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THE CRACK PROBLEM IN A SPECIALLY ORTHOTROPIC SHELL WITH DOUBLE CURVATURE(*)

by

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ABSTRACT

In this paper the crack problem of a shallow shell with two nonzero curvatures is considered. It is assumed that the crack lies in one of the principal planes of curvature and the shell is under Mode I loading condition. The material is assumed to be specially orthotropic. After giving the general formulation of the problem the asymptotic behavior of the stress state around the crack tip is examined. The analysis is based on Reissner's transverse shear theory. Thus, as in the bending of cracked plates, the asymptotic results are shown to be consistent with that obtained from the plane elasticity solution of crack problems. Rather extensive numerical results are obtained which show the effect of material orthotropy on the stress intensity factors in cylindrical and spherical shells and in shells with double curvature. Other results include the stress intensity factors in isotropic toroidal shells with positive or negative curvature ratio, the distribution of the membrane stress resultant outside the crack, and the influence of the material orthotropy on the angular distribution of the stresses around the crack tip.

1. Introduction

The crack problem in shallow shells by using a transverse shear theory [1,2] has previously been considered for cylindrical and spherical shells only [3-5]. The results given in [3-5] as well as those obtained from the classical shell theory (e.g., [6-8]) indicate that due to the curvature effects the stress intensity factors in shells may be considerably higher than that in flat plates having the same crack length and the same thickness. Also, the results given in [5] and [8] show that, unlike the infinite plate problem, the material orthotropy may have a significant influence on the stress intensity factors in shells. Therefore, from the viewpoint of practical applications it does seem to be important to study

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the problem for shell geometries other than cylinders and spheres and also to consider the effect of material orthotropy.

In this paper the basic problem of a shallow toroidal shell with two unequal and nonzero curvatures is considered. It is assumed that the material is specially orthotropic, the through crack is located in one of the principal planes of curvature, and the plane of the crack is a plane of symmetry with respect to loading as well as to the geometry of the shell. To remove the inconsistency in the asymptotic behavior of the bending and transverse shear resultants, a transverse shear theory is used in the analysis [1,2] (see the discussion given in [3-5] for shells and, for example, [9-11] for plates).

2. The Integral Equations

The geometry of the shallow shell under consideration is shown in Fig. 1. If the material is orthotropic (with x_1 and x_2 as the axes of orthotropy), the engineering material constants are defined through the following stress strain relations:

$$\varepsilon_{11} = \frac{1}{E_1}(\sigma_{11}-\nu_1\sigma_{22}) , \quad \varepsilon_{22} = \frac{1}{E_2}(\sigma_{22}-\nu_2\sigma_{11}) ,$$

$$\varepsilon_{12} = \sigma_{12}/2G_{12} , \quad \nu_1/E_1 = \nu_2/E_2 . \tag{1}$$

With the four independent material constants the differential operators arising from the formulation of the shallow shells do not seem to be factorable and consequently, the analysis becomes intractable. However, it can be shown that if the elastic constants are related through

$$G_{12} = \frac{\sqrt{E_1E_2}}{2(1+\sqrt{\nu_1\nu_2})} , \tag{2}$$

then, with a simple coordinate transformation, the orthotropic shell equations may be reduced to essentially that of isotropic shells (see, for example, [8] and [3-5]). The factorization condition (2) implies that the shell has only three independent elastic constants. Such a material (in plate or shell form) is said to be specially orthotropic. If we now define

$$E = \sqrt{E_1 E_2}\ , \quad \nu = \sqrt{\nu_1 \nu_2}\ , \quad G_{av} = E/2(1+\nu)\ , \quad c = (E_1/E_2)^{1/4}\ , \tag{3}$$

the stress-strain relations (1) become

$$\varepsilon_{11} = \frac{1}{E}\left(\frac{\sigma_{11}}{c^2} - \nu\sigma_{22}\right)\ , \quad \varepsilon_{22} = \frac{1}{E}\left(c^2\sigma_{22} - \nu\sigma_{11}\right)\ , \quad \varepsilon_{12} = \frac{\sigma_{12}}{2G_{av}} \tag{4}$$

where E and ν are the effective modulus and Poisson's ratio, c is the stiffness ratio, and from (2) and (3) it follows that in the specially orthotropic materials the measured shear modulus G_{12} is equal to the (calculated) effective shear modulus G_{av}.

Referring to Fig. 1 and the normalized quantities defined in Appendix A, and to [3-5] for details, in terms of a stress function ϕ and the (out of plane) displacement component w the shell problem may be formulated as follows:

$$\nabla^4\phi - \frac{1}{\lambda^2}\nabla_\lambda^2 w = 0\ , \tag{5}$$

$$\nabla^4 w + \lambda^2(1-\kappa\nabla^2)\nabla_\lambda^2\phi = 0\ , \tag{6}$$

$$\kappa\nabla^2\psi - \psi - w = 0\ , \tag{7}$$

$$\frac{\kappa(1-\nu)}{2}\nabla^2\Omega - \Omega = 0\ , \tag{8}$$

where

$$\nabla_\lambda^2 = \lambda_1^2\frac{\partial^2}{\partial y^2} - 2\lambda_{12}\frac{\partial^2}{\partial x\partial y} + \lambda_2^2\frac{\partial^2}{\partial x^2}\ , \tag{9}$$

$$\psi(x,y) = \kappa\left(\frac{\partial\beta_x}{\partial x} + \frac{\partial\beta_y}{\partial y}\right) - w\ , \quad \Omega(x,y) = \frac{\partial\beta_x}{\partial y} - \frac{\partial\beta_y}{\partial x}\ . \tag{10}$$

The shell parameters λ_1, λ_2, λ_{12}, and κ are defined in Appendix A, β_x and β_y are the angles of rotation of the normal to the shell surface, and the curvatures of the shell are defined by

$$\frac{1}{R_1} = -\frac{\partial^2 Z}{\partial x_1^2}\ , \quad \frac{1}{R_2} = -\frac{\partial^2 Z}{\partial x_2^2}\ , \quad \frac{1}{R_{12}} = -\frac{\partial^2 Z}{\partial x_1\partial x_2}\ , \tag{11}$$

where $Z = Z(x_1, x_2)$ is the equation of the middle surface of the shell,

x_1 and x_2 being the coordinates in the tangent plane. The normalized stress, moment, and transverse shear resultants are given by

$$N_{xx} = \frac{\partial^2\phi}{\partial y^2}, \quad N_{yy} = \frac{\partial^2\phi}{\partial x^2}, \quad N_{xy} = -\frac{\partial^2\phi}{\partial x\partial y}, \tag{12}$$

$$M_{xx} = \frac{a}{h\lambda^4}\left(\frac{\partial\beta_x}{\partial x} + \nu\frac{\partial\beta_y}{\partial y}\right), \quad M_{yy} = \frac{a}{h\lambda^4}\left(\nu\frac{\partial\beta_x}{\partial x} + \frac{\partial\beta_y}{\partial y}\right),$$

$$M_{xy} = \frac{a}{h\lambda^4}\,\frac{1-\nu}{2}\left(\frac{\partial\beta_x}{\partial y} + \frac{\partial\beta_y}{\partial x}\right), \tag{13}$$

$$V_x = \frac{\partial w}{\partial x} + \beta_x, \quad V_y = \frac{\partial w}{\partial y} + \beta_y, \tag{14}$$

where

$$\beta_x = \frac{\partial\psi}{\partial x} + \kappa\,\frac{1-\nu}{2}\,\frac{\partial\Omega}{\partial y}, \quad \beta_y = \frac{\partial\psi}{\partial y} - \kappa\,\frac{1-\nu}{2}\,\frac{\partial\Omega}{\partial x}. \tag{15}$$

The system of differential equations (5)-(8) may be solved by using the standard Fourier transforms. In this paper it is assumed that through a linear superposition the regular part of the solution has been separated and the problem is reduced to a perturbation problem in which the self-equilibrating crack surface tractions are the only nonzero external loads. Because of the assumed symmetry with respect to x_2x_3 plane in loading and shell geometry, the stress and moment resultants must satisfy the following symmetry conditions:

$$N_{xx}(x,y) = N_{xx}(-x,y), \quad N_{xy}(x,y) = -N_{xy}(-x,y),$$

$$N_{yy}(x,y) = N_{yy}(-x,y), \quad V_x(x,y) = -V_x(-x,y), \quad V_y(x,y) = V_y(-x,y),$$

$$M_{xx}(x,y) = M_{xx}(-x,y), \quad M_{yy}(x,y) = M_{yy}(-x,y), \quad M_{xy}(x,y) = -M_{xy}(-x,y), \tag{16}$$

It is, therefore, sufficient to consider one half of the shell only. Thus, in addition to the regularity conditions at infinity, the problem must be solved under the following boundary conditions:

$$N_{xy}(0,y) = 0, \quad M_{xy}(0,y) = 0, \quad V_x(0,y) = 0, \quad -\infty<y<\infty, \tag{17}$$

$$\lim_{x\to+0} N_{xx}(x,y) = F_1(y), \quad |y| < \sqrt{c}, \quad u(+0,y) = 0, \quad \sqrt{c} < |y| < \infty , \tag{18}$$

$$\lim_{x\to+0} M_{xx}(x,y) = F_2(y), \quad |y| < \sqrt{c}, \quad \beta_x(+0,y) = 0, \quad \sqrt{c} < |y| < \infty . \tag{19}$$

We note that the problem (for the half shell) is one of tenth order. By taking Fourier transforms in y it would give ten "integration constants" which are functions of the transform variable and are unknown. Five of these unknowns must be zero because of the regularity conditions at $x=\infty$, three may be eliminated by using the homogeneous conditions (17) at $x=0$, and the two mixed boundary conditions (18) and (19) would give a pair of integral equations to determine the remaining two. We also note that the integral equations of this problem would be identical to those obtained in [5] for the spherical shell. In [5], even though $R_1 = R_2$, because of the assumption of special orthotropy, the stiffness ratio $c = (E_1/E_2)^{1/4}$ is not unity and consequently $\lambda_1 \neq \lambda_2$, which is also the case in the problem considered in this paper. Thus, by defining

$$\frac{\partial}{\partial y} u(+0,y) = G_1(y), \quad \frac{\partial}{\partial y} \beta_x(+0,y) = G_2(y) , \tag{20}$$

from (18) and (19) it is seen that

$$G_j(y) = 0, \quad |y| > \sqrt{c}, \quad (j=1,2) \tag{21}$$

$$\int_{-\sqrt{c}}^{\sqrt{c}} G_j(y)dy = 0, \quad (j=1,2) . \tag{22}$$

Referring now to [5] for details the integral equations of the problem may be expressed as

$$\int_{-\sqrt{c}}^{\sqrt{c}} \frac{G_1(t)}{t-y} dt + \int_{-\sqrt{c}}^{\sqrt{c}} \sum_1^2 k_{1j}(y,t)G_j(t)dt = 2\pi F_1(y), \quad |y| < \sqrt{c} ,$$

$$\frac{1-\nu^2}{\lambda^4}\int_{-\sqrt{c}}^{\sqrt{c}} \frac{G_2(t)}{t-y} dt + \int_{-\sqrt{c}}^{\sqrt{c}} \sum_1^2 k_{2j}(y,t)G_j(t)dt = 2\pi \frac{h}{a} F_2(y), \quad |y| < \sqrt{c} , \tag{23a,b}$$

where the kernels k_{ij} are known bounded functions and are given in [5].

