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DESIGN OF A HORIZONTAL LIQUID HELIUM CRYOSTAT
FOR REFRIGERATING A FLYING SUPERCONDUCTING
MAGNET IN A WIND TUNNEL

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## 1. INTRODUCTION

The developments of new aircraft, space rockets and guided missiles require a vast amount of research, design and construction, the cost of which frequently runs into millions of pounds and many months of time. Any processes that can reduce this expenditure of time and money is, of course, vigorously pursued. The wind tunnel testing method is a convenient method for this purpose.

Most problems of experimental aerodynamics are connected with the study of motion of a body in relation to a stationary fluid. However, we can reverse the problem and study the motion of a fluid in relation to a stationary body.

Dynamical similarity is the basis of wind tunnel testing and other forms of model testing.

According to the theory of similarity, the experimental results from testing small model can be converted to the full scale object. Usually the tested model is mounted in the tunnel with the aid of different types of suspensions, supports and struts, etc. But these support systems will bring about a series of additional questions. Their influence on the flow pattern in the tunnel and around the tested model is considerable. In the general case, these effects are expressed in changes in the velocity and pressure distributions, which are noticed:

a) As changes in the average velocity in the testing section which necessitates corrections in the velocity coefficient of the tunnel.

b) As changes in the pressure gradient, which create a horizontal Archimedian force affecting the drag, then necessitating a correction in the pressure gradient.

c) As different local influences affecting boundary-layer flow, vortex formation, local flow separation etc. Especially in a supersonic wind tunnel, any support members attached to the model ahead of its most rearward point will create a shock-wave disturbance and affect the flow aft of that point within a roughly conical region.

So it is better to have a model which can be suspended in the wind tunnel without any strut.

Aeronautical scientists have achieved this technique successfully by using an electromagnetic technique.

A model which consists of a permanent magnet is placed in a wind tunnel around which there is a set of electromagnets. The permanent magnet is attracted by the electromagnets and thus can be suspended in the wind tunnel. The force that the electromagnets attract the permanent magnet is proportional to the magnetic moment per unit volume of the permanent magnet multiplied by the field gradient produced by electromagnets.

Because the magnetic moment of the permanent magnet is limited to the saturation value and large field gradients are limited by the size of aperture containing the tunnel and model, the size of the testing model is limited.

To increase the field gradient would be prohibitively expensive whereas to increase the magnetic moment per unit volume of the suspended magnet appears to be feasible through the use of a superconducting magnet to replace the permanent magnet. In practice, a combination of superconducting magnet and permanent magnet may be necessary for control purposes.

In order to keep the superconducting magnet in operation, a horizontal dewar containing liquid helium is to be designed and built for the purpose of this project.

## 2. THE STRUCTURE OF THE HORIZONTAL DEWAR

This Dewar is made up of the inner can 13, the outer can 15, the liquid helium filling tube 17 and the evaporated helium vapour vent tube 20. The superconducting magnet solenoid 12 and the superconducting magnet switch 16 are in the inner can. There is a vacuum space between the inner can and the outer can. This vacuum space is filled by the multi-layer super-insulation which consists of about 20-25 layers of Mylar and carbon-loaded paper.

The inner can and the outer can are made up of thin-walled tubes of stainless steel because the combination of reasonable strength and high toughness offered by autenistic stainless steels makes them suitable for use at low temperature. Another factor that must be considered is the weight of the Dewar. The thinner wall thickness of the stainless steel is suitable for reducing the weight of the Dewar.

The spiral tubes are used both in the filling tube and the vent tube for increasing the length of the tube and thus minimizing the heat inleak from heat conduction. Another function of the spiral tube is to maintain a horizontal liquid level when the Dewar is flying horizontally.

In order to avoid vacuum collapse, stiffening bands of aluminium are used to wrap the outer can.

There is a vacuum valve in the left-hand cap of the outer can.
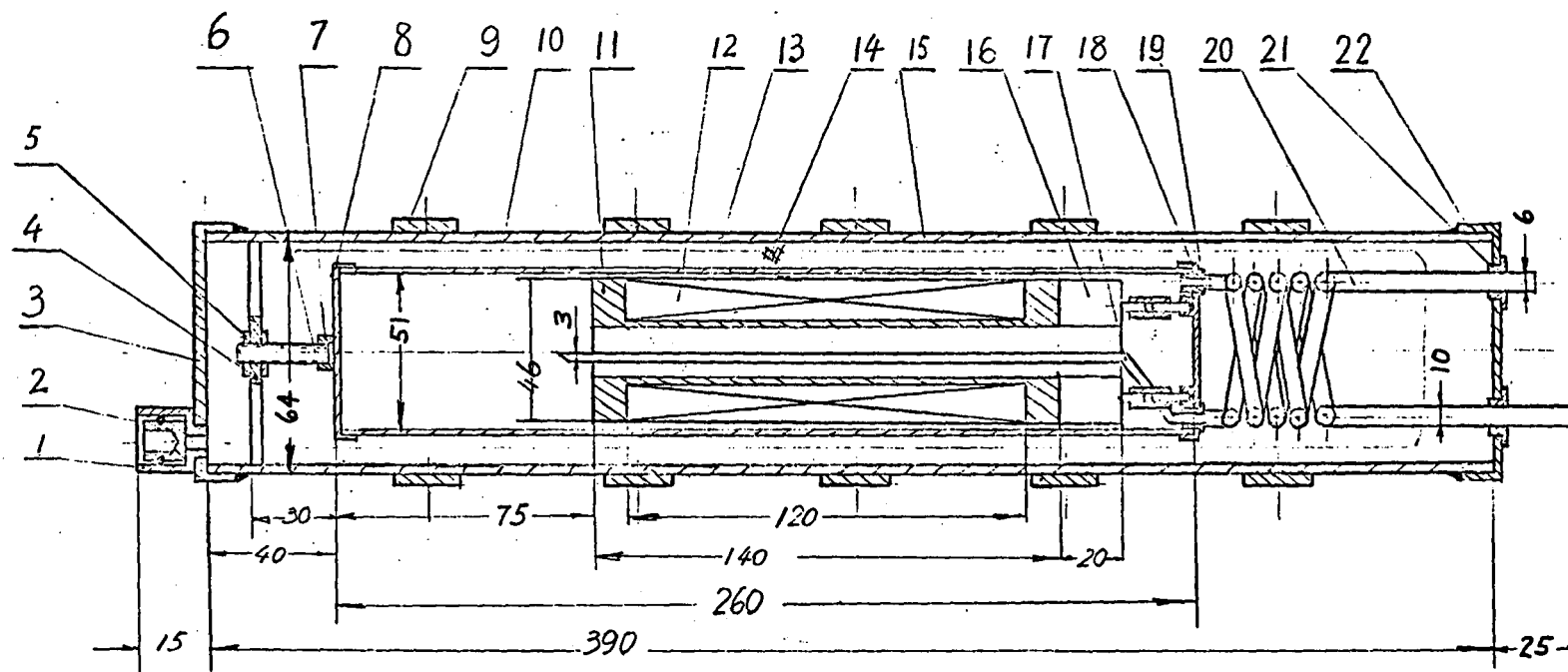
1. Bush of vacuum valve (Brass)
2. Vacuum valve (Brass)
3. Outer Can Cap(Brass)
4. Inner Can Support(Tufnon)
5. Nuts (Nylon)
6. Bolt (Nylon)
7. Bush (Brass)

8. Inner Can Cap (Brass)
9. Stiffening Band(Aluminium)
10. Shield Foil(Aluminium)
11. Former of Coil
12. Magnet Coil
13. Inner Can(StainlessSteel)
14. Super-Insulation(Al-Foil)

15. Outer Can (Stainless Steel)
16. Super-conducting magnet Switch
17. Filling Tube(Stainless Steel)
18. Inner Can Cap (Brass)
19. Bush (Brass)
20. Vent Tube (Stainless Steel)
21. Bush (Brass)
22. Outer Can Cap (Brass)

Fig.1. The Structure of the Flying Dewar.

3. CALCULATION OF THE HEAT INLEAK AND FINDING OUT THE VOLUME OF THE INNER
   CAN FOR OPERATION IN THE DESIRED TIMES.

   From the diagram of the Dewar, we note that the heat inleak into
helium is mainly caused by

   1) heat conduction throught the filling tube,
   2) heat conduction through the vent tube
   3) heat conduction through the support rod and wheel.
   4) heat radiation through the superinsulation space,

let us now consider these sources of heat inleak individually.

### 3.1. Heat Conduction Through the Filling Tube

   When we deal with the heat conduction problems in material, we get
the heat conduction differential equation (1) according to Fourier's
Law and the Law of energy conservation:

$$\dot{q}_o = \rho \, C_p \, \frac{\partial T}{\partial t} - \nabla(K \nabla T) \dots\dots\dots\dots\dots\dots(1)$$

That is $\dot{q}_o = \rho C_p \frac{\partial T}{\partial t} - \left( \frac{\partial}{\partial x} \left( K \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( K \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( K \frac{\partial T}{\partial z} \right) \right) \dots (2)$

Where $\dot{q}_o$ is the heat generated in unit volume and in unit time, Joule/sec $m^3$
      $\rho$ is the density Kg/$m^3$
      $C_p$ is the specific heat Joule/Kgmole K
      $K$ is the thermal conductivity Joule.m/sec K.

   For steady state, the temperature distribution inside the solid will
not change with the time. SO $\frac{\partial T}{\partial t} = 0$

   If there is no heat source inside the solid, $\dot{q}_o = 0$

   Thus the equation (1) reduces to $\nabla(K \nabla T) = 0 \dots\dots\dots\dots\dots(3)$

In the one-dimension problems the equation (3) reduces to

$$\frac{\partial}{\partial x} (K \frac{\partial T}{\partial x}) = 0 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(4)$$

Consider a rod or a tube which is thermal insulated in the traverse dimension. The length of the rod is L, the two end temperatures are $T_1$ and $T_2$. A coordinate system is chosen as shown in Fig.2.

When K = constant, the equation (4) reduces to $\frac{\partial^2 T}{\partial x^2} = 0 \dots\dots(5)$

The general solution of the differential equation (5) in the condition that $x = 0$, $T = T_1$ and $x = L$, $T = T_2$:

$$T = \frac{T_2 - T_1}{L} X + T_1$$

According to Fourier's Law of conduction, the heat flow rate

$$Q = -KA \frac{\partial T}{\partial x} = \frac{KA}{L} (T_1 - T_2) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(6)$$

When K is dependent to temperature we can introduce a new variable E defined by the relationship $E = \int_{T_o}^{T} K_{(T)} dT$, where $T_o$ is a reference temperature.

According to its definition we can get $\frac{\partial E}{\partial T} = K_{(T)}$

so $\frac{\partial}{\partial x} (K \frac{\partial T}{\partial x}) = \frac{\partial}{\partial x} (\frac{\partial E}{\partial T} \cdot \frac{\partial T}{\partial x}) = \frac{\partial^2 E}{\partial x^2}$

so the equation (4) reduces to $\frac{\partial^2 E}{\partial x^2} = 0 \dots\dots\dots\dots\dots\dots\dots\dots\dots(7)$

We can find that the equation (7) is very similar to the equation (5). By similarity, we get the general solution of the equation (7) when $X = 0$, $E = E_{(T_1)}$, $X = L$, $E = E_{(T_2)}$.

$$E = \frac{E_{(T_2)} - E_{(T_1)}}{L} x + E_{(T_1)}$$

We also get: $Q = \frac{KA}{L} (E_{(T_1)} - E_{(T_2)}) \dots\dots\dots\dots\dots\dots\dots\dots\dots(8)$
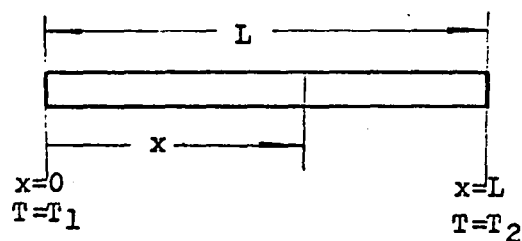
Fig.2. A rod with one dimension
heat conduction
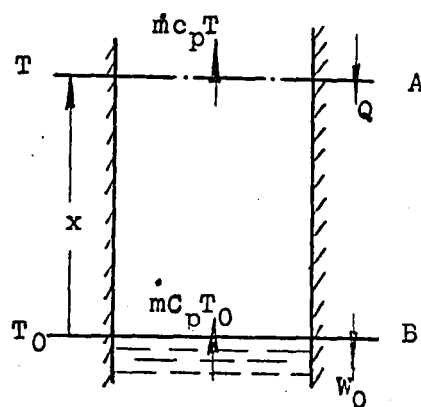


Fig.3.   Heat conduction through the vent tube.

We use the expression (6) to calculate heat flow rate in the case of one-dimensional steady-state heat conduction where the thermal conductivity is considered to be constant.

If the thermal conductivity of the material which makes up the conduction region varies significantly in the region, we use the expression (8).

At first let us outline our Dewar:

The outer diameter of the spiral tube is $\frac{1}{4}$" = 0.64 cm.

