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INCORPORATING PARTIALLY IDENTIFIED SAMPLE SEGMENTS INTO ACREAGE ESTIMATION PROCEDURES: ESTIMATES USING ONLY OBSERVATIONS FROM THE CURRENT YEAR

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16. Abstract

For the purposes of area estimation a segment is said to be completely identified if the acreage of each crop of interest can be directly estimated. A segment is said to be only partially identified if some individual crops within a family of crops cannot be differentiated and their acreages individually estimated. For example, if it estimated that 30% of a segment is spring small grains but no further differentiation into say spring wheat and other small grains is made, then the segment is only partially identified. Several methods of estimating individual crop acreages using a mixture of completely identified and partially identified segments from a single growing year are derived and discussed. A small Monte Carlo study of eight estimators is presented. The relative empirical behavior of these estimators is discussed as are the effects of segment sample size and amount of partial identification.

The principle recommendations are (1) to not exclude, but rather incorporate partially identified sample segments into the estimation procedure, (2) try to avoid having a large percentage (say 80%) of only partially identified segments, in the sample, and (3) use the maximum likelihood estimator although the weighted least squares estimator and least squares ratio estimator both perform almost as well.

In this report the family of crops which cannot always be completely identified is assumed to contain a particular crop of interest and the rest of the family is essentially considered collectively as simply a second crop. For example, the family of spring small grains might be subdivided into two crops, say barley and "other spring small grains."

TECHNICAL REPORT 21

Incorporating Partially Identified Sample Segments

Into Acreage Estimation Procedures: Estimates Using

Only Observations From The Current Year

by

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For the purposes of area estimation a segment is said to be completely identified if the acreage of each crop of interest can be directly estimated. A segment is said to be only partially identified if some individual crops within a family of crops cannot be differentiated and their acreages individually estimated. For example, if it estimated that 30% of a segment is spring small grains but no further differentiation into say spring wheat and other spring small grains is made, then the segment is only partially identified. Several methods of estimating individual crop acreages using a mixture of completely identified and partially identified segments from a single growing year are derived and discussed. A small Monte Carlo study of eight estimators is presented. The relative empirical behavior of these estimators is discussed as are the effects of segment sample size and amount of partial identification.

The principle recommendations are (1) to not exclude, but rather incorporate partially identifed sample segments into the estimation procedure, (2) try to avoid having a very large percentage (say 80%) of only partially identified segments in the sample, and (3) use the maximum likelihood estimator although the weighted least squares estimator and least squares ratio estimator both perform almost as well.

In this report the family of crops which cannot always be completely identified is assumed to contain a particular crop of interest and the rest of the family is essentially considered collectively as simply a second crop. For example, the family of spring small grains might be subdivided into two crops, say barley and "other spring small grains".

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1. Introduction

A sample of segments within a stratum is selected. Each segment is observed repeatedly through the growing seasons. The objective is to estimate for each crop of interest the proportion of the stratum's acreage corresponding to that crop's harvested acreage.

In this report, we shall assume that there are only two crops of interest. Furthermore, we shall assume that only data from the current growing year are to be used in estimating the crop at harvest proportions. The cases corresponding to more than two crops of interest and data available from more than one growing year will be considered in subsequent reports.

The sample segments are all assumed to be of the same size. No assumption is made about the stratum size or the segment size. The sampled segments are assumed to represent a random sample (without replacement) from the segments in the stratum.

Each sample segment is assumed to have been observed at least once during the growing year and possibly several times. The two crops of interest will be designated as crop A and crop B. When a sample segment is observed, the observation can have the form

$$(p_A, p_B, p_{other})$$

where

p_A = the estimated proportion of the segment which will be harvested in crop A,

p_B = the estimated proportion of the segment which will be harvested in crop B, and

$p_{other} = 1 - p_A - p_B$ = the estimated proportion of the segment which will not be harvested in either A or B.

Alternatively, estimates may not be made on A and B separately but only on A and B collectively, so that the observation can have the form

$$(p_{A+B}, \ p_{other})$$

where

p_{A+B} = the estimated proportion of the segment that will be harvested in either A or B, and

p_{other} = $1 - p_{A+B}$ = the estimated proportion of the segment that will not be harvested in A or B.

These estimates are assumed to reflect any previous observations made on this sample segment during the current growing year. If a sample segment's current observation is of the form

$$(p_{A+B}, \ p_{other}),$$

then the sample segment will be called partially identified. If its observation is of the form

$$(p_A, \ p_B, \ p_{other}),$$

then it will be called completely identified.

The proportion of the stratum harvested in crop A will be denoted P_A with P_B similarly defined. The objective is to estimate P_A and P_B using the observations on the sample segments. This estimation may have to be made at more than one time during the growing year. Of course, if there are no completely identified sampled segments, only the sum $P_A + P_B$ can be estimated on the basis of the sample segments.

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2.   Estimation Procedures

There are undoubtedly several decision makers within a stratum who collectively determine the proportional acreages (P_A, P_B, P_{other}) for the stratum. The exact decision making process is unknown. Hence, certain assumptions will be made about the outcome of this decision making process, and these assumptions will suggest a few reasonable estimation procedures for consideration. The assumptions are at best approximations and are no doubt technically not satisfied. The only role of these assumptions is to suggest estimation procedures. The worth of these procedures will not be determined relative to the underlying assumptions which suggested them, but rather determined by their behavior on real sample data.

The number of acres harvested in a segment, say $(X_A, X_B, N - X_A - X_B)$, will be assumed to follow a multinomial distribution with

$$P(X_A = x_A, X_B = x_B)$$

$$= \frac{N}{x_A! \, x_B! \, (N - x_A - x_B)!} \; P_A^{x_A} P_B^{x_B} (1 - P_A - P_B)^{N-x_A-x_B} \quad . \qquad (1)$$

This assumption would be technically correct if every decision maker acted independently and for each unit of acreage chose crop A, crop B, or other with probabilities P_A, P_B, and P_{other} respectively. The CAMS estimated segment proportions (p_A, p_B, p_{other}) are assumed to correspond to

$$\frac{X_A}{N} \, , \; \frac{X_B}{N}, \; \frac{N - X_A - X_B}{N} \quad .$$

The following notation will be used throughout this paper. The total number of segments sampled will be denoted by

$$n = \text{segment sample size} \qquad (2)$$

The sample segments which have been completely identified will be denoted by

$$(p_{Ai}, \, p_{Bi}, \, 1-p_{Ai} - p_{Bi}) \qquad i=1, 2, \ldots, I. \qquad (3)$$

Similarly, the sample segments which have only been partially identified will be

denoted by

$$[(p_A + p_B)_j, \quad 1 - (p_A - p_B)_j], \quad j=1,2, \ldots, J. \tag{4}$$

Hence

$$n = I + J. \tag{5}$$

Since no separate estimate of P_A and P_B could be made solely on the basis of the sampled segments unless some sampled segments had been completely identified, it will be assumed that

$$I > 0 \tag{6}$$

2.1 Maximum Likelihood Estimates

Under the assumed multinomial distribution (1) the joint likelihood for the CAMS estimates is

$$L = \left\{ \prod_{i=1}^{I} \frac{N!}{X_{Ai}! \, X_{Bi}! \, (N - X_{Ai} - X_{Bi})!} \, P_A^{X_{Ai}} \, P_B^{X_{Ai}} \, (1 - P_A - P_B)^{N - X_{Ai} - X_{Bi}} \right\}$$

$$\cdot \left\{ \prod_{j=1}^{J} \frac{N!}{(X_A + X_B)! \, [N-(X_A + X_B)_j]!} \right.$$

$$\left. (P_A + P_B)^{(X_A + X_B)} \, [1-(P_A + P_B)]^{N-(X_A + X_B)_j} \right\} \tag{7}$$

or, equivalently,

$$L = \left\{ \prod_{i=1}^{I} \frac{N!}{(Np_{Ai})! \, (Np_{Bi})! \, (N - Np_{Ai} - Np_{Bi})!} \right.$$

$$\left. P_A^{Np_{Ai}} \, P_B^{Np_{Bi}} \, (1-P_A-P_B)^{N-Np_{Ai}-Np_{Bi}} \right\}$$

$$\left\{ \prod_{j=1}^{J} \frac{N!}{[N(p_A + p_B)_j]! \, (N[1 - (p_A + p_B)_j])!} \right.$$

$$\left. (P_A + P_B)^{N(p_A + p_B)_j} \, (1 - P_A - P_B)^{N-N(p_A + p_B)_j} \right\} \tag{8}$$

The corresponding maximum likelihood estimates are

$$\hat{P}_A = \frac{\sum\limits_{i=1}^{I} p_{Ai}}{\sum\limits_{i=1}^{I} p_{Ai} + \sum\limits_{i=1}^{I} p_{Bi}} \left\{ \frac{\sum\limits_{i=1}^{I} p_{Ai} + \sum\limits_{i=1}^{I} p_{Bi} + \sum\limits_{j=1}^{J} (p_A + p_B)_j}{I + J} \right\}, \tag{9}$$

$$\hat{P}_B = \frac{\sum\limits_{i=1}^{I} p_{Bi}}{\sum\limits_{i=1}^{I} p_{Ai} + \sum\limits_{i=1}^{I} p_{Bi}} \left\{ \frac{\sum\limits_{i=1}^{I} p_{Ai} + \sum\limits_{i=1}^{I} p_{Bi} + \sum\limits_{j=1}^{J} (p_A + p_B)_j}{I + J} \right\}, \tag{10}$$

and

$$(1 - \hat{P}_A - \hat{P}_B) = 1 - \hat{P}_A - \hat{P}_B . \tag{11}$$

The form of these estimators is really quite intuitive. For example,

$$P_A = \left\{ \begin{array}{l} \text{Estimated Proportion} \\ \text{of Crops A and B} \\ \text{that is Crop A.} \end{array} \right\} \cdot \left\{ \begin{array}{l} \text{Estimated Proportion} \\ \text{of the Stratum that} \\ \text{is Crop A or B} \end{array} \right\} \cdot \tag{12}$$

2.2 Least Squares Estimates

If $(\hat{P}_A, \hat{P}_B, 1 - \hat{P}_A - \hat{P}_B)$ is chosen to minimize

$$\sum\limits_{i=1}^{I} (p_{Ai} - P_A)^2 + \sum\limits_{i=1}^{I} (p_{Bi} - P_B)^2 + \sum\limits_{i=1}^{I} [(1 - p_{Ai} - p_{Bi}) - (1 - P_A - P_B)]^2 +$$

$$\sum\limits_{i=1}^{J} [(p_A + p_B)_j - (P_A + P_B)]^2 + \sum\limits_{j=1}^{J} [1 - (p_A + p_B)_j] - [1 - (P_A + P_B)]^2 \tag{13}$$

with respect to P_A and P_B, then $(\hat{P}_A, \hat{P}_B, 1 - \hat{P}_A - \hat{P}_B)$, would be the least squares estimate of $(P_A, P_B, 1 - P_A - P_B)$. The resulting least squares estimates are

$$\hat{P}_A = \left\{ 2\sum_{i=1}^{I} P_{Ai} + \sum_{i=1}^{I} P_{Bi} + 2\sum_{j=1}^{J} (p_A + p_B)_j - \hat{P}_B(I + 2J) \right\} / 2(I + J) \qquad (14)$$

and

$$\hat{P}_B = \left\{ (3I + 2J)\sum_{i=1}^{I} P_{Bi} - 2J\sum_{i=1}^{I} P_{Ai} + 2I\sum_{j=1}^{J} (p_A + p_B)_j \right\} / I(3I + 4J). \qquad (14)$$

2.3 Weighted Least Squares Estimates

Since one may wish to give more weight to more precise (less variable) estimates than to less precise (more variable) estimates, a weighted least squares estimate of $(P_A, P_B, 1 - P_A - P_B)$ may be preferable. The weighted least squares estimate minimizes

$$\sum_{i=1}^{I} W_{Ai} (p_{Ai} - P_A)^2 + \sum_{i=1}^{I} W_{Bi}(p_{Bi} - P_B)^2 +$$

$$\sum_{i=1}^{I} W_{A+B,i} [(1 - p_{Ai} - p_{Bi}) - (1 - P_A - P_B)]^2 +$$

$$\sum_{j=1}^{J} W_{A+B,j} [(p_A + p_B)_j - (P_A + P_B)]^2 + \sum_{j=1}^{J} W_{A+B,j} \left\{ [1 - (p_A + p_B)_j] - \right.$$

$$\left. [1-(P_A + P_B)]^2 \right\} \qquad (15)$$

with respect to P_A and P_B. The corresponding weighted least squares estimates would be

$$\hat{P}_A = \left\{ \sum_{i=1}^{I} W_{Ai} P_{Ai} + \sum_{i=1}^{I} W_{A+B,i} (p_{Ai} + p_{Bi}) + 2\sum_{j=1}^{J} W_{A+B,j} (p_A + p_B)_j - \right.$$

$$\left. \hat{P}_B \left(\sum_{i=1}^{I} W_{A+B,i} + 2\sum_{j=1}^{J} W_{A+B,j} \right) \right\} / \left\{ \sum_{i=1}^{I} W_{Ai} + \sum_{i=1}^{I} W_{A+B,i} + 2\sum_{j=1}^{J} W_{A+B,j} \right\}$$

and

$$\hat{P}_B = \left\{ \left[\sum_{i=1}^{I} W_{Bi} P_{Bi} + \sum_{i=1}^{I} W_{A+B,i}(P_{Ai} + P_{Bi}) + 2 \sum_{j=1}^{J} W_{A+B,j}(P_A + P_B)_j \right] \cdot \right.$$

$$\left[\sum_{i=1}^{I} W_{Ai} + \sum_{i=1}^{I} W_{A+B,i} + 2 \sum_{j=1}^{J} W_{A+B,j} \right] -$$

$$\left[\sum_{i=1}^{I} W_{Ai} P_{Ai} + \sum_{i=1}^{I} W_{A+B,i}(P_{Ai} + P_{Bi}) + 2 \sum_{j=1}^{J} W_{A+Bj}(P_A + P_B)_j \right] \cdot$$

$$\left. \left[\sum_{i=1}^{I} W_{A+Bji} + 2 \sum_{j=1}^{J} W_{A+B,j} \right] \right\} / \left\{ \left[\sum_{i=1}^{I} W_{Ai} + \sum_{i=1}^{I} W_{A+B,i} + 2 \sum_{j=1}^{J} W_{A+B,j} \right] \cdot \right.$$

$$\left. \left[\sum_{i=1}^{I} W_{Bi} + \sum_{i=1}^{I} W_{A+B,i} + 2 \sum_{j=1}^{J} W_{A+B,j} \right] - \left[\sum_{i=1}^{I} W_{A+B,i} + 2 \sum_{j=1}^{J} W_{A+B,j} \right]^2 \right\} . \qquad (16)$$

If the weights, W, are all taken to be one, then the weighted least squares estimates (16) are equal to the least squares estimates (14). A reasonable alternative to equal weights, say W's = 1, is to weight each squared difference inversely in proportion to the approximate variance of the CAMS estimate it contains. Specifically,

$$W_{Ai} = 1/\hat{P}_A(1 - \hat{P}_A) \qquad i=1,2,\ldots,I,$$

$$W_{Bi} = 1/\hat{P}_B(1 - \hat{P}_B) , \qquad i=1,2,\ldots,I,$$

$$W_{A+B,i} = 1/(1 - \hat{P}_A - \hat{P}_B)_i(\hat{P}_A + \hat{P}_B)_i \qquad , \quad i=1,2,\ldots I,$$

$$W_{A+B,j} = 1/(\hat{P}_A + \hat{P}_B)_j(1 - \hat{P}_A - \hat{P}_B)_j \qquad , \quad j=1,2,\ldots,J, \qquad (17)$$

These weights put more weight on the differences where the CAMS estimates are more precise (less variable).