3. The Stress State Around Crack Tips

In the system of integral equations the interval $(-\sqrt{c},\sqrt{c})$ may be normalized by introducing the following change in variables

$$\tau = t/\sqrt{c}, \quad \eta = y/\sqrt{c}, \quad \xi = x/\sqrt{c}, \quad G_j(t) = g_j(\tau), \quad (j=1,2) . \tag{23}$$

The integral equations may thus be expressed as

$$\int_{-1}^{1} \sum_{1}^{2}\left[\frac{a_i\delta_{ij}}{\tau-\eta} + K_{ij}(\eta,\tau)\right] g_j(\tau)d\tau = f_i(\eta), \quad (i=1,2), \quad -1<\eta<1 . \tag{24}$$

The index of the singular integral equations (24) is +1 and their solution is, therefore, of the following form

$$g_i(\tau) = \frac{h_i(\tau)}{\sqrt{1-\tau^2}}, \quad (i=1,2), \quad -1<\tau<1 , \tag{25}$$

where h_1 and h_2 are bounded functions. The system of integral equations (24) subject to the additional single-valuedness conditions (see (22))

$$\int_{-1}^{1} g_i(\tau)d\tau = 0, \quad (i=1,2) \tag{26}$$

may be solved numerically by using the related Gaussian quadrature formulas [12].

After solving the integral equations (24) all the field quantities in the shell may be expressed in terms of the density functions g_1 and g_2 as finite integrals [5]. In particular, the behavior of the stress state around the crack tips may be obtained by examining the corresponding integrals asymptotically. Referring to Appendix A, the normalized membrane and bending stresses are given by

$$\sigma_{ij}^{m} = N_{ij}, \quad \sigma_{ij}^{b} = \frac{12az}{h} M_{ij}, \quad \sigma_{ij} = \sigma_{ij}^{m} + \sigma_{ij}^{b}, \quad (i,j) = (x,y) . \tag{27}$$

If we now define the polar coordinates (r,θ) at the crack tip $(\xi=0,\eta=1)$ by

$$\xi = r \sin\theta, \quad \eta-1 = r \cos\xi , \tag{28}$$

the asymptotic stress state at the crack tip may be expressed as (see [5] for details)

$$\sigma_{xx} \cong -\frac{h_1(1)+zh_2(1)}{2\sqrt{2r}} \left(\frac{5}{4}\cos\frac{\theta}{2}-\frac{1}{4}\cos\frac{5\theta}{2}\right),$$

$$\sigma_{yy} \cong -\frac{h_1(1)+zh_2(1)}{2\sqrt{2r}} \left(\frac{3}{4}\cos\frac{\theta}{2}+\frac{1}{4}\cos\frac{5\theta}{2}\right),$$

$$\sigma_{xy} \cong -\frac{h_1(1)+zh_2(1)}{2\sqrt{2r}} \left(-\frac{1}{4}\sin\frac{\theta}{2}+\frac{1}{4}\sin\frac{5\theta}{2}\right), \qquad (29a\text{-}c)$$

$$V_x \cong \left[-\frac{\kappa}{2}\, h_1(1)\lambda^2\lambda_2^2+h_2(1)\right]\sqrt{r/2}\,\sin\theta\,\cos\frac{\theta}{2}. \qquad (30)$$

Note that the asymptotic behavior of the in-plane stresses given by (29) is identical to that obtained from the plane elasticity solution of a symmetric crack problem. The transverse shear resultant given by (30) represents Mode III cleavage stress which is nonsingular for the symmetric loading under consideration.

Referring to Fig. 1, in this problem the Mode I stress intensity factor at the crack tip x_2=a is defined as follows:

$$k_1(x_3) = \lim_{x_2\to a}\sqrt{2(x_2-a)}\,\sigma_{11}(0,x_2,x_3)\,. \qquad (31)$$

From the definitions given by (23), (28) and Appendix A it can be shown that

$$\tan\theta = \frac{\xi}{\eta-1} = \frac{1}{c}\,\frac{x_1}{x_2-a}\,. \qquad (32)$$

If we define the polar coordinates in the x_1x_2 plane (at x_1=0, x_2=a) by

$$\rho\sin\alpha = x_1,\quad \rho\cos\alpha = x_2-a\,, \qquad (33)$$

from (32) it follows that

$$\tan\alpha = c\tan\theta\,. \qquad (34)$$

Similarly, by substituting from (23), (28), and Appendix A into (29) we obtain

$$\sigma_{11}(x_1,x_2,x_3) \cong -\frac{cE\sqrt{a}}{2}\,[h_1(1)+\frac{x_3}{a}h_2(1)]\,\frac{1}{\sqrt{2\rho}}\,\frac{\frac{5}{4}\cos\frac{\theta}{2}-\frac{1}{4}\cos\frac{5\theta}{2}}{\left(\cos^2\alpha+\frac{\sin^2\alpha}{c^2}\right)^{1/4}}\ . \tag{35}$$

Also if we express the (in-plane cleavage) stress σ_{11} in the close neighborhood of the crack tip (x_1=0,x_2=a) or (ρ=0) in the following standard form

$$\sigma_{11}(x_1,x_2,x_3) \cong \frac{k_1(x_3)}{\sqrt{2\rho}}\,f_{11}(\alpha)\ , \tag{36}$$

with $f_{11}(0)=1$, from (35) and (36) the Mode I stress intensity factor k_1 and the function f_{11} giving the angular distribution of the stress component σ_{11} around the crack tip may be obtained as follows

$$k_1(x_3) = -\frac{cE\sqrt{a}}{2}\,[h_1(1)+\frac{x_3}{a}h_2(1)]\ , \tag{37}$$

$$f_{11}(\alpha) = \frac{\frac{5}{4}\cos\frac{\theta}{2}-\frac{1}{4}\cos\frac{5\theta}{2}}{\left(\cos^2\alpha+\frac{\sin^2\alpha}{c^2}\right)^{1/4}}\ ,\quad \theta = \text{Arctan}(\frac{1}{c}\tan\alpha)\ . \tag{38}$$

Similarly if we let

$$\sigma_{22}(x_1,x_2,x_3) \cong \frac{k_1(x_3)}{\sqrt{2\rho}}\,f_{22}(\alpha),\quad \sigma_{12}(x_1,x_2,x_3) \cong \frac{k_1(x_3)}{\sqrt{2\rho}}\,f_{12}(\alpha), \tag{39}$$

from (29) we obtain

$$f_{22}(\alpha) = \frac{\frac{3}{4}\cos\frac{\theta}{2}+\frac{1}{4}\cos\frac{5\theta}{2}}{\left(\cos^2\alpha+\frac{\sin^2\alpha}{c^2}\right)^{1/4}}\ ,\quad f_{12}(\alpha) = \frac{-\frac{1}{4}\sin\frac{\theta}{2}+\frac{1}{4}\sin\frac{5\theta}{2}}{\left(\cos^2\alpha+\frac{\sin^2\alpha}{c^2}\right)^{1/4}}\ . \tag{40}$$

4. The Results and Discussion

The symmetric problem formulated by the system of integral equations (23) is solved under two sets of external loads, namely uniform membrane loading and uniform bending moment applied to crack surfaces. For the membrane loading it is assumed that

$$N_{11}(0,x_2) = -N_o = -h\sigma_m, \quad M_{11}(0,x_2) = 0, \quad -a < x_2 < a, \tag{41}$$

where σ_m is constant. From (41) and the Appendix A the input functions in (23) may be expressed as

$$F_1(y) = -\sigma_m/cE, \quad F_2(y) = 0, \quad -\sqrt{c} < y < \sqrt{c}. \tag{42}$$

In this problem the corresponding flat plate stress intensity factor is $k_p = \sigma_m\sqrt{a}$ and from (37) the membrane and bending components of the stress intensity factor ratio $k_1(x_3)/\sigma_m\sqrt{a}$ may be evaluated as follows:

$$k_{mm} = \frac{k_1(0)}{\sigma_m\sqrt{a}} = -\frac{cE}{2\sigma_m} h_1(1), \tag{43}$$

$$k_{bm} = \frac{k_1(h/2) - k_1(0)}{\sigma_m\sqrt{a}} = -\frac{cE}{2\sigma_m} \frac{h}{2a} h_2(1). \tag{44}$$

For the bending of the shell the external loads are

$$N_{11}(0,x_2) = 0, \quad M_{11}(0,x_2) = -M_o = -\frac{h^2}{6}\sigma_b, \quad -a < x_2 < a, \tag{45}$$

which give the input functions as follows:

$$F_1(y) = 0, \quad F_2(y) = -\frac{\sigma_b}{6cE}, \quad -\sqrt{c} < y < \sqrt{c}, \tag{46}$$

where σ_b is a known constant. For this loading condition the membrane and bending components of the stress intensity factor ratio may be defined as and evaluated from

$$k_{mb} = \frac{k_1(0)}{\sigma_b\sqrt{a}} = -\frac{cE}{2\sigma_b} h(1), \tag{47}$$

$$k_{bb} = \frac{k_1(h/2) - k_1(0)}{\sigma_b\sqrt{a}} = -\frac{cE}{2\sigma_b} \frac{h}{2a} h_2(1). \tag{48}$$

The calculated results for isotropic and specially orthotropic shells are shown in Figures 2-9 and Tables 1-11. Fig. 2 shows the comparison of the membrane component of the stress intensity factor ratios k_{mm} in a cylindrical shell containing a circumferential or an axial crack and a spherical shell containing a meridional crack subjected to uniform membrane stress $\sigma_{11} = \sigma_m$. For the same loading and curvature the Mode I stress intensity factor appears to be highest in sphere and lowest for the circumferential crack. Similar results may be observed from Fig. 3 which shows an example for the membrane stress resultant $N_{11}(0,x_2)$ for $x_2 > a$. After determining G_1 and G_2 N_{11} is obtained directly from (23a) by observing that (23a) gives the expression of $N_{xx}(0,y)$ outside (i.e., for $|y| > \sqrt{c}$) as well as inside the crack. Fig. 3 also shows the flat plate solution. Note that even though near the crack tip the stresses in the shells are greater than that in the plate, because of their greater rate of decay, away from the crack region the shell stresses fall below the stress level in the flat plate.

The results showing the effect of the curvature ratio R_1/R_2 on the Mode I stress intensity factor ratios k_{ij}, (i,j=m,b) defined by (43), (44), (47) and (48) in an isotropic shell are given in Tables 1 and 2. Table 1 shows the results for a positive curvature ratio R_1/R_2 as, for example, in the case of outside surface of pipe elbows and barrel shaped toroidal shells. The results of an example for a negative curvature ratio $R_1/R_2 = -0.5$ are given in Table 2. Fig. 4 shows the comparison of the stress intensity factors obtained from shells with positive and negative curvature ratios and from an axially cracked cylinder. Note that, as one might expect and consistent with the trends in Fig. 2, the stress intensity factor for $R_1/R_2 > 0$ is greater and that for $(R_1/R_2) < 0$ is smaller than the value for the cylinder (i.e., for $R_1/R_2 = 0$).

