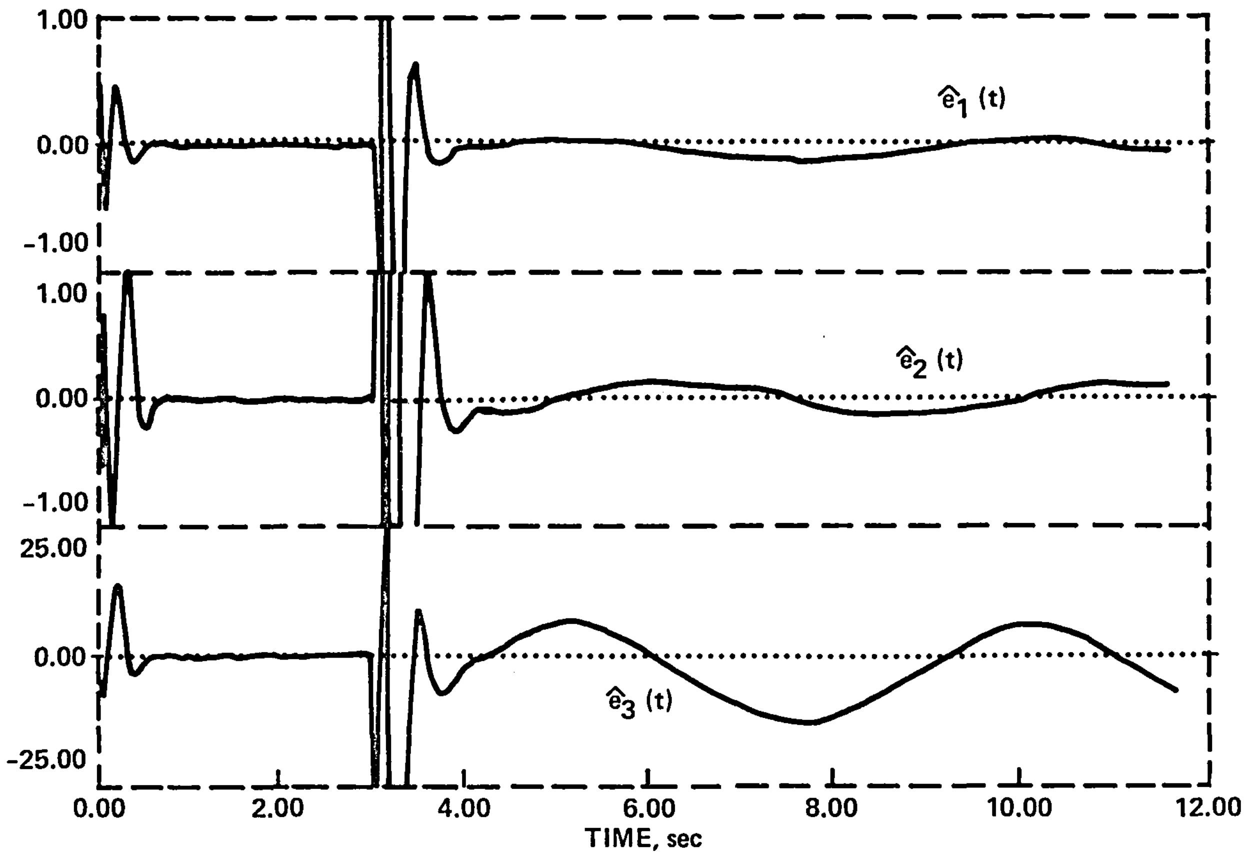

Figure 11.- Second observer output due to sensor No. 1 failure at $T_f = 3$ sec.

