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Structural Dynamics Payload Loads Estimates

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STRUCTURAL DYNAMICS PAYLOAD LOADS ESTIMATES

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FOREWORD

This Methodology Assessment Report is submitted to the National Aeronautics and Space Administration's George C. Marshall Space Flight Center, Huntsville, Alabama, in response to the contract provisions of deliverable items associated with Structural Dynamics Payload Loads Estimates, Contract Number NAS8-33556.

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TABLE OF CONTENTS

INTRODUCTION

The United States currently utilizes a rather small family of launch vehicles (boosters) to support a varied spectrum of satellite and spacecraft programs [1]. These launch vehicles have been carefully designed to accommodate a wide range of payload configurations. In general, the payload interfaces with the launch vehicle at a limited subset of candidate structural "hard" points at the payload launch vehicle separation plane. The latest example in the series is the STS (Space Transportation System).

It is important that any candidate payload be designed to withstand the load environment transmitted to the payload from within the shielded payload compartment. Such environments commonly originate from a static (steady state) vehicle acceleration, a transient or dynamic event such as rocket motor ignition, or an acoustical environment. Very often, it is the transient dynamic response behavior of the payload that constitutes payload design load profiles; hence, it is important that proper attention be given to the payload transient response characteristics as influencing major design decisions. As an example, let us consider the landing of the orbiter (= a delta-wing-airplane-like module) carrying a certain payload. Obviously, when the orbiter touches the ground it will experience reaction forces. These forces will be transmitted to the payload through the interface (i.e. through the connection points between the orbiter and its payload). The payload then, will undergo elastic displacements. The question is, will the payload be able to withstand those displacements without being damaged? The answer to this question requires a dynamic analysis of the booster/payload system as we shall see in subsequent sections. This dynamic analysis will yield the elastic displacements in the payload due to the known reaction forces on the orbiter. These displacements can then be used to calculate the internal forces in the different members of the payload which in turn leads to the calculation of stresses and strains in those members. Finallv, these stresses and strains enable the payload designer to determine whether or not the members will be damaged during the landing event.

Present analytical techniques by which such design loads are predicted are very costly and time consuming. A typical load cycle (as the above mentioned process is called) generally requires:

1. Generation of a payload model;

2. Calculation of the modal characteristics of the payload restrained at the interface;

3. Formation of a transformation to couple the payload to the booster interface;

4. Coupling of the payload to the booster and calculation of the system modal characteristics;

5. Calculation of the time response of the system to the specified forces;

6. Use of the time response results to calculate loads.

The calendar turnaround time of a given cycle usually is lengthened when the payload design organization, the booster organization and the payload integration organization are different companies. The reason for this is that a fair amount of coordination is necessary to make the transfer of information between those three organizations optimal. Unfortunately, this coordination is very difficult to establish resulting in considerable time delays. Moreover, these costs and delays repeat themselves for every load cycle (i.e., every time a change is made in the booster or payload). A typical example is the development of the Viking Orbiter System [2]. Upward of nine organizations were responsible for hardware or integration functions which directly affected the evaluation of dynamic transient loads. The number of interfaces between those organizations resulted in difficulties in arranging for the necessary analyses at each organization, in obtaining the necessary data, in establishing priorities, in establishing output requirements, and in correctly transferring data between organizations. The time duration for one load cycle ranged from three to twelve months which depended on the number of events, forcing functions per event, and complexities of the analysis. Of course, if the booster already has its final design, many of these problems can be avoided. Theoretically, only one transfer of booster data to the payload organization would be necessary.

It is clear that the need exists for a so-called "short-cut" methodology in this area. A "short-cut" method should meet three essential requirements.

1. It should take advantage of the fact that the booster stays the same from one design cycle to the next or from one flight to the next. The payload integrator should be able to reuse several previously calculated booster quantities (e.g. mass and stiffness matrices, modes etc.)

2. The "short-cut" method should avoid as much as possible the transfer of information between different organizations involved in the load analysis. Ideally, the payload organization should be able to estimate design loads to support their design activities without having to rely on other organizations. A one-time transfer of booster information should suffice if the payload integration organization is the same as the payload organization [2].

3. A "short-cut" method should be cost-effective. For example, it seems reasonable that no complete cycle (i.e. item 1-6 on previous page) is necessary if only small changes are made in the payload. A similar situation exists in the assessment of STS payload design loads. Although many of these payloads will be designed for multiple flights with moderate changes, state of the art dynamic loads prediction technology does not provide a way to avoid complete reanalysis of the booster/payload system [1].

The object of this contract then is, to develop and implement such a "short-cut" methodology. The present Assessment Report covers Study Task I of the contract. Chapter I presents the standard techniques used to analyze a payload/booster system. They are "full-scale" methods in the sense that they all require the solution of the coupled equations of motion of the booster/payload system. Chapter II identifies several "short-cut" methodologies. These already existing techniques do not require the solution of the coupled system equations. The potentials and shortcomings of each of these methods is discussed. Chapter III covers the "favored" methods accompanied by recommendations for further development, refinement and demonstrations. We also included the outline of a new approach.

CHAPTER I: STANDARD TECHNIQUES

1. INTRODUCTION

The objective of this first chapter is to identify and assess the most prominent standard techniques currently available. This will allow the introduction of the necessary background information in terms of a unified nomenclature. It will also provide us with the state-of-the-art full-scale methodology. Such a method is necessary for comparison purposes. Also, some of the features of these methods may be incorported into some of the short-cut methods. This chapter will give us the opportunity to more clearly identify the requirements of an acceptable short-cut methodology. The first section deals with the equations of motion in the discrete time domain.

2. THE EQUATIONS OF MOTION IN THE DISCRETE TIME DOMAIN [3,4]

The objective of this section is the derivation of the equations of motion of the booster/payload system. Figure 1 shows the free body diagrams of the booster B and the payload P. The booster and the payload are connected to each other through the interface. Physically, the interface is the collection of structural "hard" points which the booster and the payload have in common. Mathematically, this means that the generalized displacement vector $\left\{ x_I^B \right\}$ on the booster side of the interface must be equal to its equivalent $\left\{ x_I^P \right\}$ on the payload side. Hence

$$\left\{ x_I^B \right\} = \left\{ x_I^P \right\} , \qquad \text{for } \underline{\text{all}} \text{ times } t \tag{1}$$

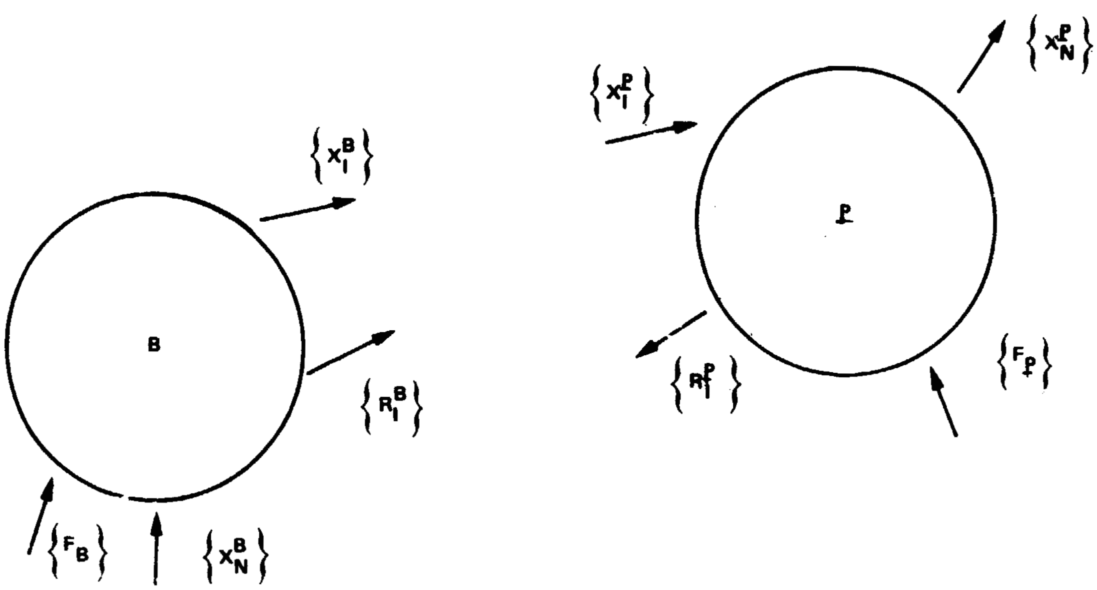

Figure 1 Free-Body Diagrams of Booster B and Pc 'load P

Similarly, the generalized reaction vectors $\left\{R_I^B\right\}$ and $\left\{R_I^P\right\}$ at the interface satisfy,

$$\left\{R_I^B\right\} = - \left\{R_I^P\right\} \quad , \quad \text{for } \underline{\text{all}} \text{ times } t \tag{2}$$

From the free body diagrams in Figure 1 we can easily write the equations of motion for the booster B and the payload P as,

$$\left[\begin{array}{c|c} M_B & \\ \hline & M_P \end{array}\right] \left\{\begin{array}{c} \ddot{x}_B \\ \hline \ddot{x}_P \end{array}\right\} + \left[\begin{array}{c|c} K_B & \\ \hline & K_P \end{array}\right] \left\{\begin{array}{c} x_B \\ \hline x_P \end{array}\right\} = \left\{\begin{array}{c} F_B \\ \hline F_P \end{array}\right\} + \left\{\begin{array}{c} 0 \\ R_I^B \\ \hline 0 \\ R_I^P \end{array}\right\} \tag{3}$$

where $\{x_B\}$ represents the generalized displacement vector of B. This vector can be partitioned according to non-interface displacements $\left\{x_N^B\right\}$ and interface displacements $\left\{x_I^B\right\}$,

$$\{x_B\} = \left\{\begin{array}{c} x_N^B \\ \hline x_I^B \end{array}\right\} \tag{4}$$

Similarly for P,

$$\{x_P\} = \left\{\begin{array}{c} x_N^P \\ \hline x_I^P \end{array}\right\} \tag{5}$$

The mass matrix $\left[M_B\right]$ and the stiffness matrix $\left[K_B\right]$ of the booster B can also be partitioned in the same manner,

$$\left[M_B\right] = \left[\begin{array}{c|c} M_{NN}^B & M_{NI}^B \\ \hline M_{IN}^B & M_{II}^B \end{array}\right], \quad \left[K_B\right] = \left[\begin{array}{c|c} K_{NN}^B & K_{NI}^B \\ \hline K_{IN}^B & K_{II}^B \end{array}\right] \tag{6}$$

Similarly for P,

$$\left[M_P\right] = \left[\begin{array}{c|c} M_{NN}^P & M_{NI}^P \\ \hline M_{IN}^P & M_{II}^P \end{array}\right], \quad \left[K_P\right] = \left[\begin{array}{c|c} K_{NN}^P & K_{NI}^P \\ \hline K_{IN}^P & K_{II}^P \end{array}\right] \tag{7}$$

Finally, the vector $\left\{F_B\right\}$ represents the externally applied forces on the booster B and can be written as

$$\left\{F_B\right\} = \left\{\begin{array}{c} F_N^B \\ \hline F_I^B \end{array}\right\} \tag{8}$$

Similarly for P,

$$\left\{F_P\right\} = \left\{\begin{array}{c} F_N^P \\ \hline F_I^P \end{array}\right\} \tag{9}$$

Using Eqs. (4), (5), (8) and (9) we can rewrite Eq. (3) as follows,

$$\left[\begin{array}{c|c} M_B & \\ \hline & M_P \end{array}\right] \left\{\begin{array}{c} \ddot{x}_N^B \\ \ddot{x}_I^B \\ \hline \ddot{x}_N^P \\ \ddot{x}_I^P \end{array}\right\} + \left[\begin{array}{c|c} K_B & \\ \hline & K_P \end{array}\right] \left\{\begin{array}{c} x_N^B \\ x_I^B \\ \hline x_N^P \\ x_I^P \end{array}\right\} = \left\{\begin{array}{c} F_N^B \\ F_I^B + R_I^B \\ \hline F_N^P \\ F_I^P + R_I^P \end{array}\right\} \tag{10}$$

Both equations (3) and (10) represent the equations of motion of the undamped booster B and the undamped payload P. In order to derive the equations of motion for the coupled system (i.e. booster/payload system), we need to eliminate the a priori unknown reactions $\left\{R_I^B\right\}$ and $\left\{R_I^P\right\}$. We shall now establish a convenient and physically meaningful way to accomplish this elimination. To this end, let us solve the third partition of Eq. (10) for the non-interface displacement vector $\left\{x_N^P\right\}$ of the payload P.

$$\left\{x_N^P\right\} = -\left[K_{NN}^P\right]^{-1}\left[K_{NI}^P\right]\left\{x_I^P\right\} + \left[K_{NN}^P\right]^{-1}\left(\left\{F_N^P\right\} - \left[M_{NN}^P\right]\left\{\ddot{x}_N^P\right\} - \left[M_{NI}^P\right]\left\{\ddot{x}_I^P\right\}\right) \tag{11}$$

It is now noted that the non-interface displacement vector $\left\{x_N^P\right\}$ consists of two parts. To understand the physical meaning of these two terms let us assume that the interface displacements are zero i.e. $\left\{x_I^P\right\} = \left\{0\right\}$. In that case it follows from Eq. (11) that the second term on the right-hand side can be interpreted as the non-interface displacement of the payload with respect to the interface. Let us denote this term by $\left\{\bar{x}_N^r\right\}$. It is then clear that the first term on the right-hand side of Eq. (11) represents the non-interface

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displacement of the payload due to the displacement $\left\{x_I^P\right\}$ of the interface only. Therefore, Eq. (11) can be written as

$$\left\{x_N^P\right\} = \left[S_P\right]\left\{x_I^P\right\} + \left\{\bar{x}_N^P\right\} \tag{12}$$

with

$$\left[S_P\right] = -\left[K_{NN}^P\right]^{-1}\left[K_{NI}^P\right] \tag{13}$$

Equations (12-13) are now used to establish the following transformation,

$$\left\{\begin{array}{c} x_B \\ \hline x_P \end{array}\right\} = \left\{\begin{array}{c} x_N^B \\ x_I^B \\ \hline x_N^P \\ x_I^P \end{array}\right\} = \left[\begin{array}{cc|c} I & 0 & 0 \\ 0 & I & 0 \\ \hline 0 & S_P & I \\ 0 & I & 0 \end{array}\right] \left\{\begin{array}{c} x_N^B \\ x_I^B \\ \hline \bar{x}_N^P \end{array}\right\} \tag{14}$$

where we used Eqs. (4), (5), (1), (12) and (13). This transformation will eliminate the redundant set of displacements $\left\{x_I^P\right\}$ in Eq. (10) and in the process it will also eliminate the unknown reactions $\left\{R_I^B\right\}$ and $\left\{R_I^P\right\}$. First, let us introduce a more convenient notation,

$$\left[\begin{array}{cc|c} I & 0 & 0 \\ 0 & I & 0 \\ \hline 0 & S_P & I \\ 0 & I & 0 \end{array}\right] = \left[\begin{array}{cc|c} I & 0 & 0 \\ 0 & I & 0 \\ \hline \multicolumn{2}{c|}{T_P} & I_P \end{array}\right] = A \tag{15}$$

with

$$\left[T_P\right] = \begin{bmatrix} 0 & S_P \\ 0 & I \end{bmatrix}, \quad \left[I_P\right] = \begin{bmatrix} I \\ 0 \end{bmatrix} \tag{16}$$

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With this notation we now substitute the transformation Eq. (14) into Eq. (10) and premultiply by A^T (T = transpose). This yields the following result,

$$\left[\begin{array}{c|c} M_B + T_P^T M_P T_P & T_P^T M_P I_P \\ \hline I_P^T M_P T_P & I_P^T M_P I_P \end{array}\right] \begin{array}{c} \ddot{x}_N^B \\ \left\{\begin{array}{c} \ddot{x}_I^B \\ \hline \ddot{x}_N^P \end{array}\right\} \end{array} + \left[\begin{array}{c|c} K_B + T_P^T K_P T_P & T_P^T K_P I_P \\ \hline I_P^T K_P T_P & I_P^T K_P I_P \end{array}\right] \begin{array}{c} x_N^B \\ \left\{\begin{array}{c} x_I^B \\ \hline x_N^P \end{array}\right\} \end{array} = \begin{array}{c} F_N^B \\ \left\{\begin{array}{c} F_I^B + F_I^P + S_P^T F_N^P \\ \hline F_N^P \end{array}\right\} \end{array} \tag{17}$$

At this point a few remarks are in order. First, we shall show that the matrix product $\left[T_P\right]^T \left[K_P\right]\left[I_P\right]$ is zero. From Eqs. (7) and (16) we have

$$\left[T_P\right]^T \left[K_P\right] \left[I_P\right] = \begin{bmatrix} 0 & 0 \\ S_P^T & I \end{bmatrix} \begin{bmatrix} K_{NN}^P & K_{NI}^P \\ K_{IN}^P & K_{II}^P \end{bmatrix} \begin{bmatrix} I \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ S_P^T K_{NN}^P + K_{IN}^P \end{bmatrix} \tag{18}$$

Substituting the expression (13) for $\left[S_P\right]$ in the lower half of Eq. (18) yields,

$$\left[S_P\right]^T \left[K_{NN}^P\right] + \left[K_{IN}^P\right] = -\left[K_{NI}^P\right]^T \left[K_{NN}^P\right]^{-1} \left[K_{NN}^P\right] + \left[K_{IN}^P\right] = \left[0\right] \tag{19}$$

because $\left[K_{IN}^P\right] = \left[K_{NI}^P\right]^T$ $\left(\left[K_P\right] \text{ is symmetric}\right)$

Secondly, the triple matrix product

$$\left[T_P\right]^T \left[K_P\right] \left[T_P\right] = \begin{bmatrix} 0 & 0 \\ S_P^T & I \end{bmatrix} \begin{bmatrix} K_{NN}^P & K_{NI}^P \\ K_{IN}^P & K_{II}^P \end{bmatrix} \begin{bmatrix} 0 & S_P \\ 0 & I \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & K_{IN}^P S_P + K_{II}^P \end{bmatrix} \tag{20}$$

will be zero for a statically determinate interface. The interface is called statically determinate when the number of interface degrees of freedom is equal to the number of rigid body degrees of freedom of the structure at hand. Otherwise, the interface is called statically indeterminate. To show that $\left[K_{IN}\right]\left[S_P\right] + \left[K_{II}\right]$ in Eq. (20) is zero for a statically determinate interface, let us first state that the numerical values of the elements of this matrix are independent of the dynamical state of the structure. More specifically, because the stiffness matrix $\left[K_P\right]$ is the same for P at rest or in motion, we can assume that the structure is in a state of static equilibrium without changing $\left[K_{IN}\right]\left[S_P\right] + \left[K_{II}\right]$. Therefore, without loss of generality we consider the equilibrium equation of P under the action of $\left\{F_N^B\right\}$ with $\left\{F_I^B\right\} = \left\{F_I^P\right\} = \left\{F_N^P\right\} = \left\{0\right\}$,

$$\left[K_P\right]\left\{x_P\right\} = \left\{\begin{matrix} 0 \\ \\ R_I^P \end{matrix}\right\} \tag{21}$$

or, using the partitioned form of $\left[K_P\right]$ Eq. (7), we can write

$$\left[K_{NN}^P\right]\left\{x_N^P\right\} + \left[K_{NI}^P\right]\left\{x_I^P\right\} = \left\{0\right\} \tag{22}$$

$$\left[K_{IN}^P\right]\left\{x_N^P\right\} + \left[K_{II}^P\right]\left\{x_I^P\right\} = \left\{R_I^P\right\} \tag{23}$$

From Eq. (22) we can solve for $\left\{x_N^P\right\}$

$$\left\{x_N^P\right\} = -\left[K_{NN}^P\right]^{-1}\left[K_{NI}^P\right]\left\{x_I^P\right\} = \left[S_P\right]\left\{x_I^P\right\} \tag{24}$$

where we used Eq. (13). Substituting Eq. (24) into Eq. (23) yields,

$$\left(\left[K_{IN}^P\right]\left[S_P\right] + \left[K_{II}^P\right]\right)\left\{x_I^P\right\} = \left\{R_I^P\right\} \tag{25}$$

At this point we should note that when the interface is statically determinate no stresses can be set up in P by the interface displacements $\left\{x_I^P\right\}$. Indeed, for a statically determinate interface the matrix $\left[S_P\right]$ becomes a rigid body transformation, transforming the interface displacements into equivalent rigid body displacemnts of the non-interface degrees of freedom of P. Because, in addition we assumed that no other forces are acting on P, it is clear that $\left\{R_I^P\right\}$ is zero in Eq. (25), from which it follows that

$$\left[K_{IN}^P\right]\left[S_P\right] + \left[K_{II}^P\right] = 0 \tag{26}$$

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This completes the proof.

