DOUBLE NODING TECHNIQUE FOR MIXED MODE CRACK PROPAGATION STUDIES

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by

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ABSTRACT

A simple dynamic finite element algorithm for analyzing a propagating mixed mode crack tip is presented. A double noding technique, which can be easily incorporated into existing dynamic finite element codes, is used together with a corrected $\hat{J}$ integral to extract modes I and II dynamic stress intensity factors of a propagating crack. The utility of the procedure is demonstrated by analyzing test problems involving a mode I central crack propagating in a plate subjected to uniaxial tension, a mixed mode I and II stationary, slanted central crack in a plate subjected to uniaxial impact loading, and a mixed mode I and II extending, slanted single edge crack in a plate subjected to uniaxial tension.

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INTRODUCTION

Most of the recent numerical studies in dynamic fracture mechanics are restricted to mode I crack propagation with the crack extending along the line of symmetry. Crack extension under such condition can be modeled with finite element method by simply releasing the crack tip node at the line of symmetry following a prescribed nodal force versus crack extension history. Mode I dynamic stress intensity factor in dynamic fracture analysis of isotropic material is justified since the crack will ordinarily propagate in the direction perpendicular to the maximum principal stress direction. There are conditions, however, when the crack will deviate from its self-similar crack extension path to curve [1] or to bifurcate [2] under specific static or dynamic loadings. Such crack extension away from the line of symmetry in a finite element mesh cannot be accomplished by the above-mentioned simple nodal release mechanism. The double-noding [3] and the nodal-grafting [4] techniques are two procedures which have been used to model off-axis crack propagation. While details of the former are not available, the latter requires a higher order isoparametric element in the dynamic finite element code. The double noding technique presented in this paper was developed for use with an implicit dynamic finite element code which utilizes a conventional isoparametric quadrilateral element.

THEORETICAL BACKGROUND

Double Noding Technique

Consider a slanted crack in a two dimensional space with a local x-y coordinate system oriented along the local crack tip region as shown in Figure 1. The dynamic equations of motion for the crack tip element with nodal displacements of $\{q\}$, nodal velocities of $\{\dot{q}\}$, nodal accelerations of

$\{\ddot{q}\}$, nodal forces of $\{F\}$, stiffness of $[K]$ and mass matrix of $[M]$ are:

$$[K]\{q\} + [M]\{\ddot{q}\} = \{F\} \tag{1}$$

If the ith and jth degrees of freedom in the above displacements, velocities and accelerations are constrained to be equal

$$q_i = q_j, \quad \dot{q}_i = \dot{q}_j, \quad \ddot{q}_i = \ddot{q}_j \tag{2}$$

through double noding, then define the relative displacement $\{q'\}$ as

$$\{q\} = [T] \ \{q'\}, \text{ etc.} \tag{3}$$

$$\{F'\} = [T]^T\{F\} \tag{4}$$

where $T_{mn} = 1$ when $m = n$

$= 1$ when $m = i$ and $m = j$

$= 0$ all other m and n

and

$$q'_i = q_i - q_j = 0 \quad \text{when } k = i$$

$$q'_k = q_k \qquad \text{when } k \neq i \tag{5}$$

Substituting Equations (3), (4) and (5) into Equation (1) yields

$$
\begin{aligned}
[K'][q'] + [M']\{\ddot{q}'\} &= \{F\} \\
&= [T]^T\{F\} \\
&= [T]^T[K]\{q\} + [T]^T[M]\{\ddot{q}\} \\
&= [T]^T[K][T]\{q'\} + [T]^T[M][T]\{\ddot{q}'\} \qquad (6)
\end{aligned}
$$

The equivalent stiffness and mass matrices in the above equivalent dynamic equations of motion are

$$[K'] = [T]^T[K][T] \qquad (7)$$

$$[M'] = [T]^T[M][T] \qquad (8)$$

Equations (3) through (8) constitutes the equivalent system for the slanted crack with two and more degrees of freedom constrained by the double noded current and future crack tip nodes. The equivalent nodal displacements are determined by applying Newmark's beta method to the dynamic equations of motion. The nodal displacements at the double nodes are then determined by Equation (3).

Nodal Force Release

Crack extension with a traction free crack surface requires elimination of the nodal forces normal, $F_{2\ell}$ and F_{2r}, and tangential, $F_{1\ell}$ and F_{1r}, to the crack surface as shown in Figure 1. The crack tip nodal force components are released simultaneously in equal increments to model a linear translation of the crack tip to its new crack tip node, $(\bar{F}_{1r}, \bar{F}_{1\ell}, \bar{F}_{2r}, \bar{F}_{2\ell}) = (\bar{F}_{1r}, \bar{F}_{1\ell}, \bar{F}_{2r}, \bar{F}_{2\ell})(1- \frac{\Delta}{d})$, where bar denotes the forces at the double node prior to crack extension, Δ is the current crack tip location and d is the crack distance to its adjacent node. Details of this release procedure as well as its comparison with others are described in Reference [5].

$\hat{J}$ Integral

The corrected $\hat{J}$ integral was used to determine the modes I and II dynamic stress intensity factors, K_I and K_{II}, respectively. The $\hat{J}$ integral as defined by Kishimoto et al [6] is

$$\hat{J} = \int_{\Gamma} [W n_1 - T_i u_{i,1}] d\Gamma + \int_A \rho \ddot{u}_i u_{i,1} dA \qquad (9)$$

where the indices refer to the local x_1 and x_2 coordinates shown in Figure 1, W is the strain energy density, n_1 is the surface normal to the integration path Γ_1 and ρ is the density.

