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STRENGTH DIAGRAMS OF FIBROUS COMPOSITES WITH UNIDIRECTIONAL STRUCTURE

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16. Abstract

The dependence of the composite strength on the volume proportion of fiber and the transfer factor is analyzed. Four types of diagrams are constructed for the strength of composites as a function of the volume proportion of fibers and the transfer factor.

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STRENGTH DIAGRAMS OF FIBROUS COMPOSITES WITH UNIDIRECTIONAL STRUCTURE

V. S. Ivanova, L. M. Ustinov

In order to solve many problems involved in the creation /176* of reinforced metals it is necessary to establish the parameters of the reinforcement: the volume proportion of the fiber, the dimension (diameter and length) of the fibers, the mechanical properties of the fiber and of the matrix and other properties consonant with a positive reinforcement effect. In this connection it is interesting to analyze the dependence of the strength of the composite [1,2] on two parameters simultaneously: the volume proportion of fiber (V_f) and the transfer factor $k = l_*/l_f$ [3], where l_f is the sought length of fiber and l_* is the critical length of fiber as defined by [1]:

$$l_* = \frac{d_f \sigma_f}{2\tau} \quad (1)$$

where d_f is the diameter of the fiber, σ_f is the breaking strength of the fiber, and τ is the flow stress in the matrix.

<u>Analysis and construction of diagrams</u>. The familiar equations for the strength of composites of unidirectional structure have the following form [1,3]:

$$\sigma_{c1} = \sigma_m(1-V_f) + k\beta V_f \sigma_f \quad \text{at } V_* \geqslant V_f \geqslant 0 \quad (2)$$

$$\sigma_{c2} = \sigma_f V_f [1 - k(1-\beta)] + \sigma_m^*(1-V_f) \quad \text{at } V_* \leqslant V_f \leqslant 1 \quad (3)$$

The "minimal" volume proportion of fiber (V_*) and the

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*Numbers in the margin indicate pagination in the foreign text.

critical volume proportion of fiber (V_{**}) are defined by the formulas:

$$V_{*} = \left[1 + \frac{\sigma_f}{\sigma_m - \sigma_m^{*}}(1-k)\right]^{-1} \tag{4}$$

$$V_{**} = \frac{\sigma_m - \sigma_m^{*}}{\sigma_f\,[1-k\,(1-\beta)] - \sigma_m^{*}} \tag{5}$$

We shall use expressions (2) and (3) hereafter in the interval of variation of the transfer factor k from 0 to 1 inclusive, i.e. under those conditions when it is possible to achieve tensile stresses equaling σ_f for the fibers introduced into the matrix upon loading of the composites, which corresponds to the most favorable case of reinforcement of the matrix.

The partial derivatives of the first and second order of expressions (2) and (3) with respect to V_f have the form:

$$\frac{\partial(\sigma_{c1})}{\partial V_f} = -\sigma_m + k\beta\sigma_f \tag{6}$$

$$\frac{\partial(\sigma_{c2})}{\partial V_f} = \sigma_f\,[1-k\,(1-\beta)] - \sigma_m^{*} \tag{7}$$

$$\frac{\partial^2(\sigma_{c1})}{\partial V_f^2} = 0 \tag{8}$$

$$\frac{\partial^2(\sigma_{c2})}{\partial V_f^2} = 0 \tag{9}$$

Since the strength of the matrix σ_m is less than σ_f, we also have $\sigma_m^*/\sigma_f < 1$, since $\sigma_m^* < \sigma_m$; the coefficient $\beta \simeq 0.5$ [4].

Derivative (6) can be:

a) < 0 when $k < \sigma_m/\beta\sigma_f$ (10)

b) = 0 when $k = \sigma_m/\beta\sigma_f$ (11)

c) > 0 when $k > \sigma_m/\beta\sigma_f$ (12)

while derivative (7) can be:

a) < 0 when $k > (1 - \dot{\sigma}_m/\sigma_f)/(1-\beta)$ (13)

b) = 0 when $k = (1 - \dot{\sigma}_m/\sigma_f)/(1-\beta)$ (14)

c) > 0 when $k < (1 - \dot{\sigma}_m/\sigma_f)/(1-\beta)$ (15)

In its turn, $\sigma_m/\beta\sigma_f$ can be:

a) < 1 when $\sigma_m/\sigma_f < \beta$ (16)

b) = 1 when $\sigma_m/\sigma_f = \beta$ (17)

c) > 1 when $\sigma_m/\sigma_f > \beta$ (18)

while the expression $(1 - \dot{\sigma}_m/\sigma_f)/(1-\beta)$ can be:

a) < 1 when $\dot{\sigma}_m/\sigma_f > \beta$ (19)

b) = 1 when $\dot{\sigma}_m/\sigma_f = \beta$ (20)

c) > 1 when $\dot{\sigma}_m/\sigma_f < \beta$ (21)