Finally, we note that in most applications of interest to this contract, the externally applied forces $\left\{F_N^P\right\}$, $\left\{F_I^P\right\}$, and $\left\{F_I^B\right\}$ are nonexistant. For example, STS payloads will be enclosed in the cargo bay and will not be exposed to external forces. Therefore, we write

$$\left\{F_N^P\right\} = \left\{F_I^B\right\} = \left\{F_I^P\right\} = \left\{0\right\} \tag{27}$$

Taking into account Eqs. (19) and (27) we can now write the final form of Eq. (17).

$$\left[\begin{array}{c|c} M_B + T_P^T M_P T_P & T_P^T M_P I_P \\ \hline I_P^T M_P T_P & I_P^T M_P I_P \end{array}\right] \left\{\begin{array}{c} \ddot{x}_N^B \\ \ddot{x}_I^B \\ \hline \ddot{\bar{x}}_N^P \end{array}\right\} + \left[\begin{array}{c|c} K_B + T_P^T K_P T_P & 0 \\ \hline 0 & I_F^T K_P I_P \end{array}\right] \left\{\begin{array}{c} x_N^B \\ x_I^B \\ \hline \bar{x}_N^P \end{array}\right\} = \left\{\begin{array}{c} F_N^B \\ 0 \\ \hline 0 \end{array}\right\} \tag{28}$$

in which

$$\left\{x_B\right\} = \left\{\begin{array}{c} x_N^B \\ x_I^B \end{array}\right\} \tag{4}$$

represents the generalized displacement vector of the free booster B. The vector $\left\{x_N^B\right\}$ contains all non-interface displacements of the booster $\left\{x_I^B\right\}$ represents the interface degrees of freedom. Furthermore, the vector $\left\{\bar{x}_N^P\right\}$ represents the non-interface displacements of the payload P with respect to the interface. The matrices $\left[M_B\right]$ and $\left[M_P\right]$ represent the mass matrices of the booster and the payload respectively and $\left[K_B\right]$ and $\left[K_P\right]$ represent the stiffness matrices. The matrix $\left[T_P\right]$ is a transformation matrix, characteristic for the payload P. In case of a statically determinate interface, $\left[T_P\right]$ represents a rigid body transformation. Finally, $\left\{F_N^B\right\}$ is the externally applied force vector acting on the booster B. The matrices $\left[T_P\right]^T \left[M_P\right] \left[T_P\right]$ and $\left[T_P\right]^T \left[K_P\right] \left[T_P\right]$ contribute only to the interface degrees of freedom as can be seen from Eq. (20) and from

$$\left[T_P\right]^T \left[M_P\right] \left[T_P\right] = \begin{bmatrix} 0 & 0 \\ S_P^T & I \end{bmatrix} \begin{bmatrix} M_{NN}^P & M_{NT}^P \\ M_{IN}^P & M_{IT}^P \end{bmatrix} \begin{bmatrix} 0 & S_P \\ 0 & I \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & M_2^P \end{bmatrix} \tag{29}$$

with

$$\left[M_2^P\right] = \left[S_P\right]^T \left(\left[M_{NN}^P\right]\left[S_P\right] + \left[M_{NI}^P\right]\right) + \left[M_{IN}^P\right]\left[S_P\right] + \left[M_{II}^P\right].$$

The matrix $\left[T_P\right]^T \left[M_P\right] \left[T_P\right]$ essentially represents the payload mass transferred to the interface. Similarly, $\left[T_P\right]^T \left[K_P\right] \left[T_P\right]$ represents the payload stiffness transferred to the interface. Note that when the interface is statically determinate no stiffness is transferred (Eq. (26)). When the interface is statically indeterminate there are what is commonly called "constraint modes" [3], i.e. the interface displacements $\left\{x_I^B\right\}$ not only induce rigid body displacements in the payload but also strains. These strains cause the triple product $\left[T_P\right]^T \left[K_P\right] \left[T_P\right]$ to be different from zero.

This concludes this section on the equations of motion of the booster/payload system. This material will be used to derive several methods of solution of the equations of motion (28).

3. SOLUTION OF THE EQUATIONS OF MOTION IN THE DISCRETE TIME DOMAIN

As stated in the Introduction, the objective of this study is to determine design loads for the payload structure. These design loads are then used to calculate stresses and strains that would exist in the structural elements that make up the payload. The stresses and strains allow the designer to determine the correct physical and geometric properties of the elements (mass, stiffness, lengths, cross sections, etc.) so that the structure does not fail when subjected to the external forces $\left[F_N^B\right]$. An element loads equation is written as

$$\left\{F_e^P\right\} = \left[k_e\right]\left[T_e\right]\left\{x_P\right\} \tag{31}$$

in which $\left\{F_e^P\right\}$ is the load vector of an individual element e of the payload P, $\left[k_e\right]$ is the stiffness matrix of the element, and $\left[T_e\right]$ is the geometric compatibility transformation. The vector $\left\{x_P\right\}$ is the time dependent displacement vector of the payload satisfying Eq. (28). Consequently, in order to determine $\left\{F_e^P\right\}$ in Eq. (31) we need to solve Eq. (28).

The most straightforward approach to determine $\left\{x_P\right\}$ is to solve Eq. (28) as a set of simultaneous second order differential equations. There are several well established response routines that handle such problems (Runge-Kutta, Newmark-Chan-Beta Numerical Integration, etc.). This direct approach has the advantage of simplicity and accuracy. The obvious drawback is the high computational cost due to the large number of degrees of freedom used to describe today's aerospace models. Furthermore, this method does not take advantage of the fact that often only small changes are made in the payload. However, this method is still useful in the context of this study because it provides us with reliable results that can be used for comparison purposes with other methods to be discussed shortly.

4. SOLUTION OF THE EQUATIONS OF MOTION BY MODAL ANALYSIS [6,7]

In this section we shall discuss a technique commonly known as modal analysis. This approach will lead us to an alternate solution method for Eq. (28) and we shall show that it has some definite advantages over the direct solution of the set of differential equations (28) as discussed in Section 3.

We start the process with the homogeneous set of equations extracted from the top row of Eq. (3),

$$\left[M_B\right]\left\{\ddot{x}_B\right\} + \left[K_B\right]\left\{x_B\right\} = \left\{0\right\} \tag{31}$$

Associated with this equation is an eigenvalue problem,

$$\left(-\omega_B^2\left[M_B\right]+\left[K_B\right]\right)\left\{\phi_B\right\}=\left\{0\right\} \qquad (32)$$

where the vector $\left\{\phi_B\right\}$ represents an eigenvector (mode shape) and ω_B an eigenvalue (natural frequency). The solution of this eigenvalue problem essentially produces a linear transformation marix $\left[\phi_B\right]$ (modal transformation matrix) in which each column represents a mode shape of the booster B. The main property of this modal transformation is that in the new normal coordinate system $\left\{q_B\right\}$, the equations of motion (31) become uncoupled, i.e. if we apply the transformation,

$$\left\{x_B\right\}=\left[\phi_B\right]\left\{q_B\right\} \qquad (33)$$

to Eq. (31), and premultiply by $\left[\phi_B\right]^T$ we obtain

$$\left[\phi_B\right]^T\left[M_B\right]\left[\phi_B\right]\left\{\ddot{q}_B\right\}+\left[\phi_B\right]^T\left[K_B\right]\left[\phi_B\right]\left\{q_B\right\}=\left\{0\right\} \qquad (34)$$

where the coefficient matrices of $\left\{\ddot{q}_B\right\}$ and $\left\{q_B\right\}$ are now diagonal,

$$\left[\phi_B\right]^T\left[M_B\right]\left[\phi_B\right]=\left[I\right],\ \left[\phi_B\right]^T\left[K_B\right]\left[\phi_B\right]=\left[\omega_B^2\right] \qquad (35)$$

where $\left[\phi_B\right]$ was normalized with respect to $\left[M_B\right]$. Equation (34) can then be written as

$$\left\{\ddot{q}_B\right\}+\left[\omega_B^2\right]\left\{q_B\right\}=\left\{0\right\} \qquad (36)$$

The obvious advantage of applying the modal transformation Eq. (33) is that Eq. (36) now represents a set of decoupled independent second order differential equations that are easily solved. The price to pay however, is the solution of eigenvalue problem (32). There are many well established eigenvalue problem "solvers" available (Jacobi, Rayleigh-Ritz, etc.) [5].

The next step is to consider the homogenous equation,

$$\left[I_P\right]^T\left[M_P\right]\left[I_P\right]\left\{\ddot{x}_N^P\right\}+\left[I_P\right]^T\left[K_P\right]\left[I_P\right]\left\{x_N^P\right\}=\left\{0\right\} \qquad (37)$$

Note that,

$$\left[I_P\right]^T\left[M_P\right]\left[I_P\right]=\left[M_{NN}^P\right],\ \left[I_P\right]^T\left[K_P\right]\left[I_P\right]=\left[K_{NN}\right] \qquad (38)$$

So that Eq. (37) becomes

$$\left[M_{NN}^{P}\right]\left\{\ddot{\bar{x}}_{N}^{P}\right\} + \left[K_{NN}^{P}\right]\left\{\bar{x}_{N}^{P}\right\} = \left\{0\right\} \tag{39}$$

In the same way as we did for Eq. (31) we can introduce a modal transformation,

$$\left\{\bar{x}_{N}^{P}\right\} = \left[\phi_{N}^{P}\right]\left\{\bar{q}_{N}^{P}\right\} \tag{40}$$

with

$$\left[\bar{\phi}_{N}^{P}\right]^{T}\left[M_{NN}^{P}\right]\left[\bar{\phi}_{N}^{P}\right] = \left\lceil I \right\rfloor, \quad \left[\bar{\phi}_{N}^{P}\right]^{T}\left[K_{NN}^{P}\right]\left[\bar{\phi}_{N}^{P}\right] = \left\lceil \bar{\omega}_{P}^{2} \right\rfloor \tag{41}$$

where we wrote $\left\lceil \bar{\omega}_{P}^{2} \right\rfloor$ instead of $\left\lceil \bar{\omega}_{N}^{P^2} \right\rfloor$ to simplify the notation. The modal matrix $\left[\bar{\phi}_{N}^{P}\right]$ has as columns the "cantilevered" mode shapes of the payload P, and $\left\lceil \omega_{P}^{2} \right\rfloor$ has the natural frequencies squared of the cantilevered payload (i.e. fixed interface) on its diagonal. Using Eqs. (40-41), we can write Eq. (39) as

$$\left\{\ddot{\bar{q}}_{N}^{P}\right\} + \left\lceil \omega_{P}^{2} \right\rfloor\left\{\bar{q}_{N}^{P}\right\} = \left\{0\right\} \tag{42}$$

Let us now apply the following transformation to Eq. (28),

$$\left\{\begin{array}{c} x_{N}^{B} \\ x_{I}^{B} \\ \hline \bar{x}_{N}^{P} \end{array}\right\} = \left\{\begin{array}{c} x_{B} \\ \hline \bar{x}_{N}^{P} \end{array}\right\} = \left[\begin{array}{c|c} \phi_{B} & \\ \hline & \bar{\phi}_{N}^{P} \end{array}\right]\left[\begin{array}{c} q_{B} \\ \hline \bar{q}_{N}^{P} \end{array}\right] \tag{43}$$

and premultiply by $\left[\begin{array}{c|c} \phi_{B} & \\ \hline & \bar{\phi}_{N}^{P} \end{array}\right]^{T}$ Taking into account Eqs. (35) and (41) we obtain,

$$\left[\begin{array}{c|c} I + \phi_{B}^{T}T_{P}^{T}M_{P}T_{P}\phi_{B} & \phi_{B}^{T}T_{P}^{T}M_{P}I_{P}\bar{\phi}_{N}^{P} \\ \hline \bar{\phi}_{N}^{P^{T}}I_{P}^{T}M_{P}T_{P}\phi_{B} & I \end{array}\right]\left\{\begin{array}{c} \ddot{q}_{B} \\ \hline \ddot{\bar{q}}_{N}^{P} \end{array}\right\} + \left[\begin{array}{c|c} \left\lceil \omega_{B}^{2} \right\rfloor + \phi_{B}^{T}T_{P}^{T}K_{P}T_{P}\phi_{B} & 0 \\ \hline 0 & \left\lceil \bar{\omega}_{P}^{2} \right\rfloor \end{array}\right]\left\{\begin{array}{c} q_{B} \\ \hline \bar{q}_{N}^{P} \end{array}\right\}$$

$$= \left\{\begin{array}{c} \phi_{B}^{T}\left\{\begin{array}{c} F_{N}^{B} \\ 0 \end{array}\right\} \\ \hline 0 \end{array}\right\} \tag{44}$$

Note that $\{q_B\}$ can be partitioned as follows

$$\{q_B\} = \left\{\begin{array}{c} q_N^B \\ \hline q_I^B \end{array}\right\} \tag{45}$$

Equation (44) as it is, probably does not yield any advantage over a direct solution of Eq. (28). However, in most practical applications there is a possibility of defining a so-called "cut-off frequency". In these cases a Fourier series expansion of the force vector $\{F_N^B\}$ shows that the energy content of the high frequency components is small compared to the energy contained in the low frequency components. Practically, this means that the response of the structure due to the high frequency content of $\{F_N^B\}$ can often be neglected. In this connection it should be noted that it is relatively difficult to excite the higher modes of the structure to any large extent, especially when $\{F_N^B\}$ only contains a few elements (i.e. only a few application points). The idea then is to only retain these modes in Eq. (44) that have a frequency smaller than the cut-off frequency. This in turn, reduces the size of Eq. (44) considerably. Experience has shown that the introduction of a cut-off frequency is a workable concept.

In conclusion, we can say that the introduction of a cut-off frequency leads to a reduction of the size of Eq. (44). Nevertheless, the solution of Eq. (44) for the modal displacement vector is still costly and again does not accommodate the special circumstances of small changes in the payload. A more serious problem however, is the representation of the interface in a model where a cut-off frequency is used. Indeed, in many cases a significant portion of the response is lost in the high frequency range due to the fact that the effect of the payload on the booster has a significant high frequency content. As will be shown in next section a need arises to improve the interface representation in the model for the booster. In sections 5 and 6 we shall discuss two methods that accomplish this.

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5. THE RESIDUAL MASS AND STIFFNESS METHOD [8, 9, 10, 11-50, 51]

As explained in Section 4. in many cases it is possible to define a cut-off frequency which enables us to truncate the high modes in Eq. (44) thereby reducing the size of this equation. Obviously, some accuracy in the response of the structure is lost due to the truncation of these high modes. This loss of accuracy is especially apparent at the interface as we shall explain shortly. The residual mass and stiffness method, instead of omitting these high modes will replace them with a set of "residual modes". The computation of these residual modes does not require any knowledge of the payload so that they represent a one-time computation effort not to be repeated as long as the booster stays the same. In order to determine the residual modes let us consider Eq. (33)

$$\left\{ x_B \right\} = \left[\phi_B \right] \left\{ q_B \right\} \tag{46}$$

which represents the modal transformation for the booster B. Assuming a cut-off frequency was determined we can partition Eq. (46) as follows

$$\left\{ x_B \right\} = \left[\phi_B^L \,\vdots\, \phi_B^H \right] \left\{ \begin{array}{c} q_B^L \\ \hline q_B^H \end{array} \right\} \tag{47}$$

where $\left[\phi_B^L \right]$ represents the modes with frequencies less than the cut-off frequency and $\left[\phi_B^H \right]$ those with higher frequencies. At this point one could neglect $\left[\phi_B^H \right]$ and calculate the response as a linear combination of the lower modes $\left[\phi_B^L \right]$ only. Usually this yields a poor accuracy in the response and the loads. The reason is that in most practical cases a significant part of the interface response is produced by the higher modes. Indeed, a typical interface is rather stiff and has little mass, i.e. that locally the interface has a high frequency content so that it responds significantly in the high frequency range. In truncating the higher modes the model does not include an adequate representation of that interface. The residual mass and stiffness method now, proposes to retain the static contribution to the response of those high modes. This leads to a much better representation of the interface. The static contribution can be obtained from the following static equation

$$\left[K_B\right]\left\{x_B\right\} = \left\{\begin{array}{c} F_N^B \\ R_I^B \end{array}\right\} \tag{48}$$

derived from Eq. (3). Substituting Eq. (46) into Eq. (48) premultiplying by $\left[\phi_B\right]^T$ and recalling Eq. (35), yields

$$\left[{}^{\nwarrow}\omega_B^{H^2}{}_{\searrow}\right]\left\{q_B^H\right\} = \left[\phi_B^H\right]^T \left\{\begin{array}{c} F_N^B \\ R_I^B \end{array}\right\} \tag{49}$$

Because we are only interested in the high frequency part, let us write Eq. (49) as

$$\left[\begin{array}{c|c} \left[{}^{\nwarrow}\omega_B^{L^2}{}_{\searrow}\right] & \\ \hline 0 & \left[{}^{\nwarrow}\omega_B^{H^2}{}_{\searrow}\right] \end{array}\right]\left\{\begin{array}{c} q_B^L \\ \hline q_B^H \end{array}\right\} = \left[\begin{array}{c} \phi_B^{L^T} \\ \hline \phi_B^{H^T} \end{array}\right]\left\{\begin{array}{c} F_N^B \\ R_I^B \end{array}\right\} \tag{50}$$

So that from the bottom row in Eq. (50) we have

$$\left[{}^{\nwarrow}\omega_B^{H^2}{}_{\searrow}\right]\left\{q_B^H\right\} = \left[\phi_B^H\right]^T \left\{\begin{array}{c} F_N^B \\ R_I^B \end{array}\right\} \tag{51}$$

Finally, let us partition $\left[\phi_B^H\right]$ in non-interface and interface partitions,

$$\left[\phi_B^H\right] = \left[\begin{array}{c} \phi_{BN}^H \\ \hline \phi_{BI}^H \end{array}\right] \tag{52}$$

Substituting Eq. (52) into Eq. (51) we obtain,

$$\left[{}^{\nwarrow}\omega_B^{H^2}{}_{\searrow}\right]\left\{q_B^H\right\} = \left[\phi_{BN}^H\right]^T\left\{F_N^B\right\} + \left[\phi_{BI}^H\right]^T\left\{R_I^B\right\} \tag{53}$$

In principal we can use Eq. (51) as it is and solve for $\left\{q_B^H\right\}$,

$$\left\{q_B^H\right\} = \left[{}^{\nwarrow}\omega_B^{H^{-2}}{}_{\searrow}\right]\left[\phi_B^H\right]^T \left\{\begin{array}{c} F_N^B \\ R_I^B \end{array}\right\} \tag{54}$$

which can be substituted in Eq. (47), yielding

$$\left\{x_B\right\} = \left[\phi_B^L \;\vdots\; \left[\phi_B^H\right]\left[\omega_B^{H^{-2}}\right]\left[\phi_B^H\right]^T\right]\left\{\begin{array}{c} q_B^L \\ \hline F_N^B \\ R_I^B \end{array}\right\} \tag{55}$$

However, it should be noted that for every force component we keep, we add a degree-of-freedom to the problem. If for example, $\left\{F_N^B\right\}$ contains many elements (i.e. many points of application) it may not pay off to use Eq.(55), i.e. we may as well keep all the modes in Eq. (46). If however $\left\{F_N^B\right\}$ contains a small number of elements (for example, in case of a landing or a rocket motor ignition) we can use Eq. (55) as it is, and obtain a much better response for few added degrees of freedom. However, because the cut-off frequency was defined in such a way that all significant frequencies of $\left\{F_N^B\right\}$ are contained in the lower frequency range L, we can state that the booster model will adequately respond to $\left\{F_N^B\right\}$ and no significant portion of the response will be lost. Therefore, we can omit the term in $\left\{F_N^B\right\}$ in Eq. (53) altogether and just keep the interface part in $\left\{R_I^B\right\}$. The latter part in $\left\{R_I^B\right\}$ is important because $\left\{R_I^B\right\}$ will usually have a significant high frequency content (after all $\left\{R_I^B\right\}$ represents the effect of the payload on the booster and as such contains a wide range of frequencies). Because the interface usually has a high frequency content (as explained before) $\left\{R_I^B\right\}$ will induce a response at the interface primarily in the high frequency range which in turn will be, transmitted to the rest of the booster.

On the other hand if $\left\{F_N^B\right\}$ contains reaction elements due to some external contstraints (e.g. a dock) we wish to retain these elements as well because they are equivalent to elements of $\left\{R_I^B\right\}$ in the sense that they represent the unknown effects of the constraints and also, the interface between the constraint and the booster usually has a high frequency content (e.g. connections between booster and dock).

Ignoring the term in $\left\{F_N^B\right\}$ in Eq. (53) and solving for $\left\{q_B^H\right\}$ yields

$$\left\{q_B^H\right\} = \left[\diagdown \omega_B^{H^{-2}} \diagdown\right] \left[\phi_{BI}^H\right]^T \left\{R_I^B\right\} \tag{56}$$

which can be substituted into Eq. (47), yielding

$$\left\{x_B\right\} = \left[\phi_B^L \;\middle|\; \left[\phi_B^H\right]\left[\diagdown \omega_B^{H^{-2}} \diagdown\right]\left[\phi_{BI}^H\right]^T\right]\left\{\begin{array}{c} q_B^L \\ \hline R_I^B \end{array}\right\} \tag{57}$$

The term $\left[\phi_B^H\right]\left[\diagdown \omega_B^{H^{-2}} \diagdown\right]\left[\phi_{BI}^H\right]^T$ represents the residual modes and they replace $\left[\phi_B^H\right]$. Also, note that these modes only involve booster quantities which makes it a one-time computational effort.