By taking a symmetric integration path with respect to the crack extension direction, as shown in Figure 1, the $\hat{J}$ integral, which contains both K_I and K_{II}, can be decomposed into two path-independent integrals $\hat{J}_I$ and $\hat{J}_{II}$ as

$$\hat{J} = \hat{J}_I + \hat{J}_{II} \qquad (10)$$

where $\hat{J}_I$ and $\hat{J}_{II}$ are only functions of K_I and K_{II}, respectively. Detailed formulations of $\hat{J}_I$ and $\hat{J}_{II}$ are described by Kishimoto et al. [6]. For a stationary crack, Ishikawa et al [7] has shown that

$$\hat{J}_I = \frac{\kappa+1}{8\mu} K_I^2 \qquad (11a)$$

$$\hat{J}_{II} = \frac{\kappa+1}{8\mu} K_{II}^2 \tag{11b}$$

where

$$\kappa = \begin{cases} 3 - 4\nu & \text{plane strain} \\ \dfrac{3-\nu}{1+\nu} & \text{plane stress} \end{cases}$$

and μ and ν are the shear modulus and Poisson's ratio, respectively. For a propagating crack, Atluri et al. [8,9] has shown that

$$\hat{J}_I = \frac{\hat{F}_I(\dot{a})}{2\mu} K_I^2 \tag{12a}$$

$$\hat{J}_{II} = \frac{\hat{F}_{II}(\dot{a})}{2\mu} K_{II}^2 \tag{12b}$$

where
$$\hat{F}_I(\dot{a}) = \frac{S_1(1-S_2^2)}{D^2} [2S_1(1+S_2) - \frac{(1+S_1)}{2S_1}(1+S_2^2)^2 - 2(S_1-S_2) \frac{(1+S_2^2)^2}{\sqrt{(1+S_1)(1+S_2)}} \tag{12c}$$

$$\hat{F}_{II}(\dot{a}) = \frac{S_2(1-S_2^2)}{D^2} [2S_2(1+S_1) - \frac{(1+S_2)}{2S_2}(1+S_2^2)^2 - 2(S_2-S_1) \frac{(1+S_2^2)^2}{\sqrt{(1+S_1)(1+S_2)}} \tag{12d}$$

$$S_1^2 = 1 - (\frac{\dot{a}}{C_1})^2 \qquad S_2^2 = 1 - (\frac{\dot{a}}{C_2})^2 \tag{12e}$$

$$D = 4S_1S_2 - (1+S_2^2)^2 \tag{12f}$$

and $\dot{a}$, C_1 and C_2 are the crack velocity, dilatational and distortional stress wave velocities, respectively.

NUMERICAL PROCEDURE

The numerical procedure consists of inputting the above double noding technique to an implicit dynamic finite element code and releasing the crack tip nodal force linearly in accordance with a prescribed crack tip motion. As shown by Equations (12), the associated dynamic stress intensity factors, K_I and K_{II}, can be determined by numerically evaluating the $\hat{J}_I$ and $\hat{J}_{II}$ integrals along predetermined symmetric contours surrounding the instantaneous crack tip. Separation of the $\hat{J}$ integral of Equation (10) into the two $\hat{J}_I$ and $\hat{J}_{II}$ integrals is accomplished by using the decomposition procedure initially developed by Ishikawa et al. [7] for a static mixed mode crack and which was later extended to a dynamically loaded mixed mode crack by Kishimoto et al. [6]. The procedure consists of first decomposing the displacements, strains, stresses and body forces into symmetric and anti-symmetric components after which Equation (9) is used to compute $\hat{J}_I$ and $\hat{J}_{II}$ integrals, respectively. In the following, three problems which utilize the above mentioned double noding technique together with the $\hat{J}_I$ and $\hat{J}_{II}$ integral technique of determining K_I and K_{II} are described. The examples involved

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steel plate under a plane strain state of stress with material properties of $\mu = 2.94 \times 10^{10}$ N/m^2, $\nu = 0.286$ and $\rho = 2.45 \times 10^3$ Kg/m^3.

EXTENDING CENTRAL CRACK IN A PLATE SUBJECTED TO UNIAXIAL TENSION

The Broberg problem [10] of a central crack extending at constant velocity in a uniaxial tension field has been used by many to verify their dynamic finite element codes. This problem yields only a mode I dynamic stress intensity factor, but it offers the only comparison between results generated by other dynamic finite element codes. Figure 2 shows the finite element break down of a half-plane as well as the seven integration paths used in this analysis. The complete specimen geometry is shown in the legend of Figure 3. Variations in the numerically determined mode I dynamic stress intensity factors with the integration paths for five crack tip locations of a/W = 0.2, 0.3, 0.4, 0.5 and 0.6 and three crack velocities of $\dot{a}/C_2$ = 0, 0.2 and 0.6 are shown in Table 1, where a is the half crack length and W is the half plate width. Maximum difference of 1 percent between the K_I computed for the various integration paths shows that the path independency is satisfied for all practical purpose.

Figure 3 shows the changes in the static and the dynamic stress intensity factors for two dynamic crack velocities. Also shown for comparison purpose are the static results of Isida [11] and the two dynamic results of Nishioka and Atluri [12,13]. The 13 percent lower K_I at the shorter crack length of a/W = 0.25 at a higher crack velocity of $\dot{a}/C_2$ = 0.6 is due in part to the coarser and conventional element used in this analysis. Otherwise, good agreements between the various results are notd.

STATIONARY SLANTED CENTRAL CRACK IN A PLATE SUBJECTED TO UNIAXIAL IMPACT

LOADING

Mixed mode dynamic stress intensity factors in a stationary slanted central crack in a plate subjected to uniaxial impact loading of σH(t) was first analyzed by Thau and Lu [14] using Wiener-Hopf technique and by Kishimoto et al. [6] using finite element analysis. This problem was also analyzed in this paper in order to compare the applicability of present finite element algorithm with the above known results. Figure 4 shows the finite element break down and the five integration paths used in this analysis. Table 2 shows the variations in the normalized K_I and K_{II} at four time intervals of the stationary crack. While the differences in K_{II} for the five integration paths is as high as 8 percent, the relatively small K_{II} values makes this difference insignificant. Figure 5 shows changes in transient K_I and K_{II} with time for the stationary crack where R_0, R_1, R_2 denoting the first arrival times at the crack tip for incident wave from leading edge, the reflected waves from upper/lower boundaries and the reflected wave from the rear edge, respectively. While agreements between the three results for the stationary crack is good at the initial stage, the finite element results differ towards the latter part of time interval of 14 ∿ 20 s. Equally puzzling is the large differences between the static stress intensity factors for the plate under static loading by these two finite element analyses. Such discrepancy may result from different finite element mesn and geometry (triangular versus quadrilateral) by Kishimoto et al. [6] and by the present computation. Nevertheless, the general results of these two analyses are very similar.

