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APPROXIMATE METHOD OF CALCULATING HEATING RATES AT GENERAL THREE - DIMENSIONAL STAGNATION POINTS DURING ATMOSPHERIC ENTRY

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INTRODUCTION

In analyzing heating on vehicles such as the space shuttle orbiter or other advanced entry configurations, it is often necessary to calculate the heating rate at the stagnation point of these vehicles (which are usually three-dimensional) or to calculate a reference heating rate which can be used to nondimensionalize the other surface heating rates experienced by the vehicle. The reference value usually chosen is the stagnation point of a sphere or other simple shape.

These stagnation point heating rates can be obtained from a solution of the full Navier-Stokes equations or from a coupled solution of the Euler and boundary-layer equations. However, either of these procedures can become very expensive if a large number of calculations are required - such as over an entire entry trajectory.

Reshotko (ref. 1) has developed a simple method of calculating the heating rate at general three-dimensional stagnation points which is applicable to cold wall flight conditions. Using this method, the heating at a general three-dimensional stagnation point can be related to the heating at an axisymmetric stagnation point by a simple algebraic expression which can be evaluated from the geometry of the stagnation region.

Although several simple methods exist for calculating the heating rates at axisymmetric stagnation points, a method developed by Fay and Riddell (ref. 2) has been widely used and has been shown to yield reasonably accurate results compared with both experimental data and more detailed calculations (refs. 1 and 3). Combining the equation developed by Fay and Riddell for axisymmetric stagnation points with the equation developed by Reshotko discussed previously, a single simple algebraic equation is obtained for calculating the heating rate at general three-dimensional stagnation points which should be sufficiently accurate for most engineering calculations.

The purpose of the present paper is to give a detailed description of the application of the method outlined previously for making stagnation point heating rate calculations for a vehicle during atmospheric entry.

SYMBOLS

$\frac{du_e}{dx}$, $\frac{dw_e}{dz}$	stagnation point velocity gradients, 1/sec
h	static enthalpy, ft^2/sec^2
h_D	dissociation enthalpy, ft^2/sec^2
H	total enthalpy, ft^2/sec^2
k	parameter defined by eq. (5)
Le	Lewis number
m	mass fraction
M	Mach number
p	static pressure, Lb/ft^2
Pr	Prandtl number
q	heating rate, BTU/ft^2-sec
R_x	radius of curvature at stagnation point in x-direction (see Fig. 1), ft
R_z	radius of curvature at stagnation point in z-direction (see fig. 1), ft
u	velocity in x-direction (see fig. 1), ft/sec
V	velocity along stagnation streamline, ft/sec
w	velocity in z-direction (see fig. 1), ft/sec
X	mole fraction
Z	compressibility parameter defined by eq. (21)
$\varepsilon_1, \varepsilon_2$	parameters defined by eqs. (22) and (23), respectively
γ	ratio of specific heats for perfect gas
μ	viscosity, slug/ft-sec
ρ	density, $slug/ft^3$

Subscripts

APPROX	approximate solution
AXI	axisymmetric
BL	boundary-layer solution
e	boundary-layer edge
s	stagnation point
sw	shock wake
w	wall
3D	three dimensional
∞	free stream

METHOD OF SOLUTION

General Three-Dimensional Stagnation Points

Consider the flow in the vicinity of a general three-dimensional stagnation point - that is, a stagnation point with different principal radii of curvature as shown in figure 1. Using the results from Reshotko (ref. 1), it can be shown that for a cold wall (which is usually the case in flight) the heating rate at a general three-dimensional stagnation point can be related to the heating rate at an axisymmetric stagnation point by the following simple expression

$$(q_s)_{3D} = \sqrt{\frac{1 + k}{2}}\ (q_s)_{AXI} \tag{1}$$

where k is a parameter that is related to the two principal velocity gradients by the following expression (see fig. 1) and $(q_s)_{AXI}$ is based upon the small radius of curvature.

$$k = \frac{(dw_e/dz)_s}{(du_e/dx)_s} \tag{2}$$

Now if the x coordinate is always identified with the smaller radius of curvature R_x, then the parameter k will always fall within the range

$$0 \leq k \leq 1 \tag{3}$$

For two-dimensional flow $R_z \quad \infty$, $\dfrac{dw_e}{dz} = 0$, and

$$k = 0 \qquad \text{(2D flow)}$$

while for axisymmetric flow $R_z = R_x$, $\dfrac{dw_e}{dz} = \dfrac{du_e}{dx}$, and

$$k = 1 \qquad \text{(axisymmetric flow)}$$

Now using a modified Newtonian flow model, the following expressions for the two principal gradients can be obtained.

