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16. Abstract

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TWO-DIMENSIONAL EFFECTS AND A COMPARISON OF THEORY AND EXPERIMENT

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Lengthwise magnetic fields are used most often to focus electrons in powerful multi-resonator klystrons with increased efficiency. By selecting the parameters of the beam focusing system and of the electron beam, conditions are created for low pulsation of the radius in the steady state (§1). The grouping of electrons in a low-pulsation beam can be described by means of a multilayered model with fixed radii of the layers (approximation of stratified layers). However, in the general case, the conditions of applicability of such a model are not fulfilled in the last drift regions. A radial movement occurs which results in dynamic defocusing. In the output resonator radial movements of retarded electrons play a large role. Some of these settle onto the walls of the resonator. /215*

Annular beam models are used to investigate two-dimensional processes in axial-symmetrical instruments. The lecture describes one of the versions of an annular model, suitable for the design of a grouper and an output resonator. The rings have an identical charge. They are infinitely thin in the lengthwise direction. The areas of the rings are finite in the transverse plane. They can be fixed or they may vary with the coordinate (be deformed), (§2,3).

For dual and multi-resonator klystrons with typical parameters a comparison is made of the results of an analysis by

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*Numbers in the margin indicate pagination in the foreign text.

various methods in a unidimensional approximation, stratified layer approximation, and with an allowance for dynamic defocusing. It is shown that under a rather large increase in the magnetic field from the cathode to the outlet resonator the principal conclusions pertaining to the multi-layered model also remain valid in the two-dimensional approximation [1].

The optimal parameters of the klystrons obtained by the analysis are comparable to the parameters of experimental models of powerful klystrons with high efficiency, described in the literature. A comparison reveals that the results of the theory developed in the lectures agree with experimental findings. /216

§1. The Formation and Focusing of an Electron Beam

In the majority of powerful multi-resonator klystrons, axial-symmetrical electron fluxes are used, focused by a uniform lengthwise magnetic field. The movement of the electrons depends on the relation between the intensities of the magnetic field and of the field of the space charge, as well as the degree of shielding of the cathode. In the steady state with no high frequency signal,the Coulomb electric field of the flux is directed along the radius. The transverse deflection of the potential, if the microperveyance of the beam is less than unity, turns out to be lower than 1.5% and does not produce a noticeable lengthwise spread in the velocities; it has a slight influence on the grouping of the electrons. At the same time, the radial-directed Coulomb force may result in pulsations of the beam radius. Only under certain conditions can the Coulomb force, the Lorentz force and the centrifugal force mutually balance out, which is a necessary condition for the existence of an equilibrium flux with no pulsations. The equilibrium radii can be obtained for any given degree of shielding of the cathode from the magnetic field. In the case of complete shielding of a flux with uniform cross section, the magnetic field intensity is a minimum. The flux rotates as a unit with angular frequency ω_B, equaling half the cyclotron frequency ω_c:

$$\omega_B = \frac{\omega_c}{2} = \frac{1}{2}\frac{|e_0|}{m_0} B_z, \tag{7.1}$$

where B_z is the lengthwise component of the magnetic induction. This is known as Brillouin focusing.

The magnetic fields which are actually used have an intensity which is 1.5-3 times larger than that of a Brillouin field. The electron fluxes are most often focused by a beam partially shielded from the magnetic field, characterized by an alignment of the magnetic lines of force and the electron trajectories in the vicinity of the cathode-anode (beam with magnetic tracking) [2]. In the space behind the anode, the magnetic field is introduced into the beam and a "twisting" of the electron trajectories takes place. In the section perpendicular to the axis the electron trajectories represent epicycloids. On the constant angular velocity of revolution there is superposed a periodic movement with respect to the radius. This results in pulsations of the diameter of the flux with a lengthwise coordinate. The pulsations depend on the relative size of the magnetic field B/B_B, the initial angle of inclination of the trajectories to the axis, and the parameter of the cathode conditions K, which varies between zero and unity: /217

$$K = \left(\frac{B_K}{B_{z0}}\right)^2 \left(\frac{r_K}{r_0}\right)^4, \tag{7.2}$$

where B_K is the magnetic field at the cathode, r_K is the radius of the cathode, r_0 is the equilibrium radius of the beam, and $B = B_{z0}$ is the lengthwise component of the magnetic induction on the beam axis.

For a typical parameter of cathode conditions K = 0.64 and a ratio $B/B_B = 1.7$, the electrons accomplish about ten oscillations in a single revolution about the axis of the system. In the case of a properly designed beam, the amplitude of the pulsations is much less than the mean radius. If the flux breaks down into layers, it is obvious that the electrons remain within the bounds of their layers over the entire extent of the grouper, except for the last cascades, where it is necessary to take into account the dynamic defocusing.

The charge density in the bunch increases in proportion to the grouping. The transverse forces of repulsion increase. The compensation of the radial forces is disturbed and a radial movement (dynamic defocusing) results. In order to reduce the defocusing in realistic systems, a magnetic field increasing from the cathode to the collector is employed. The rate of rise increases as the region of the output resonator is approached. The magnetic field intensity at the axis of the output resonator may increase the Brillouin value by 2-3 times. The increase in the magnetic field may not fully compensate for the dynamic defocusing, but only impede the settling of electrons onto the walls. /218

The analysis of the dynamic defocusing is done by means of two-dimensional models and the procedure described in the following paragraphs.

§2. Equations of the Two-Dimensional Model

The undisturbed flux is broken down into individual disks and rings exactly as is done for the multi-layered model with fixed radii, lecture 4. Along the radius the flux is divided into N_0 layers. Each segment of the layer with an extent equaling the electron wavelength is broken down into M identical thin charged rings (disks of the interior layer). The electrons are focused by a lengthwise magnetic field of varying intensity $\bar{B} = B(z)\bar{z}_0$.