The remaining results given in this paper deal with the specially orthotropic shells, that is, the orthotropic shells for which the measured in-plane elastic constants of the material approximately satisfy the factorization condition given by (2), namely

$$G_{12} \cong \frac{\sqrt{E_1 E_2}}{2(1+\sqrt{\nu_1 \nu_2})} = G_{av} . \tag{49}$$

For example, consider two graphite-epoxy (fiber reinforced) laminates consisting of 0°, ∓45°, 90° unidirectional laminae. Laminate A has 20%, 30%, and 50% of its laminae, and laminate B 20%, 50%, and 30% of its laminae oriented along 0°, ∓45°, and 90° directions, respectively. If

0° direction coincides with E_1, the measured elastic constants of the two laminates are known to be

	E_1(psi)	E_2(psi)	ν_1	ν_2	G_{12}(psi)	G_{av}(psi)
A:	6.9×10^6	12.3×10^6	0.140	0.250	2.1×10^6	3.88×10^6
B:	7.1×10^6	9×10^6	0.270	0.342	3.05×10^6	3.06×10^6

where G_{av} is calculated from (49) by using the effective modulus and the effective Poisson's ratio. Thus, it is seen that the assumption of special orthotropy $G_{12}\cong G_{av}$ is valid for laminate B but not for laminate A. Table 3 shows the elastic constants of two orthotropic materials used in the examples. The "mildly" orthotropic material is typical of rolled sheet metallic materials. Usually strongly orthotropic structural materials are fiber reinforced composites.

Tables 4-9 show the membrane and bending components of the stress intensity factor ratio k_{mm} and k_{bm} defined by (43) and (44) for three symmetric crack geometries in cylindrical and spherical shells subjected to uniform membrane loading $N_{11}=-N_0$ on the crack surfaces (Fig. 1). For completeness the results for isotropic cylinders are also given in the tables. Note that for these simple crack-shell geometries there are three length parameters, namely mean radius R, thickness h, and half crack length a. Therefore, the solution must contain two dimensionless parameters which in these examples are assumed to be a/h and λ_1 or λ_2 which contains $a/\sqrt{Rh}$ (see Appendix A). In the tables the shell is designated by E_1/E_2 and the material is oriented in such a way that the crack is parallel to E_2 axis, The two sets of orthotropic results given in the same table correspond to a 90-degree material rotation. $E_1/E_2=1$ corresponds to the isotropic shell. These tables show that the influence of the material orthotropy on the stress intensity factors could be quite significant. A graphic demonstration of this effect in a cylindrical shell with an axial crack is shown in Fig. 5(*). It is seen that the membrane component

(*)In Fig. 5 the shell parameter $\lambda_1=[12(1-\nu_1\nu_2)]^{1/4}\,a/\sqrt{Rh}$ is used as a basis in comparing k_{mm} for different shells. For an accurate comparison one should have $\sqrt{\nu_1\nu_2}=\nu$(isotropic) if λ_1 is used, otherwise one should use $a/\sqrt{Rh}$ as a basis. The difficulty in such comparisons is compounded of the stress intensity factor increases with increasing E_1/E_2. This effect is quite definite and, for higher values of E_1/E_2, is highly pronounced in cylindrical shells with an axial crack and in spherical shells. On the other hand in circumferentially cracked cylindrical shells the orthotropy effect seems to be rather insignificant. It should be pointed out that the effect of material orthotropy such as that shown in Fig. 5 is not confined to shells. For example, similar results are observed in the plane elasticity solution of an orthotropic infinite strip containing a crack parallel to its boundaries [13]. In this problem if the stress intensity factor is plotted against a/H (2a and 2H being the crack length and strip width) with E_1/E_2 as the parameter, the result would be identical (in form) to that shown in Fig. 5, that is (in the terminology of Fig. 5) for $E_1/E_2 > 1$ k_1 would be greater and for $E_1/E_2 < 1$ k_1 would be smaller than the stress intensity factor for the corresponding isotropic plate. It should, of course, be noted that the stress intensity factor is a measure of the crack driving force in fracture analysis of the structural component. In comparing two materials (or two material orientations with respect to the crack plane) one must also consider the fracture resistance of the material. For example, in fiber reinforced composites having a crack in E_2 plane for $E_1/E_2 > 1$ not only the stress intensity factor but also the fracture resistance of the material would be expected to be greater than the corresponding values for a 90° rotated material (i.e., for $E_1/E_2 < 1$). Hence from the analysis alone it is not possible to infer the fracture strength of orthotropic plates and shells.

In a mildly orthotropic shell the effect of material orthotropy on the distribution of the membrane resultant $N_{11}(0,x_2)$ outside the crack is shown in Fig. 6. Near the crack tip the results shown in Fig. 6 are consistent with that of Fig. 5. However, as in Fig. 3, away from the crack tip the trend showing the effect of E_1/E_2 on the stress distribution is

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also by the fact that $\sqrt{\nu_1\nu_2}$ or ν enters into the shell analysis independently as well as through λ_1. However, it has been shown that the effect of ν on k_{mm} is rather insignificant [4]. Also, for the three materials used in Fig. 5, namely for $(E_1/E_2) = 1, 1.38, 26.67$ the coefficient $[12(1-\nu_1\nu_2)]^{1/4}$ of $a/\sqrt{Rh}$ in λ_1 is 1.82, 1.84, 1.85, respectively. This means that in Fig. 5 even though the comparison is not made for shells with identical geometries, the error should not be very high and particularly the trend should be accurate.

reversed.

From (36)-(40) it may be seen that not only the stress intensity factor but also the angular distribution of stresses around the crack tip is influenced by the material orthotropy. Fig. 7 shows an example for this effect in a strongly orthotropic material. Note that in isotropic materials the functions $f_{ij}(\alpha)$, $(i,j=1,2)$ (see (36) and (39)) giving the angular distribution of stresses under Mode I loading condition are independent of the elastic constants (and are obtained from (36) and (39) by letting $c=1$). In the specially orthotropic materials f_{ij} seems to depend on the stiffness ratio $c=(E_1/E_2)^{1/4}$.

Tables 4-9 show the stress intensity factors for shells under membrane loading only; i.e., for $N_{11}(0,x_2)=-N_0$, $M_{11}(0,x_2)=0$. For orthotropic shells subjected to bending on the crack surfaces (i.e., for $N_{11}(0,x_2)=0$, $M_{11}(0,x_2)=-M_0$), the stress intensity factor ratios k_{bb} and k_{mb} defined by (47) and (48) do not seem to differ significantly from the corresponding results for the isotropic shells which are given in [3]-[5]. The results are, therefore, not tabulated in this paper. Some sample results are, however, shown in Figures 8 and 9 for a cylindrical shell containing an axial or a circumferential crack. The results for spherical shells may be found in [5].

Table 10 shows the principal stress intensity factor ratio k_{bb} for a plate under bending. In addition to the relative crack size a/h, the table shows the effect of the Poisson's ratio in isotropic plates and the stiffness ratio E_1/E_2 in specially orthotropic plates. Since the relative influence of the material constants on k_{bb} in plates and shells is roughly the same, these results may be used with the isotropic shell results given for a fixed Poisson's ratio (e.g., $\nu=0.3$ in most cases) to estimate the values of the k_{bb} for the isotropic and orthotropic shells.

The effect of material orthotropy on the stress intensity factor ratios k_{ij}, $(i,j=m,b)$ defined by (43), (44), (47) and (48) on a shell with principal radii of curvature R_1 and R_2 is shown in Table 11 which also includes the isotropic results for comparison. Note that the results given in Table 11 have the same trend as that shown in Fig. 5 with regard to the variation of the principal stress intensity component k_{mm} for the shell under membrane loading.

References

1. E. Reissner and F.Y.M. Wan, "On the equations of the linear shallow shell theory", Studies in Applied Mathematics, Vol. 48, pp. 132-145, 1969.

2. P.M. Nagdi, "Note on the equations of shallow elastic shells", Quart. of Appl. Math., Vol. 14, pp. 331-333, 1956.

3. S. Krenk, "Influence of transverse shear on an axial crack in a cylindrical shell", Int. Journal of Fracture, Vol. 14, pp. 123-143, 1978.

4. F. Delale and F. Erdogan, "Transverse shear effect in a circumferentially cracked cylindrical shell", Quart. of Appl. Math., Vol. 37, pp. 239-258, 1979.

5. F. Delale and F. Erdogan, "Effect of transverse shear and material orthotropy in a cracked spherical cap", Int. J. Solids Structures, Vol. 15, pp. 907-926, 1979.

6. E.S. Folias, "An axial crack in a pressurized cylindrical shell", Int. Journal of Fracture Mechanics, Vol. 1, pp. 104-113, 1965.

7. L.G. Copley and J.L. Sanders, Jr., "A longitudinal crack in a cylindrical shell under internal pressure", Int. Journal of Fracture Mechanics, Vol. 5, pp. 117-131, 1969.

8. F. Erdogan, "Crack problems in cylindrical and spherical shells", Plates and Shells with Cracks, G.C. Sih, ed., Noordhoff Int. Publ., Leyden, pp. 161-199, 1977.

9. J.K. Knowles and N.M. Wang, "On the bending of an elastic plate containing a crack", Journal of Mathematics and Physics, Vol. 39, pp. 223-236, 1960.

10. R.J. Hartranft and G.C. Sih, "Effect of plate thickness on the bending stress distribution around through cracks", Journal of Mathematics and Physics, Vol. 47, pp. 276-291, 1968.

11. F. Delale and F. Erdogan, "The effect of transverse shear in a cracked plate under skewsymmetric loading", J. Appl. Mech., Vol. 46, Trans. ASME, pp. 618-624, 1979.

12. F. Erdogan, "Mixed boundary value problems in mechanics", Mechanics Today, S. Nemat-Masser, ed., Vol. 4, Pergamon Press, Oxford, pp. 1-86, 1978.

13. A. Cinar and R. Erdogan, "The crack and wedging problem for an orthotropic strip", Technical Report, NASA Grant NGR 39-007-011, Lehigh University, Feb. 1982.