$$\left(\frac{du_e}{dx}\right)_s = \frac{1}{R_x}\sqrt{\frac{2}{\rho_s}\left(p_s - p_\infty\right)} \tag{4a}$$

$$\left(\frac{dw_e}{dx}\right)_s = \frac{1}{R_z}\sqrt{\frac{2}{\rho_s}\left(p_s - p_\infty\right)} \tag{4b}$$

Combining equations (3) and (4), the following is obtained

$$k = R_x/R_z \tag{5}$$

Both R_x and R_z can be determined once the geometry in the stagnation region is known. This is discussed in detail in reference 14.

Fay and Riddell (ref. 2) have developed the following expression for the heating rate at an axisymmetric stagnation point for a fluid in thermodynamic equilibrium

$$(q_s)_{AXI} = \frac{0.76}{(Pr)^{0.6}} (\rho_w \mu_w)^{0.1} (\rho_e \mu_e)^{0.4} \left[1 + (Le^{0.52} - 1)\frac{h_D}{H_s}\right] x$$
$$(H_s - h_w)\sqrt{\left(\frac{du_e}{dx}\right)_s} \tag{6}$$

This expression has been shown to yield results that are in good agreement with both experimental data and more detailed boundary layer calculations (refs. 2 and 3).

Combining equations (1) and (6), the following expression is obtained for the heating at a general three-dimensional stagnation point

$$(q_s)_{AXI} = \frac{0.76}{(Pr)^{0.6}} \sqrt{\frac{1 + k}{2}} (\rho_w \mu_w)^{0.1} (\rho_e \mu_e)^{0.4} \left[1 + (Le^{0.52} - 1) \frac{h_D}{H_s}\right] \times$$

$$(H_s - h_w) \sqrt{\left(\frac{du_e}{dx}\right)_s} \tag{7}$$

where the velocity gradient $(du_e/dx)_s$ is given by equation (4a). It should be noted that the effect of combining the terms $\left(\frac{1 + k}{2}\right)$ and $\left(\frac{du_e}{dx}\right)_s$ in equation 7 is to yield an effective velocity gradient which is the average of the two principal velocity gradients.

Stagnation Point Properties

The flow approaching a stagnation point first passes through a normal shock wave and then compresses until the velocity is zero (see fig. 2).

The pressure (p_{sw}) and enthalpy (h_{sw}) on the downstream side of the shock wave can be expressed in terms of the density (ρ_{sw}) as follows (ref. 5)

$$p_{sw} = p_\infty + \rho_\infty V_\infty^2 \left(1 - \frac{\rho_\infty}{\rho_{sw}}\right) \tag{8}$$

$$h_{sw} = h_\infty + \left(\frac{V_\infty^2}{2}\right)\left[1 - \left(\frac{\rho_\infty}{\rho_{sw}}\right)^2\right] \tag{9}$$

For an ideal gas, the density (ρ_{sw}) on the downstream side of the shock is given by (ref. 6):

$$\frac{\rho_{sw}}{\rho_\infty} = \frac{(\gamma + 1)M_\infty^2}{(\gamma - 1)M_\infty^2 + 2} \tag{10}$$

Using this result equations (8) and (9) can be solved directly for the pressure and enthalpy.

For equilibrium air chemistry, no simple explicit expression (such as eq. (10)) can be obtained for the density. Thus equations (8) and (9) must be solved iteratively with an equilibrium equation of state in the functional form

$$\rho = \rho(p,h) \tag{11}$$

to obtain the thermodynamic properties on the downstream of the shock wave. In the present work, the equilibrium thermodynamic properties for air have been obtained from the curve fits of Tannehill and Mugge (ref. 7).

From the conditions on the downstream side of the shock wave, the fluid on the stagnation streamline compresses isentropically until the stagnation point is reached. The conditions at the stagnation point can be obtained by integrating the following set of equations:

$$\frac{dp}{dV} = -\rho V \tag{12}$$

$$h = H - V^2/2 \tag{13}$$

along with equation (11) from conditions immediately behind the shock wave (ρ_{sw}, p_{sw}, h_{sw}, and V_{sw}) to the point where $V = 0$ (the stagnation point). The final results of this solution will produce the stagnation point conditions (ρ_s, p_s, T_s, and $h_s = H_s$).