We have two turns of the spiral tubes.

The outer diameter of the inner can is 2" = 5.08 cm.

The outer diameter of the outer can is $2\frac{1}{2}$" = 6.35 cm.

The thickness of the coiled tube is 0.01" = 0.025 cm.

The thickness of the inner can is 0.01" = 0.025 cm.

The thickness of the outer can is 0.01" = 0.025 cm.

Thus we can find out:

The cross section area of the coiled tube is

$$A = \pi \, dt = \pi \times \left(\frac{2.54}{4}\right) \times (2.54 \times 0.01) = 0.05067 \text{ cm}^2$$

The length of the coiled tube is

$$L = \pi D \cdot 2 = \pi \times (2.54 \times 2) \times 2 = 31.9 \text{ cm} \doteq 30 \text{ cm}$$

To check the thermal conductivity of stainless steel, we get

$$K = 3 \times 10^{-3} \text{ watts/cmK at } T = 4 \text{ K}$$
$$K = 150 \times 10^{-3} \text{ watts/cmK at } T = 300 \text{ K}$$

Assuming that there is a linear relation between the thermal conductivity of stainless steel and the temperatures. We find out the dependence of K to the temperature:

$$K = K_o + A(T - T_o) = 3 \times 10^{-3} + 0.5(T - 4) \times 10^{-3} \quad \dots \dots \dots \dots (9)$$

According to expression (8)

$$Q \cdot L = A \left( E_{(T_1)} - E_{(T_2)} \right) = A \left( E_{300K} - E_{4K} \right) = A \int_4^{300} \left( (K_o + b(T - T_o)) \right) dT$$

$$= \left( K_o (300-4) + \frac{b}{2} (300 - 4)^2 \right) A$$

$$= \left( 3 \times 10^{-3} \times 296 + \frac{0.5}{2} \times 296^2 \times 10^{-3} \right) A$$

$$= 22.79A$$

So $Q = 22.79 \frac{A}{L} = 22.79 \times \frac{0.05067}{30} = 38.5 \times 10^{-3}$ watts

### 3.2.    Heat conduction through the vent tube

In this case the escaping vapour maintains good thermal contact with the wall of the tube and absorbs a great deal of the heat that would otherwise enter and cause evaporation of the liquid helium.

For the control volume surrounded by the boundaries of cross section A, B and the tube wall, $W_o$ is the conducted heat which flows to the liquid helium from the wall. Q is the heat flux in the cross section A which flows into the control volume. We assume that the tube and the vapour in the same cross section have the same temperature.

Obviously, the heat which is transferred into the control volume is equal the heat which flows out from the control volume. Thus:

$$Q + \dot{m} C_p T_o = W_o + \dot{m} C_p T$$

So $Q = \dot{m} Cp (T - T_o) + W_o$ .......................................(10)

where $\dot{m}$ is the mass flow rate

Cp is the specific heat of the vapour.

According to Fourier's law of heat conduction, the heat flow in the section A is equal to

$$Q = AK \frac{dT}{dX} \quad \text{(in the case of above coordinate system)}$$

When K is dependent to the temperature, it is expressed by the expression (9).

Then $Q = A \cdot (K_o + a(T - T_o)) \frac{dT}{dX}$ ................................(11)

Upon combining Equation (10) and (11) and integrating, it is found that:

$$L/A = (\frac{1}{\dot{m}\ Cp})\ (\ a\ \Delta T + (K_o - W_o \frac{a}{\dot{m}\ Cp})\ ln\ \frac{W_o + \dot{m}\ Cp\ \Delta T}{W_o})...(12)$$

where L is the length of the tube and $\Delta T$ the temperature difference between the warm and cold ends.

In the present situation, Cp, the specific heat for helium at 1 atm is about 5.2 Joules/g.K.

$\Delta T = 300 - 4 = 296K$

$K_o = 3 \times 10^{-3}$ watts/cmK

$a = 0.5 \times 10^{-3}$ watts/cmK$^2$

According to these figures, we can plot a graph of $\dot{m}$ against L/A for various $W_o$. It is shown in Fig 4. The calculated results are shown in Table 1.

At present, we can only estimate the mass flow rate $\dot{m}$. We estimate the total inleak is about $200 \times 10^{-3}$ watts. Thus the evaporated liquid helium by absorbing this amount of heat will be:

$$\frac{200 \times 10^{-3}}{L_c} = \frac{200 \times 10^{-3}}{20.42} = 10^{-2}\ g/sec$$

where $L_c$ is the latent heat of liquid helium at 1 atm.

From $A = 0.05067$ cm$^2$ $L = 30$ cm we get L/A = 592.

Now we can check the diagram $\dot{m} \sim L/A$ for various $W_o$. We find that $W_o$ will be less than $10^{-10}$ Joule/sec.

So we can conclude that $W_o$ is nearly equal to zero, $W_o = 0$. It means that the escaping vapour will absorb almost all the heat which would be transferred to the liquid helium by heat conduction along the vent tube.

Fig 4. ṁ against L/A for various Vo.
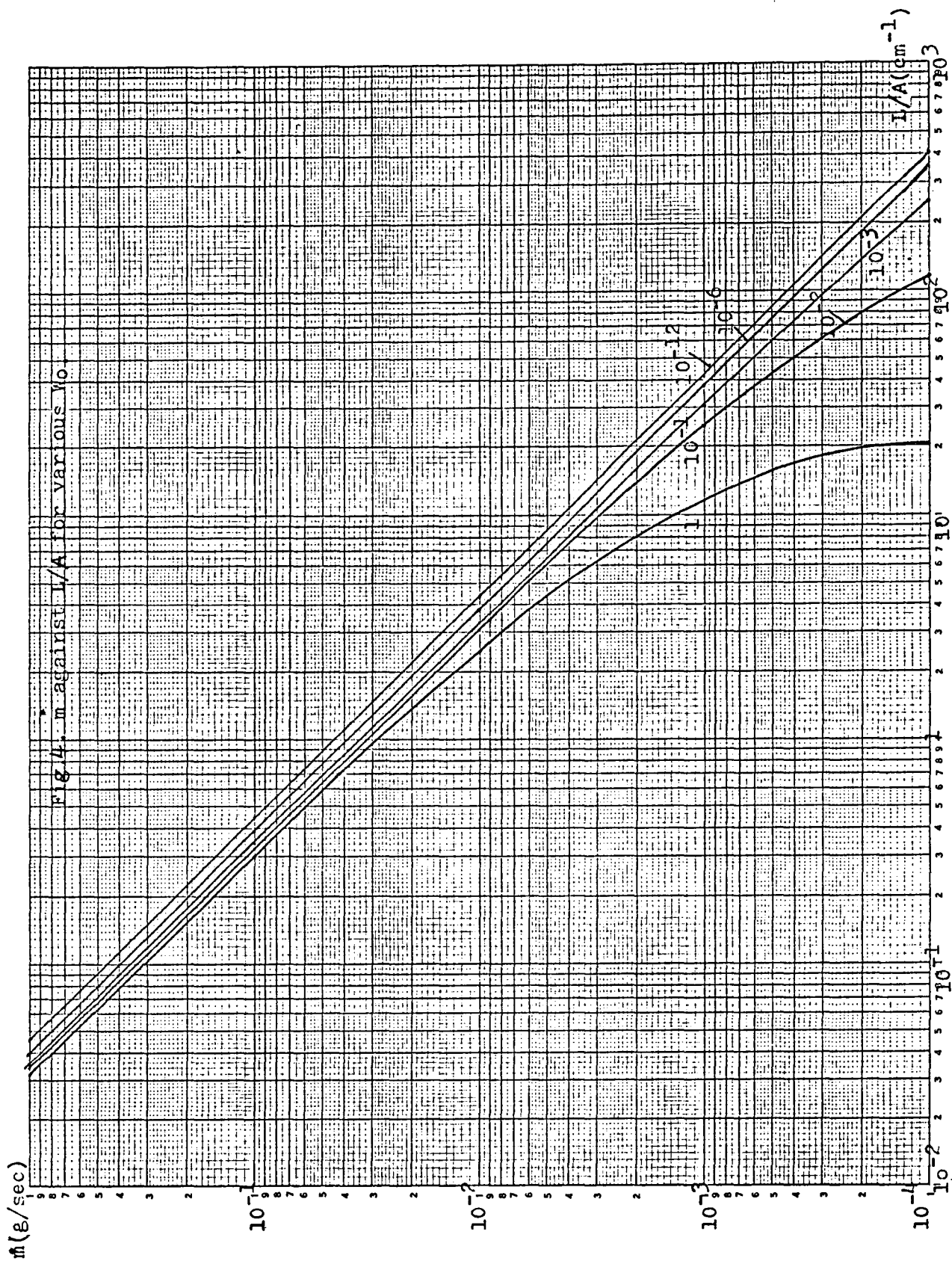
Table 1. The calculated results of $\dot{m}$ against L/A for various $W_o$ according to Equation (12).

| L/A (cm⁻¹) | | $W_o$ (Watts) | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $10^{-10}$ | $10^{-6}$ | $10^{-5}$ | $10^{-4}$ | $10^{-3}$ | $10^{-2}$ | $10^{-1}$ | $1$ |
| $\dot{m}$ (g/sec) | $10^{-4}$ | 406 | 353.5 | 340.0 | 325.6 | 304.3 | 249 | 117.6 | 20.7 |
| | $10^{-3}$ | 41.9 | 36.68 | 35.35 | 34.0 | 32.56 | 30.4 | 24.9 | 11.76 |
| | $10^{-2}$ | 4.33 | 3.80 | 3.68 | 3.53 | 3.40 | 3.25 | 3.04 | 2.49 |
| | $10^{-1}$ | 0.446 | 0.39 | 0.380 | 0.36 | 0.353 | 0.34 | 0.326 | 0.304 |
| | $1$ | 0.046 | 0.040 | 0.039 | 0.038 | 0.0367 | 0.0353 | 0.034 | 0.0325 |

### 3.3. Heat Conduction Through the Support Rod and Star-Wheel.

The supporting system is composed of nylon rod and tufnol star-wheel.

For the problems of heat conduction through the nylon rod we just simply use equation (6) but for the tufnol wheel the problem is more complicated.

At first, let us consider a long cylindrical element of uniform thermal conductivity, as shown in Fig 5. If the conduction process is in a steady state and if the conditions specified on each of the two surfaces are independent of $\theta$ and $z$ the process is one-dimensional. That is, the temperature distribution depends only upon $\gamma$. If temperature of $T_1$ and $T_2$ are specified at $\gamma_1$ and $\gamma_2$, the relevant differential equation and boundary conditions are as follows:

$$\frac{d^2 T}{d\gamma^2} + \frac{1}{r} \frac{dT}{d\gamma} = 0$$

at $\gamma = \gamma_1$, $T = T_1$

at $\gamma = \gamma_2$, $T = T_2$

The general solution is then $T = \dfrac{T_2 - T_1}{\ln \frac{r_2}{\gamma_1}} \ln \dfrac{\gamma}{\gamma_1} + T_1$

In determining the heat flow rate, we apply the Fourier equation in cylindrical form
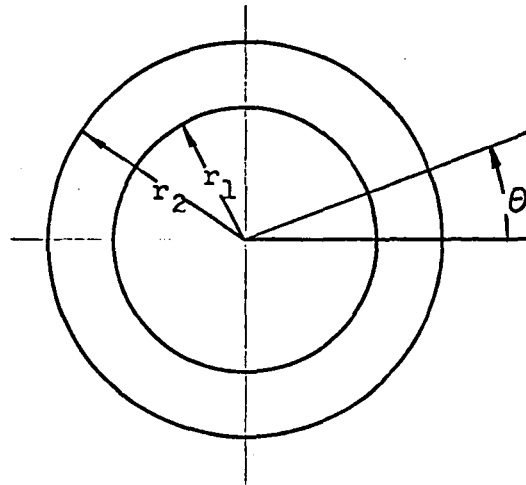
Fig.5.The diagram of the cross section
of a cylinder
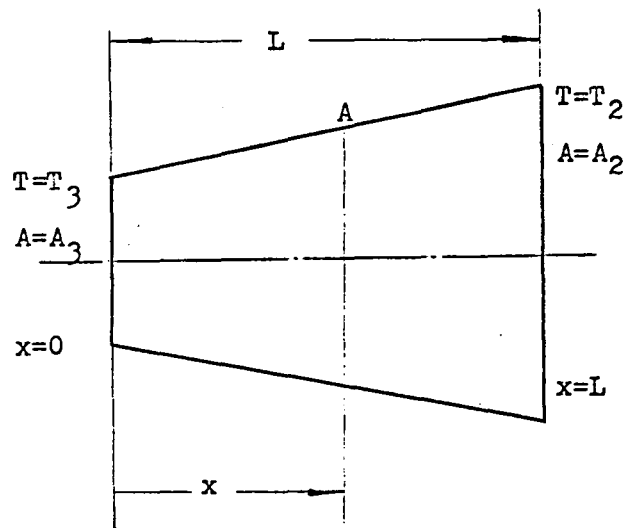


Fig.6.A rod with varying cross section area.

$$q_\gamma = -KA(\gamma) \frac{dT}{d\gamma}$$

The area in this case is equal to $2\pi\gamma.L$. (L is the length of the cylinder)
Then we obtain

$$q_\gamma = \frac{2\pi KL}{\ln \frac{\gamma_2}{\gamma_1}} (T_2 - T_1) = \frac{T_2 - T_1}{\frac{1}{2\pi KL} \ln \frac{\gamma_2}{\gamma_1}} \quad \dots\dots\dots\dots\dots (13)$$

We can express equation (13) as following form:

$$\frac{T_2 - T_1}{q_\gamma} = \frac{1}{2\pi KL} \ln \frac{\gamma_2}{\gamma_1} = R \text{ cylinder} \dots\dots\dots\dots (14)$$

Now we define $R = \dfrac{T_2 - T_1}{q_\gamma}$ as the thermal resistance of the Cylinder.
$\Delta T = T_2 - T_1$ is the temperature difference in analogy with the electric
voltage. $q_\gamma$ is the heat flow rate in analogy with the electric current.

