NASA CR-165,992

NASA Contractor Report 165992

NASA-CR-165992
19830004253

# An Analysis of Thermal Response Factors & How to Reduce Their Computational Time Requirement

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Prepared for
Langley Research Center
under Contract NAS1-16078
August 1982



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Space Administration

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# SUMMARY

This paper presents an in-depth study of the Thermal Response Factor Method enhanced by G. P. Mitalas and J. G. Arseneault. A concise analysis of the theoretical mathematics and physical interpretation of response factors will be presented. It will be shown how a new root calculating procedure was developed to substantially reduce the computational time requirement of response factors by an average of 88.5%. Comparison testing between the original and the new procedures will be presented to validate the new root procedure.

CONTENTS

# INTRODUCTION

The Thermal Response Factor Method is recognized by the American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) as the current state-of-the-art technique to use for the computer simulation of hourly transient heat conduction analysis. This method calculates hourly heat transfer coefficients (response factors) by using the material properties of a particular planar construction. These response factors are then combined with an appropriate temperature history curve to yield a series solution for the heat flux at a given time. This Response Factor Method is the heart of many current energy analysis programs (such as NASA's ENERGY COST ANALYSIS PROGRAM - NECAP).

This paper is an in-depth study of the Response Factor Method enhanced by G. P. Mitalas and J. G. Arseneault, and of the RESPONSE FACTOR PROGRAM (RESFAC[1]). As this research progressed, it became quite apparent that the calculation of thermal response factors was being handled in a 'black box' manner. To rectify this, a concise analysis of the theoretical mathematics and physical interpretation will be presented.

The second part of this paper will present an investigation of the response factor's computational time requirement which has been criticized for taking too long. In particular, the root finding procedure has been singled out as the main culprit. To determine the validity of this criticism, the RESFAC code was analyzed and test cases consisting of various planar constructions were run. No basis for the computational time criticism could be found in the test cases' results. (See TABLE I)

However, the analysis of the FORTRAN code revealed several portions of inefficient coding. Moreover, it was determined that a faster, more efficient, and more accurate root calculating procedure could be developed. Upon cleaning up the code and installing a new root procedure, the modified RESFAC was benchmarked using the test cases mentioned previously. The validation testing shows a substantial reduction in RESFAC's conputational time requirement. (see TABLE I)

## NOMENCLATURE

c : specific heat (BTU/LB-°F)

$D_i$ : density of material layer i ($LB/FT^3$)

f : heat flux function ($BTU/HR-FT^2$)

$\overline{F}$ : Laplace transform of heat flux function

k : conductivity (BTU/HR-FT-°f)

NRT : number of response factor terms calculated

P : Laplace transform parameter

QI : inside surface heat flux ($BTU/HR-FT^2$)

QO : outside surface heat flux ($BTU/HR-FT^2$)

$R_i$ : thermal resistance of material layer i ($HR-FT^2$-°F/BTU)

RN : number of real, positive roots calculated

$R_t$ : total thermal resistance of all material layers ($HR-FT^2$-°F/BTU)

$SH_i$ : specific heat of material layer i (BTU/LB-°F)

t : time

TI : inside surface temperature (°F)

TO : outside surface temperature (°F)

u : Heaviside unit function

U : overall heat transfer coefficient ($BTU/HR-FT^2$-°F)

v : temperature function (°F)

$\overline{V}$ : Laplace transform of temperature function

W : real, positive roots of $B(W_k)=0.0$, k=1, 2, ... NRT

X, Y, Z : thermal response factor series ($BTU/HR-FT^2$-°F)

$XK_i$ : conductivity of material layer i (BTU/HR-FT-°F)

$XL_i$ : thickness of material layer i (FT)

$\alpha$ : diffusivity ($FT^2/HR$)

$\eta$ : number of material layers

$\phi$ : pulse function

$\bar{\phi}$ : Laplace transform of pulse function

$\delta$ : time increment

$\rho$ : density (LB/FT$^3$)

$BETA_i$ : reciprocal of the square root of Fourier number for material layer i

$$BETA_i = XL_i * \left( \frac{D_i * SH_i}{XK_i * \delta} \right)^{1/2}$$

A, B, C, D: overall conduction matrix elements where

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} * \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} * \ldots * \begin{pmatrix} a_\eta & b_\eta \\ c_\eta & d_\eta \end{pmatrix};$$

$\eta$ = number of material layers; $a_i$, $b_i$, $c_i$, and $d_i$ are the conduction matrix elements for each material layer $i = 1, 2, \ldots, \eta$

dA, dB, dC, dD: overall derivatives of the conduction matrix elements where

$$d\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} da_1 & db_1 \\ dc_1 & dd_1 \end{pmatrix} * \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} * \ldots * \begin{pmatrix} a_n & b_n \\ c_n & d_n \end{pmatrix} +$$

$$\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} * \begin{pmatrix} da_2 & db_2 \\ dc_2 & dd_2 \end{pmatrix} * \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix} * \ldots * \begin{pmatrix} a_n & b_n \\ c_n & d_n \end{pmatrix} + \ldots +$$

$$\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} * \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} * \ldots * \begin{pmatrix} a_{n-1} & b_{n-1} \\ c_{n-1} & d_{n-1} \end{pmatrix} * \begin{pmatrix} da_n & db_n \\ dc_n & dd_n \end{pmatrix}$$

# NOTES

## ANALYZING RESPONSE FACTORS

### Physical Interpretation

Thermal response factors can be defined as time dependent heat transfer coefficients which depict an object's action upon a particular outside surface/inside surface temperature difference over a period of time. This action is composed of three terms--the X, Y, and Z response factors.

Using the heat flux equation $Q = U * \Delta T$ along with the response factors, the time dependent heat flux values for the planar construction shown in Figure 1 can be obtained.

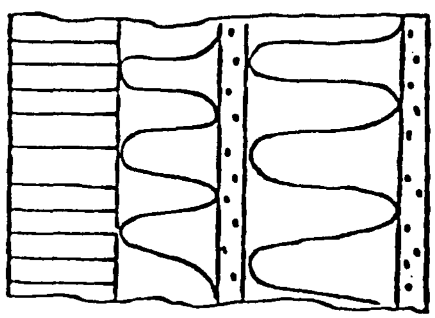


FIGURE 1

The outside surface heat flux (QO) at any time t is calculated by[9]:

$$QO_t = \sum_{i=0}^{NRT}(TO_{t-i} * X_i) - \sum_{i=0}^{NRT}(TI_{t-i} * Y_i)$$

where NRT is the number of response factor terms calculated, $TO_{t-i}$ and $TI_{t-i}$ are outside surface and inside surface temperatures at times t-i hour.

The inside surface heat flux (QI) at any time t is calculated by[9]:

$$QI_t = \sum_{i=0}^{NRT}(TO_{t-i} * Y_i) - \sum_{i=0}^{NRT}(TI_{t-i} * Z_i)$$

In order for the steady state heat conduction situation[9]

$$QO_t = QI_t = U(TO - TI)$$

$$TO = TO_t \quad \text{for any time } t$$

$$TI = TI_t \quad \text{for any time } t$$

to be satisfied, the individual summation of the X, Y, and Z response factors must equal the overall heat transfer coefficient U[9]:

$$U = \left|\sum_{i=1}^{\infty} X_i\right| = \left|\sum_{i=1}^{\infty} Y_i\right| = \left|\sum_{i=1}^{\infty} Z_i\right|$$

## Mathematical Derivation

For a homogeneous, isotropic solid whose thermal conductivity is independent of temperature, the one dimensional equation of conduction of heat with no heat generation is:

$$(1) \quad \frac{\partial^2 v}{\partial x^2} - \frac{1}{\alpha}\frac{\partial v}{\partial t} = 0$$

where $\alpha$, the diffusivity, $= \frac{k}{\rho c}$ and k is the conductivity, $\rho$ is density, and c is specific heat.

For a planar construction that experiences a steady, periodic temperature v(t), the boundary conditions are:

$v(t) = v_0\phi(t)$ for the surface at $x = 0$

$v(t) = v_L\phi(t)$ for the surface at $x = L$

where L is the thickness of the construction.

$\phi(t)$ is a pulse function of period $2\delta$ ($\delta$ = discrete hourly time interval) and has the values:

$$(2) \begin{cases} \phi = \frac{1}{\delta} & ,0 < t \leq \delta \\ \phi = \frac{1}{\delta} - \frac{2}{\delta}\,[u(t-\delta)] & ,\delta < t \leq 2\delta \\ \phi = \frac{1}{\delta} - \frac{2}{\delta}\,[u(t-\delta)] + \frac{1}{\delta}\,[u(t-2\delta)] & ,t > 2\delta \end{cases}$$

where u is the Heaviside unit function.[3]

The heat flux can then be expressed as:

$$(3) \quad f_x = -\frac{1}{R}\phi(t)\frac{\partial v}{\partial x}$$

where R, the resistivity, $= \frac{1}{k}$.

The Laplace transforms of equations (1), (2), and (3) are now needed. Using the Laplace transform equation[2]

$\bar{S} = \int_0^\infty e^{-Pt} S(t)\,dt$, where P is the Laplace transform parameter, the solutions are:

$$(4)\quad \frac{d^2\overline{V}}{dx^2} - \frac{P}{\alpha}\overline{V} = 0$$

$$(5)\quad \begin{cases} \overline{\phi} = \dfrac{1}{\delta P} & ,0 < t \leq \delta \\ \overline{\phi} = \dfrac{1}{\delta P} - \dfrac{2e^{-P\delta}}{\delta P} & ,\delta < t \leq 2\delta \\ \overline{\phi} = \dfrac{1}{\delta P} - \dfrac{2e^{-P\delta}}{\delta P} + \dfrac{e^{-2P\delta}}{\delta P} & ,t > 2\delta \end{cases}$$

$$(6)\quad \overline{F} = -\frac{1}{R}\,\overline{\phi}\,\frac{d\overline{V}}{dx}$$

The solutions (only the real parts) to equations (4) and (6) are:

$$(7)\quad \begin{cases} \overline{V}_x\overline{\phi} = \overline{V}_0\overline{\phi}\ \mathrm{COS}qx + \dfrac{\overline{F}_0 R\ \mathrm{SIN}qx}{q} \\ \overline{F}_x = -\dfrac{\overline{V}_0\overline{\phi}q\ \mathrm{SIN}qx}{R} + \overline{F}_0\ \mathrm{COS}qx \end{cases}$$

where $q = \sqrt{\frac{P}{\alpha}}$. This solution can then be expressed in matrix form:

$$(8)\quad \begin{pmatrix} \overline{V}_x\overline{\phi} \\ \overline{F}_x \end{pmatrix} = \begin{pmatrix} \mathrm{COS}qx & \frac{R}{q}\ \mathrm{SIN}qx \\ -\frac{q}{R}\ \mathrm{SIN}qx & \mathrm{COS}qx \end{pmatrix} \begin{pmatrix} \overline{V}_0\overline{\phi} \\ \overline{F}_0 \end{pmatrix}.$$

In terms of heat flux, the matrix can be manipulated into:

$$(9)\quad \begin{pmatrix} \overline{F}_0 \\ \overline{F}_x \end{pmatrix} = \begin{pmatrix} \dfrac{-q\ \mathrm{COS}qx}{R\ \mathrm{SIN}qx} & \dfrac{q}{R\ \mathrm{SIN}qx} \\ \dfrac{-q}{R\ \mathrm{SIN}qx} & \dfrac{q\ \mathrm{COS}qx}{R\ \mathrm{SIN}qx} \end{pmatrix} \begin{pmatrix} \overline{\phi V}_0 \\ \overline{\phi V}_x \end{pmatrix}$$

