"Turbulent Transport Modelling
of Separating and Reattaching Shear Flows"

Final Report
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ABSTRACT

The report summarizes the research undertaken at the University of California Davis from 1977-1980 aimed at improving present capabilities for computer simulation of turbulent recirculating flows. Attention has been limited to two-dimensional flows and principally to statistically stationary motion.

The work has been of two types: research aimed at improving the turbulence model and work on the development of the numerical solution procedure. The research on turbulence modelling has explored separately the treatment of the near-wall sublayer and the exterior (fully turbulent region) working within the framework of turbulence closures requiring the solution of transport equations for the turbulence energy and its dissipation rate. The work on the numerical procedure, which has been based on the Gosman-Pun program TEACH, has addressed the problems of incorporating the turbulence model as well as such matters as the extension to time-dependent flows, the incorporation of a 3rd-order approximation of convective transport and the treatment of non-orthogonal boundaries.

Most of the work has already been documented in the open literature either as journal articles or as computer program guides. The present report does not attempt to duplicate these documents but instead provides a summarizing review of the work which may serve both to capture the flavor of the project as a whole and to provide a guide to the literature for those intending a deeper examination.

1. Introduction

The work summarized in this report covers the period August 1977-January 31st 1981. The research has been aimed at developing a reliable, tested method for calculating turbulent flows involving separated-flow regions. Although the study has been confined to two-dimensional situations several different questions have been addressed. Broadly these fall into one of two categories relating either to how the turbulent mixing should be approximated (or "modelled") or to how the posed set of differential equations could be accurately solved. Section 2 of this report considers the latter question while Section 3 deals with the former. No attempt is made to duplicate already documented work; where the research results are published only an outline summary is provided. Passages of the report that contain new (i.e. hitherto unpublished) results or which provide a new perspective on the published information are presented more fully.

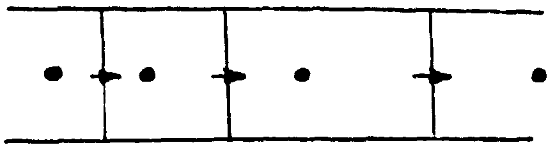
2. Numerical Procedure for Elliptic Flow

2.1 The Starting Point and a Summary of the Work

The computations of recirculating flows have been founded on the Gosman-Pun two-dimensional, steady, elliptic solver TEACH. In its basic form it solves, by finite volume discretization, the Reynolds equations for plane two-dimensional or axisymmetric flow employing primitive variables, using an upwind approximation of convective transport and the SIMPLE algorithm (Patankar [1]). The turbulence model built into the code is the high-Reynolds-number form of the Jones-Launder [2] $k \sim \varepsilon$ Boussinesq viscosity model matched to wall functions broadly (but not entirely) in accord with the proposals of Launder and Spalding [3].

In the work undertaken for NASA the following corrections, adaptations and extensions to the basic TEACH code have been introduced.

(i) The original TEACH Code was inconsistent in the assumptions made about the relative location of the velocity and pressure nodes. The velocity nodes are defined as lying midway between pressure nodes. The program also presumes, conversely, that the pressure nodes are situated mid-way between velocity nodes. For a uniform mesh the converse statement is obviously true but not otherwise.

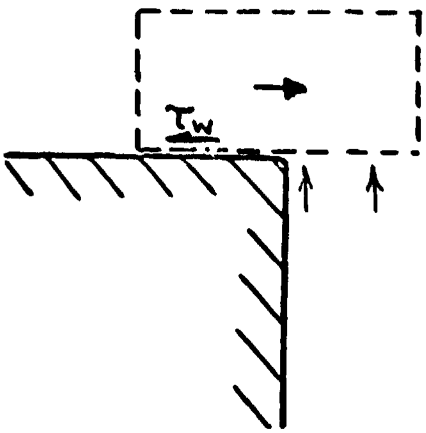


• pressure nodes

→ x-directed velocity nodes

The program was adapted to remove the presumption. It turned out that this modification did not significantly modify the results obtained, at any rate for the fairly modest expansion ratios (~ 1.2:1) employed.

(ii) Although TEACH was set up for the backward-step geometry we found a number of inconsistencies in the treatment of corner control volumes where, in certain cases, flow can occur through only part of a cell.