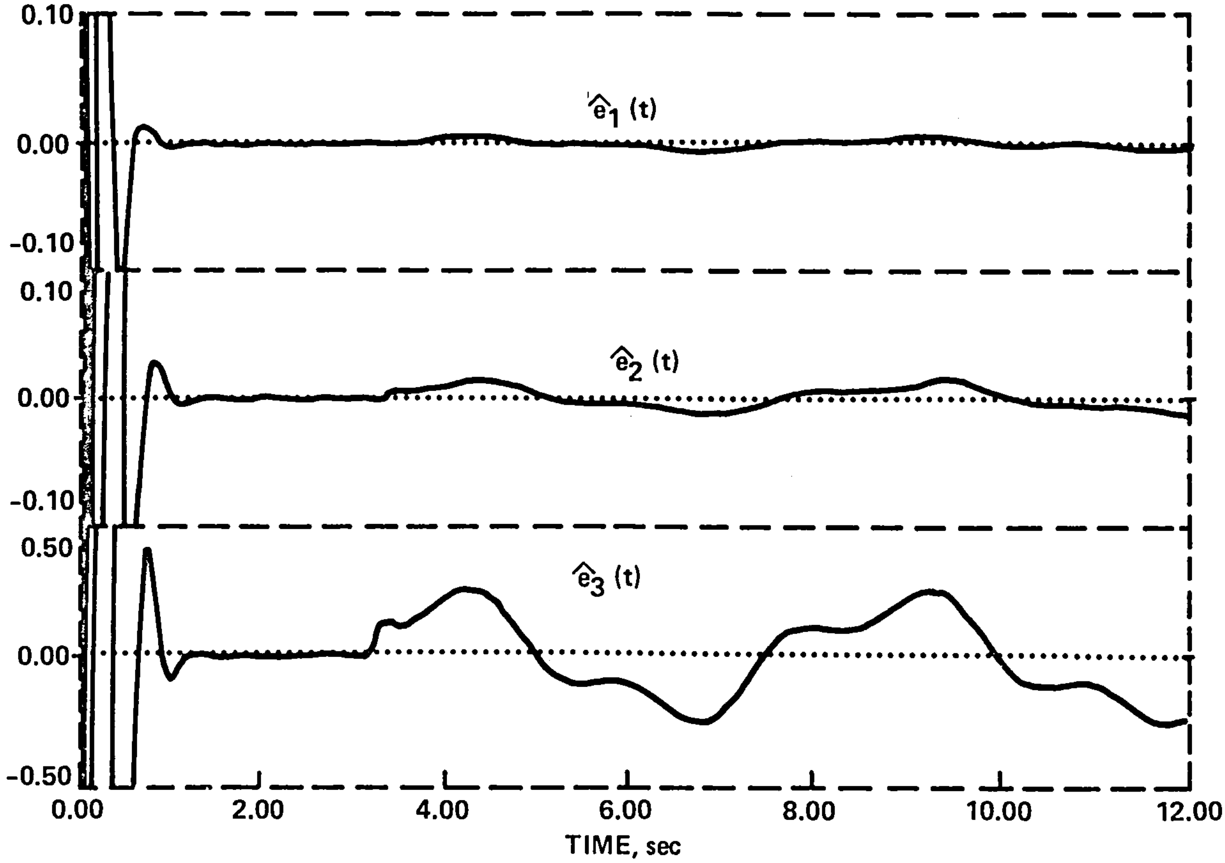

Figure 12.- The effect of actuator failure at T_f = 3 sec on $\underline{\hat{e}}(t)$.