Let us now derive the modally coupled equations of motion for the booster/payload system. First, we substitute Eq. (57) into the top row of Eq. (3) and then we premultiply by $\left[\phi_B^L \;\middle|\; \left[\phi_B^H\right]\left[\diagdown \omega_B^{H^{-2}} \diagdown\right]\left[\phi_{BI}^H\right]^T\right]^T$, yielding

$$\left[\begin{array}{c|c} I & 0 \\ \hline 0 & \underbrace{\left[\phi_{BI}^H\right]\left[\diagdown \omega_B^{H^{-4}} \diagdown\right]\left[\phi_{BI}^H\right]^T}_{\text{residual mass}} \end{array}\right]\left\{\begin{array}{c} \ddot{q}_B^L \\ \hline \ddot{R}_I^B \end{array}\right\} + \left[\begin{array}{c|c} \left[\diagdown \omega_B^{L^2} \diagdown\right] & 0 \\ \hline 0 & \underbrace{\left[\phi_{BI}^H\right]\left[\diagdown \omega_B^{H^{-2}} \diagdown\right]\left[\phi_{BI}^H\right]^T}_{\text{residual flexibility}} \end{array}\right]\left\{\begin{array}{c} q_B^L \\ \hline R_I^B \end{array}\right\}$$

$$= \left[\begin{array}{c} \left[\phi_B^L\right]^T \\ \hline \left[\phi_{BI}^H\right]\left[\diagdown \omega_B^{H^{-2}} \diagdown\right]\left[\phi_B^H\right]^T \end{array}\right]\left\{\begin{array}{c} F_N^B \\ \hline R_I^B \end{array}\right\} \tag{58}$$

Before proceeding, let us consider the homogeneous equation extracted from the lower half of Eq. (58)

$$\left[\phi_{BI}^H\right]\left[\diagdown \omega_B^{H^{-4}} \diagdown\right]\left[\phi_{BI}^H\right]^T\left\{\ddot{R}_I^B\right\} + \left[\phi_{BI}^H\right]\left[\diagdown \omega_B^{H^{-2}} \diagdown\right]\left[\phi_{BI}^H\right]^T\left\{R_I^B\right\} = 0 \tag{59}$$

and solve the following eigenvalue problem

$$\left(-\omega_R^2\left[\phi_{BI}^H\right]\left[\diagdown \omega_B^{H^{-4}} \diagdown\right]\left[\phi_{BI}^H\right]^T + \left[\phi_{BI}^H\right]\left[\diagdown \omega_B^{H^{-2}} \diagdown\right]\left[\phi_{BI}^H\right]^T\right)\left\{\phi_R\right\} = \left\{0\right\} \tag{60}$$

leading to the modal transformation,

$$\left[R_I^B\right] = \left[\phi_R\right]\left\{q_R\right\} \tag{61}$$

and the properties

$$\left[\phi_R\right]^T \left[\phi^H_{BI}\right]\left[\omega^H_B\right]^{-4} \left[\phi^H_{BT}\right]^T \left[\phi_R\right] = \left[I\right], \left[\phi_R\right]^T \left[\phi^H_{BI}\right]^{-2} \left[\phi^H_{BT}\right]^T \left[\phi_R\right] = \left[\omega^2_R\right] \qquad (62)$$

We shall make use of these properties in deriving the modally coupled equations of motion of the booster/payload system. To this end let us write Eq. (28),

$$\left[\begin{array}{c|c} M_B + T^T_P M_P T_P & T^T_P M_P I_P \\ \hline I^T_P M_P T_P & I^T_P M_P I_P \end{array}\right] \left\{\begin{array}{c} \ddot{x}_B \\ \hline \ddot{\bar{x}}^P_N \end{array}\right\} + \left[\begin{array}{c|c} K_B + T^T_P K_P T_P & 0 \\ \hline 0 & I^T_P K_P I_P \end{array}\right] \left\{\begin{array}{c} x_B \\ \hline \bar{x}^P_N \end{array}\right\} = \left\{\begin{array}{c} \\ 0 \end{array}\right\} \qquad (63)$$

Let us now introduce the following notations

$$\left\{q_B\right\} = \left\{\begin{array}{c} q^L_B \\ \hline q_R \end{array}\right\}, \quad \left[\phi_B\right] = \left[\phi^L_B \;\middle|\; \left[\phi^H_B\right]\left[\omega^H_B\right]^{-2}\left[\phi^H_{BI}\right]^T\left[\phi_R\right]\right] \qquad (64)$$

so that, combining Eqs. (57), (61) and (64) we can write,

$$\left\{x_B\right\} = \left[\phi_B\right]\left\{q_B\right\} \qquad (65)$$

We now define a transformation similar to Eq. (43)

$$\left\{\begin{array}{c} x_B \\ \hline \bar{x}^P_n \end{array}\right\} = \left[\begin{array}{c|c} \phi_B & \\ \hline & \bar{\phi}^P_N \end{array}\right] \left\{\begin{array}{c} q_B \\ \hline \bar{q}^P_N \end{array}\right\} \qquad (66)$$

where this time $\left\{q_B\right\}$ and $\left[\phi_B\right]$ are given by Eq. (64)

Applying this transformation to Eq. (63) and premultiplying by $\begin{bmatrix} \phi_B & 0 \\ 0 & \bar{\phi}_N^P \end{bmatrix}^T$ yields,

$$\begin{bmatrix} \phi_B^T M_B \phi_B + \phi_B^T T_P^T M_P T_P \phi_B & \phi_B^T T_P^T M_P I_P \bar{\phi}_N^P \\ \bar{\phi}_N^{P^T} I_P^T M_P T_P \phi_B & \bar{\phi}_N^{P^T} I_P^T M_P I_P \bar{\phi}_N^P \end{bmatrix} \begin{Bmatrix} \ddot{q}_B \\ \ddot{\bar{q}}_N^P \end{Bmatrix} + \begin{bmatrix} \phi_B^T K_B \phi_B + \phi_B^T T_P^T K_P T_P \phi_B & \\ & \bar{\phi}_N^{P^T} I_P^T K_P I_P \bar{\phi}_N^P \end{bmatrix} \begin{Bmatrix} q_B \\ \bar{q}_N^P \end{Bmatrix}$$

$$= \begin{Bmatrix} \phi_B^T \begin{Bmatrix} F_N^B \\ 0 \end{Bmatrix} \\ 0 \end{Bmatrix} \tag{67}$$

Using Eqs. (35), (40) and (62) we can show that

$$\left[\phi_B\right]^T \left[M_B\right]\left[\phi_B\right] = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}, \quad \left[\phi_B\right]^T \left[K_B\right]\left[\phi_B\right] = \begin{bmatrix} \left[\omega_B^L\right]^2 & \\ 0 & \left[\omega_R^2\right] \end{bmatrix} \tag{68}$$

and

$$\left[\bar{\phi}_N^P\right]^T \left[I_P\right]^T \left[M_P\right] \left[I_P\right]\left[\bar{\phi}_N^P\right] = \left[I\right], \quad \left[\bar{\phi}_N^P\right]^T \left[I_P\right]^T \left[K_P\right] \left[I_P\right] \left[\bar{\phi}_N^P\right] = \left[\omega_P^2\right] \tag{69}$$

so that Eq. (67) becomes

$$\begin{bmatrix} \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} + \phi_B^T T_P^T M_P T_P \phi_B & \phi_B^T T_P^T M_P I_P \bar{\phi}_N^P \\ \bar{\phi}_N^{P^T} I_P^T M_P T_P \phi_B & I \end{bmatrix} \begin{Bmatrix} \ddot{q}_B \\ \ddot{\bar{q}}_N^P \end{Bmatrix} + \begin{bmatrix} \begin{bmatrix} \left[\omega_B^L\right]^2 & \\ 0 & \left[\omega_R^2\right] \end{bmatrix} + \phi_B^T T_P^T K_P T_P \phi_B & 0 \\ 0 & \left[\omega_P^2\right] \end{bmatrix} \begin{Bmatrix} q_B \\ \bar{q}_N^P \end{Bmatrix}$$

$$= \begin{Bmatrix} \phi_B^T \begin{Bmatrix} F_N^B \\ 0 \end{Bmatrix} \\ 0 \end{Bmatrix} \tag{70}$$

The residual mass and stiffness technique essentially improves the interface representation in the model where the model is subjected to a frequency cut-off.

The result Eq.(57) can then be used in any type of modal synthesis technique such as result Eq. (70). Due to this interface improvement it is now possible to truncate the high booster modes while still obtaining an acceptable accuracy. As we shall discuss in section 9, the residual mass and stiffness method turns out to be the most efficient full-scale method currently available in the literature.

6. THE MASS AND STIFFNESS LOADING TECHNIQUE [52,10,53]

Another way of improving the interface representation in the booster model subject to frequency cut-off is given by a technique developed by Hruda and Benfield and is based on Eq. (28) which we repeat here for convenience,

$$\left[\begin{array}{c|c} M_B + T_P^T M_P T_P & T_P^T M_P I_P \\ \hline I_P^T M_P T_P & I_P^T M_P I_P \end{array}\right] \left\{\begin{array}{c} \ddot{x}_B \\ \hline \ddot{x}_N^P \end{array}\right\} + \left[\begin{array}{c|c} K_B + T_P^T K_P T_P & 0 \\ \hline 0 & I_P^T K_P I_P \end{array}\right] \left\{\begin{array}{c} x_B \\ \hline x_N^P \end{array}\right\} = \left\{\begin{array}{c} F_N^B \\ 0 \\ \hline 0 \end{array}\right\} \tag{71}$$

Instead of solving eigenvalue problem (32) Hruda and Benfield propose to solve the following eigenvalue problem,

$$\left(-\omega'^2_B \left[M_B + T_P^T M_P T_P\right] + \left[K_P + T_P^T K_P T_P\right]\right)\left\{\phi'_B\right\} = \left\{0\right\} \tag{72}$$

yielding the modal transformation

$$\left\{x_B\right\} = \left[\phi'_B\right]\left\{q'_B\right\} \tag{73}$$

Again, the modal transformations (40) and (73) can be combined in

$$\left\{\begin{array}{c} x_N^B \\ x_I^B \\ \hline \bar{x}_N^P \end{array}\right\} = \left\{\begin{array}{c} x_B \\ \hline \bar{x}_N^P \end{array}\right\} = \left[\begin{array}{c|c} \phi'_B & 0 \\ \hline 0 & \bar{\phi}_N^P \end{array}\right] \left\{\begin{array}{c} q'_B \\ \hline \bar{q}_N^P \end{array}\right\} \tag{74}$$

Substituting Eq. (74) into Eq. (71) and premultiplying by $\left[\begin{array}{c|c} \phi'_B & \\ \hline 0 & \bar{\phi}_N^P \end{array}\right]^T$ yields,

$$\left[\begin{array}{c|c} I & \phi'^T_B T_P^T M_P T_P \bar{\phi}_N^P \\ \hline \bar{\phi}_N^{P^T} I_P^T M_P T_P \phi'_B & I \end{array}\right] \left\{\begin{array}{c} \ddot{q}'_B \\ \hline \ddot{\bar{q}}_N^P \end{array}\right\} + \left[\begin{array}{c|c} \left[\diagdown \omega'^2_B \diagdown\right] & \\ \hline & \left[\diagdown \bar{\omega}_P^2 \diagdown\right] \end{array}\right] \left\{\begin{array}{c} q'_B \\ \hline \bar{q}_N^P \end{array}\right\} = \left\{\begin{array}{c} \phi'^T_B \left\{\begin{array}{c} F_N^B \\ 0 \end{array}\right\} \\ \hline 0 \end{array}\right\} \tag{75}$$

where we used the properties

$$\left[\phi'_B\right]^T \left[M_B + T_P^T M_P T_P\right] \left[\phi'_B\right] = \left[\diagdown I \diagdown\right], \quad \left[\phi'_B\right]^T \left[K_B + T_P^T K_P T_P\right] \left[\phi'_B\right] = \left[\diagdown \omega'^2_B \diagdown\right] \tag{76}$$

Equation (75) now replaces Eq. (44). The main difference lies in the fact that in solving eigenvalue problem (72), the booster interface is mass and stiffness loaded by $\left[T_P^T M_P T_P\right]$ and $\left[T_P^T K_P T_P\right]$ respectively; i.e. the booster interface is loaded with approximate dynamic effects of the payload. In doing so, the new modes $\left[\phi'_B\right]$ and frequencies $\left[\diagdown \omega'^2_B \diagdown\right]$ will include a good representation of the interface. This allows us to reduce the number of booster modes in Eq. (75) according to the predetermined cut-off frequency. The disadvantage of this method in connection with the present study is that eigenvalue problem (72) is dependent on the payload. This means that for every change in the payload we must solve this eigenvalue problem again although the booster does not change. This makes the Hruda/Benfield technique less suitable for our purposes. However, if the changes in P are small, we can use the old booster modes as a first estimate to calculate the new booster modes in a Raleigh-Ritz type eigenvalue problem solver.

7. THE COUPLED BASE MOTION TECHNIQUE [54,55,56,57]

The coupled base motion technique as presented in this section does not yield any immediate advantages over the methods presented in previous sections. However, it can be used as a starting point for possible short-cut methods (These possibilities will be investigated in Chapter II). In addition, this section will give us the opportunity to develop an alternative set of equations for Eq. (4). Indeed, we shall not only use "cantilevered" displacements for the payload P but also for the booster B, while only the interface will be free. The derivation is very similar to the one in section 2. Let us define a transformation similar to Eq. (12) but now for the booster B,

$$\left\{x_N^B\right\} = \left[S_B\right]\left\{x_I^B\right\} + \left\{\bar{x}_N^B\right\} \tag{77}$$

with

$$\left[S_B\right] = -\left[K_{NN}^B\right]^{-1}\left[K_{NI}^B\right] \tag{78}$$

Equation (14) can now be replaced by

$$\left\{\begin{array}{c} x_B \\ \hline x_P \end{array}\right\} = \left\{\begin{array}{c} x_N^B \\ x_I^B \\ \hline x_N^P \\ x_I^P \end{array}\right\} = \left[\begin{array}{c|c|c} I & S_B & 0 \\ 0 & I & 0 \\ \hline 0 & S_P & I \\ 0 & I & 0 \end{array}\right]\left\{\begin{array}{c} \bar{x}_N^B \\ \hline x_I^B \\ \hline \bar{x}_N^P \end{array}\right\} ; \tag{79}$$

Again, this transformation will eliminate the redundant set of displacements $\left\{x_I^P\right\}$ in Eq. (10) and in the process it will also eliminate the unknown reations $\left\{R_I^B\right\}$ and $\left\{R_I^P\right\}$

Introducing the notations

$$\left[\begin{array}{c|c|c} I & S_B & 0 \\ 0 & I & 0 \\ \hline 0 & S_P & I \\ 0 & I & 0 \end{array}\right] = \left[\begin{array}{c|c|c} I_B & T_B & 0 \\ & & 0 \\ \hline 0 & & \\ 0 & T_P & I_P \end{array}\right] = A \tag{80}$$

where this time,

$$\begin{bmatrix} T_B \end{bmatrix} = \begin{bmatrix} S_B \\ I \end{bmatrix}, \quad \begin{bmatrix} T_P \end{bmatrix} = \begin{bmatrix} S_P \\ I \end{bmatrix}, \quad \begin{bmatrix} I_B \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix}, \quad \begin{bmatrix} I_P \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix} \tag{81}$$

Note that $\begin{bmatrix} I_B \end{bmatrix}$ and $\begin{bmatrix} I_P \end{bmatrix}$ have different dimensions.

Substituting transformation (79) into Eq. (10) and premultiplying by A^T yields,

$$\left[\begin{array}{c|c|c} I_B^T M_B I_B & I_B^T M_B T_B & 0 \\ \hline T_B^T M_B I_B & T_B^T M_B T_B + T_P^T M_P T_P & T_P^T M_P I_P \\ \hline 0 & I_P^T M_P T_P & I_P^T M_P I_P^T \end{array}\right] \left\{\begin{array}{c} \ddot{\bar{x}}_N^B \\ \hline \ddot{x}_I^B \\ \hline \ddot{\bar{x}}_N^P \end{array}\right\} + \left[\begin{array}{c|c|c} I_B^T K_B I_B & 0 & 0 \\ \hline 0 & T_B^T K_B T_B + T_P^T K_P T_P & 0 \\ \hline 0 & 0 & I_P^T K_P I_P \end{array}\right] \left\{\begin{array}{c} \bar{x}_N^B \\ \hline x_I^B \\ \hline \bar{x}_N^P \end{array}\right\}$$

$$= \left\{\begin{array}{c} I_B^T F_B \\ \hline T_B^T F_B \\ \hline 0 \end{array}\right\} \tag{82}$$

Equation (82) replaces Equation (28).

The basic idea for a base drive method is the seperation of the booster response into two sepatate parts

$$\left\{\begin{array}{c} \bar{x}_N^B \\ x_I^B \end{array}\right\} = \left\{\begin{array}{c} \bar{x}_N^B \\ x_I^B \end{array}\right\}^F + \left\{\begin{array}{c} \bar{x}_N^B \\ x_I^B \end{array}\right\}^R \tag{83}$$

where

$$\begin{Bmatrix} \ddot{x}_N^B \\ x_I^B \end{Bmatrix}^F = \text{part due to the action of } \left\{ F_B \right\} \text{ only} \tag{84}$$

and

$$\begin{Bmatrix} \ddot{x}_N^B \\ x_I^B \end{Bmatrix}^R \doteq \text{part due to the presence of the payload (=feedback)} \tag{85}$$

It is clear that vector (82) satisfies

$$\begin{bmatrix} I_B^T M_B I_B & I_B^T M_B T_B \\ T_B^T M_B I_B & T_B^T M_B T_B \end{bmatrix} \begin{Bmatrix} \ddot{x}_N^B \\ \ddot{x}_I^B \end{Bmatrix}^F + \begin{bmatrix} I_B^T K_B I_B & 0 \\ 0 & T_B^T K_B T_B \end{bmatrix} \begin{Bmatrix} \ddot{x}_N^B \\ x_I^B \end{Bmatrix}^F = \begin{Bmatrix} I_B^T F_B \\ T_B^T F_B \end{Bmatrix} \tag{86}$$

The solution of Eq. (86) is a one-time computional effort because it only involves booster quantities. If we now substitute Eq. (83) into Eq. (82) and take into account Eq. (86) we obtain the following new set of equations

$$\begin{bmatrix} I_B^T M_B I_B & I_B^T M_B T_B & 0 \\ T_B^T M_B I_B & T_B^T M_B T_B + T_P^T M_P T_P & T_P^T M_P I_P \\ 0 & I_P^T M_P T_P & I_P^T M_P I_P \end{bmatrix} \begin{Bmatrix} \ddot{x}_N^{BR} \\ \ddot{x}_I^{BR} \\ \ddot{x}_N^P \end{Bmatrix} + \begin{bmatrix} I_B^T K_B I_B & 0 & 0 \\ 0 & T_B^T K_B T_B + T_P^T K_P T_P & 0 \\ 0 & 0 & I_P^T K_P I_P \end{bmatrix} \begin{Bmatrix} \ddot{x}_N^{BR} \\ x_I^{BR} \\ \ddot{x}_N^P \end{Bmatrix}$$

$$= - \begin{Bmatrix} 0 \\ T_P^T M_P T_P \ddot{x}_I^{BF} \\ I_P^T M_P T_P \ddot{x}_I^{BF} \end{Bmatrix} - \begin{Bmatrix} 0 \\ T_P^T K_P T_P x_I^{BF} \\ 0 \end{Bmatrix} \tag{87}$$

where $\{\ddot{x}_I^{BF}\}$ and $\{x_I^{BF}\}$ are known from Eq. (86)

The second idea of a base drive method is to consider the bottom partition of Eq. (87) and write it in the following form,

$$[I_P^T M_P I_P]\{\ddot{\bar{x}}_N^P\} + [I_P^T K_P I_P]\{\bar{x}_N^P\} = -[I_P^T M_P T_P]\left(\{\ddot{x}_I^{BF}\} + \{\ddot{x}_I^{BR}\}\right) \tag{88}$$

If one is only interested in the design of the payload, Eqs. (88), (12) and (83) is all we need, to determine the response of the payload. If $\{\ddot{x}_I^{BR}\}$ is known we can "base drive" the payload by the terms on the right-hand side of Eq. (88) to obtain $\{\ddot{x}_N^P\}$. Of course, $\{\ddot{x}_I^{BR}\}$ is coupled into the booster equations in Eq. (87).

As mentioned before Eq. (87) does not yield any immediate advantages, but as will be discussed in the next chapter, Eq. (88) becomes very useful if $\{\ddot{x}_I^{BR}\}$ (= feedback) is small.

Equation (87) can also be written in terms of normal coordinates. To this end let us introduce the following transformation

$$\left\{\begin{array}{c} \bar{x}_N^{BR} \\ \hline x_I^{BR} \\ \hline x_N^P \end{array}\right\} = \left[\begin{array}{c|c|c} \bar{\phi}_N^B & 0 & 0 \\ \hline 0 & \phi_I^B & 0 \\ \hline 0 & 0 & \phi_N^P \end{array}\right] \left\{\begin{array}{c} \bar{q}_N^{BR} \\ \hline q_I^{BR} \\ \hline q_N^P \end{array}\right\} \tag{89}$$

where $[\bar{\phi}_N^B]$ and $[\phi_I^B]$ are obtained from solving the following eigenvalue problems,

$$\left[-\bar{\omega}_B^2 [I_B^T M_B I_B] + [I_B^T K_B I_B]\right]\{\bar{\phi}_N^B\} = \{0\} \tag{90}$$

$$\left[-\omega_I^2 [T_B^T M_B T_B + T_P^T M_P T_P] + [T_B^T K_B T_B + T_P^T K_P T_P]\right]\{\phi_I^B\} = \{0\} \tag{91}$$

with

$$[\bar{\phi}_N^B]^T [I_B^T M_B I_B][\bar{\phi}_N^B] = [I], \; [\bar{\phi}_N^B]^T [I_B^T K_B I_B][\bar{\phi}_N^B] = [\bar{\omega}_B^2] \tag{92}$$

$$[\phi_I^B]^T\,[T_B^T M_B T_B + T_P^T M_P T_P]\,[\phi_I^B] = [I]\,,\quad [\phi_I^B]^T\,[T_B^T K_B T_B + T_P^T K_P T_P]\,[\phi_I^B] = [\omega_I^2] \tag{93}$$

where again we used the simpler notations $[\bar{\omega}_B^2]$ and $[\omega_I^2]$ instead of $[\bar{\omega}_N^{B^2}]$ and $[\omega_I^{B^2}]$.