Note: flow occurs through only a portion of lower face of control volume and wall stress is likewise applied to only a portion of this face.

This modification had only a small influence on the computed flow field f r the backward step though Durst and Rastogi [4] found important differences for flow over an isolated rectangular sectioned block resting on a plane surface. (In the latter case there are strong y-direction velocities just in front of the rib and the x-direction velocity is also large).

(iii) Adaptations to incorporate the modified set of wall functions due to Chieng and Launder [13].

(iv) Adaptations to include an algebraic stress model (ASM) as an alternative to the $k - \varepsilon$ BVM.

(v) Inclusion of time-dependent terms.

(vi) Adaptation to allow computation of confined backward-facing steps with non parallel walls.

The ensemble of changes noted above are incorporated in a program VAST-STEP (Viscous and Algebraic-Stress Turbulent Simulations with a Teach Elliptic Program).

We summarize briefly in the following sections the work entailed in (iii) - (vi). More comprehensive accounts are given by Sindir [5], [6].

Before turning to these topics we mention two items which, though explored during our research, were finally not included in the computer program for elliptic flows. As the program of work neared completion, papers began to appear which made it clear that, in a number of recirculating flows, the use of upwind differencing of convective terms led to unacceptably large numerical errors. The use of quadratic upstream differencing of Leonard [7], formally of 3rd order accuracy, was found to give uniformly better results. An initial attempt was therefore made to include this form following the recommendations of Han et al. [8]. It was found however that for turbulent flow, stable results could not be obtained. Now, Han et al. [8] had shown for the abrupt pipe expansion (a very similar flow to the backstep) that upwind and quadratic-upwind differencing produced nearly the same results (the reason being, apparently, that these are "thin" recirculating flows where, apart from a small patch around the reattachment point, the mean streamlines are oriented at only a small angle with the mesh thus keeping false diffusion to unimportant levels*. Accordingly, since there seemed no reason to doubt that the numerical results obtained in our study with a 42 x 42 mesh were sensibly grid independent [6] the original upwind approximation was retained.**

In the final year of the project a "multiscale" version of the backstep code was created by Professor Hanjalic. The incorporation of this model led to essentially the same computed behaviour as with the single-scale scheme (due to the fact that nearly all the turbulent energy finds itself in the "production" range - a problem developed and explained in Section 3). For this reason therefore no formal 'guide' to this version has been developed.

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*The false-diffusion coefficient is generally estimated as proportional to the sine of twice the angle made by the flow with respect to the grid line.

**Subsequently, the winter's research group has entirely abandoned upwind differencing. Of the various alternatives tested (power-law differencing [9], skew upwind differencing [10], locally analytic differencing scheme [11]) the quadratic differencing appears to give, overall, the best results [12].

2.2 Incorporation of New Wall Functions

The term "wall function" refers to the special formulae applied in the 'finite-volume' equations for the wall-adjacent control volumes for the turbulence variables and the velocity component parallel to the wall. The wall functions attempt to include, in rather simple form, the physics of the viscosity-affected sublayer and near-wall region where properties vary so rapidly with distance. Strictly wall functions ought perhaps to be discussed under section 3. They are presented here, however, because the underlying physical model of the wall functions developed by Chieng and Launder [13] is the same as the standard version [3]; it is in the way the physics are translated to formulae for the near-wall cells that differences emerge.

The principal differences occur in the balance equation for kinetic energy. The kinetic energy level near the wall is dominated by generation and destruction processes which appear as sources and sinks in the conservation equation. In local equilibrium they are the only terms of any significance and, outside the viscous layer, they both vary as the reciprocal of distance from the wall. Since a finite volume method obtains the 'difference' equations by integration of the conservation equation over the control volume it follows that cell-averaged values of the production and dissipation are needed. In the original TEACH wall function a cell-averaged value of production is obtained but a point (i.e. the nodal) value of energy dissipation rate is used. This inconsistent treatment is clearly undesirable. A further serious (though, as it turns out, partly compensating) error is that the original wall functions did not distinguish the different physical significance of the total shear stress, τ, and the turbulent shear stress $-\rho\overline{uv}$. In the viscous layer the former stays finite but the latter goes to zero. Now, apart from minor terms, the turbulence energy generation rate for a point near the wall is $-\rho uv\partial U/\partial y$ which clearly vanishes when $\overline{uv}$ becomes zero. Because they used τ in place of $\rho\overline{uv}$ the original wall functions greatly over estimated the turbulence energy generation in the near-wall cell (typically by a factor of 3 or 4).