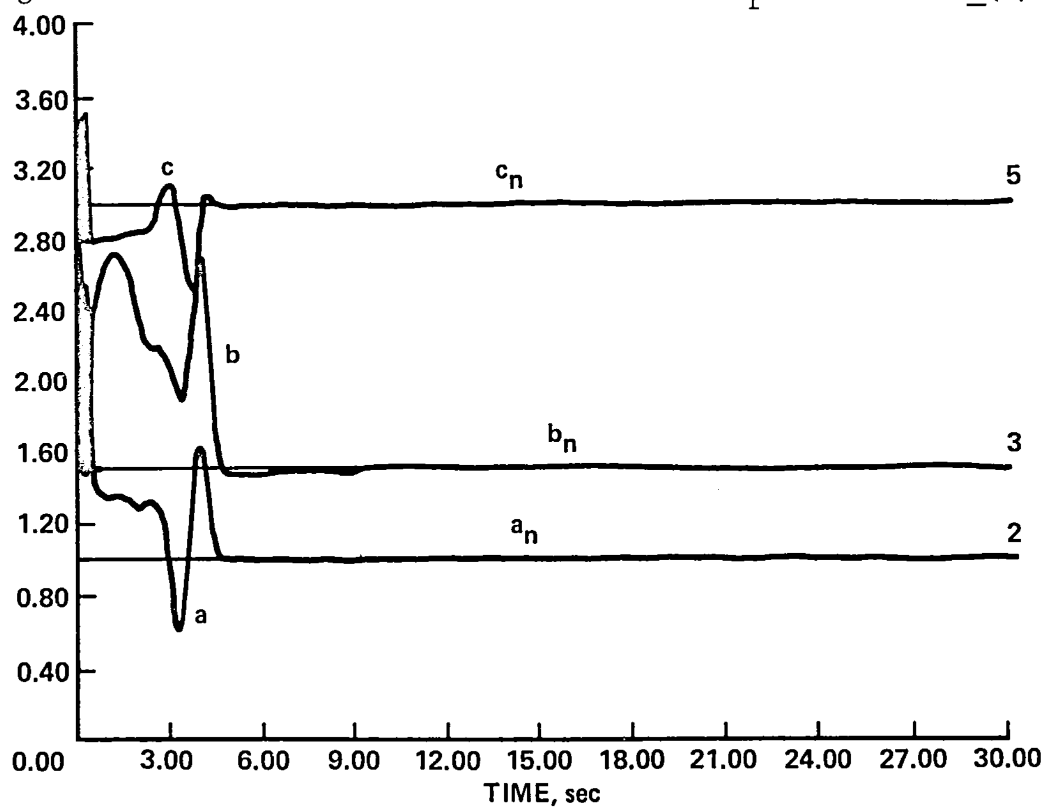

Figure 13.- Adaptation process and identification of observer parameters.