The equation of movement of the m-th charged element of the i-th layer in the output resonator is written in an immovable system of coordinates $(\vec{r}, t)$:

$$\frac{d^2 r_{mi}}{dt^2} - r_{mi}\frac{d^2 \varphi_{mi}}{dt^2} = \eta\,(E_{rB} + E_{rq})_{mi}$$

$$r_{mi}^2 \frac{d^2\varphi_{mi}}{dt^2} + 2\frac{dr_{mi}}{dt}\frac{d\varphi_{mi}}{dt} = \eta\, r_{mi}(E_{\varphi B} + E_{\varphi q})_{mi} \tag{7.3}$$

$$\frac{d^2 z_{mi}}{dt^2} = \eta\,(E_{zB} + E_{zq})_{mi},$$

where $\vec{E}_B$ is the intensity of the vortical electric field of the resonator and $\vec{E}_q$ is the intensity of the Coulomb field, $\eta = \frac{e_0}{m_0}$.

The theory of Bouche is applicable for description of the focusing [2]: /219

$$\frac{d\varphi_{mi}}{dt} = \frac{\omega_c}{2}\left[1 - \sqrt{K}\,\frac{r_{0mi}^2}{r_{mi}^2}\right]. \tag{7.4}$$

where r_{0mi} is the equilibrium radius of the i-th layer of the beam; K is the parameter of cathode conditions, given by formula (7.2).

The movement of the charge in the grouper is considered in a phase system with time as the dependent variable. The phase is given by the expression:

$$\varphi_{mi} = \omega t_{mi}(y, \varphi_{0mi}, R_{0mi}) - y, \tag{7.5}$$

where $y = \frac{\omega}{v_0} z$ is the independent variable, ϕ_{0mi} and R_{0mi} are the initial values of the phase and of the non-dimensional radius, and $R_{0mi} = \frac{r_{0mi}}{r_r}$, r_r is the radius of an equivalent tube of drift.

The equations of movement in the phase system of coordinates have the appearance:

$$\frac{d^2\varphi_{mi}}{dy^2} = \left(1+\frac{d\varphi_{mi}}{dy}\right)^3 \frac{\eta}{\omega v_0}\left(E_{z\delta}+E_{zq}\right)_{mi}$$

$$\frac{d^2R_{mi}}{dy^2} = \frac{d^2\varphi_{mi}}{dy^2}\left(1+\frac{d\varphi_{mi}}{dy}\right)^{-1}\frac{dR_{mi}}{dy} - \left(1+\frac{d\varphi_{mi}}{dy}\right)^2\frac{\eta}{r_T\omega^2}\left(E_{r\delta}+E_{rq}\right) - \tag{7.6}$$

$$-\frac{1}{4}\left(1+\frac{d\varphi_{mi}}{dy}\right)^2\left(\frac{\omega_c}{\omega}\right)^2 R_{mi}\left[1-K\left(\frac{R_{0mi}}{R_{mi}}\right)^4\right].$$

In deriving (7.6) the Bouche theorem (7.4) was used $R_{mi}=\frac{r_{mi}}{r_T}$.

The intensity of the Coulomb field $\vec{E}_q$ in equations (7.3) and (7.6) is found to an electrostatic approximation by solving the Poisson equation for the scalar potential ϕ. The components of the force of interaction between two elements of charge are found after an averaging over the cross sections of the elements. For the lengthwise component,the averaging is similar to that described in lecture 4.

The components of the intensity of the electric field - that acting on the m-th charge of the i-th layer from the direction of the n-th charge of the j-th layer - are expressed in terms of the functions of influence F_{zminj}, F_{rminj}: /220

$$E_{zq} = -\frac{\partial\varphi}{\partial z} = -\frac{\sigma}{\varepsilon_0}F_{zminj}$$

$$E_{rq} = -\frac{\partial\varphi}{\partial r} = -\frac{\sigma}{\varepsilon_0}F_{rminj}, \tag{7.7}$$

where σ is the surface density of the charge, depending on the current and area of the beam, the accelerating potential, and the number of times the beam is broken down into rings.

After integrating over the cross sections of the interacting elements, the following expression is found for the lengthwise function of influence:

$$F_{zmi\,nj} = \frac{2}{R_{Hmi}^2 - R_{Bmi}^2} \sum_{\ell=1}^{\infty} e^{-\frac{K_0 \mathscr{x}_\varphi}{2} \nu_e |\Delta\bar{\varphi}|} \frac{\left[R_{Hnj} J_1(\nu_e R_{Hnj}) - R_{Bnj} J_1(\nu_e R_{Bnj})\right]}{\nu_e J_1(\nu_e)} \cdot \tag{7.8}$$
$$\frac{\left[R_{Hmi} J_1(\nu_e R_{Hmi}) - R_{Bmi} J_1(\nu_e R_{Bmi})\right]}{\nu_e J_1(\nu_e)} \operatorname{sgn}(\Delta\bar{\varphi}),$$

where $\Delta\varphi = (\varphi_{mi} - \varphi_{nj})$

ν_e is the root of the Bessel function:

$$J_0(\nu_e) = 0;$$

R_{Hmi} and R_{Bmi} are the outer and inner radius of the layer.