Dissociation Enthalpy

For Lewis numbers other than 1, calculation of the stagnation point heating rate from equation (7) requires the dissociation enthalpy for the fluid at the edge of the boundary layer at the stagnation point which is given by

$$(h_D)_s = m(O)(h_D)_O + m(N)(h_D)_N \tag{14}$$

The dissociation energies ($(h_D)_O$ and $(h_D)_N$) are readily available, but the mass fractions ($m(O)$ and $m(N)$) must be determined for the stagnation conditions of interest.

The mass fractions of atomic oxygen and atomic nitrogen can be determined from the mole fractions as follows

$$m(O) = \frac{16\ X(O)}{16\ X(O) + 32\ X(O_2) + 14\ X(N) + 28\ X(N_2)} \tag{15}$$

$$m(N) = \frac{14\ X(N)}{16\ X(O) + 32\ X(O_2) + 14\ X(N) + 28\ X(N_2)} \tag{16}$$

Hansen (ref. 8) gives the following approximate equations for determining the mole fractions (neglecting ionization)

$$x(O) = 2\varepsilon_1/Z \tag{17}$$

$$x(O_2) = (0.2 - \varepsilon_1)/Z \tag{18}$$

$$x(N) = 2\varepsilon_2/Z \tag{19}$$

$$x(N_2) = (0.8 - \varepsilon_2)/Z \tag{20}$$

$$Z = 1 + \varepsilon_1 + \varepsilon_2 \tag{21}$$

$$\varepsilon_1 = \frac{-0.8 + \sqrt{0.64 + 0.81\ (1 + 4p_s/Kp_1)}}{2(1 + 4p_s/Kp_1)} \tag{22}$$

$$\varepsilon_2 = \frac{-0.4 + \sqrt{0.16 + 3.84(1 + 4p_s/Kp_2)}}{2(1 + 4p_s/Kp_2)} \tag{23}$$

Values of $\ell n(Kp_1)$ and $\ell n(Kp_2)$ are given in Table III of reference 8 as functions of temperature.

The value of the dissociation enthalpy obtained by the procedures outlined in this section are approximate but should be sufficiently accurate for the heating calculations described in this paper.

RESULTS AND DISCUSSION

The ratio of the heating rate at a general three-dimensional stagnation point ratioed to that at an axisymmetric stagnation point can be determined from equation (1) as a function of the parameter k which is the ratio of the principal radii of curvature (R_x/R_z). The ratio $(q_s/q_s)_{AXI}$ is plotted as a function of k in figure 3. In the discussion that follows, it is assumed that R_x is held fixed and k is varied by varying the value of R_z. It is easily seen that the heating rate is the highest when the principal radii of curvature are equal (i.e., k = 1). Thus the largest value of heating occurs at an axisymmetric stagnation point. As R_z increases, k decreases and the stagnation point heating decreases. For k = 0, a two-dimensional stagnation point, the heating rate is approximately 70 percent of the value at an axisymmetric stagnation point.

While no detailed calculations are known to be available for the heating at a general three-dimensional stagnation point, it is possible to make detailed calculations for the limiting cases of a sphere and a cylinder. These calculations have been performed for several points along the entry trajectory of the second space shuttle flight (STS-2). The points selected are listed in Table I and are the same as those used in the heating analysis of reference 9. The calculations were performed for the following set of fixed conditions using the transport properties from reference 8.

$$T_w = 2000°R$$

$$Pr = 0.72$$

$$R_x = 1 \text{ ft.}$$

$$Le = 1 \text{ and } 1.4$$

The heating rates obtained from the present approximate method are compared with heating rates obtained from detailed boundary-layer calculations that were made by the present author using a method similar to that described in reference 10.

The results for the sphere are tabulated in Table II. It should be noted that at the higher Mach numbers the heating rates calculated by the present method for a Lewis number of 1.4 are higher than those for a Lewis number of 1.0. The difference is due approximately to the diffusional contribution to the heating because of the presence of relatively large amounts of dissociated oxygen and nitrogen at these conditions. The value of 1.4 was chosen for the Lewis number because it has been used extensively by other investigators (for example, refs. 2 and 3). At lower Mach numbers where dissociation is less important, the diffusional contribution to heating decreases. Results for the present method for a Lewis number of 1.4 are compared with the boundary layer calculations (which include diffusional effects) in figure 4. For the portion of the STS-2 entry trajectory considered, the present approximate method is in good agreement with the boundary layer solutions. Results for the cylinder, which are listed in Table III and shown in figure 5, are similar to those for the sphere except that the heating rates are lower (as would be expected).