When the planar construction is composed of varying material layers, the matrix (8) would be represented as:

$$(10)\quad \begin{pmatrix} \bar{V}_L\bar{\Phi} \\ \bar{F}_L \end{pmatrix} = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} \cdots \begin{pmatrix} a_n & b_n \\ c_n & d_n \end{pmatrix} \begin{pmatrix} \bar{V}_0\bar{\Phi} \\ \bar{F}_0 \end{pmatrix}$$

$$= \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \bar{V}_0\bar{\Phi} \\ \bar{F}_0 \end{pmatrix}$$

where $\begin{pmatrix} a_i & b_i \\ c_i & d_i \end{pmatrix}$ for i = 1, 2, ... n (number of layers)

is each layers 2*2 matrix and L is the thickness of the construction. Note that this solution assumes that there is no contact resistance between the layers. Matrix (9) can be represented by:

$$(11)\quad \begin{pmatrix} \bar{F}_0 \\ \bar{F}_L \end{pmatrix} = \begin{pmatrix} \frac{-A}{B} & \frac{1}{B} \\ \frac{-1}{B} & \frac{D}{B} \end{pmatrix} \begin{pmatrix} \overline{\phi V}_0 \\ \overline{\phi V}_L \end{pmatrix}$$

The solution to matrix (11) is obtained by applying the Inversion Theorem for Laplace transforms:

$$F = \frac{1}{2\pi i} \lim_{\gamma\to\infty} \int_{\varepsilon-i\gamma}^{\varepsilon+i\gamma} e^{\lambda t}\, \bar{F} d\lambda.$$

In order to simplify this calculation, express the transformed flux $\bar{F}$ as a generalized equation:

$$(12)\quad \bar{F} = \frac{\bar{\phi} M(P)}{B(P)}$$

where $\bar{\phi}$ is the Laplace transform of the pulse function $\phi$, $B(P) = \frac{R_t \ \mathrm{SIN} qx}{q}$, and M(P) is either 1, A, or D. Since $R_t$ is the total resistivity of the planar construction and $U = \frac{1}{R_t}$, then $B(P) = \frac{\mathrm{SIN}qx}{Uq}$.

To find the inverse of equation (12), the Calculus of Residues[2] will be utilized:

1) determine which components in (12) will make the equation undefined --

   a. Since $\overline{\phi}$ is composed of element $\frac{1}{P}$ (refer equation (5)), if P = 0 then (12) becomes undefined

   b. Since (12) is composed of element $\frac{1}{B(P)}$, if B(P) = 0 then (12) becomes undefined

2) dealing with the $\overline{\phi}$ part first --

   a. cancel out the element $\frac{1}{P}$ in $\overline{\phi}$ and obtain '$\overline{\phi}$':

$$
(13)\begin{cases}
'\overline{\phi}' = \frac{1}{\delta} & ,0 < t \leq \delta \\
'\overline{\phi}' = \frac{1}{\delta} - \frac{2e^{-P\delta}}{\delta} & ,\delta < t \leq 2\delta \\
'\overline{\phi}' = \frac{1}{\delta} - \frac{2e^{-P\delta}}{\delta} + \frac{e^{-2P\delta}}{\delta} & ,t > 2\delta
\end{cases}
$$

   b. By the Calculus of Residues, part I of the answer is therefore:

$$
(14)\quad \lim_{P\to 0} \frac{d}{dP}\left(\frac{e^{Pt}M(P)\,'\overline{\phi}'}{B(P)}\right) =
$$

$$
\left[UM(P) + \frac{U}{\delta}\frac{dM}{dP} - \frac{U^2M(P)}{\delta}\frac{dB}{dP}\right]_{P=0} \qquad ,0 < t \leq \delta
$$

$$
\left[\frac{U^2M(P)}{\delta}\frac{dB}{dP} - \frac{U}{\delta}\frac{dM}{dP}\right]_{P=0} \qquad ,\delta < t \leq 2\delta
$$

$0,\ t > 2\delta$

3) for the $B(P) = 0$ part --

a. By using the Heaviside Expansion Theorem,[3] part II of the answer is:

$$(15) \quad \sum_{r=1}^{RN}\left[\frac{e^{-W_r t}\ \bar{\phi}M(W_r)}{W_r}\ \frac{1}{\frac{dB}{dW}}\right] =$$

$$\sum_{r=1}^{RN}\left[\frac{e^{-W_r\delta}M(W_r)}{\delta W_r^2}\ \frac{1}{\frac{dB}{dW}}\right] \qquad , t = \delta$$

$$\sum_{r=1}^{RN}\left[\frac{M(W_r)}{\delta W_r^2}\ \left(e^{-2W_r\delta} - 2e^{-W_r\delta}\right)\frac{1}{\frac{dB}{dW}}\right] \qquad , t = 2\delta$$

$$\sum_{r=1}^{RN}\left[\frac{M(W_r)}{\delta W_r^2}\ \left(e^{-W_r\delta j} - 2e^{-W_r\delta(j-1)} + e^{-W_r\delta(j-2)}\right)\frac{1}{\frac{dB}{dW}}\right] ,$$

$t > 2\delta$, $j = 3, 4, \ldots, n$ = number of hours

In the above equations, RN is the number of roots of $B(P) = 0$ and $W_r$ is the negative of each root ($W_r = -P$).

When equations (14) and (15) are combined, when $\delta$ is set to 1 hour, and when M is replaced by D, 1, and A, and evaluated, the equations for the X, Y, and Z (respectfully) response factors are obtained:

The equations for the X response factors are:

$$X_0 = U + \left[U * \left(\frac{dD}{dW}\right)_{W\to 0.0}\right] - \left[U^2 * \left(\frac{dB}{dW}\right)_{W\to 0.0}\right] + \sum_{k=1}^{RN}\left[\left(\frac{1}{W_k^2}\right) * D_{W_k} * \left(\frac{1}{\frac{dB}{dW}}\right)_{W_k} * e^{-(\delta W_k)}\right]$$

$$X_1 = \left[U * \left(\frac{dD}{dW}\right)_{W\to 0.0}\right] + \left[U^2 * \left(\frac{dB}{dW}\right)_{W\to 0.0}\right] + \sum_{k=1}^{RN}\left[\left(\frac{1}{W_k^2}\right) * D_{W_k} * \left(\frac{1}{\frac{dB}{dW}}\right)_{W_k} * \left(1-2e^{(\delta W_k)}\right) * e^{-(2\delta W_k)}\right]$$

$$X_i = \sum_{k=1}^{RN}\left[\left(\frac{1}{W_k^2}\right) * D_{W_k} * \left(\frac{1}{\frac{dB}{dW}}\right)_{W_k} * \left(1-e^{(\delta W_k)}\right)^2 * e^{-[(i+1)\delta W_k]}\right]$$

i=2, 3, 4...NRT

The equations for the Y response factors are:

$$Y_0 = U - \left[U^2 * \frac{dB}{dW}_{W\to 0.0}\right] + \sum_{k=1}^{RN}\left[\left(\frac{1}{W_k^2}\right) * \left(\frac{1}{\frac{dB}{dW}}\right)_{W_k}\right]$$

$$Y_1 = \left[U^2 * \left(\frac{dB}{dW}\right)_{W\to 0.0}\right] + \sum_{k=1}^{RN}\left[\left(\frac{1}{W_k^2}\right) * \left(\frac{1}{\frac{dB}{dW}}\right)_{W_k} * \left(1-e^{-(\delta W_k)}\right) * e^{-(2\delta W_k)}\right]$$

$$Y_i = \sum_{k=1}^{RN}\left[\left(\frac{1}{W_k^2}\right) * \left(\frac{1}{\frac{dB}{dW}}\right)_{W_k} * \left(1-e^{(\delta W_k)}\right)^2 * e^{-[(i+1)\delta W_k]}\right]$$

i=2, 3, 4...NRT

And the equations for the Z response factors are:

$$Z_0 = U + \left[U * \left(\frac{dA}{dW}\right)_{W\to 0.0}\right] - \left[U^2 * \left(\frac{dB}{dW}\right)_{W\to 0.0}\right] + \sum_{k=1}^{RN}\left[\left(\frac{1}{W_k^2}\right) * A_{W_k} * \left(\frac{1}{\frac{dB}{dW}}\right)_{W_k} * e^{-(\delta W_k)}\right]$$

$$Z_1 = \left[U * \frac{dA}{dW}_{W\to 0.0}\right] + \left[U^2 * \frac{dB}{dW}_{W\to 0.0}\right] + \sum_{k=1}^{RN}\left[\left(\frac{1}{W_k^2}\right) * A_{W_k} * \left(\frac{1}{\frac{dB}{dW}}\right) * \left(1 - 2e^{(\delta W_k)}\right) * e^{-(\delta W_k)}\right]$$

$$Z_i = \sum_{k=1}^{RN}\left[\left(\frac{1}{W_k^2}\right) * A_{W_k} * \left(\frac{1}{\frac{dB}{dW}}\right)_{W_k} * \left(1-e^{(\delta W_k)}\right)^2 * e^{-[(i+1)]\delta W_k}\right]$$

i=2, 3, 4...NRT

Conduction Matrix Element B

To begin with, define a purely arbitrary wall composed of eight inch thick concrete block (outside surface), air space, gypsum board, and inside air. Using the material characteristics described in APPENDIX A, the equation for the overall conduction matrix element B will be obtained.

The equation for B is determined by the following matrix algebra:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} * \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} * \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix} * \begin{pmatrix} a_4 & b_4 \\ c_4 & d_4 \end{pmatrix}$$

$$\Rightarrow \quad B = (a_1*a_2 + b_1*c_2) * (a_3*b_4 + b_3*d_4) + (a_1*b_2 + b_1*d_2) * (c_3*b_4 + a_3*d_4)$$

As generalized equations, each layer's matrix values are determined by:

NON-AIR LAYER

for W>0.0:

$$a_i = \mathrm{COS}(\sqrt{W} * \mathrm{BETA}_i)$$
$$b_i = R_i*\sin(\sqrt{W} * \mathrm{BETA}_i)/(\sqrt{W} * \mathrm{BETA}_i)$$
$$c_i = -(\sqrt{W} * \mathrm{BETA}_i)*\sin(\sqrt{W} * \mathrm{BETA}_i)/R_i$$
$$d_i = a_i$$

$$da_i = \frac{BETA_i * \sin(\sqrt{W} * BETA_i)}{2.0*\sqrt{W}}$$

$$db_i = \left(\frac{R_i}{2.0*W}\right) * \left(\frac{\sin(\sqrt{W} * BETA_i)}{\sqrt{W} * BETA_i} - \cos(\sqrt{W} * BETA_i)\right)$$

$$dc_i = \left(\frac{BETA_i}{2*R_i}\right) * \left(\frac{\sin(\sqrt{W} * BETA_i)}{\sqrt{W}} + BETA_i*\cos(\sqrt{W} * BETA_i)\right)$$

$$dd_i = da_i$$

as $W \to 0.0$

$A = 1.0$

$B = R_t$

$C = 0.0$

$D = 1.0$

$$da_i = BETA_i*BETA_i/2.0$$

$$db_i = R_i*BETA_i*BETA_i/6.0$$

$$dc_i = BETA_i*BETA_i/R_i$$

$$dd_i = da_i$$

AIR LAYER

$a_i = 1.0$

$b_i = R_i$

$c_i = 0.0$

$d_i = a_i$

Upon substituting the appropriate expressions for the a's, b's, c's, and d's (keeping in mind that the second and fourth layers are air), the B equation then becomes:

$$
\begin{aligned}
B = \Big[ & COS(\sqrt{W}*BETA_1) * \big[R_4*COS(\sqrt{W}*BETA_3) + \\
& R_3*(\sqrt{W}*BETA_3)^{-1}*SIN(\sqrt{W}*BETA_3)\big]\Big] + \\
& \Big[\big[R_2*COS(\sqrt{W}*BETA_1) + \\
& R_1*(\sqrt{W}*BETA_1)^{-1}*SIN(\sqrt{W}*BETA_1)\big] * \\
& \big[-R_4*(R_3)^{-1}*\sqrt{W}*BETA_3*SIN(\sqrt{W}*BETA_3) + \\
& COS(\sqrt{W}*BETA_3)\big]\Big]
\end{aligned}
$$