The weights (17) use estimated variances corresponding to the multinomial distribution assumption. The actual calculation of the weighted least squares estimates would be an iterative procedure. Initially, the weights would be set

equal to one, then $\hat{P}_A$ and $\hat{P}_B$ determined from (16). These initial estimates $\hat{P}_A$ and $\hat{P}_B$ would be substituted into (17) to determine the weights for the second iteration. The evaluation of (16) now yields the second set of estimates $\hat{P}_A$ and $\hat{P}_B$. After a moderate number of iterations, say 4, the $\hat{P}_A$ and $\hat{P}_B$ should change very little and will be taken as the weighted least squares estimates.

2.4 The Combination of a Least Squares Ratio Estimate and a Maximum Likelihood Proportion Estimate

An alternative approach is to use only the completely identified sample segments to estimate

$$R_A = P_A/(P_A + P_B) \tag{18}$$

where R_A is the relative proportion of crop A among the acreage proportion for crop A or B. Similarly,

$$R_B = P_B/(P_A + P_B) = 1 - R_A. \tag{19}$$

Then, having estimated R_A and hence R_B, estimate the overall stratum proportions P_A and P_B by

$$\hat{P}_A = \hat{R}_A \quad \cdot \quad \hat{P}_{A+B}$$

and

$$\hat{P}_B = \hat{R}_B \quad \cdot \quad \hat{P}_{A+B} \tag{20}$$

where $\hat{P}_{A+B}$ is an estimate for the combined acreage proportion

$$P_{A+B} = P_A + P_{\bar{B}} \quad . \tag{21}$$

The combined acreage proportion P_{A+B} could be estimated by the maximum likelihood estimate

$$\hat{P}_{A+B} = \left\{ \sum_{i=1}^{I} P_{Ai} + \sum_{i=1}^{I} P_{Bi} + \sum_{j=1}^{J} (P_A + P_B)_j \right\} / (I + J) \quad . \tag{22}$$

Other estimates of P_{A+B} could be used.

The relative proportion R_A can be estimated by minimizing

$$\sum_{i=1}^{I} [P_{Ai} - R_A(P_{Ai} + P_{Bi})]^2 + \sum_{i=1}^{I} [P_{Bi} - (1 - R_A)(P_{Ai} + P_{Bi})]^2 \tag{23}$$

with respect to R_A. The resulting least squares estimator of R_A is

$$\hat{R}_A = \sum_{i=1}^{I} p_{Ai}(p_{Ai} + p_{Bi}) / \sum_{i=1}^{I} (p_{Ai} + p_{Bi})^2 \ .$$

(24)

3. Empirical Behavior of Some Estimation Procedures

Someone once said, "The proof of the pudding is in the tasting." Similarly, here the proof of an estimation procedure's value lies in its actual performance on real data. To learn about the empirical behavior of the four alternative estimation procedures

(1) maximum likelihood estimators, (9) - (11),

(2) least squares estimators, (14),

(3) weighted least squares estimators, (16), and

(4) the combination of a least squares ratio estimator, (24), and the maximum likelihood proportion estimator, (22),

a Monte Carlo study was performed. The structure of the study is discussed in sub-sections 3.2 and 3.3. The computer output from the study is reproduced in Appendices A and B. The conclusions reached from the study are given in subsection 3.4.

3.1 Potential for Bias

When some of the sampled segments are only partially identified, it is possible for the estimators of $(P_A, P_B, 1-P_A-P_B)$ to be biased.

To demonstrate this potential for bias, the following simple example is given. The stratum is composed of two segments. Segment one has $(P_A, P_B, 1-P_A-P_B) = $ (.5, .5, 0), and segment two has $(P_A, P_B, 1-P_A-P_B) = (.2, .6, .2)$. Hence, the stratum has $(P_A, P_B, 1-P_A-P_B) = (.35, .55, .1)$. Suppose that the maximum likelihood estimator (9) - (11) is to be used. One segment is to be partially identified and one completely identified. If segment one is the one completely identified, then

$$\hat{P}_A = (\frac{.5}{.5 + .5})(\frac{.5 + .5 + .8}{1 + 1}) = .45 \quad .$$

If segment two is the one completely identified, then

$$\hat{P}_A = (\frac{.2}{.2 + .6})(\frac{.2 + .6 + 1}{1 + 1}) = .225 \quad .$$

Thus, the expected value of $\hat{P}_A$ is

$$E[\hat{P}_A] = .5(.45) + .5(.225) = .3375 .$$

Hence $\hat{P}_A$ is biased since .3375 $\neq$ .35.

3.2 Monte Carlo Study

The simulation of the behavior of the estimation procedures was based around two real sets of CAMS estimates. The 1977 North Dakota CAMS estimates of

crop A = spring wheat

and

crop B = spring small grains other than spring wheat

formed one set. The second set was the 1978 North Dakota CAMS estimates of

crop A = barley

and

crop B = spring small grains other than barley.

There were 83 segments comprising the first set and 76 segments in the second set. All simulations were done on each data set separately.

To obtain a single observation on $(\hat{P}_A, \hat{P}_B, 1-\hat{P}_A-\hat{P}_B)$ for each estimator the following procedure was performed:

1) Simulate a stratum by sampling without replacement segments from the data set.

ii) Select a sample of segments from the simulated stratum by sampling without replacement segments from the simulated stratum.

iii) Calculate all estimators.

This procedure was repeated 1000 times for each data set, stratum size, segment sample size, and fraction of the segment sample that would be only partially identified.

Stratums of size 40 segments and 20 segments were considered. The segment sample sizes were 5, 10, and 15 when the stratum size was 40 segments and were 5 and 10 when

the stratum size was 20 segments. For each stratum size and segment sample size, 20%, 40%, 60%, and 80% of the segments were partially identified.

3.3 Evaluation Criteria

There are several possible ways to measure the sample behavior of the estimators. For each estimator and each of $\hat{P}_A$, $\hat{P}_B$, and $1-\hat{P}_A-\hat{P}_B$ the following measures were calculated for each data set:

(i) average absolute error = the average over 1000 simluations of $|\hat{P} - P_{Stratum}|$ where $P_{stratum}$ represents the actual crop proportion in the particular simulated stratum,

(ii) average squared error = the average over 1000 simulations of $(\hat{P} - P_{stratum})^2$

(iii) bias of average estimate = the difference between the average $(\hat{P})$ in 1000 simulations and P_{set} where P_{set} is the actual crop proportion in the entire set of segments, and

(iv) sample variance of the estimator.

Some information is, of course, lost when some segments are only partially identified. To assess this loss, the maximum likelihood estimators were also calculated using the complete identification for all sampled segments. Since these estimators have complete information for the entire sample of n segments instead of complete information on only some of the n segments and partial information on the remainder, these latter estimators should perform better.

To show that the inclusion of the partially identified segments into the estimation procedure is better than simply ignoring them, the maximum likelihood estimators, least squares estimators, and weighted least squares estimators were also calculated using only the subset of the n sample segments corresponding to the completely identified segments.

The observed behavior of each of these eight estimators is detailed in the computer outputs reproduced in Appendix A for the 1977 spring wheat data set and Appendix B for the 1978 barley data set.

3.4 Indications of the Monte Carlo Study

Summaries of the observed average absolute errors for the better estimators are given in Tables 1 and 2 for the 1977 and 1978 data sets respectively. The average over the entire data sets of $(p_A, p_B, 1 - p_A - p_B)$ are

$$1977: \quad (.235, .073, .692)$$
$$1978: \quad (.053, .251, .696).$$

Hence it is not surprising to observe that the behaviors of the estimators for A in 1977 are very similar to those for B in 1978 and, correspondingly, the estimators for B in 1977 and for A in 1978 have nearly identical behavior. Also, the behaviors of the estimators of $1 - P_A - P_B$ in 1977 are nearly the same as the corresponding behaviors in 1978.

The unweighted least squares estimators given in (14) are substantially inferior to the other estimators, and hence their behavior is not summarized in Tables 1 and 2. There are much smaller differences among the remaining estimators.

When both the partially and completely identified sample segment estimates are incorporated into the maximum likelihood estimator (MLE) given in (9)-(11), the weighted least squares estimator (WLS) given in (16), and the combination of the least squares ratio estimator (LSR) given in (24) and the maximum likelihood proportional acreage estimator given in (22), all three estimators have similar behavior. Overall, the MLE is slightly better than the WLS, and the WLS is slightly better than the LSR. The differences are only slight since the average absolute error for LSR is never more than 110% of that for MLE. The average squared errors tell the same story. There are no significant differences among the biases or sample variances for the MLE, WLS, and LSR.

The effects of the sample size and amount of partial identification are essentially the same for MLE, WLS, and LSR. When the underlying data set proportional acreage is nearly 25% as it is for P_A in 1977 and P_B in 1978, then the average absolute error decreases as the segment sample size, n, increases roughly as

follows

.04 for n = 5,
.03 for n = 10, and
.02 for n = 15.

When the underlying data set proportional acreage is nearly 5% as it is for P_B in 1977 and P_A in 1978, then the average absolute decreases with increasing segment sample size roughly as follows

.02 for n = 5,
.015 for n = 10, and
.01 for n = 15.

There is roughly a quadratic increase in average absolute error as the amount of partial identification increases from 20% to 40% to 60% to 80%. The average absolute error increases around twice as much when the amount of partial identification increases from 60% to 80% as it does when the amount of partial identification increases from 40% to 60%.

All of the estimators seem to slightly (½%) overestimate the combined proportional acreage for A and B when the segment sample size, n, is 5. This bias drops to ¼% when n = 10 and essentially disappears at n = 15.

The WLS does not seem to be as good as either MLE or LSR when there is only one completely identified sample segment.

Tables 3 and 4 indicate the slight increase in average absolute error caused by only partially identifying a segment's acreages instead of completely identifying them. Only the behavior of MLE is summarized since WLS and LSR should behave similarly. Naturally the increase in the average absolute error increases as the amount of partial identification increases. For less than 80% partial identification there is always an increase of less than .01. For 80% the largest increase is less than .017. At 80% partial identification the average absolute error can double. Of course, if there is 100% partial identification, then there are no individual crop estimates.

Tables 5 and 6 indicate the average absolute errors both with and without the partially identified sample segments being included in the estimators MLE and WLS. These tables are presented to exemplify the value of at least partially identifying a sample segment instead of simply disregarding it if it can't be completely identified. The percentage increase in the average absolute errors when the underlying proportional acreage is nearly 25% is usually at least

5%	–	10%	for	20%	partial identification,
10%	–	25%	for	40%	partial identification,
25%	–	50%	for	60%	partial identification,
50%	–	100%	for	80%	partial identification.

The percentage increases are somewhat smaller when the underlying proportional acreage is around 5%.

4. Recommendations

On the basis of the limted Monte Carlo study the following recommendations are made:

1) As long as there are some completely identified sample segments, it is reasonable to estimate the individual crop proportions in the stratum.

2) It is prudent to avoid having a large percentage (say 80%) of only partially identified sample segments.

3) It is much better to incorporate the partially identified sample segments into the estimators than it is to disregard the partially identified sample segments.

4) The maximum likelihood estimators seem to be the best estimators, but they are not greatly superior to weighted least squares estimators or the use of a least squares ratio estimator.

Table 1. A Summary of the Average Absolute Error for the Maximum Likelihood Estimator (MLE), the Weighted Least Squares Estimator (WLS), and the Combination of Least Squares Ratio Estimator (LSR) with the Maximum Likelihood Proportional Acreage Estimator on the 1977 North Dakota Spring Wheat Data Set

Stratum Sample Size	Segment Sample Size	Segments Only Partially Identified	$\hat{P}_A$			$\hat{P}_B$			$1-\hat{P}_A-\hat{P}_B$		
		Estimator:	MLE	WLS	LSR	MLE	WLS	LSR	MLE	WLS	LSR
40	5	1	.0424	.0424	.0431	.0156	.0157	.0156	.0512	.0512	.0512
		2	.0433	.0435	.0442	.0177	.0176	.0181	.0516	.0516	.0516
		3	.0442	.0452	.0450	.0210	.0208	.0213	.0510	.0510	.0510
		4	.0491	.0621	.0491	.0281	.0410	.0281	.0505	.0507	.0505
	10	2	.0276	.0276	.0281	.0106	.0106	.0104	.0334	.0334	.0334
		4	.0288	.0288	.0296	.0120	.0122	.0120	.0343	.0343	.0343
		6	.0298	.0299	.0305	.0144	.0145	.0149	.0342	.0342	.0342
		8	.0328	.0334	.0338	.0194	.0199	.0202	.0339	.0339	.0339
	15	3	.0206	.0206	.0210	.0079	.0079	.0080	.0248	.0248	.0248
		6	.0207	.0207	.0212	.0093	.0095	.0096	.0247	.0248	.0247
		9	.0216	.0217	.0222	.0108	.0110	.0114	.0247	.0247	.0247
		12	.0249	.0251	.0260	.0160	.0160	.0170	.0251	.0252	.0251
20	5	1	.0387	.0387	.0395	.0149	.0150	.0149	.0473	.0472	.0473
		2	.0385	.0387	.0394	.0172	.0173	.0176	.0463	.0463	.0463
		3	.0417	.0425	.0426	.0199	.0199	.0202	.0477	.0476	.0477
		4	.0464	.0610	.0464	.0287	.0439	.0287	.0472	.0474	.0472
	10	2	.0231	.0231	.0236	.0085	.0086	.0087	.0272	.0272	.0272
		4	.0232	.0233	.0240	.0105	.0108	.0107	.0271	.0271	.0271
		6	.0241	.0242	.0247	.0131	.0132	.0136	.0269	.0270	.0269
		8	.0287	.0290	.0296	.0185	.0192	.0193	.0279	.0278	.0279