The present approximate method for calculating the heating rate at a general three-dimensional stagnation point has been shown to agree well with boundary layer calculations for the special limiting cases of a sphere and cylinder. Thus, this method should provide a reasonably accurate method of making equilibrium stagnation point heating calculations for "engineering type" design and analysis applications.

CONCLUDING REMARKS

An approximate method has been developed for calculating the heating rate at general three-dimensional stagnation points during atmospheric entry. Although no other experimental or calculated heating rates are known to be available for general three-dimensional stagnation points, the present method has been shown to agree closely with boundary layer calculations for the special limiting cases of a sphere and a cylinder. Thus, it should provide a reasonably accurate method of making equilibrium stagnation point heating calculations for "engineering type" design and analysis applications.

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TABLE I. - STS-2 TRAJECTORY POINTS.

CASE NO.	ALT KFT	M_∞	V_∞ FT/SEC	P_∞ LB/FT^2	T_∞ °R
1	246.0	25.5	23620.	.0454	357
2	219.2	19.9	19860.	.1677	414
3	198.7	15.7	16420.	.3973	455
4	173.8	11.9	12680.	1.0635	472
5	156.4	9.15	9710.	2.0950	468
6	141.5	7.19	7540.	3.8000	457
7	129.8	6.25	6460.	6.1590	445

TABLE II. - STAGNATION POINT HEATING RATE ON A SPHERE

CASE NO.	$(q_s)_{APPROX}$ BTU/FT^2 - SEC		$(q_s)_{BL}$ BTU/FT^2 - SEC
	Le = 1	Le = 1.4	
1	55.7	63.1	60.6
2	58.9	65.0	63.8
3	48.4	52.7	51.2
4	33.8	36.3	34.9
5	19.0	19.7	19.3
6	9.87	9.92	9.65
7	6.35	6.35	6.13

TABLE III. - STAGNATION POINT HEATING RATE ON A CYLINDER

CASE NO.	$(q_s)_{APPROX}$ BTU/FT^2 - SEC		$(q_s)_{BL}$ BTU/FT^2 - SEC
	Le = 1	Le = 1.4	
1	39.4	44.6	44.4
2	41.7	46.0	46.7
3	34.2	37.3	37.5
4	23.8	25.7	25.7
5	13.4	14.0	14.3
6	6.98	7.02	7.12
7	4.44	4.44	4.53