Secondly, let us consider a rod of varying cross section area as
shown in Fig 6. The boundary conditions are as follows,
   at $x = 0$, $A = A_3$, $T = T_3$
   at $x = L$, $A = A_2$, $T = T_2$
The relation of the cross section area to X is a linear relation. So we
can obtain $A_{(x)} = A_3 + \frac{A_2 - A_3}{L} X$. The heat flow rate is obtained by applying
Fourier Law of heat conduction.

$$q' = -KA_{(x)} \frac{dT}{dX} \quad \text{(q is constant for steady state heat conduction)}$$

Separating variables and integrating, we obtain that:

$$\frac{T_3 - T_2}{q'} = \frac{L}{\frac{A_2 - A_3}{\ln \frac{A_2}{A_3}} \cdot K} = R_{v-rod}$$

The total thermal resistance of six rods of varying cross section area
as shown in Fig 7 is one-sixth of the $R_{v-rod}$
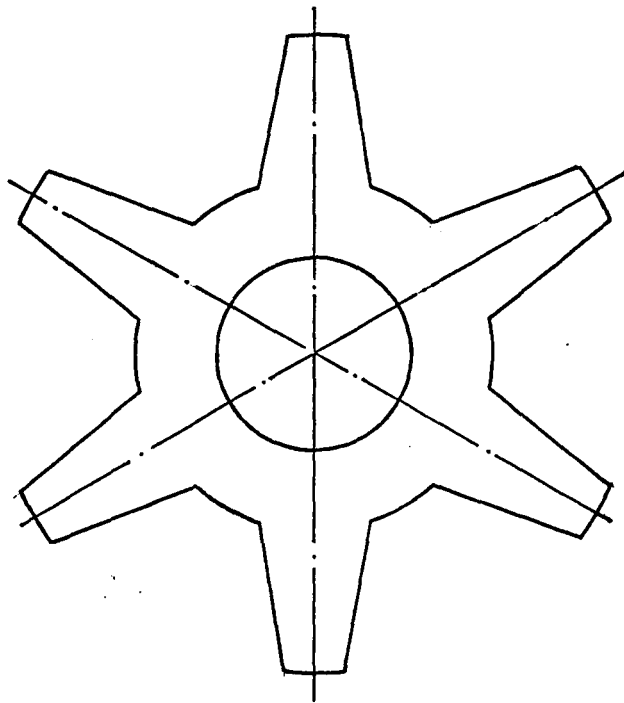
Fig.7. The diagram of the Star-wheel.

so: $\dfrac{T_3 - T_2}{q} = \dfrac{L}{\dfrac{A_2 - A_3}{\ln \dfrac{A_2}{A_3}} 6K} = R_{total\ rod}$ ......................... (15)

Finally we use equation (6) to get the thermal resistance of nylon bolt.

$\dfrac{T_1 - T_0}{q} = \dfrac{L_{nylon}}{K\,A_{nylon}} = R_{nylon}$ ............................ (16)

Upon addition of equation (14), (15) and (16), the unknown temperatures $(T_2, T_1)$ are eliminated.

so $q = \dfrac{T_3 - T_0}{R_{total}}$ ................................................... (17)

Equation (17) is the expression for the heat flow through the supporting system in terms of the overall temperature difference.

Let us pre-determine that:

The length of the nylon bolt is 2.0cm

The outer diameter of the bolt is 0.6 cm.

The inner diameter of the bolt is 0.3 cm

The outer diameter of the disc is 2 cm

The inner diameter of the disc is 0.6 cm

The thickness of the plastic plate is   0.2 cm

For the rod of varying cross section area, its thickness is 2mm

$A_3$ = 0.2cm x 0.2cm   $A_2$ = 0.4cm x 0.2cm

The length of each rod is about 20mm

The thermal conductivity of plastic plate is considered to be constant, K = $3 \times 10^{-3}$ watts/cmK

The thermal conductivity of nylon bolt is considered to be constant K = $3 \times 10^{-3}$ watts/cmK.

$R_{nylon} = \dfrac{L_{nylon}}{K\,A_{nylon}} = \dfrac{2.0}{3 \times 10^{-3}\ \dfrac{\pi}{4}(0.6^2 - 0.3^2)} = 3143$ K/Watts

$R_{cylinder} = \dfrac{1}{2\pi\,KL} \ln \dfrac{r_2}{r_1} = \dfrac{1}{2\pi\ 3 \times 10^3\ 0.2} \ln \dfrac{1}{a^3} = 319.36$ K/watts

$$R_{v\text{-}rod} = \frac{L}{\frac{A_2 - A_3}{\ln \frac{A_2}{A_3}} 6K} = \frac{2}{(0.4-0.2) \times 0.2 \times 6 \times 3 \times 10^{-3}} \ln \frac{4 \times 2}{2 \times 2} = 1925 \text{ K/watts}$$

So $R_{total} = \Sigma R_n = 3929 + 319.36 + 1925 = 5387.5$ K/watts

So $Q = \frac{\Delta T}{R_{total}} = \frac{300-4}{5387.5} = 54.9 \times 10^{-3}$ watts

3.4. <u>The Transfer of Heat by Radiation Through the Superinsulation Space</u>

The major heat flow through an evacuated insulation is radiation.

The fundamental equation for heat transfer by radiation from a black body (a perfect radiator) is the Stefan-Boltzmann Law.

$$q_\gamma = \sigma A T^4$$

where $q_\gamma$ = rate of emission of radiant energy

A = surface area of the emitter

T = surface temperature of the emitter

$\sigma$ = Stefan-Boltzmann constant, $5.67 \cdot 10^{-8}$ wm$^{-2}$ K$^{-4}$

Most bodies are not perfect radiators and emit radiation at a lower rate than a black body thus

$$q_\gamma = \sigma e A T^4$$

where e, the emissivity of the surface, is generally a function of T. If a body at a temperature equal to that of a black body emits a constant fraction of the black body emission at each wavelength the body is called a grey body and e is a constant for that body.

If a black body area $A_1$ at a temperature $T_1$ is completely surrounded by another black body at a higher temperature $T_2$ then the net rate of radiant heat transfer to the inner body will be

$$q_\gamma = \sigma A_1 (T_2^4 - T_1^4)$$

If neither of the bodies is a black body and the one body does not necessarily surround the other then the above equation is modified as follows,

$$q_r = \sigma A_1 \frac{e_1 e_2}{e_2 + (\frac{A_1}{A_2})(1-e_2)e_1}(T_2^4 - T_1^4)$$

If $A_1$ is nearly equal to $A_2$ and $e_1 = e_2$, for n thermally floating reflecting surfaces with emissivity $\hat{e}$ the radiation heat transfer between parallel surfaces at temperatures $T_1$ and $T_2$ is given by

$$q_r = \frac{1}{n+1} \sigma A \frac{e}{2-e}(T_2^4 - T_1^4)$$

An effective mean thermal conductivity $\bar{K}$ can be defined using the equation (6).

$$\frac{q}{A} = \bar{K} \frac{T_2 - T_1}{t} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (18)$$

where t is the thickness of the insulation
then

$$\bar{K} = \frac{t}{n+1} \frac{e\sigma}{2-e} \frac{T_2^4 - T_1^4}{T_2 - T_1} \quad \text{watts/mK}$$

It is known that $\bar{K} = 0.52 \times 10^{-6}$ watt/cmK for the multi-layer super-insulation which is composed by aluminium foil $12_\mu$ and $12_\mu$ Dexter paper.
It is also known that,

the length of the superconducting magnet is 16cm.
We assume that the length of the inner can is (16+L)cm.
Thus the surface area of the inner can is

$$A = \frac{\pi}{2} d^2 + (16+L)\pi d$$
$$= \frac{\pi}{2} \times 5.08^2 + (16+L)\pi \times 5.08 = 40.5 + 255.3 + 15.96L$$
$$= 295.8 + 15.96L \text{ cm}^2$$
$$t = \frac{d_2 - d_1}{2} = \frac{6.35 - 5.08}{2} = 0.635 \text{ cm}$$

According to equation 18,

$$q = \bar{K} \frac{T_2 - T_1}{t} = 0.52 \times 10^{-6} \times \frac{300 - 4.2}{0.635} \times (295.8 + 15.96L)$$

$$= (71.6 + 3.86L) \times 10^{-3} \text{ watts}$$

Now we summarize that

the heat flow through the filling tube is $38.5 \times 10^{-3}$ watts,

the heat flow through the vent tube is nearly zero,

the heat flow through the support system is $54.9 \times 10^{-3}$ watts

the heat flow through the evacuated space is $(71.6 + 3.86L) \times 10^{-3}$ watts.

Thus the total heat leakage into the liquid helium from the surroundings is

$$q = \Sigma \, q_i = (38.5 + 54.9 + 71.6 + 3.86L) \times 10^{-3} = (165 + 3.86L) \times 10^{-3} \text{ watts}$$

## 4. THE DETERMINATION OF THE VOLUME OF THE INNER CAN BY HEAT BALANCE

### 4.1. The Volume of Liquid Helium Evaporated for One Milliwatt Heat Inleak

Checking the table of thermophysical properties of Helium-4 we got that at 4.22 K and one atmosphere,

the enthalpy of the liquid helium $H_L$ = 9.711 (J/g)

the enthalpy of the vapour of helium $H_V$ = 30.13 (J/g)

the density of the liquid helium $\rho_L$ = 0.125 (g/cc)

the density of the vapour of helium $\rho_V$ = 0.01689 (g/cc).

Thus the latent heat of the helium at above states is

$$\lambda = 30.13 - 9.711 = 20.42 \ (J/g)$$

If the heat inleak is 1 milliwatt, the volume of liquid helium that could be evaporated is:

$$V = \frac{q}{\rho_L \ \lambda} = \frac{1 \times 10^{-3}}{20.42 \times 0.125} = 0.3917 \times 10^{-3} \ cm^3/sec$$

that is 1.410 $cm^3$/hr.

For our design, we intend to operate the superconducting magnet for half an hour. During that time, one milliwatt heat inleak could make 0.705cc of liquid helium evaporate.

### 4.2. Heat Balance

Apart from the volume which is occupied by the superconducting magnet the volume of the inner can which will hold the liquid helium is

$$V_L = \frac{\pi}{4} \ D^2 \ L = \frac{\pi}{4} \times 5.03^2 \times L = 19.87L \ (cm^3)$$

We have found out that the total heat inleak is $(165+3.86L) \times 10^{-3}$ watts.

For half an hour of operating time, the volume of the liquid helium which would evaporate with that amount of heat inleak is $(165+3.86L) \times 0.705 \ (cm^3)$

so we obtain that

$(165+3.86L) \times 0.705 = 19.87L$

solving this equation we obtain that

L = 6.78 cm

According to calculation, the length of inner can will be 16+L = 22.78cm.

In order to allow a margin, we shall consider the length of the inner can to be 26cm. It means that we adopt L = 10cm.