Table 2. A Summary of the Average Absolute Error for the Maximum Likelihood Estimator (MLE), the Weighted Least Squares Estimator (WLS), and the Combination of Least Squares Ratio Estimator (LSR) with the Maximum Likelihood Proportional Acreage Estimator on the 1978 North Dakota Barley Data Set

Stratum Sample Size	Segment Sample Size	Segments Only Partially Identified	$\hat{P}_A$			$\hat{P}_B$			$1 - \hat{P}_A - \hat{P}_B$		
		Estimator:	MLE	WLS	LSR	MLE	WLS	LSR	MLE	WLS	LSR
50	5	1	.0183	.0182	.0194	.0426	.0426	.0433	.0508	.0509	.0508
		2	.0205	.0203	.0218	.0446	.0448	.0455	.0523	.0523	.0523
		3	.0239	.0232	.0252	.0463	.0465	.0470	.0516	.0516	.0516
		4	.0333	.0347	.0333	.0515	.0536	.0515	.0514	.0515	.0514
	10	2	.0116	.0116	.0125	.0294	.0294	.0302	.0341	.0342	.0341
		4	.0136	.0136	.0146	.0282	.0283	.0290	.0332	.0332	.0332
		6	.0163	.0160	.0178	.0316	.0319	.0326	.0337	.0337	.0337
		8	.0229	.0227	.0242	.0348	.0355	.0356	.0335	.0335	.0335
	15	3	.0092	.0091	.0100	.0219	.0219	.0225	.0260	.0259	.0260
		6	.0103	.0102	.0112	.0225	.0225	.0234	.0262	.0262	.0262
		9	.0129	.0125	.0139	.0231	.0231	.0241	.0262	.0261	.0262
		12	.0173	.0169	.0187	.0263	.0265	.0271	.0260	.0260	.0260
20	5	1	.0169	.0168	.0179	.0408	.0408	.0414	.0486	.0486	.0486
		2	.0190	.0189	.0201	.0414	.0415	.0422	.0482	.0483	.0482
		3	.0233	.0227	.0246	.0441	.0445	.0447	.0492	.0493	.0492
		4	.0327	.0356	.0327	.0488	.0521	.0488	.0478	.0480	.0478
	10	2	.0100	.0100	.0109	.0239	.0239	.0245	.0274	.0275	.0275
		4	.0123	.0123	.0133	.0238	.0238	.0243	.0271	.0271	.0271
		6	.0149	.0147	.0165	.0254	.0253	.0263	.0265	.0266	.0265
		8	.0217	.0220	.0232	.0305	.0306	.0316	.0277	.0278	.0277

Table 3. The Loss From Only Partially Identifying Sample Segments: A Summary of the Average Absolute Errors for the Maximum Likelihood Estimator Both With All Sample Segments Completely Identified and With Only Some Completely Identified and the Remainder Partially Identified (1977 North Dakota Spring Wheat Data Set)

Stratum Sample Size	Segment Sample Size	Segments Only Partially Identified	$\widehat{P}_A$		$\widehat{P}_B$	
			All	Some	All	Some
40	5	1	.0416	.0424	.0147	.0156
		2	.0425	.0433	.0146	.0177
		3	.0414	.0442	.0148	.0210
		4	.0413	.0491	.0145	.0281
	10	2	.0271	.0276	.0098	.0106
		4	.0278	.0288	.0099	.0120
		6	.0278	.0298	.0098	.0144
		8	.0274	.0328	.0100	.0194
	15	3	.0203	.0206	.0072	.0079
		6	.0200	.0207	.0074	.0093
		9	.0200	.0216	.0072	.0108
		12	.0205	.0249	.0073	.0160
20	5	1	.0383	.0387	.0137	.0149
		2	.0378	.0385	.0139	.0172
		3	.0389	.0417	.0140	.0199
		4	.0385	.0464	.0137	.0287
	10	2	.0227	.0231	.0076	.0085
		4	.0222	.0232	.0081	.0105
		6	.0221	.0241	.0078	.0131
		8	.0229	.0286	.0080	.0185

Table 4. The Loss From Only Partially Identifying Sample Segments: A Summary of the Average Absolute Errors for the Maximum Likelihood Estimator Both With All Sample Segments Completely Identified and With Only Some Completely Identified and the Remainder Partially Identified (1978 North Dakota Barley Data Set)

Stratum Sample Size	Segment Sample Size	Segments Only Partially Identified	$\hat{P}_A$ All	$\hat{P}_A$ Some	$\hat{P}_B$ All	$\hat{P}_B$ Some
40	5	1	.0170	.0183	.0421	.0426
		2	.0170	.0205	.0431	.0446
		3	.0167	.0239	.0426	.0463
		4	.0170	.0333	.0426	.0515
	10	2	.0106	.0116	.0288	.0294
		4	.0108	.0136	.0276	.0282
		6	.0104	.0163	.0288	.0316
		8	.0108	.0229	.0281	.0348
	15	3	.0081	.0092	.0215	.0219
		6	.0082	.0103	.0216	.0225
		9	.0080	.0129	.0216	.0231
		12	.0081	.0173	.0215	.0263
20	5	1	.0154	.0169	.0399	.0408
		2	.0154	.0190	.0403	.0414
		3	.0157	.0233	.0403	.0441
		4	.0154	.0327	.0394	.0488
	10	2	.0087	.0099	.0233	.0239
		4	.0091	.0123	.0228	.0230
		6	.0085	.0149	.0223	.0254
		8	.0091	.0217	.0234	.0305

Table 5. A Summary of the Average Absolute Errors for the Maximum Likelihood Estimator
(MLE), and the Weighted Least Squares Estimator (WLS), both With and Without
the Partially Identified Sample Segments on the 1977 North Dakota Spring Wheat
Data Set

Stratum Sample Size	Segment Sample Size	Segments Only Partially Identified	$\hat{P}_A$				$\hat{P}_B$			
			MLE		WLS		MLE		WLS	
Estimator:			With	Without	With	Without	With	Without	With	Without
40	5	1	.0424	.0476	.0424	.0476	.0156	.0166	.0157	.0166
		2	.0433	.0566	.0435	.0566	.0177	.0201	.0176	.0201
		3	.0442	.0693	.0452	.0693	.0210	.0254	.0208	.0254
		4	.0491	.1030	.0621	.1030	.0281	.0365	.0410	.0365
	10	2	.0276	.0324	.0276	.0324	.0106	.0115	.0106	.0115
		4	.0288	.0381	.0288	.0288	.0120	.0135	.0122	.0135
		6	.0298	.0491	.0299	.0491	.0144	.0176	.0145	.0176
		8	.0328	.0725	.0334	.0725	.0194	.0248	.0199	.0248
	15	3	.0206	.0245	.0206	.0245	.0079	.0086	.0079	.0086
		6	.0207	.0289	.0207	.0289	.0093	.0107	.0095	.0107
		9	.0216	.0386	.0217	.0386	.0108	.0140	.0110	.0140
		12	.0249	.0578	.0251	.0578	.0160	.0204	.0160	.0204
20	5	1	.0387	.0443	.0387	.0443	.0149	.0163	.0150	.0163
		2	.0385	.0526	.0387	.0526	.0172	.0194	.0173	.0194
		3	.0417	.0682	.0425	.0682	.0199	.0242	.0199	.0242
		4	.0464	.1005	.0610	.1005	.0287	.0359	.0439	.0359
	10	2	.0231	.0278	.0231	.0278	.0085	.0096	.0086	.0096
		4	.0232	.0338	.0233	.0338	.0105	.0121	.0108	.0121
		6	.0241	.0453	.0242	.0453	.0131	.0163	.0132	.0163
		8	.0286	.0706	.0394	.0706	.0185	.0241	.0192	.0241

Table 6. A Summary of the Average Absolute Errors for the Maximum Likelihood Estimator (MLE), and the Weighted Least Squares Estimator (WLS), both With and Without the Partially Identified Sample Segments on the 1978 North Dakota Barley Data Set

Stratum Sample Size	Segment Sample Size	Segments Only Partially Identified	$\hat{P}_A$				$\hat{P}_B$			
			MLE		WLS		MLE		WLS	
Estimator:			With	Without	With	Without	With	Without	With	Without
40	5	1	.0183	.0189	.0182	.0189	.0426	.0475	.0426	.0475
		2	.0205	.0220	.0203	.0220	.0446	.0579	.0448	.0579
		3	.0239	.0273	.0232	.0273	.0463	.0689	.0465	.0689
		4	.0333	.0389	.0345	.0389	.0515	.0103	.0536	.0103
	10	2	.0116	.0120	.0116	.0120	.0294	.0326	.0294	.0326
		4	.0136	.0149	.0136	.0149	.0282	.0380	.0283	.0380
		6	.0163	.0187	.0160	.0187	.0316	.0501	.0319	.0501
		8	.0229	.0276	.0227	.0276	.0348	.0689	.0355	.0689
	15	3	.0092	.0098	.0091	.0098	.0219	.0264	.0219	.0264
		6	.0103	.0119	.0102	.0119	.0225	.0324	.0225	.0324
		9	.0129	.0156	.0125	.0156	.0231	.0403	.0231	.0403
		12	.0173	.0216	.0179	.0216	.0263	.0590	.0265	.0590
20	5	1	.0169	.0176	.0168	.0176	.0408	.0466	.0408	.0466
		2	.0190	.0203	.0189	.0203	.0414	.0540	.0415	.0540
		3	.0233	.0265	.0227	.0265	.0441	.0679	.0445	.0679
		4	.0327	.0385	.0356	.0385	.0488	.0996	.0521	.0996
	10	2	.0099	.0105	.0100	.0105	.0239	.0281	.0239	.0281
		4	.0123	.0137	.0123	.0137	.0238	.0339	.0238	.0339
		6	.0149	.0176	.0147	.0176	.0254	.0455	.0253	.0455
		8	.0217	.0260	.0220	.0260	.0305	.0646	.0306	.0646

APPENDIX A

MONTE CARLO STUDY USING THE

1977 SPRING WHEAT DATA SET

THE NUMBER OF COMMENTS TO BE PRINTED =   49

NASA
PARTIAL IDENTIFICATION PROBLEM
SIMULATOR FOR SINGLE YEAR ESTIMATION PROCEDURES

TWO CROPS OF INTEREST

DECEMBER 1981

EIGHT ESTIMATION PROCEDURES ARE CONSIDERED

1. MAXIMUM LIKELIHOOD ESTIMATION USING BOTH PARTIALLY
   AND COMPLETELY IDENTIFIED SAMPLE SEGMENTS

2. MAXIMUM LIKELIHOOD ESTIMATION USING ONLY THE
   COMPLETELY IDENTIFIED SAMPLE SEGMENTS

3. MAXIMUM LIKELIHOOD ESTIMATION PRETENDING THAT ALL
   SAMPLE SEGMENTS WERE COMPLETELY IDENTIFIED

4. UNWEIGHTED LEAST SQUARES ESTIMATION USING BOTH
   PARTIALLY AND COMPLETELY IDENTIFIED SAMPLE SEGMENTS

5. UNWEIGHTED LEAST SQUARES ESTIMATION USING ONLY THE
   COMPLETELY IDENTIFIED SAMPLE SEGMENTS

6. WEIGHTED LEAST SQUARES ESTIMATION USING BOTH
   PARTIALLY AND COMPLETELY IDENTIFIED SAMPLE SEGMENTS
   AND WEIGHTS INVERSELY PROPORTIONAL TO THE VARIANCES

7. WEIGHTED LEAST SQUARES ESTIMATION USING ONLY THE
   COMPLETELY IDENTIFIED SAMPLE SEGMENTS
   AND WEIGHTS INVERSELY PROPORTIONAL TO THE VARIANCES

8. THE LEAST SQUARES ESTIMATOR OF THE RATIO OF CROP 1'S
   ACREAGE PROPORTION TO CROP 1 AND 2'S ACREAGE PROPORTION
   TIMES THE MAXIMUM LIKELIHOOD ESTIMATOR OF THE
   COMBINED ACREAGE PROPORTION FOR CROPS 1 AND 2

THE FINITE SUPER POPULATION IS THE CAMS ESTIMATES FOR
NORTH DAKOTA IN 1977

CROP 1 IS SPRING WHEAT
CROP 2 IS SPRING SMALL GRAINS LESS SPRING WHEAT

THE NUMBER OF SEGMENTS IN THE FINITE SUPER POPULATION IS   83

THE SEGMENT PERCENTAGES IN THE SUPER POPULATION ARE AS FOLLOWS: SEGMENT NUMBER, PROPORTIONS

Segment					
1	10.300	6.200	83.500	10.300	6.200
2	33.900	8.100	58.000	33.900	8.100
3	28.600	9.200	62.200	28.600	9.200
4	32.600	13.400	54.000	32.600	13.400
5	29.000	3.000	68.000	29.000	3.000
6	27.300	13.300	59.400	27.300	13.300
7	22.800	14.500	62.700	22.800	14.500
8	38.500	14.700	46.800	38.500	14.700
9	23.600	3.900	72.500	23.600	3.900
10	27.100	9.800	63.100	27.100	9.800
11	40.100	5.900	54.000	40.100	5.900
12	20.100	15.900	64.000	20.100	15.900
13	34.300	4.700	61.000	34.300	4.700
14	27.900	10.200	61.900	27.900	10.200
15	23.900	2.100	74.000	23.900	2.100
16	28.600	6.400	65.000	28.600	6.400
17	12.100	4.900	83.000	12.100	4.900
18	13.600	5.400	81.000	13.600	5.400
19	15.400	5.600	79.000	15.400	5.600
20	29.000	9.900	61.100	29.000	9.900
21	19.200	7.400	73.400	19.200	7.400
22	3.800	1.200	95.000	3.800	1.200
23	22.900	8.700	68.400	22.900	8.700
24	25.800	10.100	64.100	25.800	10.100
25	37.500	5.860	56.760	37.500	5.860
26	28.200	11.500	60.300	28.200	11.500
27	37.600	13.500	48.900	37.600	13.500
28	37.200	10.300	52.000	37.200	10.300
29	17.600	7.500	74.900	17.600	7.500
30	28.000	7.900	64.100	28.000	7.900
31	42.500	7.400	50.100	42.500	7.400
32	23.300	5.700	71.000	23.300	5.700
33	45.000	13.000	42.000	45.000	13.000
34	16.700	3.600	79.700	16.700	3.600
35	0.001	0.001	100.000	0.001	0.001
36	17.600	7.500	74.900	17.600	7.500
37	8.400	2.600	89.000	8.400	2.600
38	15.800	6.000	78.200	15.800	6.000
39	14.000	5.000	81.000	14.000	5.000
40	7.400	0.600	92.000	7.400	0.600
41	39.600	3.900	56.500	39.600	3.900
42	25.300	5.500	69.200	25.300	5.500
43	13.500	3.500	83.600	13.500	3.500