The only unknown value in the B equation is the variable W. When 0.0 < W < ∞, B graphs out as:

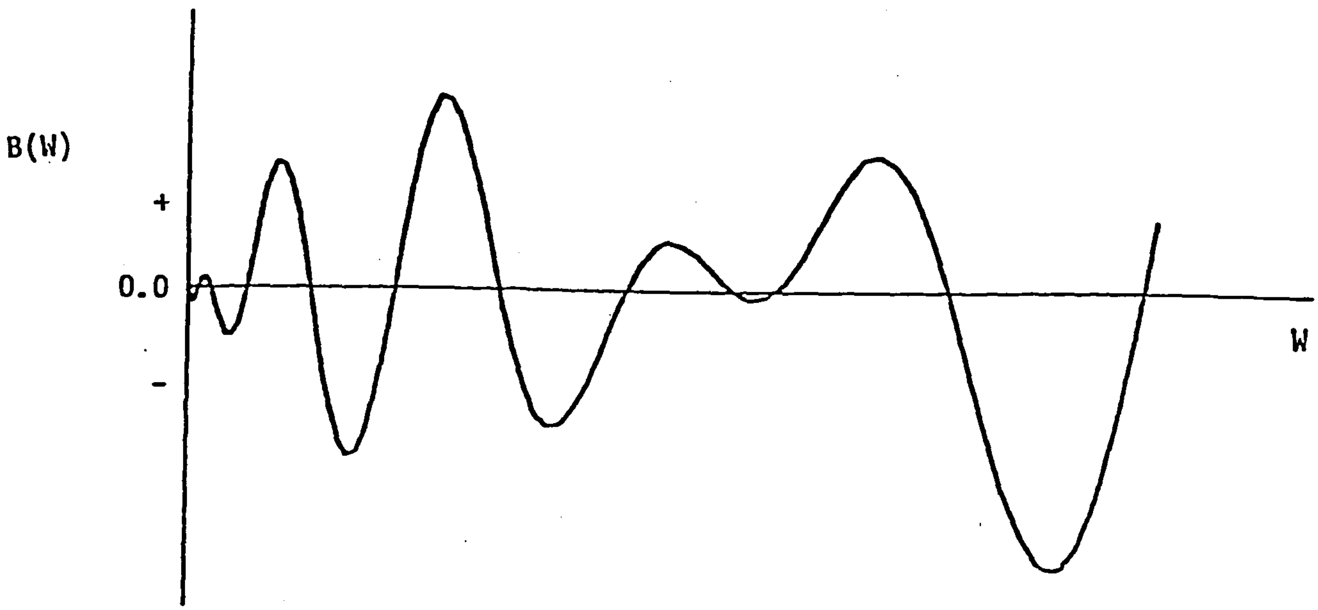


FIGURE 2

In order to calculate the thermal response factors (X, Y, and Z) for this planar construction, the values of W which makes B(W)=0.0 must be determined. As can be seen in Figure 2, this complicated wave has no predictable period. Therefore, a mathematical root finding technique must be used to solve B(W)=0.0.

## CALCULATING ROOTS

### Original Root Procedure

The calculation sequence for this method is:

1. Calcualte first root >30.0 using sequence 3; and set ROOT(1) = to this root; set LAST = 1

2. Determine root intervals for locating all roots <30.0 but >0.0

   a) if LAST = 1 then set W1 = 0.0001
   else W1 = ROOT(LAST - 1)

   b) W2 = ROOT(LAST)

   c) W2 = W2 - 0.0001
   W1 = W1 + 0.0001

   d) DELTA = (W2 - W1)/20.0
   J = 1
   IC = 0

   e) W3 = W1 + (J*DELTA)

   f) calculate B(W1) and the derivative of B, DB(W1)

   g) calculate B(W3) and the derivative of B, DB(W3)

   h) if B(W1) * B(W3) <0.0 then have a root interval so set W2 = W3 and go to 3

   i) if DB(W1) * DB(W3) <0.0, set IC = IC + 1

   j) if IC - 2>=0.0 then have a root interval so set W2 = W3 and go to 3

   k) J = J + 1

   l) if J >19 then no root interval so set LAST = LAST - 1 and go to 4

   m) B(W1) = B(W3)
   DB(W1) = DB(W3)
   W3 = W1 + (J*DELTA)
   go to 2g

3. Determine the root that is in the interval (W1, W2)

a) N = 1
   J = 1
   BP3 = W1

b) calculate B(W1)
   B2 = B(W1)

c) DELTA = (W2 - W1)/(20.0*N)

d) W3 = W1 + (J*DELTA)

e) if W3 - W2 >0.0 go to 3i

f) N = N + 1

g) if N >25 then there is no root <30.0 so go to 4

h) set J = 1 and go to 3c

i) calculate B(W3)
   B3 = B(W3)

j) if B(W1) * B(W3) <0.0 go to 3l

k) J = J + 1
   B2 = B3
   BP3 = W3
   go to 3d

l) A = BP3
   B = W3
   FA = B2
   FB = B3

m) perform Bisection Method on interval (A, B) until root is located (i.e. ABS(B(A) - B(B)) $\leq$1.0E-8), go to 4

```
4.  LAST = LAST + 1
    ROOT(LAST) = ROOT(LAST - 1)
    ROOT(LAST - 1) = W3

5.  Go to 2

6.  End
```

Although the Bisection Method guarantees a root once the root interval is found, it requires a significant amount of execution time to actually locate the root. Compounded with the time needed to locate each root interval, this method is quite inefficient.

## New Root Procedure

The basis for reducing execution time and cost lies in two categories: originating a faster method to locate the root intervals and using a more efficient algorithm for determining the roots.

### Root Intervals:

As was shown , the temperature flux matrix element B is mainly composed of sines and cosines. Taken individually, each non-air layer's sine and cosine equations have calculatable roots. Therefore, this result will be used to segment the B wave (Figure 2) into possible root intervals.

For each non-air layer, use its BETA value to determine the segmenter roots (i.e. $\sqrt{W}$).

$$RVC = (i-0.5)*\pi/BETA \qquad \text{(for cosine)}$$

$$RVS = i*\pi/BETA \qquad \text{(for sine)}$$

$$\text{for } i = 1,2,3,4,...$$

Upon doing this for each non-air layer and then sorting all of the RVC and RVS values in ascending order, an array V is obtained that can be used for root interval checking.

$$V \equiv (V_1, V_2, V_3, \ldots.) \qquad , V_1 < V_2 < V_3 \ldots.$$

where $\{V_1, V_2\}$ is root interval 1

$\{V_2, V_3\}$ is root interval 2

etc.

## Determining the Roots of B :

Having determined the root interval checking array, the actual roots can then be found.

The calculation sequence can be summerized as:

a) set $i=1$

b) set $G1=V_i$

c) set $G2=V_{i+1}$

d) calculate B(G1)

e) calculate B(G2)

f) if ABS(B(G1)) or ABS(B(G2)) $\leq$ 1.0E-8, a root has been found so go to h.

g) else check the possible root interval at one-tenth segments:

1) set D = G2-G1, FD = 0.0

2) FK = FK+1.0, if FK >10.0 then the entire interval has been checked so increment i and go to b

3) else calculate segment end point:
GNEW = $PINT_i$ + (D*FK/10.0)

4) calculate FNEW = B(GNEW)

5) check sign of FNEW against sign of B(G1):
if FNEW*B(G1) >0.0 then no root in this segment, so reset segment front point, G1 = GNEW and B(G1) = FNEW then gc to g2

6) else there is a root in this segment; store segment front point values: GX = G1 and FX = B(G1), go to h

h) attempt to locate the root by first using the Secant method; if the root 'guess' ever exceeds the end points of the segment being checked, then this method failes so use the more time consuming Bisection method on the segment end points (i.e. GX and GNEW)to locate the root; go to i

i) the root value obtained was actually the square root of the root value so compute the actual root (i.e. ROOT = root value *root value); if ROOT >30.0 go to k

j) reset segment front point: G1 = GNEW and B(G1) = FNEW; go to g2 to check next segment in this interval

k) end

## VALIDATION OF NEW ROOT PROCEDURE

Ten test cases were run to compare the original RESFAC against the new program. Input requirements and output for both programs are in APPENDIX A. Planar construction ranged from a simple, two material layer wall to a complex, ten material layer roof. A wide sampling was used for comparison testing and execution timing. The data input and both program's output for the test cases are given in APPENDIX B.

All testing was done on a high speed, Control Data Corp. CYBER 173 computer. Both programs use single precision FORTRAN code and were run on a time sharing, batch mode. TABLE I lists the execution times and costs for the test cases.

TABLE I

EXECUTION TIME & COST COMPARISON*

| TEST CASE | EXECUTION** TIME | | % REDUCTION | EXECUTION*** COST | | % REDUCTION |
|---|---|---|---|---|---|---|
| | OLD | NEW | | OLD | NEW | |
| 1 | 0.965 | 0.228 | 76.4 | 0.57 | 0.45 | 21.0 |
| 2 | 5.642 | 0.131 | 97.7 | 0.61 | 0.45 | 26.2 |
| 3 | 1.712 | 0.292 | 82.9 | 0.58 | 0.45 | 22.4 |
| 4 | 3.566 | 0.415 | 88.4 | 0.60 | 0.45 | 25.0 |
| 5 | 2.181 | 0.344 | 84.2 | 0.58 | 0.45 | 22.4 |
| 6 | 17.584 | 1.341 | 92.4 | 0.74 | 0.46 | 37.8 |
| 7 | 6.767 | 0.696 | 89.7 | 0.63 | 0.45 | 28.6 |
| 8 | 5.136 | 0.533 | 89.6 | 0.61 | 0.45 | 26.2 |
| 9 | 5.618 | 0.601 | 89.3 | 0.62 | 0.45 | 27.4 |
| 10 | 17.314 | 1.034 | 94.0 | 0.73 | 0.46 | 40.0 |

Average reduction in execution time was 88.5%

Average reduction in execution cost was 27.7%

* All runs were made on a Control Data Corp. CYBER-173

** Execution time is in Central Processor (CP) seconds

*** Execution cost is in System Resource Units (SRU)

## CONCLUDING REMARKS

From the analysis performed, it was found that thermal response factors are time dependent heat transfer coefficients whose derivations require complex mathematics. In deriving the response factor equations, it was necessary to calculate the Laplace transforms and the inverse Laplace transforms for the heat conduction equation and the heat flux equation - both of which are acted upon by a pulse function comprised of the Heaviside unit function. Furthermore, the keys to obtaining the inverses were found to be the Calculus of Residues Theorem and the Heaviside Expansion Theorem.