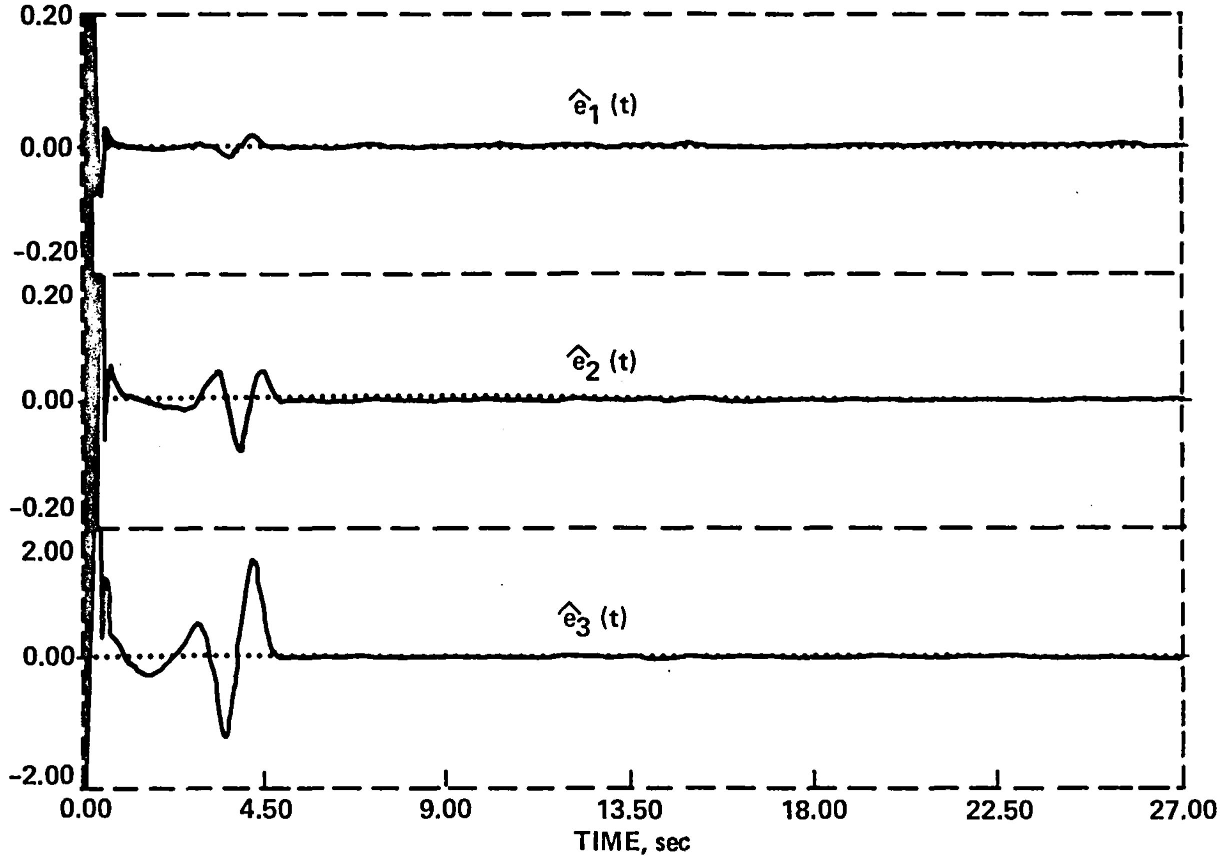

Figure 14.- The components of the vector $\underline{\hat{e}}(t)$ during and after the adaptation process.

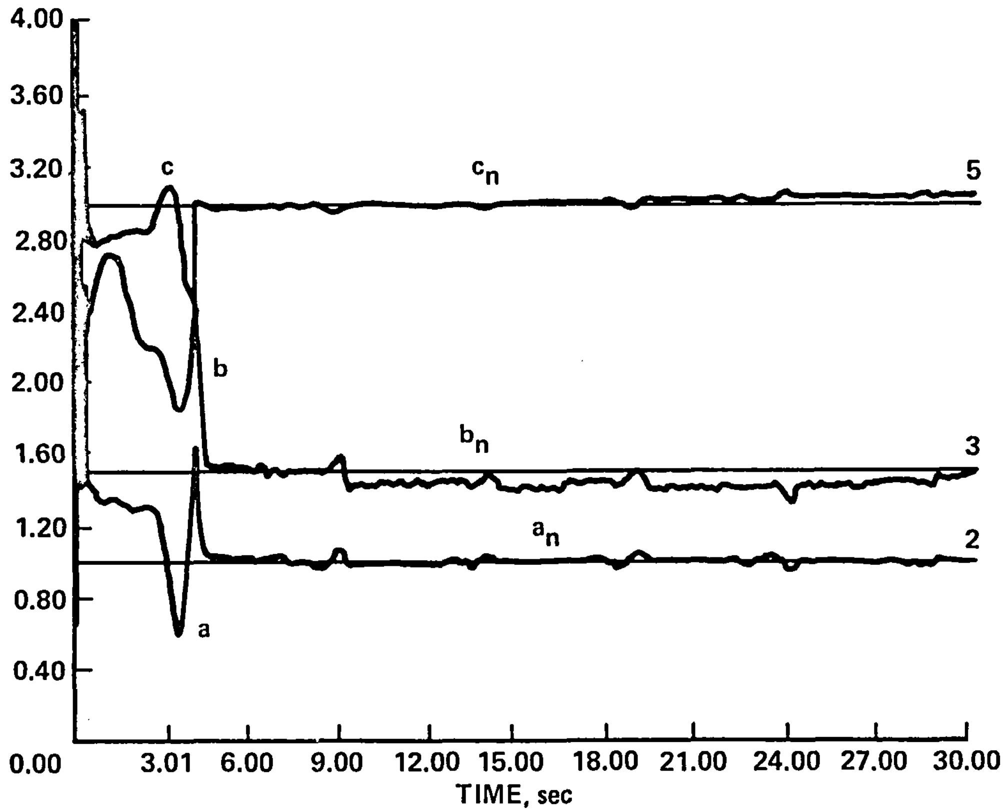

Figure 15.- Adaptation process and identification of observer parameters, with measurement noise.