Appendix A

The dimensionless and normalized quantities used in the analysis

$$x = x_1/a\sqrt{c}\,, \quad y = x_2\sqrt{c}/a\,, \quad z = x_3/a\,; \tag{A.1}$$

$$u = u_1\sqrt{c}/a\,, \quad v = u_2/a\sqrt{c}\,, \quad w = u_3/a\,; \tag{A.2}$$

$$\beta_x = \beta_1\sqrt{c}\,, \quad \beta_y = \beta_2/\sqrt{c}\,, \quad \phi(x,y) = F(x_1,x_2)/Eha^2\,; \tag{A.3}$$

$$\sigma_{xx} = \sigma_{11}/Ec\,, \quad \sigma_{yy} = \sigma_{22}\,c/E\,, \quad \sigma_{xy} = \sigma_{12}/E\,; \tag{A.4}$$

$$N_{xx} = N_{11}/Ehc\,, \quad N_{yy} = cN_{22}/Eh\,, \quad N_{xy} = N_{12}/Eh\,; \tag{A.5}$$

$$M_{xx} = M_{11}/Ech^2\,, \quad M_{yy} = cM_{22}/Eh^2\,, \quad M_{xy} = M_{12}/Eh^2\,; \tag{A.6}$$

$$V_x = V_1/hB\sqrt{c}\,, \quad V_y = V_2\sqrt{c}/Bh\,; \tag{A.7}$$

$$\lambda_1^4 = 12(1-\nu^2)a^4c^2/h^2R_1^2\,, \quad \lambda_2^4 = 12(1-\nu^2)a^4/c^2h^2R_2^2\,,$$

$$\lambda_{12}^4 = 12(1-\nu^2)a^4/h^2R_{12}^2\,, \quad \lambda^4 = 12(1-\nu^2)a^2/h^2\,, \quad \kappa = E/B\lambda^4\,; \tag{A.8}$$

$$E = \sqrt{E_1E_2}\,, \quad \nu = \sqrt{\nu_1\nu_2}\,, \quad (\nu_1/E_1 = \nu_2/E_2)\,, \quad B = 5E/12(1+\nu)\,. \tag{A.9}$$

Table 1. Stress intensity factor ratios in an isotropic shell with double curvature under uniform membrane load $N_{11} = N_0$, $M_{11} = 0$, (k_{mm}, k_{bm}) or uniform bending moment $M_{11} = M_0$, $N_{11} = 0$, (k_{bb}, k_{mb}) (the crack in the R_2 plane, $\nu = 0.3$, $a/h = 10$)

R_1/R_2 \ $a/\sqrt{hR_2}$	0.1	0.25	0.50	0.75	1.0	1.5	2.0
				k_{mm}			
0.2	1.040	1.216	1.681	2.246	2.854	4.145	5.556
1/3	1.025	1.141	1.471	1.897	2.378	3.434	4.590
0.5	1.018	1.100	1.349	1.686	2.078	2.969	3.965
2	1.005	1.035	1.130	1.277	1.460	1.917	2.471
3	1.004	1.027	1.101	1.219	1.368	1.740	2.195
5	1.003	1.021	1.078	1.168	1.286	1.577	1.934
				k_{bm}			
0.2	0.046	0.134	0.199	0.142	-0.038	-0.742	-1.899
1/3	0.034	0.109	0.188	0.181	0.081	-0.392	-1.221
0.5	0.028	0.092	0.174	0.189	0.132	-0.207	-0.843
2	0.015	0.055	0.123	0.163	0.166	0.062	-0.182
3	0.013	0.050	0.112	0.153	0.161	0.084	-0.104
5	0.011	0.045	0.102	0.142	0.153	0.097	-0.044
				k_{bb}			
0.2	0.641	0.615	0.555	0.495	0.441	0.356	0.293
1/3	0.643	0.621	0.566	0.508	0.455	0.368	0.305
0.5	0.644	0.624	0.573	0.516	0.463	0.377	0.313
2	0.645	0.630	0.586	0.532	0.480	0.393	0.331
3	0.645	0.630	0.587	0.535	0.482	0.395	0.334
5	0.645	0.631	0.589	0.536	0.484	0.397	0.336
				k_{mb}			
0.2	0.011	0.033	0.064	0.086	0.100	0.115	0.122
1/3	0.008	0.026	0.054	0.074	0.089	0.104	0.111
0.5	0.006	0.022	0.047	0.066	0.079	0.095	0.102
2	0.003	0.013	0.030	0.044	0.055	0.067	0.073
3	0.003	0.011	0.027	0.040	0.050	0.061	0.066
5	0.003	0.010	0.024	0.037	0.045	0.054	0.058

Table 2. Stress intensity factors in a saddle-shaped shell under uniform membrane loading $N_{11} = N_o$, $M_{11} = 0$, (k_{mm}, k_{bm}). or uniform bending $M_{11} = M_o$, $N_{11} = 0$, (k_{bb}, k_{mb}) (the crack in R_2 plane, $R_1/R_2 = -0.5$, $a/h = 10$, $\nu = 0.3$)

$a/\sqrt{R_2 h}$	0.1	0.25	0.5	0.75	1.0	1.5	2.0
k_{mm}	1.014	1.079	1.261	1.480	1.699	2.098	2.455
k_{bm}	-0.015	-0.045	-0.066	-0.039	0.025	0.195	0.335
k_{bb}	0.645	0.633	0.594	0.546	0.498	0.414	0.353
k_{mb}	-0.003	-0.011	-0.019	-0.020	-0.016	-0.004	0.006

Table 3. Elastic constants of the orthotropic materials used in the examples ($\nu_1/E_1 = \nu_2/E_2$, $G_{av} = E/2(1+\nu)$, $E = \sqrt{E_1 E_2}$, $\nu = \sqrt{\nu_1 \nu_2}$)

	Mildly orthotropic material (titanium)	Strongly orthotropic material (graphite-epoxy)
E_1 N/m^2(psi)	1.039×10^{11} (1.507×10^7)	1.034×10^{10} (1.5×10^6)
E_2 N/m^2(psi)	1.434×10^{11} (2.08×10^7)	2.758×10^{11} (4×10^7)
ν_1	0.1966	0.0075
ν_2	0.2714	0.2000
G_{12} N/m^2(psi)	4.675×10^{10} (6.78×10^6)	2.758×10^{10} (4.0×10^6)
G_{av} N/m^2(psi)	4.930×10^{10} (7.15×10^6)	2.572×10^{10} (3.73×10^6)

Table 4. Stress intensity factors in a mildly orthotropic and in an isotropic cylindrical shell containing an axial through crack and subjected to uniform membrane loading $N_{11} = N_o$ ($\nu = 0.3$ for the isotropic shell $\lambda_1 = [12(1-\nu^2)]^{1/4}\, a/\sqrt{Rh}$, $\nu = \sqrt{\nu_1 \nu_2}$)

a/h	10			5			2			1		
E_1/E_2	0.725	1.380	1	0.725	1.380	1	0.725	1.380	1	0.725	1.380	1
λ_1						k_{mm}						
0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.5	1.048	1.064	1.057	1.048	1.065	1.057	1.049	1.066	1.058	1.051	1.069	1.061
1.0	1.171	1.226	1.199	1.172	1.228	1.200	1.178	1.236	1.208	1.197	1.261	1.233
1.5	1.341	1.442	1.394	1.344	1.446	1.398	1.362	1.468	1.420			
2.0	1.539	1.687	1.618	1.545	1.695	1.625	1.580	1.738	1.668			
3.0	1.976	2.213	2.105	1.991	2.230	2.122						
4.0	2.430	2.748	2.603	2.458	2.780	2.634						
5.0	2.887	3.283	3.096	2.934	3.338	3.146						
6.0	3.347	3.827	3.580									
8.0	4.327		4.515									
						k_{bm}						
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.041	0.051	0.054	0.042	0.052	0.055	0.044	0.053	0.057	0.046	0.056	0.060
1.0	0.095	0.113	0.120	0.095	0.112	0.119	0.098	0.113	0.121	0.102	0.117	0.124
1.5	0.138	0.158	0.167	0.137	0.154	0.164	0.137	0.150	0.160			
2.0	0.168	0.185	0.194	0.164	0.176	0.185	0.159	0.165	0.175			
3.0	0.186	0.184	0.189	0.173	0.159	0.166						
4.0	0.154	0.117	0.116	0.127	0.073	0.076						
5.0	0.076	-0.008	-0.014	0.034	-0.073	-0.070						
6.0	-0.041	-0.183	-0.187									
8.0	-0.388		-0.634									
10.0	---	---	-1.183									

Table 5. Stress intensity factors in a strongly orthotropic and in an isotropic ($\nu = 0.3$) cylindrical shell containing an axial crack and subjected to uniform membrane loading $N_{11} = N_o$.

a/h	10			5			2			1		
E_1/E_2	0.037	26.67	1	0.037	26.67	1	0.037	26.67	1	0.037	26.67	1
λ_1	k_{mm}											
0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.5	1.012	1.238	1.057	1.012	1.238	1.057	1.012	1.242	1.058	1.012	1.253	1.061
1.0	1.043	1.714	1.199	1.043	1.717	1.200	1.045	1.736	1.208	1.048	1.795	1.233
1.5	1.093	2.244	1.394	1.093	2.252	1.398	1.047	2.298	1.420			
2.0	1.155	2.779	1.618	1.156	2.794	1.625	1.164	2.877	1.668			
3.0	1.309	3.826	2.105	1.313	3.865	2.122						
4.0	1.490	4.852	2.603	1.498	4.955	2.634						
5.0	1.686	---	3.096	1.699	---	3.146						
6.0	1.891	---	3.580									
8.0	2.311	---	4.515									
10.0	2.737	---	5.422									
	k_{bm}											
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.008	0.075	0.054	0.008	0.075	0.055	0.009	0.075	0.057	0.010	0.077	0.060
1.0	0.023	0.150	0.120	0.023	0.145	0.119	0.024	0.139	0.121	0.027	0.137	0.124
1.5	0.039	0.186	0.167	0.040	0.172	0.164	0.042	0.151	0.160			
2.0	0.056	0.175	0.194	0.056	0.148	0.185	0.059	0.105	0.175			
3.0	0.089	0.017	0.189	0.088	-0.045	0.166						
4.0	0.117	-0.287	0.116	0.115	-0.395	0.076						
5.0	0.141	---	-0.014	0.136	---	-0.070						
6.0	0.158	---	-0.187									
8.0	0.169	---	-0.634									
10.0	0.148	---	-1.183									

Table 6. Stress intensity factors in a mildly orthotropic and in an isotropic cylindrical shell containing a circumferential through crack and subjected to uniform membrane loading $N_{11} = N_o$, ($\nu = 1/3$ for the isotropic shell, $\lambda_2 = [12(1-\nu^2)]^{1/4} a/\sqrt{Rh}$, $\nu = \sqrt{\nu_1 \nu_2}$)

a/h	10			5			2			1		
E_1/E_2	0.725	1.380	1	0.725	1.380	1	0.725	1.380	1	0.725	1.380	1
λ_2	k_{mm}											
0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.5	1.012	1.012	1.012	1.012	1.012	1.012	1.013	1.013	1.012	1.013	1.013	1.013
1.0	1.047	1.047	1.048	1.047	1.047	1.048	1.049	1.049	1.050	1.053	1.053	1.055
1.5	1.100	1.099	1.102	1.101	1.100	1.103	1.105	1.104	1.108			
2.0	1.164	1.164	1.168	1.166	1.165	1.169	1.174	1.173	1.179			
3.0	1.308	1.308	1.314	1.312	1.311	1.317						
4.0	1.457	1.456	1.462	1.462	1.461	1.467						
5.0	1.599	1.598	1.604									
	k_{bm}											
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.036	0.036	0.041	0.036	0.036	0.041	0.038	0.038	0.043	0.040	0.040	0.044
1.0	0.081	0.082	0.092	0.081	0.081	0.092	0.082	0.082	0.092	0.084	0.083	0.093
1.5	0.110	0.111	0.123	0.108	0.109	0.119	0.104	0.105	0.114			
2.0	0.115	0.117	0.125	0.110	0.111	0.119	0.100	0.101	0.107			
3.0	0.072	0.075	0.071	0.059	0.062	0.057						
4.0	-0.009	-0.006	-0.024	-0.026	-0.024	-0.042						
5.0	-0.100	-0.096	-0.126									