Testing of the original Response Factor Program revealed that its computational time requirement was not exorbitant. Thus, no basis was found for the criticism expounded against it. However, an analysis of the program code did reveal inefficiencies in the root calculating procedure for conduction matrix element B. This warranted the development of a new root calculating procedure.

From an analysis of the response factor derivation, it was decided that each material layers SIN and COSINE components could be utilized in an effective manner. Since these components are periodic functions, their zero roots are easily calculated. Once these component roots are obtained for each layer and then ascendingly sorted, a root interval checking array can be obtained. Segments from this array (values at array positions i and i+1) can then be sectionalized into ten parts and each of these mini-segments can be checked to determine if it contains a root for matrix element B = 0. This approach to a new root calculating procedure proved to be very appropriate.

In comparison testing, the new Response Factor Program executed more efficiently and more accurately than the original RESFAC. The new program's computational time requirement was much less that that of the original program - a substantial 88.5% average reduction.

NOTES

APPENDIXES

# APPENDIX A

## PROGRAM INPUT/OUTPUT

Input to the Response Factor Program (FTN 4.8 extended) consists of:

a) number of material layers

b) for each material layer (arranged-from outside to inside), its:

   i) thickness (FT); if air layer then input 0.0

   ii) thermal conductivity (BTU/HR.FT.F), if air layer then input 0.0

   iii) density ($LB/FT^3$), if air layer then input 0.0

   iv) specific heat (BTU/LB.F); if air layer then input 0.0

   v) thermal resistance ($HR.FT^2.F/BTU$); if NON air layer then input 0.0

   vi) alpha-numeric description (30 characters)

Output from the Response Factor Program consists of:

a) echo of the input

b) thermal conductance of planar construction

c) response factors series X, Y, and Z

d) number of hours to reach common ration

e) number of response factors per set

f) common ratio

The common ratio is defined as the common rate at which the response factors tend to decrease:

$$\frac{X_{i+1}}{X_i} = \frac{Y_{i+1}}{Y_i} = \frac{Z_{i+1}}{Z_i}$$

# APPENDIX B

# TEST CASES

TABLE 1 (Cont.)

ROOTS

| ROOT # | OLD | NEW |
|---|---|---|
| 1 | 0.32755449581283 | 0.32755450010252 |
| 2 | 1.93392856508856 | 1.93392855643586 |
| 3 | 5.10625948098138 | 5.10625947586985 |
| 4 | 9.86100164604460 | 9.86100166757376 |
| 5 | 16.19980403179670 | 16.19980403077625 |
| 6 | 24.12301443118963 | 24.12301442902663 |
| 7 | 33.63074066210538 | 33.63074065640694 |

B(ROOT)

| FOR ROOTS # | OLD | NEW |
|---|---|---|
| 1 | 1.191627E-8 | -8.96300E-11 |
| 2 | 7.966970E-9 | 8.20000E-13 |
| 3 | -2.796240E-9 | 4.00000E-14 |
| 4 | -1.886340E-9 | 6.49869E-9 |
| 5 | -3.086200E-9 | 5.00000E-14 |
| 6 | 5.348400E-10 | -7.00000E-14 |
| 7 | 1.191860E-9 | 1.00000E-15 |

TABLE 2

DESCRIPTION OF CONSTRUCTION

| LAYER NUMBER | THICKNESS FT | CONDUCTIVITY BTU PER (HR)(FT)(F) | DENSITY LB PER CU FT | SPECIFIC HEAT BTU PER (LB)(F) | RESISTANCE (HR)(SQ FT)(F) PER BTU | |
|---|---|---|---|---|---|---|
| 1 | .0050 | 26.000 | 480.0 | .1000 | 0.0000 | SHEET METAL |
| 2 | 0.0000 | 0.000 | 0.0 | 0.0000 | .6853 | INSIDE AIR |

OLD RESFAC

RESPONSE FACTORS

| HOUR | X | Y | Z |
|---|---|---|---|
| 0 | 1.6987382517 | 1.4587719164 | 1.4588055812 |
| 1 | -.2399326768 | .0000336584 | -.0000000063 |
| 2 | 0.0000000000 | 0.0000000000 | -.0000000000 |

NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 2
NUMBER OF RESPONCE FACTORS PER SET = 3
COMMON RATIO = .0000000000

NEW RESFAC

RESPONSE FACTORS

| HOUR | X | Y | Z |
|---|---|---|---|
| 0 | 1.6987382517 | 1.4587719164 | 1.4588055812 |
| 1 | -.2399326758 | .0000336584 | -.0000000063 |
| 2 | 0.0000000000 | 0.0000000000 | -.0000000000 |

NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 2
NUMBER OF RESPONCE FACTORS PER SET = 3
COMMON RATIO = 0.0000000000

ROOTS

| OLD | NEW |
|---|---|
| "NO ROOT IN FALSE" | "NO ROOT <30.0" |

TABLE 3

DESCRIPTION OF CONSTRUCTION

| LAYER NUMBER | THICKNESS FT | CONDUCTIVITY BTU PER (HR)(FT)(F) | DENSITY LB PER CU FT | SPECIFIC HEAT BTU PER (LB)(F) | RESISTANCE (HR)(SQ FT)(F) PER BTU | |
|---|---|---|---|---|---|---|
| 1 | .0833 | .400 | 116.0 | .2000 | 0.0000 | STUCCO |
| 2 | .6670 | .600 | 61.0 | .2000 | 0.0000 | CONCRETE BLOCK |
| 3 | 0.0000 | 0.000 | 0.0 | 0.0000 | .6850 | INSIDE AIR |

OLD RESFAC

RESPONSE FACTORS

| HOUR | X | Y | Z |
|---|---|---|---|
| 0 | 3.3351989142 | .0047349047 | 1.0268924803 |
| 1 | -2.0357518186 | .0774691384 | -.2325472853 |
| 2 | -.3181873898 | .1220914743 | -.1039496668 |
| 3 | -.1673243788 | .0956263776 | -.0644821910 |
| 4 | -.1057356979 | .0658090798 | -.0423797458 |
| 5 | -.0697646918 | .0441981869 | -.0281976595 |
| 6 | -.0464561062 | .0295390350 | -.0188093241 |
| 7 | -.0309938423 | .0197221399 | -.0125533739 |
| 8 | -.0206860498 | .0131650601 | -.0083790413 |
| 9 | -.0138074819 | .0087876637 | -.0055929090 |
| 10 | -.0092163408 | .0058657058 | -.0037332161 |
| 11 | -.0061518258 | .0039153118 | -.0024918902 |

NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 11
NUMBER OF RESPONCE FACTORS PER SET = 12
COMMON RATIO = .6674917204

NEW RESFAC

RESPONSE FACTORS

| HOUR | X | Y | Z |
|---|---|---|---|
| 0 | 3.3351989055 | .0047349117 | 1.0268924801 |
| 1 | -2.0357518095 | .0774691303 | -.2325472859 |
| 2 | -.3181873891 | .1220914743 | -.1039496663 |
| 3 | -.1673243787 | .0956263777 | -.0644821907 |
| 4 | -.1057356980 | .0658090799 | -.0423797457 |
| 5 | -.0697646920 | .0441981870 | -.0281976594 |
| 6 | -.0464561064 | .0295390352 | -.0188093241 |
| 7 | -.0309938425 | .0197221400 | -.0125533739 |
| 8 | -.0206860499 | .0131650602 | -.0083790413 |
| 9 | -.0138074820 | .0087876638 | -.0055929090 |
| 10 | -.0092163409 | .0058657058 | -.0037332161 |
| 11 | -.0061518259 | .0039153119 | -.0024918903 |

NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 11
NUMBER OF RESPONCE FACTORS PER SET = 12
COMMON RATIO = .6674917214

TABLE 3 (Cont.)

ROOTS

| ROOT # | OLD | NEW |
|---|---|---|
| 1 | 0.40422829270580 | 0.40422829118816 |
| 2 | 1.98888907867067 | 1.98888908214629 |
| 3 | 4.95990390912746 | 4.95990391749302 |
| 4 | 9.33689576656178 | 9.33689576507368 |
| 5 | 15.23955713347846 | 15.23955713476266 |
| 6 | 22.76863355467356 | 22.76863355381067 |
| 7 | 31.86645567696542 | 31.86645567723178 |

B(ROOTS)

| FOR ROOTS # | OLD | NEW |
|---|---|---|
| 1 | -4.60508E-9 | -2.500E-13 |
| 2 | -3.28262E-9 | -7.000E-14 |
| 3 | 4.50591E-9 | -7.000E-14 |
| 4 | 5.74390E-10 | -3.000E-14 |
| 5 | 3.98570E-10 | -1.100E-13 |
| 6 | 2.42400E-10 | 1.857E-11 |
| 7 | 5.67000E-11 | -7.600E-13 |

TABLE 4

DESCRIPTION OF CONSTRUCTION

| LAYER NUMBER | THICKNESS FT | CONDUCTIVITY BTU PER (HR)(FT)(F) | DENSITY LB PER CU FT | SPECIFIC HEAT BTU PER (LB)(F) | RESISTANCE (HR)(SQ FT)(F) PER BTU | |
|---|---|---|---|---|---|---|
| 1 | 1.0000 | .470 | 61.0 | .2000 | 0.0000 | CONCRETE |
| 2 | 0.0000 | 0.000 | 0.0 | 0.0000 | .9100 | AIR SPACE |
| 3 | .0625 | .420 | 100.0 | .2000 | 0.0000 | GYPSUM BOARD |
| 4 | 0.0000 | 0.000 | 0.0 | 0.0000 | .6850 | INSIDE AIR |

OLD RESFAC

RESPONSE FACTORS

| HOUR | X | Y | Z |
|---|---|---|---|
| 0 | 2.7019910060 | .0000045494 | .9028094952 |
| 1 | -1.5827901353 | .0013717141 | -.3592850806 |
| 2 | -.2604623224 | .0104498496 | -.0959141188 |
| 3 | -.1353416110 | .0217968584 | -.0425646423 |
| 4 | -.0878924779 | .0269294928 | -.0267846451 |
| 5 | -.0643224585 | .0269628958 | -.0198978947 |
| 6 | -.0504462651 | .0246772257 | -.0158734504 |
| 7 | -.0411015290 | .0216900394 | -.0130731950 |
| 8 | -.0341619995 | .0187069269 | -.0109294008 |
| 9 | -.0286814781 | .0159940218 | -.0092032465 |
| 10 | -.0242006794 | .0136146009 | -.0077768730 |
| 11 | -.0204699917 | .0115652090 | -.0065827591 |
| 12 | -.0173351451 | .0096144694 | -.0055766122 |
| 13 | -.0146889411 | .0083247083 | -.0047261531 |
| 14 | -.0124502106 | .0070594147 | -.0040061772 |
| 15 | -.0105541392 | .0059857498 | -.0033962047 |
| 16 | -.0089474249 | .0050750956 | -.0028792386 |
| 17 | -.0075855568 | .0043028691 | -.0024410192 |
| 18 | -.0064310775 | .0036480965 | -.0020695194 |
| 19 | -.0054523455 | .0030929417 | -.0017545676 |

NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 19
NUMBER OF RESPONCE FACTORS PER SET = 20
COMMON RATIO = .8478183024

## TABLE 4 (Cont.)

NEW RESFAC

RESPONSE FACTORS

| HOUR | X | Y | Z |
|---|---|---|---|
| 0 | 2.7019910343 | .0000045273 | .9028094952 |
| 1 | -1.5827901625 | .0013717369 | -.3592850790 |
| 2 | -.2604623235 | .0104498499 | -.0959141192 |
| 3 | -.1353416117 | .0217968587 | -.0425646427 |
| 4 | -.0978924783 | .0269294930 | -.0267846454 |
| 5 | -.0643224589 | .0269628958 | -.0198978949 |
| 6 | -.0504462653 | .0246772257 | -.0158734506 |
| 7 | -.0411015290 | .0216900394 | -.0130731952 |
| 8 | -.0341619995 | .0187089269 | -.0109294008 |
| 9 | -.0285814781 | .0159940218 | -.0092032465 |
| 10 | -.0242006794 | .0136146008 | -.0077768730 |
| 11 | -.0204699916 | .0115652089 | -.0065827591 |
| 12 | -.0173351450 | .0098144694 | -.0055766122 |
| 13 | -.0146889410 | .0083247082 | -.0047251531 |
| 14 | -.0124502105 | .0070594145 | -.0040061772 |
| 15 | -.0105541391 | .0059857497 | -.0033962047 |
| 16 | -.0089474248 | .0050750955 | -.0028792385 |
| 17 | -.0075855567 | .0043028690 | -.0024410192 |
| 18 | -.0064310774 | .0036480965 | -.0020695194 |
| 19 | -.0054523455 | .0030929417 | -.0017545676 |

NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 19
NUMBER OF RESPONCE FACTORS PER SET = 20
COMMON RATIO = .8478183011

TABLE 4 (Cont.)