The conjuncture of conditions (16) - (21) produces five possible paired combinations of these, to each of which there should correspond, as will be shown below, its own diagram of σ_c as a function of V_f and k, different in principle from the other four. These are the following combinations:

a) $\sigma_m/\sigma_f < \beta$ and $\dot{\sigma}_m/\sigma_f < \beta$ (22)

under conditions (8) - (12), 15;

b) $\sigma_m/\sigma_f = \beta$ and $\dot{\sigma}_m/\sigma_f < \beta$ (23)

under conditions (8) - (11), 15;

c) $\sigma_m/\sigma_f > \beta$ and $\dot{\sigma}_m/\sigma_f > \beta$ (24)

under conditions (8) - (10), (13) - (15);

d) $\sigma_m/\sigma_f > \beta$ and $\sigma_m^*/\sigma_f = \beta$ (25)

under conditions (8) - (10), (14) - (15);

e) $\sigma_m/\sigma_f > \beta$ and $\sigma_m^*/\sigma_f < \beta$ (26)

under conditions (8) - (10), (15).

Figure 1 shows three-dimensional diagrams of the strength of composites as a function of the volume proportion of fibers and the transfer factor k, corresponding to conditions (22) - (25). /178

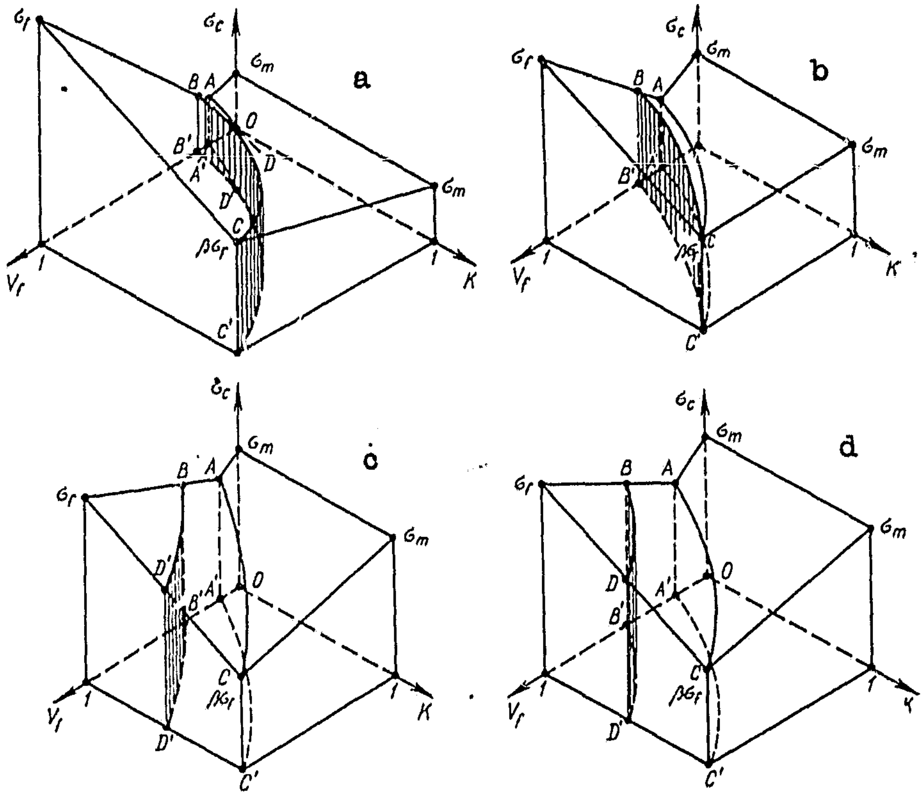

Fig. 1. Diagrams of the strength of composites (σ_c) as a function of the volume proportion of fibers (V_f) and the transfer factor $k = l_*/l_f$.

For a more clear representation we can use the method of cross sections of the diagrams by a plane perpendicular to the axis of variation of the transfer factor k. In this case each diagram has a set of characteristic cross sections intrinsic only to itself, representing a series of two-dimensional composite strength diagrams as functions of the volume proportion of fibers for various constant values of the transfer factor k (Fig. 2).