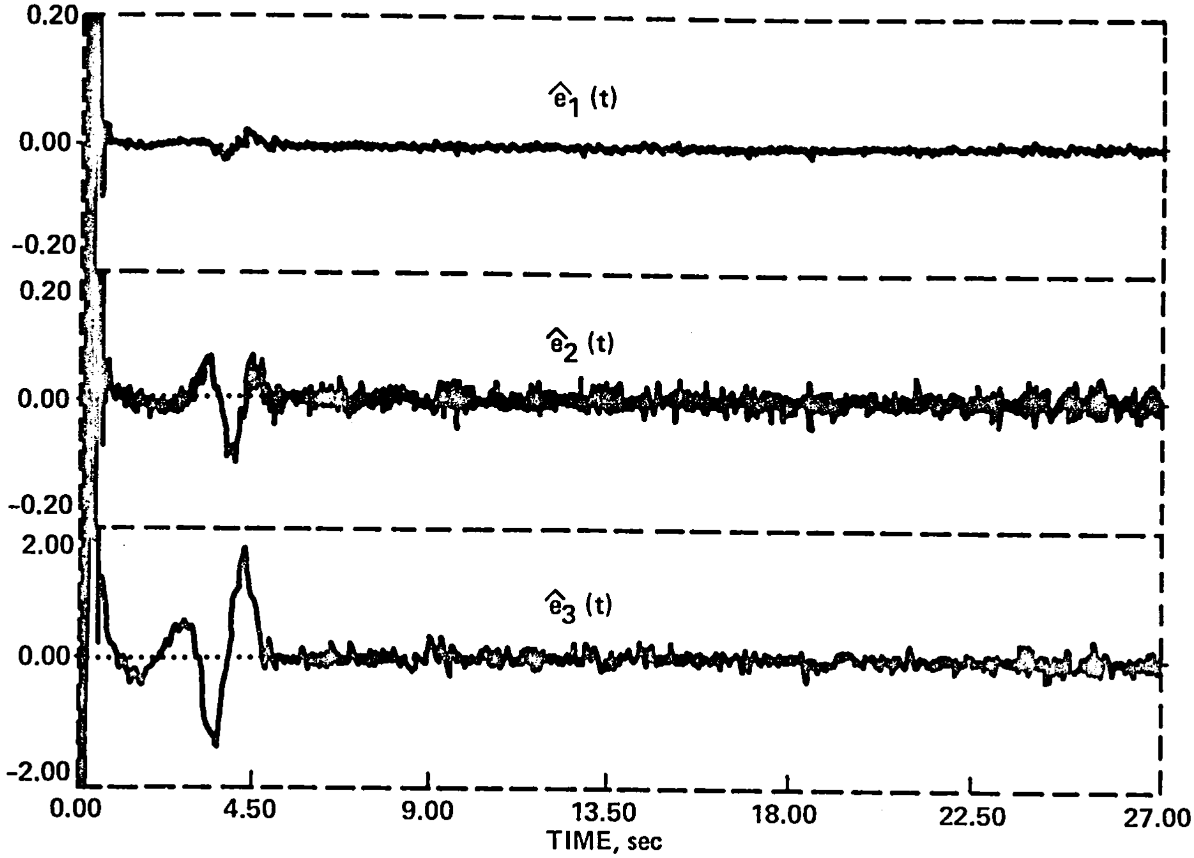

Figure 16.- The components of the vector $\underline{\hat{e}}(t)$ during and after adaptation process, with noisy measurements.