45	48.700	15.300	36.000
46	49.000	8.100	42.900
47	7.200	7.800	85.000
48	50.900	9.100	40.000
49	12.900	6.100	81.000
50	15.400	5.400	79.200
51	19.000	15.500	65.500
52	16.700	5.900	77.400
53	5.800	2.500	91.700
54	20.400	4.500	75.100
55	20.100	4.200	75.700
56	14.300	11.700	74.000
57	16.100	10.100	73.800
58	5.000	0.000	95.000
59	36.500	13.100	50.400
60	32.600	15.300	52.100
61	26.300	7.700	66.000
62	28.300	4.700	67.000
63	23.300	13.700	63.000
64	30.900	6.500	62.600
65	26.900	11.800	61.300
66	33.900	3.000	63.100
67	3.200	0.900	95.900
68	43.700	2.700	53.600
69	0.001	0.001	100.000
70	48.600	10.400	41.000
71	12.400	2.500	85.100
72	7.400	1.600	91.000
73	7.000	0.000	93.000
74	9.100	10.900	80.000
75	35.700	16.900	47.400
76	27.400	10.800	61.800
77	4.900	0.000	95.100
78	11.300	2.700	86.000
79	16.200	2.800	81.000
80	20.600	4.400	75.000
81	18.900	16.000	65.100
82	33.700	4.100	62.200
83	20.800	9.500	69.700

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ORIGINAL PAGE IS
OF POOR QUALITY

THERE ARE 2 STRATUM SIZES TO BE CONSIDERED.
THE DIFFERENT STRATUM SIZES ARE
   40   20

FOR STRATA OF THE 1-TH SIZE, THE FOLLOWING SEGMENT SAMPLE SIZES ARE CONSIDERED:
   5   15

FOR STRATA OF THE 2-TH SIZE, THE FOLLOWING SEGMENT SAMPLE SIZES ARE CONSIDERED:
   5   10

FOR STRATA OF THE 1-TH SIZE AND SEGMENT SAMPLES OF SIZE 5
THE FOLLOWING NUMBERS OF PARTIALLY IDENTIFIED SEGMENTS ARE CONSIDERED:
   1   2   3   4

FOR STRATA OF THE 1-TH SIZE AND SEGMENT SAMPLES OF SIZE 10
THE FOLLOWING NUMBERS OF PARTIALLY IDENTIFIED SEGMENTS ARE CONSIDERED:
   2   4   6   8

FOR STRATA OF THE 1-TH SIZE AND SEGMENT SAMPLES OF SIZE 15
THE FOLLOWING NUMBERS OF PARTIALLY IDENTIFIED SEGMENTS ARE CONSIDERED:
   3   6   9   12

FOR STRATA OF THE 2-TH SIZE AND SEGMENT SAMPLES OF SIZE 5
THE FOLLOWING NUMBERS OF PARTIALLY IDENTIFIED SEGMENTS ARE CONSIDERED:
   1   2   3   4

FOR STRATA OF THE 2-TH SIZE AND SEGMENT SAMPLES OF SIZE 10
THE FOLLOWING NUMBERS OF PARTIALLY IDENTIFIED SEGMENTS ARE CONSIDERED:
   2   4   6   8

1000 SEGMENT SAMPLES ARE TO BE TAKEN FOR EACH AMOUNT OF PARTIAL IDENTIFICATION.

THE TRUE SUPER POPULATION PROPORTIONS WERE :   0.23457   0.07301   0.69242

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 5 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 1

ESTIMATOR NO.   OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

Estimator No.			
1	0.04242	0.01563	0.05118
2	0.04761	0.01663	0.05835
3	0.04164	0.01467	0.05118
4	0.04272	0.01900	0.05173
5	0.04761	0.01663	0.05835
6	0.04242	0.01571	0.05122
7	0.04761	0.01663	0.05835
8	0.04306	0.01562	0.05118

AVERAGE SQUARED ERROR

Estimator No.			
1	0.00279	0.00039	0.00408
2	0.00354	0.00044	0.00530
3	0.00271	0.00034	0.00408
4	0.00283	0.00057	0.00413
5	0.00354	0.00044	0.00530
6	0.00279	0.00040	0.00408
7	0.00354	0.00044	0.00530
8	0.00288	0.00039	0.00408

BIAS OF AVERAGE ESTIMATE

Estimator No.			
1	0.00249	0.00222	-0.00471
2	0.00321	0.00233	-0.00554
3	0.00285	0.00185	-0.00471
4	0.00269	0.00181	-0.00450
5	0.00321	0.00233	-0.00554
6	0.00233	0.00237	-0.00470
7	0.00321	0.00233	-0.00554
8	0.00360	0.00111	-0.00471

SAMPLE VARIANCE OF ESTIMATOR

Estimator No.			
1	0.05491	0.02047	0.06676
2	0.06153	0.02169	0.07548
3	0.05429	0.01923	0.06676
4	0.05534	0.02441	0.06709
5	0.06153	0.02169	0.07548
6	0.05491	0.02058	0.06672
7	0.06153	0.02169	0.07548
8	0.05570	0.02049	0.06676

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 5 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 2

ESTIMATOR NO.   OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.04334	0.01773	0.05164
2	0.05663	0.02010	0.06935
3	0.04250	0.01462	0.05164
4	0.04457	0.02368	0.05205
5	0.05663	0.02010	0.06935
6	0.04353	0.01761	0.05164
7	0.05663	0.02010	0.06935
8	0.04423	0.01809	0.05164

AVERAGE SQUARED ERROR

1	0.00291	0.00049	0.00411
2	0.00489	0.00063	0.00733
3	0.00280	0.00033	0.00411
4	0.00305	0.00090	0.00419
5	0.00489	0.00063	0.00733
6	0.00293	0.00050	0.00412
7	0.00489	0.00063	0.00733
8	0.00302	0.00051	0.00411

BIAS OF AVERAGE ESTIMATE

1	0.00369	0.00053	-0.00422
2	0.00524	0.00073	-0.00598
3	0.00376	0.00045	-0.00422
4	0.00421	-0.00030	-0.00391
5	0.00524	0.00073	-0.00598
6	0.00301	0.00125	-0.00426
7	0.00524	0.00073	-0.00598
8	0.00430	-0.00008	-0.00422

SAMPLE VARIANCE OF ESTIMATOR

1	0.05642	0.02267	0.06682
2	0.07209	0.02549	0.08798
3	0.05532	0.01885	0.06682
4	0.05770	0.03017	0.06729
5	0.07209	0.02549	0.08798
6	0.05665	0.02269	0.06684
7	0.07209	0.02549	0.08798
8	0.05742	0.02308	0.06682

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 5 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 3

ESTIMATOR NO. OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.04422	0.02096	0.05104
2	0.06930	0.02537	0.08514
3	0.04138	0.01475	0.05104
4	0.04655	0.02999	0.05168
5	0.06930	0.02537	0.08514
6	0.04515	0.02079	0.05104
7	0.06930	0.02537	0.08514
8	0.04501	0.02128	0.05104

AVERAGE SQUARED ERROR

1	0.00297	0.00070	0.00402
2	0.00710	0.00096	0.01079
3	0.00265	0.00034	0.00402
4	0.00331	0.00140	0.00409
5	0.00710	0.00096	0.01079
6	0.00309	0.00070	0.00402
7	0.00710	0.00096	0.01079
8	0.00310	0.00072	0.00402

BIAS OF AVERAGE ESTIMATE

1	0.00199	0.00267	-0.00466
2	0.00339	0.00255	-0.00595
3	0.00268	0.00198	-0.00466
4	0.00268	0.00184	-0.00452
5	0.00339	0.00255	-0.00595
6	-0.00031	0.00515	-0.00484
7	0.00339	0.00255	-0.00595
8	0.00242	0.00224	-0.00466

SAMPLE VARIANCE OF ESTIMATOR

1	0.05704	0.02694	0.06664
2	0.08628	0.03154	0.10644
3	0.05403	0.01930	0.06664
4	0.06004	0.03764	0.06705
5	0.08628	0.03154	0.10645
6	0.05898	0.02665	0.06657
7	0.08628	0.03154	0.10645
8	0.05824	0.02730	0.06664

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 5 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 4

ESTIMATOR NO.   OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.04912	0.02814	0.05050
2	0.10303	0.03650	0.12561
3	0.04131	0.01451	0.05050
4	0.05710	0.04408	0.05101
5	0.10303	0.03650	0.12561
6	0.06211	0.04096	0.05071
7	0.10303	0.03650	0.12561
8	6.04912	0.02814	0.05050

AVERAGE SQUARED ERROR

1	0.00372	0.00131	0.00399
2	0.01554	0.00193	0.02304
3	0.00267	0.00033	0.00399
4	0.00494	0.00302	0.00402
5	0.01554	0.00193	0.02304
6	0.02812	0.02764	0.00403
7	0.01554	0.00193	0.02304
8	0.00372	0.00131	0.00399

BIAS OF AVERAGE ESTIMATE

1	0.00025	0.00251	-0.00275
2	0.00365	0.00096	-0.00461
3	0.00188	0.00087	-0.00275
4	0.00267	-0.00002	-0.00266
5	0.00365	0.00096	-0.00461
6	-0.01665	0.02032	-0.00367
7	0.00365	0.00096	-0.00461
8	0.00025	0.00251	-0.00275

SAMPLE VARIANCE OF ESTIMATOR

1	0.06348	0.03625	0.06566
2	0.12617	0.04409	0.15313
3	0.05389	0.01892	0.06566
4	0.07263	0.05310	0.06591
5	0.12617	0.04409	0.15313
6	0.16780	0.16497	0.06591
7	0.12617	0.04409	0.15313
8	0.06748	0.03675	0.06566

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 10 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 2

ESTIMATOR NO. OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.02759	0.03341
2	0.03237	0.03963
3	0.02713	0.03341
4	0.02789	0.03357
5	0.03237	0.03963
6	0.02758	0.03338
7	0.03237	0.03963
8	0.02808	0.03341

AVERAGE SQUARED ERROR

1	0.00116	0.00171
2	0.00156	0.00236
3	0.00113	0.00171
4	0.00118	0.00174
5	0.00156	0.00236
6	0.00116	0.00171
7	0.00156	0.00236
8	0.00121	0.00171

BIAS OF AVERAGE ESTIMATE

1	0.00152	-0.00208
2	0.00125	-0.00167
3	0.00147	-0.00208
4	0.00151	-0.00218
5	0.00125	-0.00167
6	0.00145	-0.00213
7	0.00125	-0.00167
8	-0.00282	-0.00074

SAMPLE VARIANCE OF ESTIMATOR

1	0.03707	0.04546
2	0.04232	0.05239
3	0.03665	0.04546
4	0.03740	0.04568
5	0.04232	0.05239
6	0.03707	0.04545
7	0.04232	0.05239
8	0.03778	0.04546

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 10 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 4

ESTIMATOR NO. OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.02876	0.01198	0.03428
2	0.03813	0.01353	0.04651
3	0.02781	0.00988	0.03428
4	0.02945	0.01684	0.03471
5	0.03813	0.01353	0.04651
6	0.02878	0.01219	0.03431
7	0.03813	0.01353	0.04651
8	0.02958	0.01201	0.03428

AVERAGE SQUARED ERROR

1	0.00127	0.00023	0.00180
2	0.00223	0.00028	0.00332
3	0.00120	0.00015	0.00180
4	0.00134	0.00045	0.00185
5	0.00223	0.00028	0.00332
6	0.00128	0.00024	0.00181
7	0.00223	0.00028	0.00332
8	0.00134	0.00023	0.00180

BIAS OF AVERAGE ESTIMATE

1	0.00175	0.00106	-0.00281
2	0.00136	0.00074	-0.00215
3	0.00178	0.00103	-0.00281
4	0.00178	0.00116	-0.00294
5	0.00136	0.00074	-0.00210
6	0.00152	0.00138	-0.00290
7	0.00136	0.00074	-0.00210
8	0.00307	-0.00026	-0.00281

SAMPLE VARIANCE OF ESTIMATOR

1	0.03861	0.01592	0.04584
2	0.04943	0.01752	0.06009
3	0.03737	0.01330	0.04584
4	0.03939	0.02171	0.04643
5	0.04943	0.01752	0.06009
6	0.03865	0.01621	0.04591
7	0.04943	0.01752	0.06009
8	0.03947	0.01589	0.04584

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 10 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 6

ESTIMATOR NO.   OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.02976	0.01442	0.03420
2	0.04910	0.01763	0.06039
3	0.02779	0.00982	0.03420
4	0.03215	0.02152	0.03466
5	0.04910	0.01763	0.06039
6	0.02986	0.01448	0.03422
7	0.04910	0.01763	0.06039
8	0.03045	0.01492	0.03420

AVERAGE SQUARED ERROR

1	0.00138	0.00033	0.00182
2	0.00363	0.00048	0.00550
3	0.00121	0.00015	0.00182
4	0.00157	0.00072	0.00185
5	0.00363	0.00048	0.00550
6	0.00139	0.00034	0.00182
7	0.00363	0.00048	0.00550
8	0.00145	0.00035	0.00182

BIAS OF AVERAGE ESTIMATE

1	0.00147	0.00065	-0.00213
2	0.00122	0.00039	-0.00161
3	0.00179	0.00034	-0.00213
4	0.00150	0.00068	-0.00218
5	0.00122	0.00039	-0.00161
6	0.00066	0.00156	-0.00223
7	0.00122	0.00039	-0.00161
8	-0.00023	0.00236	-0.00213

SAMPLE VARIANCE OF ESTIMATOR

1	0.03958	0.01888	0.04593
2	0.06100	0.02248	0.07528
3	0.03759	0.01314	0.04593
4	0.04162	0.02722	0.04649
5	0.06100	0.02248	0.07528
6	0.03973	0.01900	0.04599
7	0.06100	0.02248	0.07528
8	0.04051	0.01939	0.04593

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 10 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 8

ESTIMATOR NO.   OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.03277	0.01942	0.03385
2	0.07251	0.02475	0.08752
3	0.02741	0.00995	0.03385
4	0.03978	0.03190	0.03385
5	0.07251	0.02475	0.08752
6	0.03346	0.01985	0.03385
7	0.07251	0.02475	0.08752
8	0.03382	0.02024	0.03385

AVERAGE SQUARED ERROR

1	0.00166	0.00061	0.00173
2	0.00804	0.00095	0.01186
3	0.00114	0.00015	0.00173
4	0.00240	0.00153	0.00175
5	0.00804	0.00095	0.01186
6	0.00171	0.00064	0.00173
7	0.00804	0.00095	0.01186
8	0.00177	0.00066	0.00173