EXTENDING SLANTED SINGLE EDGE CRACK IN A PLATE SUBJECTED TO UNIAXIAL TENSION

As a further study in mixed mode dynamic crack, an extending slanted

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single edge crack in a plate subjected to uniaxial tension, which is the dynamic counterpart of the static solution of Bowie [15], was analyzed. This crack was extended along its original crack direction at two crack velocities of $\dot{a}/C_2 = 0.2$ and 0.6. Figure 6 shows the finite element breakdown as well as the four integration paths used in this analysis. Table 3 shows the variations in K_I and K_{II} at five crack tip locations for the static and two propagating cracks. Unlike the previous case, little variation in K_{II} are noted, possibly due to the relatively larger values of K_{II} in this problem. Figure 7 shows the changes in K_I and K_{II} with crack extension. Also shown for comparison is the static results of Bowie [15] which are within 4% of the present analysis. While one would not expect self-similar crack propagation under this loading condition, it is interesting to note the closeness of K_I and K_{II} values at the higher crack velocity of $\dot{a}/C_2 = 0.6$.

CONCLUSIONS

A double noding technique suitable for dynamic finite element analysis of a crack extending under mixed mode loading condition is presented. Mixed mode dynamic stress intensity factors are computed by using the $\hat{J}$ integrals as modified by Atluri et al. [8,9] for the extending crack.

The procedure was used to determine the dynamic stress intensity factors of an extending central crack in a uniaxially loaded plate, the dynamic stress intensity factors of a stationary slanted central crack in an uniaxially impacted plate and the static and dynamic stress intensity factors of an extending slanted, single edge crack in a uniaxially loaded plate. The computed values by this procedure were generally in good agreement with known results.

When modified by appropriate criteria of dynamic crack curving [1] and

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branching [2], this simple procedure can be used to determine the dynamic fracture parameters associated with such problems.

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TABLE 2
PATH INDEPENDENCE OF $\hat{J}$-INTEGRAL FOR
EXTENDING CENTRAL CRACK PROBLEM

$\dot{a}/C_2$	a/W	$K_I/\sigma\sqrt{\pi W}$ Path 1	2	3	4	5	6	7
	0.2	0.4530	0.4602	0.4607	0.4593	0.4582	0.4576	0.4571
	0.3	0.5974	0.5987	0.6008	0.6012	0.5999	0.5988	0.5978
0	0.4	0.7493	0.7500	0.7514	0.7526	0.7516	0.7500	0.7485
	0.5	0.8316	0.8325	0.8339	0.8350	0.8338	0.8319	0.8302
	0.6	0.9195	0.9207	0.9223	0.9230	0.9211	0.9188	0.9170
	0.3	0.4560	0.4617	0.4617	0.4620	0.4606	0.4601	0.4597
	0.4	0.5630	0.5702	0.5713	0.5690	0.5701	0.5682	0.5673
0.2	0.5	0.7278	0.7335	0.7318	0.7281	0.7310	0.7286	0.7275
	0.6	0.9099	0.9251	0.9215	0.9108	0.9144	0.9108	0.9103
	0.3	0.2459	0.2487	0.2553	0.2598	0.2588	0.2580	0.2574
	0.4	0.2951	0.3098	0.3084	0.3079	0.3106	0.3097	0.3088
0.6	0.5	0.3423	0.3572	0.3563	0.3558	0.3556	0.3563	0.3569
	0.6	0.3703	0.3878	0.3889	0.3881	0.3878	0.3869	0.3860

a is the half crack length.
W is the half plate width.
σ is the applied uniaxial load.

TABLE 2
PATH INDEPENDENCE OF $\hat{J}$ INTEGRAL FOR
STATIONARY SLANTED CENTRAL CACK PROBLEM

	$K_I/\sigma\sqrt{\pi W}$					$K_{II}/\sigma\sqrt{\pi W}$				
TIME	Path 1	2	3	4	5	Path 1	2	3	4	5
5μs	0.1625	0.1619	0.1616	0.1615	0.1596	0.0825	0.0857	0.0895	0.0896	0.0888
10	C ’706	0.3730	0.3740	0.3711	0.3686	0.2837	0.2833	0.2776	0.2770	0.2809
15	1.0422	1.0492	1.0552	1.0532	1.0512	0.6190	0.6171	0.6089	0.6067	0.6050
20	1.1699	1.1769	1.1779	1.1729	1.1739	0.8234	0.8190	0.8092	0.8071	0.8098

W is the half plate width.
σ is the applied uniaxial impact load intensity.