The radial function of influence is given by the formula:

$$F_{rmi\,nj} = \frac{\pi}{R_{Hmi}^2 - R_{Bmi}^2} \sum_{\ell=1}^{\infty} e^{-\frac{K_0 \mathscr{x}_\varphi}{2} \nu_e |\Delta\bar{\varphi}|} \frac{\left[R_{Hnj} J_1(\nu_e R_{Hnj}) - R_{Bnj} J_1(\nu_e R_{Bnj})\right]}{\nu_e^2 J_1^2(\nu_e)} \times$$
$$\times \Big\{ R_{Hmi} \left[J_1(\nu_e R_{Hmi}) H_0(\nu_e R_{Hmi}) - J_0(\nu_e R_{Hmi}) H_1(\nu_e R_{Hmi}) \right] - \tag{7.9}$$
$$- R_{Bmi} \left[J_1(\nu_e R_{Bmi}) H_0(\nu_e R_{Bmi}) - J_0(\nu_e R_{Bmi}) H_1(\nu_e R_{Bmi}) \right] \Big\},$$

where H_0 and H_1 are Struve functions.

The function of influence and the difference $\Delta\bar{\phi}$ are calculated at the given instant of time t = const. The phases in equations (7.6) are found in the given cross section y. In order to convert from the phase difference $\Delta\phi$ to the difference $\Delta\bar{\phi}$, we use an expansion in a Taylor series in terms of the small parameter, similar to the expansion in lecture 1. The processes in the fixed cross section are period in time. In order to solve the equations of movement in the regions of drift we must /221 know the phase coordinates of the charges present at one starting period. The starting conditions at the entry to the grouper are the phases ϕ_{0mi} and radii R_{0mi}. The conditions are identical to those in the procedure for analysis of the stratifications.

As the electrons travel through the gaps of the resonators of the grouper they are influenced by the variable potential of the resonance mode of the vortical electric field E_B. The flux acquires an additional lengthwise and radial modulation of velocity. In the majority of cases the radial modulation is negligibly small. The lengthwise modulation is found similar to the procedure described in lecture 4.

The electron flux at the outlet of the grouper is characterized by known values of the phases, radii and velocities of the charged particles. These are used as the initial conditions at the entry to the output resonator. The movement of the electrons in the output resonator is calculated by means of the set of equations (7.3). The number of equations is variable. The backward and oscillatory movement of the electrons are taken into account. The calculation procedure is largely similar to the method of analysis of the influence of stratification in the output resonator (lecture 6) and differs from it by allowing for the radial movement of the rings and the radial components of the Coulomb forces. There are additional terms in the expression for the power of the interaction and the electron conductance, depending on the radial velocity of the electrons. The distribution of the fields in the gap is taken from measurement data of an electrolysis bath or from measurements by the test body method.

In the grouper and in the exit resonator, the Coulomb forces are calculated for the charges in the actual or equivalent tube of drift. The equivalent tube has a radius somewhat larger than that of the tubes of the gaps of the resonator. The m-th charge of the i-th layer is influenced by electrons situated at a distance on the order of the range of action of the Coulomb forces $1/R_0$.

In finding the acceleration we usually add the influence of the electrons in the particular period and in periods adjacent with respect to the starting phase: /222

$$\begin{cases} \dfrac{\eta}{\omega v_0} E_{zmi} = -\dfrac{8\pi}{MN_0}\left(\dfrac{\omega_p}{\omega}\right)^2 \sum_{j=1}^{N_0} \sum_{n=-M}^{M} \dfrac{\varkappa_0^2}{R_{Hnj}^2 - R_{Bnj}^2} F_{zminj}; \\ \dfrac{\eta}{\omega^2} E_{rmi} = -\dfrac{8\pi}{MN_0} \dfrac{K_0 \varkappa_0}{2}\left(\dfrac{\omega_p}{\omega}\right)^2 \sum_{j=1}^{N_0} \sum_{n=-M}^{M} \dfrac{\varkappa_0^2}{R_{Hnj}^2 - R_{Bnj}^2} F_{rminj}, \end{cases} \tag{7.10}$$

where $\varkappa_0 = \frac{r_{n0}}{r_T}$ is the space factor at the entry to the region of interaction;

$K_0 = \frac{2}{\omega/v_0\, r_{n0}}$ is the radius of action of the Coulomb forces in an undisturbed beam;

$\frac{\omega_p}{\omega}$ is the ratio between the non-reduced plasma frequency and the frequency of the signal at the entry to the region of interaction.

§3. Beam Models

In finding the Coulomb forces of interaction between charged cross sections as functions of the particular problem one of the three varieties of the beam model is used: the model of deformable rings, that of constant-area rings, or that of infinitely thin rings. In the model of constant-area rings the surface density of the charge does not change, and the movement of the boundaries of the individual ring or disk is determined solely by the movement of the center of mass. Radial movement leads to the appearance of breaks in the distribution of charge and to overlap of the areas of the individual rings.

When using a different model - that of deformable disks and rings - the surface density of the charge is a variable. The movement of the boundaries of an individual ring agrees with the movement of the adjacent rings. If at the entry to the region of interaction we use the beam cross section as the element of charge and break it down into separate rings, then in the process of movement the flat charged disk is initially deformed - expanding and bending. This process is described by lengthwise and radial displacement of the centers of mass and boundaries of the rings. The radii of adjacent elements always coincide. The model of deformable disks and rings is convenient to describe pulsations of the beam in a magnetic field. It can describe powerful pulsations, accompanied by a change in the distribution of charge density over the cross section. When investigating dynamic processes with great nonlinearity, characterized by a large number of intersections of trajectories, the program provides for a conversion from the model of deformable rings to the model of constant-area rings. In a number of cases this conversion allows a simplified analysis. /223

A partial case of the deformable ring model (DRM) is the deformable disk model (DDM) or unilayered model of deformable rings. Computations by these models are distinguished by reduced computer time and they are convenient for investigating dynamic corrections to the results of the unidimensional theory, introduced by allowing for radial movement of the charges. The deformable disk model is similar to the model used in the theory of a klystron [3], differing from it by the method of averaging the Coulomb forces.