BIAS OF AVERAGE ESTIMATE

1	0.00154	0.00120	-0.00273
2	0.00042	0.00001	-0.00042
3	0.00175	0.00098	-0.00273
4	0.00163	0.00122	-0.00286
5	0.00042	0.00001	-0.00042
6	-0.00222	0.00516	-0.00294
7	0.00042	0.00001	-0.00042
8	0.00178	0.00096	-0.00273

SAMPLE VARIANCE OF ESTIMATOR

1	0.04337	0.02525	0.04536
2	0.09070	0.03151	0.11044
3	0.03657	0.01369	0.04536
4	0.05104	0.03937	0.04549
5	0.09070	0.03151	0.11044
6	0.04383	0.02523	0.04535
7	0.09070	0.03151	0.11044
8	0.04467	0.02639	0.04536

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 15 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 3

ESTIMATOR NO.   OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

Estimator			
1	0.02056	0.00787	0.02482
2	0.02446	0.00863	0.02989
3	0.02029	0.00724	0.02482
4	0.02089	0.00982	0.02489
5	0.02446	0.00863	0.02989
6	0.02055	0.00791	0.02479
7	0.02446	0.00863	0.02989
8	0.02104	0.00795	0.02482

AVERAGE SQUARED ERROR

Estimator			
1	0.00068	0.00010	0.00098
2	0.00092	0.00012	0.00138
3	0.00066	0.00008	0.00098
4	0.00069	0.00016	0.00099
5	0.00092	0.00012	0.00138
6	0.00068	0.00010	0.00098
7	0.00092	0.00012	0.00138
8	0.00071	0.00010	0.00098

BIAS OF AVERAGE ESTIMATE

Estimator			
1	-0.00020	0.00001	0.00019
2	-0.00065	-0.00015	0.00080
3	-0.00018	-0.00001	0.00019
4	-0.00027	0.00023	0.00004
5	-0.00065	-0.00015	0.00081
6	-0.00024	0.00010	0.00014
7	-0.00065	-0.00015	0.00081
8	0.00101	-0.00120	0.00019

SAMPLE VARIANCE OF ESTIMATOR

Estimator			
1	0.02960	0.01083	0.03536
2	0.03341	0.01163	0.04057
3	0.02920	0.01015	0.03536
4	0.02980	0.01321	0.03561
5	0.03341	0.01163	0.04057
6	0.02960	0.01091	0.03536
7	0.03341	0.01163	0.04057
8	0.03021	0.01090	0.03536

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 15 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 6

ESTIMATOR NO.   OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.02073	0.00932	0.02473
2	0.02889	0.01070	0.03560
3	0.02002	0.00739	0.02473
4	0.02125	0.01344	0.02542
5	0.02889	0.01070	0.03560
6	0.02074	0.00952	0.02484
7	0.02889	0.01070	0.03560
8	0.02124	0.00962	0.02473

AVERAGE SQUARED ERROR

1	0.00067	0.00014	0.00095
2	0.00132	0.00018	0.00199
3	0.00062	0.00008	0.00095
4	0.00071	0.00029	0.00099
5	0.00132	0.00018	0.00199
6	0.00068	0.00014	0.00095
7	0.00132	0.00018	0.00199
8	0.00070	0.00015	0.00095

BIAS OF AVERAGE ESTIMATE

1	0.00024	-0.00006	-0.00018
2	-0.00039	-0.00031	0.00070
3	0.00009	0.00009	-0.00018
4	0.00013	0.00021	-0.00034
5	-0.00039	-0.00031	0.00070
6	0.00008	0.00018	-0.00026
7	-0.00039	-0.00031	0.00070
8	0.00140	-0.00122	-0.00018

SAMPLE VARIANCE OF ESTIMATOR

1	0.02980	0.01291	0.03594
2	0.03867	0.01454	0.04780
3	0.02896	0.01064	0.03594
4	0.03032	0.01775	0.03661
5	0.03867	0.01454	0.04780
6	0.02983	0.01312	0.03602
7	0.03867	0.01454	0.04780
8	0.03047	0.01304	0.03594

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 15 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 9

ESTIMATOR NO. OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

Estimator No.	P(1)	P(2)	P(3)
1	0.02160	0.01083	0.02471
2	0.03862	0.01404	0.04775
3	0.02000	0.00723	0.02471
4	0.02356	0.01704	0.02501
5	0.03862	0.01404	0.04775
6	0.02172	0.01098	0.02474
7	0.03862	0.01404	0.04775
8	0.02216	0.01143	0.02471

AVERAGE SQUARED ERROR

Estimator No.	P(1)	P(2)	P(3)
1	0.00074	0.00019	0.00094
2	0.00228	0.00031	0.00349
3	0.00062	0.00008	0.00094
4	0.00087	0.00045	0.00097
5	0.00228	0.00031	0.00349
6	0.00074	0.00020	0.00094
7	0.00228	0.00031	0.00349
8	0.00078	0.00021	0.00094

BIAS OF AVERAGE ESTIMATE

Estimator No.	P(1)	P(2)	P(3)
1	-0.00036	-0.00609	0.00045
2	-0.00297	-0.00099	0.00396
3	-0.00059	0.00314	0.00045
4	-0.00102	0.00096	0.00006
5	-0.00297	-0.00099	0.00396
6	-0.00087	0.00056	0.00031
7	-0.00297	-0.00099	0.00396
8	0.00045	-0.00090	0.00045

SAMPLE VARIANCE OF ESTIMATOR

Estimator No.	P(1)	P(2)	P(3)
1	0.03026	0.01475	0.03491
2	0.04975	0.01834	0.06166
3	0.02829	0.01045	0.03491
4	0.03252	0.02172	0.03519
5	0.04975	0.01834	0.06166
6	0.03036	0.01489	0.03492
7	0.04975	0.01834	0.06166
8	0.03116	0.01525	0.03491

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 15 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 12

ESTIMATOR NO. OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

Estimator			
1	0.02490	0.01599	0.02513
2	0.05783	0.02037	0.07038
3	0.02047	0.00734	0.02513
4	0.02984	0.02571	0.02544
5	0.05783	0.02037	0.07038
6	0.02511	0.01603	0.02518
7	0.05783	0.02037	0.07038
8	0.02596	0.01698	0.02513

AVERAGE SQUARED ERROR

Estimator			
1	0.00097	0.00040	0.00099
2	0.00499	0.00064	0.00747
3	0.00067	0.00008	0.00099
4	0.00139	0.00101	0.00101
5	0.00499	0.00064	0.00747
6	0.00099	0.00041	0.00100
7	0.00499	0.00064	0.00747
8	0.00105	0.00045	0.00099

BIAS OF AVERAGE ESTIMATE

Estimator			
1	0.00026	0.00050	-0.00076
2	-0.00105	-0.00054	0.00159
3	0.00079	-0.00004	-0.00076
4	0.00019	0.00070	-0.00088
5	-0.00105	-0.00054	0.00159
6	-0.00152	0.00240	-0.00088
7	-0.00105	-0.00054	0.00159
8	0.00023	0.00052	-0.00076

SAMPLE VARIANCE OF ESTIMATOR

Estimator			
1	0.03436	0.02037	0.03641
2	0.07221	0.02566	0.08856
3	0.02976	0.01035	0.03641
4	0.04004	0.03196	0.03668
5	0.07221	0.02566	0.08856
6	0.03458	0.02059	0.03645
7	0.07221	0.02566	0.08856
8	0.03567	0.02154	0.03641

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 20 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 5 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 1

ESTIMATOR NO.   OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

No.		
1	0.03868	0.01493	0.04725
2	0.04432	0.01628	0.05502
3	0.03831	0.01370	0.04725
4	0.03897	0.01822	0.04752
5	0.04432	0.01628	0.05502
6	0.03869	0.01496	0.04722
7	0.04432	0.01628	0.05502
8	0.03952	0.01486	0.04725

AVERAGE SQUARED ERROR

No.		
1	0.00232	0.00035	0.00343
2	0.00304	0.00041	0.00465
3	0.00228	0.00030	0.00343
4	0.00236	0.00052	0.00348
5	0.00304	0.00041	0.00465
6	0.00232	0.00036	0.00343
7	0.00304	0.00041	0.00465
8	0.00241	0.00035	0.00343

BIAS OF AVERAGE ESTIMATE

No.		
1	0.00174	0.00229	-0.00403
2	0.00208	0.00232	-0.00440
3	0.00216	0.00188	-0.00403
4	0.00185	0.00209	-0.00394
5	0.00208	0.00232	-0.00440
6	0.00159	0.00246	-0.00404
7	0.00208	0.00232	-0.00440
8	0.00286	0.00117	-0.00403

SAMPLE VARIANCE OF ESTIMATOR

No.		
1	0.05416	0.02058	0.06615
2	0.06099	0.02202	0.07541
3	0.05369	0.01929	0.06615
4	0.05459	0.02416	0.06637
5	0.06099	0.02202	0.07541
6	0.05417	0.02067	0.06611
7	0.06099	0.02202	0.07541
8	0.05504	0.02054	0.06615

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 20 SEGMENTS,
THE SEGMENT SAMPLE SIZF IS 5 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 2

ESTIMATOR NO.   OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.03852	0.01719	0.04629
2	0.05255	0.01942	0.06508
3	0.03779	0.01385	0.04629
4	0.03970	0.02361	0.04686
5	0.05255	0.01942	0.06508
6	0.03970	0.01726	0.04634
7	0.05255	0.01942	0.06508
8	0.03937	0.01761	0.04629

AVERAGE SQUARED ERROR

1	0.00237	0.00046	0.00338
2	0.00425	0.00059	0.00644
3	0.00225	0.00029	0.00338
4	0.00247	0.00088	0.00348
5	0.00425	0.00059	0.00644
6	0.00238	0.00048	0.00339
7	0.00425	0.00059	0.00644
8	0.00247	0.00049	0.00338

BIAS OF AVERAGE ESTIMATE

1	0.00170	0.00145	-0.00315
2	0.00214	0.00119	-0.00333
3	0.00233	0.00081	-0.00315
4	0.00203	0.00108	-0.00312
5	0.00214	0.00119	-0.00333
6	0.00103	0.00223	-0.00326
7	0.00214	0.00119	-0.00333
8	0.00228	0.00087	-0.00315

SAMPLE VARIANCE OF ESTIMATOR

1	0.05557	0.02279	0.05599
2	0.07176	0.02550	0.08762
3	0.05435	0.01888	0.06599
4	0.05691	0.03048	0.06642
5	0.07176	0.02550	0.08762
6	0.05574	0.02288	0.05600
7	0.07176	0.02550	0.08762
8	0.05662	0.02329	0.06599

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 20 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 5 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 3

ESTIMATOR NO. OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.04167	0.01989	0.04768
2	0.06821	0.02423	0.08368
3	0.03894	0.01398	0.04768
4	0.04444	0.02949	0.04808
5	0.06821	0.02423	0.08368
6	0.04247	0.01991	0.04762
7	0.06821	0.02423	0.08368
8	0.04255	0.02020	0.04768

AVERAGE SQUARED ERROR

1	0.00265	0.00064	0.00347
2	0.00697	0.00089	0.01047
3	0.00232	0.00030	0.00347
4	0.00302	0.00137	0.00352
5	0.00697	0.00089	0.01047
6	0.00275	0.00065	0.00346
7	0.00697	0.00089	0.01047
8	0.00276	0.00066	0.00347

BIAS OF AVERAGE ESTIMATE

1	0.00179	0.00150	-0.00330
2	0.00262	0.00103	-0.00364
3	0.00202	0.00128	-0.00330
4	0.00242	0.00083	-0.00326
5	0.00262	0.00103	-0.00364
6	-0.00045	0.00395	-0.00350
7	0.00262	0.00103	-0.00364
8	0.00200	0.00130	-0.00330

SAMPLE VARIANCE OF ESTIMATOR

1	0.05775	0.02695	0.06750
2	0.08753	0.03133	0.10764
3	0.05482	0.01958	0.06750
4	0.06093	0.03797	0.06788
5	0.08753	0.03133	0.10764
6	0.05870	0.02673	0.06746
7	0.08753	0.03133	0.10764
8	0.05894	0.02726	0.06750

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 20 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 5 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 4

ESTIMATOR NO. OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

Estimator			
1	0.04639	0.02874	0.04719
2	0.10052	0.03585	0.12135
3	0.03850	0.01366	0.04719
4	0.05433	0.04491	0.04793
5	0.10052	0.03585	0.12135
6	0.06095	0.04389	0.04743
7	0.10052	0.03585	0.12135
8	0.04639	0.02874	0.04719

AVERAGE SQUARED ERROR

Estimator			
1	0.00329	0.00138	0.00350
2	0.01476	0.00191	0.02179
3	0.00232	0.00029	0.00350
4	0.00446	0.00310	0.00356
5	0.01476	0.00191	0.02179
6	0.03216	0.03223	0.00353
7	0.01476	0.00191	0.02179
8	0.00329	0.00138	0.00350

BIAS OF AVERAGE ESTIMATE

Estimator			
1	0.00090	0.00365	-0.00455
2	0.00385	0.00160	-0.00546
3	0.00309	0.00146	-0.00455
4	0.00337	0.00112	-0.00450
5	0.00385	0.00160	-0.00545
6	-0.02031	0.02618	-0.00587
7	0.00385	0.00160	-0.00545
8	0.00090	0.00365	-0.00455

SAMPLE VARIANCE OF ESTIMATOR

Estimator			
1	0.06352	0.03725	0.06642
2	0.12537	0.04408	0.15162
3	0.05456	0.01870	0.06642
4	0.07259	0.05594	0.06682
5	0.12537	0.04409	0.15162
6	0.18089	0.17748	0.06636
7	0.12537	0.04408	0.15162
8	0.06152	0.03725	0.06642

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 20 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 10 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 2

ESTIMATOR NO. OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.02312	0.00852	0.02721
2	0.02779	0.00960	0.03379
3	0.02269	0.00758	0.02721
4	0.02341	0.01145	0.02759
5	0.02779	0.00960	0.03379
6	0.02312	0.00859	0.02722
7	0.02779	0.00960	0.03379
8	0.02362	0.00869	0.02721

AVERAGE SQUARED ERROR

1	0.00082	0.00012	0.00116
2	0.00119	0.00014	0.00176
3	0.00079	0.00009	0.00116
4	0.00084	0.00021	0.00118
5	0.00119	0.00014	0.00176
6	0.00082	0.00012	0.00116
7	0.00119	0.00014	0.00176
8	0.00085	0.00012	0.00116