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TABLE 3

PATH INDEPENDENCE OF $\hat{J}$ INTEGRAL FOR
EXTENDING SLANTED SINGLE EDGE CRACK PROBLEM

		$K_I/\sigma\sqrt{\pi W}$				$K_{II}/\sigma\sqrt{\pi W}$			
$\dot{a}/C_2$	a/W	Path 1	2	3	4	Path 1	2	3	4
	0.2	0.4314	0.4437			0.2243	0.2220		
	0.3	0.6583	0.6638	0.6726	0.6691	0.3226	0.3212	0.3198	0.3270
0	0.4	1.0079	1.0111	1.0167	1.0245	0.4498	0.4485	0.4455	0.4405
	0.5	1.5784	1.5829	1.5897	1.5972	0.6134	0.6102	0.6036	0.5865
	0.6	2.5244	2.5451	2.5226	2.5223	0.8462	0.8230	0.7749	0.7488
	0.3	0.4205	0.4252	0.4316	0.4254	0.2708	0.2708	0.2708	0.2749
	0.4	0.5497	0.5550	0.5563	0.5613	0.3122	0.3111	0.3099	0.3108
0.2	0.5	0.7549	0.7590	0.7583	0.7627	0.4163	0.4168	0.4167	0.4142
	0.6	0.8831	0.9010	0.8902	0.9001	0.5579	0.5387	0.5227	0.5263
	0.3	0.1762	0.1863	0.1914	0.1774	0.2166	0.2178	0.2161	0.2252
0.6	0.4	0.2095	0.2240	0.2187	0.2190	0.2287	0.2322	0.2360	0.2326
	0.5	0.2211	0.2347	0.2321	0.2351	0.2661	0.2648	0.2641	0.2639
	0.6	0.2571	0.2793	0.2750	0.2759	0.2282	0.2834	0.2772	0.2783

a is the crack length.
W is the plate width.
σ is the applied uniaxial load.

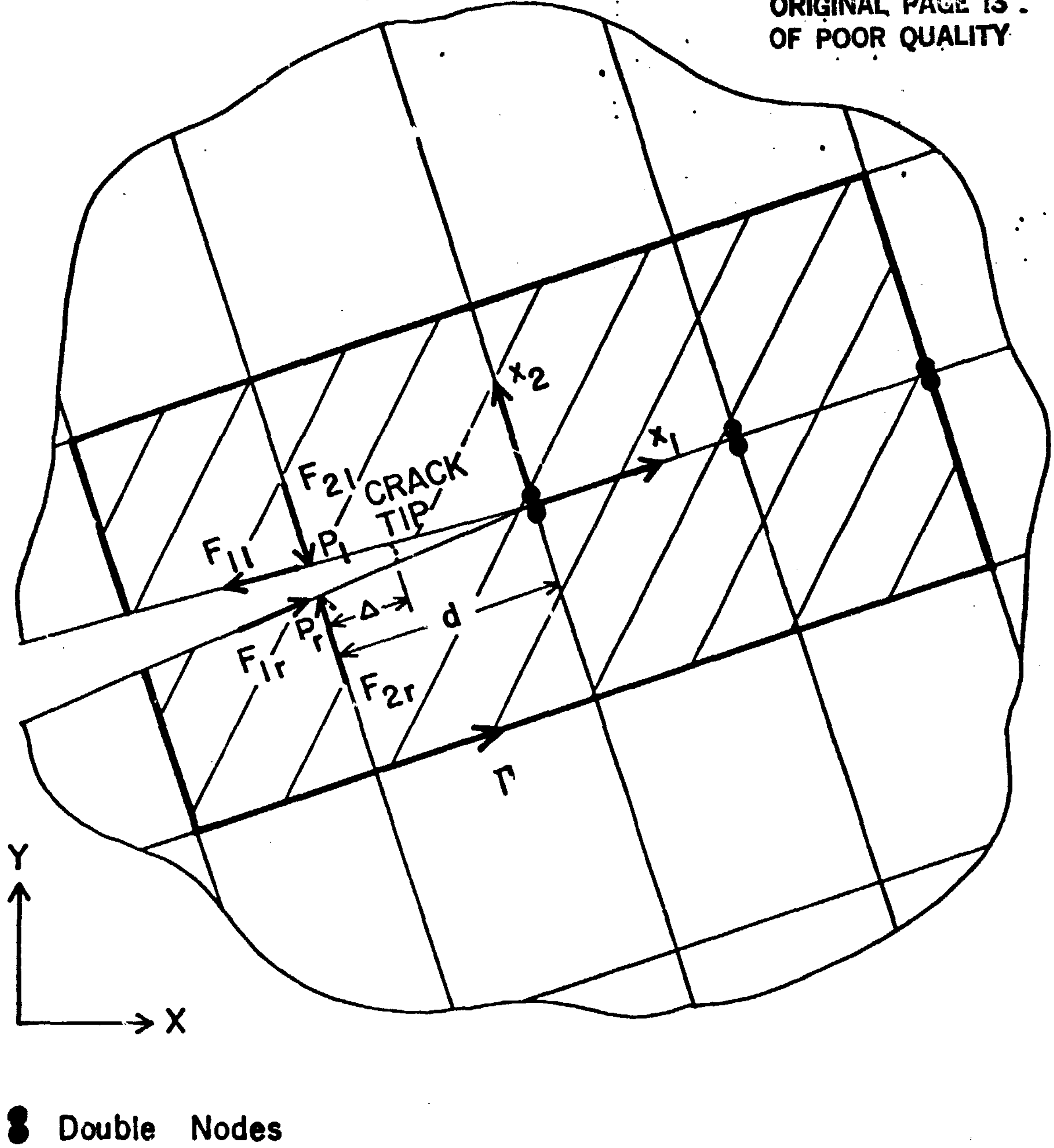

FIGURE 1. DOUBLE NODING, NODAL FORCE RELEASE AND $\hat{J}$ INTEGRAL PATH, Γ.