Substituting transformation (89) into Eq. (87) and premultiplying by the transpose of the square transformation matrix in Eq. (89) yields the modal form,

$$\left[\begin{array}{c:c:c} I & \bar{\phi}_N^{B^T} I_B^T M_B T_B \phi_I^B & 0 \\ \hdashline \phi_I^{B^T} T_B^T M_B I_B \bar{\phi}_N^B & I & \phi_I^{B^T} T_P^T M_P I_P \bar{\phi}_N^P \\ \hdashline 0 & \bar{\phi}_N^{P^T} I_P^T M_P T_P \phi_I^B & I \end{array}\right] \left\{\begin{array}{c} \ddot{\bar{q}}_N^{BR} \\ \hdashline \ddot{q}_I^{BR} \\ \hdashline \ddot{\bar{q}}_N^{P} \end{array}\right\} + \tag{94}$$

$$\left[\begin{array}{c:c:c} [\bar{\omega}_B^2] & 0 & 0 \\ \hdashline 0 & [\omega_I^2] & 0 \\ \hdashline 0 & 0 & [\bar{\omega}_P^2] \end{array}\right] \left\{\begin{array}{c} \bar{q}_N^{BR} \\ \hdashline q_I^{BR} \\ \hdashline \bar{q}_N^{P} \end{array}\right\} = - \left\{\begin{array}{c} 0 \\ \hdashline \phi_I^{B^T} T_P^T M_P T_P \ddot{x}_I^{BF} \\ \hdashline \bar{\phi}_N^T I_P^T M_P T_P \ddot{x}_I^{BF} \end{array}\right\} - \left\{\begin{array}{c} 0 \\ \hdashline \phi_I^{B^T} T_P^T K_P T_P x_I^{BF} \\ \hdashline 0 \end{array}\right\}$$

8. A LOAD TRANSFORMATION CONSISTENT WITH MODAL SYNTHESIS TECHNIQUES [53].

Before presenting the assessment section it will prove productive to discuss the topic of determining the internal structural loads in the payload members. As discussed in section 3, the reason for solving the equations of motion of the booster/payload system is the determination of the displacement vector $\{x_P\}$ so that we can substitute this vector into Eq. (31),

$$\{F_e^P\} = [k_e]\,[T_e]\,\{x_P\} \tag{95}$$

in order to obtain the internal structural loads $\{F_e^P\}$ on an individual member e of the payload P. In principal Eq. (95) could be used as it is, but this "displacement" approach turns out to be very sensitive to inaccuracies in

$\{x_P\}$; e.g. truncating high frequency modes as we did in section 5 could very easily lead to erratic loads $\{F_e^P\}$. Heuristically speaking, $\{x_P\}$ contains three parts, the static displacement, the rigid body displacement and the "vibrational" displacement.

Therefore, if one has an error in $\{x_P\}$ one necessarily affect the accuracy of all three parts. For this reason one prefers the so called "acceleration method". Basically this approach is capable of seperating the static and rigid body parts from the "vibrational" displacement. As a consequence one only makes errors in the "vibrational" part which often is the smallest part of the displacement vector $\{x_P\}$. Such an "acceleration" approach which is consistent with modal synthesis techniques was developed by Hruda and Jones.[53]

Recalling Eq. (79) we can write

$$\{x_P\} = [T_P]\{x_I^B\} + [I_P]\{\bar{x}_N^P\} \tag{96}$$

so that from Eq. (95) we obtain

$$\{F_e^P\} = [k_e][T_e][T_P]\{x_I^B\} + [k_e][T_e][I_P]\{\bar{x}_N^P\} \tag{97}$$

From the bottom row of Eq. (82) we obtain

$$\{\bar{x}_N^P\} = [I_P^T K_P I_P]^{-1}\left(-[I_P^T M_P T_P]\{\ddot{x}_I^B\} - [I_P^T M_P I_P]\{\ddot{\bar{x}}_N^P\}\right) \tag{98}$$

and from the second row of eq. (82) we obtain

$$\{x_I^B\} = [T_B^T K_B T_B + T_P^T K_P T_P]^{-1}\left([T_B]^T\{F_B\} - [T_B^T M_B T_B]\{\ddot{\bar{x}}_N^B\} - [T_P^T M_P T_P]\{\ddot{\bar{x}}_N^P\}\right) \tag{99}$$

Expressions (98) and (99) can now be substituted into Eq. (97) yielding an equation for $\{F_e^P\}$ in terms of accelerations. Many of the matrix multiplications involved in Eqs. (97-99) can be simplified by using a unit load solution, which is a feature of most finite element programs [53]

9. ASSESSMENT

The methods as discussed in sections 1-8 are believed to be the currently most prominent full-scale methods. Some of these are improvements or adaptations of previously existing approaches (Hurty, MacNeal, Bamford, Craig/Bampton, etc.) A study by R. Hruda [11-50] showed that the residual mass and flexibility approach is the most effective in terms of cost and convenience. As a test structure, Hruda used two planar trusses cc - ed together by a statically indeterminate interface (Figure 2.) Five different techniques were compared to the exact solution, i.e. the solution in the discrete time domain as discussed in section 3.:

1. Hruda/Benfield Technique (section 6): inertial coupling of truss-2 constrained modes onto free-free modes of truss-1 which was mass and stiffness loaded at its truss-2 interface degrees-of-freedom by the interface properties of truss-2.(IMSL)

2. Craig/Bampton Technique (modal version of Eq. (82)): inertial coupling of truss-1 and truss-2 constrained modes onto a free-free modal representation of the interface degrees-of-freedom.(I/F)

3. MacNeal Technique: residual flexibility approach of coupling truss-2 constrained modes onto free-free modes of truss-1 which creates stiffness coupling (residual mass not included).(RFSWOM)

4. Rubin Technique (the residual mass and flexibility technique - section 5): coupling of truss-2 constrained modes onto free-free modes of truss-1 which yields only inertial coupling, and, by consistent application to the mass and stiffness terms in the equations of motion, yields both residual stiffness and residual mass terms.(RFIWM)

5. <u>Rubin Technique</u> but without residual mass contribution for truss-1.

The truss problem as illustrated in Figure 2. represents a planar problem with three rigid body degrees of freedom (two translational, one rotational). Each pinned joint has two translational degrees of freedom. The interface is statically indeterminate because there are six interface degrees of freedom. The heavy masses (asymmetric with respect to interface) are added to produce interface distortion. The forcing function is a ramp function.(RFIWOM)

The "exact" results, against which all comparisons were made, were obtained by extracting eigenvalues, eigenvectors, and loads directly from a finite element discrete/physical model using no modes at all.

Five different cases we investigated

EXACT: Discrete modal 70 DOF

CASE A: Modally coupled, 70 modes retained (=100%)

CASE B: Modally coupled, 50 modes retained (=71%)

CASE C: Modally coupled, 19 modes retained (=46%)

CASE F: Modally coupled, 19 modes retained (=27%)

Hruda used the following comparison values:

Frequencies: percent error against the "exact" solution.

Modes: An error vector is formed ($\phi_N - \phi_E$) and its norm is calculated (which is defined as the Root Square Sum of the elements of the vector); the comparison value is then defined as the norm of the error divided by the norm of the base/exact mode. Note that the norms are based on the modal amplitudes of <u>all</u> degrees of freedom from the coupled system.

Loads: Loads were calculated at the truss interface on both the truss-1 and truss-2 joints. A percent error of the absolute value of the largest (either maximum or minimum) value from a given case against the absolute value of the largest value from the exact solution.

i.e.

$$\text{Frequency comparison value} = \frac{\omega_N - \omega_E}{\omega_E} \times 100$$

$$\text{Mode comparison value} = \frac{\text{RSS} \; (\phi_N - \phi_E)}{\text{RSS} \; \phi_E} \times 100$$

$$\text{Load comparison value} = \frac{L_N - L_E}{L_E} \times 100$$

where E=Exact, and N=Case being compared.

The results are presented in Tables 1-12

For the 100% case-A, the MacNeal technique requires the inversion of the residual flexibility matrix to obtain a "residual stiffness". When attempting to retain all (100%) of the modes, this residual flexibility matrix is a function of the interface highest frequency modal amplitudes which can cause an ill-conditioned matrix (as in the present case). Since this is an unrepresentative case, it should not be deduced that this is an unacceptable technique. As can be seen in succeeding cases, where more residual modes are available, the MacNeai technique falls into line with other techniques. Note that in cases B,C, and F, in both the frequency and mode shape comparisons, that the MacNeal and the Rubin technique without residual mass are identical, thereby numerically supporting R.

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Coppolino's contention that these two techniques are equivalent for modal synthesis. In comparing the loads it is seen that the MacNeal column for case-A reflects the propagation of the ill-conditioning mentioned earlier. Loads were calculated by the modal acceleration technique (section 8) for all methods except for the MacNeal technique.

Due to the stiffness coupling involved in the MacNeal method, a complete modal acceleration technique for calculating loads could not be used, therefore, the modal displacement techniques of calculating loads was used. Because of this, the larger loads inaccuracies for this method must be attributed to the method of loads calculation and not to the method itself.

In conclusion we can state that methods 1 through 5 are acceptable. However, the Rubin Technique (Residual Mass and Stiffness Approach) seems to outweigh the other approaches in terms of cost and convenience. Again, it should be noted that this method does not require any knowledge of payload properties which makes it very valuable for analysis of STS-applications.

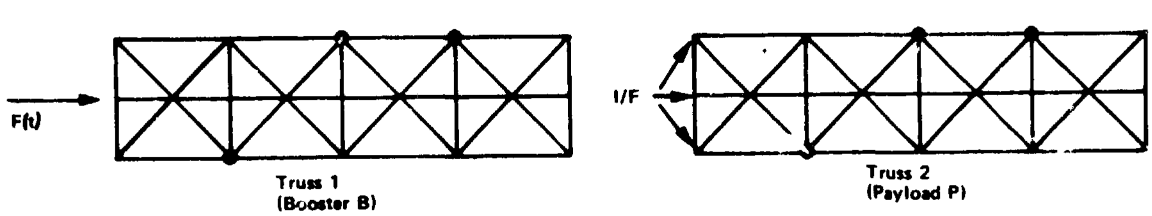

Figure 2 Structure Used for Comparing Coupling Techniques

Table 1: Frequency Comparison
Case A = 100% of Available Modes
% Diff for Various Modal Coupling Tech

Mode No	Exact Freq (Hz)	IMSL	I/F	RFSWOM	RFIWM	RFIWOM
4	.76	-.00	-.00	-49.17	-.00	-.00
5	1.75	-.00	-.00	6.75	-.00	.00
6	2.84	-.00	-.00	-21.81	-.00	-.00
7	3.08	-.00	-.00	-3.63	-.00	.00
8	3.80	-.00	-.00	-9.24	-.00	.00
9	4.62	-.00	-.00	-16.35	-.00	.00
10	5.11	-.00	-.00	-24.08	-.00	.00
11	5.50	-.00	-.00	-6.02	-.00	.00
12	5.81	-.00	-.00	-3.31	-.00	.00
13	7.69	-.00	-.00	-6.08	-.00	.00
14	8.69	-.00	-.00	-.60	-.00	.00
15	9.14	-.00	-.00	-3.32	-.00	.00
16	9.42	-.00	-.00	-1.76	-.00	.00
17	9.73	-.00	-.00	-.92	-.00	.00
18	9.85	-.00	-.00	-.98	-.00	-.00
19	10.36	-.00	-.00	-1.36	-.00	.00
20	10.43	-.00	-.00	-.35	-.00	.00
21	10.79	-.00	-.00	-2.99	-.00	.00
22	10.90	-.00	-.00	-2.55	-.00	.00
23	11.37	-.00	-.00	-1.89	-.00	.00
24	11.49	-.00	-.00	.80	-.00	.00
25	11.78	-.00	-.00	.39	-.00	.00
26	11.96	-.00	-.00	-.12	-.00	.00
27	12.03	-.00	-.00	.19	-.00	.00
28	12.20	-.00	-.00	.40	-.00	.00
29	12.43	-.00	-.00	.12	-.00	.00
30	12.50	-.00	-.00	5.58	-.00	.00
31	12.75	-.00	-.00	4.00	-.00	.00
32	13.29	-.00	-.00	2.54	-.00	.00
33	13.51	-.00	-.00	3.12	-.00	.00
34	14.22	-.00	-.00	-.19	-.00	.00
35	14.53	-.00	-.00	.31	-.00	.00
36	14.86	-.00	-.00	-.11	-.00	.00
37	15.19	-.00	-.00	1.59	-.00	.00
38	15.54	-.00	-.00	1.23	-.00	.00
39	15.69	-.00	-.00	2.50	-.00	.00
40	16.16	-.00	-.00	.55	-.00	.00
41	16.17	-.00	-.00	.66	-.00	.00
42	16.28	-.00	-.00	3.34	-.00	.00
43	16.86	-.00	-.00	.46	-.00	.00
44	17.07	-.00	-.00	.21	-.00	.00
45	17.17	-.00	-.00	5.12	-.00	.00
46	18.17	-.00	-.00	.03	-.00	.00
47	18.17	-.00	-.00	10.37	-.00	.00

Table 1: Frequency Comparison (Concl)
Case A = 100% of Available Modes
% Diff for Various Modal Coupling Tech

Mode No	Exact Freq (Hz)	IMSL	I/F	RFSWOM	RFIWM	RFIWOM
48	20.11	-.00	-.00	.81	-.00	.00
49	20.27	-.00	-.00	3.02	-.00	.00
50	21.11	-.00	-.00	.14	-.00	.00
51	21.15	-.00	-.00	.08	-.00	.00
52	21.26	-.00	-.00	.07	-.00	.00
53	21.29	-.00	-.00	.53	-.00	.00
54	21.45	-.00	-.00	.02	-.00	.00
55	21.47	-.00	-.00	2.91	-.00	.00
56	22.06	-.00	-.00	.45	-.00	.00
57	22.15	-.00	-.00	2.18	-.00	.00
58	22.53	-.00	-.00	5.09	-.00	.00

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Table 2: Frequency Comparison
Case B = 71% of Available Modes
% Diff for Various Modal Coupling Tech

Mode No	Exact Freq (Hz)	IMSL	I/F	RFSWOM	RFIWM	RFIWOM
4	.76	.00	.00	.00	.00	[illegible]
5	1.75	.00	.00	.00	.00	[illegible]
6	2.84	.00	.00	.00	.00	.02
7	3.08	.00	.00	.00	.00	.06
8	3.80	.00	.00	.00	.00	.00
9	4.62	.00	.00	.04	.00	.04
10	5.11	.00	.00	.00	.00	.00
11	5.50	.00	.00	.05	.00	.05
12	5.81	.04	.05	.03	.03	.03
13	7.60	.00	.00	.21	.00	.21
14	8.69	.02	.02	.03	.01	.03
15	9.14	.04	.06	.12	.04	.12
16	9.42	.01	.01	.01	.00	.01
17	9.73	.01	.02	.02	.01	.02
18	9.85	.03	.04	.03	.03	.03
19	10.36	.01	.01	.06	.01	.06
20	10.43	.00	.00	.01	.00	.01
21	10.79	.03	.05	.04	.02	.04
33	10.90	.03	.03	.04	.02	.04
23	11.37	.00	.00	.03	.00	.03
24	11.49	.00	.01	.05	.00	.05
25	11.78	.03	.03	.08	.03	.08
26	11.96	.08	.09	.08	.07	.08
27	12.03	.02	.03	.04	.02	.04
28	12.20	.13	.13	.10	.10	.10
29	12.43	.01	.01	.02	.00	.02
30	12.50	.03	.09	.07	.03	.07
31	12.75	.07	.07	.08	.06	.08
32	13.29	.04	.05	.06	.02	.06
33	13.51	.08	.07	.07	.05	.07
34	14.22	.13	.13	.78	.09	.78
35	14.53	.09	.13	.29	.07	.29
36	14.86	.01	.01	.17	.01	.17
37	15.19	.06	.07	.11	.05	.11
38	15.54	.01	.03	1.17	.04	1.17
39	15.69	.02	.02	1.61	.03	1.61
40	16.16	.01	.02	.13	.09	.13
41	16.17	.07	.08	2.68	.12	2.68
42	16.28	.37	.44	3.88	.34	3.88
43	16.86	.13	.12	7.73	.25	7.73
44	17.07	.06	.10	18.51	1.52	18.51
45	17.17	1.00	1.00	66.94	5.86	66.94
46	18.17	.02	.02	72.29	9.54	72.29

Table 2: Frequency Comparison (Concl)
Case B = 71% of Available Modes
% Diff for Various Modal Coupling Tech

Mode No	Exact Freq (Hz)	IMSL	I/F	RFSWOM	RFIWM	RFIWOM
47	18.17	.03	.04	122.22	11.51	122.22
48	20.11	.07	.09	473.05	7.55	473.05
49	20.27	.31	.3	962.70	8.65	902.70
50	21.11	4.07	4.20	536.67	19.40	536.67

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Table 3: Frequency Comparison
Case C = 46% of Available Modes
% Diff for Various Modal Coupling Tech

Mode No	Exact Freq (Hz)	IMSL	I/F	RFSWOM	RFIWM	RFIWOM
4	.76	.00	.00	.02	.00	.02
5	1.75	.02	.00	.00	.00	.00
6	2.84	.04	.02	.12	.01	.12
7	3.08	.01	.00	.28	.00	.28
8	3.80	.02	.00	.00	.00	.00
9	4.62	.12	.05	.26	.02	.28
10	5.1[illegible]	.02	.01	.01	.00	.01
11	5.50	.26	.10	.35	.05	.35
12	5.81	.16	.24	.12	08	.12
13	7.69	.03	.04	1.55	.06	1.55
14	8.69	.14	.22	.17	.08	.17
15	9.14	.32	.33	.41	.17	.41
16	9.42	.05	.07	.07	.01	.07
17	9.73	.13	.18	.34	.08	.34
18	9.85	.30	.27	.31	.18	.31
19	10.36	.11	.11	.51	.12	.51
20	10.43	.09	.04	.45	.02	.45
21	10.79	.17	.33	.73	.11	.73
22	10.90	.41	.42	3.43	.55	3.43
23	11.37	.06	.04	3.23	1.49	3.23
24	11.49	.04	.11	5.14	3.66	5.14
25	11.78	.17	.17	5.52	2.76	5.52
26	11.96	.40	.90	4.79	3.70	4.79
27	12.03	.30	3.16	53.65	4.71	53.65
28	12.20	1.73	2.35	77.63	3.91	77.63
29	12.43	.41	4.66	80.30	14.58	80.30
30	12.50	1.94	12.32	143.04	22.38	143.04
31	12.75	3.52	14.90	167.76	29.91	167.76
32	13.29	11.88	61.19	630.91	63.27	630.19

Table 4: Frequency Comparison
Case F = 27% of Available Modes
% Diff for Various Modal Coupling Tech

Mode No	Exact Freq (Hz)	IMSL	I/F	RFSWOM	RFIWM	RFIWOM
4	.76	.00	.00	.05	.00	.05
5	1.75	.02	.00	.02	.00	.02
6	2.84	.06	.06	.31	.03	.31
7	3.08	.07	.00	4.32	.19	4.32
8	3.80	.03	.01	.03	.00	.03
9	4.62	.18	.14	.71	.08	.71
10	5.11	.05	.03	.03	.01	.03
11	5.50	.46	.34	1.64	.49	1.64
12	5.81	1.03	1.13	23.29	4.70	23.29
13	7.69	.14	.16	15.39	8.20	15.39
14	8.69	1.05	1.14	45.28	1.30	45.28
15	9.14	1.20	6.85	84.48	8.49	84.48
16	9.42	.37	27.17	98.00	27.57	98.00
17	9.73	2.23	36.67	97.08	36.41	97.08
18	9.85	18.25	37.17	203.74	37.43	203.74
19	10.36	22.07	103.62	279.52	105.23	279.52

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Table _: Mode Shape Comparison
Case A = 100% of Available Modes
% Diff for Various Modal Coupling Tech.