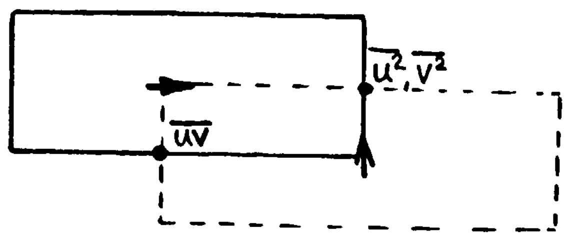
The Chieng-Launder [13] wall functions incorporate a consistent cell averaged treatment for both generation and dissipation including, for the latter, an exact expression for the rate of energy dissipation in the viscous sublayer. The other difference between the original TEACH wall functions and those incorporated in VAST STEP is that the characteristic turbulent velocity scale for the near wall region is taken as that at the outer edge of the viscous sublayer rather than that at the node of the near-wall cell. The latter choice was physically undesirable because it meant that the characteristic velocity scale depended (greatly, in the region of reattachment) on the size of the near-wall cell. In the Chieng-Launder scheme the kinetic energy at the sublayer edge was found by linear extrapolation of the values at the two nodes closest to the wall. It must be admitted that this modification, while logical, did not, from a practical point of view, reduce the sensitivity of the near-wall flow properties to the size of the near-wall cell; indeed it rendered the results somewhat <u>more</u> sensitive. The reasons for this are numerous though seem to be principally due to the assumed logarithmic law for velocity from which the wall shear stress is obtained. Research continues on this problem at UMIST and interim results are documented in [14].

2.3 Incorporation of Algebraic Stress Model

The introduction of the algebraic stress model due to Gibson and Launder [15] (omitting buoyant terms) in place of the Boussinesq viscosity stress strain law initially caused divergence of the solution procedure. Several approaches were tried before a convergent treatment was evolved. The initial practice of putting all terms relating to the Reynolds stresses into one of two source arrays proved spectacularly unsuccessful. The next step was to evaluate the Reynolds stresses on a staggered array of points as shown in the diagram (for simplicity, for the case of a uniform mesh).

—— U control volume

---- V control volume



The staggering ensured that the shear stresses were evaluated at exactly the positions they were needed when formulating a cell momentum balance for both the U and V cells. This arrangement, however, still did not produce a convergent field when used in connection with the source terms. Finally the following successful iteration sequence was developed. Firstly the four dimensionless stress ratios, $\overline{u^2}/k$, $\overline{uv}/k$ etc. were obtained <u>simultaneously</u> (and at the same points) by solving the 4 x 4 matrix. These ratios were used as the basis for finding the new stresses at the next cycle of iteration. In obtaining these values as much as possible of the stress-strain connection was absorbed into an effective viscosity. For example, in a free shear flow the form of the ASM adopted gives:

$$-\overline{uv} = \left(\frac{1 - c_2}{c_1 - 1 + P/\varepsilon}\right) \frac{k^2}{\varepsilon} \left[\frac{\overline{v^2}}{k} \frac{\partial U}{\partial y} \; \frac{\overline{u^2}}{k} \frac{\partial V}{\partial x}\right]$$

Thus, in analysing the x-momentum equation, we treat

$$\left(\frac{1 - c_2}{c_1 - 1 + P/\varepsilon}\right) \left(\frac{\overline{v^2}}{k}\right) \frac{k^2}{\varepsilon}$$

as an effective viscosity and the remainder of the stress strain relation, involving $\partial V/\partial x$, is absorbed as a source. By proceeding in this way unerringly convergent iteration was achieved.