The multi-layered deformable ring model also differs from the model used to design the exit resonator [4] by the fact that in [4] deformable charge elements of finite extent in the z-direction are considered. In our view, the use of charge elements deformable with respect to z provides no absolute advantages whatever, although it considerably increases the analysis time. /224 The experience in using the disk and multi-layered models, which employ infinitely thin charge elements, allows us to state that the distances between the elements usually turn out to be less than the characteristic dimensions of a physically infinitely small volume. Further refinement of the distribution of charge among the elements or balancing out of this provides no new results. On the other hand, the model of z-deformable rings has an advantage over the model of rings with fixed lengthwise dimensions. In the model of deformable elements, as well as the model of thin rings, the precision in calculating the charge density in the bunch depends on the distance between the elements, which can be very small. The precision increases in proportion to the degree of grouping. At the same time the precision of an identical calculation for a model of rings which are rigid in respect of z depends on their lengthwise dimensions and diminishes in proportion to the grouping of electrons in the bunch [5].

Radial division is almost always less frequent than lengthwise. The radial movement only provides corrections to the basic process of lengthwise grouping. The use of the cross section in computations with rings of finite area increases the precision and reduces the analysis time. Control calculations of versions with finite and infinitely thin rings in respect of z and r (the model of [6]) have shown that, for a proper description of the Coulomb forces, in the latter model it is necessary to increase the initial number of beam layers by almost one order of magnitude.

The model of a beam with rings of constant area allows the use of previously-computed values of the functions of influence of the given radial and lengthwise subdivision, available in the form of tables. The conversion to the actual distribution is done by means of interpolation formulas. Such a procedure considerably shortens the analysis time.

When using a model with deformable rings the functions of influence are computed at each step from the current values of the radius. The computer time for the analysis is increased by 2-3 times. /225

As the control calculations have shown, when the beam pulsations are small the model of constant-area rings and the deformable ring model produce almost identical results.

In a two-dimensional approximation, as well as a uni-dimensional approximation or in the procedure of stratification, the grouping quality factor can be used as a qualitative characteristic of the grouping. The quality factor is close to the electron efficiency in the two-dimensional approximation as well, if the distribution of the field in the outlet gap is close to the rectangular and the corrections associated with the transverse movement of slow electrons are small [4].

§4. Dynamic Defocusing in the Grouper of a Two-Resonator Klystron

Dynamic defocusing in the drift region of a two-resonator klystron was calculated without allowing for the transverse component of the high-frequency field of the gap. It is presumed that the retarding effect of the radial forces in one half of the gap is compensated by their acceleration in the other half.

Figure 1 shows functions of the mean values of the amplitude of the first current harmonic for the various discrete beam models. The computations were done with the following parameters, typical for a powerful klystron: $\omega_p/\omega = 0.24$; $K_0 = 4$; $B_z/B_6 = 2$; $x_0 = 0{,}6$. the parameter of cathode conditions is automatically computed from the condition of absence of pulsations in the steady state.

The lengthwise modulation of the velocity of the electron beam is converted into a density modulation as the grouping proceeds, forming regions of congestion and rarefaction. In the regions of congestion the radial force exceeds the static value, the compensation of the Coulomb forces by the Lorentz force is disturbed, and the beam radius increases. The change /227 in the radius of the beam for analysis in a uni-layer approximation is shown in Fig. 2. Since there are more particles in the regions of congestion than the regions of rarefaction, on the average the beam radius increases. As the grouping proceeds, the radii of particles belonging to the center of the bunch increase, while those of particles in the regions of rarefaction somewhat diminish. The increase in the mean beam radius when the change in the parameter of the Coulomb forces is taken into account results in a decrease in the mean repulsive force, which promotes a further approach of the electrons. The comparison of the phase trajectories of "large" particles for models of disks with variable radii and rigid disks testifies to the improved grouping into a bunch for a transverse dynamic beam defocusing. A comparison of curves 1 and 2 of Fig. 1 reveals that the amplitude of the first current harmonic somewhat increases when defocusing is present.

Calculations with a multi-layered beam model have revealed that the pattern of radial trajectories turns out to depend on the number of layers (Fig. 3). Under the influence of the interior layers,the average radii of the rings of the outer layer increase. The spread in radii of the outer layer is larger than the spread in the interior layers.