BIAS OF AVERAGE ESTIMATE

1	0.00226	0.00031	-0.00257
2	0.00224	0.00026	-0.00250
3	0.00226	0.00031	-0.00257
4	0.00228	0.00030	-0.00258
5	0.00224	0.00026	-0.00250
6	0.00219	0.00039	-0.00258
7	0.00224	0.00026	-0.00250
8	0.00354	-0.00097	-0.00257

SAMPLE VARIANCE OF ESTIMATOR

1	0.03809	0.01434	0.04648
2	0.04313	0.01537	0.05324
3	0.03768	0.01347	0.04648
4	0.03843	0.01694	0.04657
5	0.04313	0.01538	0.05324
6	0.03809	0.01440	0.04645
7	0.04313	0.01537	0.05324
8	0.03876	0.01413	0.04648

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 20 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 10 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 4

ESTIMATOR NO.   OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

Estimator No.			
1	0.02323	0.01051	0.02712
2	0.03384	0.01210	0.04135
3	0.02215	0.00809	0.02712
4	0.02406	0.01584	0.02747
5	0.03384	0.01210	0.04135
6	0.02325	0.01078	0.02712
7	0.03384	0.01210	0.04135
8	0.02399	0.01068	0.02712

AVERAGE SQUARED ERROR

Estimator No.			
1	0.00084	0.00017	0.00116
2	0.00182	0.00023	0.00270
3	0.00077	0.00010	0.00116
4	0.00091	0.00039	0.00120
5	0.00182	0.00023	0.00270
6	0.00084	0.00018	0.00116
7	0.00182	0.00023	0.00270
8	0.00090	0.00018	0.00116

BIAS OF AVERAGE ESTIMATE

Estimator No.			
1	0.00156	0.00076	-0.00232
2	0.00139	0.00052	-0.00190
3	0.00129	0.00103	-0.00232
4	0.00163	0.00076	-0.00240
5	0.00139	0.00052	-0.00190
6	0.00135	0.00106	-0.00240
7	0.00139	0.00052	-0.00190
8	0.00286	-0.00053	-0.00232

SAMPLE VARIANCE OF ESTIMATOR

Estimator No.			
1	0.03789	0.01613	0.04535
2	0.04869	0.01768	0.05946
3	0.03672	0.01363	0.04536
4	0.03868	0.02181	0.04596
5	0.04869	0.01768	0.05946
6	0.03791	0.01641	0.04543
7	0.04869	0.01768	0.05946
8	0.03873	0.01612	0.04536

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 20 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 10 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 6

ESTIMATOR NO.   OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.02407	0.01306	0.02694
2	0.04533	0.01629	0.05568
3	0.02206	0.00784	0.02694
4	0.02688	0.02077	0.02739
5	0.04533	0.01629	0.05568
6	0.02419	0.01324	0.02695
7	0.04533	0.01629	0.05568
8	0.02476	0.01362	0.02694

AVERAGE SQUARED ERROR

1	0.00092	0.00027	0.00117
2	0.00315	0.00041	0.00478
3	0.00077	0.00010	0.00117
4	0.00111	0.00066	0.00120
5	0.00315	0.00041	0.00478
6	0.00093	0.00028	0.00117
7	0.00315	0.00041	0.00478
8	0.00098	0.00029	0.00117

BIAS OF AVERAGE ESTIMATE

1	0.00088	0.00061	-0.00149
2	0.00031	0.00020	-0.00051
3	0.00100	0.00049	-0.00149
4	0.00085	0.00075	-0.00159
5	0.00031	0.00020	-0.00051
6	0.00009	0.00151	-0.00160
7	0.00031	0.00020	-0.00051
8	0.00186	-0.00037	-0.00149

SAMPLE VARIANCE OF ESTIMATOR

1	0.03923	0.01871	0.04574
2	0.06086	0.02221	0.07509
3	0.03741	0.01300	0.04574
4	0.04141	0.02704	0.04622
5	0.06086	0.02221	0.07509
6	0.03935	0.01885	0.04579
7	0.06086	0.02221	0.07509
8	0.04009	0.01920	0.04574

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 20 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 10 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 8

ESTIMATOR NO. OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.02864	0.01852	0.02786
2	0.07057	0.02414	0.08563
3	0.02289	0.00799	0.02786
4	0.03621	0.03107	0.02783
5	0.07057	0.02414	0.08563
6	0.02895	0.01918	0.02782
7	0.07057	0.02414	0.08563
8	0.02961	0.01930	0.02786

AVERAGE SQUARED ERROR

1	0.00128	0.00055	0.00120
2	0.00761	0.00091	0.01128
3	0.00080	0.00010	0.00120
4	0.00202	0.00148	0.00121
5	0.00761	0.00091	0.01128
6	0.00131	0.00059	0.00120
7	0.00761	0.00091	0.01128
8	0.00137	0.00060	0.00120

BIAS OF AVERAGE ESTIMATE

1	0.00179	0.00108	-0.00286
2	0.00150	0.00029	-0.00180
3	0.00207	0.00079	-0.00286
4	0.00206	0.00085	-0.00292
5	0.00150	0.00029	-0.00179
6	-0.00207	0.00512	-0.00305
7	0.00150	0.00029	-0.00179
8	0.00195	0.00091	-0.00286

SAMPLE VARIANCE OF ESTIMATOR

1	0.04350	0.02539	0.04580
2	0.09067	0.03173	0.11043
3	0.03698	0.01375	0.04580
4	0.05127	0.03944	0.04591
5	0.09057	0.03173	0.11043
6	0.04400	0.02523	0.04579
7	0.09067	0.03173	0.11043
8	0.04463	0.02540	0.04580

APPENDIX B

MONTE CARLO STUDY USING THE

1978 BARLEY DATA SET

THE NUMBER OF COMMENTS TO BE PRINTED =   49

NASA
PARTIAL IDENTIFICATION PROBLEM
SIMULATOR FOR SINGLE YEAR ESTIMATION PROCEDURES

TWO CROPS OF INTEREST

DECEMBER 1981

EIGHT ESTIMATION PROCEDURES ARE CONSIDERED

1. MAXIMUM LIKELIHOOD ESTIMATION USING BOTH PARTIALLY
    AND COMPLETELY IDENTIFIED SAMPLE SEGMENTS

2. MAXIMUM LIKELIHOOD ESTIMATION USING ONLY THE
    COMPLETELY IDENTIFIED SAMPLE SEGMENTS

3. MAXIMUM LIKELIHOOD ESTIMATION PRETENDING THAT ALL
    SAMPLE SEGMENTS WERE COMPLETELY IDENTIFIED

4. UNWEIGHTED LEAST SQUARES ESTIMATION USING BOTH
    PARTIALLY AND COMPLETELY IDENTIFIED SAMPLE SEGMENTS

5. UNWEIGHTED LEAST SQUARES ESTIMATION USING ONLY THE
    COMPLETELY IDENTIFIED SAMPLE SEGMENTS

6. WEIGHTED LEAST SQUARES ESTIMATION USING BOTH
    PARTIALLY AND COMPLETELY IDENTIFIED SAMPLE SEGMENTS
    AND WEIGHTS INVERSELY PROPORTIONAL TO THE VARIANCES

7. WEIGHTED LEAST SQUARES ESTIMATION USING ONLY THE
    COMPLETELY IDENTIFIED SAMPLE SEGMENTS
    AND WEIGHTS INVERSELY PROPORTIONAL TO THE VARIANCES

8. THE LEAST SQUARES ESTIMATOR OF THE RATIO OF CROP 1'S
    ACREAGE PROPORTION TO CROP 1 AND 2'S ACREAGE PROPORTION
    TIMES THE MAXIMUM LIKELIHOOD ESTIMATOR OF THE
    COMBINED ACREAGE PROPORTION FOR CROPS 1 AND 2

THE FINITE SUPER POPULATION IS THE CAMS ESTIMATES FOR
NORTH DAKOTA IN 1978

CROP 1 IS BARLEY
CROP 2 IS SPRING SMALL GRAINS LESS BARLEY

THE NUMBER OF SEGMENTS IN THE FINITE SUPER POPULATION IS 76

THE SEGMENT PERCENTAGES IN THE SUPER POPULATION ARE AS FOLLOWS: SEGMENT NUMBER, PROPORTIONS

1	0.000	47.000	53.000
2	2.700	40.300	57.000
3	4.200	22.100	73.700
4	12.000	31.000	57.000
5	2.000	33.000	65.000
6	9.000	32.000	59.000
7	4.000	32.000	64.000
8	3.100	30.900	66.000
9	4.200	14.300	81.500
10	1.000	29.000	70.000
11	15.200	26.700	58.100
12	6.000	48.100	45.900
13	7.700	47.900	44.400
14	17.300	22.600	60.100
15	13.700	17.800	68.500
16	3.700	30.000	66.300
17	14.000	18.000	68.000
18	12.000	36.700	51.300
19	10.000	26.200	63.800
20	18.000	35.000	47.000
21	8.400	49.900	41.700
22	7.800	38.700	53.500
23	6.000	31.000	63.000
24	1.500	13.500	85.000
25	4.000	42.000	54.000
26	7.000	41.000	52.000
27	11.000	17.000	72.000
28	0.000	34.000	66.000
29	0.600	21.600	77.800
30	13.000	40.000	47.000
31	5.000	19.000	76.000
32	0.000	19.000	81.000
33	2.800	47.200	50.000
34	3.000	29.000	68.000
35	0.000	7.000	93.000
36	9.500	31.400	59.100
37	4.000	38.000	58.000
38	15.900	40.000	44.100
39	0.600	9.300	90.100
40	3.000	33.000	64.000
41	0.900	7.100	92.000
42	2.200	19.800	78.000
43	1.200	3.600	95.200
44	8.000	23.000	69.000
45	2.000	41.000	57.000
46	14.000	36.000	50.000

47	17.300	23.800	58.900
48	4.400	16.000	79.600
49	0.000	9.600	90.400
50	0.000	19.000	81.000
51	0.000	9.600	90.400
52	4.300	15.000	80.700
53	5.000	27.000	68.000
54	1.000	3.000	96.000
55	1.600	3.600	94.600
56	7.600	25.000	67.400
57	4.000	35.000	61.000
58	1.700	15.300	83.000
59	5.000	26.000	69.000
60	8.000	18.000	74.000
61	10.000	24.000	66.000
62	4.500	39.300	56.200
63	4.000	36.000	60.000
64	1.700	13.000	85.300
65	0.000	10.000	90.000
66	0.600	15.000	84.400
67	0.000	12.900	87.100
68	6.100	21.700	72.200
69	0.000	9.000	91.000
70	2.000	23.700	74.300
71	5.000	20.000	75.000
72	4.000	5.000	91.000
73	2.000	17.000	81.000
74	5.000	19.000	76.000
75	1.000	17.000	82.000
76	4.000	29.000	67.000

THERE ARE 2 STRATUM SIZES TO BE CONSIDERED.
THE DIFFERENT STRATUM SIZES ARE
40   20

FOR STRATA OF THE 1-TH SIZE, THE FOLLOWING SEGMENT SAMPLE SIZES ARE CONSIDERED:
5   15

FOR STRATA OF THE 2-TH SIZE, THE FOLLOWING SEGMENT SAMPLE SIZES ARE CONSIDERED:
5   10

FOR STRATA OF THE 1-TH SIZE AND SEGMENT SAMPLES OF SIZE 5
THE FOLLOWING NUMBERS OF PARTIALLY IDENTIFIED SEGMENTS ARE CONSIDERED:
1   2   3   4

FOR STRATA OF THE 1-TH SIZE AND SEGMENT SAMPLES OF SIZE 10
THE FOLLOWING NUMBERS OF PARTIALLY IDENTIFIED SEGMENTS ARE CONSIDERED:
2   4   6   8

FOR STRATA OF THE 1-TH SIZE AND SEGMENT SAMPLES OF SIZE 15
THE FOLLOWING NUMBERS OF PARTIALLY IDENTIFIED SEGMENTS ARE CONSIDERED:
3   6   9   12

FOR STRATA OF THE 2-TH SIZE AND SEGMENT SAMPLES OF SIZE 5
THE FOLLOWING NUMBERS OF PARTIALLY IDENTIFIED SEGMENTS ARE CONSIDERED:
1   2   3   4

FOR STRATA OF THE 2-TH SIZE AND SEGMENT SAMPLES OF SIZE 10
THE FOLLOWING NUMBERS OF PARTIALLY IDENTIFIED SEGMENTS ARE CONSIDERED:
2   4   6   8

1000 SEGMENT SAMPLES ARE TO BE TAKEN FOR EACH AMOUNT OF PARTIAL IDENTIFICATION.

THE TRUE SUPER POPULATION PROPORTIONS WERE :   0.05279   0.25134   0.69587

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 5 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 1

ESTIMATOR NO. OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.01825	0.04259	0.05082
2	0.01885	0.04747	0.05655
3	0.01697	0.04214	0.05082
4	0.02150	0.04291	0.05138
5	0.01885	0.04747	0.05655
6	0.01821	0.04260	0.05087
7	0.01885	0.04747	0.05655
8	0.01936	0.04333	0.05082

AVERAGE SQUARED ERROR

1	0.00653	0.00279	0.00389
2	0.00055	0.00338	0.00479
3	0.00045	0.00271	0.00389
4	0.00072	0.00281	0.00399
5	0.00055	0.00338	0.00479
6	0.00053	0.00279	0.00390
7	0.00055	0.00338	0.00479
8	0.00058	0.00288	0.00389

BIAS OF AVERAGE ESTIMATE

1	0.00161	0.00615	-0.00776
2	0.00170	0.00593	-0.00763
3	0.00154	0.00622	-0.00776
4	0.00178	0.00631	-0.00779
5	0.00170	0.00593	-0.00763
6	0.00181	0.00598	-0.00779
7	0.00170	0.00593	-0.00763
8	0.00335	0.00441	-0.00776

SAMPLE VARIANCE OF ESTIMATOR

1	0.02388	0.05423	0.06427
2	0.02448	0.06019	0.07183
3	0.02209	0.05358	0.06427
4	0.02747	0.05464	0.06477
5	0.02448	0.06019	0.07183
6	0.02390	0.05426	0.06430
7	0.02448	0.06019	0.07183
8	0.02492	0.05529	0.06427

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 5 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 2

ESTIMATOR NO. OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

Estimator No.		
1	0.02047	0.05229
2	0.02200	0.05785
3	0.01699	0.04307
4	0.02661	0.05276
5	0.02200	0.05785
6	0.02025	0.04477
7	0.02200	0.05785
8	0.02176	0.04548

AVERAGE SQUARED ERROR

Estimator No.		
1	0.00065	0.00416
2	0.00074	0.00711
3	0.00044	0.00416
4	0.00107	0.00423
5	0.00074	0.00711
6	0.00064	0.00417
7	0.00074	0.00711
8	0.00072	0.00416