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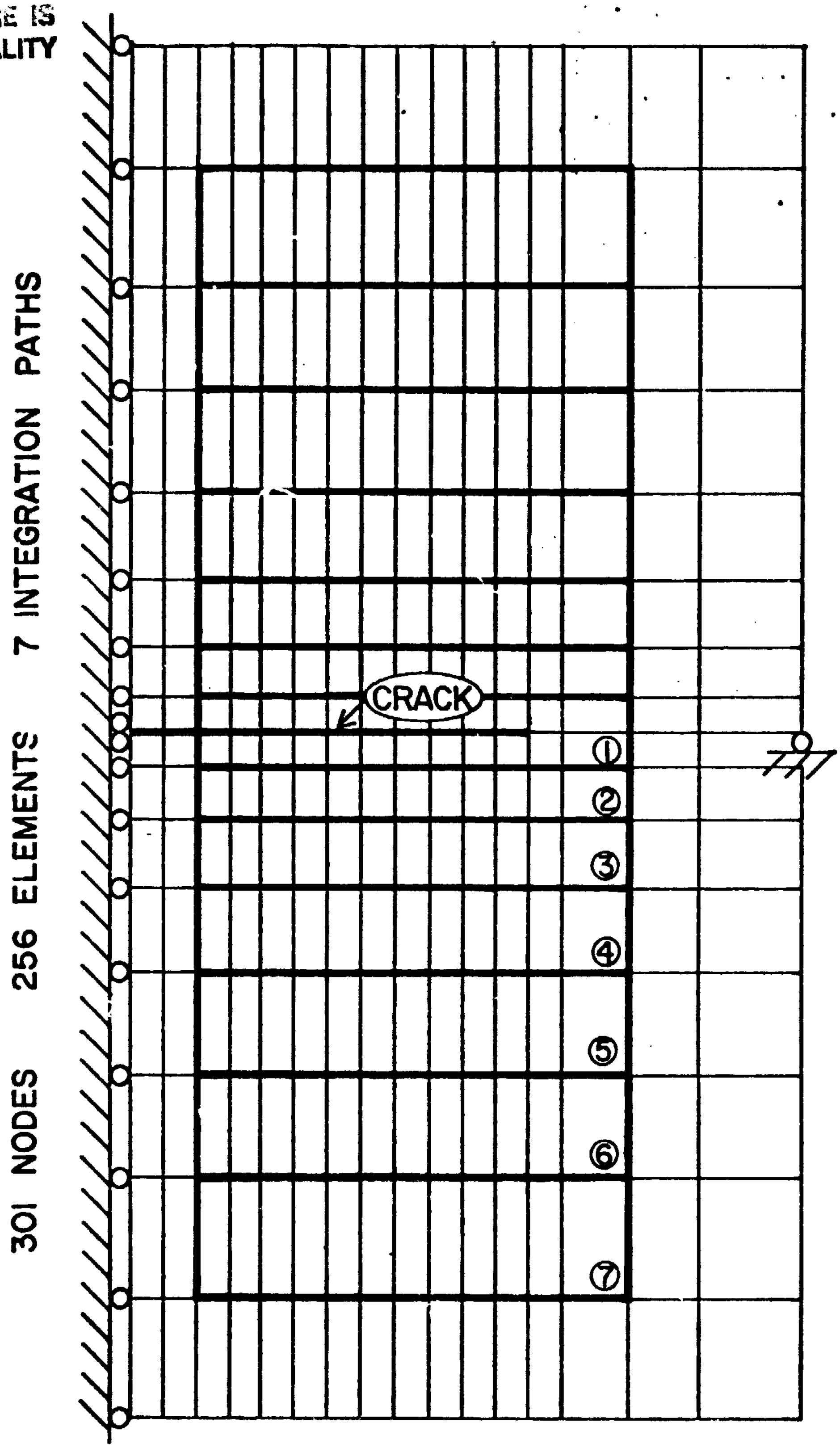

FIGURE 2. FINITE ELEMENT BREAKDOWN OF A HALF PLANE OF AN EXTENDING CENTRAL CRACK.