5.   ESTIMATE THE TOTAL WEIGHT OF THE CRYOSTAT

   5.1.   The Weight of Outer Can

      According to the length of inner can, we determine the length of outer
can to be 39 cm.

      the thickness of the outer can t is 0.01" = 0.025 cm,

      so the volume of the outer can is

$$V = \pi d_1 L_{outer} \, t = \pi \times 6.3 \times 39 \times (2.54 \times 0.01) = 19.6 \text{ cm}^3$$

      the density of stainless steel is 10 g/cm$^3$

      so $W_{stainless} = 19.6 \times 10 = 196g$

      The volume of two caps of the outer can is

$$V_{cap} = 2(\frac{\pi}{4} D^2 \times t_{cap}) + 2(\pi \, Dtl) = 2 \times (\frac{\pi}{4} \times 6.3^2 \times 0.1) + 2(\pi \times 6.3 \times 0.1 \times 0.5)$$
$$= 8.213 \text{ cm}^3$$

      The weight of two caps is

$$W_2 = \rho V_{cap} = 8.6 \times 8.215 = 70.6(g)$$

      The total weight of outer can is

$$W = W_1 + W_2 = 70.6 + 196 = 266.6(g)$$

   5.2.   The Weight of Inner Can

      The volume of stainless steel shell is

$$V = \pi d_2 l_2 t_2 = \pi \times 5.08 \times 26 \times (2.54 \times 0.01) = 10.54 \text{ cm}^3$$

      The weight of stainless steel shell is

$$W = \rho_s V = 10 \times 10.54 = 105.4 \text{ (g)}$$

      The weight of two caps of inner can is

$$W = \rho_{cap} V_{cap} = \rho_{cap} \frac{\pi}{2} d_2^2 \times t + 2(\pi d \, t \, 1) = (\frac{\pi}{2} \times 5.08^2 \times 0.1 + 2(\pi \times 5.08 \times 0.1 \times 0.5)) = 48.5 \text{ (g)}$$

The total weight of the inner can is

$$W = W_{shell} + W_{caps} = 105.4 + 48.5 = 153.9 (g)$$

## 5.3. The Weight of Stiffening Bands

The thickness of each band is 3mm ( $t$ = 3mm)

The width of each band is 20mm ( $b$ = 20mm)

These bands are made of aluminium of which the density is 2.67 g/cm$^3$

So the total weight of five bands is 2.67 g/cm$^3$

$$W_{S.B} = \rho_{AL} \cdot V_{AL} = \rho_{sl}(5 \pi Dlt) = 2.67 \times (5 \pi \times 6.35 \times 2 \times 0.3) = .159.7g$$

## 5.4. The Weight of Both Filling Tube and Vent Tube

Either filling tube or vent tube is composed of coiled part and straight part.

i) Coiled tubes of both filling tube and vent tube

the diameter of the tube d = ¼" = 0.64 cm

the length of each tube L = 30cm

the thickness of the wall of the tube t = 0.01" = 0.025 cm

so its weight is

$$W_c = 2 \times (\pi dtL) \rho_{stainless} = 2 \times (\pi \times \frac{2.54}{4} \times 0.01 \times 2.54 \times 30) \times 10 = 30.4g$$

ii) The straight part of filling tube

the diameter of it is d = 3/8" = 0.95 cm

the length of it is L = 7.5cm

the thickness of the wall is t = 0.01" = 0.025 cm

so its weight is

$$W_{f.s} = \pi dtl \rho = \pi \times \frac{3}{8} \times 2.54 \times 0.01 \times 2.54 \times 7.5 \times 10 = 5.70g$$

iii) The straight part of vent tube

the diameter of it is d = ¼" = 0.64 cm

the length of it is L = 6.0cm

the thickness of the wall is t = 0.01" = 0.025 cm

so its weight is

$$W_{v.s} = \pi dtl \rho = \pi \times \frac{1}{4} \times 2.54 \times 0.01 \times 2.54 \times 6.0 \times 10 = 3.04 (g)$$

## 5.5.  Vacuum Valve System

It is made of brass.

Its outer diameter $d_1$ = 1.2cm. The thickness of the wall t = 0.15cm

Its length 1 = 1.5cm

Its weight is

$$W = \pi \, dt1 \rho = \pi \times 1.2 \times 0.15 \times 1.5 \times 8.6 = 7.29 \ (g)$$

For slide-block. Its outer diameter $d_1'$ = 0.8cm. Its inner diameter $d_2'$ = 0.3cm. Its length $\ell$ = 1cm.

Its weight will be

$$W' = \frac{\pi}{4} (d_1'^2 - d_2'^2) \ell \rho = \frac{\pi}{4} \times (0.8^2 - 0.3^2) \times 1 \times 8.6 = 3.71 \ (g)$$

so the weight of the vacuum system is

$$W = 3.71 + 7.29 = 11(g)$$

5.6.  Bushes of the Cap of the Outer Can is about 5g.  Bushes of the cap of inner can is also about 5g.

Thus we obtain that the total weight of the whole cryostat is about

$$W = W_{\substack{outer \\ can}} + W_{\substack{inner \\ can}} + W_{\substack{stiffening \\ bands}} + W_{tubes} + W_{v.s.} + W_{bushes}$$

$$= 266.6 + 153.9 + 159.7 + (30.4 + 5.70 + 3.04) + 11 + 10$$

$$= 640.34 \ (g)$$

6.   SETTING THE CENTRE OF GRAVITY OF THE CRYOSTAT NEAR THE CENTRE OF MAGNET

According to the demand of operation we should set the centre of gravity of the Cryostat near the centre of gravity of the magnet as close as possible.

Now the structure of the outer can and the inner can are fixed, the position of the inner can where it is inside the outer can is also fixed. So the only adjustment is the position of the magnet inside the inner can.

Let us obtain the position of the centre of gravity of the Cryostat. We just simply use the Lever-rule.  We assume that the centre of gravity of the Cryostat (B) is X cm away from the middle point (A) of the outer can and on the right of the latter.

the distance of W valve is $(202.5+X)$mm

the distance of W(inner can) is $(25+X)$mm

the distance of W(coiled tubes)isabout $(125-X)$mm

the distance of W(straight tubes) is about $(175-X)$mm

the distance of W(outer can)  is Xmm

the distance of W (bushes of inner can)is $(105-X)$cm

All forces and distances relative to the point B are shown in the Fig 8. According to the lever-rule, the total torque in clockwise direction to the point B must be equal to the total torque in anti-clockwise direction. So we get

$$11 \times (202.5+X) + 153.9 \times (25+X) + (266.6+159.7) \times X$$
$$= 30.4 \times (125-X) + 8.74 \times (175-X) + 5 \times (195-X) + 5 \times (105-X)$$
$$640.34X = 754.5$$
$$X = 1.18mm$$

So the centre of gravity of the Cryostat is 1.18mm away from the middle point of the outer can and to the right of the latter.

Thus we can determine the position of the superconducting magnet inside the inner can.

If we assume the centre of gravity of magnet is in the middle point of the magnet that is 80mm away from the left end of the magnet.

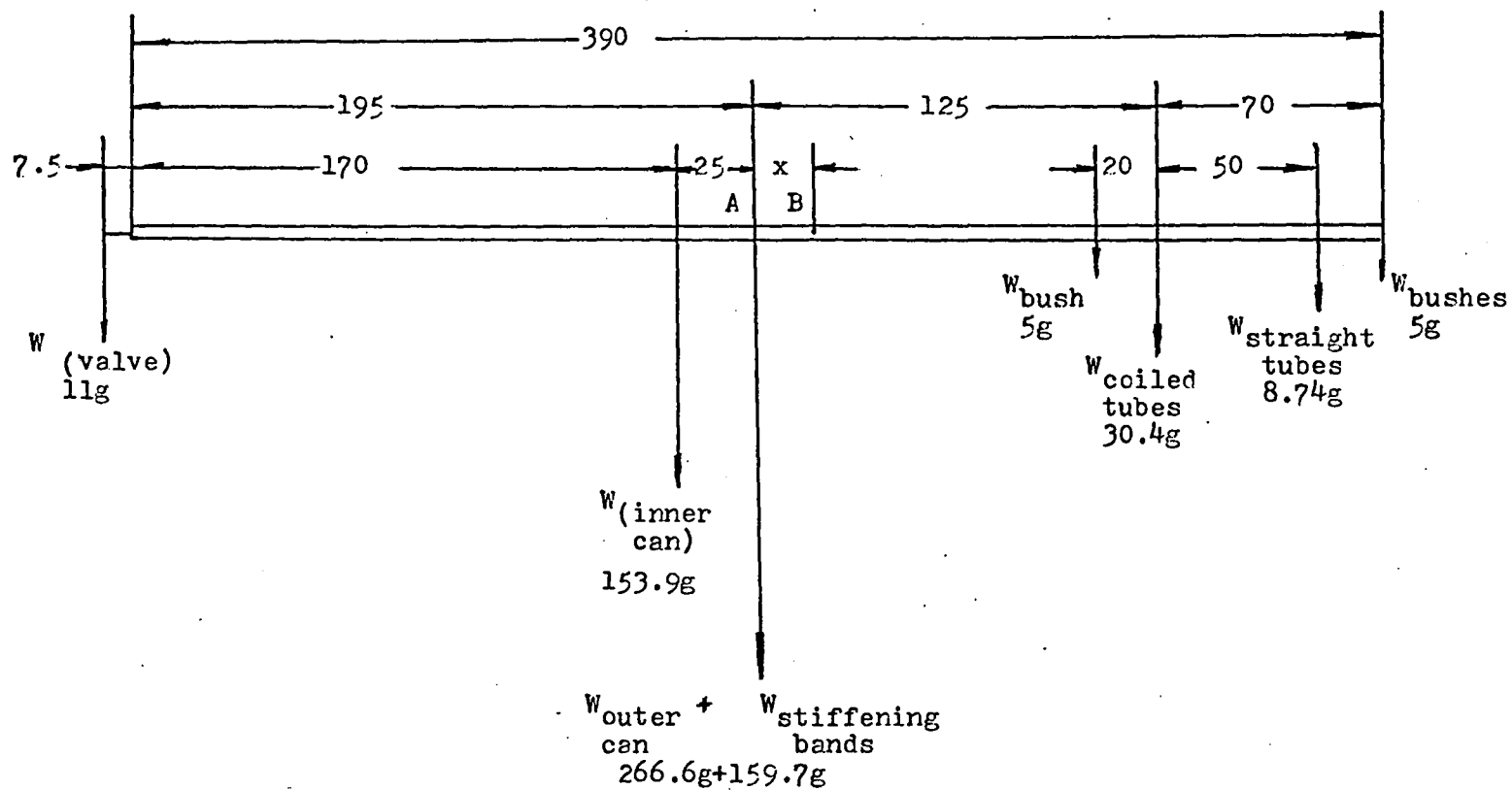As shown in Fig 9, the position of the magnet inside the inner can

Fig.8. The sketch for calculating the position of
the centre of gravity of Dewar.
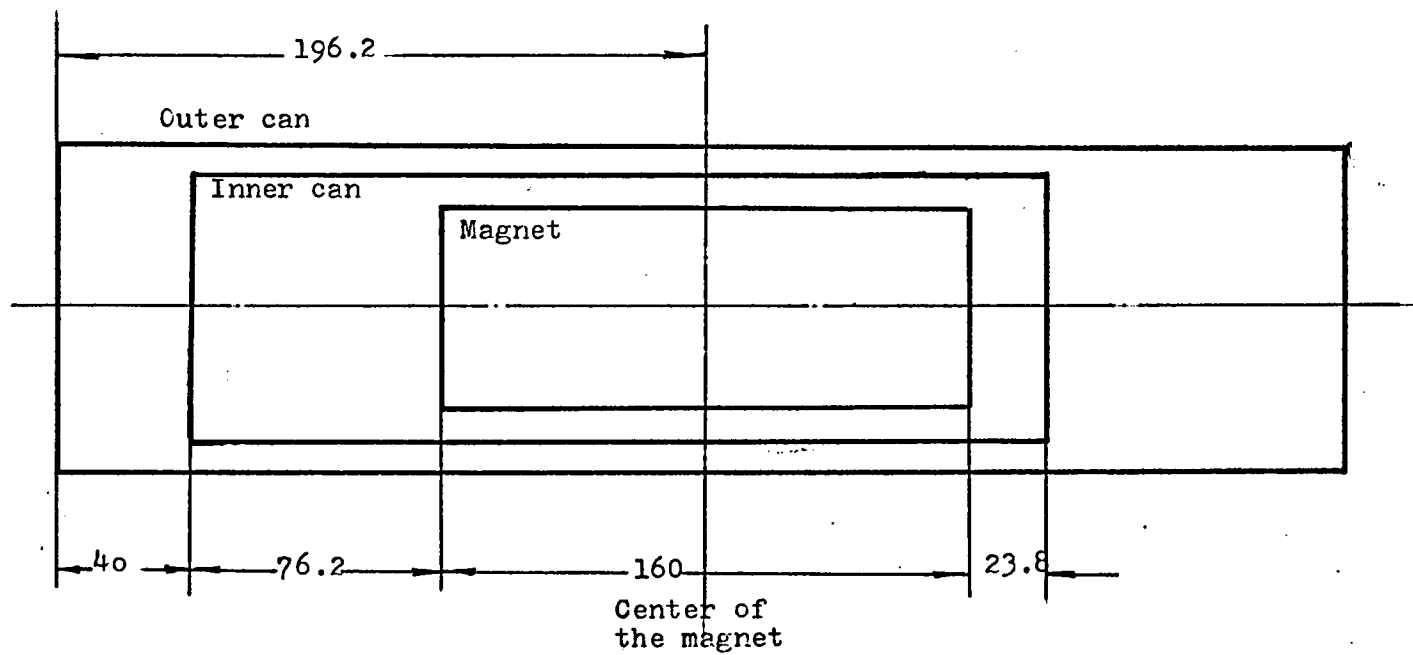
Fig.9. The position of the magnet inside the inner can.

is 76.2 mm away from the left end of the inner can or 23.8mm away from the right end of the inner can.