ROOTS

| ROOT # | OLD | NEW |
|---|---|---|
| 1 | 0.16508893214859 | 0.16508893366130 |
| 2 | 0.88663631632084 | 0.88663631571600 |
| 3 | 1.82111532969845 | 1.82111532712176 |
| 4 | 2.70897699922766 | 2.70897699132630 |
| 5 | 4.87369736475012 | 4.87369737239763 |
| 6 | 7.89216455071565 | 7.89216455605310 |
| 7 | 11.68507789151147 | 11.68507788855550 |
| 8 | 16.24256497751424 | 16.24256497291390 |
| 9 | 21.56196345126659 | 21.56196345365345 |
| 10 | 27.64206980186293 | 27.64206980859319 |
| 11 | 34.48177223186940 | 34.48177223524772 |

B(ROOTS)

| FOR ROOTS # | OLD | NEW |
|---|---|---|
| 1 | 2.128693E-8 | -3.80000E-13 |
| 2 | 1.763430E-9 | 7.00000E-14 |
| 3 | -3.889470E-9 | -6.41180E-10 |
| 4 | 1.059484E-8 | -2.67600E-11 |
| 5 | 2.079299E-8 | 7.00000E-14 |
| 6 | -1.933347E-8 | 1.03077E-9 |
| 7 | -1.325572E-8 | -2.71100E-11 |
| 8 | 2.175127E-8 | 7.33800E-11 |
| 9 | 1.086137E-8 | -1.09000E-12 |
| 10 | -2.711985E-8 | 1.61000E-12 |
| 11 | 1.036738E-8 | -4.57000E-10 |

# TABLE 5

DESCRIPTION OF CONSTRUCTION

| LAYER NUMBER | THICKNESS FT | CONDUCTIVITY BTU PER (HR)(FT)(F) | DENSITY LB PER CU FT | SPECIFIC HEAT BTU PER (LB)(F) | RESISTANCE (HR)(SQ FT)(F) PER BTU | |
|---|---|---|---|---|---|---|
| 1 | .0417 | .830 | 55.0 | .4000 | 0.0000 | STONE |
| 2 | .0313 | .110 | 70.0 | .4000 | 0.0000 | FELT |
| 3 | .1670 | .025 | 2.0 | .2000 | 0.0000 | CELLULAR GLASS |
| 4 | .0050 | 26.000 | 480.0 | .1000 | 0.0000 | SHEET METAL |
| 5 | 0.0000 | 0.000 | 0.0 | 0.0000 | .6850 | INSIDE AIR |

## OLD RESFAC

RESPONSE FACTORS

| HOUR | X | Y | Z |
|---|---|---|---|
| 0 | 1.8974134526 | .0743183181 | .3484881084 |
| 1 | -1.7660938687 | .0549331620 | -.2179800359 |
| 2 | -.0014456694 | .0006165926 | -.0006354967 |
| 3 | -.0000034153 | .0000024075 | -.0000020801 |
| 4 | -.0000000105 | .0000000084 | -.0000000070 |
| 5 | -.0000000000 | .0000000000 | -.0000000000 |

NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 5
NUMBER OF RESPONCE FACTORS PER SET = 6
COMMON RATIO = .0033832881

## NEW RESFAC

RESPONSE FACTORS

| HOUR | X | Y | Z |
|---|---|---|---|
| 0 | 1.8974134525 | .0743183181 | .3484881084 |
| 1 | -1.7660938686 | .0549331620 | -.2179800359 |
| 2 | -.0014456694 | .0006165926 | -.0006354967 |
| 3 | -.0000034153 | .0000024075 | -.0000020801 |
| 4 | -.0000000105 | .0000000084 | -.0000000070 |
| 5 | -.0000000000 | .0000000000 | -.0000000000 |

NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 5
NUMBER OF RESPONCE FACTORS PER SET = 6
COMMON RATIO = .0033832881

TABLE 5 (Cont.)

ROOTS

| ROOT # | OLD | NEW |
|---|---|---|
| 1 | 5.68890723428157 | 5.68890723527994 |
| 2 | 7.23930079653397 | 7.23930076296122 |
| 3 | 23.82487390640392 | 23.82487391298810 |
| 4 | 46.56416592840105 | 46.56416592806750 |

B(ROOTS)

| FOR ROOTS # | OLD | NEW |
|---|---|---|
| 1 | 1.4505E-9 | 4.42000E-12 |
| 2 | 4.4640E-9 | -3.36164E-9 |
| 3 | 9.4499E-9 | -9.11880E-10 |
| 4 | 1.0243E-9 | 6.80000E-13 |

# TABLE 6

DESCRIPTION OF CONSTRUCTION

| LAYER NUMBER | THICKNESS FT | CONDUCTIVITY BTU PER (HR)(FT)(F) | DENSITY LB PER CU FT | SPECIFIC HEAT BTU PER (LB)(F) | RESISTANCE (HR)(SQ FT)(F) PER BTU | |
|---|---|---|---|---|---|---|
| 1 | .0400 | .400 | 116.0 | .2000 | 0.0000 | STUCCO |
| 2 | .0630 | .015 | 74.0 | .2000 | 0.0000 | INSULATION |
| 3 | .0400 | .400 | 116.0 | .2000 | 0.0000 | STUCCO |
| 4 | 1.0000 | .380 | 106.0 | .2000 | 0.0000 | MASONARY |
| 5 | .1250 | .250 | 5.7 | .2000 | 0.0000 | RIGID INSULATION |
| 6 | .1250 | .420 | 100.0 | .2000 | 0.0000 | GYPSUM |
| 7 | 0.0000 | 0.000 | 0.0 | 0.0000 | .6850 | INSIDE AIR |

## OLD RESFAC

RESPONSE FACTORS

| HOUR | X | Y | Z |
|---|---|---|---|
| 0 | 1.4133452868 | -.0000016845 | 1.0234028097 |
| 1 | -1.1675788059 | .0000016986 | -.2922661790 |
| 2 | -.0287241362 | .0000006888 | -.1269399552 |
| 3 | -.0094665212 | .0000158459 | -.0696275927 |
| 4 | -.0058867493 | .0000965331 | -.0445554251 |
| 5 | -.0045961430 | .0002893197 | -.0318517154 |
| 6 | -.0038760191 | .0005614340 | -.0245613444 |
| 7 | -.0033688850 | .0009223559 | -.0199316112 |
| 8 | -.0030284883 | .0012630118 | -.0167516158 |
| 9 | -.0027478187 | .0015717229 | -.0144353344 |
| 10 | -.0025216815 | .0018335561 | -.0126736589 |
| 11 | -.0023349639 | .0020448307 | -.0112905141 |
| 12 | -.0021778771 | .0022080835 | -.0101785182 |
| 13 | -.0020437032 | .0023286877 | -.0092681270 |
| 14 | -.0019276198 | .0024128978 | -.0085119026 |
| 15 | -.0018260430 | .0024668386 | -.0078760512 |
| 16 | -.0017362397 | .0024960470 | -.0073356433 |
| 17 | -.0016560803 | .0025033151 | -.0068717849 |
| 18 | -.0015836905 | .0024986835 | -.0064698635 |
| 19 | -.0015183328 | .0024795039 | -.0061164099 |
| 20 | -.0014583340 | .0024505270 | -.0058083259 |
| 21 | -.0014030280 | .0024139934 | -.0055323389 |
| 22 | -.0013517122 | .0023717167 | -.0052846038 |
| 23 | -.0013038151 | .0023251678 | -.0050604034 |
| 24 | -.0012588698 | .0022755169 | -.0048559191 |
| 25 | -.0012164939 | .0022237162 | -.0046680514 |
| 26 | -.0011763732 | .0021705143 | -.0044942787 |
| 27 | -.0011382486 | .0021165137 | -.0043325451 |
| 28 | -.0011019055 | .0020621967 | -.0041811707 |
| 29 | -.0010671655 | .0020079211 | -.0040387794 |
| 30 | -.0010338796 | .0019539995 | -.0039042409 |
| 31 | -.0010019227 | .0019006553 | -.0037766241 |
| 32 | -.0009711891 | .0018480652 | -.0036551595 |
| 33 | -.0009415890 | .0017963638 | -.0035392082 |
| 34 | -.0009130455 | .0017456515 | -.0034282382 |
| 35 | -.0008854922 | .0016960017 | -.0033218035 |
| 36 | -.0008586716 | .0016474659 | -.0032195287 |
| 37 | -.0008331331 | .0016000783 | -.0031210956 |
| 38 | -.0008082321 | .0015538592 | -.0030262333 |
| 39 | -.0007841290 | .0015088160 | -.0029347082 |
| 40 | -.0007607860 | .0014649553 | -.0028463187 |
| 41 | -.0007381769 | .0014222651 | -.0027608889 |

| | | | |
|---|---|---|---|
| 42 | -.0007162661 | .0013807359 | -.0026782639 |
| 43 | -.0006950287 | .0013403523 | -.0025983063 |
| 44 | -.0006744394 | .0013010956 | -.0025208934 |
| 45 | -.0006544751 | .0012629448 | -.0024459142 |
| 46 | -.0006351140 | .0012258772 | -.0023732683 |
| 47 | -.0006163353 | .0011898689 | -.0023028634 |
| 48 | -.0005981199 | .0011548952 | -.0022346145 |
| 49 | -.0005804492 | .0011209307 | -.0021684428 |
| 50 | -.0005633057 | .0010879499 | -.0021042747 |
| 51 | -.0005466728 | .0010559273 | -.0020420411 |
| 52 | -.0005305344 | .0010248373 | -.0019816770 |
| 53 | -.0005149752 | .0009946547 | -.0019231205 |
| 54 | -.0004996804 | .0009653545 | -.0018663132 |

NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 54
NUMBER OF RESPONCE FACTORS PER SET = 55
COMMON RATIO = .9705098791