2.4 Time-Dependent Explorations

After an initial phase of exploration with the elliptic program we asked ourselves whether the turbulent flow downstream of a backstep should really be treated as steady. There seemed to us two possible situations that would necessitate a time dependent approach: first, the obvious case where the flow was periodic and second the case where the final state was steady but where two or more solutions existed for the given boundary conditions and where, therefore, we had to march through a true time transient to discover which solution 'Nature' chose.

Our concern was stimulated by laser Doppler studies of laminar flow in a double-step expansion [16], which showed both asymmetries and time dependencies above certain limiting Reynolds numbers. There was also felt to be a curious behaviour in the heat transfer data downstream of an abrupt expansion reported by Zemanick and Dougall [17]. For a diameter ratio of 0.43:1 the maximum surface heat transfer rate (that occurs close to the position of reattachment) appeared to occur two diameters further downstream at a Reynolds number of 40000 than at 20000. Such a change suggested some definite alteration in the flow pattern; the calculated distribution of Nusselt number, however, was essentially the same at the two values of R_e.

After discussions with the sponsor it was agreed that some exploration of the uniqueness and/or periodic nature of the backstep flow be investigated for up to six months. The code was initially transformed to an implicit time-dependent solver by sim·· replacing $\frac{\partial \phi}{\partial t}$ by $\frac{\phi}{\delta t} - \frac{\phi_{old}}{\delta t}$ and inserting into the appro···ate source arrays; all other terms in the equation were evaluated at the 'new' time level. Following the experience of a group at G.E. Schenectady (Dr M Lubert, private communication) that this implicit treatment entirely suppressed the formation of a Karman vortex street behind a square sectioned cylinder, the code was adapted so that the terms other than the time-dependent one could be evaluated at any fraction α between the new time state ($\alpha = 1$; implicit) and the old ($\alpha = 0$; explicit). The adaptations required to the code are entirely straightforward but substantial and are documented in detail by Sindir [5], [6]. In practice the value $\alpha = 0.5$ was adopted for all the subsequent calculations.

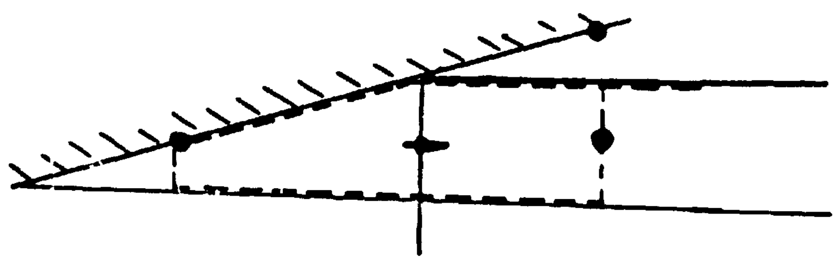
There followed several months of effort searching for periodic and or asymmetric flow behaviour in the double backstep flow. The approach followed was to start from rest and gradually raise to its asymptotic level the inlet velocity (several channel heights upstream of the backstep). The resultant flow certainly showed a degree of oscillation but the amplitude of these oscillations diminished with time. A variant was to apply a slightly asymmetric velocity at inlet thereby producing a correspondingly non-symmetric flow field in the interior. When this had reached a stationary state the asymmetry in the inlet profile was removed and the computation continued to see whether a symmetric flow was re-established. On one occasion a spectacularly asymmetric flow had been established which remained after the symmetric boundary conditions were restored. Because of the relatively coarse mesh used however the flow evidently suffered from false diffusion. Subsequent attempts with a finer mesh did not succeed in reproducing such a flow.

After the budgeted period had been spent it was felt that the questions of non-uniqueness and periodicity were ones that could easily absorb years of work and a very large computer budget. We could afford neither and so we decided to proceed on the assumption that a steady flow model was a reasonable one for examining the backward facing step. (Both time-dependent and steady flow versions of VAST STEP are listed in [6], however.)