Mode No	Exact Freq (HZ)	IMSL	I/F	RFSWOM	RFIWM	RFIWOM
4	14.84	.00	.00	104.09	.00	.00
5	13.95	.00	.00	92.76	.00	.01
6	13.45	.00	.00	104.91	.00	.01
7	13.70	.00	.00	94.49	.00	.02
8	12.23	.00	.00	132.95	.00	.02
9	11.21	.00	.00	100.73	.00	.03
10	9.54	.00	.00	153.35	.00	.05
11	8.75	.00	.00	104.39	.00	.12
12	13.25	.00	.00	129.05	.00	.08
13	10.51	.00	.00	186.06	.00	.18
14	11.49	.00	.00	96.78	.00	.04
15	11.24	.00	.00	102.21	.00	.16
16	14.5	.00	.00	71.19	.00	.07
17	13.16	.00	.00	141.30	.00	.09
18	13.99	.00	.00	90.01	.00	.02
19	15.47	.00	.00	81.01	.00	.12
20	13.49	.00	.00	99.38	.00	.05
21	12.74	.00	.00	136.40	.00	.13
22	15.25	.00	.00	138.19	.00	.15
23	16.56	.00	.00	132.20	.00	.15
24	18.06	.00	.00	100.18	.00	.25
25	17.70	.00	.00	132.72	.00	.18
26	17.29	.00	.00	142.58	.00	.17
27	16.94	.00	.00	68.01	.00	.09
28	17.58	.00	.00	69.89	.00	.03
29	17.06	.00	.00	41.89	.00	.19
30	16.31	.00	.00	138.22	.00	.28
31	15.86	.00	.00	149.30	.00	.01
32	18.16	.00	.00	138.85	.00	.08
33	16.92	.00	.00	148.49	.00	.11
34	15.21	.00	.00	91.84	.00	.84
35	18.67	.00	.00	75.41	.00	.52
36	18.48	.00	.00	49.75	.00	.36
37	17.23	.00	.00	89.32	.00	.16
38	18.14	.00	.00	145.84	.00	1.36
39	18.12	.00	.00	146.47	.00	.72
40	18.82	.00	.00	160.19	.00	.51
41	19.22	.00	.00	126.88	.00	.56
42	15.63	.00	.00	162.42	.00	.90
43	18.47	.00	.00	124.22	.00	1.12
44	19.44	.00	.00	61.05	.00	.45
45	16.54	.00	.00	135.41	.00	.84
46	18.17	.00	.00	54.79	.00	.32

Table 5: Mode Shape Comparison (Concl)
Case A = 100% of Available Modes
% Diff for Various Modal Coupling Tech.

Mode No	Exact Freq (HZ)	IMSL	I/F	RFSWOM	RFIWM	RFIWOM
47	18.33	.00	.00	142.35	.00	.30
48	18.25	.00	.00	145.79	.00	2.22
49	19.16	.00	.00	139.28	.00	.27
50	19.29	.00	.00	65.342	.00	2.16
51	19.51	.00	.00	62.72	.00	1.22
52	19.17	.00	.00	52.42	.00	3.43
53	18.89	.00	.00	105.50	.00	4.69
54	19.52	.00	.00	38.21	.00	1.48
55	19.13	.00	.00	103.39	.00	2.35
56	11.17	.00	.00	161.11	.00	5.68
57	19.81	.00	.00	11.29	.00	.17
58	19.81	.00	.00	135.33	.00	.09

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Table 6: Mode Shape Comparison
Case B = 71% of Available Modes
% Diff for Various Modal Coupling Tech.

Mode No	Exact Freq (HZ)	IMSL	I/F	RFSWOM	RFIWM	RFIWOM
4	14.84	.00	.00	.00	.00	.00
5	13.05	.02	.01	.01	.01	.01
6	13.45	.04	.03	.18	.02	.18
7	13.70	.07	.01	.23	.01	.23
8	12.23	.08	.06	.06	.05	.06
9	11.21	.12	.08	.33	.06	.33
10	9.54	.16	.18	.16	.15	.16
11	8.75	.22	.22	.66	.14	.66
12	13.25	.80	.89	.76	.70	.76
13	10.51	.30	.41	1.78	.27	1.78
14	11.49	.91	1.21	.88	.57	.88
15	11.24	1.75	2.05	2.53	1.67	2.53
16	14.55	.61	.71	.95	.34	.95
17	13.16	1.02	1.10	1.02	.70	1.02
18	13.99	1.52	1.67	1.63	1.50	1.63
19	15.47	1.28	1.03	1.86	.77	1.86
20	13.49	1.30	1.03	1.21	.75	1.21
21	12.74	1.54	2.02	2.20	1.23	2.20
22	15.25	1.93	1.84	1.98	1.50	1.98
23	16.56	.38	.62	3.31	.12	3.31
24	18.06	.32	.81	3.56	.31	3.56
25	17.70	3.44	3.63	4.94	3.43	4.94
26	17.29	9.71	9.84	8.91	8.36	8.91
27	16.94	8.37	8.24	7.18	7.04	7.18
28	17.58	7.16	7.15	6.09	5.94	6.09
29	17.06	1.11	2.22	3.50	.79	3.50
30	16.31	3.83	5.48	5.80	3.52	5.80
31	15.88	3.90	3.89	4.95	3.86	4.95
32	18.16	2.81	2.72	4.45	2.02	4.45
33	16.92	4.06	3.29	3.91	2.92	3.91
34	15.21	6.36	4.34	25.34	4.43	25.34
35	18.67	5.53	5.12	22.71	4.42	22.71
36	18.48	1.64	1.62	16.67	1.79	16.67
37	17.23	4.65	5.05	12.42	3.98	12.42
38	18.14	1.43	2.71	71.43	6.38	71.43
39	18.12	2.45	2.06	189.04	4.32	189.04
40	18.82	35.48	19.60	104.85	60.42	104.85
41	19.22	39.52	27.36	128.81	60.10	128.81
42	15.63	26.78	27.93	126.75	28.17	126.75
43	18.47	9.51	9.90	140.52	38.29	140.52
44	19.44	14.36	15.85	144.04	126.68	144.04
45	16.54	24.08	24.65	213.55	142.78	213.55
46	18.17	3.25	4.86	150.22	123.17	150.22

Table 6: Mode Shape Comparison (Concl)
Case B = 71% of Available Modes
% Diff for Various Modal Coupling Tech.

Mode No	Exact Freq (HZ)	IMSL	I/F	RFSWOM	RFIWM	RFIWOM
47	18.33	4.54	5.77	250.51	144.13	250.51
48	18.25	62.46	70.50	553.15	77.72	553.15
49	19.16	57.97	66.55	965.00	130.07	985.90
50	19.29	114.23	116.94	930.68	128.01	930.68

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Table 7: Mode Shape Comparison
Case C = 46% of Available Modes
% Diff for Various Modal Coupling Tech.

Mode No	Exact Freq (HZ)	IMSL	I/F	RFSWOM	RFIWM	RFIWOM
4	14.84	.02	.02	.03	.02	.03
5	13.95	.18	.04	.03	.03	.03
6	13.45	.47	.45	.49	.31	.49
7	13.70	.31	.10	.55	.13	.55
8	12.23	.54	.23	.18	.16	.18
9	11.21	.1.59	1.30	2.00	.90	2.00
10	9.54	1.10	.74	.41	.36	.41
11	8.75	2.79	3.01	3.61	1.79	3.61
12	13.25	2.50	3.48	2.42	1.79	2.42
13	10.51	1.92	2.35	9.62	2.83	9.62
14	11.49	4.84	7.14	5.19	3.88	5.19
15	11.24	8.72	9.37	8.63	5.92	8.63
16	14.55	3.33	4.77	3.71	1.62	3.71
17	13.16	9.03	90.46	13.85	5.90	13.85
18	13.99	11.17	10.37	13.28	7.67	13.28
19	15.47	12.62	11.13	68.73	9.12	68.73
20	13.49	13.23	10.81	81.34	7.28	81.34
21	12.74	7.56	16.01	46.68	10.40	46.68
22	14.25	16.60	19.11	54.62	20.68	54.62
23	16.56	6.01	5.57	123.28	97.95	123.28
24	18.06	4.87	9.79	129.15	145.42	129.15
25	17.70	17.82	16.45	135.79	155.65	135.79
26	17.29	68.71	127.43	150.09	124.74	150.09
27	16.94	72.98	144.11	134.45	120.45	134.45
28	17.58	103.74	98.65	152.44	150.49	152.44
29	17.06	81.82	139.33	166.49	128.91	166.49
30	16.31	110.88	130.70	145.30	111.59	145.30
31	15.86	95.31	153.24	243.62	141.77	243.62
32	18.16	101.15	122.19	695.80	128.54	695.89

Table 8: Mode Shape Comparison
Case F = 27% of Available Modes
% Diff for Various Modal Coupling Tech.

Mode No	Exact Freq (HZ)	IMSL	I/F	RFSWOM	RFIWM	RFIWOM
4	14.84	.04	.05	.06	.03	.06
5	13.95	.24	.07	.26	.06	.26
6	13.45	.71	.89	2.63	.71	2.63
7	13.70	1.02	.20	8.69	2.36	8.69
8	12.23	.74	.39	1.48	.42	1.48
9	11.21	2.58	2.75	4.62	1.96	4.62
10	9.54	2.84	2.19	1.40	.98	1.40
11	8.75	10.05	9.34	38.19	23.43	38.19
12	13.25	10.96	11.05	65.52	28.35	65.52
13	10.51	7.03	7.59	128.49	53.35	128.49
14	11.49	29.41	33.79	159.45	43.11	159.45
15	11.24	33.59	66.95	167.22	88.27	167.22
16	14.55	30.24	113.08	161.12	119.44	161.12
17	13.16	111.59	138.72	161.33	140.08	161.33
18	13.99	123.87	130.93	148.55	132.90	148.55
19	15.47	148.78	127.29	379.09	127.01	379.09

Table 9
Comparisons of Maximum Absolute Values of Interface Loads
Case A = 100% of Available Modes

		Percent Difference - ABS. Max. Loads				
Load No.	Exact Load (lbs)	IMSL	I/F	RFSWOM	RFIWM	RFIWOM
1	-481.999	0.	-.00	3419.56	.00	-.05
3	-202.138	0.	-.00	10154.67	.00	-.08
5	-498.819	0.	.00	3957.31	-.00	.11
7	474.713	0.	-.00	2208.71	.00	-.03
9	191.901	0.	-.00	22733.86	.00	-.02
11	486.870	0.	.00	17322.92	.00	.03

IMSL = Inertial Coupling W/ Mass and Stiffness Loading
I/F = Interface Method of Inertial Coupling
RFSWOM = Residual Flexibility with Stiffness Coupling, without Residual Mass
RFIWM = Residual Flexibility with Inertial Coupling, with Residual Mass
RFIWOM = Residual Flexibility with Inertial Coupling, without Residual Mass

Table 10
Comparisons of Maximum Absolute Values of Interface Loads
Case B = 71% of Available Modes

		Percent Difference - ABS. Max. Loads				
Load No.	Ex ct Load (lbs)	IMSL	I/F	RFSWOM	RFIWM	RFIWOM
1	-481.999	.10	-.00	2.08	.10	-.38
3	-202.138	.26	.27	2.62	.27	.26
5	-498.819	.12	.13	-.13	.14	-.20
7	474.713	.17	.15	-1.36	.18	.22
9	191.901	.15	.18	.26	.13	-.24
11	486.870	.19	.19	2.00	.24	-.10

IMSL = Inertial Coupling W/ Mass and Stiffness Loading
I/F = Interface Method of Inertial Coupling
RFSWOM = Residual Flexibility with Stiffness Coupling, without Residual Mass
RFIWM = Residual Flexibility with Inertial Coupling, with Residual Mass
RFIWOM = Residual Flexibility with Inertial Coupling, without Residual Mass

Table 11
Comparisons of Maximum Absolute Values of Interface Loads
Case C = 46% of Available Modes

Load No.	Exact Load (lbs)	IMSL	I/F	RFSWOM	RFIWM	RFIWOM
		Percent Difference - ABS. Max. Loads				
1	-481.999	-1.32	-1.29	-4.07	-1.09	.29
3	-202.138	2.34	2.33	3.10	2.43	3.19
5	-498.819	.25	.39	-7.33	.45	.17
7	474.713	-1.48	-1.43	2.93	-1.36	-.34
9	191 901	.00	-.12	-25.44	-.07	-1.48
11	486.870	-.85	-.77	-3.88	-.78	-.73

IMSL = Inertial Coupling W/ Mass and Stiffness Loading
I/F = Interface Method of Inertial Coupling
RFSWOM = Residual Flexibility with Stiffness Coupling, without Residual Mass
RFIWM = Residual Flexibility with Inertial Coupling, with Residual Mass
RFIWOM = Residual Flexibility with Inertial Coupling, without Residual Mass

Table 12
Comparisons of Maximum Absolute Values of Interface Loads
Case F = 27% of Available Modes

Load No.	Exact Load (lbs)	IMSL	I/F	RFSWOM	RFIWM	RFIWOM
		Percent Difference - ABS. Max. Loads				
1	-481.999	-2.45	-[illegible].88	6.38	-6.66	-2.60
3	-202.138	3.04	-2.85	5.68	-10.99	.27
5	-498.819	-2.24	-2.61	8.02	-4.49	-1.85
7	474.713	-3.82	-4.44	10.81	-8.79	-10.31
9	191.901	-2.79	-.54	21.52	-7.54	-14.78
11	486.870	-.08	-.48	17.06	-4.28	-7.60

IMSL = Inertial Coupling W/ Mass and Stiffness Loading
I/F = Interface Method of Inertial Coupling
RFSWOM = Residual Flexibility with Stiffness Coupling, without Residual Mass
RFIWM = Residual Flexibility with Inertial Coupling, with Residual Mass
RFIWOM = Residual Flexibility with Inertial Coupling, without Residual Mass

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CHAPTER II: SHORT-CUT METHODOLOGIES

1. INTRODUCTION

In Chapter I we discussed several modal coupling techniques. All these techniques necessitate the solution of the coupled booster/payload equations, i.e. they are "full-scale" methods. As we pointed out before, this solution is quite expensive, especially if it has to be repeated several times e.g. during a design effort. Although mass and stiffness changes during a design effort are often small, current practices used to design payload structures require a new "full-scale" solution every time such small changes in the payload are made. A similar situation exists in the case of payloads that are designed for multiple flights with moderate configuration changes.

A need exists for the development of "short-cut" methods. The term "short-cut" method implies that the method should be able to evaluate small changes in the payload in a relatively short time. First, a short-cut method should avoid the direct solution of the coupled equations of the booster/payload system. Secondly, it should avoid as much as possible the interfacing between different organizations. This means that one should strife towards as much independence for the payload design organization as possible.

The objective then of Chapter II is to present several of the most promising of these short-cut methods. Also an assessment of their strengths and weaknesses will be made. The first of these methods will be discussed in the next section.

2. THE PERTURBATION TECHNIQUE

In this section we shall discuss a short-cut method which is based on a well known perturbation technique. We shall first discuss the perturbation technique in general terms and then apply it to the particular problem of a booster/payload system.

Let us consider a set of equations of motion of a certain structure,

$$[M_0]\{\ddot{x}\} + [K_0]\{x\} = \{F\} \tag{100}$$

where $[M_0]$ and $[K_0]$ are the mass and stiffness matrix of the structure, respectively. The vectors $\{x\}$ and $\{F\}$ are the generalized discrete displacement and force vectors. The eigenvalue problem associated with Eq. (100) can be written as

$$\left(-\omega_0^2\ [M_0] + [K_0]\right)\{\phi_0\} = \{0\} \tag{101}$$

The solution of this eigenvalue problem yields a modal matrix $[\phi_0]$ and a diagonal eigenvalue matrix $[\omega_0^2]$ satisfying

$$[\phi_0]^T[M_0][\phi_0] = [I]\ ,\quad [\phi_0]^T[K_0][\phi_0] = [\omega_0^2] \tag{102}$$

Next, let us assume that the elements of $[M_0]$ and $[K_0]$ undergo small changes, so that the new system of equations can be written as follow

$$[M]\{\ddot{x}\} + [K]\{x\} = \{F\} \tag{103}$$

with

$$[M] = [M_0] + \varepsilon[M_1]\ ,\quad [K] = [K_0] + \varepsilon[K_1] \tag{104}$$

where ϵ is a small parameter such that

$$\epsilon [M_1] = [M] - [M_0], \quad \epsilon [K_1] = [K] - [K_0] \tag{105}$$

Note that the matrix differences on the right-hand sides of Eqs. (105) are small, so that it is easy to determine a small ϵ so that $[M_1]$ and $[K_1]$ are of the same order of magnitude as $[M]$, $[M_0]$ and $[K]$, $[K_0]$.

The objective of the perturbation technique is to obtain a solution for the new eigenvalue problem associated with Eq. (103)

$$\left(-\omega^2 [M] + [K]\right)\{\phi\} = \{0\} \tag{106}$$

without actually solving Eq.(106). To this end, let us write

$$\{x\} = \{x_0\} + \epsilon \{x_1\} + \epsilon^2 \{x_2\} + \dots \tag{107}$$

$$\{\phi\} = \{\phi_0\} + \epsilon \{\phi_1\} + \epsilon^2 \{\phi_2\} + \dots \tag{108}$$

$$\omega = \omega_0 + \epsilon \omega_1 + \epsilon^2 \omega_2 + \dots \tag{109}$$

This can be done because of the small changes in $[M_0]$ and $[K_0]$ as expressed in Eq. (104). Also,

$$\{q\} = \{q_0\} + \epsilon \{q_1\} + \epsilon^2 \{q_2\} + \dots \tag{110}$$

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where $\{q_0\}$ and $\{q\}$ are the normal coordinate vectors of the unperturbed and perturbed system, respectively, i.e.

$$\{x_0\} = [\phi_0]\{q_0\} \tag{111}$$

$$\{x\} = [\phi]\{q\} \tag{112}$$

with

$$[\phi]^T[M][\phi] = [1], \quad [\phi]^T[K][\phi]^T[K][\phi] = [\omega^2] \tag{113}$$

First, let us substitute Eq. (112) into Eq. (103) and premultiply by $[\phi]^T$, yielding,

$$\{\ddot{q}\} + [\omega^2]\{q\} = [\phi]^T\{F\} \tag{114}$$

Substituting Eqs. (108), (109) and (110) into Eq. (114) and equating coefficents of like powers of ε, we can write,

$$\{\ddot{q}_0\} + [\omega_0^2]\{q_0\} = [\phi_0]^T\{F\} \tag{115}$$

$$\{\ddot{q}_1\} + [\omega_0^2]\{q_1\} = [\phi_1]^T\{F\} - 2[\omega_0][\omega_1]\{q_0\} \tag{116}$$

It is now possible to solve Eqs. (115, 116, etc.) in sequence. The first Eq. (115) represents the unperturbed equation of motion, i.e., the modal form of Eq. (100). This solution is available or can be determined.

Once the vector $\{q_0\}$ is determined one can solve Eq. (116) if $[\phi_1]$ and $[\omega_1]$ are known. The determination of these matrices is the subject of next paragraph.