## 7. THE CALCULATION OF THE HELIUM GAS PRESSURE IN THE INNER CAN

### 7.1. The Released Magnetic Energy

After all the liquid helium in the inner can has evaporated, the temperature of the superconducting magnet will rise and exceed its transition temperature. At that time it will be driven into normal state. Its resistance-less property will be lost. The electric current of the superconducting solenoid will decrease in a very short time. So does the magnetic field of the solenoid. During that time, the induced emf due to the decrease of the flux inside the solenoid will drive the current. The stored magnetic energy in the space inside the solenoid will be released as a form of heat energy. It is known that the electric current inside the solenoid is 20A, and the inductance of the solenoid is 9 Henry. So the released heat energy is

$$E = \tfrac{1}{2} L I^2 = \tfrac{1}{2} \times 9 \times 20^2 = 1800 \text{ Joules}$$

### 7.2 The Specific Heat of the Magnet

As we have known, the winding wire of the magnet solenoid is composed of Niobium-Titanium filaments and the substrate of copper. Because copper has a much higher thermal conductivity than Niobium-Titanium, when the quench occurs, the heat generated can transfer to the liquid helium quickly and thus prevent damage to the operation of superconductivity magnet.

When we calculate the increase of the temperatures of both superconducting magnet and the helium gas, at first we must consider the specific heat of the superconducting magnet and of the helium gas.

There are two contributions to the specific heat of a metal. Heat raises the temperature both of the crystal lattice and of the conduction electrons.

According to solid-state physics, we can write the specific heat of a metal as

$$C = C_{latt} + C_{eL} = \frac{12}{5} \pi^4 NK_B \left(\frac{T}{\theta}\right)^3 + rT = 234R\left(\frac{T}{\theta}\right)^3 + rT \ldots\ldots\ldots(19)$$

where R is the Universal gas constant

     N is Avogadro's number

     $K_B$ is Boltzmann's constant

     $\gamma$ is a measure of the density of the electron states at the Fermi
                                                   surface

     $\theta$ is the Debye temperature.

    The specific heat of a metal in superconducting state is more complicated than in normal state.

    As we have known the superconducting state is one of the thermodynamic states of metal. When the temperatures of the metal decreases below the transition temperature in the absence of any applied magnetic field there will be a phase transition. The lowest energy state of the metal will become a superconducting state. This phase transition is known as a second-order phase transition. At the transition there is no latent heat, and there is a jump in the specific heat. It is true for type I superconductor that there is a sudden change in the specific heat as the metal goes from the superconducting into the normal state. (When the phase transition occurs in a constant applied magnetic field, the transition is of the first order, and latent heat is absorbed). There are three thermodynamic states for type II superconductors, namely superconducting state, mixed state and normal state. When the metal goes from the superconducting into the mixed state, there will be a large narrow peak in the curve of specific heat against T more like a "$\lambda$ -type" specific heat anomaly. When the metal goes from the mixed state into the normal state, there will be a sudden drop in the specific heat. (Both transitions occur in a constant applied magnetic field).

    For our calculation, we only consider the specific heat of the metal above the transition temperature.

    For simplicity of calculation, we assume that the winding wire is fully made of copper.

1) At first, let us calculate the specific heat of copper at 4.2 K and at 20 K by using the expression (19)

It is known that

Debye temperature of copper is 310 K

the atomic weight of copper is 64

Total weight of magnet is 0.85 kg

So the number of moles of copper is 13.28 gmoles

The specific heat of copper contributed by Lattice vibration at 4.2K is:

$$Cv_{latt} = 234R(\frac{T}{\theta})^3 = 234 \times 8.3143 \times (\frac{4.2}{310})^3 = 4.838 \times 10^{-3} \text{ J/g mole K.}$$

The specific heat of copper contributed by the conducting elections is: ($r = 7.4 \times 10^{-4}$ J/mole K)

$$C_{v \text{ electron gas}} = rT = 7.4 \times 10^{-4} \times 4.2 = 3.108 \times 10^{-3} \text{ J/gmole K.}$$

The specific heat of copper by lattice at 20K is

$$C_{v \text{ latt}} = 234 R(\frac{T}{\theta})^3 = 234 \times 8.3143 \times (\frac{20}{310})^3 = 0.522 \text{ J/gmole K}$$

The specific heat of copper by electron at 20K is

$$C_{v \text{ electron}} = rT = 7.4 \times 10^{-4} \times 20 = 0.0148 \text{ J/gmole K}$$

So the heat capacity of copper at 4.2K is

$$C = N(C_{v \text{ latt}} + C_{v \text{ el}}) = 13.28 \times (4.838 + 3.108) \times 10^{-3} = 0.105 \text{ J/K}$$

The heat capacity of copper at 20K is

$$C = N(C_{v \text{ lat}} + C_{v \text{ el}}) = 13.28 \times (0.522 + 0.0148) = 7.128 \text{ J/K}$$

So we can see that even at 20K the heat capacity of copper is still rather small. For calculating the specific heat of a metal, the expression (19) is only applied in the low temperature region ($T/\theta < 0.1$). So for higher temperature region we must use the diagram (10) to calculate the temperature rise of copper.

2) Estimation of the temperature increase of copper by assuming that all the released heat energy is absorbed by the copper.

We can see from the diagram (11) that either $C_v$ or E/T changes linearly with $T/\theta$ in the region $T/\theta = 0.1 \sim 0.3$.

Checking the diagram, we get:

3) Calculation of the final temperature of both copper and helium vapour.
At first, we calculate the heat absorbed by the helium vapour and
copper when the temperatures of the helium vapour and copper change
from 4.2K to 31 K.

Since the specific heat of copper in the temperature range of 31 K→
62 K changes linearly with $T/\theta$ we can get the average specific heat of
copper in this range. The specific heat of helium vapour in this range
is about a constant. Thus we can obtain the average heat capacity of
copper plus helium vapour. Then we calculate how many degrees the
temperatures of copper and helium vapour can increase due to the
absorbed heat energy starting from 31K. At last we can obtain the
final temperature.

a) According to T = 31 K and $\rho$ = 0.01689 g/cc, we get from the
diagram of helium properties or from the table:

$$E_{31 \ K} = 108 \ J/g$$

The absorbed heat energy by helium vapour when its temperature chances
from 4.2K to 31K is:

$$\Delta E_{vapour} = (108-24.13) \times 3.834 = 321.56 \ joules$$

The absorbed heat energy by copper:
when $T/\theta = 0.1$ (T=31K), $\frac{E}{T} = 0.481$ (Checking the table (II))
We consider the internal energy of copper at 4.2K as zero, so the
absorbed heat energy $\Delta E$ is considered to be equal to E.

$$\Delta E = E = 0.481 \ T = 0.481 \times 0.1 \times 310 = 14.91 \ Joules/gmole$$
$$\Delta E_{copper} = 14.91 \times 13.28 = 198 \ joules$$

So the absorbed heat energy both by copper and helium vapour when
the temperature of the system changes from 4.2 to 31K is

$$\Delta E_{total} = \Delta E_{vapour} + \Delta E_{copper} = 198 + 321.56 = 519.5 \ joules$$

b) The average specific heat of copper in the region ($T/\theta = 0.1 → 0.167$)

when $T/\theta = 0.1$.  $\frac{E}{T} = 0.481$.  $E = 14.91$ Joule/gmole

when $T/\theta = 0.167$, $\frac{E}{T} = 1.93$.  $E = 1.93 \times 0.167 \times 310 = 99.92$ Joule/gmole

$C_{v\,average} = \frac{\Delta E}{\Delta T} = \frac{99.92 - 14.91}{310 \times (0.167 - 0.1)} = 4.09$ joule/gmole K

The heat capacity of copper

$C_{copper} = 4.09 \times 13.28 = 54.32$ joules/K

The heat capacity of helium vapour is 12 joules/K

The total heat capacity is

$C_{total} = C_{copper} + C_{vapour} = 12 + 54.315 = 66.32$ joules/K

c) The released heat energy is 1800 Joules.

When the temperature of the system goes from 4.2K to 31 K, both copper and helium vapour absorbed 519.5 joules of the heat energy.

The heat energy to be absorbed is $\Delta E = 1800 - 519.5 = 1280.5$ joules
The increase of the temperature will be

$$\Delta T = \frac{\Delta E}{C_{total}} = \frac{1280.5}{66.32} = 19.3$$

So $T = 31 + 19.3 = 50.3$ K.

d) Checking the calculated results.

For $T = 50.3$ K. $T/\theta = \frac{50.3}{310} = 0.162$

When $T/\theta = 0.167$.  $\frac{E}{T} = 1.93$

$T/\theta = 0.154$  $\frac{E}{T} = 1.59$

By interpolation, when $T/\theta = 0.162$, $\frac{E}{T} = 1.80$

So $\Delta E_{copper} = 1.80 \times 0.162 \times 310 \times 13.28 = 1200$ Joules

Check the table of thermodynamic properties of helium, we get the internal energy of the helium at $T = 50.3$K and $\rho = 0.01689$ g/cc
$E = 167.6$.

so $\Delta E_{vapour\ total} = (167.6 - 24.13) \times 3.834 = 550$ joules

$$\Delta E_{total} = 550 + 1200 = 1750 \text{ joules.}$$

There is only a very small error of the calculation.
Thus we get the final temperatures of the magnet and the helium vapour
to be about 50 K.

## 7.3. The Pressure of the Helium Gas in the Inner Can

We assume that the quench of the superconducting magnet is so fast
that the helium gas which has absorbed the released heat energy and has
been in the high pressure can't escape from the inner can during that very
short time.

So we consider that the density of the helium vapour in the whole
process will not change.

According to $T = 50.3K$ and $\rho = 0.01689$ g/cc, checking "the
thermophysical properties of helium - 4", we obtain:

$$P = 18 \text{ atm.}$$

## 7.4. The Bursting Pressure of the Inner Can

The equation for calculating the bursting pressure is

$$P = 2S \frac{t}{D}$$

where S is the tensile strength
t is the thickness of the tube
D is the diameter of the tube

It is known that,
the tensile strength of stainless steel is $150 \times 10^3$ lb/in$^2$ = 10555.5 kg/cm$^2$
the thickness of the wall is 0.01" = 0.025 cm
the diameter of the tube is 2" = 5.08 cm

$$S = 150 \times 10^3 \text{ lb/in}^2 = 10555.5 \text{ Kg/cm}^2$$

$$P = 2S.\frac{t}{D} = \frac{2 \times 10555.5 \times 0.005}{1.0336} = 102 \text{ atm.}$$

We adopt the safety factor as 5.5

So the allowed working pressure is

$$P_{working} = \frac{102}{5.5} = 18.5 \text{ atm.}$$

So the pressure of the helium gas does not exceed the allowed working pressure.

Thus we conclude that it is safe for the magnet to operate.

## 8. THE BASIC PRINCIPLES OF MAGNETIC SUSPENSION SYSTEM

The principles of magnetic suspension system can be described by using the idea of the interaction of the magnetic poles. It can be explained by the theory of magnetic field too.

### 8.1. Description Based on the Magnetic Pole Theory

According to classical magnetic theory, the force between two magnetic poles varies inversely as the square of the distance apart and is directly proportional to the 'strength' of the poles. This is Coulomb's law.

In order to suspend a model in a wind tunnel horizontally, a strong permanent magnet is mounted inside the fuselage of the model. When the high speed air stream passes through the wind tunnel, there will be three directions of forces and three rotation moments acting on the model. (We adopt a system of orthogonal axes centred on the model, with one axis along the tunnel centre-line, a vertical axis, and a second horizontal axis transverse to the tunnel, say x,y,z axes res- pectively). They are x-direction force, y direction force, z direction force, moment about x axis, moment about y axis and moment about z axis. Hence the model will have transition movements in the x,y,z directions. The rotation about x axis called 'roll', the rotation about y axis called 'yaw' and the rotation about z axis called 'pitch' also occur. In order to keep the model in equilibrium during the testing process, the magnetic suspension system should provide forces and moments to counteract these forces and moments. Furthermore those forces and moments acting on the model should be able to be measured.

·Consider a pair of similar electro-magnets arranged around the wind tunnel as shown schematically in Fig.11(a). Each magnet is separately energised. If the electro-magnets have equal energisation producing equal pole-strengths, the net force on a model positioned symmetrically below the electromagnets will act vertically through the centre O of the model. In this way the gravitational force and the Y direction force acting on the model by the stream can be counteracted. The net vertical force will be proportional to the sum of the currents in the coils of the two electromagnets.