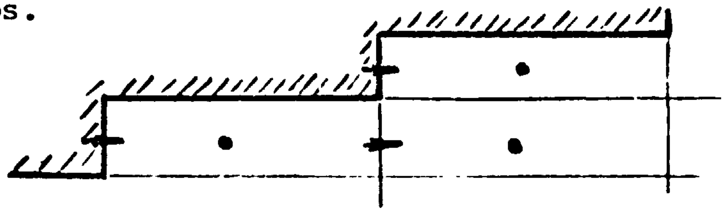
As a footnote to this part of our study we note that three groups contributed computations of the turbulent backstep to the 1981 Stanford Conference using time-dependent procedures. Ha Minh et al. [18] and Mansour and Morel [19] using the $k - \varepsilon$ Boussinesq turbulence model both obtained steady flow results. Mellor and Celenligil [20] using a Reynolds stress closure, however, found a periodic behaviour. Clearly the question is still open.

2.5 Adaptation to allow computation of non-rectilinear walls

We desired to extend the capabilities of the numerical procedure to allow the computation of separated flow in weakly converging or diverging passages. Three possibilities were considered for achieving this: a) an orthogonal curvi-linear coordinate system; b) a non-orthogonal mesh in which one set of coordinate lines was straight parallel lines at right angles to the main flow; c) a Cartesian grid where special balance equations were developed for the edge control volumes which, along at least one of the walls were of complex form as indicated in the sketch.



Having regard for the fact that an algebraically cumbersome turbulence model was to be used (the ASM) it quickly seemed that the last option was the best for that retained the relative simplicity of the Cartesian mesh over the interior flow. However we met no success in obtaining convergent behaviour even for roof angles as small as 4 degrees. Finally, following a suggestion by Patankar [9] the sloping roof was treated as simply a succession of steps.



This simplified approach certainly converged for both ASM and BVM treatments and it is this which has been built into VAST STEP. It appears to be satisfactory provided that the angles of expansion are small (no more than about 8 degrees) and provided that the flow region of interest is not along the "stepped" wall itself. These conditions were met in the Stanford Test Cases that were tried. The view that the approach was "satisfactory" is based on the fact that the computations [6], [21] show a similar disagreement with the data as were shown for the case of parallel walls.

The limitations of the method are evident however and there is clearly a need, in the near future, to embed an ASM into an arbitrary orthogonal coordinate system.

3. Turbulence Modelling Research

3.1 The Overall Plan:

In the first year of work attention was confined to thin shear flows. Moreover only the fully turbulent region was considered for in the boundary layers in question the near-wall viscosity-affected region was well described by the "universal" semi-logarithmic velocity profile. The work was initially aimed at placing on a firmer footing than hitherto the "multi-scale" modelling approach which had been developed by Launder & Schiestel [22, 23]. Section 3.2 outlines the principal features of this work. It was discovered, however, that most of the benefits that we felt accrued from the multi-scale treatment could in fact also be obtained with a single scale treatment provided appropriate modifications were introduced to the dissipation rate equation. In the second year attention shifted to turbulent separated flows. The handling of the near-wall region under conditions far removed from local equilibrium via wall functions has already been discussed in Section 2.2. Likewise, work on incorporating an algebraic stress closure into the recirculating flow procedure posed mainly numerical problems and was discussed in Section 2.3. The third year's work combined the exploration of new closure ideas and their evaluation in both recirculating and thin shear flows. Several faults of the turbulence model were identified and attempts made to remove them; these are discussed in Section 3.

3.2 The Multi-Scale Turbulence Closure - and its Single scale derivative

The multi-scale approach was evolved to cope with the fact that the local rate of energy dissipation, ε , is not as directly connected with the mean flow and energetic motions as any of the conventional rate equations for ε suggested. The energy containing part of the spectrum was thus divided into two portions as indicated in figure 3.1: a production region and a transfer region. This move naturally brought into focus the quantity ε_p, the transfer rate of energy from the former region to the latter. The multi-scale model of Launder and Schiestel [22, 23] provided independent transport equations for finding both ε_p and (quantities that would be equal if spectral equilibrium prevailed). While ε_p is associated with large-scale interactions and is directly responsive to the mean strain field, ε is held to feel these influences only indirectly through their effect on the energy transfer rate, ε_p. These ideas are broadly in accord with the accepted view of the dynamics of the turbulent energy spectrum.

Work undertaken by Dr Hanjalic during his period of support as a visiting research engineer was directed at improving the generality of this model. While the framework of the model remained intact, the detailed form underwent considerable change in order to provide, so far as possible, an internally consistent physical picture. Moreover the range of flows was extended from the three equilibrium shear flows considered by Launder and Schiestel to include boundary layers and distorted grid turbulence. The out-come of this study is reported in Hanjalic, Launder and Schiestel [24].