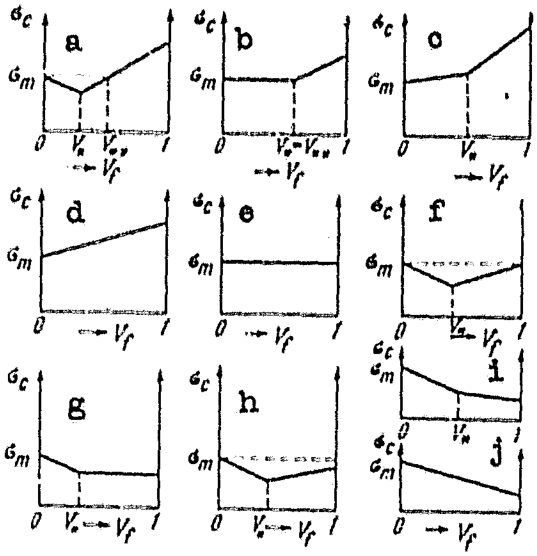

Fig. 2. Characteristic cross sections of strength diagrams (Fig. 1) for various values of k = const.

The diagram of Fig. 1a has four characteristic sections (Fig. 2): $1 \leq k < \sigma_m/\beta\sigma_f$ (Fig. 2a); $k = \sigma_m/\beta\sigma_f$ (Fig. 2b); $\sigma_m/\beta\sigma_f < k < 1$ (Fig. 2c) and k = 1 (Fig. 2d).

The diagram of Fig. 1b has two characteristic sections: $0 \leq k < 1$ (Fig. 2a) and k = 1 (Fig. 2e).

The diagram of Fig. 1c has six characteristic sections: $0 \leq k < (1-\sigma_m/\sigma_f)/(1-\beta)$ (Fig. 2a); $k = (1-\sigma_m/\sigma_f)/(1-\beta)$ (Fig. 2f); $(1-\sigma_m/\sigma_f)(1-\beta) = k$ (Fig. 2g); $(1-\sigma_m/\sigma_f)/(1-\beta) < k < (1-\sigma_m/\sigma_f)/(1-\beta)$ (Fig. 2h); $(1-\sigma_m/\sigma_f)/(1-\beta) < k < 1$ (Fig. 2i) and k = 1 (Fig. 2j).

The diagram of Fig. 1d has four characteristic sections: $0 \leq k < (1-\sigma_m/\sigma_f)/(1-\beta)$ (Fig. 2a); $k = (1-\sigma_m/\sigma_f)/(1-\beta)$ (Fig. 2f); $1-\sigma_m/\sigma_f)/(1-\beta) < k < 1$ (Fig. 2h) and $k = 1$ (Fig. 2j).

Combinations (25) and (26) in principle produce identical diagrams, as the latter have absolutely identical sets of characteristic sections. In the final analysis we obtain four basically different three-dimensional diagrams: a, b, c and d (Fig. 1). /179

An analysis of the diagrams and their characteristic sections reveals that each of these has regions of values V_f and k for which the strength of the composite σ_c exceeds that of the matrix, i.e. all four diagrams have strengthening regions. The strengthening region is situated beyond the line BD, corresponding to the dependence of V_{**} on k, in the direction of increasing values of V_f. The only exception in this case is the diagram of Fig. 1a, according to which a strengthening is also possible for all values of V_f from 0 to 1 under the condition that: $k > \sigma_m/\beta\sigma_f$.

In view of the diminishing effect of reinforcement all these diagrams can be arranged in the following sequence:

1) the diagram of Fig. 1a [3], according to which the inequality $\sigma_c > \sigma_m$ can be fulfilled when $V_f > V_{**}$, if $k \leq \sigma_m/\beta\sigma_f$, and when $0 < V_f \leq 1$, if $k > \sigma_m/\beta\sigma_f$;

2) the diagram of Fig. 1b. Here $\sigma_c > \sigma_m$ is possible when $V_f > V_{**}$ and $0 < k \leq 1$;

3) the diagrams of Fig. 1c and d, according to which $\sigma_c > \sigma_m$ is possible only in the relatively narrow region of values of V_f and k, when $V_f > V_{**}$ and $k < (1-\sigma_m/\sigma_f)/(1-\beta)$.

The limits of applicability of the diagrams. The constructed diagrams allow for the variation of V_f in the closed interval from 0 to 1. However we must realize that the fibers used in the composite as a rule have a circular cross section. But this means that even for their greatest packing density V_f cannot exceed 0.907. Therefore we must consider the constructed diagrams in the intervals of V_f values from 0 to 0.907.

Moreover we must allow for the fact that at a high volume proportion the experimental strength values may differ considerably from the calculated, since formula (3) has been derived on the basis of the law of additivity and is valid only in a certain interval of V_f.

Conclusions. 1. On the basis of the existing expressions /180
for the strength of composites of unidirectional structure of the system "soft metal matrix - strong and rigid fibers" by means of the combinatorial method four possible types of diagrams have been constructed for the strength of composites (σ_c) as a function of the volume proportion of fibers (V_f) and the transfer factor k.

2. Using the method of characteristic cross sections, conditions are established for positive reinforcement of a soft metal matrix by rigid fibers as a function of the mechanical properties of the matrix, the volume proportion of fibers, and the dimension of the fibers.

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