BIAS OF AVERAGE ESTIMATE

Estimator No.		
1	0.00077	-0.00420
2	0.00069	-0.00206
3	0.00078	-0.00420
4	0.00195	-0.00458
5	0.00069	-0.00206
6	0.00170	-0.00434
7	0.00069	-0.00206
8	0.00246	-0.00420

SAMPLE VARIANCE OF ESTIMATOR

Estimator No.		
1	0.02612	0.06662
2	0.02791	0.08706
3	0.02173	0.05515
4	0.03305	0.05869
5	0.02791	0.08700
6	0.02592	0.05713
7	0.02791	0.08700
8	0.02742	0.05794

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 5 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 3

ESTIMATOR NO. OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.02393	0.04626	0.05160
2	0.02727	0.06885	0.08351
3	0.01666	0.04260	0.05160
4	0.03111	0.04836	0.05204
5	0.02727	0.06885	0.08351
6	0.02316	0.04650	0.05161
7	0.02727	0.06985	0.08351
8	0.02524	0.04702	0.05160

AVERAGE SQUARED ERROR

1	0.00087	0.00324	0.00402
2	0.00113	0.00699	0.01021
3	0.00044	0.00280	0.00402
4	0.00149	0.00356	0.00406
5	0.00113	0.00699	0.01021
6	0.00085	0.00329	0.00401
7	0.00113	0.00699	0.01021
8	0.00098	0.00332	0.00402

BIAS OF AVERAGE ESTIMATE

1	0.00077	0.00658	-0.00734
2	0.00101	0.00109	-0.00211
3	0.00177	0.00557	-0.00734
4	0.00392	0.00400	-0.00792
5	0.00101	0.00109	-0.00211
6	0.00351	0.00407	-0.00758
7	0.00101	0.00109	-0.00211
8	0.00220	0.00515	-0.00734

SAMPLE VARIANCE OF ESTIMATOR

1	0.03023	0.05824	0.06531
2	0.03411	0.08488	0.10245
3	0.02162	0.05465	0.06531
4	0.03901	0.06126	0.06564
5	0.03411	0.08488	0.10245
6	0.02965	0.05890	0.06529
7	0.03411	0.08488	0.10245
8	0.03199	0.05914	0.06531

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 5 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 4

ESTIMATOR NO. OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.03330	0.05145	0.05142
2	0.03893	0.10254	0.12268
3	0.01704	0.04262	0.05142
4	0.04530	0.05901	0.05163
5	0.03893	0.10254	0.12268
6	0.03469	0.05360	0.05150
7	0.03893	0.10254	0.12268
8	0.03330	0.05145	0.05142

AVERAGE SQUARED ERROR

1	0.00165	0.00407	0.00400
2	0.00226	0.01443	0.02104
3	0.00045	0.00276	0.00400
4	0.00314	0.00506	0.00404
5	0.00226	0.01443	0.02104
6	0.00424	0.00658	0.00402
7	0.00226	0.01443	0.02104
8	0.00165	0.00407	0.00400

BIAS OF AVERAGE ESTIMATE

1	-0.00519	0.01184	-0.00665
2	-0.00429	-0.00917	0.01345
3	0.00123	0.00542	-0.00665
4	0.00629	0.00141	-0.00771
5	-0.00429	-0.00917	0.01345
6	0.00970	-0.00236	-0.00734
7	-0.00429	-0.00917	0.01345
8	-0.00519	0.01184	-0.00665

SAMPLE VARIANCE OF ESTIMATOR

1	0.04041	0.06457	0.06516
2	0.04752	0.12068	0.14524
3	0.02188	0.05425	0.06516
4	0.05603	0.07261	0.06540
5	0.04752	0.12068	0.14524
6	0.06451	0.08247	0.06522
7	0.04752	0.12068	0.14524
8	0.04041	0.06457	0.06516

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 10 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 2

ESTIMATOR NO.   OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.01161	0.02941	0.03410
2	0.01201	0.03257	0.03798
3	0.01064	0.02880	0.03410
4	0.01428	0.02935	0.03464
5	0.01201	0.03257	0.03798
6	0.01164	0.02941	0.03416
7	0.01201	0.03257	0.03798
8	0.01245	0.03017	0.03410

AVERAGE SQUARED ERROR

1	0.00021	0.00135	0.00181
2	0.00023	0.00168	0.00229
3	0.00018	0.00129	0.00181
4	0.00032	0.00137	0.00186
5	0.00023	0.00168	0.00229
6	0.00021	0.00135	0.00182
7	0.00023	0.00168	0.00229
8	0.00024	0.00142	0.00181

BIAS OF AVERAGE ESTIMATE

1	0.00094	0.00219	-0.00313
2	0.00078	0.00140	-0.00217
3	0.00089	0.00224	-0.00313
4	0.00138	0.00200	-0.00337
5	0.00078	0.00140	-0.00217
6	0.00107	0.00212	-0.00319
7	0.00078	0.00140	-0.00217
8	0.00280	0.00033	-0.00313

SAMPLE VARIANCE OF ESTIMATOR

1	0.01584	0.03979	0.04628
2	0.01634	0.04435	0.05200
3	0.01483	0.03903	0.04628
4	0.01880	0.04013	0.04654
5	0.01634	0.04435	0.05200
6	0.01589	0.03979	0.04628
7	0.01634	0.04435	0.05200
8	0.01666	0.04075	0.04628

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SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 10 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 4

ESTIMATOR NO.   OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.01361	0.02824	0.03318
2	0.01488	0.03800	0.04567
3	0.01084	0.02759	0.03318
4	0.01794	0.02915	0.03361
5	0.01488	0.03800	0.04567
6	0.01356	0.02828	0.03322
7	0.01488	0.03800	0.04567
8	0.01458	0.02897	0.03318

AVERAGE SQUARED ERROR

1	0.00029	0.00124	0.00165
2	0.00035	0.00223	0.00323
3	0.00019	0.00117	0.00165
4	0.00049	0.00131	0.00168
5	0.00035	0.00223	0.00323
6	0.00029	0.00124	0.00165
7	0.00035	0.00223	0.00323
8	0.00033	0.00130	0.00165

BIAS OF AVERAGE ESTIMATE

1	-0.00016	0.00282	-0.00266
2	-0.00102	-0.00261	0.00363
3	0.00046	0.00220	-0.00266
4	0.00268	0.00109	-0.00377
5	-0.00102	-0.00261	0.00363
6	0.00041	0.00247	-0.00288
7	-0.00102	-0.00261	0.00363
8	0.00172	0.00094	-0.00266

SAMPLE VARIANCE OF ESTIMATOR

1	0.01862	0.03868	0.04465
2	0.01945	0.05015	0.06011
3	0.01490	0.03764	0.04465
4	0.02257	0.03980	0.04488
5	0.01945	0.05015	0.06011
6	0.01797	0.03870	0.04468
7	0.01945	0.05015	0.06011
8	0.01962	0.03956	0.04465

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 10 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 6

ESTIMATOR NO. OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.01625	0.03156	0.03368
2	0.01874	0.05009	0.05918
3	0.01037	0.02879	0.03368
4	0.02282	0.03398	0.03398
5	0.01874	0.05009	0.05918
6	0.01599	0.03185	0.03369
7	0.01874	0.05009	0.05918
8	0.01784	0.03256	0.03368

AVERAGE SQUARED ERROR

1	0.00040	0.00156	0.00177
2	0.00052	0.00381	0.00527
3	0.00017	0.00128	0.00177
4	0.00083	0.00178	0.00182
5	0.00052	0.00381	0.00527
6	0.00040	0.00158	0.00178
7	0.00052	0.00381	0.00527
8	0.00047	0.00165	0.00177

BIAS OF AVERAGE ESTIMATE

1	0.00090	0.00156	-0.00246
2	0.00069	-0.00148	0.00079
3	0.00084	0.00162	-0.00246
4	0.00250	0.00033	-0.00283
5	0.00069	-0.00148	0.00079
6	0.00211	0.00049	-0.00261
7	0.00069	-0.00148	0.00079
8	0.00257	-0.00010	-0.00246

SAMPLE VARIANCE OF ESTIMATOR

1	0.02134	0.04209	0.04618
2	0.02403	0.06407	0.07572
3	0.01462	0.03900	0.04618
4	0.02957	0.04498	0.04651
5	0.02403	0.06407	0.07572
6	0.02110	0.04235	0.04620
7	0.02403	0.06407	0.07572
8	0.02297	0.04323	0.04618

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 10 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 8

ESTIMATOR NO. OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

0.02294	0.03478	0.03352
0.02759	0.06985	0.08265
0.01084	0.02809	0.03352
0.03183	0.04001	0.03388
0.02759	0.06885	0.08265
0.02272	0.03546	0.03354
0.02759	0.06885	0.08265
0.02422	0.03561	0.03352

AVERAGE SQUARED ERROR

0.00078	0.00187	0.00171
0.00111	0.00719	0.01032
0.00019	0.00123	0.00171
0.00160	0.00242	0.00175
0.00111	0.00719	0.01032
0.00083	0.00199	0.00172
0.00111	0.00719	0.01032
0.00088	0.00195	0.00171

BIAS OF AVERAGE ESTIMATE

-0.00128	0.00398	-0.00270
-0.00095	-0.00348	0.00443
0.00076	0.00194	-0.00270
0.00280	0.00028	-0.00308
-0.00095	-0.00348	0.00443
0.00354	-0.00062	-0.00293
-0.00095	-0.00348	0.00443
0.00023	0.00247	-0.00270

SAMPLE VARIANCE OF ESTIMATOR

0.02848	0.04555	0.04527
0.03388	0.08663	0.10384
0.01487	0.03836	0.04527
0.04028	0.05172	0.04551
0.03388	0.08663	0.10384
0.02906	0.04679	0.04526
0.03388	0.08663	0.10384
0.03028	0.04652	0.04527

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 15 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 3

ESTIMATOR NO.   OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.00915	0.02185	0.02597
2	0.00981	0.02639	0.03188
3	0.00806	0.02147	0.02597
4	0.01081	0.02230	0.02583
5	0.00981	0.02639	0.03188
6	0.00908	0.02186	0.02591
7	0.00981	0.02639	0.03188
8	0.00983	0.02248	0.02597

AVERAGE SQUARED ERROR

1	0.00013	0.00072	0.00101
2	0.00015	0.00105	0.00153
3	0.00010	0.00069	0.00101
4	0.00018	0.00075	0.00100
5	0.00015	0.00105	0.00153
6	0.00013	0.00072	0.00101
7	0.00015	0.00105	0.00153
8	0.00015	0.00076	0.00101

BIAS OF AVERAGE ESTIMATE

1	-0.00004	-0.00011	0.00015
2	-0.00056	-0.00292	0.00348
3	-0.00004	-0.00011	0.00015
4	0.00152	-0.00084	-0.00068
5	-0.00056	-0.00292	0.00348
6	0.00016	-0.00016	0.00001
7	-0.00056	-0.00292	0.00348
8	0.00165	-0.00180	0.00015

SAMPLE VARIANCE OF ESTIMATOR

1	0.01285	0.03072	0.03679
2	0.01349	0.03553	0.04305
3	0.01157	0.03039	0.03679
4	0.01473	0.03122	0.03670
5	0.01349	0.03553	0.04305
6	0.01282	0.03073	0.03676
7	0.01349	0.03553	0.04305
8	0.01365	0.03139	0.03679

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 15 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 6

ESTIMATOR NO. OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

Estimator No.			
1	0.01032	0.02250	0.02624
2	0.01185	0.03243	0.03952
3	0.00816	0.02161	0.02624
4	0.01406	0.02382	0.02623
5	0.01185	0.03243	0.03952
6	0.01019	0.02254	0.02619
7	0.01185	0.03243	0.03752
8	0.01117	0.02342	0.02624

AVERAGE SQUARED ERROR

Estimator No.			
1	0.00016	0.00078	0.00105
2	0.00021	0.00160	0.00234
3	0.00010	0.00072	0.00105
4	0.00031	0.00085	0.00106
5	0.00021	0.00160	0.00234
6	0.00016	0.00078	0.00104
7	0.00021	0.00160	0.00234
8	0.00019	0.00084	0.00105

BIAS OF AVERAGE ESTIMATE

Estimator No.			
1	-0.00037	-0.00062	0.00099
2	-0.00101	-0.00442	0.00544
3	0.00003	-0.00102	0.00099
4	0.00160	-0.00180	0.00020
5	-0.00101	-0.00442	0.00544
6	0.00002	-0.00085	0.00083
7	-0.00101	-0.00442	0.00544
8	0.00131	-0.00230	0.00099

SAMPLE VARIANCE OF ESTIMATOR

Estimator No.			
1	0.01401	0.03141	0.03658
2	0.01559	0.04240	0.05115
3	0.01174	0.03036	0.03658
4	0.01861	0.03256	0.03669
5	0.01559	0.04240	0.05115
6	0.01395	0.03141	0.03654
7	0.01559	0.04240	0.05115
8	0.01517	0.03224	0.03658

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 15 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 9

ESTIMATOR NO. OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

Estimator	P(1)	P(2)	P(3)
1	0.01285	0.02311	0.02616
2	0.01558	0.04031	0.04930
3	0.00799	0.02157	0.02616
4	0.01804	0.02572	0.02612
5	0.01558	0.04031	0.04930
6	0.01249	0.02312	0.02610
7	0.01558	0.04031	0.04930
8	0.01389	0.02407	0.02616

AVERAGE SQUARED ERROR

Estimator	P(1)	P(2)	P(3)
1	0.00026	0.00080	0.00103
2	0.00037	0.00251	0.00378
3	0.00010	0.00070	0.00103
4	0.00051	0.00099	0.00102
5	0.00037	0.00251	0.00378
6	0.00025	0.00081	0.00102
7	0.00037	0.00251	0.00378
8	0.00030	0.00088	0.00103

BIAS OF AVERAGE ESTIMATE

Estimator	P(1)	P(2)	P(3)
1	-0.00020	-0.00056	0.00075
2	-0.00070	-0.00549	0.00619
3	-0.00009	-0.00067	0.00075
4	0.00232	-0.00247	0.00015
5	-0.00070	-0.00549	0.00619
6	0.00061	-0.00124	0.00063
7	-0.00070	-0.00549	0.00619
8	0.00143	-0.00218	0.00075

SAMPLE VARIANCE OF ESTIMATOR

Estimator	P(1)	P(2)	P(3)
1	0.01691	0.03195	0.03697
2	0.02018	0.05204	0.06421
3	0.01155	0.03054	0.03697
4	0.02298	0.03473	0.03689
5	0.02018	0.05204	0.06421
6	0.01654	0.03192	0.03691
7	0.02018	0.05204	0.06421
8	0.01807	0.03300	0.03697

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 40 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 15 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 12

