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Scientific and Technical Facility
Baltimore-Washington International Airport, MD 21240

FROM: Andrew Hargrove, Principal Investigator
NASA Research Grant NG 1-172

SUBJECT: Semiannual Report

Enclosed herewith is the semiannual report of NASA Research Grant NG 1-172, "Solution of Nonlinear Multivariable Constrained Systems Using A Gradient Projection Digital Algorithm That Is Insensitive To The Initial State."

Enclosure

(NASA-CR-169752) SOLUTION OF NONLINEAR MULTIVARIABLE CONSTRAINED SYSTEMS USING A GRADIENT PROJECTION DIGITAL ALGORITHM THAT IS INSENSITIVE TO THE INITIAL STATE Semiannual Report (Tuskegee Inst.) 8 p
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SOLUTION OF NONLINEAR MULTIVARIABLE

CONSTRAINED SYSTEMS USING A GRADIENT PROJECTION

DIGITAL ALGORITHM THAT IS INSENSITIVE TO THE INITIAL STATE

Semiannual Status Report

Research Grant NG 1-172

Submitted to
National Aeronautics and Space Administration
Scientific and Technical Information Facility
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INTRODUCTION

Tuskegee Institute was awarded a research grant (NG1-172, starting date May 1, 1981) to make a study of possible improvements in optimal digital control of nonlinear multivariable constrained systems. The optimal controller in the form of an algorithm developed by the principal investigator in an earlier study was to be independent of the system's initial operating level and was to be improved and refined by reducing running time and storage requirements. A particularly difficult system of nine nonlinear state variable equations was chosen as a test problem for analyzing and improving the controller.

Lengthy analysis, modeling, computing and optimization were accomplished using the NASA-Langley digital computer. A remote interactive teletype terminal was installed at Tuskegee Institute and connected by telephone line to the NASA computing facility. Analysis requiring computer usage of short duration was accomplished using Tuskegee's VAX 11/750 system.

Specific objectives of the investigation were recorded as a "Statement of Work" in the original proposal.

STATEMENT OF WORK

Extensive reprogramming of the recursive scheme that computes new values of control and state variables must be accomplished.

A completely new algorithm that computes cost function and gradient must be written, tested, and coded. The Hamming Predictor-Corrector algorithm used previously to allow for longer values of time increment must be reexamined and improved.

Computing running time and storage requirements for the algorithm are excessive and must be reduced. The "TRENDX" algorithm that operates on the initial state is now a separate program and must be combined with the main program. A new algorithm that linearizes constant coefficient nonlinear systems must be written, coded, tested and incorporated into the overall program.

A user's guide must be written and published to assure that the algorithm can be used by the average scientist.

Additional tasks added at the request of the technical monitor were to reduce the complexity of the algorithm to a level that could be used by the average scientist.

THEORY OF THE METHOD

Definition of Symbols Used

x	operating level or "state" of the system at any time, t
x_{ss}	desired or "steady state" operating level of the system
t_f	time required to minimize the cost functional
u	level of input to the system
u_{max}	maximum allowable input to the system
u_{min}	minimum allowable input to the system
t	time
Δt	constant time interval

ORIGINAL PAGE IS OF POOR QUALITY

The controller optimizes the system's operation by minimizing critical errors in the digital model. To accomplish this optimization, the controller minimizes a cost function of the type:

$$J = \int_0^{t_f} Q\left[x(t) - x_{ss}\right]^2 dt + \int_0^{t_f} u^2 R\, dt + H\left[x\left(t_f\right) - x_{ss}\right]^2$$

where $\left[x(t) - x_{ss}\right]$ is the error to be minimized and Q, R and H are scaling constants. x, the state of the system and u the input to the system are related by the differential equation

$$\frac{dx}{dt} = Ax + Bu + g(x) + d$$

Here A, B and d are constants that describe the physical plant and $g(x)$ represents a combination of plant constants and nonlinear combinations of x.

For use in digital computing the differential equation is converted first to

$$dx = (Ax + Bu + g(x) + d)\, dt,$$

then to the discrete equation

$$x(k + 1) - x(k) = [Ax(k) + Bu(k) + g(x(k)) + d]\ \Delta t$$

where $k = t_f/\Delta t$ and finally to the difference equation

$$x(k + 1) = x(k) + [A(x(k)) + Bu(k) + G(x(k)) + d] \Delta t$$

Using a known initial state $x(k)$, and estimated initial input $u(k)$, and a chosen constant time interval Δt, the state $x(k + 1)$ of the system at any future time $(k + 1)$ is computed. The cost function $J = \int_0^{t_f} (x(t) - x_{ss})^2 \, dt$ is observed after each trial minimization time interval t_f and the value of u is changed in a manner that will cause J to decrease. u must at all times have a value between the maximum (u_{max}) and minimum (u_{min}) allowable inputs. The iteration method that accomplishes this minimization is based on the gradient projection method first developed by Rosen in 1960 and described by Hargrove. (See reference.) The value of input u that causes the minimization is called the "optimal control" and when physically applied to the system will cause the system to perform at its optimal level.

The actual convergence criteria for minimization were twofold:

1. Successive computed values of the cost functional J were less than 0.000005 apart, and
2. the normalized value of the "gradient projection" $dJ/du + G$ was less than 0.000005. Here dJ/du is a very small change in the cost function J as compared to the input u, and G is a very small fraction of the normalized maximum (u_{max}) or minimum (u_{min}) allowable value of the input.

A unique feature of the gradient projection algorithm is that the cost function, J, must be converted to a function of the input, u, alone. This conversion is done for each minimization trial. That input, u, computed during the final trial, at which time the cost function is found to be minimum, is the "optimal control" and is to be used on a microprocessor to actual direct and control the process.

The digital algorithm, herein called a "digital optimal controller" will be a valuable tool in the analysis, design and testing of a wide variety of nonlinear constrained multivariable aeronautical systems.

The basic gradient projection algorithm has been recoded, tested and debugged. The program is functioning well. Many improvements have been accomplished as noted in the last report.

SENSITIVITY TO INITIAL INPUT ESTIMATE

An important remaining problem in the algorithm is its sensitivity to the inital control (input) guess. The input, computed on the final minimization trial, is the "optimal control" and is to be used to direct the optimization process in an actual system. This optimal control was found to be completely dependent on the initial control guess. Much analysis and comparison of the model used in this research with the original digital model by Kunster and Mize[1] and with the digital solution of the Kirk[2] problem was performed. Selected portions of the three models were compared using the Tuskegee Institute VAX 11/750 computer. The results thus far are inconclusive.

RECOMMENDATIONS FOR FUTURE STUDY

An original algorithm TRENDX was developed earlier by the principal investigator that would accept any initial system operating condition and convert it to a condition acceptable to the minimization algorithm. Much progress was made toward interposing TRENDX between the system input and the rest of the system. Additional research should be done to make the two programs compatible and to complete the interface. This accomplishment will allow the minimization algorithm to accept any initial system operating condition. The universal current shortcoming of all minimization algorithms is that they are extremely selective as to which initial states that they will accept.

Extensive recoding of subroutines SYSTEM-2 and POINT must be accomplished in order to cause the control variables to be placed on the boundary of the admissible region if constraints are violated. This is a crucial task that will probably require more time than is left in the research grant period.

The "optimal control" should be independent of the initial control estimate. Further study is needed to assure this independence.

An extension of the research grant is needed.

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