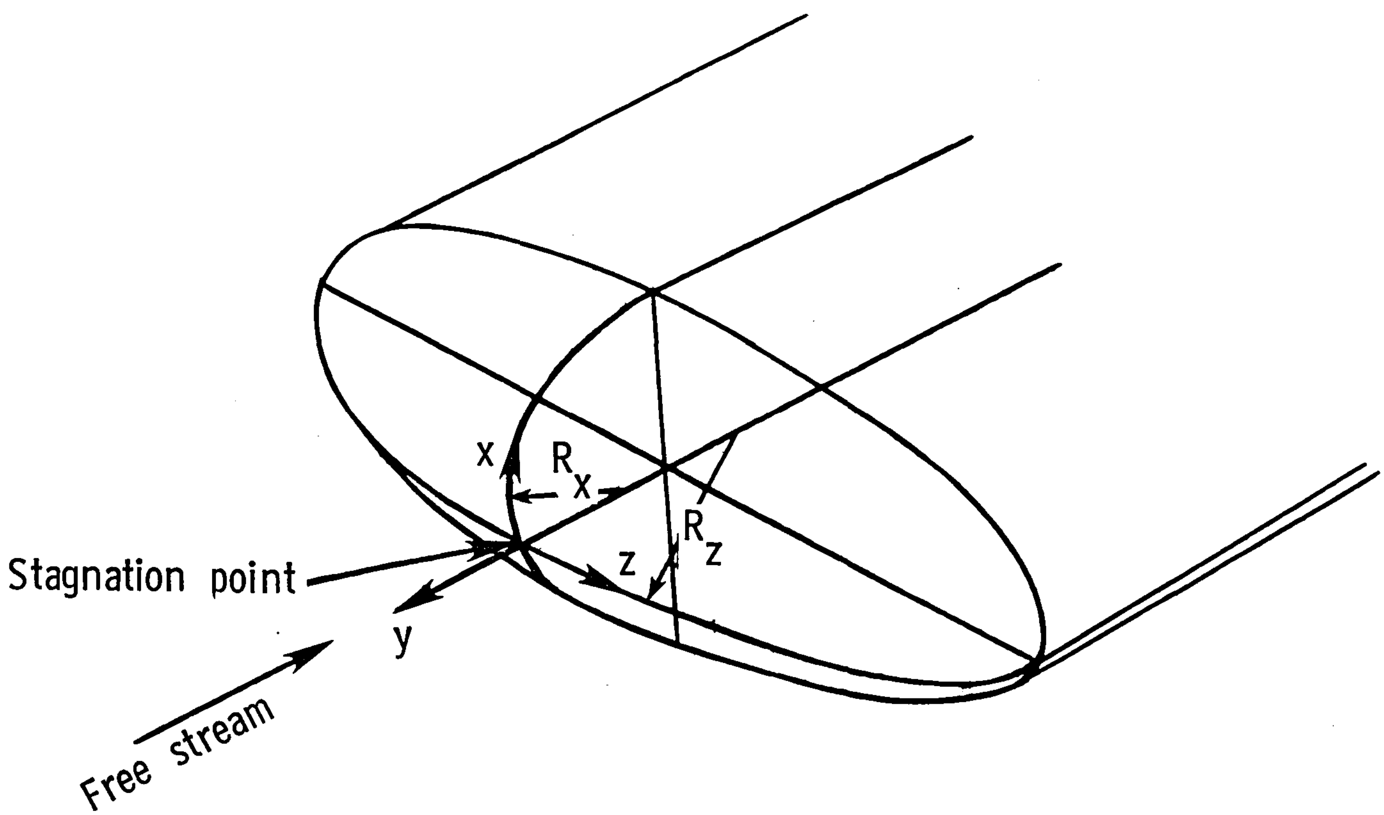

Figure 1. - Coordinate system for general three-dimensional stagnation point.