First, it is always possible to write $[\phi_1]$ as a linear combination of the eigenvectors $[\phi_0]$,

$$[\phi_1] = [\phi_0] [\alpha] \tag{117}$$

because $[\phi_0]$ is a complete set of vectors (i.e. they can be used as a basis for a vector space). Note that $[\alpha]$ represents the coefficient matrix of $[\phi_0]$ in the linear combination and must still be determined. To this end, let us introduce Eqs. (104),(108), (109) and (117) into Eqs.(113), and only keep terms in ε^0 and ε^1,

$$\left([\phi_0] + \varepsilon [\phi_0] [\alpha]\right)^T \left([M_0] + \varepsilon [M_1]\right) \left([\phi_0] + \varepsilon [\phi_0] [\alpha]\right) = [I] \tag{118}$$

$$\left([\phi_0] + \varepsilon [\phi_0] [\alpha]\right)^T \left([K_0] + \varepsilon [K_1]\right) \left([\phi_0] + \varepsilon [\phi_0] [\alpha]\right) = \left([\omega_0] + \varepsilon[\omega_1]\right)^2 \tag{119}$$

Equating coefficents of like powers in ε^0 and ε^1 we obtain from Eqs. (118-119),

$$[\alpha] + [\alpha]^T = - [\phi_0]^T [M_1] [\phi_0] \tag{120}$$

$$2 [\omega_0] [\omega_1] = [\omega_0^2][\alpha] + [\alpha]^T [\omega_0^2] + [\phi_0]^T [K_1] [\phi_0] \tag{121}$$

Equations (120-121) can now be solve for $[\phi_1]$ and $[\omega_1]$. This erables us to solve Eq. (116) for $\{q_1\}$, so that from Eq. (110) we obtain the first order approximation

$$\{q\} = \{q_0\} + \varepsilon \{q_1\} \tag{122}$$

and from Eq. (112) we obtain $\{x\}$ where we use

$$[\phi] = [\phi_0] + \varepsilon[\phi_1] \tag{123}$$

This perturbation technique can now be applied to a booster/payload situation. The assumption is that only changes of order ε are made in the payload, i.e.

$$[M_P] = [M_{PO}] + \varepsilon[M_{P1}] , [K_P] = [K_{PO}] + \varepsilon[K_{P1}] \tag{124}$$

where $[M_{PO}]$ and $[K_{PO}]$ are the mass and stiffness matrices of the unperturbed payload PO. Let us write the mass and stiffness matrices in Eq. (28) for the perturbed payload P,

$$\left[\begin{array}{c|c} M_B + T_P^T M_P T_P & T_P^T M_P I_P \\ \hline I_P^T M_P T_P & I_P^T M_P I_P \end{array}\right] , \left[\begin{array}{c|c} K_B + T_P^T K_P T_P & 0 \\ \hline 0 & I_P^T K_P I_P \end{array}\right] \tag{125}$$

with

$$[T_P] = \begin{bmatrix} 0 & S_P \\ 0 & I \end{bmatrix} , [S_P] = -\left[K_{NN}^P\right]^{-1}\left[K_{NI}^P\right] \tag{126}$$

Using Equation (124) we can write $[S_P]$ as

$$[S_P] = -\left([K_{NN}^{PO}] + \varepsilon[K_{NN}^{P1}]\right)^{-1}\left([K_{NI}^{PO}] + \varepsilon[K_{NI}^{P1}]\right) \tag{127}$$

and, keeping terms in ε only, we obtain

$$[S_P] = -\left([K_{NN}^{P0}]^{-1} - \varepsilon\,[K_{NN}^{P0}]^{-1}[K_{NN}^{P1}][K_{NN}^{P0}]\right)\left([K_{NN}^{P0}] + \varepsilon\,[K_{NI}^{P1}]\right) \tag{128}$$

$$= -[K_{NN}^{P0}]^{-1}[K_{NI}^{P0}] - \varepsilon\left([K_{NN}^{P0}]^{-1}[K_{NI}^{P1}] - [K_{NN}^{P0}]^{-1}[K_{NN}^{P1}][K_{NN}^{P0}]^{-1}[K_{NI}^{P0}]\right)$$

or

$$[S_P] = [S_{P0}] + \varepsilon\,[S_{P1}] \tag{129}$$

with

$$[S_{P0}] = -[K_{NN}^{P0}]^{-1}[K_{NI}^{P0}] \tag{130}$$

$$[S_{P1}] = -[K_{NN}^{P0}]^{-1}[K_{NI}^{P1}] + [K_{NN}^{P0}]^{-1}[K_{NN}^{P1}][K_{NN}^{P0}]^{-1}[K_{NI}^{P0}] \tag{131}$$

It is now possible to write $[T_P]$ as

$$[T_P] = [T_{P0}] + \varepsilon\,[T_{P1}] \tag{132}$$

with

$$[T_{P0}] = \begin{bmatrix} 0 & S_{P0} \\ 0 & 0 \end{bmatrix}, \quad [T_{P1}] = \begin{bmatrix} 0 & S_{P1} \\ 0 & I \end{bmatrix} \tag{133}$$

Finally we can write the mass and stiffness matrices of the coupled booster/payload system as follows

$$[M_C] = [M_{C0}] + \epsilon [M_{C1}] \;,\; [K_C] = [K_{C0}] + \epsilon [K_{C1}] \tag{134}$$

where C indicates Coupled. Matrices $[M_C]$ and $[K_C]$ are given by expressions (125). Taking into account Eqs. (124) and (132) we can write

$$[M_{C0}] = \left[\begin{array}{c|c} M_B + T_{P0}^T M_{P0} T_{P0} & T_{P0}^T M_{P0} I_{P0} \\ \hline I_{P0}^T M_{P0} T_{P0} & I_{P0}^T M_{P0} I_{P0} \end{array}\right] \;,\; [K_{C0}] = \left[\begin{array}{c|c} K_B + T_{P0}^T K_{P0} T_{P0} & 0 \\ \hline 0 & I_{P0}^T K_{P0} I_{P0} \end{array}\right] \tag{135}$$

and

$$[M_{C1}] = \left[\begin{array}{c|c} T_{P0}^T M_{P0} T_{P1} + T_{P0}^T M_{P1} T_{P0} + T_{P1}^T M_{P0} T_{P0} & (T_{P0}^T M_{P1} + T_{P1}^T M_{P0})\, I_{P0} \\ \hline I_{P0}^T \; (M_{P0} T_{P1} + M_{P1} T_{P0}) & I_{P0}^T M_{P1} I_{P0} \end{array}\right] \tag{136}$$

$$[K_{C1}] = \left[\begin{array}{c|c} T_{P0}^T K_{P0} T_{P1} + T_{P0}^T K_{P1} T_{P0} + T_{P1}^T K_{P0} T_{P0} & 0 \\ \hline 0 & I_{P0}^T K_{P1} I_{P0} \end{array}\right] \tag{137}$$

where $[T_{P0}]$ and $[T_{P1}]$ are given by Eq. (133). Equation (134) is now equivalent to Eq. (104) and the perturbation technique can be applied.

Note that theoretically one can also obtain the higher-order perturbations ϵ^2, ϵ^3, etc. but for all practical purposes only ϵ perturbations are included. The question then is, how important ϵ^2, ϵ^3, etc. terms are. It is evident that Eq. (107) is only valid as long as the asymptotic expansion in ϵ does not break down, i.e. as long as $\epsilon \{x_1\} \ll \{x_0\}$, $\epsilon^2 \{x_2\} \ll \epsilon \{x_1\}$, etc. There are indeed cases where such an asymptotic expansion breaks down. It is then necessary to introduce other perturbation techniques (e.g. Lighthill, multiple scales, etc). In this connnection it is important to recognize the fact that small changes in the mass and stiffness of the payload produce small changes in the eigenvalues and eigenvectors, but not necessarily small changes in the response. These perturbation techniques show some promise and will be investigated further as part of Task II Methodology Development.

3. THE BASE DRIVE TECHNIQUE

In this section we shall discuss the Base Drive Technique as developed by W. Holland, A. Devers and H.Harcrow. Let us first recall Eq. (87) in partitioned form,

$$\left[I_B^T M_B I_B\right]\left\{\ddot{\bar{x}}_N^{BR}\right\} + \left[I_B^T K_B I_B\right]\left\{\bar{x}_N^{BR}\right\} = -\left[I_B^T M_B T_B\right]\left\{\ddot{x}_I^{BR}\right\} \tag{138}$$

$$\left[I_P^T M_P I_P\right]\left\{\ddot{\bar{x}}_N^{P}\right\} + \left[I_P^T K_P I_P\right]\left\{\bar{x}_N^{P}\right\} = -\left[I_P^T M_P T_P\right]\left(\left\{\ddot{x}_I^{BF}\right\} + \left\{\ddot{x}_I^{BR}\right\}\right) \tag{139}$$

$$\begin{aligned}\left\{\ddot{x}_I^{BR}\right\} = \left[T_B^T M_B T_B + T_P^T M_P T_P\right]^{-1}\Big(& -\left[T_P^T M_P T_P\right]\left\{\ddot{x}_I^{BF}\right\} - \left[T_P^T K_P T_P\right]\left\{x_I^{BF}\right\} \\ & -\left[T_P^T M_P I_P\right]\left\{\ddot{\bar{x}}_N^{P}\right\} - \left[T_B^T M_B I_B\right]\left\{\ddot{\bar{x}}_N^{BR}\right\} - \left[T_B^T K_B T_B + T_P^T K_P T_P\right]\left\{x_I^{BR}\right\}\Big)\end{aligned} \tag{140}$$

where we solved for $\left\{\ddot{x}_I^{BR}\right\}$ in Eq. (140).

The payload designer is primarily interested in predicting the response of the payload (see Chapter I, section 8.) which is given by

$$\left\{\begin{array}{c} x_N^P \\ \hline x_I^P \end{array}\right\} = \left[\begin{array}{c|c} I & S_P \\ \hline 0 & I \end{array}\right]\left\{\begin{array}{c} \bar{x}_N^P \\ \hline x_I^P \end{array}\right\} \tag{141}$$

with

$$\left\{\begin{array}{c} \bar{x}_N^P \\ \hline x_I^P \end{array}\right\} = \left\{\begin{array}{c} \bar{x}_N^P \\ \hline x_I^{BF} + x_I^{BR} \end{array}\right\} \tag{142}$$

where $\left\{\ddot{x}_N^P\right\}$ and $\left\{\ddot{x}_I^{BR}\right\}$ must be computed from, Eqs. (138-140) and $\left\{\ddot{x}_I^{BF}\right\}$ from Eq. (86).

The idea of a base drive short-cut method is to approximate $\left\{\ddot{x}_I^R\right\}$ in Eq. (140) in such a way as to avoid the solution of the complete set (138-140). To evaluate a particular short-cut method, the approximation of $\left\{\ddot{x}_I^R\right\}$ must be compared to the exact value given by Eq. (140).

A significant simplification of Eq. (140) occurs when the interface is statically determinate, i.e., when

$$\left[T_B^T K_B T_B\right] = \left[T_P^T K_P T_P\right] = \left[0\right] \tag{143}$$

This eliminates the dependence of $\left\{\ddot{x}_I^R\right\}$ on $\left\{x_I^{BR}\right\}$ and $\left\{x_I^{BF}\right\}$ and $\left\{\ddot{x}_I^R\right\}$ becomes,

$$\left\{\ddot{x}_I^{BR}\right\} = \left[T_B^T M_B T_B + T_P^T M_P T_P\right]^{-1} \left(- \left[T_P^T M_P T_P\right] \left\{\ddot{x}_I^{BF}\right\} - \left[T_P^T M_P I_P\right] \left\{\ddot{x}_N^P\right\} - \left[T_B^T M_B I_B\right] \left\{\ddot{x}_N^{BR}\right\} \right) \tag{144}$$

A first possibility is to assume that the presence of the payload has no effect on the response of the booster, i.e. $\left\{\ddot{x}_I^{BR}\right\} = \left\{0\right\}$. This approach is called the Direct Base Drive Technique. Indeed if $\left\{\ddot{x}_I^{BR}\right\} = \left\{0\right\}$ then Eq. (139) becomes

$$\left[I_P^T M_P I_P\right] \left\{\ddot{x}_N^P\right\} + \left[I_P^T K_P I_P\right] \left\{x_N^P\right\} = - \left[I_P^T M_P T_P\right] \left\{\ddot{x}_I^{BF}\right\} \tag{145}$$

which means that the payload is "directly" driven at its base (i.e. its interface with the booster B) by the force on the right hand side of Eq. (145).

Theoretically, the Direct Base Drive Technique assumes there is no coupling at all between the booster and the payload. Practically, it means that $\{\ddot{x}_I^{BR}\} << \{\ddot{x}_I^{BF}\}$ or that the feedback of the payload is negligible. The conditions under which such an approximation is valid is still an unanswered question. The development of a criterion of validity of the use of the Direct Base Drive Technique should be part of Task II: Methodology Development. This topic together with some other considerations will be discussed in Chapter III.

4. THE IMPEDANCE TECHNIQUE [71]

In this section we shall discuss yet another approach to the solution of the equations of motion of the booster/payload system. The Impedance Technique as developed by K. Payne is basically a full-scale method in the sense that it does not make any assumptions concerning the size of the payload nor the extent of the changes made in the payload. However, the method does avoid a full-scale solution of the coupled booster/payload equations of motion and is particularly suited to deal with small changes in the payload. The Impedance Technique is essentially a Base Drive method (see section 3.). It differs from the approach in section 3. in the manner in which the interface accelerations $\{\ddot{x}_I^{BR}\}$ are computed. Indeed, the interface accelerations will be computed in the frequency domain instead of the discrete time domain thereby essentially converting a set of differential equations into a set of algebraic equations.

Let us now derive the necessary equations. First, recall Eq. (3).

$$\left[\begin{array}{c|c} M_B & 0 \\ \hline 0 & M_P \end{array}\right]\left\{\begin{array}{c} \ddot{x}_B \\ \hline \ddot{x}_P \end{array}\right\} + \left[\begin{array}{c|c} K_B & \\ \hline & K_P \end{array}\right]\left\{\begin{array}{c} x_B \\ \hline x_P \end{array}\right\} = \left\{\begin{array}{c} F_B \\ \hline F_P \end{array}\right\} + \left\{\begin{array}{c} 0 \\ R_I^B \\ \hline 0 \\ R_I^P \end{array}\right\} \tag{146}$$

and write the top and bottom partitions separately,

$$\left[M_B\right]\{\ddot{x}_B\} + \left[K_B\right]\{x_B\} = \{F_B\} + \left\{\begin{array}{c} 0 \\ R_I^B \end{array}\right\} \tag{147}$$

$$\left[M_P\right]\{\ddot{x}_P\} + \left[K_P\right]\{x_P\} = \left\{\begin{array}{c} 0 \\ -R_I^P \end{array}\right\} \tag{148}$$

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where we invoked Eq. (2) and assumed the external payload forces to be absent $\left(\{F_P\} = \{0\}\right)$. Next, we introduce an equation similar to Eq. (83),

$$\{x_B\} = \{x_B\}^F + \{x_B\}^R \tag{149}$$

where the F vector represents the booster response due to the externally applied force vector $\{F_B\}$ and therefore satisfies,

$$\left[M_B\right]\{\ddot{x}_B\}^F + \left[K_B\right]\{x_B\}^F = \{F_B\} \tag{150}$$

and the R vector represents the response of the booster due to the feedback of the payload and satisfies

$$\left[M_B\right]\{\ddot{x}_B\}^R + \left[K_B\right]\{x_B\}^R = \left\{\begin{array}{c} 0 \\ -R_I^B \end{array}\right\} \tag{151}$$

Applying transformation (33) to Eq. (150) yields,

$$\{\ddot{q}_B\}^F + \left[\omega_B^2\right]\{q_B\}^F = \left[\phi_B\right]^T \{F_B\} \tag{152}$$

We now consider Eq. (149) and recall Eq. (4),

$$\left\{\begin{array}{c} x_N^B \\ \hline x_I^B \end{array}\right\} = \left\{\begin{array}{c} x_N^B \\ \hline x_I^B \end{array}\right\}^F + \left\{\begin{array}{c} x_N^B \\ \hline x_I^B \end{array}\right\}^R \tag{153}$$

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Keeping the bottom partition in Eq. (153) yields

$$\left\{x_I^B\ (t)\right\} = \left\{x_I^B\ (t)\right\}^F + \left\{x_I^B\ (t)\right\}^R \tag{154}$$

where we now included the dependence of the vectors on the discrete time t. Taking the Laplace Transform (with zero initial conditions) of both sides of Eq. (154) and introducing the transformation $s = j\Omega$.(with s = Laplace variable; Ω_i = the ith input frequency; and $j = \sqrt{-1}$ we can write

$$\left\{x_I^B\ (j\Omega_i)\right\} = \left\{x_I^B\ (j\Omega_i)\right\}^F + \left\{x_I^B\ (j\Omega_i)\right\}^R, \quad i = 1,2,\ldots.. \tag{155}$$

(= number of input frequencie

which represents Eq. (154) in the frequency domain. Taking the second time derivative of Eq. (155) yields

$$\left\{\ddot{x}_I^B\ (j\Omega_i)\right\} = \left\{\ddot{x}_I^B\ (j\Omega_i)\right\}^F + \left\{\ddot{x}_I^B\ (j\Omega_i)\right\}^R, \quad i = 1,2,\ldots \tag{156}$$

The basic idea of the Impedance Technique is to calculate the interface accelerations $\left\{\ddot{x}_I^B\ (j\Omega_i)\right\}$ in the frequency domain and then transform them back to the discrete time domain. The two terms on the righthand side of Eq. (156) will be replaced by algebraic matrix expressions so that the calculation of $\left\{\ddot{x}_I^B\ (j\Omega_i)\right\}$ does not involve the solution of a set of differential equations. Let us start with the first term on the right-hand side of Eq. (156). To this end let us convert Eq. (152) to the frequency domain,

$$\left\{q_B\ (j\Omega_i)\right\}^F = \left[\frac{1}{\omega_B^2 - \Omega_i^2}\right] \left[\phi_B\right]^T \left\{F_B\ (j\Omega_i)\right\}, \quad i = 1,2,\ldots. \tag{157}$$

or, taking the second time derivative.

$$\left\{\ddot{q}_B\ (j\Omega_i)\right\}^F = \left[\frac{\Omega_i^2}{\Omega_i^2 - \omega_B^2}\right] \left[\phi_B\right]^T \left\{F_B\ (j\Omega_i)\right\}, \quad i = 1,2,\ldots \tag{158}$$

Let us write Eq. (33) as follows,

$$\left\{\begin{array}{c} x_N^B \\ \hline x_I^B \end{array}\right\} = \left[\begin{array}{c} \phi_N^B \\ \hline \phi_I^B \end{array}\right] \left\{q_B\right\} \tag{159}$$

with

$$\left[\phi_B\right] = \left[\begin{array}{c} \phi_N^B \\ \hline \phi_I^B \end{array}\right] \tag{160}$$

Then the bottom partition of Eq. (159) reads,

$$\left\{x_I^B\right\} = \left[\phi_I^B\right] \left\{q_B\right\} \tag{161}$$

Premultiplying Eq. (158) by $\left[\phi_I^B\right]$ and invoking Eq. (161) yields

$$\left\{\ddot{x}_I^B\ (j\Omega_i)\right\}^F = \left[\phi_I^B\right] \left[\frac{\Omega_i^2}{\Omega_i^2 - \omega_B^2}\right] \left[\phi_B\right]^T \left\{F_B\ (j\Omega_i)\right\}, \quad i = 1,2,\ldots. \tag{162}$$

or

$$\left\{\ddot{x}_I^B\ (j\Omega_i)\right\}^F = \left[A\ (j\Omega_i)\right] \left\{F_B\ (j\Omega_i)\right\}, \quad i = 1,2,\ldots. \tag{163}$$

with

$$\left[A\ (j\Omega_i)\right] = \left[\phi_I^B\right]\left[\frac{\Omega_i^2}{\Omega_i^2 - \omega_B^2}\right]\left[\phi_B\right]^T, \quad i = 1,2,\ldots. \tag{164}$$

Equation (163) then, yields the first term on the right-hand side of Eq. (156). The matrix $A\ (j\Omega_i)$ in Eq. (164) is the transfer admittance from the points of application of the external forces $\left\{F_B\right\}$ to the interface accelerations.

Similarily, Eq. (151) can be transformed into

$$\left\{\ddot{x}_I\ (j\Omega_i)\right\}^R = \left[B\ (j\Omega_i)\right]\left\{R_I^P\ (j\Omega_i)\right\}, \quad i = 1,2,\ldots. \tag{165}$$

where this time

$$\left[B\ (j\Omega_i)\right] = \left[\phi_I^B\right]\left[\frac{\Omega_i^2}{\Omega_i^2 - \omega_B^2}\right]\left[\phi_I^B\right]^T, \quad i = 1,2,\ldots. \tag{166}$$

is the matrix of coefficients for the point admittance for the booster at the interface. Equation (165) yields the second term on the right-hand side of Eq (156). However, the reaction vector $\left\{R_I^P\ (j\Omega_i)\right\}$ is not known a priori. To determine this vector let us consider Eq. (148) which represents the payload equations of motion, and write it as

$$\left[\begin{array}{c|c} M_{NN}^P & M_{NI}^P \\ \hline M_{IN}^P & M_{II}^P \end{array}\right]\left[\begin{array}{c} \ddot{x}_N^P \\ \hline \ddot{x}_I^P \end{array}\right] + \left[\begin{array}{c|c} K_{NN}^P & K_{NI}^P \\ \hline K_{IN}^P & K_{II}^P \end{array}\right]\left\{\begin{array}{c} x_N^P \\ \hline x_I^P \end{array}\right\} = \left\{\begin{array}{c} 0 \\ \hline -R_I^P \end{array}\right\} \tag{167}$$

where we used Eqs. (5) and (7). Introducing the modal transformation (40) and taking into account the properties (41) we obtain from Eq. (167)

$$\left[\begin{array}{c|c} I & \bar{\phi}_N^{P^T} M_1^P \\ \hline M_1^{P^T} \bar{\phi}_N^P & M_2^P \end{array}\right] \left\{\begin{array}{c} \ddot{\bar{q}}_N^P \\ \hline \ddot{x}_I^P \end{array}\right\} + \left[\begin{array}{c|c} \bar{\omega}_P^2 & 0 \\ \hline 0 & K_2^P \end{array}\right] \left\{\begin{array}{c} \bar{q}_N^P \\ \hline x_I^P \end{array}\right\} = \left\{\begin{array}{c} 0 \\ \hline -R_I^P \end{array}\right\} \tag{168}$$

with

$$\left[M_1^P\right] = \left[M_{NN}^P\right]\left[S_P\right] + \left[M_{NI}^P\right] \tag{169}$$

$$\left[K_2^P\right] = \left[K_{IN}^P\right]\left[S_P\right] + \left[K_{II}^P\right] \tag{170}$$

and $\left[M_2^P\right]$ given by Eq. (30). Note that $\left[K_2^P\right]$ is zero when the interface is statically determinate.