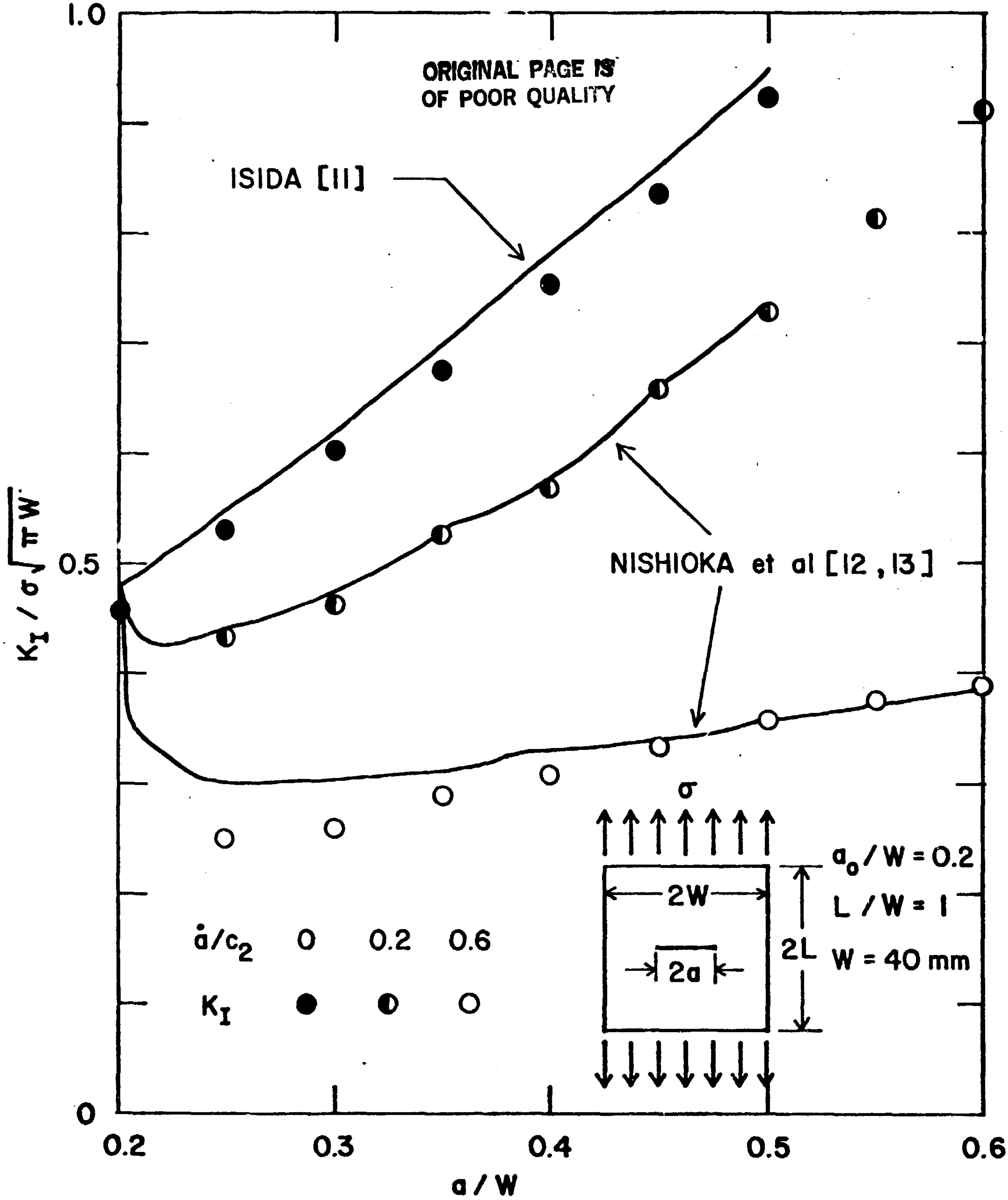

FIGURE 3. NORMALIZED STRESS INTENSITY FACTORS OF EXTENDING CENTRAL CRACK.

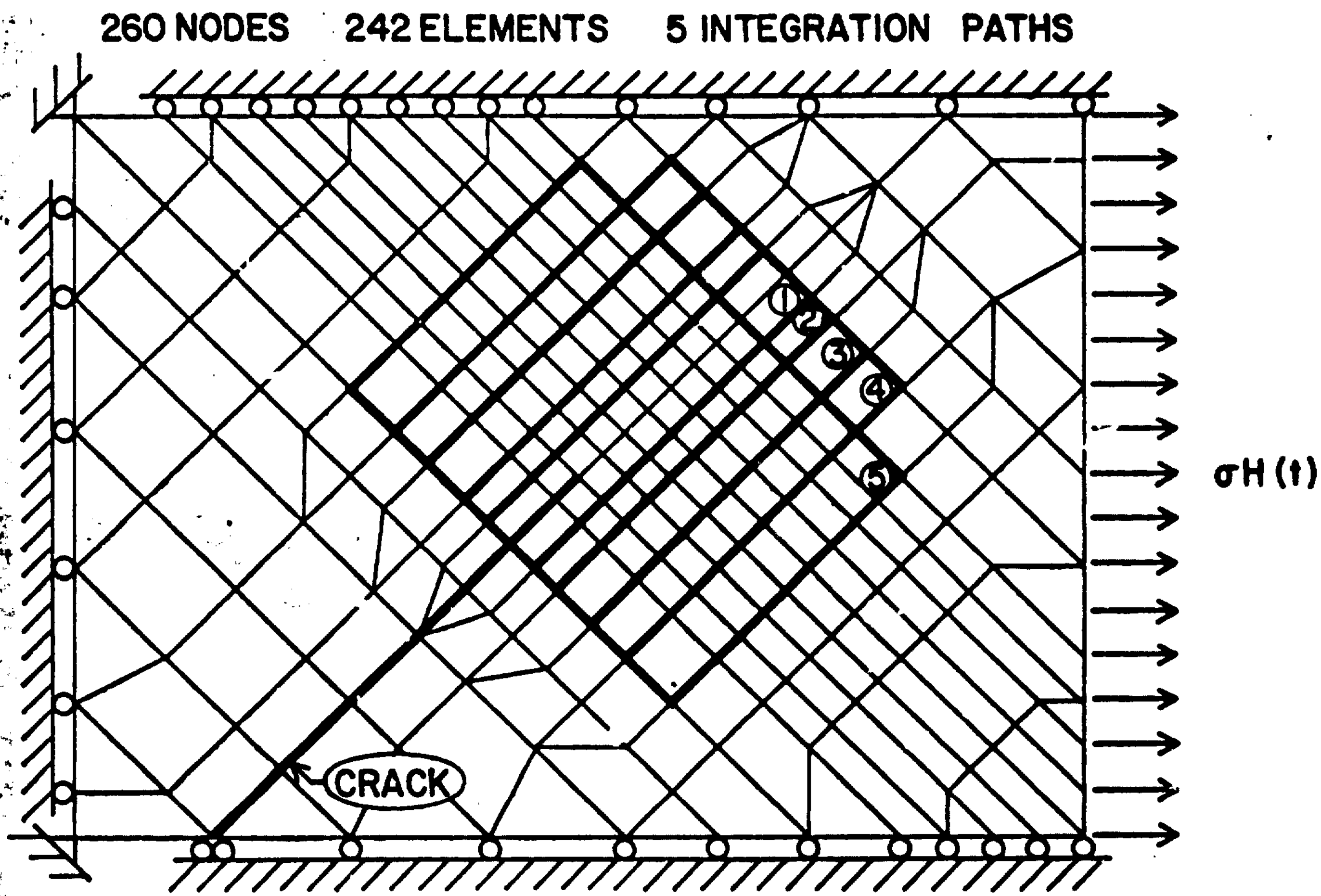

FIGURE 4. FINITE ELEMENT BREAKDOWN OF STATIONARY SLANTED CENTRAL CRACK.