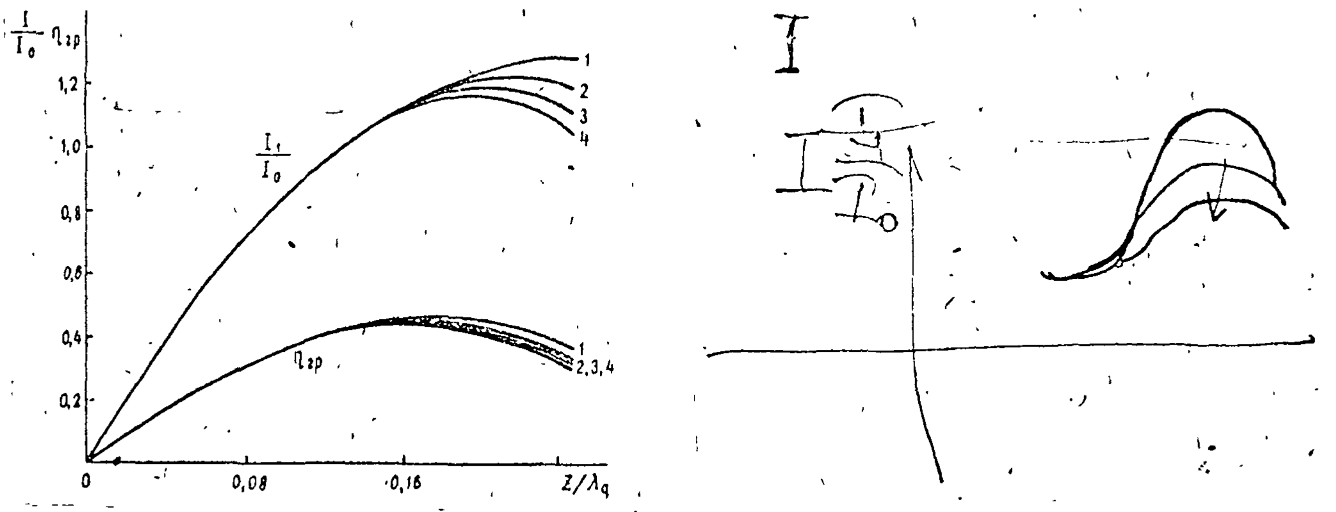

Fig. 1. 1 - deformable disks;
2 - rigid disks; 3 - DRM; 4 - stratified
layers.

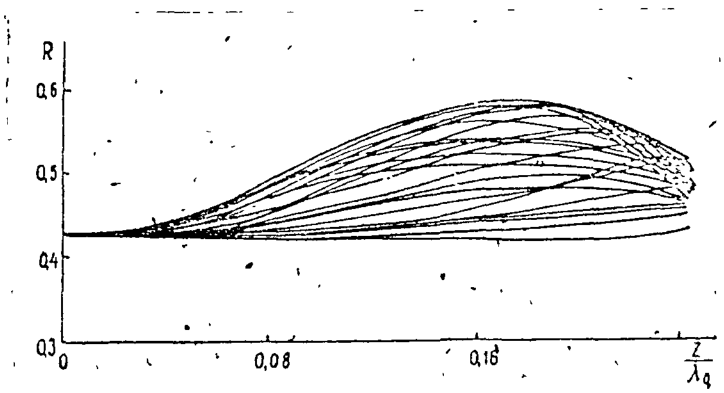

Fig. 2.

Since the radial forces acting on the outer layer are larger than the forces acting on the interior layers, the dynamic defocusing increases with the number of layers. Correspondingly, the amplitudes of the first current harmonic of the various layers change differently - the larger amplitude corresponding to the outer layer. The maxima of the amplitude of the first current harmonic occur at different values of the coordinates. The effect is that of a two-dimensional dynamic stratification of the beam. However, as shown by a comparison of curves 3 and 4 (Fig. 1), the expansion of the beam in the radial direction promotes an improved grouping in comparison with that in a multi-layered beam in the approximation of stratified layers. /230

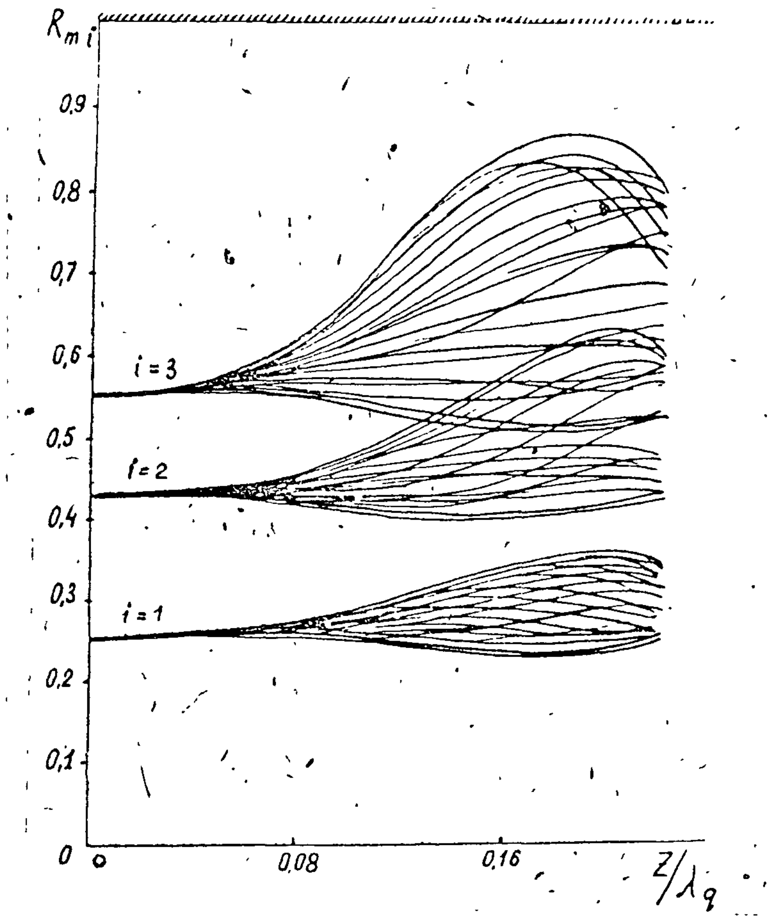

Fig. 3.

In considering the process of grouping, an important parameter of the system is the space factor κ_0. If the space factor is large, under dynamic defocusing the electrons may settle onto the drift tube, resulting in reduced efficiency of the instrument. Figure 4a shows the minimum value of the focusing magnetic field B_{min}/B_b for which there is still a settling of the beam onto the drift tube, as a function of the space factor. This graph was obtained from the results of an analysis for a two-dimensional model of disks of variable radius and optimal values of the amplitude of the alternating potential in the entry gap of the grouper of a two-resonator klystron.