Coil 1                    Coil 2

S                         N

N₁ I₁                     N₂ I₂

N        S

O
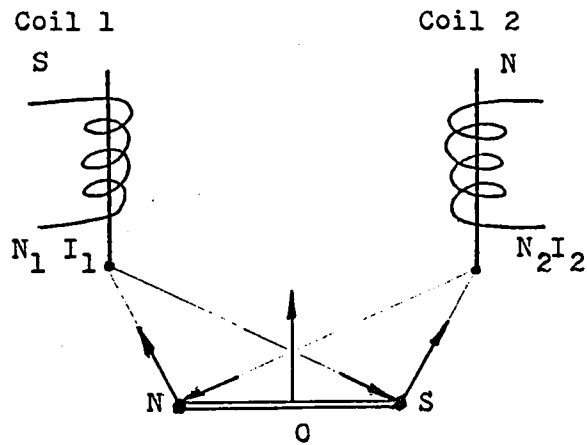
Fig.11(a)   Four force components acting on a model

Coil 1                    Coil 2

$F_{yaxis}$

$M_z$        $F_{x_{axis}}$
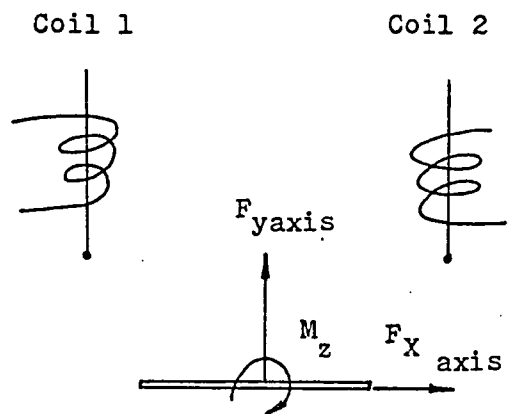
Fig.11(b) Resolution of forces into two components
and a moment

Coil 3          A          Coil 4

Z

$M_y$                     X          Coil 7

Coil 5                    Coil 6
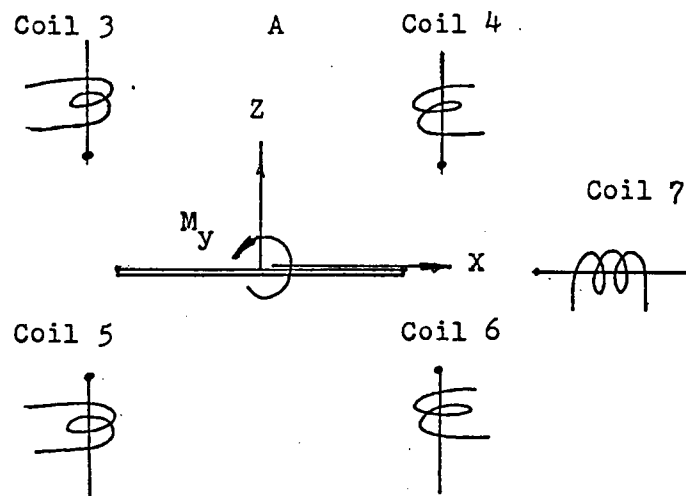
Fig.11(c) Lateral magnet and drag magnet

$$F_y = K_y \; (N_1 I_1 + N_2 I_2)$$

where $K_y$ is a constant determined by experiment.

If the electro-magnets are energised to different levels, the force on the model provided by the interaction between electromagnet 1 and the model magnet will be different from the force of the interaction between electromagnet 2 and the model magnet. These forces can be resolved into vertical force $F_y$, horizontal force $F_x$ and a moment about Z-axis $M_z$, see Fig. 11(b). The moment $M_z$ and the force $F_x$ would be proportional to the difference between the strengths of the magnets (i.e. ampere-turns per metre of each magnet)

$$M_z = K_{mz} \; (N_1 I_1 - N_2 I_2)$$

$$F_x' = K_{fx}' \; (N_2 I_2 - N_1 I_1)$$

where $K_{mz}$ and $K_{fx}'$ are constants determined by experiment.

The force components in the y direction, as described before, would be proportional to the sum of the strengths of electromagnet 1 and 2.

So we can control the force in y direction and the moment about z axis by changing the currents in electromagnet 1 and 2.

(The x component force produced here is much smaller than the y direction force.)

When the model moves away from its symmetrical position related to coil 1 and coil 2, a relatively large effect on the x direction force will be produced (see Appendix A).

This can be used to resist the applied drag force if the drag force is rather small in the case of low speed wind tunnel.

For a high speed wind tunnel, the electromagnet 7 is used to provide a force in the upstream direction (x direction) opposing the drag force. This force is proportional to the strength of the electromagnet 7.

$$F_x = K_{fx} \; N_7 I_7$$

where $K_{fx}$ is constant determined by the experiment.

The lateral electromagnets are shown in Fig. 11(c). The diagram represents a horizontal section through the wind tunnel axis. A force component along the Z axis and a moment about y axis are provided by the electromagnets. Similarly the z direction force is proportional to the sum of the strengths of electromagnets 3 and 4 less the sum of the strengths of electromagnets 5 and 6. The moment about y axis is proportional to the sum of the strengths of electromagnets 5 and 4 less the sum of the strengths of the electromagnets 3 and 6.

$$F_z = K_{fz} \; (N_3 I_3 + N_4 I_4 - N_5 I_5 - N_6 I_6)$$

$$M_y = K_{my} \; (N_5 I_5 + N_4 I_4 - N_3 I_3 - N_6 I_6)$$

where $K_{fz}$ and $K_{my}$ are constants determined by experiment.

There are special arrangements for controlling the roll about x axis. Methods for roll control can be found in the reference material (1).

## 8.2. The Description Based on the Theory of Magnetic Field

### 1. Moment and force on a current loop

A magnetic dipole experiences a torque in a uniform magnetic field

$$\vec{T} = \vec{m} \times \vec{B}$$

where $\vec{m}$ is the magnetic dipole moment of a current loop, $|\vec{m}| = IA$
I is the current flowing in the current loop.
A is the area surrounded by the current loop.
$\vec{B}$ is the intensity of the uniform magnetic field.

In a uniform field, the net force on a current loop is zero:

$$\vec{F} = \oint I(d\vec{\ell} \times \vec{B}) = I \oint d\vec{\ell} \times \vec{B} = I(\oint d\vec{\ell}) \times \vec{B} = 0;$$

the constant $\vec{B}$ comes outside the integral, and the net displacement $\oint d\vec{\ell}$ around a closed loop vanishes.

In a non-uniform field, this is no longer the case. For example, suppose a circular wire of radius R, carrying a current I is suspended above a short solenoid in the "fringing" region (Fig.12). Here $\vec{B}$ has a radial component, and there is a net downward force F on the loop (Fig 13), $F = 2\pi \, IRB \cos \theta$.

For an infinitesimal loop of dipole moment $\vec{m}$ in a field $\vec{B}$ the force is:

$$\vec{F} = (\vec{m} \cdot \nabla) \, \vec{B}$$

It is proved in reference (5)
It may be written in the form:

$$F_x = \vec{m} \cdot \frac{\partial \vec{B}}{\partial x} \, , \quad F_y = \vec{m} \cdot \frac{\partial \vec{B}}{\partial y} \, , \quad F_z = \vec{m} \cdot \frac{\partial B}{\partial z} \dots\dots\dots\dots\dots\dots\dots (20)$$

2.  Force on a magnetic substance

When placed in a magnetic field, a magnetic substance becomes magnetized. An element of volume dV of the magnetic substance acquires a magnetic moment.

$$\vec{dm} = \vec{M} \, dV,$$

where $\vec{m}$ is the magnetisation vector. According to equation (20), the force on this volume element of the magnetic substance is equal to

$$dF_x = \vec{M} \cdot \frac{\partial \vec{B}}{\partial x} \, dv, \quad dF_y = \vec{M} \cdot \frac{\partial \vec{B}}{\partial y} \, dv, \quad dF_z = \vec{M} \cdot \frac{\partial \vec{B}}{\partial z} \, dv \dots\dots\dots\dots (21)$$

The integral of equation (21) over the whole volume of the magnetic substance will be the total force on that magnetic substance.

3.  Magnetic moments and forces on the model

The actual magnetic moments or torques that can be applied to the model containing a magnetic substance or a permanent magnet are:

$$\vec{T} \simeq K \, V \, \underline{\vec{M}} \times \underline{\vec{B}}_a \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \dots\dots (22)$$

where K is a constant
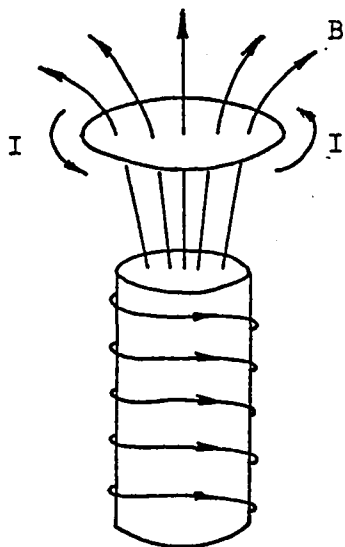     V is the volume of the magnetic substance
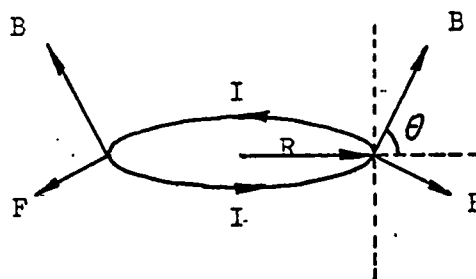
Fig.12. A current loop in a nonuniform field



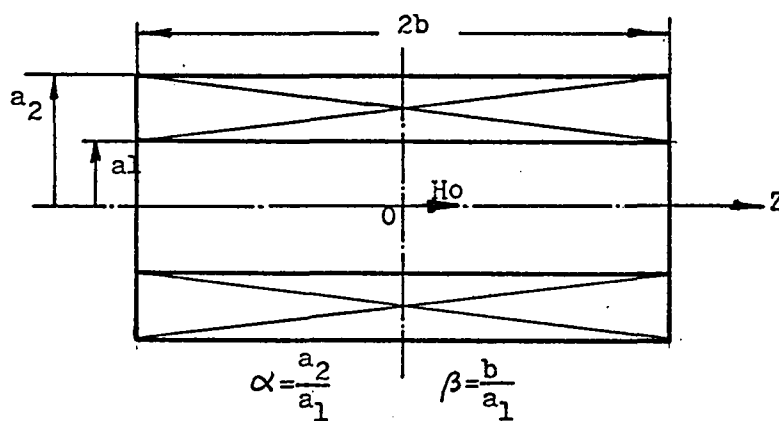Fig.13. A net downward force on the current loop in a nonuniform field



Fig.14. The sketch of the superconducting magnet

$\overset{>}{\underset{\sim}{M}}$ is the average magnetization vector of the magnet.

$\overset{>}{\underset{\sim}{B}}_a$ is the average applied magnetic field intensity.

The magnetic forces are applied to the magnetized model by means of a set of controlled gradients in the magnetic fields. The general equation describing these magnetic forces is as follows:

$$\overset{>}{\underset{\sim}{F}} \;\underset{\sim}{=}\; KV(\overset{>}{\underset{\sim}{M}} \cdot \nabla)\,\overset{>}{\underset{\sim}{B}}_a$$

This can be expanded as follows:

$$F_x \doteq KV \left( \underline{M}_x \frac{\partial \underline{B}_x}{\partial x} + \underline{M}_y \frac{\partial \underline{B}_y}{\partial x} + \underline{M}_z \frac{\partial \underline{B}_z}{\partial x} \right)$$

$$F_y \doteq KV \left( \underline{M}_x \frac{\partial \underline{B}_x}{\partial y} + \underline{M}_y \frac{\partial \underline{B}_y}{\partial y} + \underline{M}_z \frac{\partial \underline{B}_z}{\partial y} \right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(23)$$

$$F_z \doteq KV \left( \underline{M}_x \frac{\partial \underline{B}_x}{\partial z} + \underline{M}_y \frac{\partial \underline{B}_y}{\partial z} + \underline{M}_z \frac{\partial \underline{B}_z}{\partial z} \right)$$

where $\dfrac{\partial \underline{B}_x}{\partial y}$ is the average value of $\dfrac{\partial \underline{B}_x}{\partial y}$ over model magnet volume.

Others such as $\dfrac{\partial \underline{B}_x}{\partial x}, \dfrac{\partial \underline{B}_z}{\partial x} \quad \dfrac{\partial \underline{B}_z}{\partial y} \quad \dfrac{\partial \underline{B}_y}{\partial y} \quad \dfrac{\partial \underline{B}_z}{\partial y} \quad \dfrac{\partial \underline{B}_x}{\partial z} \quad \dfrac{\partial \underline{B}_y}{\partial z}$ and $\dfrac{\partial \underline{B}_z}{\partial z}$

are similar as $\dfrac{\partial \underline{B}_x}{\partial y}$ .

By assuming the magnetization lies along the wind tunnel direction (that is $\overset{>}{M}_y = o$, $\overset{>}{M}_z = o$) we obtain

$$F_x \doteq KV \, \underline{M}x \frac{\partial \underline{B}_x}{\partial x}$$

$$F_y \doteq KV \, \underline{M}x \frac{\partial \underline{B}_x}{\partial y} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(24)$$

$$F_z \doteq KV \, \underline{M}x \frac{\partial \underline{B}_x}{\partial z}$$

Thus, according to equation (24), clearly independent control of the gradient of the magnetic field along and normal to the magnetization allows the force components to be controlled independently, no matter what the direction of $\overset{>}{M}$. This is the approach that has been taken in the prototype system in fact. It is possible to

provide independent control of the three gradient components and
at the same time maintain the magnetizing field in the vicinity of
the model approximately constant.