ESTIMATOR NO. OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

Estimator			
1	0.01728	0.02625	0.02596
2	0.02160	0.05899	0.07095
3	0.00810	0.02149	0.02596
4	0.02579	0.03219	0.02599
5	0.02160	0.05899	0.07095
6	0.01690	0.02646	0.02595
7	0.02160	0.05899	0.07095
8	0.01867	0.02710	0.02596

AVERAGE SQUARED ERROR

Estimator			
1	0.00046	0.00108	0.00103
2	0.00069	0.00515	0.00737
3	0.00010	0.00071	0.00103
4	0.00107	0.00159	0.00104
5	0.00069	0.00515	0.00737
6	0.00044	0.00109	0.00103
7	0.00069	0.00515	0.00737
8	0.00053	0.00115	0.00103

BIAS OF AVERAGE ESTIMATE

Estimator			
1	-0.00130	0.00045	0.00085
2	-0.00210	-0.00754	0.00964
3	-0.00003	-0.00083	0.00085
4	0.00252	-0.00291	0.00039
5	-0.00210	-0.00754	0.00964
6	0.00128	-0.0(77	0.00069
7	-0.00210	-0.00754	0.00964
8	-0.00037	-0.00048	0.00085

SAMPLE VARIANCE OF ESTIMATOR

Estimator			
1	0.02196	0.03639	0.03708
2	0.02663	0.07287	0.08710
3	0.01181	0.03075	0.03708
4	0.03324	0.04257	0.03718
5	0.02663	0.07287	0.08710
6	0.02161	0.03645	0.03708
7	0.02663	0.07287	0.08710
8	0.03755	0.03771	0.03768

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 20 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 5 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 1

ESTIMATOR NO.   OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.01685	0.04080	0.04862
2	0.01758	0.04659	0.05538
3	0.01542	0.03990	0.04862
4	0.02000	0.04125	0.04891
5	0.01758	0.04659	0.05538
6	0.01679	0.04079	0.04862
7	0.01758	0.04659	0.05538
8	0.01793	0.04144	0.04144

AVERAGE SQUARED ERROR

1	0.00045	0.00257	0.00352
2	0.00049	0.00324	0.00457
3	0.00038	0.00247	0.00352
4	0.00063	0.00261	0.00358
5	0.00049	0.00324	0.00457
6	0.00045	0.00257	0.00352
7	0.00049	0.00324	0.00457
8	0.00050	0.00265	0.00352

BIAS OF AVERAGE ESTIMATE

1	0.00213	0.00527	-0.06740
2	0.00230	0.00525	-0.00756
3	0.00205	0.00535	-0.00740
4	0.00221	0.00516	-0.00736
5	0.00230	0.00525	-0.00756
6	0.00230	0.00510	-0.00740
7	0.00230	0.00525	-0.00756
8	0.00377	0.00363	-0.06740

SAMPLE VARIANCE OF ESTIMATOR

1	0.02360	0.05508	0.06472
2	0.02442	0.06165	0.07326
3	0.02179	0.05430	0.06472
4	0.02690	0.05567	0.06495
5	0.02442	0.06165	0.07326
6	0.02365	0.05510	0.06470
7	0.02442	0.06165	0.07326
8	0.02472	0.05619	0.06472

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 20 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 5 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 2

ESTIMATOR NO.   OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.01995	0.04140	0.04822
2	0.02026	0.05402	0.06451
3	0.01541	0.04028	0.04822
4	0.02533	0.04241	0.04880
5	0.02026	0.05402	0.06451
6	0.01886	0.04145	0.04826
7	0.02026	0.05402	0.06451
8	0.02009	0.04219	0.04822

AVERAGE SQUARED ERROR

1	0.00057	0.00261	0.00352
2	0.00066	0.00436	0.00623
3	0.00037	0.00244	0.00352
4	0.00098	0.00273	0.00361
5	0.00066	0.00436	0.00623
6	0.00057	0.00252	0.00353
7	0.00066	0.00436	0.00623
8	0.00064	0.00270	0.00352

BIAS OF AVERAGE ESTIMATE

1	0.00078	0.00616	-0.00694
2	0.00076	0.00425	-0.00501
3	0.00095	0.00599	-0.00694
4	0.00189	0.00538	-0.00728
5	0.00076	0.00425	-0.00506
6	0.00163	0.00543	-0.00706
7	0.00076	0.00425	-0.00501
8	0.00261	0.00433	-0.00694

SAMPLE VARIANCE OF ESTIMATOR

1	0.02604	0.05600	0.06552
2	0.02775	0.07128	0.08533
3	0.02174	0.05444	0.06552
4	0.03263	0.05759	0.06589
5	0.02775	0.07128	0.08533
6	0.02589	0.05617	0.06554
7	0.02775	0.07128	0.08533
8	0.02776	0.05713	0.06552

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 20 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 5 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 3

ESTIMATOR NO. OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.02327	0.04414	0.04924
2	0.02651	0.06786	0.08205
3	0.01568	0.04027	0.04924
4	0.03144	0.04669	0.04970
5	0.02651	0.06786	0.08205
6	0.02268	0.04447	0.04926
7	0.02651	0.06786	0.08205
8	0.02459	0.04467	0.0492

AVERAGE SQUARED ERROR

1	0.00085	0.00301	0.00370
2	0.00109	0.00688	0.01000
3	0.00039	0.00255	0.00370
4	0.00151	0.00334	0.00376
5	0.00109	0.00688	0.01000
6	0.00083	0.00305	0.00370
7	0.00109	0.00688	0.01000
8	0.00095	0.00307	0.00370

BIAS OF AVERAGE ESTIMATE

1	0.00057	0.00542	-0.00600
2	0.00053	-0.00122	0.00069
3	0.00155	0.00445	-0.00600
4	0.00424	0.00250	-0.00674
5	0.00053	-0.00122	0.00069
6	0.00362	0.00268	-0.00630
7	0.00053	-0.00122	0.00069
8	0.00178	0.00422	-0.00600

SAMPLE VARIANCE OF ESTIMATOR

1	0.03072	0.05912	0.06695
2	0.03434	0.08612	0.10377
3	0.02204	0.05545	0.06695
4	0.04010	0.06222	0.06735
5	0.03434	0.08612	0.10377
6	0.03067	0.05978	0.06694
7	0.03434	0.08612	0.10377
8	0.03230	0.05987	0.06695

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 20 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 5 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 4

ESTIMATOR NO.   OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.03269	0.04881	0.04782
2	0.03854	0.09759	0.11980
3	0.01539	0.03939	0.04782
4	0.04462	0.05483	0.04814
5	0.03854	0.09759	0.11980
6	0.03561	0.05212	0.04801
7	0.03854	0.09759	0.11980
8	0.03269	0.04881	0.04782

AVERAGE SQUARED ERROR

1	0.00160	0.00361	0.00342
2	0.00225	0.01361	0.02013
3	0.00039	0.00236	0.00342
4	0.00307	0.00447	0.00348
5	0.00225	0.01361	0.02013
6	0.00673	0.00824	0.00345
7	0.00225	0.01361	0.02013
8	0.00160	0.00361	0.00342

BIAS OF AVERAGE ESTIMATE

1	-0.00482	0.01274	-0.00792
2	-0.00299	-0.00608	0.00908
3	0.00167	0.06625	-0.00792
4	0.00595	0.00286	-0.00881
5	-0.00299	-0.00608	0.00908
6	0.01000	-0.00145	-0.00855
7	-0.00299	-0.00608	0.00908
8	-0.00482	0.01274	-0.00792

SAMPLE VARIANCE OF ESTIMATOR

1	0.04072	0.06249	0.06451
2	0.04847	0.11925	0.14548
3	0.02192	0.05357	0.06451
4	0.05572	0.07054	0.06470
5	0.04847	0.11925	0.14548
6	0.08185	0.09365	0.06463
7	0.04847	0.11925	0.14548
8	0.04072	0.06249	0.06451

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 20 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 10 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 2

ESTIMATOR NO.   OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.00993	0.02387	0.02745
2	0.01052	0.02810	0.03253
3	0.00868	0.02333	0.02745
4	0.01282	0.02409	0.02801
5	0.01052	0.02810	0.03253
6	0.00995	0.02386	0.02749
7	0.01052	0.02810	0.03253
8	0.01088	0.02447	0.02745

AVERAGE SQUARED ERROR

1	0.00015	0.00088	0.00116
2	0.00017	0.00120	0.00167
3	0.00012	0.00083	0.00116
4	0.00025	0.00089	0.00120
5	0.00017	0.00120	0.00167
6	0.00015	0.00088	0.00116
7	0.00017	0.00120	0.00167
8	0.00018	0.00093	0.00116

BIAS OF AVERAGE ESTIMATE

1	0.00088	0.00176	-0.00265
2	0.00077	0.00110	-0.00187
3	0.00073	0.00192	-0.00265
4	0.00125	0.00159	-0.00284
5	0.00077	0.00110	-0.00187
6	0.00100	0.00169	-0.00269
7	0.00077	0.00110	-0.00187
8	0.00268	-0.00004	-0.00265

SAMPLE VARIANCE OF ESTIMATOR

1	0.01596	0.03966	0.04624
2	0.01658	0.04454	0.05251
3	0.01465	0.03903	0.04624
4	0.01880	0.04005	0.04641
5	0.01658	0.04454	0.05251
6	0.01597	0.03966	0.04623
7	0.01658	0.04454	0.05251
R	0.01675	0.04071	0.04624

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 20 SEGMENTS, THE SEGMENT SAMPLE SIZE IS 10 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 4

ESTIMATOR NO. OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

ESTIMATOR NO.			
1	0.01233	0.02381	0.02706
2	0.01372	0.03391	0.04094
3	0.00908	0.02276	0.02706
4	0.01681	0.02442	0.02778
5	0.01372	0.03391	0.04094
6	0.01225	0.02379	0.02713
7	0.01372	0.03391	0.04094
8	0.01330	0.02429	0.02706

AVERAGE SQUARED ERROR

ESTIMATOR NO.			
1	0.00023	0.00091	0.00112
2	0.00029	0.00178	0.00256
3	0.00013	0.00080	0.00112
4	0.00043	0.00094	0.00118
5	0.00029	0.00128	0.00256
6	0.00023	0.00090	0.00113
7	0.00029	0.00178	0.00256
8	0.00027	0.00094	0.00112

BIAS OF AVERAGE ESTIMATE

ESTIMATOR NO.			
1	0.00021	0.00270	-0.00291
2	-0.00045	-0.00159	0.00204
3	0.00090	0.00202	-0.00203
4	0.00246	0.00132	-0.00339
5	-0.00045	-0.00159	0.00204
6	0.00073	0.00237	-0.00310
7	-0.00045	-0.00159	0.00204
8	0.00228	0.00063	-0.00291

SAMPLE VARIANCE OF ESTIMATOR

ESTIMATOR NO.			
1	0.01791	0.03919	0.04520
2	0.01936	0.05053	0.06040
3	0.01484	0.03824	0.04520
4	0.02265	0.04029	0.04544
5	0.01936	0.05053	0.06040
6	0.01785	0.03921	0.04520
7	0.01936	0.05053	0.06040
8	0.01887	0.03998	0.04520

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 20 SEGMENTS,
THE SEGMENT SAMPLE SIZE IS 10 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 6

ESTIMATOR NO.   OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

1	0.01491	0.02535	0.02650
2	0.01763	0.04549	0.05449
3	0.00847	0.02233	0.02650
4	0.02218	0.02755	0.02721
5	0.01763	0.04549	0.05449
6	0.01471	0.02533	0.02560
7	0.01763	0.04549	0.05449
8	0.01650	0.02625	0.02650

AVERAGE SQUARED ERROR

1	0.00034	0.00103	0.00108
2	0.00047	0.00316	0.00450
3	0.00011	0.00077	0.00108
4	0.00075	0.00119	0.00114
5	0.00047	0.00316	0.00450
6	0.00033	0.00103	0.00108
7	0.00047	0.00315	0.00450
8	0.00041	0.00110	0.00108

BIAS OF AVERAGE ESTIMATE

1	0.00059	0.00199	-0.00258
2	0.00016	-0.00291	0.00275
3	0.00060	0.00198	-0.00258
4	0.00312	0.00005	-0.00317
5	0.00016	-0.00291	0.00275
6	0.00186	0.00388	-0.00274
7	0.00016	-0.00291	0.00275
8	0.00223	0.00035	-0.00258

SAMPLE VARIANCE OF ESTIMATOR

1	0.02117	0.04129	0.04556
2	0.02415	0.06439	0.07687
3	0.01487	0.03825	0.04556
4	0.02893	0.04452	0.04570
5	0.02415	0.06439	0.07687
6	0.02086	0.04146	0.04555
7	0.02415	0.06439	0.07687
8	0.02281	0.04242	0.04556

SUMMARY: WHEN THE SIMULATED STRATUM CONTAINS 20 SEGMENTS, THE SEGMENT SAMPLE SIZE IS 10 AND THE NUMBER OF PARTIALLY IDENTIFIED SEGMENTS IS 8

ESTIMATOR NO. OBSERVED BEHAVIOR OF PROPORTIONS P(1), P(2), P(3), ETC.

AVERAGE ABSOLUTE ERROR

Estimator			
1	0.02168	0.03046	0.02770
2	0.02599	0.06455	0.07736
3	0.00905	0.02338	0.02770
4	0.03221	0.03534	0.02826
5	0.02599	0.06455	0.07736
6	0.02198	0.03062	0.02777
7	0.02599	0.06455	0.07736
8	0.02319	0.03151	0.02770

AVERAGE SQUARED ERROR

Estimator			
1	0.00073	0.00145	0.00118
2	0.00102	0.00646	0.00919
3	0.00013	0.00084	0.00118
4	0.00162	0.00189	0.00124
5	0.00102	0.00646	0.00919
6	0.00081	0.00152	0.00119
7	0.00102	0.00646	0.00919
8	0.00083	0.00153	0.00118

BIAS OF AVERAGE ESTIMATE

Estimator			
1	-0.00084	0.00356	-0.00272
2	-0.00046	-0.00297	0.00342
3	0.00091	0.00181	-0.00272
4	0.00278	0.00027	-0.00305
5	-0.00046	-0.00297	0.00342
6	0.00416	-0.00120	-0.00297
7	-0.00046	-0.00297	0.00342
8	0.00072	0.00200	-0.00272

SAMPLE VARIANCE OF ESTIMATOR

Estimator			
1	0.02844	0.04570	0.04611
2	0.03342	0.08527	0.10176
3	0.01488	0.03896	0.04611
4	0.04118	0.05100	0.04661
5	0.03342	0.08527	0.10176
6	0.02901	0.04676	0.04615
7	0.03342	0.08527	0.10176
8	0.03044	0.04671	0.04611