Table 7. Stress intensity factors in a strongly orthotropic and in an isotropic ($\nu = 1/3$) cylindrical shell containing a circumferential crack and subjected to uniform membrane loading $N_{11} = N_o$.

a/h	10			5			2			1		
E_1/E_2	0.037	26.67	1	0.037	26.67	1	0.037	26.67	1	0.037	26.67	1
λ_1	k_{mm}											
0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.5	1.012	1.011	1.012	1.012	1.011	1.012	0.013	0.012	0.012	1.013	1.012	1.013
1.0	1.046	1.046	1.048	1.047	1.046	1.048	1.049	1.047	1.050	1.056	1.048	1.055
1.5	1.097	1.095	1.102	1.099	1.096	1.103	1.105	1.097	1.108			
2.0	1.159	1.157	1.168	1.162	1.157	1.169	1.175	1.161	1.179			
3.0	1.302	1.299	1.314	1.307	1.300	1.317						
4.0	1.450	1.446	1.462	1.458	1.448	1.467						
5.0	1.593	1.588	1.604									
	k_{bm}											
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.028	0.028	0.041	0.029	0.028	0.041	0.030	0.029	0.043	0.033	0.030	0.044
1.0	0.066	0.065	0.092	0.066	0.065	0.092	0.067	0.065	0.092	0.069	0.066	0.093
1.5	0.090	0.091	0.123	0.088	0.089	0.119	0.084	0.087	0.114			
2.0	0.096	0.101	0.125	0.090	0.099	0.119	0.080	0.090	0.107			
3.0	0.065	0.077	0.071	0.051	0.067	0.057						
4.0	0.001	0.019	-0.024	-0.017	0.004	-0.042						
5.0	-0.072	-0.050	-0.126									

Table 8. Stress intensity factors in a mildly orthotropic and in an isotropic ($\nu = 1/3$) spherical shell containing a meridional crack and subjected to uniform membrane loading $N_{11} = N_o$ ($\lambda_2 = [12(1-\nu^2)]^{1/4} a/\sqrt{Rh}$, $\nu = \sqrt{\nu_1 \nu_2}$)

a/h	10			5			2			1		
E_1/E_2	0.725	1.380	1	0.725	1.380	1	0.725	1.380	1	0.725	1.380	1
λ_2	k_{mm}											
0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.25	1.016	1.021	1.018	1.016	1.021	1.018	1.016	1.021	1.019	1.017	1.022	1.019
0.50	1.060	1.077	1.069	1.060	1.077	1.069	1.062	1.079	1.071	1.065	1.084	1.076
0.75	1.129	1.164	1.149	1.130	1.165	1.150	1.135	1.171	1.156	1.148	1.187	1.173
1.0	1.220	1.276	1.252	1.222	1.279	1.255	1.232	1.291	1.268	1.262	1.327	1.305
1.5	1.450	1.555	1.512	1.456	1.561	1.519	1.486	1.596	1.556			
2.0	1.733	1.891	1.828	1.744	1.903	1.841	1.807	1.976	1.918			
2.5	2.056	2.270	2.186	2.076	2.291	2.208						
3.0	2.415	2.685	2.579	2.446	2.718	2.615						
3.5	2.806	3.132	3.004	2.852	3.183	3.058						
4.0	3.226	3.611	3.460	3.296	3.687	3.539						
	k_{bm}											
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.25	0.025	0.029	0.033	0.026	0.030	0.034	0.027	0.031	0.035	0.029	0.033	0.037
0.50	0.065	0.073	0.084	0.066	0.074	0.084	0.068	0.075	0.086	0.071	0.079	0.090
0.75	0.104	0.115	0.130	0.104	0.115	0.130	0.106	0.115	0.130	0.109	0.118	0.133
1.0	0.135	0.148	0.165	0.134	0.145	0.162	0.133	0.142	0.158	0.134	0.141	0.157
1.5	0.165	0.175	0.187	0.158	0.164	0.174	0.145	0.146	0.155			
2.0	0.146	0.145	0.142	0.129	0.121	0.117	0.099	0.083	0.077			
2.5	0.077	0.054	0.031	0.047	0.015	-0.010						
3.0	-0.044	-0.099	-0.146	-0.090	-0.158	-0.206						
3.5	-0.219	-0.317	-0.390	-0.283	-0.398	-0.471						
4.0	-0.448	-0.602	-0.701	-0.533	-0.709	-0.807						

Table 9. Stress intensity factors in a strongly orthotropic and in an isotropic ($\nu = 1/3$) spherical shell containing a meridional crack and subjected to uniform membrane loading $N_{11} = N_0$.

a/h	10			5			2			1		
E_1/E_2	0.037	26.67	1	0.037	26.67	1	0.037	26.67	1	0.037	26.67	1
λ_2	k_{mm}											
0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.25	1.006	1.073	1.018	1.006	1.073	1.018	1.006	1.074	1.019	1.006	1.075	1.019
0.50	1.024	1.250	1.069	1.024	1.251	1.069	1.024	1.255	1.071	1.026	1.267	1.076
0.75	1.051	1.491	1.149	1.052	1.493	1.150	1.053	1.504	1.156	1.059	1.540	1.173
1.0	1.089	1.767	1.252	1.090	1.771	1.255	1.095	1.794	1.268	1.108	1.863	1.305
1.5	1.190	2.377	1.512	1.193	2.387	1.519	1.207	2.445	1.556			
2.0	1.318	3.030	1.828	1.324	3.050	1.841	1.354	3.165	1.918			
2.5	1.470	3.713	2.186	1.480	3.748	2.208						
3.0	1.643	4.423	2.579	1.658	4.484	2.615						
3.5	1.835	5.164	3.004	1.857	5.271	3.058						
4.0	2.044	5.948	3.460	2.075	6.145	3.539						
	k_{bm}											
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.25	0.012	0.037	0.033	0.012	0.037	0.034	0.013	0.038	0.035	0.014	0.040	0.037
0.50	0.033	0.088	0.084	0.033	0.087	0.084	0.035	0.087	0.086	0.038	0.089	0.090
0.75	0.055	0.133	0.130	0.056	0.130	0.130	0.058	0.126	0.130	0.061	0.125	0.133
1.0	0.076	0.165	0.165	0.076	0.158	0.162	0.077	0.147	0.158	0.079	0.141	0.157
1.5	0.104	0.172	0.187	0.101	0.151	0.174	0.096	0.114	0.155			
2.0	0.111	0.084	0.142	0.103	0.041	0.117	0.091	-0.035	0.077			
2.5	0.099	-0.108	0.031	0.085	-0.179	-0.010						
3.0	0.069	-0.404	-0.146	0.048	-0.510	-0.206						
3.5	0.022	-0.804	-0.390	-0.006	-0.957	-0.471						
4.0	-0.041	-1.314	-0.701	-0.074	-1.538	-0.807						

Table 10. The stress intensity factor ratio k_{bb} in isotropic and specially orthotropic plates under bending (the crack along the x_2 axis, $\nu = \sqrt{\nu_1 \nu_1}$, $E_1/E_2 = 1.38$, 0.725 for titanium, $E_1/E_2 = 26.67$, 0.037 for graphite epoxy)

E_1/E_2	ν	a/h = 10	a/h = 5	a/h = 2	a/h = 1
1	0	0.598	0.613	0.651	0.702
1	0.1	0.616	0.631	0.669	0.719
1	0.2	0.632	0.647	0.685	0.734
1	0.3	0.644	0.662	0.699	0.747
1	1/3	0.652	0.667	0.704	0.752
1	0.4	0.661	0.675	0.712	0.760
1	0.5	0.673	0.688	0.724	0.771
0.725	0.231	0.632	0.651	0.691	0.742
1.38	0.231	0.634	0.650	0.687	0.735
0.037	0.037	0.615	0.635	0.686	0.751
26.67	0.037	0.599	0.611	0.639	0.677

Table 11. Stress intensity factor ratios in an isotropic and in a specially orthotropic shell with principal radii of curvature R_1 and R_2 having a through crack in R_2 plane (material: titanium, $R_1/R_2 = 1/3$, $a/h = 1/10$, for the isotropic shell $\nu = 0.3$)

		$a/\sqrt{R_2 h}$						
	E_1/E_2	0.1	0.25	0.50	0.75	1.0	1.5	2.0
k_{mm}	0.725	1.022	1.122	1.412	1.795	2.233	3.210	4.294
	1.38	1.029	1.159	1.521	1.981	2.492	3.604	4.824
	1.0	1.025	1.141	1.471	1.897	2.378	3.434	4.590
k_{bm}	0.725	0.027	0.089	0.165	0.177	0.118	-0.233	-0.893
	1.38	0.032	0.102	0.178	0.175	0.085	-0.367	-1.177
	1.0	0.034	0.109	0.188	0.181	0.081	-0.392	-1.221
k_{bb}	0.725	0.632	0.613	0.566	0.514	0.464	0.381	0.319
	1.38	0.634	0.614	0.564	0.510	0.459	0.376	0.314
	1.0	0.643	0.621	0.566	0.508	0.455	0.368	0.305
k_{mb}	0.725	0.006	0.021	0.045	0.064	0.078	0.095	0.103
	1.38	0.007	0.024	0.050	0.071	0.085	0.102	0.109
	1.0	0.008	0.026	0.054	0.074	0.089	0.104	0.111