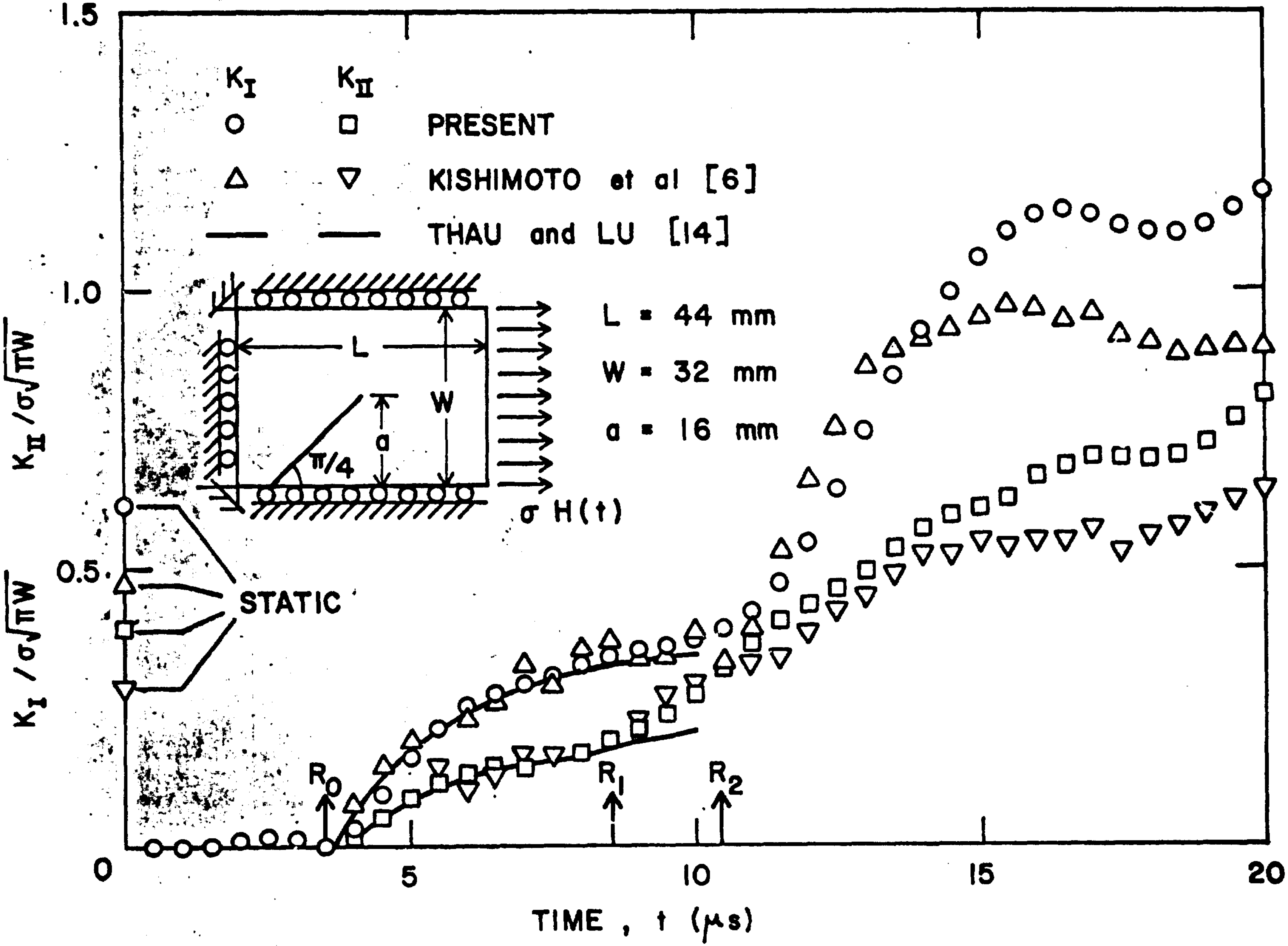

FIGURE 5. NORMALIZED DYNAMIC STRESS INTENSITY FACTORS OF STATIONARY SLANTED CENTRAL CRACK.