Equation (22) provides a means of estimating the magnetic torques
generated through the interaction of the magnetized body and the
applied field. It is a matter of little importance whether $\vec{M}$ is
due to a permanent magnet or is induced by a magnetizing field or
is due to a equivalent superconducting magnet. The result is the
same. Consider the first. From equation (22) in component form:

$$T_x = KV\ (\underline{M}y\ \underline{B}z - \underline{M}z\ \underline{B}y),$$

$$T_y = KV\ (\underline{M}z\ \underline{B}x - \underline{M}x\ \underline{B}z)$$

$$T_z = KV\ (\underline{M}x\ \underline{B}y - \underline{M}y\ \underline{B}x)$$

Similarly assuming the magnetization is aligned with the x-axis,
($\underline{M}y= o$, $\underline{M}z = o$) we also obtain:

$$T_x = o$$

$$T_y = KV\ \underline{M}x\ \underline{B}z$$

$$T_z = KV\ \underline{M}x\ \underline{B}y$$

In other words if the magnetization vector lies along the x or
wind tunnel axis, a pitching moment or torque is developed by a
vertical field and a yawing torque is developed by a lateral field.
These two moments can be developed independently as required. It
means that these two moments can be controlled.

The roll torque depends upon the difference in the curvature of
$B_y$ and $B_z$ in the x direction. It is described in reference (3),
Stephens and Goodyer indicated the methods for generating an
independent roll torque. The details are in the reference (1) and (2).

9.  CALCULATION OF THE SUPERCONDUCTING MAGNET

The structure of the superconducting magnet is just like a solenoid coil magnet, but the wire is made of superconducting filamentary material $N_b$- $T_i$ in a cupronickel matrix. The length of the coil is 2b = 120mm. Its outer diameter is 2 $a_2$ = 46mm. Its inner diameter is 2 $a_1$= 18.9mm. The sketch of it is shown in Fig.14.

NIOMAX CN A61/25 is chosen for winding the coil. It is an intrinsically stable superconductor. It is suitable for small coils without inter-layer cooling. Its diameter is 0.25 mm. If the external field is 6 tesla, its critical current is 27 amperes. The operating current is about 21 amperes. The use of a twisted array of 61 niobium-titanium filaments in a high resistivity matrix permits very rapid changes of current and magnetic field without quenching.

The formula for calculating the central field $H_o$ is given as follows: (see ref.4 ).

$$H_o = J \, \lambda \, a_1 \, \beta \, \ln \frac{\alpha + (\alpha^2 + \beta^2)^{\frac{1}{2}}}{1 + (1 + \beta^2)^{\frac{1}{2}}}$$

where $H_o$ is the magnetic strength in central point (ampere/metre)

$a_1$ is the inner radius of the coil, (m)

$\beta$ is defined as $\frac{b}{a_1}$

$\alpha$ is defined as $\frac{a_2}{a_1}$

$\lambda$ is the space factor, is defined as

$$\lambda = \frac{\text{active section of the winding}}{\text{total section of the winding}}$$

J is the current density in the conductor (ampere/m²)

So we obtain:

$a_1 = 9.45 \times 10^{-3}$m, $\alpha$ = 2.4338, $\beta$ = 6.35

$$F_{(\alpha \, \beta)} = \beta \ln \frac{\alpha + (\alpha^2 + \rho^2)^{\frac{1}{2}}}{1 + (1 + \beta^2)^{\frac{1}{2}}} = 1.378$$

The cross section of the wire is $A = \dfrac{\pi}{4} d^2$

The effective cross section area $A' = \dfrac{\pi d^2}{4\lambda}$

The number of turns of wires per $m^2$ is $n$ which equals

$$n = \frac{4\lambda}{\pi d^2} = \frac{3}{\pi \times (0.25 \times 10^{-3})^2} = 1.52 \times 10^7$$

According to definitions of $J$ and $\lambda$, $\lambda J = n I$.

where $I$ is the operating current which equals 21.19 amperes.

so $B = \mu_o H = \mu_o n I a_1 F(\alpha, \beta)$

$\quad = (4\pi \times 10^{-7}) \times 1.527 \times 10^7) \times 21.19 \times (9.45 \times 10^{-3}) \times 1.378$

$\quad = 5.29$ (Tesla)

The volume of the wire winding space is

$v = 2\pi \beta (\alpha^2 - 1) a_1^3$

$\quad = 2\pi \times 6.35 \times (2.4338^2 - 1) \times (9.45 \times 10^{-3})^3$

$\quad = 1.65 \times 10^{-4} \, m^3$

The total length of the wire in the coil is equal to

$$\ell = n.v. = 1.527 \times 10^7 \times 1.65 \times 10^{-4} = 2521 \, m$$

The number of turns of wires in the coil is equal to

$N = 2a_1^2 (\alpha - 1) \beta n$

$\quad = 2 \times (9.45 \times 10^{-3})^2 \times (2.4338 - 1) \times 6.35 \times 1.527 \times 10^7 = 24845$ (turns)

The inductance of the coil is equal to

$$L = \frac{\mu_o N^2 A}{\ell}$$

Where A is the cross section area through which the flux of the magnetic field passes.

If we adopt $A = \pi a_2^2$

$$L = \frac{4\pi \times 10^{-7} \times 2.4845^2 \times 10^8 \times \pi \times (23 \times 10^{-3})^2}{120 \times 10^{-3}} = 10.74 \text{ (henry)}$$

If A is the average cross section area of the coil

$$A = \frac{\pi}{4} (a_1 + a_2)^2 = \frac{\pi}{4} \times (23 + 9.45)^2 \times 10^{-6} = 827 \times 10^{-6} \ m^2$$

$$L = 5.34 \ (henry)$$

The results of calculation are summarized as follows:

The central field is 5.29 Tesla when operating current is 21.19(amperes)

The total length of the wire is 2521(m)

The number of turns of the wire is 24845(turns)

The inductance of the coil is 10.74(henry)

The actual values of the superconducting magnet made by Oxford Instruments Ltd., are as follows:

Central field is 5.3 Tesla

Current for central field is 21.19 ampere

Inductance of the coil is 9 henry.

10. THE EQUIVALENT AVERAGE MAGNETIC MOMENT PER UNIT VOLUME OF THE
SUPERCONDUCTING MAGNET.

### 10.1. Uniformly Magnetized Rod and Equivalent Air Field Solenoid

It was Ampere's theory that the pronounced magnetic effects of an
iron bar occur when large numbers of atomic-sized magnets associated
with the iron atoms are oriented in the same direction so that their
effects are additive.  They may be regarded as tiny magnets or as
miniature current loops.  We can describe them by their magnetic moment,
which can be expressed as $IA'$. $A'$ is the area of a miniature current loop.
$I$ is the current flowing in the loop.  The effect of the atomic magnets
can be conveniently described by a quantity called the magnetization $M$,
which is defined as the magnetic dipole moment per unit volume.

A long bar magnet which consists of miniature current loops is
shown in Fig. 15.  The magnetic moment of each loop is $IA'$.

Assuming that there are $n$ loops in a single cross section of the
rod.  We have $nA' = A$, where $A'$ = area of elemental loop.  $A$ = cross
sectional area of rod.

Further, let us assume that there are $N$ such sets of loops in the
length of the rod, then $nN = N'$
where $n$ = number of loops in a cross section or rod

   $N$ = number of such sets of loops

   . $N'$ = total number of loops in rod
It follows that the magnetization $M$ of the rod is given by

$$M = \frac{m}{v} = \frac{N' \, I \, A'}{LA} = \frac{NI}{L} \frac{nA'}{A} = \frac{NI}{L} = K'$$

where $K'$ = equivalent sheet current density on outside surface of rod,
   $A \, m^{-1}$

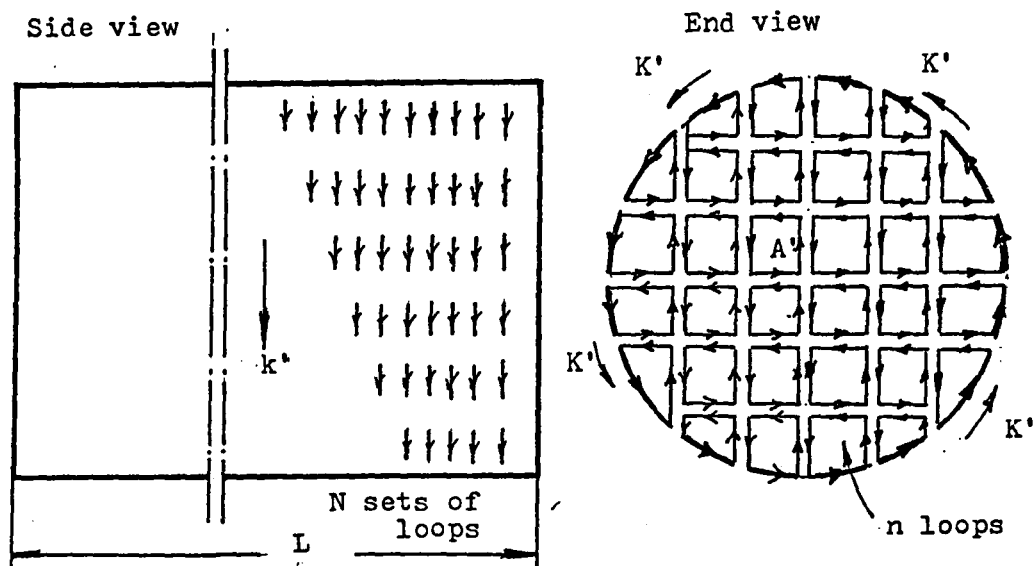   $L$ = Length of rod, m.

50.



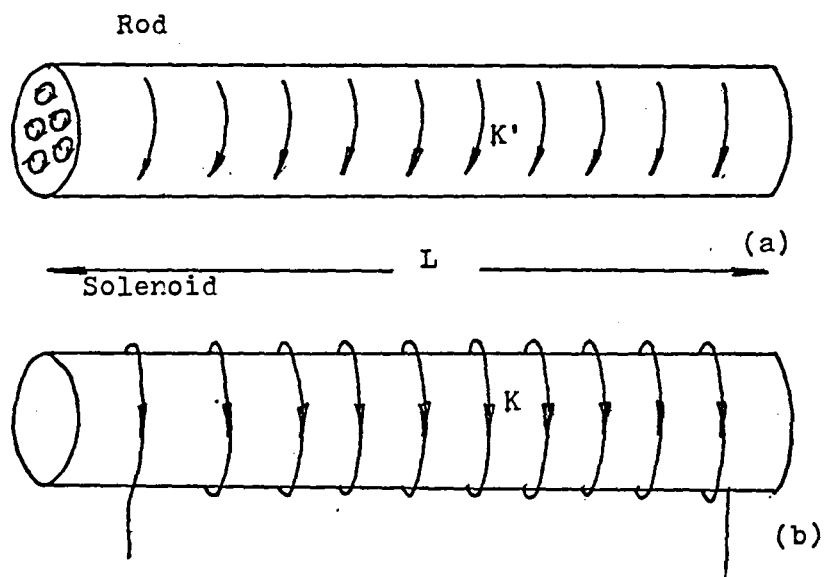Fig.15. Uniformly magnetized rod with elemental current loops.



Fig.16. (a) Uniformly magnetized rod
(b) Equivalent solenoid

From the end view of the rod, we can see that there are equal and oppositely directed currents wherever loops are adjacent, so that the currents have no net effect with the exception of the currents at the periphery of the rod. As a result there is the equivalent of a current sheet flowing around the rod.

This type of a current sheet is just like the case of a solenoid with many turns of fine wire, as in Fig.16.

The actual sheet-current density K for the solenoid is

$$K = \frac{NI}{L} \ (A \ m^{-1})$$

where N = number of turns in solenoid,

    I = current through each turn, A

    L = length of solenoid, m.

If the solenoid of Fig.16 is the same length and diameter as the rod of Fig.16 , and K = K', the solenoid is the magnetic equivalent of the rod.

We can obtain the magnetic field in the centre of the solenoid as follows (in the air).

$$B = \mu_o \ \frac{NI}{L} = \mu_o K = \mu_o H \ (Tesla)$$

At the centre of the rod, $B = \mu_o K' = \mu_o M (Tesla)$

So we can also say that a uniformly magnetized bar magnet with magnetization M is the magnetic equivalent of the solenoid with a sheet current density $K = \frac{NI}{L} = M$ provided the rod is the same length and diameter as the solenoid.

In this case, the total magnetic moment of the solenoid is

$$M_{solenoid} = NIA = \frac{NI}{L} \ . \ (LA) = K.V,$$

and total magnetic moment of the rod is

$$M_{rod} = M.V = K.V = M_{solenoid}$$

So the total magnetic moment of the rod is the same as the solenoid which is the magnetic equivalent of that rod.

So if a solenoid is presented, its total magnetic moment divided by the whole volume will be the magnetization M of the equivalent bar magnet.