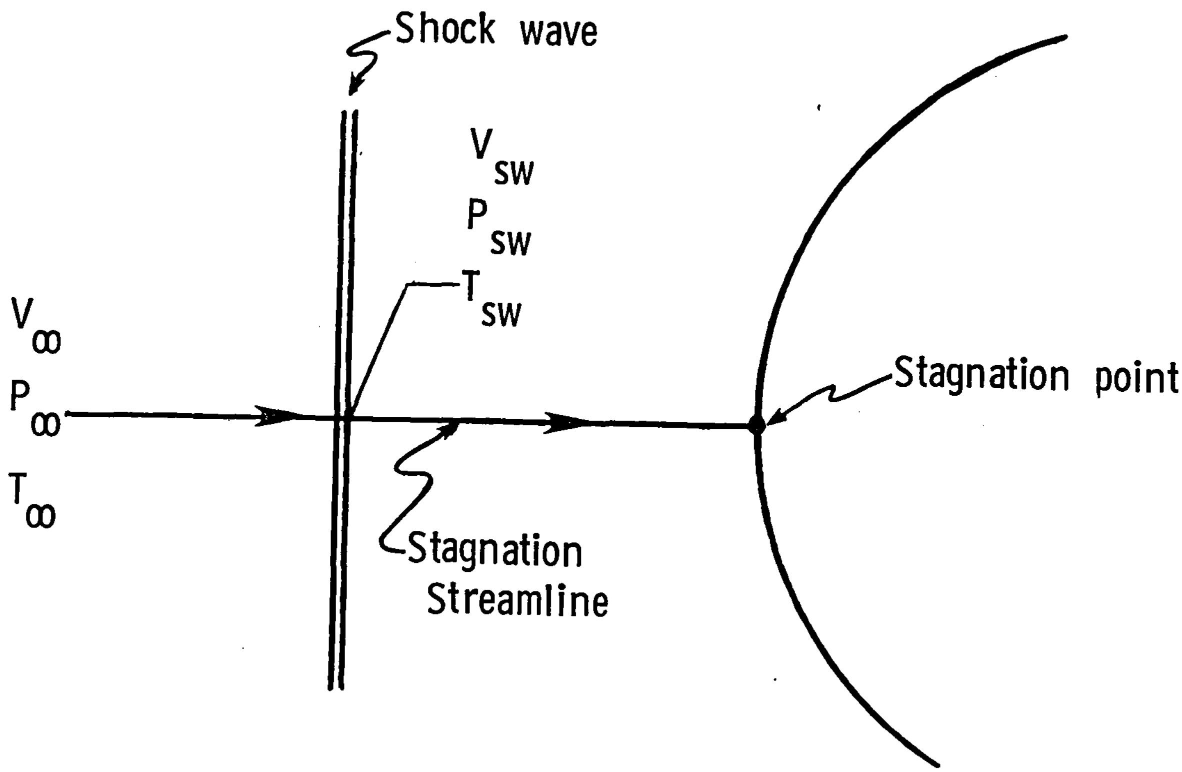

Figure 2. - Flow approaching stagnation point.

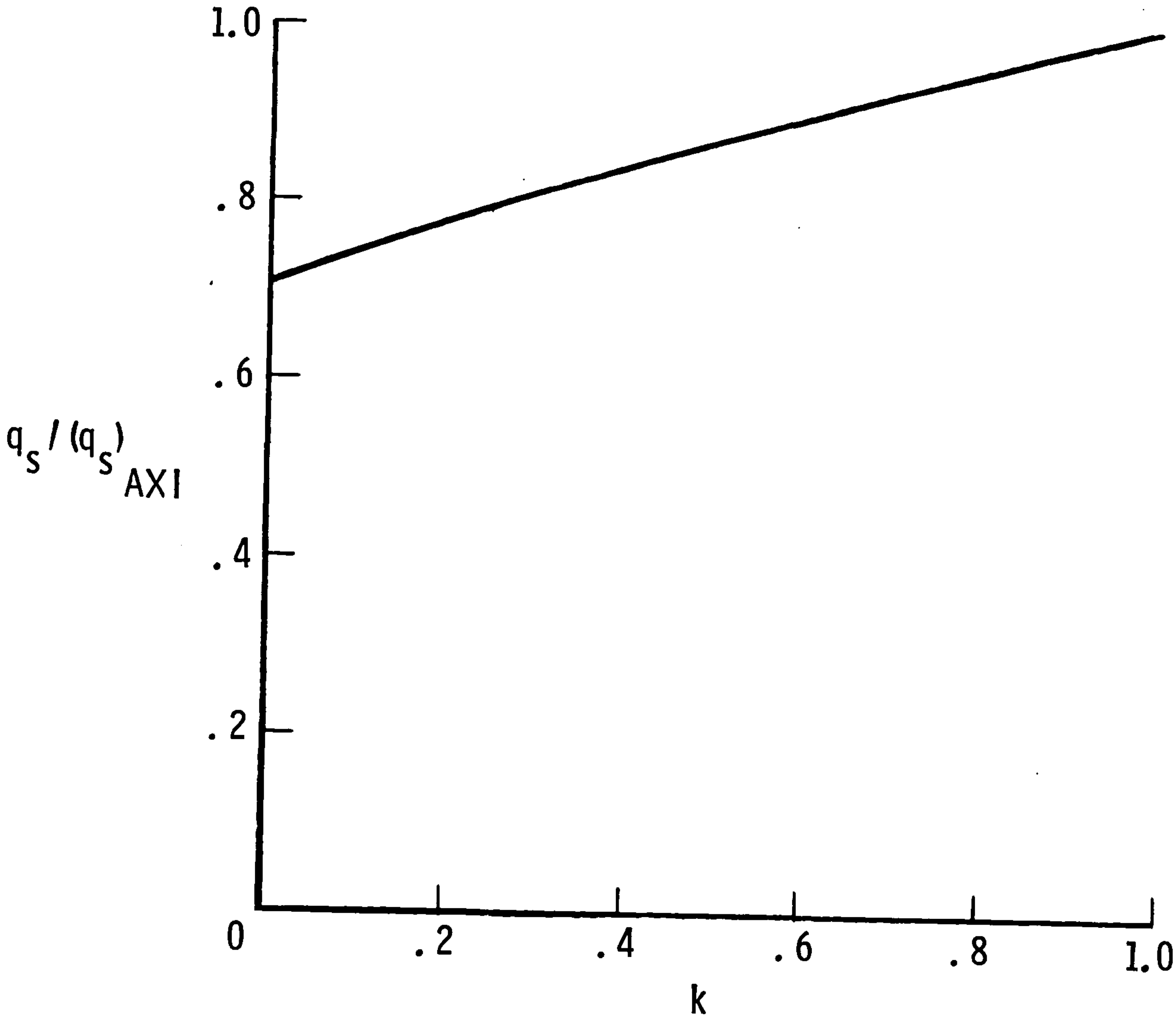

Figure 3. - Heating rate at a general three-dimensional stagnation point as a function of k.