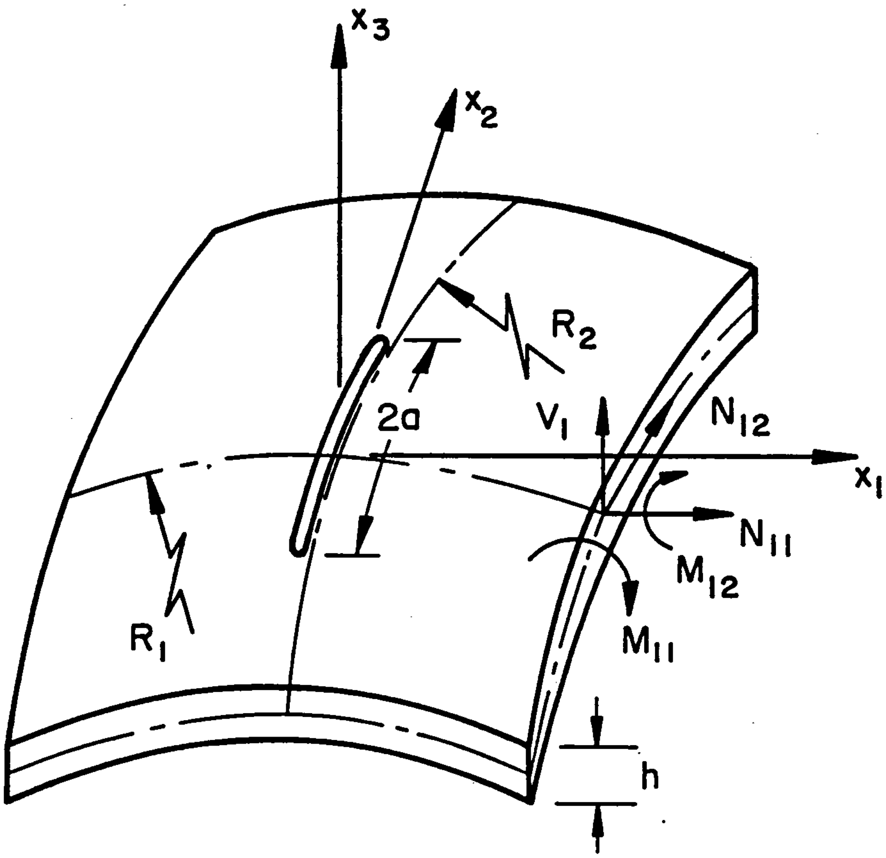

Fig. 1 The geometry and the notation for a cracked shell

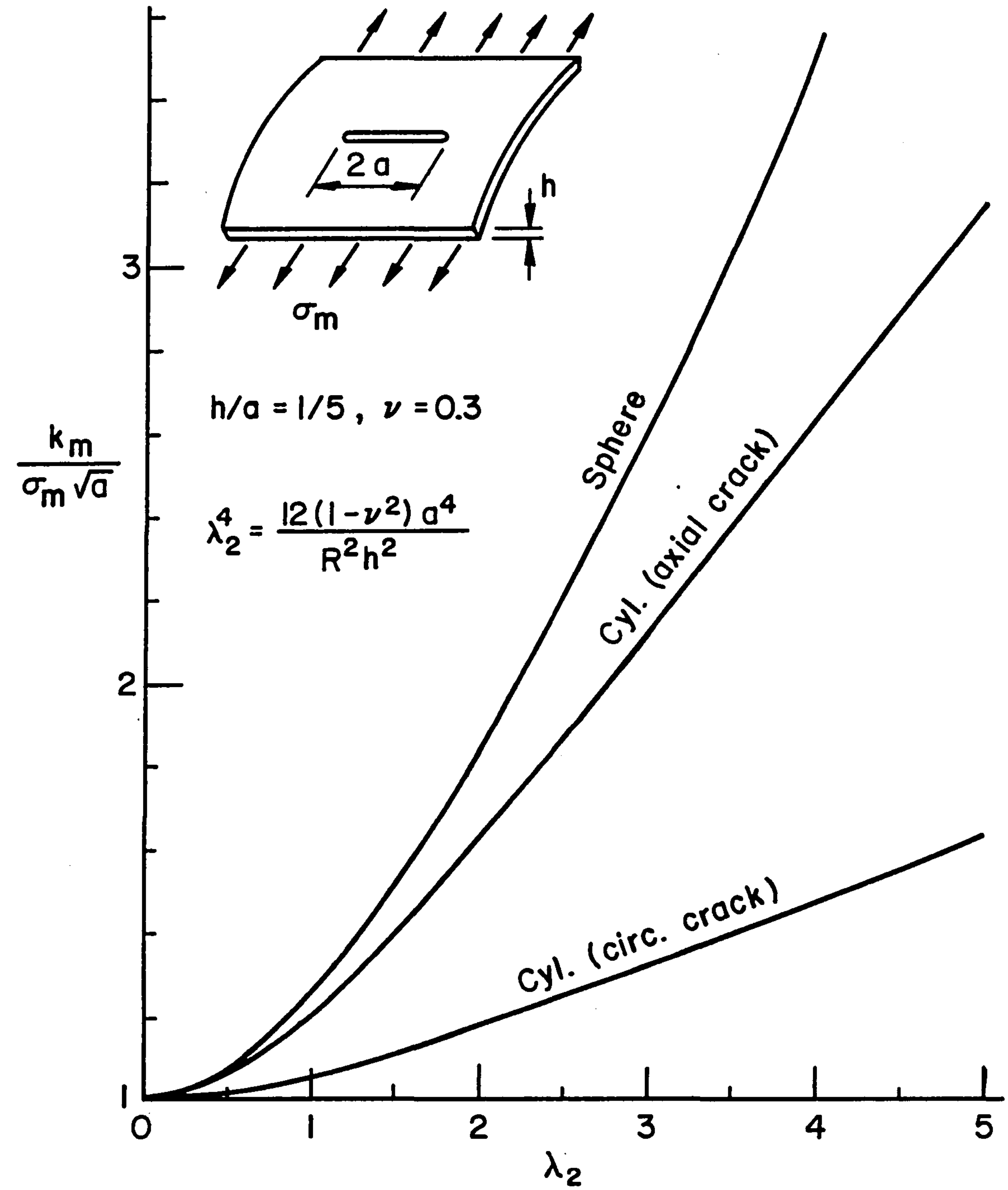

Fig. 2 The comparison of the membrane components of the stress intensity ratio $k_{mm} = k_m/\sigma_m\sqrt{a}$ for a cylindrical shell with a circumferential or an axial crack and a spherical shell with a meridional crack all under uniform membrane stress σ_m.

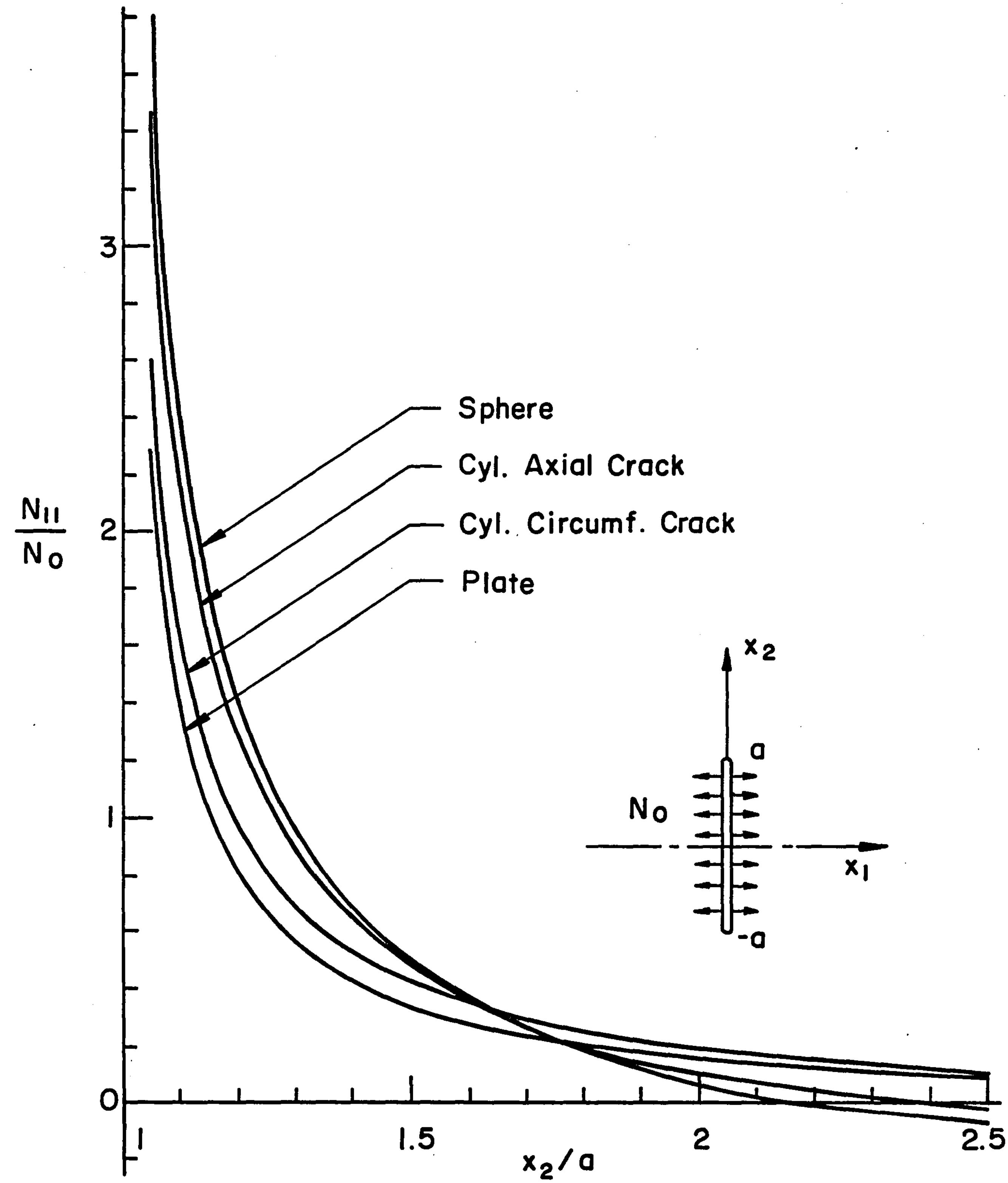

Fig. 3 Distribution of the membrane resultant N_{11} outside the crack in isotropic spherical and cylindrical shells and flat plates $\nu = 0.3$, $a/h = 5$, $\lambda = [12(1-\nu^2)]^{1/4}\, a/\sqrt{Rh} = 1.5$ in shells and $\lambda = 0$ in plates.