NEW RESFAC

RESPONSE FACTORS

| HOUR | X | Y | Z |
|---|---|---|---|
| 0 | 1.4133454204 | .0000000000 | 1.0234016119 |
| 1 | -1.1675789617 | .0000000016 | -.2922650222 |
| 2 | -.0287241355 | .0000006864 | -.1269399500 |
| 3 | -.0094665199 | .0000158453 | -.0696275884 |
| 4 | -.0058867490 | .0000965324 | -.0445554213 |
| 5 | -.0045961418 | .0002893190 | -.0318517119 |
| 6 | -.0038760181 | .0005814334 | -.0245613414 |
| 7 | -.0033888841 | .0009223553 | -.0199316084 |
| 8 | -.0030284874 | .0012630113 | -.0167516133 |
| 9 | -.0027478179 | .0015717225 | -.0144353321 |
| 10 | -.0025214807 | .0018335557 | -.0126736568 |
| 11 | -.0023349631 | .0020448303 | -.0112905121 |
| 12 | -.0021778763 | .0022080832 | -.0101785164 |
| 13 | -.0020437025 | .0023786874 | -.0097691253 |
| 14 | -.0019276171 | .0024178975 | -.0085119010 |
| 15 | -.0018260424 | .0024668383 | -.0078760497 |
| 16 | -.0017362390 | .0024960468 | -.0073356418 |
| 17 | -.0016560797 | .0025053149 | -.0068717935 |
| 18 | -.0015838900 | .0024986833 | -.0064698622 |
| 19 | -.0015183322 | .0024795038 | -.0061184088 |
| 20 | -.0014583335 | .0024505269 | -.0059093248 |
| 21 | -.0014030275 | .0024139934 | -.0055323378 |
| 22 | -.0013517118 | .0023717166 | -.0052846028 |
| 23 | -.0013038146 | .0023251678 | -.0050604025 |
| 24 | -.0012588693 | .0022755189 | -.0048559183 |
| 25 | -.0012164935 | .0022237163 | -.0046680506 |
| 26 | -.0011763728 | .0021705143 | -.0044942779 |
| 27 | -.0011382482 | .0021165138 | -.0043325444 |
| 28 | -.0011019051 | .0020621908 | -.0041811700 |
| 29 | -.0010671651 | .0020079212 | -.0040387788 |
| 30 | -.0010338733 | .0019539996 | -.0039042403 |
| 31 | -.0010019224 | .0019006554 | -.0037756236 |
| 32 | -.0009711888 | .0018480653 | -.0036551890 |
| 33 | -.0009415887 | .0017963639 | -.0035392078 |
| 34 | -.0009130452 | .0017456517 | -.0034282378 |
| 35 | -.0008854919 | .0016960019 | -.0033218031 |
| 36 | -.0008588713 | .0016474661 | -.0032195284 |
| 37 | -.0008331328 | .0016000785 | -.0031210955 |
| 38 | -.0008082319 | .0015538594 | -.0030262330 |
| 39 | -.0007841288 | .0015088182 | -.0029347079 |
| 40 | -.0007607878 | .0014649555 | -.0028463185 |
| 41 | -.0007381767 | .0014222653 | -.0027608887 |

TABLE 6 (Cont.)

| | | | |
|---|---|---|---|
| 42 | -.0007162659 | .0013907361 | -.0026782637 |
| 43 | -.0006950285 | .0013403525 | -.0025983062 |
| 44 | -.0006744393 | .0013010955 | -.0025208932 |
| 45 | -.0006544750 | .0012629450 | -.0024459141 |
| 46 | -.0006351138 | .0012258774 | -.0023732682 |
| 47 | -.0006163352 | .0011898691 | -.0023028633 |
| 48 | -.0005981198 | .0011548554 | -.0022346144 |
| 49 | -.0005804491 | .0011209309 | -.0021684428 |
| 50 | -.0005633056 | .0010879501 | -.0021042747 |
| 51 | -.0005466727 | .0010559275 | -.0020420411 |
| 52 | -.0005305343 | .0010248375 | -.0019816770 |
| 53 | -.0005148751 | .0009946549 | -.0019231205 |
| 54 | -.0004996803 | .0009653547 | -.0018663132 |

NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 54
NUMBER OF RESPONCE FACTORS PER SET = 55
COMMON RATIO = .9705098860

ROOTS

| ROOT # | OLD | NEW |
|---|---|---|
| 1 | 0.02993369702080 | 0.02993368988454 |
| 2 | 0.21242985580688 | 0.21242984890035 |
| 3 | 0.61496372900272 | 0.61496373599719 |
| 4 | 1.09618246581550 | 1.09618246155674 |
| 5 | 1.66638739781747 | 1.66638739147555 |
| 6 | 2.32695013035205 | 2.32695012724621 |
| 7 | 2.96406703559819 | 2.96406703639735 |
| 8 | 4.26934120771537 | 4.26934121302924 |
| 9 | 5.96470442943337 | 5.96470443677839 |
| 10 | 7.90782511376565 | 7.90782511203173 |
| 11 | 9.38101582073091 | 9.38101581368824 |
| 12 | 10.74572406709456 | 10.74572406078107 |
| 13 | 13.20137627111041 | 13.20137627948367 |
| 14 | 15.14241773577010 | 15.14241774014755 |
| 15 | 16.58063080041711 | 16.58063079648036 |
| 16 | 19.19008477162163 | 19.19008477625005 |
| 17 | 20.55331112918213 | 20.55331113165494 |
| 18 | 23.66926094969699 | 23.66926094966527 |
| 19 | 27.32398281315591 | 27.32398280793188 |
| 20 | 29.01735456492725 | 29.01735456421909 |
| 21 | 32.16377783473581 | 32.16377783283110 |

TABLE 6 (Cont.)

| | B(ROOTS) | |
|---|---|---|
| FOR ROOTS # | OLD | NEW |
| 1 | -1.46805208E-6 | -1.00000E-14 |
| 2 | 4.47713380E-7 | -9.00000E-14 |
| 3 | 1.88247280E-7 | -1.40000E-13 |
| 4 | 5.84924700E-8 | 2.24335E-9 |
| 5 | -5.49342000E-8 | -1.27855E-9 |
| 6 | 2.11869900E-8 | -1.40000E-13 |
| 7 | 7.39351000E-9 | 3.80000E-13 |
| 8 | -9.94726500E-8 | -1.34000E-12 |
| 9 | 1.35020560E-7 | -3.19200E-10 |
| 10 | 1.47768100E-8 | 4.58000E-12 |
| 11 | -3.02108600E-8 | 7.20000E-13 |
| 12 | 2.44352100E-8 | -4.60000E-13 |
| 13 | 2.34856100E-8 | -2.11340E-10 |
| 14 | -5.98533000E-9 | 8.00000E-14 |
| 15 | -4.85016000E-9 | 7.00000E-13 |
| 16 | -4.95923000E-9 | 1.48200E-11 |
| 17 | 3.20458000E-9 | 1.74200E-10 |
| 18 | 2.62937000E-9 | 2.54531E-92 |
| 19 | -8.93764000E-9 | -8.12000E-12 |
| 20 | 1.14600000E-9 | -8.40000E-12 |
| 21 | -6.36227000E-9 | -2.40000E-13 |

TABLE 7

DESCRIPTION OF CONSTRUCTION

| LAYER NUMBER | THICKNESS FT | CONDUCTIVITY BTU PER (HR)(FT)(F) | DENSITY LB PER CU FT | SPECIFIC HEAT BTU PER (LB)(F) | RESISTANCE (HR)(SQ FT)(F) PER BTU | |
|---|---|---|---|---|---|---|
| 1 | .3330 | .770 | 125.0 | .2200 | 0.0000 | FACE BRICK |
| 2 | 0.0000 | 0.000 | 0.0 | 0.0000 | .9100 | AIR SPACE |
| 3 | .0633 | .070 | 37.0 | .6000 | 0.0000 | SHEATHING BOARD |
| 4 | .3330 | .025 | 5.7 | .2000 | 0.0000 | INSULATION |
| 5 | .0625 | .420 | 100.0 | .2000 | 0.0000 | GYPSUM BOARD |
| 6 | 0.0000 | 0.000 | 0.0 | 0.0000 | .6850 | INSIDE AIR |

OLD RESFAC

RESPONSE FACTORS

| HOUR | X | Y | Z |
|---|---|---|---|
| 0 | 5.1759961919 | .0000002162 | .8567490859 |
| 1 | -3.2693245014 | .0001735606 | -.5063325817 |
| 2 | -.7627360815 | .0019875908 | -.1769113994 |
| 3 | -.3954482090 | .0053818003 | -.0661057275 |
| 4 | -.2236660860 | .0077112856 | -.0258009737 |
| 5 | -.1372336219 | .0082363141 | -.0106121783 |
| 6 | -.0902026585 | .0075773934 | -.0047087431 |
| 7 | -.0624053982 | .0064230232 | -.0023146240 |
| 8 | -.0446903989 | .0051940987 | -.0012765386 |
| 9 | -.0327070454 | .0040853314 | -.0007867634 |
| 10 | -.0242524535 | .0031605263 | -.0005255624 |
| 11 | -.0181221793 | .0024210054 | -.0003703670 |
| 12 | -.0136018523 | .0018436046 | -.0002693425 |
| 13 | -.0102351670 | .0013969911 | -.0001993224 |
| 14 | -.0077130398 | .0010593995 | -.0001488973 |
| 15 | -.0058172472 | .0008012596 | -.0001117858 |
| 16 | -.0043895017 | .0006055852 | -.0000841465 |
| 17 | -.0033130658 | .0004575046 | -.0000634302 |
| 18 | -.0025009878 | .0003455492 | -.0000478500 |
| 19 | -.0018881264 | .0002609535 | -.0000361113 |
| 20 | -.0014255162 | .0001970521 | -.0000272583 |
| 21 | -.0010762808 | .0001487916 | -.0000205781 |
| 22 | -.0008126172 | .0001123476 | -.0000155360 |
| 23 | -.0006135507 | .0000848287 | -.0000117298 |
| 24 | -.0004632519 | .0000640498 | -.0000088563 |
| 25 | -.0003497721 | .0000483605 | -.0000066867 |

NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 25
NUMBER OF RESPONCE FACTORS PER SET = 26
COMMON RATIO = .7550396065

TABLE 7 (Cont.)

NEW RESFAC

RESPONSE FACTORS

| HOUR | X | Y | Z |
|---|---|---|---|
| 0 | 5.1759967596 | .0000002177 | .8567491015 |
| 1 | -3.2693245749 | .0001735577 | -.5063326014 |
| 2 | -.7627360829 | .0019875912 | -.1769113981 |
| 3 | -.3954482093 | .0053818007 | -.0661057268 |
| 4 | -.2236660857 | .0077112659 | -.0258009732 |
| 5 | -.1372336213 | .0082363143 | -.0106121779 |
| 6 | -.0902026578 | .0075773935 | -.0047087428 |
| 7 | -.0624053974 | .0064230733 | -.0023146237 |
| 8 | -.0446903981 | .0051940988 | -.0012785364 |
| 9 | -.0327070448 | .0040853314 | -.0007867633 |
| 10 | -.0242524529 | .0031605263 | -.0005255623 |
| 11 | -.0181221788 | .0024210053 | -.0003703670 |
| 12 | -.0136018519 | .0018436045 | -.0002693425 |
| 13 | -.0102351666 | .0013989911 | -.0001993223 |
| 14 | -.0077130395 | .0010593995 | -.0001488973 |
| 15 | -.0058172469 | .0008012595 | -.0001117857 |
| 16 | -.0043895015 | .0006055851 | -.0000841464 |
| 17 | -.0033130656 | .0004575046 | -.0000634302 |
| 18 | -.0025009877 | .0003455492 | -.0000479500 |
| 19 | -.0018981263 | .0002609535 | -.0000361113 |
| 20 | -.0014255152 | .0001970521 | -.0000272583 |
| 21 | -.0010762807 | .0001487916 | -.0000205781 |
| 22 | -.0008176172 | .0001123476 | -.0000155360 |
| 23 | -.0006135507 | .0000848287 | -.0000117298 |
| 24 | -.0004632518 | .0000640498 | -.0000088563 |
| 25 | -.0003497721 | .0000483605 | -.0000066867 |

NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 25
NUMBER OF RESPONCE FACTORS PER SET = 26
COMMON RATIO = .7550396037

TABLE 7 (Cont.)