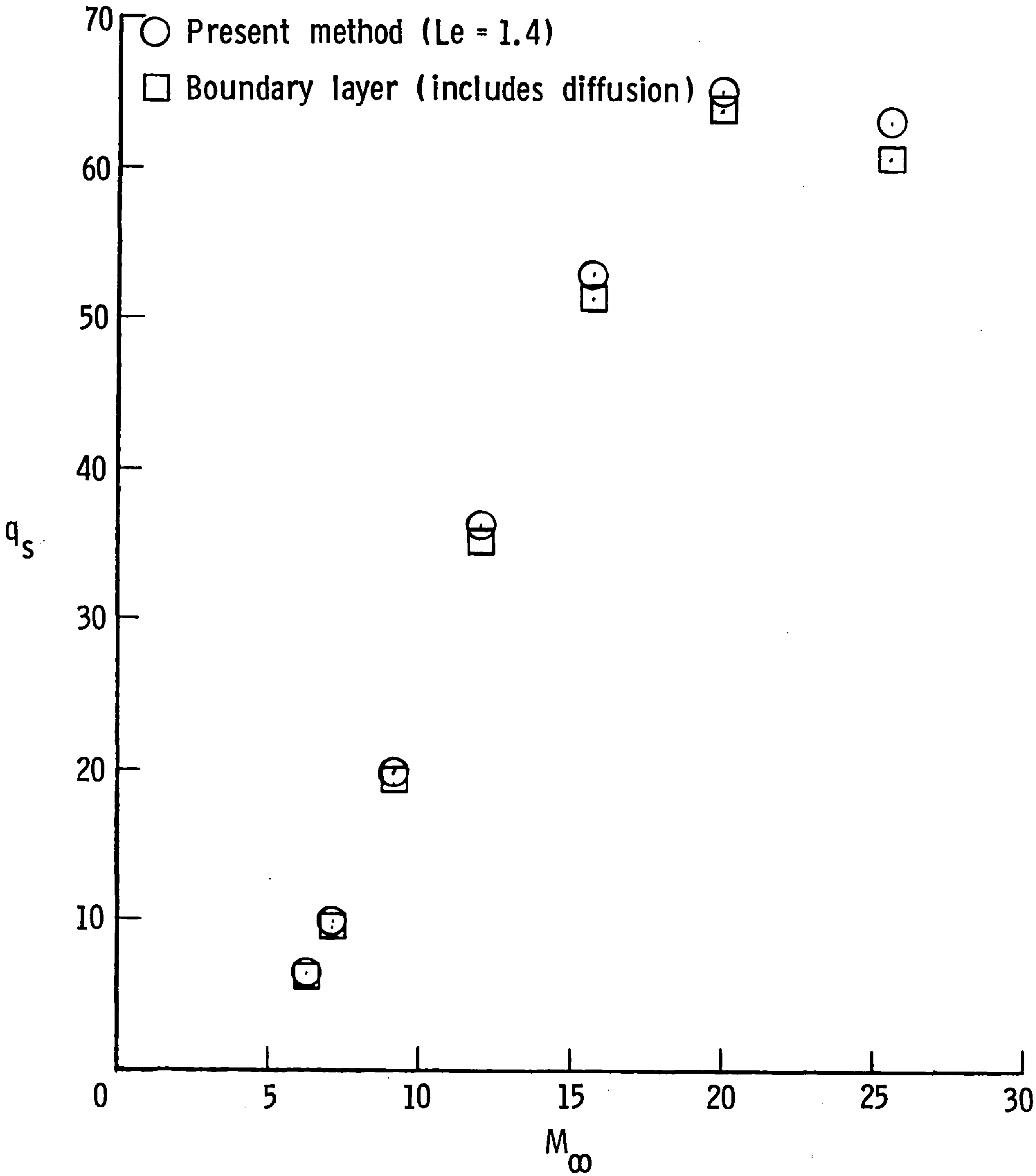

Figure 4. - Comparison of calculated stagnation point heating rates for a sphere for the STS-2 trajectory.