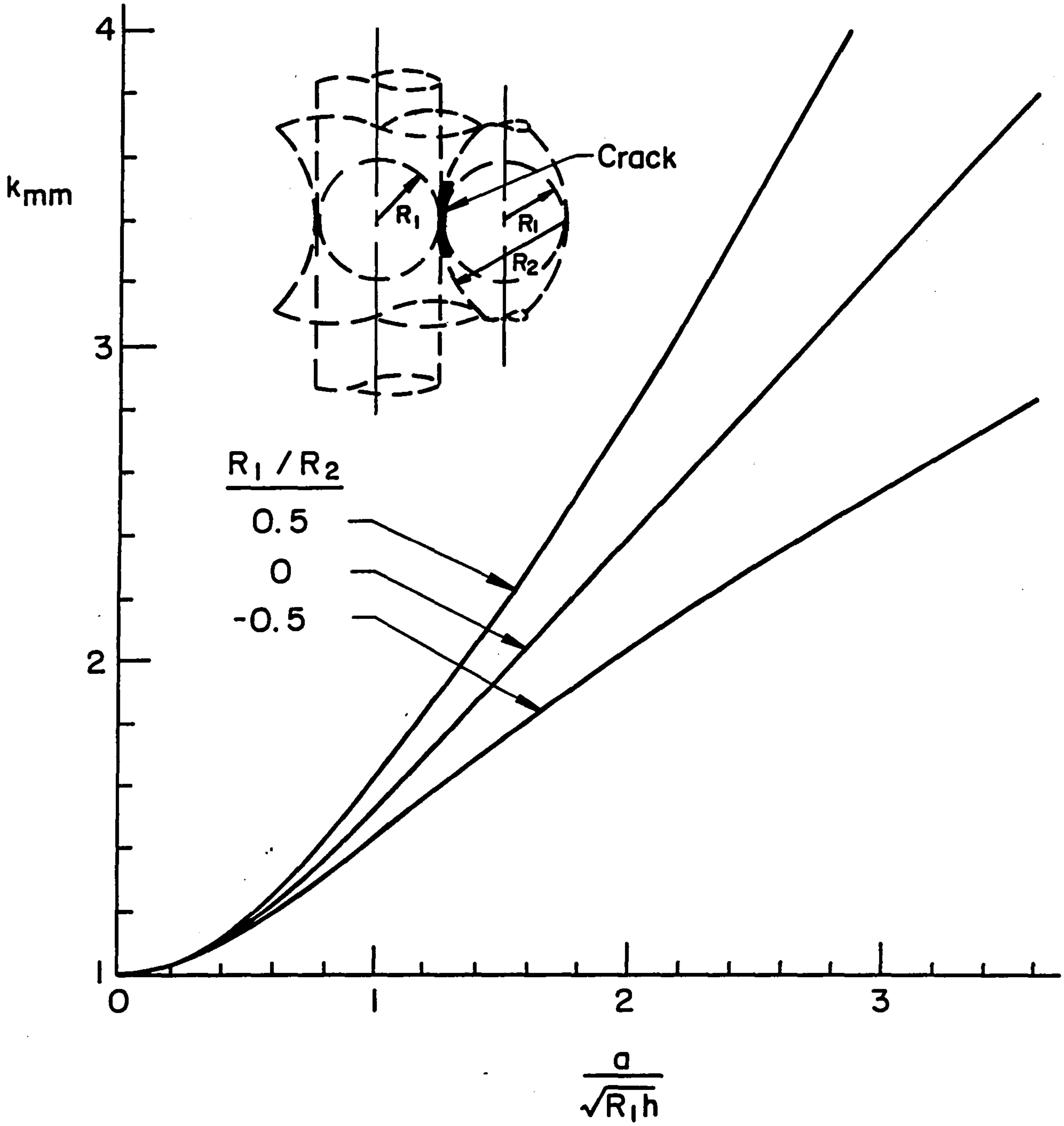

Fig. 4 Influence of the curvature ratio R_1/R_2 on the membrane component of the stress intensity factor in isotropic shells; (crack in R_2 plane, $a/h = 10$, $\nu = 0.3$)

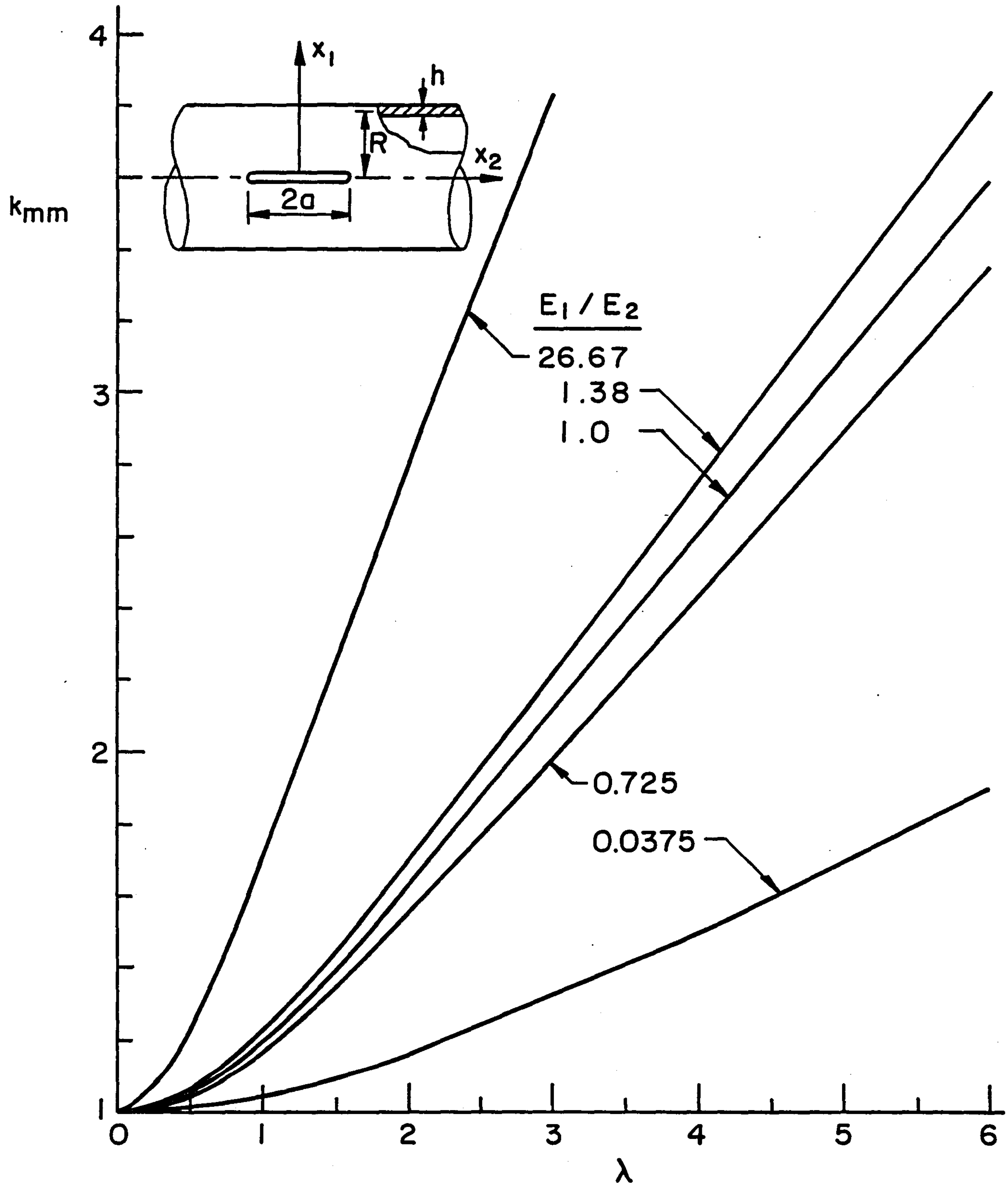

Fig. 5 The influence of material orthotropy on the membrane component of the stress intensity factor in a cylindrical shell containing an axial crack; ($a/h = 10$, $\nu = 0.3$ for the isotropic shell)

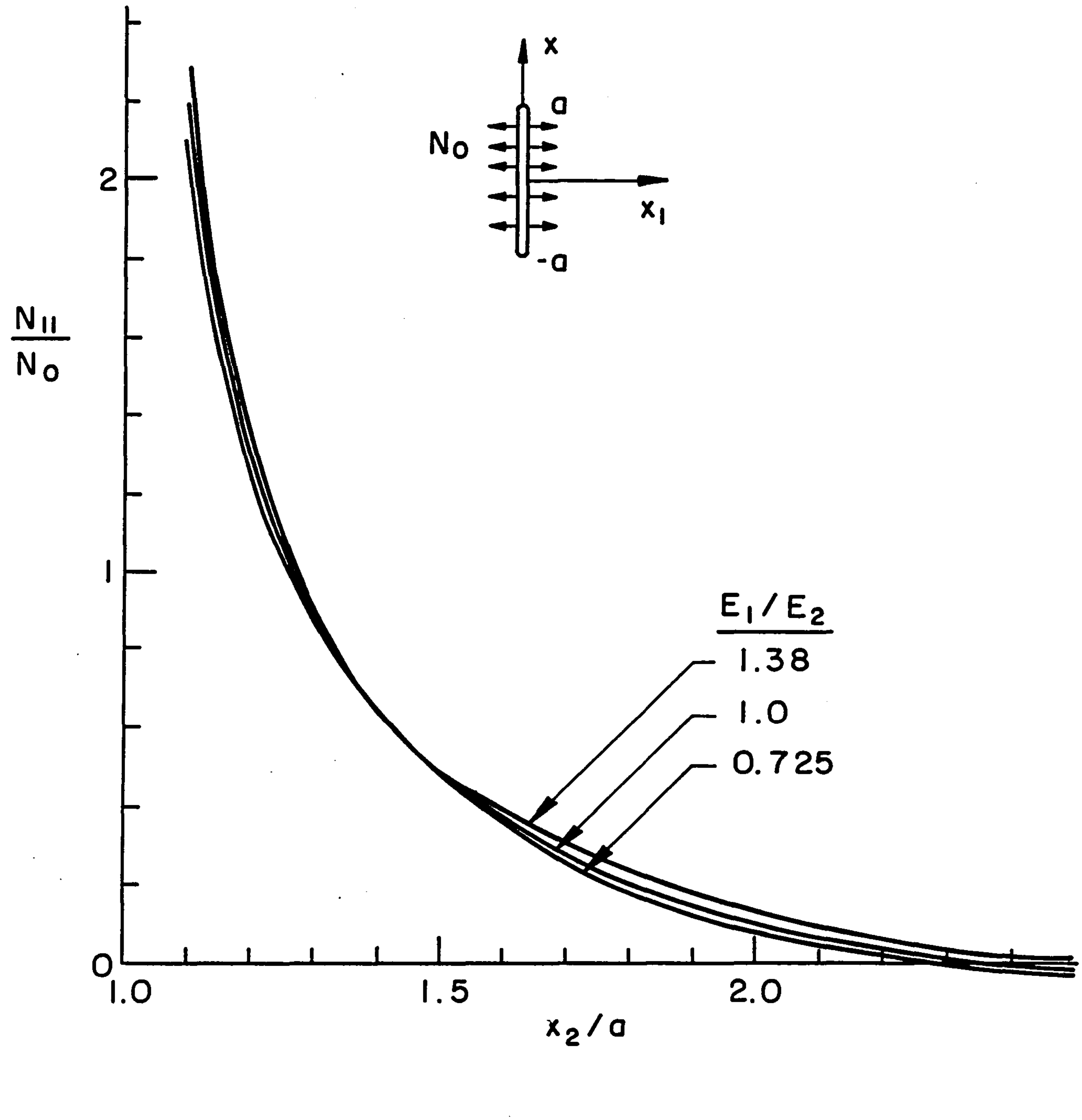

Fig. 6 The influence of the material orthotropy on the distribution of membrane resultant $N_{11}(0,x_2)$ outside the crack in a cylindrical shell containing an axial crack; ($a/h = 5$, $\lambda_1 = 1.5$, $\nu = 0.3$ for the isotropic shell)