The top and bottom partitions of Eq. (168) can be written as,

$$\left\{\ddot{\bar{q}}_N^P\right\} + \left[\bar{\omega}_P^2\right]\left\{\bar{q}_N^P\right\} = -\left[\bar{\phi}_N^P\right]^T \left[M_1^P\right]\left\{\ddot{x}_I^P\right\} \tag{171}$$

$$\left[M_1^P\right]^T \left[\bar{\phi}_N^P\right]\left\{\ddot{\bar{q}}_N^P\right\} + \left[M_2^P\right]\left\{\ddot{x}_I^P\right\} + \left[K_2^P\right]\left\{x_I^P\right\} = -\left\{R_I^P\right\} \tag{172}$$

We shall now assume that the interface is statically determinate ($\left[K_2^P\right] = \left[0\right]$) and calculate an expression for $\left\{R_I^P\right\}$ from Eqs. (171-172). First, we transform Eq. (171) to the frequency domain,

$$\left\{\ddot{\bar{q}}_N^P \quad (j\Omega_i)\right\} = -\left[\frac{\Omega_i^2}{\Omega_i^2 - \bar{\omega}_P^2}\right]\left[\bar{\phi}_N^P\right]^T \left[M_1^P\right]\left\{\ddot{x}_I^P \quad (j\Omega_i)\right\}, \quad i = 1,2,\ldots \tag{173}$$

and from Eq. (172) we obtain,

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$$\left[M_1^P\right]^T \left[\bar{\phi}_N^P\right]\left\{\ddot{q}_N^P \quad (j\Omega_i)\right\} + \left[M_2^P\right]\left\{\ddot{x}_I^P \quad (j\Omega_i)\right\} = -\left\{R_I^P \ (j\Omega_i)\right\} \tag{174}$$

$$i = 1,2,\ldots\ldots$$

Substituting Eq. (173) into Eq. (174) yields

$$-\left[M_1^P\right]^T \left[\bar{\phi}_N^P\right]\left[\frac{\Omega_i^2}{\Omega_i^2 - \bar{\omega}_P^2}\right]\left[\bar{\phi}_N^P\right]^T\left[M_1^P\right]\left\{\ddot{x}_I^P \quad (j\Omega_i)\right\}$$

$$+ \left[M_2^P\right]\left\{\ddot{x}_I^P \quad (j\Omega_i)\right\} = -\left\{R_I^P \ (j\Omega_i)\right\}, \quad i=1,2,\ldots \tag{175}$$

from which we obtain the following expression for $\left\{R_I^P \quad (j\Omega_i)\right\}$

$$\left\{R_I^P \ (j\Omega_i)\right\} = \left[C \ (j\Omega_i)\right]\left\{\ddot{x}_I^P \ (j\Omega_i)\right\}, \qquad i = 1,2,\ldots. \tag{176}$$

where

$$\left[C \ (j\Omega_i)\right] = \left[M_1^P\right]^T \left[\bar{\phi}_N^P\right]\left[\frac{\Omega_i^2}{\Omega_i^2 - \bar{\omega}_P^2}\right]\left[\bar{\phi}_N^P\right]^T\left[M_1^P\right] - \left[M_2^P\right] \tag{177}$$

is the impedance matrix of the payload at the payload/booster interface. Finally, we substitute Eq. (176) into Eq. (165),

$$\left\{\ddot{x}_I \ (j\Omega_i)\right\}^R = \left[B \ (j\Omega_i)\right]\left[C \ (j\Omega_i)\right]\left\{\ddot{x}_I^P \ (j\Omega_i)\right\}, \quad i = 1,2,\ldots\ldots \tag{178}$$

Combining Eqs. (156),(163) and (178) yields

$$\left(\left[I\right] - \left[B \ (j\Omega_i)\right]\left[C \ (j\Omega_i)\right]\right)\left\{\ddot{x}_I^B \ (j\Omega_i)\right\} = \left[A \ (j\Omega_i)\right]\left\{F_R \ (j\Omega_i)\right\} \tag{179}$$

$$i = 1,2,\ldots.$$

where we also used Eq. (1)

The coefficient matrix of $\left\{\ddot{x}_I^B(j\Omega_i)\right\}$ in Eq. (179) represents the coupled impedance of the booster/payload system, and the right hand side represents a pseudo generalized force. The interface acceleration can now be computed from Eq. (179) with relative ease.

If we now consider a new payload on the same booster and with the same force $\left\{F_B\right\}$, the right hand side of Eq. (179) does not change so that,

$$\left\{\ddot{x}_I^B(j\Omega_i)\right\}^{(2)} = \left(\left[I\right] - \left[B(j\Omega_i)\right]^{(2)}\left[C(j\Omega_i)\right]^{(2)}\right)^{-1}\left(\left[I\right] - \left[B(j\Omega_i)\right]^{(1)}\left[C(j\Omega_i)\right]^{(1)}\right) \times \left\{\ddot{x}_I(j\Omega_i)\right\}^{(1)} \qquad (180)$$

provided the interface does not change.

The interface accelerations $\left\{\ddot{x}_I(t)\right\}$ in the discrete time domain can now be derived from Eq. (179) or Eq. (180), using the inverse Laplace Transform. The payload response then follows from Eq. (171).

The approximation involved in the Impedance Technique is imbedded in the transformation to and from the frequency domain. If these transformations were exact, the method of determining $\left\{\ddot{x}_I(t)\right\}$ would be exact. Therefore, one of the objectives of Task II. Methodology Development, should be a detailed investigation of these transformations. There are also problems pertaining to the modal damping when working in the frequency domain.

Although Eqs. (179) and (180) were derived for an undamped statically determinate system, it is clear that damping and statically indeterminate interfaces can be included. For an indeterminate system it becomes necessary to keep track not only of the interface accelerations but also of the velocities and displacements at the interface. The use of the Fast Fourier Transform in obtaining the spectral data to be used in Eqs. (179-180) also presents some problems. In general, however, enough correlation with the exact time domain solution is apparent to warrant further investigation into possible improvements.

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OF POOR QUALITY

5. THE GENERALIZED SHOCK SPECTRUM TECHNIQUE [72, 73]

The approach presented in this section is a generalized version of the shock spectrum technique as developed by Bamford [72].

In order to explain the basic ideas underlying this technique let us recall equation (44)

$$\left[\begin{array}{c:c} I+\phi_B^T T_P^T M_P T_P \phi_B & \phi_B^T T_P^T M_P I_P \bar{\phi}_N^P \\ \hdashline \bar{\phi}_N^{P\,T} I_P^T M_P T_P \phi_B & I \end{array}\right] \left\{\begin{array}{c} \ddot{q}_B \\ \hdashline \ddot{\bar{q}}_N^P \end{array}\right\} + \left[\begin{array}{c:c} \left[\omega_B^2\right] + \phi_B^T T_P^T K_P T_P \phi_B & 0 \\ \hdashline 0 & \left[\bar{\omega}_P^2\right] \end{array}\right] \left\{\begin{array}{c} q_B \\ \hdashline \bar{q}_N^P \end{array}\right\} = \left\{\begin{array}{c} \phi_B^T \left\{\begin{array}{c} F_N^B \\ 0 \end{array}\right\} \\ \hdashline 0 \end{array}\right\} \tag{181}$$

and let us assume we retained M modes for the booster (ie $\left[\phi_B\right]$ is an MxM matrix) and N modes for the payload (ie $\left[\bar{\phi}_N^P\right]$ is an NxN matrix)

The basic idea of the shock spectrum technique is to determine load maxima without having to solve Eq. (181). To reach this goal, a new model both for the coupled system and the forcing function $\left\{F_B\right\}$ is introduced.

First, the (N + M) modally coupled Eqs.(181) are replaced by (N x M) sets of two simultaneous equations each of which represents the coupling of one payload mode with one booster mode, as follows,

$$\left[\begin{array}{c:c} 1 + \left\{\phi_B\right\}_i^T I_P^T M_P T_P \left\{\phi_B\right\}_i & \left\{\phi_B\right\}_i^T T_P^T M_P I_P \left\{\bar{\phi}_N^P\right\}_j \\ \hdashline \left\{\bar{\phi}_N^P\right\}_j^T I_P^T M_P T_P \left\{\phi_B\right\}_i & 1 \end{array}\right] \left\{\begin{array}{c} \ddot{q}_{Bi} \\ \hdashline \ddot{\bar{q}}_{Nj}^B \end{array}\right\}$$

$$+ \left[\begin{array}{c:c} \omega_{Bi}^2 & 0 \\ \hdashline 0 & \bar{\omega}_{Pj}^2 \end{array}\right] \left\{\begin{array}{c} q_{Bi} \\ \hdashline \bar{q}_{Nj}^B \end{array}\right\} = \left\{\begin{array}{c} \left\{\phi_B\right\}_i^T \left\{\begin{array}{c} F_N^B \\ 0 \end{array}\right\} \\ \hdashline 0 \end{array}\right\} \qquad \begin{array}{l} i=1,2,\dots M \\ j=1,2,\dots N \end{array} \tag{182}$$

where we assumed that the interface is statically determinate (ie $\left[T_P^T K_P T_P\right] = \left[0\right]$)

Secondly, a bound q_{BP} on each of the (N x M) modal responses of the payload is established. This is done by introducing a new model for the forcing function in Eq. (182). The rather complicated forcing function is replaced by a much simpler function (e.g. an impulsive force) which produces the same maximum response peak as the original force. Therefore, an analytical solution for both the response and maximum response of Eq. (182) is possible (after some additional simplifications). Finally, a bound q_P on the total payload response is constructed by summing over all the individual modal bounds q_{BP} (over absolute values or in a root-sum-square sense that allows for phase weighting). Payload member loads are obtained by adding the contributions of all payload modal loads.

As stated above, the forcing function in Eq. (182) is replaced by a modal delta function of a certain magnitude F_B . This magnitude F_B is evaluated from an already existing transient analysis of the booster with or without a dummy payload.

The main objections that can be raised against the Shock Spectrum Approach are:

1) No critical evaluation is available on the validity of replacing model (181) by model (182). What effect does this replacement have on the load bounds? This change of model could not only result in a too conservative design but also in an unconservative one. Model (182) not only ignores the coupling between the B-modes due to rigid body feedback of P , but more importantly it ignores the effects that the coupling of one B-mode with one P-mode has on all the other P -modes.

2) The manner in which F_S is calculated again leaves the question of whether or not the envelope values are conservative or not and by how much.

3) The method appears rather complicated and is not simple to use. This can lead to misinterpretation and confusion when the method is applied. More rigor in the mathematical formulation is desirable.

For these and some other minor reasons we do not favor further investigation of the Shock Spectrum Technique unless a better version appears that answers our basic objections.

CHAPTER III: PROPOSED IMPROVEMENTS OF EXISTING SHORT CUT METHODS - A NEW APPROACH

1. INTRODUCTION

In Chapter I we reviewed and assessed four prominent "full-scale" methodologies. This allowed us to introduce the necessary background material in terms of a unified nomenclature. After careful evaluation we can state that all four methods have their merits. However, the Residual Mass and Stiffness Method appears to be the most efficient general purpose method, as discussed in section 9 of Chapter I. It is the full-scale method which best describes the booster structure in terms of a minimum number of modes, given a certain cut-off frequency for the externally applied force $\left\{F_B\right\}$. The fact that no payload information is required to obtain the booster model is a very convenient feature in connection with the present study. Therefore, the same booster model can be used as long as the booster does not change.

The Residual Mass and Stiffness Method can be used for comparison purposes when future short-cut methods are evaluated. Moreover, most short-cut methods require a full-scale "start-solution" before they can be applied, which demonstrates the need for an efficient full-scale method. Also, some of the short-cut methods (e.g. Base Drive Method) are based on their full-scale parent method.

In Chapter II four general short-cut methods have been discussed and evaluated. Although each of these methods has its own merits, it is believed that none of them is acceptable in their present stage of development to function as a standard short-cut method for general use. It is the purpose of this Chapter III to propose several possible improvements of these techniques. In addition, we also wish to present a new approach. Although still being developed, this new approach shows great promise.

2. THE BASE MOTION TECHNIQUE

The Coupled Base Motion Technique as explained in Chapter I-7, leads to the fundamental set of equations (138-140), which we repeat here for clarity of presentation,

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$$\left[I_B^T M_B I_B\right]\left\{\ddot{\bar{x}}_N^{BR}\right\} + \left[I_B^T K_B I_B\right]\left\{\bar{x}_N^{BR}\right\} = -\left[I_B^T M_B T_B\right]\left\{\ddot{x}_I^{BR}\right\} \tag{183}$$

$$\left[I_P^T M_P I_P\right]\left\{\ddot{\bar{x}}_N^{P}\right\} + \left[I_P^T K_P I_P\right]\left\{\bar{x}_N^{P}\right\} = -\left[I_P^T M_P T_P\right]\left(\left\{\ddot{x}_I^{BF}\right\} + \left\{\ddot{x}_I^{BR}\right\}\right) \tag{184}$$

$$\begin{aligned}\left\{\ddot{x}_I^{BR}\right\} = \left[T_B^T M_B T_B + T_P^T M_P T_P\right]^{-1} \Big(& -\left[T_P^T M_P T_P\right]\left\{\ddot{x}_I^{BF}\right\} - \left[T_P^T K_P T_P\right]\left\{x_I^{BF}\right\} \\ & - \left[T_P^T M_P I_P\right]\left\{\ddot{\bar{x}}_N^{P}\right\} - \left[T_B^T M_B I_B\right]\left\{\ddot{\bar{x}}_N^{BR}\right\} - \left[T_B^T K_B T_B + T_P K_P T_P\right]\left\{x_I^{BR}\right\}\Big)\end{aligned} \tag{185}$$

As mentioned in Chapter I-8, the payload designer is primarily interested in the response of the payload i.e.,

$$\begin{Bmatrix} x_N^P \\ \hline x_I^P \end{Bmatrix} = \left[\begin{array}{c|c} I & S_P \\ \hline 0 & I \end{array}\right] \begin{Bmatrix} \bar{x}_N^P \\ \hline x_I^P \end{Bmatrix} \tag{186}$$

with

$$\begin{Bmatrix} \bar{x}_N^P \\ \hline x_I^P \end{Bmatrix} = \begin{Bmatrix} \bar{x}_N^P \\ \hline x_I^{BF} + x_I^{BR} \end{Bmatrix} \tag{187}$$

where $\left\{\bar{x}_N^P\right\}$ and $\left\{x_I^{BR}\right\}$ must be computed from Eqs. (183-185) and $\left\{x_I^{BF}\right\}$ from Eq. (86). The base drive method focuses on Eq. (184) which yields $\left\{\bar{x}_N^P\right\}$ provided $\left\{\ddot{x}_I^{BR}\right\}$ on the right hand side of Eq. (184) is known. The idea is to produce an expression for $\left\{\ddot{x}_I^{BR}\right\}$ without actually solving the coupled set of equations (183-185).

First, consider the coefficients of $\left\{\ddot{\bar{x}}_N^P\right\}$ and $\left\{\ddot{\bar{x}}_I^{BF}\right\}$ in Eq. (185). These coefficients represent the ratio of the payload mass and the total vehicle mass. In many STS applications this ratio will be rather small (<10%). Therefore, a first possibility is to ignore these terms in Eq. (185). Secondly, in many applications we can assume a statically determinate interface, i.e. $\left[T_B^T K_B T_B\right] = \left[T_P^T K_P T_P\right] = \left[0\right]$, so that Eq. (185) becomes

$$\left\{\ddot{x}_I^{BR}\right\} = -\left[T_B^T M_B T_B + T_P^T M_P T_P\right]^{-1} \left[T_B^T M_B I_B\right]\left\{\ddot{\bar{x}}_N^{BR}\right\} \tag{188}$$

Ordinarily, the coefficient matrix of $\left\{\ddot{\bar{x}}_N^{BR}\right\}$ in Eq. (188) is not small and cannot be ignored. A first possibility is to assume that $\left\{\ddot{\bar{x}}_N^{BR}\right\}$ is small and can be ignored. This means that the feedback of the payload is not important. This can be a realistic assumption because the payload is usually small compared to the booster. In this case we can completely ignore $\left\{\ddot{x}_I^{BR}\right\}$ in Eq. (184) and write

$$\left[I_P^T M_P I_P\right]\left\{\ddot{x}_N^{P}\right\} + \left[I_P^T K_P I_P\right]\left\{\bar{x}_N^{P}\right\} = -\left[I_P^T M_P T_P\right]\left\{\ddot{x}_I^{BF}\right\} \tag{189}$$

Equation (189) is now effectively decoupled from Eqs. (183) and (185). Physically, ignoring the feedback of the payload means that payload and booster are not modally coupled. The question is, when such an approach is valid. This is one of the points to be investigated in the future.

A second possibility is to scale the vector $\left\{\ddot{\bar{x}}_N^{BR}\right\}$ in Eq. (188). Indeed, let us assume a full-scale solution is available for some payload P_1 Now, some relatively small changes are made in the payload P_1 to generate payload P. The assumption now, is that $\left\{\ddot{\bar{x}}_N^{BR}\right\}$ is not much different from $\left\{\ddot{\bar{x}}_N^{BR}\right\}_1$ i.e.

$$\left\{\ddot{\bar{x}}_N^{BR}\right\} \simeq \left\{\ddot{\bar{x}}_N^{BR}\right\}_1 \tag{190}$$

Equation (188) for payload P1 can be written as

$$\left\{\ddot{x}_I^{BR}\right\}_1 = -\left[T_B^T M_B T_B + T_{P1}^T M_{P1} T_{P1}\right]^{-1} \left[T_B^T M_B I_B\right]\left\{\ddot{\bar{x}}_N^{BR}\right\}_1 , \tag{191}$$

or

$$\left[T_B^T M_B I_B\right]\left\{\ddot{\bar{x}}_N^{BR}\right\}_1 = -\left[T_B^T M_B T_B + T_{P1}^T M_{P1} T_{P1}\right]\left\{\ddot{x}_I^{BR}\right\}_1 \tag{192}$$

Taking into account Eq. (190) it follows from Eqs. (192) and (188) that

$$\left\{\ddot{x}_I^{BR}\right\} = \left[T_B^T M_B T_B + T_P^T M_P T_P\right]^{-1}\left[T_B^T M_B T_B + T_{P1}^T M_{P1} T_{P1}\right]\left\{\ddot{x}_I^{BR}\right\}_1 \tag{193}$$

which yields a scaled value for $\left\{\ddot{x}_I^{BR}\right\}$ to be used in Eq. (184). Again, one should investigate when such an approach is valid.

One way to improve the Direct Base Drive approach is to use a perturbation technique such as the one discussed in Chapter II-2. This could at the same time reveal when a Direct Base Drive is valid or not. Indeed, an asymptotic expansion of $\left\{\bar{x}_N^P\right\}$ could reveal the magnitudes of the terms in ε, ε^2 etc. It should then be possible to decide if and when the zero-order term is sufficient or not to represent the response of the payload.

Another possible route of investigation is given by the modal form of Eqs. (87), namely Eqs. (94),

$$\left\{\ddot{\bar{q}}_N^{BR}\right\} + \left[\bar{\omega}_B^2\right]\left\{\bar{q}_N^{BR}\right\} = -\left[\bar{\phi}_N^{B^T} I_B^T M_B T_B \phi_I^B\right]\left\{\ddot{q}_I^{BR}\right\} \tag{194}$$

$$\left\{\ddot{\bar{q}}_N^{P}\right\} + \left[\omega_P^2\right]\left\{\bar{q}_N^{P}\right\} = -\left[\bar{\phi}_N^{P^T} I_P^T M_P T_P\right]\left(\left\{\ddot{x}_I^{BF}\right\} + \left[\phi_I^B\right]\left\{\ddot{q}_I^{BR}\right\}\right) \tag{195}$$

$$\begin{aligned}\left\{\ddot{q}_I^{BR}\right\} = &-\left[\omega_I^2\right]\left\{q_I^{BR}\right\} - \left[\phi_I^{B^T} T_B^T M_B I_B \bar{\phi}_N^B\right]\left\{\ddot{\bar{q}}_N^{BR}\right\} - \left[\phi_I^{B^T} T_P^T M_P I_P \bar{\phi}_N^P\right]\left\{\ddot{\bar{q}}_N^{P}\right\} \\ &- \left[\phi_I^{B^T} T_P^T M_P T_P\right]\left\{\ddot{x}_I^{BF}\right\} - \left[\phi_I^{B^T} T_P^T K_P T_P\right]\left\{x_I^{BF}\right\}\end{aligned} \tag{196}$$

Once $\{\ddot{q}_I^{BR}\}$ is known in Eqs. (194) and (195) we have a decoupled system of equations which can be easily solved. The idea is to start an iterative process with $\{\ddot{q}_I^{BR}\}=\{0\}$ (i.e. Direct Base Drive) and solve for $\{\bar{q}_N^{BR}\}$ and $\{\bar{q}_N^{P}\}$ in Eqs. (194) and (195) which would be easy to do as mentioned above. These vectors are now used to compute a new value for $\{\ddot{q}_I^{BR}\}$ from Eq. (196). This value is then used in Eqs. (194-195) and the process is repeated until a satisfactory solution is obtained. Hopefully, a fast convergence scheme can be developed. In the meantime, further research showed that no iteration process is necessary and a direct solution is possible. We shall discuss this approach in a future report.

In conclusion, it can be said that the set of Eqs. (183-185) or any equivalent set offer several possibilities for the development of an adequate short-cut method. The main problem is the development of criteria of validity for the several assumptions that must be made.

3. THE IMPEDANCE TECHNIQUE

As stated in Chapter II-4, the Impedance Technique is basically a Base Motion Technique. A set of suitable interface accelerations are given by Eq. (179),

$$\left\{\ddot{x}_I^B\ (j\Omega_i)\right\} = \left(\left[I\right] - \left[B\ (j\Omega_i)\right]\left[C(j\Omega_i)\right]\right)^{-1}\left[A(j\Omega_i)\right]\left\{F_B(j\Omega_i)\right\}_1 \tag{197}$$

or, when the booster, booster forces and interface do not change we also have Eq. (180),

$$\left\{\ddot{x}_I^B\ (j\Omega_i)\right\}^{(2)} = \left(\left[I\right] - \left[B(j\Omega_i)\right]^{(2)}\left[C(j\Omega_i)\right]^{(2)}\right)^{-1} \left(\left[I\right] - \left[B(j\Omega_i)\right]^{(1)}\left[C(j\Omega_i)\right]^{(1)}\right)\left\{\ddot{x}_I\ (j\Omega_i)\right\}^{(1)} \tag{198}$$

The Impedance Technique is an exact method, in the sense that the interface accelerations given by Eqs. (197-198) are exact. The approximation lies in the transformations from the discrete time domain to the frequency domain and back again. Indeed, problems were encountered with regard to the modeling of damping and the application of the Fast Fourier Transform.