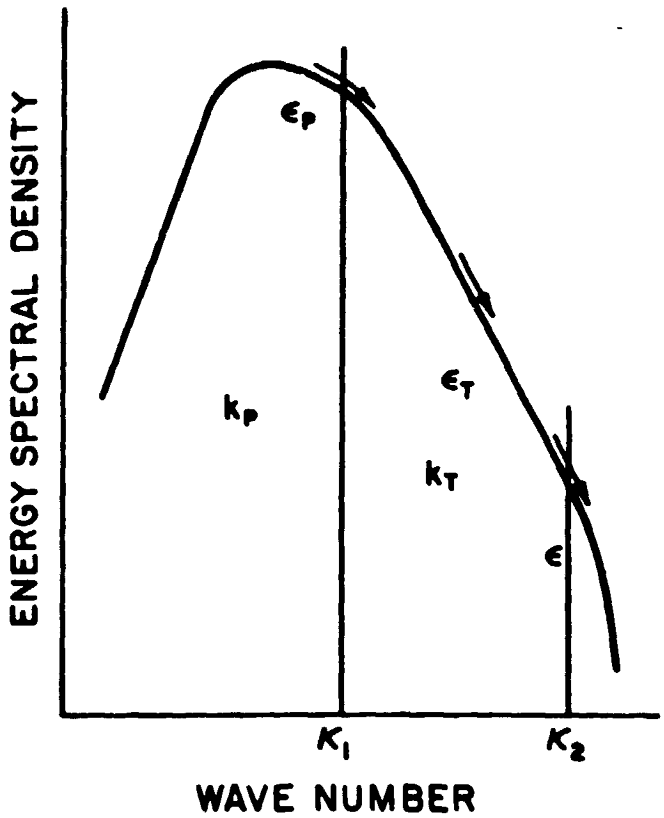


Fig. 3.1 Division of Turbulent Energy Spectrum in Multi-Scale Model

Towards the end of the development work on the multi-scale approach it was found (Hanjalic and Launder [25]) that practically as good results were obtainable with the single scale scheme if corresponding modifications to those devised for the multi-scale closure were introduced to the (single) equation for the energy transfer/dissipation rate. Only for the case of strongly accelerated grid turbulence was there a decisive advantage from using the multi-scale model and this example is far removed in kind from most practically interesting shear flows.

The failure to achieve significantly better results with the multi-scale model is believed to be due to the fact that, as presently constructed, the energy in the "transfer" range amounts to only about 10% of the total kinetic energy for a typically shear flow. There is thus only marginal advantage to be gained from recognizing the difference between the rate that energy is fed to this transfer range and the rate that it leaves it through viscous dissipation. The best way forward appears to be to reformulate the model placing a greater proportion of the total energy into the transfer range. This would necessitate that the assumption of isotropic turbulence in the transfer range be discarded, however; instead, an algebraic stress model for the transfer-range stresses would be appropriate.

3.3 Final Explorations in Turbulence Modelling

By late 1979 we had thus concluded that provided we introduced a new source term proportional to the square of the mean vorticity [24, 25] we could achieve essentially as good agreement in thin shear flows with a single-scale model as with the multi-scale scheme. The new term had a vital effect for it made the flow more sensitive to normal-strain production than shear strain. The modification allowed much improved prediction of the axisymmetric jet and the turbulent boundary layer in adverse pressure gradients. In each case, due to the flow deceleration, the levels of viscosity were reduced producing, for the former, lower rates of spread and, for the latter, a diminution in wall friction [25]. It was therefore decided to make this single-scale model the basis for the recirculating flow studies.

When, however, the small-modifications to the ε equation were introduced into the backstep code (which, at that time, contained only the BVM treatment) it was found, against all expectations, that the computed reattachment length was substantially shortened, indicating a pronounced <u>increase</u> in turbulent viscosity. This was a quite unacceptable result, for the standard $k-\varepsilon$ model already gave significantly too short reattachment lengths. The cause was traced to the fact that, due to the invariant form of the new source term it had the unfortunate property of strongly modifying ε levels in flows with streamline curvature. The sign of the effect was the opposite of what is found to occur in practice, e.g. when the fluid's angular momentum increased with radius (stable stratification) the model strongly enhanced viscosity levels.