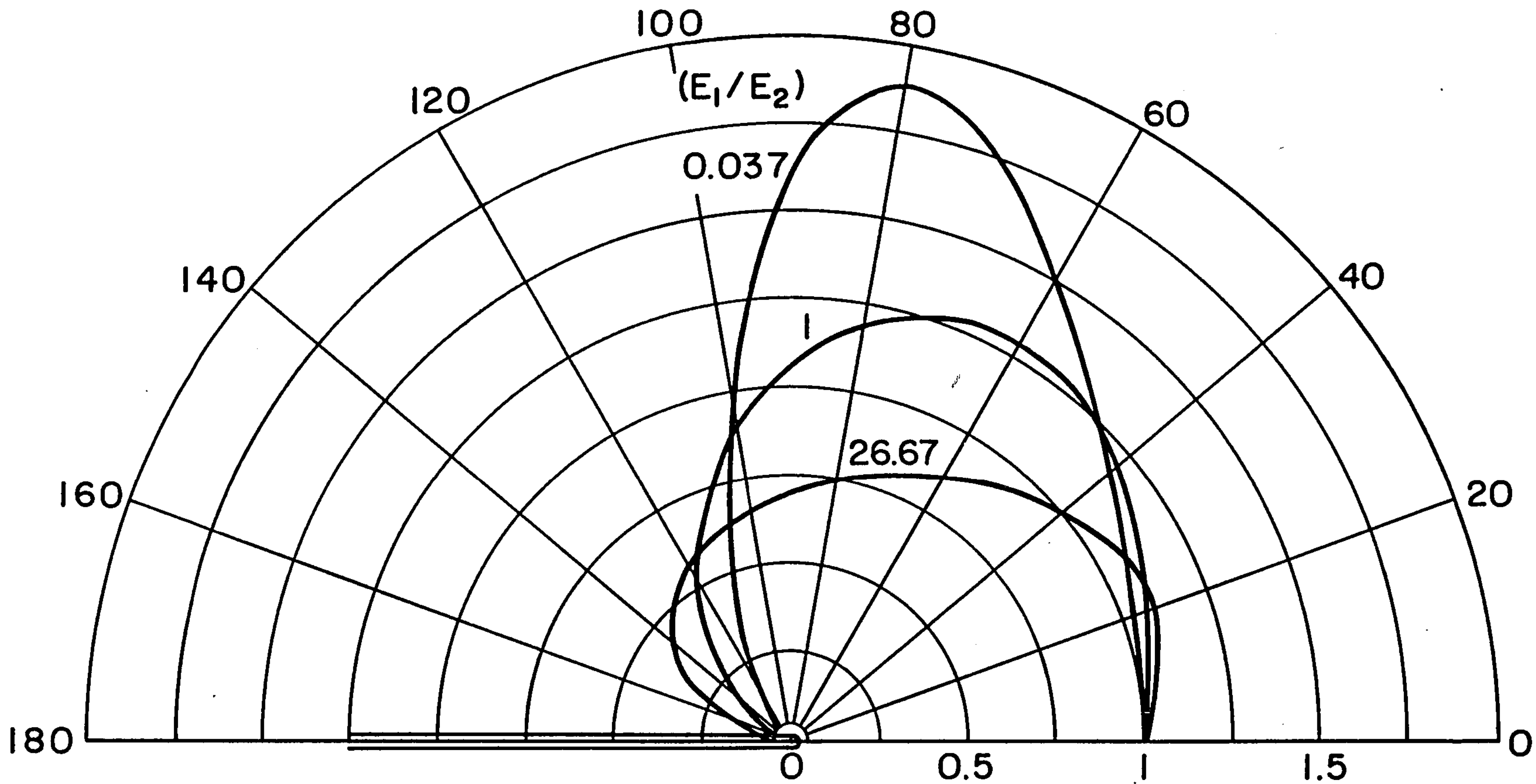

Fig. 7 The influence of the material orthotropy on the angular distribution $f_{11}(\alpha)$ of the stress component σ_{11} for Mode I loading condition (the crack lies along the x_2 axis)

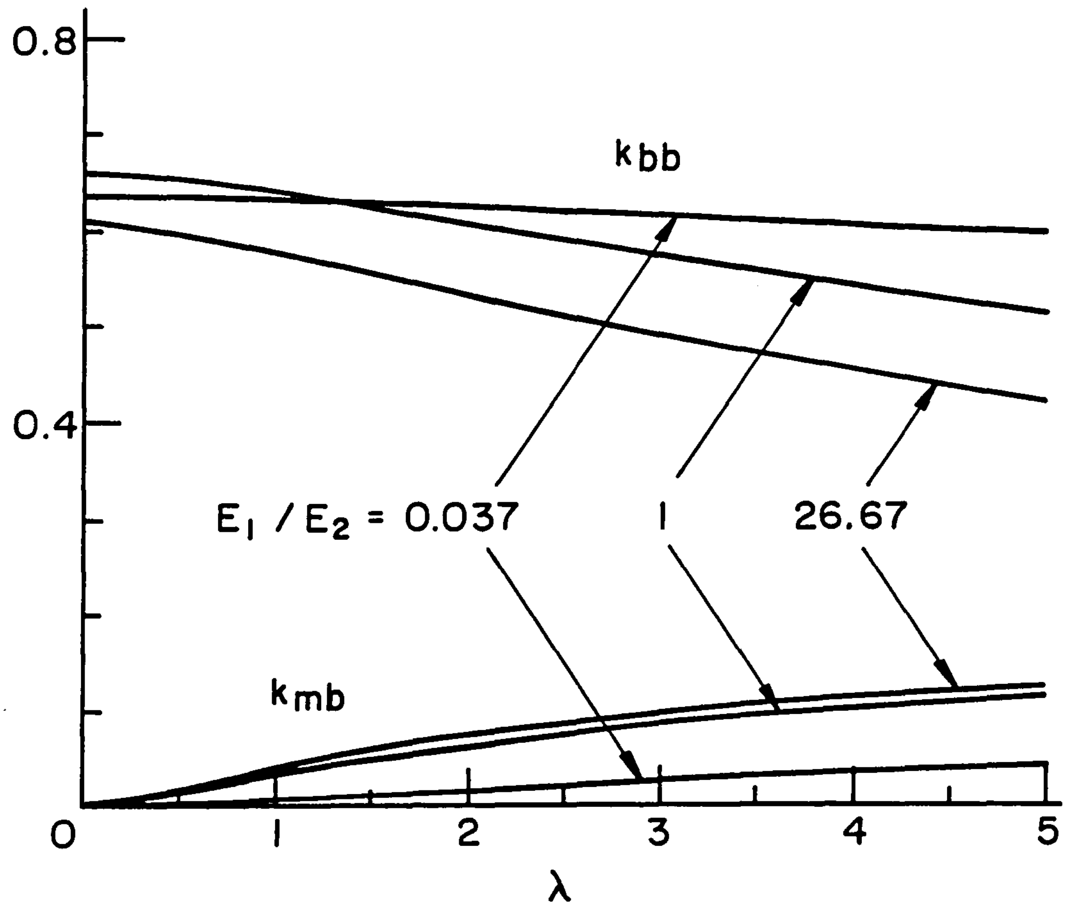

Fig. 8 The influence of the material orthotropy on the membrane and bending components of the stress intensity factor in a cylindrical shell containing an axial crack and subjected to uniform bending $M_{11}(0,x_2) = -M_0$ on the crack surfaces; ($a/h = 5$, $\nu = 0.3$ for the isotropic shell, $\lambda = [12(1-\nu^2)]^{1/4}\, a/\sqrt{Rh}$)

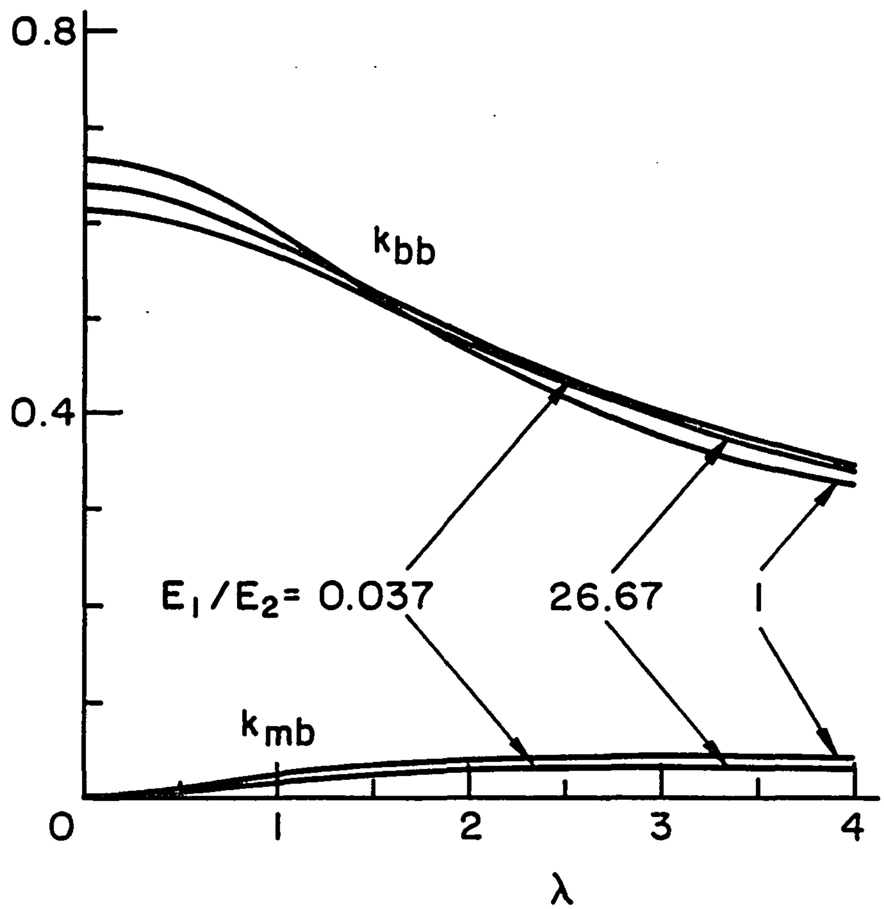

Fig. 9 Same as Fig. 8 for a cylindrical shell with a circumferential crack; ($a/h = 5$, $\nu = 1/3$ for the isotropic shell)

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16. Abstract

In this paper the crack problem of a shallow shell with two nonzero curvatures is considered. It is assumed that the crack lies in one of the principal planes of curvature and the shell is under Mode I loading condition. The material is assumed to be specially orthotropic. After giving the general formulation of the problem, the asymptotic behavior of the stress state around the crack tip is examined. The analysis is based on Reissner's transverse shear theory. Thus, as in the bending of cracked plates, the asymptotic results are shown to be consistent with that obtained from the plane elasticity solution of crack problems. Rather extensive numerical results are obtained which show the effect of material orthotropy on the stress intensity factors in cylindrical and spherical shells and in shells with double curvature. Other results include the stress intensity factors in isotropic toroidal shells with positive or negative curvature ratio, the distribution of the membrane stress resultant outside the crack, and the influence of the material orthotropy on the angular distribution of the stresses around the crack tip.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
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