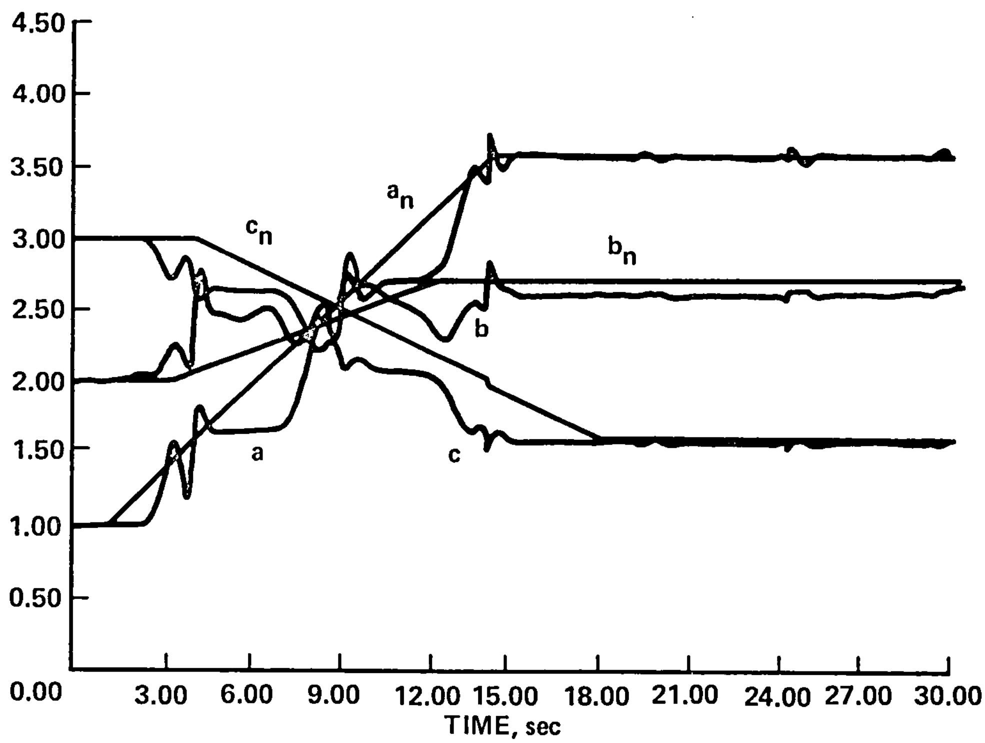

Figure 17.- The adaptation/tracking process of the adaptive primary/secondary observer pair.

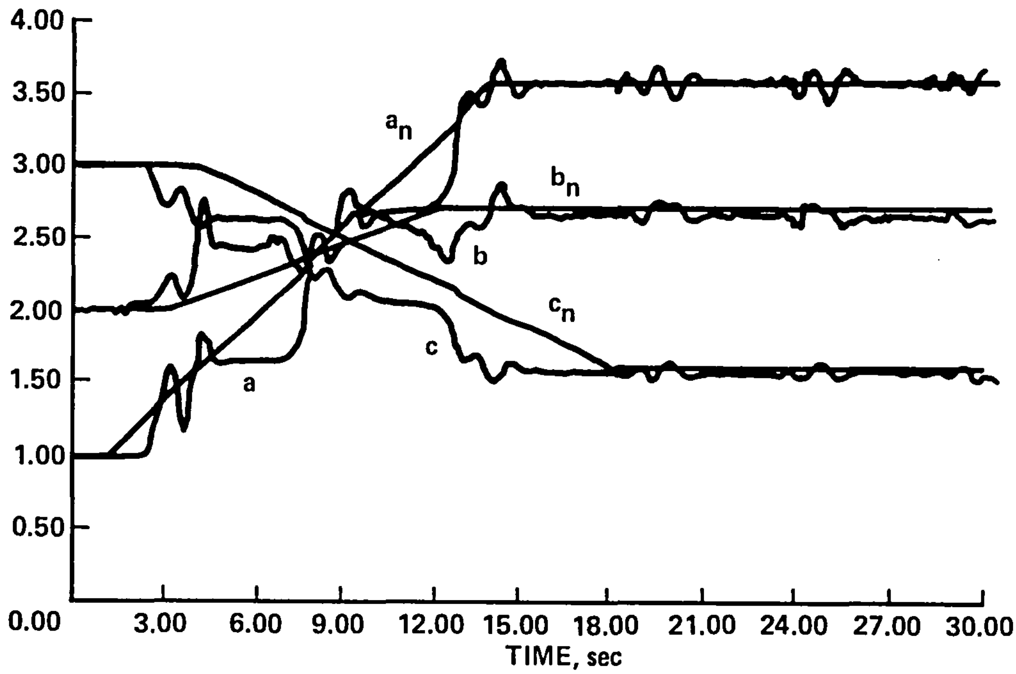

Figure 18.- The adaptation/tracking process of the adaptive primary/secondary observer with noisy measurements.