ROOTS

| ROOT # | OLD | NEW |
|---|---|---|
| 1 | 0.28098507216704 | 0.28098507582694 |
| 2 | 0.84228454393237 | 0.84228454471546 |
| 3 | 1.02137107973186 | 1.02137107926540 |
| 4 | 2.17346651520421 | 2.17346650715298 |
| 5 | 5.11260893999551 | 5.11260894227845 |
| 6 | 6.05598433406738 | 6.05598433479236 |
| 7 | 8.22009552003982 | 8.22009552583131 |
| 8 | 15.68213748921056 | 15.68213749230597 |
| 9 | 16.65349741351815 | 16.65349740592023 |
| 10 | 20.10396604661435 | 20.10396604274922 |
| 11 | 30.63091142382473 | 30.63091143841234 |

B(ROOTS)

| FOR ROOTS # | OLD | NEW |
|---|---|---|
| 1 | 6.780432E-8 | 2.87200E-11 |
| 2 | 6.798700E-10 | -2.00281E-9 |
| 3 | -7.311400E-10 | 3.30000E-13 |
| 4 | 5.621767E-8 | -7.90000E-13 |
| 5 | 8.845090E-9 | -4.60900E-11 |
| 6 | -3.960310E-9 | -1.52034E-9 |
| 7 | 5.605072E-8 | 4.30000E-13 |
| 8 | -9.308060E-9 | -1.15000E-12 |
| 9 | -1.537846E-8 | 3.25792E-9 |
| 10 | 3.210414E-8 | -1.76000E-12 |
| 11 | 5.580300E-10 | -9.69469E-9 |

TABLE 8

DESCRIPTION OF CONSTRUCTION

| LAYER NUMBER | THICKNESS FT | CONDUCTIVITY BTU PER (HR)(FT)(F) | DENSITY LB PER CU FT | SPECIFIC HEAT BTU PER (LB)(F) | RESISTANCE (HR)(SQ FT)(F) PER BTU | |
|---|---|---|---|---|---|---|
| 1 | .0100 | 2.300 | 70.0 | .3500 | 0.0000 | ASPHALT SHINGLE |
| 2 | .0420 | .067 | 34.0 | .2900 | 0.0000 | PLYWOOD SHEATHING |
| 3 | .2500 | .025 | .5 | .1600 | 0.0000 | INSULATION |
| 4 | 0.0000 | 0.000 | 0.0 | 0.0000 | .9100 | AIR SPACE |
| 5 | .0420 | .067 | 34.0 | .2900 | 0.0000 | PLYWOOD |
| 6 | .5000 | .025 | .5 | .1600 | 0.0000 | INSULATION |
| 7 | .0625 | .420 | 100.0 | .2000 | 0.0000 | GYPSUM BOARD |
| 8 | 0.0000 | 0.000 | 0.0 | 0.0000 | .6850 | INSIDE AIR |

OLD RESFAC

RESPONSE FACTORS

| HOUR | X | Y | Z |
|---|---|---|---|
| 0 | .7251569790 | .0003756736 | .8374457518 |
| 1 | -.6562617879 | .0036660986 | -.5326726953 |
| 2 | -.0101950075 | .0055020236 | -.1759137625 |
| 3 | -.0074798669 | .0050052262 | -.0596083780 |
| 4 | -.0055072144 | .0039969156 | -.0211851227 |
| 5 | -.0040565343 | .0030453759 | -.0082261789 |
| 6 | -.0029985457 | .0022765117 | -.0036634230 |
| 7 | -.0022019154 | .0016879889 | -.0019206027 |
| 8 | -.0016223972 | .0012472018 | -.0011622678 |
| 9 | -.0011954206 | .0009200953 | -.0007742682 |
| 10 | -.0008908205 | .0006783189 | -.0005438346 |
| 11 | -.0006490160 | .0004999254 | -.0003920535 |
| 12 | -.0004782159 | .0003683998 | -.0002860650 |
| 13 | -.0003523651 | .0002714615 | -.0002098686 |
| 14 | -.0002596341 | .0002000258 | -.0001543414 |
| 15 | -.0001913070 | .0001473869 | -.0001136275 |
| 16 | -.0001409613 | .0001086000 | -.0000836932 |
| 17 | -.0001038649 | .0000800202 | -.0000616578 |
| 18 | -.0000765311 | .0000589616 | -.0000454282 |
| 19 | -.0000563906 | .0000434448 | -.0000334719 |
| 20 | -.0000415505 | .0000320116 | -.0000246629 |
| 21 | -.0000306158 | .0000235872 | -.0000181723 |

NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 21
NUMBER OF RESPONCE FACTORS PER SET = 22
COMMON RATIO = .7368329969

TABLE 8 (Cont.)

NEW RESFAC

RESPONSE FACTORS

| HOUR | X | Y | Z |
|---|---|---|---|
| 0 | .7251569715 | .0003756761 | .8374457390 |
| 1 | -.6562617794 | .0036660957 | -.5326726811 |
| 2 | -.0101950074 | .0055020236 | -.1759137619 |
| 3 | -.0074798671 | .0050052263 | -.0596083779 |
| 4 | -.0055072146 | .0039969156 | -.0211851230 |
| 5 | -.0040565345 | .0030453759 | -.0082261792 |
| 6 | -.0029885459 | .0022765118 | -.0036634233 |
| 7 | -.0022019155 | .0016879889 | -.0019206029 |
| 8 | -.0016223973 | .0012472019 | -.0011672680 |
| 9 | -.0011954207 | .0009200953 | -.0007742684 |
| 10 | -.0008808705 | .0006783189 | -.0005438347 |
| 11 | -.0006490160 | .0004999255 | -.0003920536 |
| 12 | -.0004782159 | .0003683998 | -.0002860651 |
| 13 | -.0003523651 | .0002714615 | -.0002098687 |
| 14 | -.0002596342 | .0002000259 | -.0001543415 |
| 15 | -.0001913070 | .0001473870 | -.0001136275 |
| 16 | -.0001409613 | .0001086000 | -.0000836932 |
| 17 | -.0001038649 | .0000800202 | -.0000616578 |
| 18 | -.0000765311 | .0000589616 | -.0000454282 |
| 19 | -.0000563907 | .0000434449 | -.0000334719 |
| 20 | -.0000415505 | .0000320116 | -.0000246629 |
| 21 | -.0000306158 | .0000235872 | -.0000181723 |

NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 21
NUMBER OF RESPONCE FACTORS PER SET = 22
COMMON RATIO = .7368330005

TABLE 8 (Cont.)

ROOTS

| ROOT # | OLD | NEW |
| --- | --- | --- |
| 1 | 0.30539401103140 | 0.30539400613318 |
| 2 | 1.12468525501743 | 1.12468526089836 |
| 3 | 9.41824542784383 | 9.41824542612801 |
| 4 | 12.27380260558755 | 12.27380260257945 |
| 5 | 33.85678581614047 | 33.85678582246646 |

B(ROOTS)

| FOR ROOTS # | OLD | NEW |
| --- | --- | --- |
| 1 | -3.4314850E-7 | 2.93000E-12 |
| 2 | -2.9825105E-7 | -1.36000E-12 |
| 3 | -5.2242660E-8 | -7.50152E-9 |
| 4 | 7.2314770E-8 | -1.06420E-10 |
| 5 | 6.6737980E-8 | 3.69000E-12 |

# TABLE 9

## DESCRIPTION OF CONSTRUCTION

| LAYER NUMBER | THICKNESS FT | CONDUCTIVITY BTU PER (HR)(FT)(F) | DENSITY LB PER CU FT | SPECIFIC HEAT BTU PER (LB)(F) | RESISTANCE (HR)(SQ FT)(F) PER BTU | |
|---|---|---|---|---|---|---|
| 1 | .0417 | .830 | 55.0 | .4000 | 0.0000 | STONE |
| 2 | .0313 | .110 | 70.0 | .4000 | 0.0000 | FELT |
| 3 | .1670 | .025 | 2.0 | .2000 | 0.0000 | CELLULAR GLASS |
| 4 | .3330 | .100 | 40.0 | .2000 | 0.0000 | CONCRETE |
| 5 | .0050 | 26.000 | 480.0 | .1000 | 0.0000 | METAL PAN |
| 6 | 0.0000 | 0.000 | 0.0 | 0.0000 | .9100 | AIR SPACE |
| 7 | .0625 | .420 | 100.0 | .2000 | 0.0000 | GYPSUM BOARD |
| 8 | 0.0000 | 0.000 | 0.0 | 0.0000 | .9100 | AIR SPACE |
| 9 | .0625 | .035 | 30.0 | .2000 | 0.0000 | ACOUSTICAL TILE |
| 10 | 0.0000 | 0.000 | 0.0 | 0.0000 | .6850 | INSIDE AIR |

## OLD RESFAC

### RESPONSE FACTORS

| HOUR | X | Y | Z |
|---|---|---|---|
| 0 | .7578139001 | .0027676836 | .8455636399 |
| 1 | -.3264110204 | .0257121522 | -.3394234703 |
| 2 | -.0551369293 | .0378860569 | -.1224280305 |
| 3 | -.0406467194 | .0340939470 | -.0561552755 |
| 4 | -.0303621399 | .0272297798 | -.0321965209 |
| 5 | -.0227925210 | .0209383902 | -.0214183746 |
| 6 | -.0171418960 | .0158870616 | -.0153359574 |
| 7 | -.0129010796 | .0119958160 | -.0113254352 |
| 8 | -.0097119218 | .0090413992 | -.0084651492 |
| 9 | -.0073118274 | .0068100785 | -.0063561928 |
| 10 | -.0055050614 | .0051281546 | -.0047808134 |
| 11 | -.0041448055 | .0038612698 | -.0035981829 |
| 12 | -.0031206732 | .0029072631 | -.0027087421 |
| 13 | -.0023495961 | .0021889357 | -.0020393435 |
| 14 | -.0017690432 | .0016480851 | -.0015354205 |
| 15 | -.0013319373 | .0012408677 | -.0011560313 |

NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 15
NUMBER OF RESPONCE FACTORS PER SET = 16
COMMON RATIO = .7529140452

NEW RESFAC

RESPONSE FACTORS

| HOUR | X | Y | Z |
|---|---|---|---|
| 0 | .7578138940 | .0027677014 | .8455636324 |
| 1 | -.3264110153 | .0257171323 | -.3394234638 |
| 2 | -.0551369295 | .0378860570 | -.1224280295 |
| 3 | -.0406467189 | .0340939471 | -.0561552749 |
| 4 | -.0303621396 | .0272297800 | -.0371965207 |
| 5 | -.0227925708 | .0209383903 | -.0214183745 |
| 6 | -.0171418959 | .0158870618 | -.0153359574 |
| 7 | -.0129010796 | .0119958162 | -.0113254352 |
| 8 | -.0097119219 | .0090413994 | -.0084651493 |
| 9 | -.0073118275 | .0068100787 | -.0063561929 |
| 10 | -.0055050615 | .0051281548 | -.0047808135 |
| 11 | -.0041448056 | .0038612700 | -.0035981930 |
| 12 | -.0031206733 | .0029072632 | -.0027087422 |
| 13 | -.0023495962 | .0021889358 | -.0020393435 |
| 14 | -.0017690433 | .0016480851 | -.0015354206 |
| 15 | -.0013319373 | .0012408677 | -.0011560314 |

NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 15
NUMBER OF RESPONCE FACTORS PER SET = 16
COMMON RATIO = .7529140463

ROOTS

| ROOT # | OLD | NEW |
|---|---|---|
| 1 | 0.28380420751416 | 0.28380420343971 |
| 2 | 1.27316441620625 | 1.27316442200970 |
| 3 | 6.08916372121456 | 6.08916372270650 |
| 4 | 16.59531217201106 | 16.59531217589722 |
| 5 | 31.14535162877291 | 31.14535163507094 |

B(ROOTS)

| FOR ROOTS # | OLD | NEW |
|---|---|---|
| 1 | -4.681876E-8 | 1.800E-12 |
| 2 | -4.410708E-8 | -3.000E-14 |
| 3 | 1.978496E-8 | -1.238E-11 |
| 4 | -5.435033E-8 | -8.071E-11 |
| 5 | 3.951397E-8 | 1.320E-12 |

# TABLE 10

DESCRIPTION OF CONSTRUCTION

| LAYER NUMBER | THICKNESS FT | CONDUCTIVITY BTU PER (HR) (FT) (F) | DENSITY LB PER CU.FT | SPECIFIC HEAT BTU PER (LB) (F) | RESISTANCE (HR) (SQ FT) (F) PER BTU | |
|---|---|---|---|---|---|---|
| 1 | .0050 | 26.000 | 480.0 | .1000 | 0.0000 | METAL SIDING |
| 2 | .0420 | .025 | 2.0 | .2000 | 0.0000 | INSULATION PANELS |
| 3 | .0833 | .400 | 116.0 | .2000 | 0.0000 | STUCCO |
| 4 | .0313 | .110 | 70.0 | .4000 | 0.0000 | FELT |
| 5 | .0420 | .067 | 34.0 | .2900 | 0.0000 | PLYWOOD SHEATHING |
| 6 | 0.0000 | 0.000 | 0.0 | 0.0000 | .9100 | AIR SPACE |
| 7 | .0625 | .420 | 100.0 | .2000 | 0.0000 | GYPSUM BOARD |
| 8 | .0104 | .070 | 37.0 | .6000 | 0.0000 | PANELING |
| 9 | 0.0000 | 0.000 | 0.0 | 0.0000 | .6850 | INSIDE AIR |

OLD RESFAC

RESPONSE FACTORS

| HOUR | X | Y | Z |
|---|---|---|---|
| 0 | 1.8963060277 | .0000004210 | .4011426131 |
| 1 | -1.7737519020 | .0001551499 | -.1512127477 |
| 2 | -.0093253891 | .0012842006 | -.0295006032 |
| 3 | -.0055369512 | .0028719285 | -.0201942048 |
| 4 | -.0043045688 | .0038685829 | -.0152586280 |
| 5 | -.0035601171 | .0042707124 | -.0122500417 |
| 6 | -.0030526487 | .0043131968 | -.0102674603 |
| 7 | -.0026786892 | .0041663052 | -.0088693871 |
| 8 | -.0023856690 | .0039289601 | -.0078185899 |
| 9 | -.0021447051 | .0036553284 | -.0069833746 |
| 10 | -.0019393478 | .0033741921 | -.0062892735 |
| 11 | -.0017599490 | .0031003295 | -.0056933503 |
| 12 | -.0016006431 | .0028408834 | -.0051701766 |
| 13 | -.0014576938 | .0025988771 | -.0047041173 |
| 14 | -.0013285809 | .0023751441 | -.0042850716 |
| 15 | -.0012114942 | .0021693853 | -.0039061168 |
| 16 | -.0011050518 | .0019807441 | -.0035621987 |
| 17 | -.0010081407 | .0018081172 | -.0032494010 |
| 18 | -.0009198274 | .0016503210 | -.0029645329 |
| 19 | -.0008393047 | .0015061780 | -.0027048935 |
| 20 | -.0007658611 | .0013745599 | -.0024681341 |
| 21 | -.0006988606 | .0012544076 | -.0022521756 |
| 22 | -.0006377307 | .0011447383 | -.0020551558 |
| 23 | -.0005819529 | .0010446462 | -.0018753946 |
| 24 | -.0005310563 | .0009532999 | -.0017113698 |

NUMBER OF HOURS REQUIRED TO REACH COMMON RATIO = 24
NUMBER OF RESPONCE FACTORS PER SET = 25
COMMON RATIO = .9125489730

TABLE 10 (Cont.)

NEW RESFAC

RESPONSE FACTORS

| HOUR | X | Y | Z |
|---|---|---|---|
| 0 | 1.8963060503 | .0000003904 | .4011426325 |
| 1 | -1.7737519229 | .0001551815 | -.1512127661 |
| 2 | -.0093253903 | .0012842006 | -.0299006033 |
| 3 | -.0055369520 | .0028719285 | -.0201942051 |
| 4 | -.0043045692 | .0036685829 | -.0152586282 |
| 5 | -.0035601173 | .0042707124 | -.0122500420 |
| 6 | -.0030526498 | .0043131968 | -.0102674605 |
| 7 | -.0026786893 | .0041663052 | -.0088693873 |
| 8 | -.0023856690 | .0039289601 | -.0078185901 |
| 9 | -.0021447051 | .0036553284 | -.0069833747 |
| 10 | -.0019393478 | .0033741921 | -.0062892736 |
| 11 | -.0017599499 | .0031003295 | -.0056933503 |
| 12 | -.0016006431 | .0028408834 | -.0051701767 |
| 13 | -.0014576938 | .0025988770 | -.0047041173 |
| 14 | -.0013285808 | .0023751441 | -.0042850717 |
| 15 | -.0012114942 | .0021693853 | -.0039051168 |
| 16 | -.0011050517 | .0019807440 | -.0035621987 |
| 17 | -.0010081407 | .0018081172 | -.0032494010 |
| 18 | -.0009198273 | .0016503210 | -.0029645329 |
| 19 | -.0008393047 | .0015061780 | -.0027048935 |
| 20 | -.0007658610 | .0013745599 | -.0024681341 |
| 21 | -.0006988606 | .0012544075 | -.0022521756 |
| 22 | -.0006377307 | .0011447382 | -.0020551558 |
| 23 | -.0005819529 | .0010446462 | -.0018753946 |
| 24 | -.0005310563 | .0009532999 | -.0017113698 |

NUMBER OF HOURS RECUIRED TO REACH COMMON RATIO = 24
NUMBER OF RESPONCE FACTORS PER SET = 25
COMMON RATIO = .9125469711

TABLE 10 (Cont.)

ROOTS

| ROOT # | OLD | NEW |
|---|---|---|
| 1 | 0.09151352592139 | 0.09151352798700 |
| 2 | 0.59670697196906 | 0.59670696614259 |
| 3 | 1.87826524925806 | 1.87826525339801 |
| 4 | 4.46784784597435 | 4.46784784772299 |
| 5 | 5.21288150152188 | 5.21288150553909 |
| 6 | 6.84838752278688 | 6.84838751809579 |
| 7 | 9.19016020960470 | 9.19016020792799 |
| 8 | 15.55378495246231 | 15.55378495460405 |
| 9 | 21.54078931780691 | 21.54078932206482 |
| 10 | 25.48150421908758 | 25.48150421445916 |
| 11 | 26.15232440552245 | 26.15232441059038 |
| 12 | 36.39218806754798 | 36.39218806540566 |

B(ROOTS)

| FOR ROOTS # | OLD | NEW |
|---|---|---|
| 1 | 2.4113701E-7 | 1.50000E-13 |
| 2 | 2.6229038E-7 | -2.40000E-13 |
| 3 | 1.0058861E-7 | -9.31998E-9 |
| 4 | -1.3448400E-9 | 4.28628E-9 |
| 5 | -8.6910700E-9 | 1.92000E-12 |
| 6 | 1.8604440E-8 | -2.90600E-11 |
| 7 | -1.0867190E-8 | 7.27528E-9 |
| 8 | -4.9799730E-8 | 1.23000E-12 |
| 9 | 2.9246420E-8 | -2.29313E-9 |
| 10 | 5.6764700E-9 | 9.40000E-13 |
| 11 | 7.1909600E-9 | 2.68070E-10 |
| 12 | 3.9220780E-8 | -5.25500E-11 |

## APPENDIX C

## BISECTION METHOD FOR LOCATING ROOTS

Given a continuous function f on the interval [a, b], where f(a) and f(b) have opposite signs. To find a solution to f(x) = 0.0:

1) set $a_1 = a$ and $b_1 = b$

2) set $i = 1$

3) set $p_i = \frac{1}{2}(a_i + b_i)$

4) if ABS(f($p_i$)≤1.0E-8 go to 6, else go to 5

5) if f($p_i$)*f($a_i$)>0,set $a_{i+1}=p_i$ and $b_{i+1}=b_i$;

   if f($p_i$)*f($a_i$)<0,set $a_{i+1}=a_i$ and $b_{i+1}=p_i$;

   add 1 to i and go to 3

6) end

## APPENDIX D

## SECANT METHOD FOR LOCATING ROOTS

Given a continous function f on the interval $[p_0, p_1]$, where $p_0$ and $p_1$ are initial root approximations such that $p_0 \neq p_1$. To find a solution to $f(x) = 0.0$:

1) set $i = 2$
2) set $p_i = p_{i-1} - \dfrac{f(p_{i-1})*(p_{i-1}-p_{i-2})}{f(p_{i-1})-f(p_{i-2})}$
3) if $ABS(f(p_i)) \leq 1.0E-8$ go to 4, else add 1 to i and go to 2
4) end

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| 1. Report No | 2. Government Accession No | 3. Recipient's Catalog No |
|---|---|---|
| NASA CR-165992 | | |

| 4. Title and Subtitle | 5. Report Date |
|---|---|
| AN ANALYSIS OF THERMAL RESPONSE FACTORS AND HOW TO REDUCE THEIR COMPUTATIONAL TIME REQUIREMENT | August 1982 |
| | 6. Performing Organization Code |
| 7. Author(s) | 8. Performing Organization Report No. |
| Michael R. Wiese | |
| 9. Performing Organization Name and Address | 10. Work Unit No. |
| Computer Sciences Corporation<br>Hampton, Virginia | 11. Contract or Grant No.<br>NAS1-16078 |
| 12. Sponsoring Agency Name and Address | 13. Type of Report and Period Covered<br>Contractor Report<br>1981-1982 |
| National Aeronautics and Space Administration<br>Washington, D.C. 20546 | 14. Sponsoring Agency Code |

15. Supplementary Notes

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16. Abstract


The RESFAC2 version of the Thermal Response Factor Program (RESFAC) is the result of numerous modifications and additions to the original RESFAC. These modifications and additions have significantly reduced the program's computational time requirement. As a result of this work, the program is more efficient and its code is both readable and understandable.

This report describes what a thermal response factor is; analyzes the original matrix algebra calculations and root finding technique; presents a new root finding technique and streamlined matrix algebra; supplies ten validation cases and their results.


| 17. Key Words (Suggested by Author(s)) | 18. Distribution Statement |
|---|---|
| ENERGY<br>ENERGY ANALYSIS<br>HEAT TRANSFER | Subject Category 44<br>Unclassified-Unlimited |

| 19. Security Classif. (of this report) | 20. Security Classif. (of this page) | 21. No. of Pages | 22. Price |
|---|---|---|---|
| Unclassified | Unclassified | 64 | A04 |




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