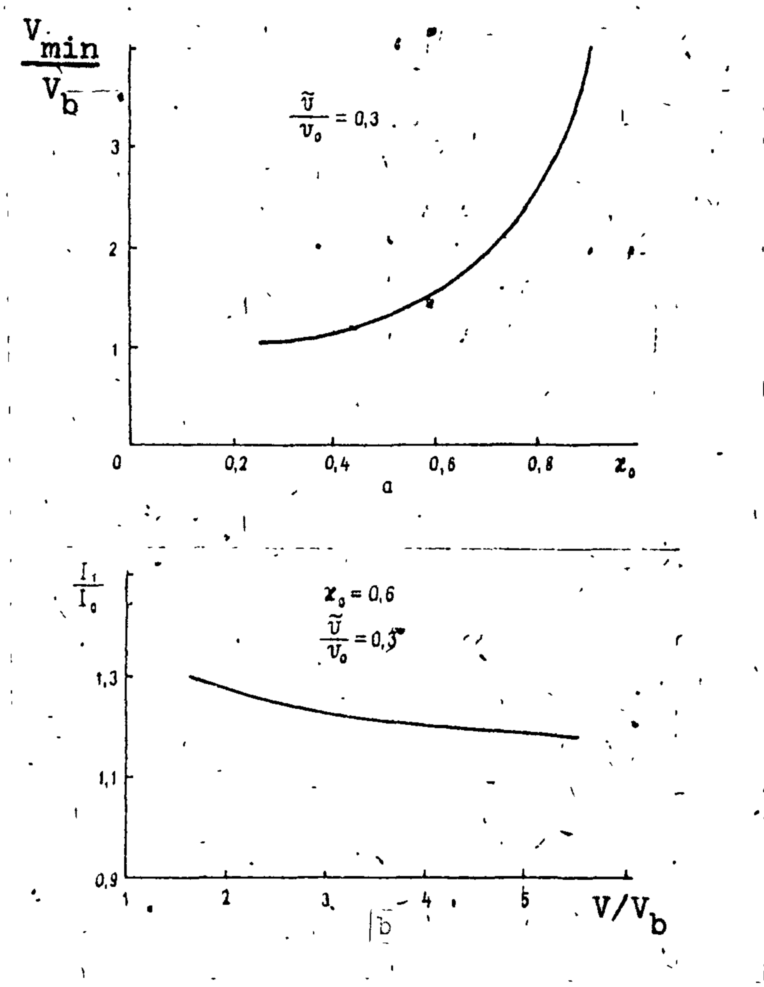

Fig. 4.

As follows from the preceding remarks, in the case of the model with disks of variable radii,the amplitude of the first current harmonic should increase as the focusing magnetic field diminishes down to the zero value, at which the beam begins to settle onto the drift tube (Fig. 4b).

§5. Dynamic Beam Defocusing in a Multi-Resonator Grouper

The processes of grouping of electrons in a multi-resonator klystron, just as those in a two-resonator klystron, are largely determined by the action of the space charge forces. Calculations in a uni-dimensional approximation and in an approximation of stratified layers have revealed that optimal beam parameters, optimal lengths of the drift regions, and optimal gap potentials exist for which at the end of the grouper, bunches are formed with small velocity spread (lecture 2-5). We can expect that in the two-dimensional approximation with sufficiently large focusing of the field, the discovered optimal parameters of the instrument will basically remain the same. In order to examine /231 the influence of dynamic defocusing on the processes of grouping in a multi-resonator klystron, typical parameters of a three-resonator grouping section of a five-resonator klystron including 2-4 resonators were taken. The first stage is used to amplify the weak signal and is omitted from the succeeding analysis. The distribution of lengths and potentials was taken to be optimal in terms of the uni-dimensional theory (lecture 2):

$$\frac{\ell_2}{\lambda_q} = 0{,}25; \quad \frac{\ell_3}{\lambda_q} = 0{,}17; \quad \frac{\ell_4}{\lambda_q} = 0{,}07; \quad \frac{V_2}{V_0} = 0{,}16; \quad \frac{V_3}{V_0} = 0{,}32.$$

The potential on the fourth resonator, in keeping with the recommendations of the theory of stratification, was taken to be less than that of the uni-dimensional theory $V_4/V_0 = 0.45$.

Let us examine the results of the analysis for the two-dimensional uni-layered model of disks of variable radius (Fig. 5). As in the case of the two-resonator klystron, during the grouping of the flux, at first the radii of the disks in the center of the bunches increase. As the Coulomb repulsion decreases, the beam expansion results in an improved grouping. Under the action of the high-frequency fields of the following resonators the velocity modulation of the beam is intensified and the diameter of the flux again increases. In the last drift region, the dynamic defocusing may lead to a settling of the electrons onto the drift tube or walls of the exit resonator. Settling occurs when the value of the magnetic field intensity is less than $3B_b$.