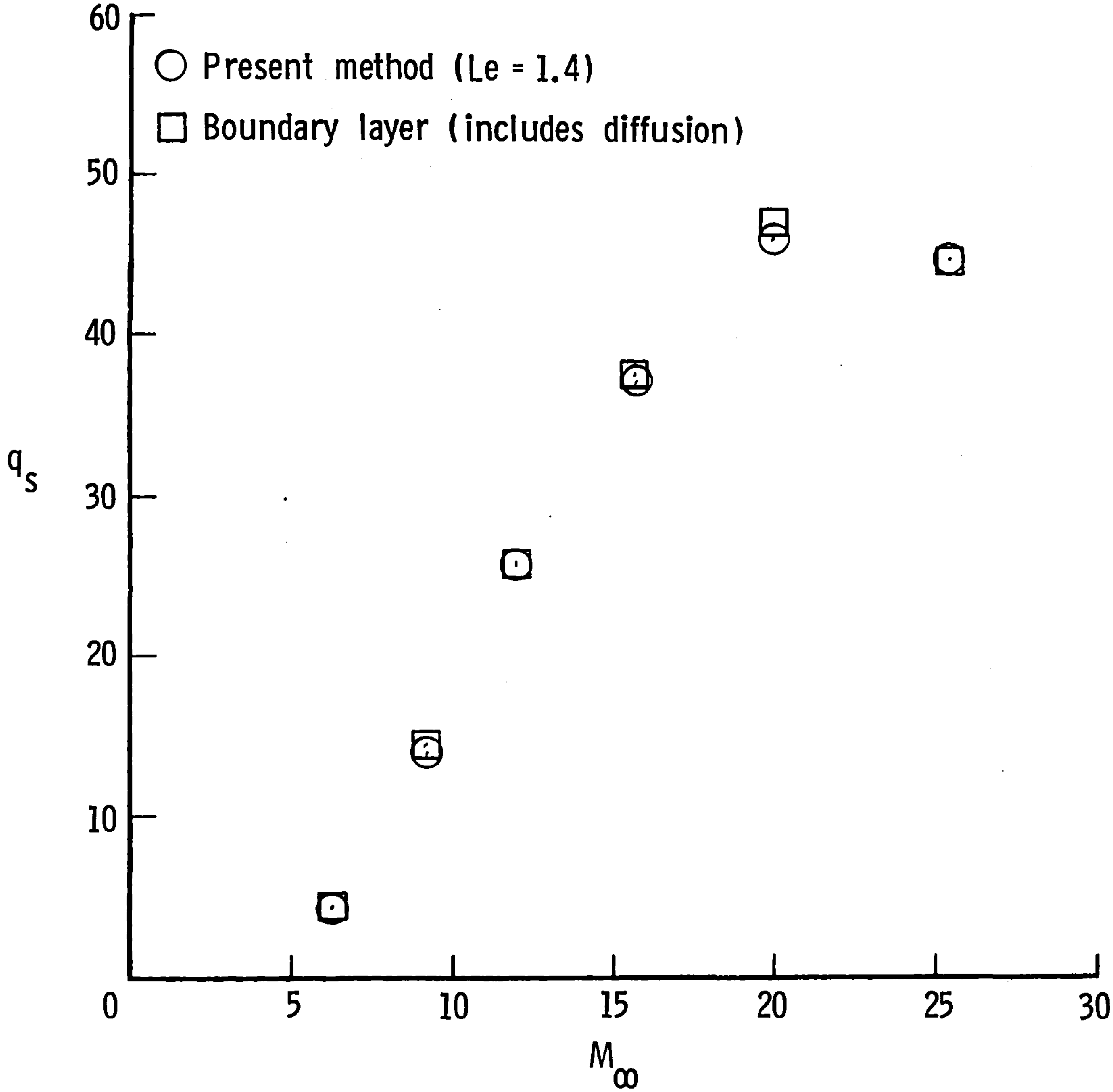

Figure 5. - Comparison of calculated stagnation point heating rates for a cylinder for STS-2 trajectory.

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15. Supplementary Notes

16. Abstract

An approximate method is presented for calculating heating rates at general three-dimensional stagnation points. This paper gives a detailed description of the application of the method for making stagnation point heating calculations during atmospheric entry. Comparisons with results from boundary layer calculations indicate that it should provide a reasonably accurate method for "engineering type" design and analysis applications.

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