## 10.2. The ‵Average Magnetic Moment Per Unit Volume of the Superconducting Magnet.

Our superconducting magnet is just like a multilayer solenoid. A representative current element $i = J\lambda\, dz\, dr$ is shown in Fig. 17, where J is the current density, $\lambda$ is the filling factor.

The area of the current loop which i flows through is

$$A = \pi r^2$$

So the magnetic moment of the current loop is

$$dm = i\, A = \pi r^2\, J\lambda\ dz\, dr$$

Integrating above equation over the entire cross section, we can obtain the total magnetic moment of the magnet.

$$m = \int_o^L \int_{r_1}^{rm} J\,\lambda\,\pi r^2\ dr\ dZ = \int_o^L \int_{r_1}^{rm} n\, I\, \pi\, r^2 dr\ dZ. \quad (\lambda\, J = n\, I)$$

So the average magnetic moment per unit volume of the superconducting magnet is:

$$\overline{M} = \frac{m}{v} = \frac{\int_o^L \int_{r1}^{rm} n\, I\, \pi r^2 dr\ dZ}{\pi\, r^2 m\, .\, L} = \frac{n\, I(rm^3 - r_1^3)}{3\, r^2 m}$$

It is known that

$$I = 21.19 \text{ ampere}$$
$$n = 1.527 \times 10^7 \text{ turns/m}^2$$
$$r_1 = 9.45 \times 10^{-3} m. \quad r_m = 23 \times 10^{-3}\ m$$

So we obtain:

$$\mu_o \overline{M} = \frac{\mu_o n\, I(rm^3 - r_1^3)}{3\ rm^2}$$

$$= \frac{(4\pi \times 10^{-7}) \times (1.527 \times 10^7) \times 21.19 \times (23^3 - 9.45^3) \times 10^{-9}}{3 \times (23^3 \times 10^{-6})}$$
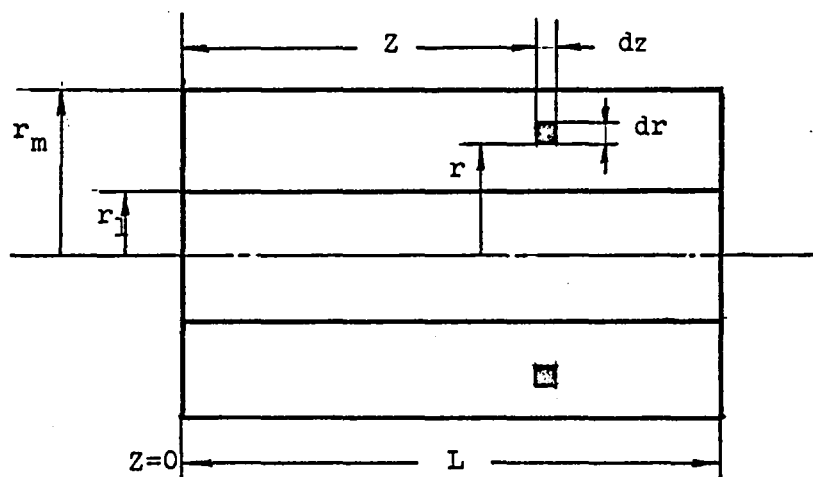
$$= 2.902\ (\text{Tesla})$$

Fig. 17. The current element of the superconducting·magnet
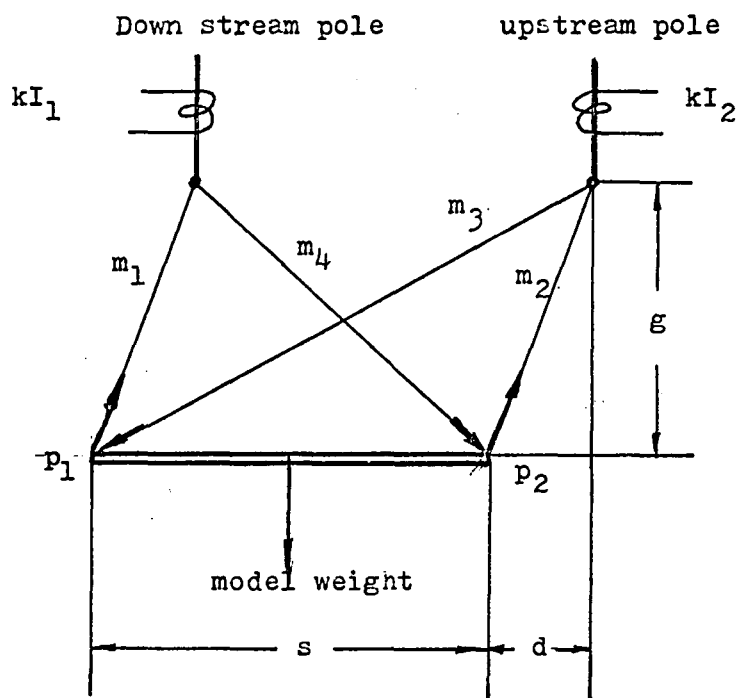


Fig.18. The geometrical arrangement of a magnetically
suspended model resisting a drag force   by
means of horizontal force components   from
the suspension magnet

So the magnetic equivalent of the superconducting magnet is a permanent magnet with magnetization $\mu_0 M = 2.902$ Tesla.

It is not clear whether the magnetic moment of the superconducting magnet will vary with variations in the gradient of the suspension fields. If it does, then position control may be more difficult than with a permanent magnet. If the moment does not vary to a first order of approximation, then the control will be similar to that using a permanent magnet. Only an experimental test will reveal the answer. This will be carried out at a later date when the magnetic suspension system is ready in the Department of Aeronautics.

If control proves to be difficult, the central hole will allow a high permeability ferro-magnet to be used for control, while the super-conducting magnet is used for suspension.

11. THE EXPERIMENTAL RESULTS OF THE BOIL-OFF RATE OF LIQUID NITROGEN AND
    LIQUID HELIUM IN THE CRYOSTAT.

The results are as follows:

|  | Boil-off gas flow (1 atm, room temp) (litre) | Time required (second) | Cryostat position |
|---|---|---|---|
| liquid nitrogen | 0.5 | 360 | vertical |
| liquid nitrogen | 0.5 | 340 | horizontal |
| liquid helium | 5 | 115 | vertical |
| liquid helium | 5 |  | horizontal |

From above experimental results we can see that when the cryostat
is in horizontal position the boil-off rate of liquid nitrogen or liquid
helium is only slightly larger than that when it is in vertical position.

When the cryostat is horizontal the temperature difference between
the outer can cap and the inner can cap (that is between the ends of
the spiral-tubes) is equal to the room temperature minus liquid helium
temperature or liquid nitrogen temperature. When the cryostat is vertical,
the temperature difference between the ends of the spiral-tubes will be
less than the former temperature difference because part of the stainless
steel wall above the liquid level will become one part of the heat flow
path and provide additional heat-flow resistance. Thus the heat inleak
is reduced.

According to calculations from the experimental results, the heat
inleak with liquid helium in the cryostat is $144 \times 10^{-3}$ watt. The super-
conducting magnet can be maintained to operate at least 50 minutes. It
has been proved in the experiment.

So we can conclude that our cryostat is satisfactory for the require-
ment of half an hour of operation time.

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# APPENDIX A

It is assumed that a model has a pole length S equal to the distance between the poles of the suspension electromagnets. It can only move horizontally in a plane beneath the electromagnets at a constant distance g, as shown in Fig. 18.

It is also assumed that the pole strength of the suspension magnet is proportional to the current flowing in the coil of that magnet. Hence the forces between poles are given by

$$f_1 = K \frac{I_1 P}{m_1^2}, \quad f_2 = K \frac{I_2 P}{m_2^2}, \quad f_3 = K \frac{I_2 P}{m_3^2}, \quad f_4 = K \frac{I_1 P}{m_4^2}$$

where K is a constant, $m_1^2 = m_2^2 = d^2 + g^2$, $m_3^2 = (s+d)^2 + g^2$.

$m_4^2 = (s-d)^2 + g^2$. P is the magnetic strength of permanent magnet inside the model.

By taking vertical components of $f_1$ and $f_3$, the vertical force on $P_1$ which should oppose half weight of the model is equal to,

$$Kpg \left( \frac{I_1}{m_1^3} - \frac{I_2}{m_3^3} \right) = \tfrac{1}{2} w$$

Similarly the net vertical force on $P_2$ is equal to

$$Kpg \left( \frac{I_2}{m_2^3} - \frac{I_1}{m_4^3} \right) = \tfrac{1}{2} w$$

Solving above two equations we get

$$I_2 = \frac{s^2 w (M_1^3 + M_4^3)}{2KP \, G((M_4/M_1)^3 - (M_1/M_3)^3)}$$

where M is non-dimensional form of m given by M = m/s, similarly G = g/s.

and

$$I_1 = \frac{s^2 w}{2KP} \left( \frac{M_1^3}{G} + (\frac{M_1}{M_3})^3 I_2' \right)$$

where $I'_2 = \frac{2Kp}{s^2 w} I_2$. Similarly one can define $I'_1 = \frac{2Kp}{s^2 w} I_1$

Both $I'_1$ and $I'_2$, the non-dimensional currents in the coil 1 and coil 2, are functions only of the geometry of the system.

By solving the force components $f_1$ to $f_4$ in a horizontal direction, a non-dimensional axial force due to model displacement can be obtained in terms of non-dimensional distance and currents.

$$f = \frac{w}{2} \left( \frac{DI'}{M_1^3} + \frac{DI_2'}{M_1^3} + \frac{(1-D)I_1'}{M_4^3} - \frac{(1+D)I_2'}{M_3^3} \right)$$

where $D = \dfrac{d}{s}$

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RESULTS OF EXPERIMENTS ON THE LIQUID HELIUM CRYOSTAT WITH A SUPER-
CONDUCTING MAGNET PRIOR TO MAGNETIC LEVITATION

We have carried out the following experiments:

1.  Cooldown of the magnet to the normal boiling point temperature
    of liquid nitrogen (77 K)

2.  Measuring the boil off of liquid nitrogen inside the cryostat,
    with the cryostat in vertical standing attitude and . horizontal
    flying attitude respectively.

3.  Fill cryostat with liquid helium after precooling with liquid nitrogen.

4.  Measure the boil off liquid helium, with the cryostat in vertical
    standing attitude and horizontal flying attitude respectively.

5.  Energise the superconducting magnet and put into a persistent mode.
    Then allow magnet to quench as the liquid helium was totally evap-
    orated.

1.  The boil off of liquid nitrogen and liquid helium was:

| | Boil off gas flow Litres at NTP | Time required (second) | Heat in leak mW | Cryostat attitude |
|---|---|---|---|---|
| Liquid nitrogen | 0.5 | 360 | 335 | vertical |
| | 0.5 | 340 | 355 | horizontal |
| liquid helium | 5 | 120 | 144 | vertical |
| | 5 | 108 | 160 | horizontal |

It was noted that when the cryostat is in horizontal attitude the boil-off
of liquid nitrogen or liquid helium is only slightly larger than that when
it is in vertical attitude.

2. The boil off rates of liquid nitrogen when the vacuum space is pumped for different times:

|  | Boil off gas flow (1 atm. room temp) (litre) | Time required (second) | Heat in leak mW |
|---|---|---|---|
| pumping 2 days | 0.5 | 260 | 465 |
| pump 5 days | 0.5 | 340 | 355 |

It was noted that the boil off of either liquid nitrogen or liquid helium is sensitive to the pumping time. After the cryostat is assembled, the boil off of either liquid nitrogen or liquid helium is lower, when the time to pump the vacuum space of the cryostat is longer. The reason is that the out-gassing process can be carried out more thoroughly via a long pumping time.

In any case, we should maintain a vacuum in vacuum space and not expose it to atmosphere or we should have to pump it again for a long time.

3. The time required for filling up the cryostat with liquid helium is 80 minutes.

The amount of liquid helium required in the transfer process is 4 litres.

4. When we energise the magnet, the boil off is increased to 5 litres per minute of helium gas flow. This is twice as much as that in the normal operating process.

It is suggested that we should energise the magnet before we stop the transfer of liquid helium.

5. According to calculations from the experimental results, the heat inleak with liquid helium in the cryostat is about $150 \times 10^{-3}$ watt, equivalent to a boil-off rate of 215cc of liquid helium per hour. So we can conclude that the cryostat is satisfactory for the requirement of half an hour of operation time.

6. The boil off of liquid helium with the cryostat at flying attitude is not affected by vibration of the cryostat.

7. When we energise the magnet, the maximum current is 15 amperes. The maximum charging rate is 6 amperes per minute.

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| | | 10. Work Unit No. |
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16. Abstract

The design of a horizontal liquid helium cryostat for refrigerating a flying super-conducting magnet in a wind tunnel is presented.  The basic principles of magnetic suspension theory are described.  Theoretical calculation of the superconducting magnet is given.

The helium cryostat has been made in the Institute of Cryogenics at the University of Southampton.  The experimental results of the boil-off of liquid nitrogen and liquid helium in the cryostat are reported.

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