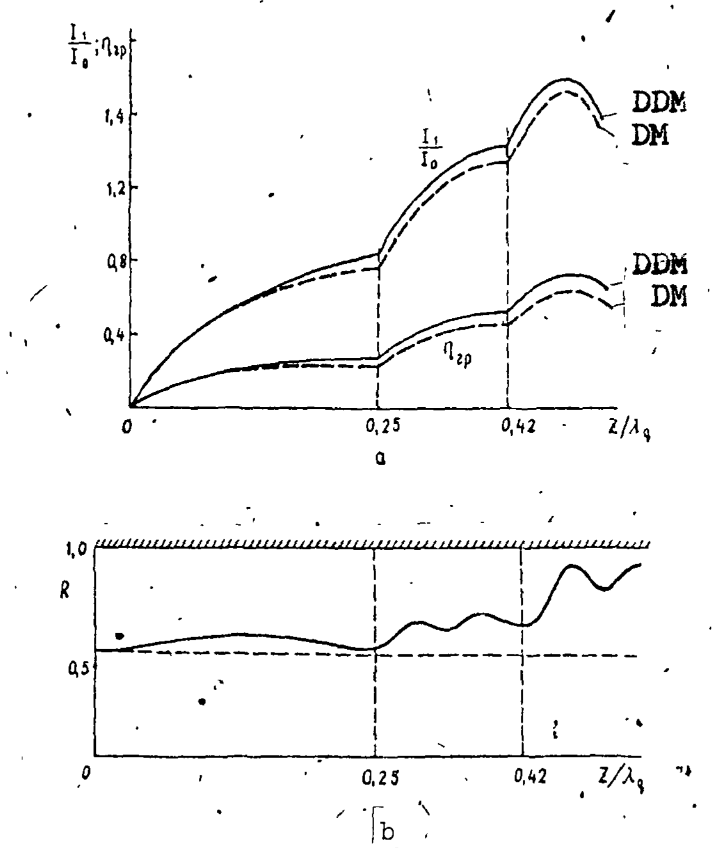

Fig. 5.

In the case of a multi-resonator klystron, the obtained relation of the minimum-permissible magnetic field intensity as a function of the space factor (Fig. 4a) produces depressed results. The amplitude of the first current harmonic and the quality function for the model with two-dimensional disks, as in the case of a two-resonator klystron, increase more quickly than the corresponding values in the uni-dimensional approximation

(Fig. 5). In order to prevent settling, the magnetic field induction is varied along the flux. In the first drift regions /233 it equals 2.5 B_b. In the last drift region it is increased to $B_z = 3\ B_b$. The raising of the magnetic field toward the exit end of the instrument is typical for modern high-power multi-resonator instruments.

As can be seen from Fig. 5b, the electron flux enlarges toward the exit resonator, although there is no settling. The dynamic defocusing does not produce a noticeable change in the values or position of the maxima of the amplitude of the first current harmonic I_1/I_0 or the grouping quality factor $\eta_{\text{гр}}$

In [4] two-dimensional processes were considered in the exit resonator for a given shape of the exciting bunch. The parameters of the beam and the exit resonator were varied. The grouper was not examined. It was presumed that optimal grouping is possible in a broad range of variation of the beam parameters and that two-dimensional effects were insignificant. Such an assumption is only partially correct. In actuality, the shape of the bunch depends on the beam parameters. As follows from the preceding material, at the exit of the grouper dense bunches with small spread are obtained when completely determined optimal conditions are imposed on the plasma frequency and the beam radius. The variation of the parameters in the limits discussed in [4] would displace the grouper from its optimal mode even in the uni-dimensional approximation, to say nothing of when stratification and dynamic defocusing are taken into account. In a joint analysis of grouper and exit resonator a maximum should be found on the curve of the efficiency as a function of the reduced radius, and not the polytone increase in efficiency with decrease in this radius as found in [4].

§6. Comparison of the Results of Theory and Experiment

The theoretical investigations of which the results are presented in these lectures indicate the important role of the space charge forces and the possibility of their use for optimization of the parameters of klystrons. /234

A general characteristic of the Coulomb forces in the steady state is the microperveyance of the beam P_μ. If $P_\mu \lesssim 1$, the transverse deflection of the potential is small and does not affect the grouping. However the choice of $P_\mu \ll 1$ in powerful instruments is not desirable, as there occurs an unjustifiable increase in the weight of the instrument and focusing system. In experimental instruments with high efficiency, microperveyances of 0.5-1.0 are usually employed. For a given adjusted radius (parameter K), the decrease in P_μ from 1 to 0.5 corresponds to a change in the relative value of the plasma frequency ω_p/ω by $\sqrt{2}$ times:

$$\frac{\omega_p}{\omega} = K\sqrt{\frac{P_\mu}{132}}$$

As follows from the contents of lecture 2, under such a decrease in P_μ the efficiency increases by 2-3%. Approximately the same increase in efficiency is found in experiments when P_μ is changed from 1 to 0.5 [7].

An optimization of the grouping of electrons is possible by choosing a range of action of the Coulomb forces 1/K on the order of the dimensions of the front of the bunch. A calculation reveals that the parameter K should be within limits of 2.5-5 (lecture 2). Precisely such values of the parameter are characteristic for the majority of powerful klystrons.

The groupers of powerful klystrons are distinguished by excellent passage of current. The ratio between the beam radius and the radius of the drift tube $\varkappa = \frac{r_n}{r_T}$ is taken within limits of 0.5-0.8. Most often values of 0.6-0.7 are used. In this case, the good current conduction in the steady state is accompanied by small settling of the current in the dynamic mode for moderate magnetic fields (lecture 7). The majority of the analyses of which the results are given in these lectures have been performed for beams with typical space factor κ = 0.6-0.7. /235

Theoretical investigations have demonstrated the possibility of constructing klystrons with two optimal designs of grouper. In klystrons with resonators tuned to the fundamental frequency, the first drift tubes of the grouper should be "elongated". The lengths of the drift segments should diminish as their number increases by a given law (lecture 2). In klystrons with additional resonators tuned to the frequency of the second signal harmonic, the lengths of the drift tubes are shortened and also obey a certain correspondence (lecture 3). A uni-dimensional treatment reveals that at the exit of optimized groupers there are formed bunches with a high content of the first current harmonic and diminished spread in the velocities of the electrons. These are characterized by an elevated value of the grouper quality factor.