We finally concluded that streamline curvature was such an abiding feature of turbulent shear flows that it would be intolerable to retain the proposed form of dissipation equation. Instead we decided to include a term of the same type but with the sign reversed. In fact for simplicity the form adopted with both the ASM and BVM was

$$\text{Source of } \varepsilon = c_{\varepsilon 1} c_\mu \frac{k^2}{\varepsilon} \left(\frac{\partial U_i}{\partial x_j}\right)^2 \qquad (3.1)$$

For the case of a simple shear flow the source takes the standard BVM form. Computations were reported using eq (3.1) for the various backward-step cases selected for the Stanford Conference [21]. The ASM computations used in conjunction with the above ε source gave the best agreement with experiment in the separated-flow region of any of the contributed results for this case.

A further fault of the original ε equation was diagnosed in the course of our work. In near-wall turbulence it is found that the lengthscale, $k^{3/2}/\varepsilon$, produced by that equation became too large in the vicinity of the wall as the flow approaches separation. It is believed to be this property which is mainly responsible for the failure to predict separation in various supersonic flows [26] and for the excessive levels of heat transfer predicted downstream of an abrupt pipe expansion [17] when the low-Reynolds-number version of the BVM is used [12]. Although this property of the ε equation had been known for some years [3] it was only with the detailed measurements of East and Sawyer [27] that it became fully apparent that this was an <u>un</u>desired characteristic. Their experiments of equilibrium turbulent boundary layers in adverse pressure gradients indicated that the near-wall values of $k^{3/2}/\varepsilon$ were, to a close approximation, only a function of distance from the wall and thus independent of the turbulence energy generation to dissipation rate. We therefore attempted to find a further source term for the ε equation that would counteract the tendency for the predicted length scale to become too large. Various source terms were explored involving typically spatial gradients of the kinetic energy. While we succeeded in devising fairly successful forms for boundary layer flows these uniformly led to disastrous predictions in free shear flows. An alternative approach we considered was to let the turbulent Prandtl number for ε diffusion become a function of the turbulence energy generation to dissipation rate. An analytical study for the case where P/ε was zero led to the idea that one might adopt the form

$$\sigma_\varepsilon = 1.3 \; \frac{c_{\varepsilon 2} - c_{\varepsilon 1}}{c_{\varepsilon 2} - P/\varepsilon c_{\varepsilon 1}} \tag{3.2}$$

This version did indeed lead to a small overall improvement in the backstep computations without apparently harming the prediction of free flows. However the introduction of P/ε made it so difficult to achieve converged numerical results that we finally did not include the modification.

3.4 Closing Remarks

Despite the considerable effort expended in turbulence model development over the course of the research grant it must be acknowledged that no fully satisfactory form of length-scale determining equation was developed. We found no invariant way of removing the tendency for the near-wall length to grow too large without giving the equation the wrong sensitivity to streamline curvature. At present the writer does not feel that a satisfactory solution cannot be devised but clearly more thought and careful testing is needed. The current practice of his group is merely to overwrite the near-wall length scale at the value found under local-equilibrium conditions (i.e. $P = \varepsilon$) should the dissipation equation return length scales larger than that. We would be interested to learn the experience of others with such a modification.

The multi-scale approach, while of definite advantage in certain rapidy distorted flows, does not, as currently constructed, have any sufficiently marked practical benefits to justify its use in shear flows (entailing, as it does, the solution of two extra transport equations). The potential usefulness of the approach is widely acknowledged but a formulation needs to be developed that gives a more even distribution of energy between the two regions. At present several groups are trying to understand the physical processes of energy transfer across the spectrum more quantitatively as a preliminary to further model development.

Acknowlegements

We take this final opportunity of thanking the authorities at NASA Ames for its support of our research on turbulent separated flows. The many interactions we have had with the staff of the Experimental Fluid Mechanics Branch have always been supportive of our efforts and we are grateful for the very tangible help and encouragement that this has provided.

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