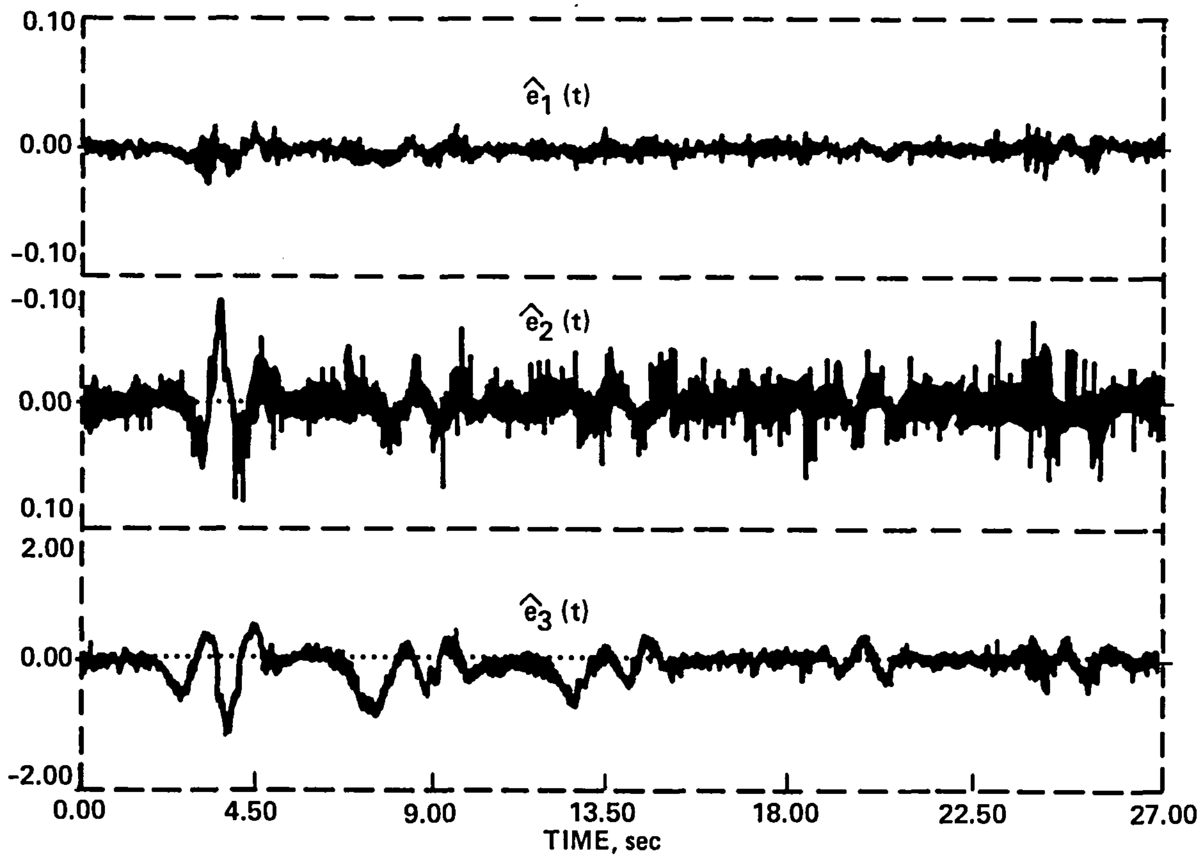

Figure 19.- The second observer output vector during adaptation, with noisy measurements.

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16. Abstract

The problem of designing analytical failure-detection systems (FDS) for sensors and actuators, using observers, is addressed. These failure-detection systems can be applied to linear, constant, and possibly time-varying, multi-input, multi-output systems with measurement noise. The use of observers in FDS is related to the examination of the n-dimensional observer error vector which carries the necessary information on possible failures. The problem is that in practical systems, in which only some of the components of the state vector are measured, one has access only to the m-dimensional observer-output error vector, with $m \leq n$. In order to cope with these cases, a secondary observer is synthesized to reconstruct the entire observer-error vector from the observer output error vector. This approach leads toward the design of highly sensitive and reliable FDS, with the possibility of obtaining a unique fingerprint for every possible failure (abrupt or soft). The use of the secondary observers allows us also to solve the measurement noise problem in a very efficient way. Further, in order to keep the observer's (or Kalman filter) false-alarm rate (FAR) under a certain specified value, it is necessary to have an acceptable matching between the observer (or Kalman filter) models and the system parameters. Only properly designed adaptive observers are able to detect abrupt changes in the system (actuator, sensor failures, etc.) with adequate reliability and FAR. A previously developed adaptive observer algorithm is used here to maintain the desired system-observer model matching, despite initial mismatching or system parameter variations. Conditions for convergence for the adaptive process are obtained, leading to a simple adaptive law (algorithm) with the possibility of an a priori choice of fixed adaptive gains. Simulation results show good tracking performance with small observer output errors, while accurate and fast parameter identification, in both deterministic and stochastic cases, is obtained.

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