Powerful groupers use resonators with gridless gaps. The radial distribution of the high-frequency fields results in a discrepancy in the grouping of the electrons over the layers - beam stratification. In a grouper with elongated tubes, the stratification is somewhat moderated on account of the specific process of preferential excitation of the fundamental mode of the waves of the space charge. There is no complete compensation for the stratification (lectures 4,5). To achieve this, it is necessary to employ special systems of resonators with gaps of different extent (lecture 4), which have not yet been used in klystrons. A five-resonator klystron with efficiency ∿65% (P_μ = 1.2) has a triple-resonator optimized grouper with length distribution $1/\lambda_q = 0.27, 0.15, 0.07$ [8]. This distribution is practically the same as the distribution recommended in lecture 2 for the corresponding grouper $1/\lambda_q = 0.24, 0.16, 0.07$. Evidently with compensation for the stratification the instrument of [8] could have a higher efficiency level. The same can be said of the instrument with efficiency of 63% [9]. /236

The best grouping of electrons with almost complete compensation for stratification can be observed in systems with auxiliary resonators, tuned to the frequency of the second harmonic. A six-resonator klystron with one resonator tuned to the frequency 2ω, fourth in the sequence, has a efficiency of 57% [3]. The instrument has a length distribution $1/\lambda_q = 0.106$, 0.128, 0.145, 0.078, 0.128. As shown by analysis of the grouper in terms of the uni-dimensional theory, conducted in lecture 3, by placing an auxiliary resonator of the second harmonic in the region of drift with length 0.268 [3] a maximum of the quality factor is achieved at the exit of the grouper, although the conditions for compensation of the stratification are absent. The influence of the stratification resulted in decreased efficiency.

The best conditions of compensation for stratification while preserving excellent grouping are achieved by placing a dual frequency resonator in the third drift region from the end. The length of the region should be twice as small as the optimal length found by analysis of a single-frequency grouping, $1/\lambda_q = 0.14$ (lecture 3,5). A six-resonator instrument with third resonator tuned to the frequency 2ω allowed an efficiency of 70% [7]. In the report on this instrument the lengths are given only in values of $\frac{\omega}{v_0}\ell$. However a comparison of the data of different figures allows us to presume that the typical angle of flight in the first drift region is equal to 120° (pertaining to a value $\omega l_1/v_0 = 0.24$). In this case, a ratio of reduced plasma frequency to signal frequency $\omega_p/\omega = 0.087$ is obtained. Let us assume that in the six-resonator instrument the first stage is a linear booster and the following stages form the grouper. Knowing the values of $\omega l/v_0$ and ω_q/ω, we can find the sequence of lengths of the grouper in l/λ_q: 0.87, 0.067, 0.114, 0.07. This turns out to be close to the optimal, considered in lecture 5, l/λ_q = 0.07, 0.07, 0.11, 0.07. Thus with proper tuning of the resonators in the instrument of [7], a mode of optimal grouping with compensation for stratification is achieved. /237

Increasing the number of resonators of the grouper N_p increases the quality factor and efficiency of the instrument (lecture 2). The increase is first rapid and then when N_p achieves $\sim$5, the dependence of the anticipated efficiency on N_p reaches saturation. As follows from the results of lecture 2, the variation of N_p from 3 to 4 increases the efficiency by about 5%. In an eight-resonator instrument with two resonators tuned to the frequency of the second harmonic an efficiency level of 75% is achieved [7]. We can presume that the role of the additional resonators consists in creating a possibility of compensation for the stratification and production of an optimal grouping. Since the introduction of an additional modulation at 2ω is equivalent to the formation of twin-hump bunches in elongated drift tubes, we may conclude that the grouper of the eight-resonator klystron is equivalent to a four-resonator grouper with elongated tubes, while the grouper of the six-resonator klystron is equivalent to a three-resonator grouper. An increase in the number of resonators by one will produce an instant increase in efficiency by 5%.

The theoretical results given in the lectures and their comparison with experimental findings reveal that the nonlinear theory produces reliable recommendations for the design of klystrons with high efficiency. The analysis of the klystrons should make use of the entire complex of methods and programs.

It is desirable to begin the analysis with the analytic method and then employ the uni-dimensional procedure, the stratification procedure, and finally two-dimensional computations. At the first stage the grouper is designed. The maximum of the grouping quality factor is found. Then the exit resonator is designed. An investigation of the processes in the exit resonator in a uni-dimensional approximation and in an approximation of stratification with allowance for Coulomb forces and return movement of the electrons has established that the quality factor is close to the electron efficiency of transformation of energy if the parameters of the exit resonator are optimal (lecture 6). /238 The identical conclusion follows from the results of the analysis of the exit resonator in a two-dimensional approximation, allowing for the settling of electrons [4].

The exit resonator should be designed with smallest possible value of the reduced radius of the drift tube [4]. The use of smaller $\frac{\omega}{v_0} r_T$ enables a decrease in the marginal effects. Conditions are created for removal of slow electrons from the interaction region. Further lengthwise acceleration of the slow electrons is the reason for the lowering of the efficiency of the exit resonator as compared with the value of the quality factor. This decrease is pronounced in the calculations with the multi-layered model (lecture 6). An increase in the efficiency in proportion to decrease in the reduced radius of the tubes of the exit resonator was pointed out previously in the experimental study [7].

The angle of flight in the gap of the single-gap exit resonator in modern klystrons is taken to be sufficiently small, for example 42° [4], which agrees with the analysis results (lecture 6). Additional possibilities of increasing the efficiency of the klystron are opened up by using double-gap resonators (lectures 4,6).

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