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The outstanding feature of the Impedance Technique is that no assumption is made as to the size of the payload. Also, the relative ease with which the interface accelerations are determined is a very attractive property of this technique. However, further research is needed in the area of modeling the damping and in the area of conversion from the time domain to the frequency domain and vice versa. Also, we would like to investigate the possible application of a perturbation approach in case the payload and/or the changes in the payload are small.

4. A NEW TECHNIQUE [74]

In the course of our investigation and evaluation of several short-cut methods it was noted that many methods involve assumptions and approximations leading to either doubtful or cumbersome results. In addition, it is often very difficult to assess the effects of those assumptions on the response and the loads of the booster/payload system.

The basic problem is to somehow deal with the coupling effects between booster B and payload P without solving an eigenvalue problem pertaining to the coupled booster/payload system. This is a difficult problem indeed. Each of the short-cut methods discussed in Chapter II addresses this problem in a different way. However, the proposed solutions invariably lead to cumbersome mathematics and program coding. This observation led us to the development of a more direct approach which we think shows great promise. This new approach is easy to understand and easy to implement. It is based on the work of C. W. White and B. D. Maytum [74]. Although the theory is still being developed we already can present the basic philosophy.

Let us recall Eq. (28)

$$\left[\begin{array}{c|c} M_B+T_P^TM_PT_P & T_P^TM_PI_P \\ \hline I_P^TM_PT_P & I_P^TM_PI_P \end{array}\right]\left\{\begin{array}{c} \ddot{x}_B \\ \hline \ddot{x}_N^P \end{array}\right\} + \left[\begin{array}{c|c} K_B+T_P^TK_PT_P & 0 \\ \hline 0 & I_P^TK_PI_P \end{array}\right]\left\{\begin{array}{c} x_B \\ \hline x_N^P \end{array}\right\} = \left\{\begin{array}{c} \overset{F_N^B}{0} \\ \hline 0 \end{array}\right\} \qquad (199)$$

which represents the set of equations of motion of the coupled booster/payload system. It is now assumed that a cut-off frequency is defined based on a Fourier series expansion of $\left\{F_N^B\right\}$. Furthermore, we also assume that e.g. the Residual Mass and Stiffness Method was used to construct the following set of modally coupled equations.

$$\left[\begin{array}{c|c} I+\phi_B^T T_P^T M_P T_P \phi_B & \phi_B^T T_P^T M_P T_P \bar{\phi}_N^P \\ \hline \bar{\phi}_N^{P\,T} T_P^T M_P T_P \phi_B & I \end{array}\right]\left\{\begin{array}{c} \ddot{q}_B \\ \hline \ddot{\bar{q}}_N^P \end{array}\right\} + \left[\begin{array}{c|c} \left[\omega_B^2\right] + \phi_B^T T_P^T K_P T_P \phi_B & \\ \hline 0 & \left[\bar{\omega}_P^2\right] \end{array}\right]\left\{\begin{array}{c} q_B \\ \hline \bar{q}_N^P \end{array}\right\}$$

$$= \left\{\begin{array}{c} \phi_B^T \left\{\begin{array}{c} F_N^B \\ 0 \end{array}\right\} \\ \hline 0 \end{array}\right\} \tag{200}$$

where

$$\left\{x_B\right\} = \left[\phi_B\right]\left\{q_B\right\}, \quad \left\{\bar{x}_N^P\right\} = \left[\bar{\phi}_N^P\right]\left\{\bar{q}_N^P\right\} \tag{201}$$

and the cut-off frequency was used to determine the size of $\left[\phi_B\right]$ and $\left[\bar{\phi}_N^P\right]$ according to e.g. Chapter I-5. In other words, the size of Eq. (200) is already much less than the size of Eq. (199). Due to e.g. the Residual Mass and Stiffness Method, the reduced Eq. (200) still represents an acceptable model for the coupled booster/payload system.

The first step of the present approach is to solve the eigenvalue problem associated with Eq. (200), namely

$$\left(-\Omega^2 \left[\begin{array}{c|c} I+\phi_B^T T_P^T M_P T_P \phi_B & \phi_B^T T_P^T M_P T_P \phi_N^P \\ \hline \bar{\phi}_N^{P\,T} T_P^T M_P T_P \phi_B & I \end{array}\right] + \left[\begin{array}{c|c} \left[\omega_B^2\right] + \phi_B^T T_P^T K_P T_P \phi_B & \\ \hline 0 & \left[\bar{\omega}_B^2\right] \end{array}\right]\right)\left\{\psi\right\} = \left\{0\right\} \tag{202}$$

yielding a set of modes $\left[\psi\right]$ and a set of frequencies $\left[\Omega^2\right]$ satisfying

$$\begin{Bmatrix} q_B \\ \bar{q}_N^P \end{Bmatrix} = [\Psi]\{u\} \tag{203}$$

and

$$[\Psi]^T \left[\begin{array}{c:c} I+\phi_B^T T_P^T M_P T_P \phi_B & \phi_B^T T_P^T M_P I_P \bar{\phi}_N^P \\ \hdashline \bar{\phi}_N^{P\,T} I_P^T M_P T_P \phi_B & I \end{array}\right] [\Psi] = \left\lceil I \right\rfloor \tag{204}$$

$$[\Psi]^T \left[\begin{array}{c:c} \left\lceil \omega_B^2 \right\rfloor + \phi_B^T T_P^T K_P T_P \phi_B & \\ \hdashline 0 & \left\lceil \bar{\omega}_P^2 \right\rfloor \end{array}\right] [\Psi] = \left\lceil \Omega^2 \right\rfloor \tag{205}$$

where $\{u\}$ are the new normal coordinates. Substituting transformation (203) into Eq. (200) and premultiplying by $[\Psi]^T$ and using Eqs. (204-205) we obtain the uncoupled set of equations,

$$\{\ddot{u}\} + \left\lceil \Omega^2 \right\rfloor \{u\} = [\Psi]^T \begin{Bmatrix} \phi_B^T \begin{Bmatrix} F_N^B \\ 0 \end{Bmatrix} \\ \hdashline 0 \end{Bmatrix} \tag{206}$$

The modal matrix $[\Psi]$ and the frequency matrix $\left\lceil \Omega^2 \right\rfloor$ represent the modal information of the coupled booster/payload system. The idea now is to change the payload and calculate the changes in $[\Psi]$ and $\left\lceil \Omega^2 \right\rfloor$. In other words, we use the full-scale solution of Eq. (200) as a "start-solution". This approach is taken in most short-cut methods and as such does not detract from the present approach. For example, this full-scale solution could be determined at the beginning of a design effort and would stay the same for all subsequent design cycles of a particular payload.

Let us now consider a new payload P1, with mass matrix $[K_{P1}]$ and stiffness matrix $[M_{P1}]$. This payload P1 could be totally new or just a modification of the nominal payload P, as long as we have the same degrees of freedom for both payloads P and P1. For this new payload P1, we replace Eq. (199) by

$$\left[\begin{array}{c|c} M_B+T_{P1}^T M_{P1} T_{P1} & T_{P1}^T M_{P1} I_{P1} \\ \hline I_{P1}^T M_{P1} T_{P1} & I_{P1}^T M_{P1} I_{P1} \end{array}\right] \left\{\begin{array}{c} \ddot{x}_{B1} \\ \hline \ddot{\bar{x}}_N^{P1} \end{array}\right\} + \left[\begin{array}{c|c} K_B+T_{P1}^T K_{P1} T_{P1} & 0 \\ \hline 0 & I_{P1}^T K_{P1} I_{P1} \end{array}\right] \left\{\begin{array}{c} x_{B1} \\ \hline \bar{x}_N^{P1} \end{array}\right\} = \left\{\begin{array}{c} F_N^{B1} \\ 0 \\ \hline 0 \end{array}\right\} \tag{207}$$

where

$$\left[S_{P1}\right] = -\left[K_{NN}^{P1}\right]^{-1}\left[K_{NI}^{P1}\right] \tag{208}$$

is the new transformation matrix and

$$\left\{\begin{array}{c} x_{B1} \\ \hline \bar{x}_N^{P1} \end{array}\right\} \tag{209}$$

is the new system displacement vector.

Let us now writs Eq. (207) as follows

$$\left[\begin{array}{c|c} M_B+T_P^T M_P T_P+m_{BB} & T_P^T M_P I_P+m_{BP} \\ \hline I_P^T M_P T_P+m_{BP}^T & I_P^T M_P I_P+m_{PP} \end{array}\right] \left\{\begin{array}{c} \ddot{x}_{B1} \\ \hline \ddot{\bar{x}}_N^{P1} \end{array}\right\} + \left[\begin{array}{c|c} K_B+T_P^T K_P T_P+k_{BB} & 0 \\ \hline 0 & I_P^T K_P I_P+k_{PP} \end{array}\right] \left\{\begin{array}{c} x_{B1} \\ \hline \bar{x}_N^{P1} \end{array}\right\}$$

$$= \left\{\begin{array}{c} F_N^{B1} \\ 0 \\ \hline 0 \end{array}\right\} \tag{210}$$

where

$$\left[m_{BB}\right] = \left[T_{P1}^T M_{P1} T_{P1}\right] - \left[T_P^T M_P T_P\right] \tag{211}$$

$$\left[m_{BP}\right] = \left[T_{P1}^T M_{P1} I_{P1}\right] - \left[T_P^T M_P I_P\right] = \left[(T_{P1}^T M_{P1} - T_P^T M_P)\, I_P\right] \tag{212}$$

$$\left[m_{PP}\right] = \left[I_{P1}^T M_{P1} I_{P1}\right] - \left[I_P^T M_P I_P\right] = \left[I_P^T\ (M_{P1} - M_P)\ I_P\right] \tag{213}$$

$$\left[k_{BB}\right] = \left[T_{P1}^T K_{P1} T_{P1}\right] - \left[T_P^T K_P T_P\right] \tag{214}$$

$$\left[k_{PP}\right] = \left[I_{P1}^T K_{P1} I_{P1}\right] - \left[I_P^T K_P I_P\right] = \left[I_P\ (K_{P1} - K_P)\ I_P\right] \tag{215}$$

Note that in case the interface is statically determinate $\left[k_{BB}\right] = \left[0\right]$ and if in addition the geometry of the payload is not changed then $\left[T_{P1}\right] = \left[T_P\right]$ and

$$\left[m_{BB}\right] = \left[T_P^T\ (M_{P1} - M_P)\ T_P\right] \tag{216}$$

$$\left[m_{BP}\right] = \left[T_P^T\ (M_{P1} - M_P)\ I_P\right] \tag{217}$$

$$\left[m_{PP}\right] = \left[I_P^T\ (M_{P1} - M_P)\ I_P\right] \tag{218}$$

$$\left[k_{BB}\right] = \left[0\right] \tag{219}$$

$$\left[k_{PP}\right] = \left[I_P\ (K_{P1} - K_P)\ I_P\right] \tag{220}$$

Also, note that if no changes are made in the mass the right hand sides of Eqs. (213) and (216-219) become $[0]$ and similarly, if no stiffness changes are made we have from Eq. (208) that $\left[T_P\right] = \left[T_{P1}\right]$ and consequently Eqs. (216-219) are valid while Eq (220) becomes $\left[k_{PP}\right] = [0]$ although the interface can still be statically indeterminate.

Next, let us define the following transformation

$$\left\{\begin{array}{c} x_{B1} \\ \hline \bar{x}_N^{P1} \end{array}\right\} = \left[\begin{array}{c|c} \phi_B & 0 \\ \hline 0 & \bar{\phi}_N^{P} \end{array}\right] \left\{\begin{array}{c} q_{B1} \\ \hline \bar{q}_N^{P1} \end{array}\right\} = \left[A\right] \left\{\begin{array}{c} q_{B1} \\ \hline \bar{q}_N^{P1} \end{array}\right\} \tag{221}$$

After substituting Eq (221) into Eq. (210) and premultiplying by $[A]^T$ we obtain the set of equations that now replaces the set Eq. (200)

$$\left[\begin{array}{c|c} I+\phi_B^T T_P^T M_P T_P \phi_B + \phi_B^T m_{BB} \phi_B & \phi_B^T T_P^T M_P I_P \bar{\phi}_N^P + \phi_B^T m_{BP} \bar{\phi}_N^P \\ \hline \phi_N^{P^T} I_P M_P T_P \phi_B + \bar{\phi}_N^{P^T} m_{BP}^T \phi_B & I + \bar{\phi}_N^{P^T} m_{PP} \bar{\phi}_N^P \end{array}\right] \left\{\begin{array}{c} \ddot{q}_{B1} \\ \hline \ddot{\bar{q}}_N^{P1} \end{array}\right\}$$

$$+ \left[\begin{array}{c|c} \left[\omega_B^2\right] + \phi_B^T T_P^T K_P T_P \phi_B + \phi_B^T k_{BB} \phi_B & 0 \\ \hline 0 & \left[\bar{\omega}_P^2\right] + \bar{\phi}_N^{P^T} k_{PP} \bar{\phi}_N^P \end{array}\right] \left\{\begin{array}{c} q_{B1} \\ \hline \bar{q}_N^{P1} \end{array}\right\} = \left\{\begin{array}{c} \phi_B^T \left\{\begin{array}{c} F_N^{B1} \\ 0 \end{array}\right\} \\ \hline 0 \end{array}\right\} \tag{222}$$

The next step is to define the transformation

$$\left\{\begin{array}{c} q_{B1} \\ \hline \bar{q}_N^{P1} \end{array}\right\} = \left[\Psi\right] \left\{u_1\right\} \tag{223}$$

The transformation (223) is now substituted into Eq. (222) after which we premultiply by $\left[\Psi\right]^T$ and invoke properties (204-205), yielding

$$\left(\left[I\right] + \left[\Psi\right]^T \left[\begin{array}{c|c} \phi_B^T m_{BB} \phi_B & \phi_B^T m_{BP} \bar{\phi}_N^P \\ \hline \bar{\phi}_N^{P\,T} m_{BP}^T & \bar{\phi}_N^{P\,T} m_{PP} \bar{\phi}_N^P \end{array}\right] \left[\Psi\right] \right) \left\{ \ddot{u}_1 \right\}$$

$$+ \left(\left[\Omega^2\right] + \left[\Psi\right]^T \left[\begin{array}{c|c} \phi_B^T k_{BB} \phi_B & \\ \hline 0 & \bar{\phi}_N^{P\,T} k_{PP} \bar{\phi}_N^P \end{array}\right] \left[\Psi\right] \right) \left\{ u_1 \right\} = \left[\Psi\right]^T \left\{ \begin{array}{c} \phi_B^T \left\{ \begin{array}{c} F_N^{B1} \\ 0 \end{array} \right\} \\ \hline 0 \end{array} \right\} \tag{224}$$

This Eq (224) replaces Eq (206). For convenience, let us denote

$$\left[m\right] = \left[\Psi\right]^T \left[\begin{array}{c|c} \phi_B^T m_{BB} \phi_B & \phi_B^T m_{BP} \bar{\phi}_N^P \\ \hline \bar{\phi}_N^{P\,T} m_{BP}^T \phi_B & \bar{\phi}_N^{P\,T} m_{PP} \bar{\phi}_N^P \end{array}\right] \left[\Psi\right] \tag{225}$$

$$\left[k\right] = \left[\Psi\right]^T \left[\begin{array}{c|c} \phi_B^T k_{BB} \phi_B & 0 \\ \hline 0 & \bar{\phi}_N^{P\,T} k_{PP} \bar{\phi}_N^P \end{array}\right] \left[\Psi\right] \tag{226}$$

Matrices $[M]$ and $[k]$ represent the perturbations in the mass and stiffness matrices $[I]$ and $[\Omega^2]$ of system (206). At this point, several observations can be made. First, it should be noted that it is very possible that certain changes in the payload will only affect a limited number of modes and frequencies. This means that several columns in $[\Psi]$ and corresponding elements in $[\Omega^2]$ will not change after the changes in the payload are made. This reduces the size of Eq. (224). Secondly, in solving the eigenvalue problem associated with Eq. (224) it is possible to use a Rayleigh-Ritz approach with $[I]$ as the estimated start modes. The smaller the changes in the payload the better estimate $[I]$ will be and the less iterations will be necessary to produce the new modes and frequencies of the perturbed booster

B/Payload P1 system. An even better starting set of modes could be the solution to the perturbed eigenvalue problem with all off-diagonal terms equal to zero (this is equivalent to the first term in a Taylor series expansion of the perturbed system modes and frequency). Thirdly, we wish to investigate the possibility of truncating modes in $[\Psi]$ according to the initially defined cut-off frequency. If this was possible Eq. (224) could be reduced in size by approximately 50% compared to the already reduced system Eq. (200). This reduction would be in addition to the one due to unaffected modes as mentioned above. However, this question must still be carefully investigated. Finally, it is also possible that the modes are grouped in subsets which show very little or no coupling between each other. This means that the eigenvalue problem associated with Eq. (224) can be replaced by two or more smaller eigenvalue problems, which of course reduces the computation time.

There are additional advantages to this method: simplicity of use; accuracy of results (e.g. this method could even be used as a full-scale method); possibility of using engineering judgement and experience; the possibility to identify changes required to meet certain frequency requirements; the possibility to change branch frequencies to decouple the load problem leading to smaller eigenvalue problems, the potential for significant computational time savings.

Note that it is also possible to solve eigenvalue problem Eq. (202) for the perturbed system, using the coupled modes of the unperturbed system as a first guess in a Rayleigh-Ritz type solution.

Finally, we will investigate the possible combination of this technique with the Base Motion Approach.

5. CONCLUSION

The purpose of this report is to define existing methodologies, evaluate their effectiveness in analyzing dynamically coupled structural systems, and to define an approach where a "short-cut" methodology may be derived in Study Task II. This goal was defined within our proposal:

"Our approach to validation of existing "short-cut" methodologies will be to disect the accepted state of the art mathematical description of a coupled payload/booster system to identify the various interactive forces that arise as a result of that coupling. The merits of each methodology will be judged upon the degree to which they represent these interactions and in relation to the costs (in elapsed time and computer dollars) required by the method".

The previous chapters have definitized the problem by reviewing full scale methodologies (Chapter I), assessment of strengths and weaknesses of defined "short-cut" methods (Chapter II) and our proposed approaches to define an accurate and useable "short-cut" methodology.

The following critical comments reflect detailed evaluations of the reviewed techniques. It should be noted only sensory mathematical studies were completed on these methods. During evaluation of derived techniques, Study Task III, these methods will be reassessed with respect to the newly derived techniques.

Perturbation Technique

The driving principal of this technique is that small changes in the mass and stiffness results in small changes in the modal characteristics. However, this is not a sufficient mathematical premise to assure small changes in the response. This method shows some promise and will be included within the methodology development of Study Task II.

Base Drive Technique

Analytically the Base Drive Technique assumes no coupling between the payload and launch vehicle. The principal short coming of this method is a lack of adequate definition that this "structural feedback" is negligible. The development of this criteria is planned in Study Task II.

Impedance Technique

This technique is similar to a base drive method with the unknown criteria involving frequency transformations. This technique will be checked out through application to a perturbation technique in Study Task III, but will not be seriously considered during Study Task II.

Generalized Shock Spectrum Technique

The assumption that the total response for a single coupled mode can be related by coupling a single payload mode with an associate booster mode is questionable. Although the applied forces are replaced by modal delta functions, it is not clear to the author that this would assure conservativeness. In fact, our opinion is that these model changes could result in either a too conservative or unconservative design. The application of a generalized force even though based on historical data is difficult to assure proper conservativeness. Utilizing these forces may eliminate frequency dependence of the force but what direct influence this has on the dynamic response is unanswered. Additionally, due to the numerous mathematical assumptions, more rigor in the mathematical formulation is required. The generalized shock spectrum technique could be useful in situations where weight considerations are not crucial (e.g., static buildings in an earthquake analysis) or dynamic situations, as a first approximation in conjunction with a more sophisticated method. This technique will be reassessed during Study Task III, but will not be seriously considered during Study Task II.

Planned Activity

The detailed study plan has been revised and is included as Figure 3.

FIGURE 3. STRUCTURAL DYNAMICS PAYLOAD LOADS ESTIMATE STUDY PLAN

1980 — 1981

M A M J J A S O N D J F M A M J J A S O N D J

TASK III 2 METHODOLOGY DEVELOPMENT

- ESTABLISH PROGRAM PLAN
- DEFINITION OF SIMPLIFIED STRUCTURAL MODEL
- DERIVATION OF THE MODIFIED BASE DRIVE TECHNIQUE (MOBAT)
- DERIVATION OF THE MODIFIED MODAL SYNTHESIS TECHNIQUE (MOMOST)
- PROGRAM CODE AND CHECKOUT
- SAMPLE PROBLEM DEMONSTRATION

TASK III.3 METHODOLOGY VALIDATION

- INVESTIGATION OF IMPEDANCE PERTURBATION METHOD
- STRUCTURAL DYNAMIC MODELS
  - SIMPLIFIED IUS
  - COMPLEX FREQUENCY COUPLER
- RESOLUTION OF EXACT SOLUTION
- RE-ASSESSMENT OF PRINCIPAL METHODS
  - PERTURBATION
  - BASE DRIVE
  - IMPEDANCE
  - GENERALIZED SHOCK SPECTRUM
- SHORT CUT METHODOLOGY DEMONSTRATION
- FINAL PROGRAM PRESENTATION

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