274 NODES 252 ELEMENTS 4 INTEGRATION PATHS

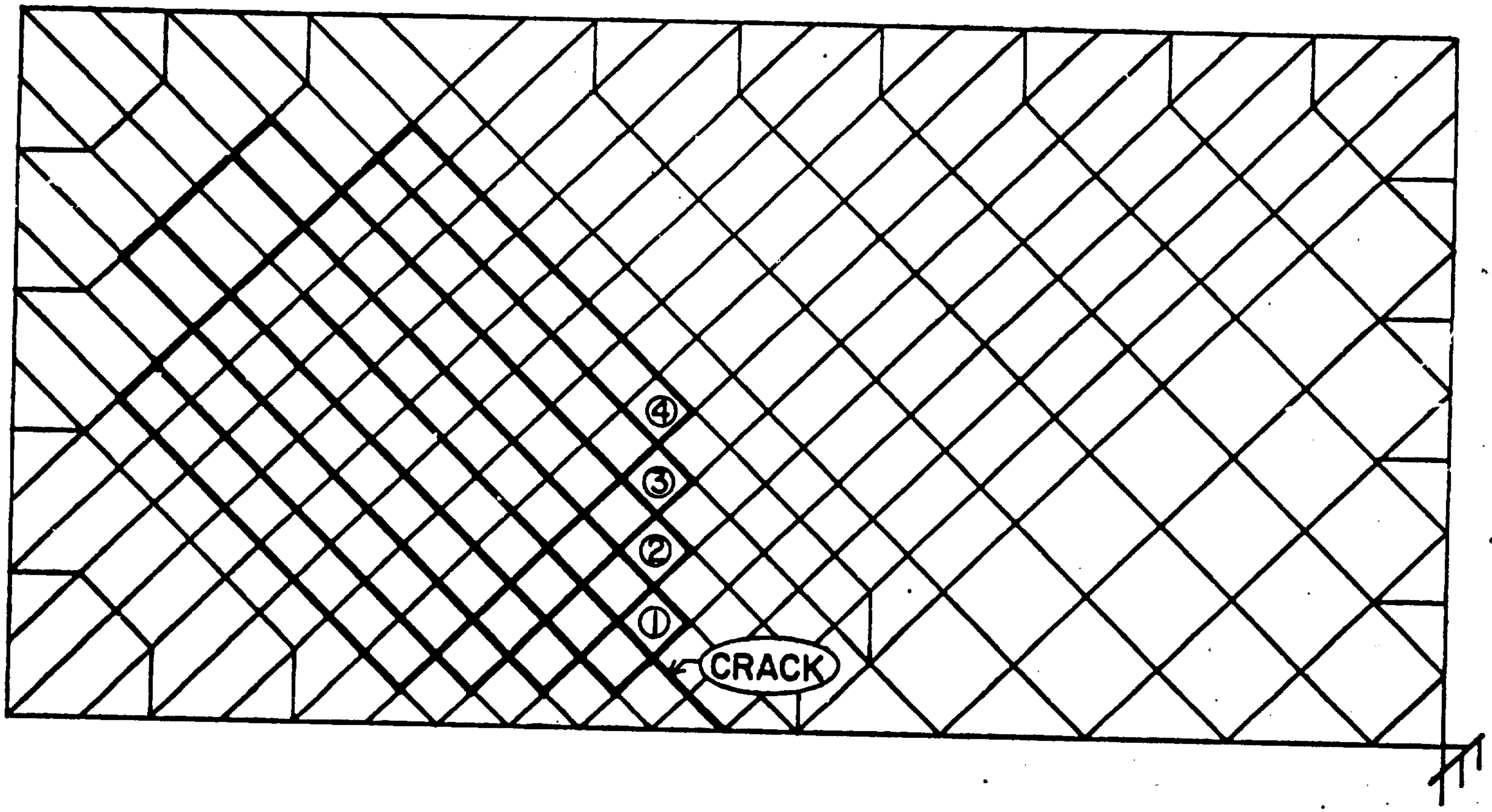

FIGURE 6. FINITE ELEMENT BREAKDOWN OF EXTENDING SLANTED SINGLE EDGE CRACK.

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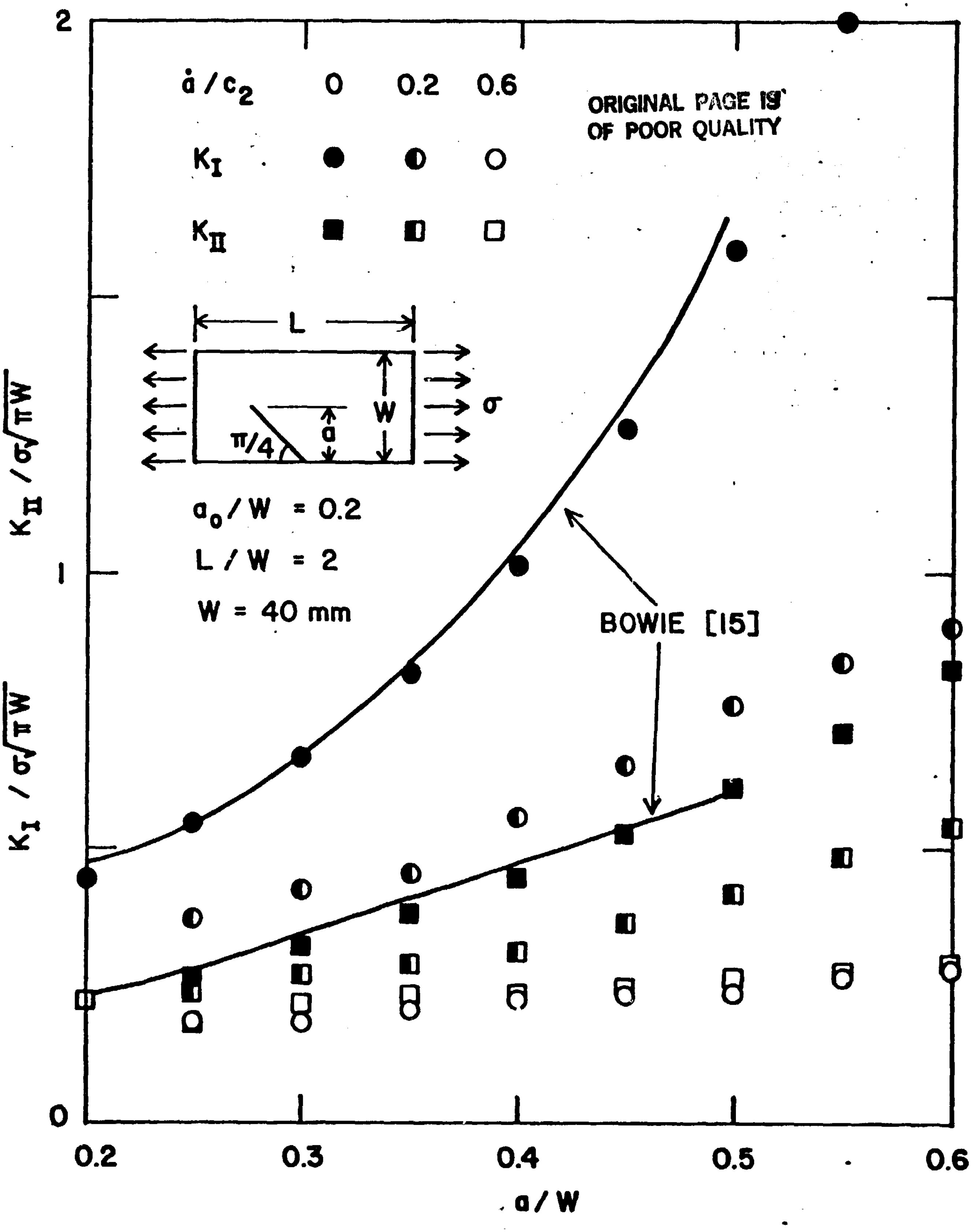

FIGURE 7. NORMALIZED STRESS INTENSITY FACTORS OF EXTENDING SLANTED SINGLE EDGE CRACK.