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CONTROL POLE PLACEMENT RELATIONSHIPS

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ABSTRACT

A Closed Form Pole Placement Scheme (CFPPS) is currently being used to synthesize control systems for the Large Space Structure (LSS) ground test, which is located at MSFC, and the Solar Array Flight Experiment II (SAFE II), which will be flown in 1987. Since most LSS possess a very low frequency fundamental and a dense modal pattern, designing a LSS control system without modal interaction is extremely difficult. The CFPPS is a very efficient control design tool relative to LSS modal constraints, but the migration of the unconstrained poles is an enigma.

Using a simplified LSS model, a technique has been developed which gives algebraic relationships for the unconstrained poles. The relationships, which were obtained by this technique, are functions of the structural characteristics and the control gains. Extremely interesting relationships evolve for the case when the structural damping is zero. If the damping is zero, the constrained poles are uncoupled from the structural mode shapes. These relationships, which are derived for structural damping and without structural damping, provide new insight into the migration of the unconstrained poles for the CFPPS.

INTRODUCTION

The CFPPS uses techniques from both the frequency domain and the state space analysis to synthesize control systems. An experienced control engineer can take advantage of the frequency design methods and place the appropriate poles of an open loop system such that the closed loop system has the desired performance goals. In using the state space techniques, the CFPPS masks off the migration of the system poles which are not placed. Herein lies the research objectives which are to determine the migration characteristics of these free poles by algebraic relationships which involve the system structural model data.

THE CLOSED FORM POLE PLACEMENT SCHEME (CFPPS)

The system description used to derive the pole placement scheme is

$$\dot{x} = Ax + Bu, \tag{1}$$

$$y = Cx, \tag{2}$$

$$\dot{z} = Dz + Ey, \text{ and} \tag{3}$$

$$u = Fz \tag{4}$$

where

x : n x 1 plant state vector,

u : r x 1 control state vector,

z : p x 1 filter states,

y : m x 1 measurement vector,

D : p x p specified filter matrix, and

E : p x m specified matrix.

After some algebriac matrix manipulations[1], the pole placement constraint relationships are

$$Q = |\ I_p - P(\lambda_i)F\ |_{j^*} = 0 \tag{5}$$

where

F : r x p unknown control gain matrix,

$$P(\lambda) = (\lambda I_p - D)^{-1} EC(\lambda I_n - A)^{-1} B,$$

λ_i : placed characteristic root, and

j* : the jth row of (5).

This procedure yields at least p independent relationships [2], so p poles of the system described by (1) - (4) can be arbitrarily placed.

LARGE SPACE STRUCTURE MODAL DESCRIPTION

Almost all vibrational data for a Large Space Structure is given in modal coordinate form. For a scalar input, a proportional-derivative (PD) controller and no filter, the modal coordinate representation for (1) - (4) is

$$A = \begin{bmatrix} A_1 & 0_1 & \cdots & 0_1 \\ 0_1 & A_2 & \cdots & 0_1 \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ 0_1 & 0_1 & \cdots & A_n \end{bmatrix}$$

$$B^t = \begin{bmatrix} 0 & s_1 & 0 & s_2 & \cdots & 0 & s_n \end{bmatrix}$$

$$C = \begin{bmatrix} t_1 & 0 & \cdots & t_n & 0 \\ 0 & v_1 & \cdots & 0 & v_n \end{bmatrix}$$

where $\quad 0_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$,

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},$$

$$A_i = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -\lambda \zeta_i \omega_i \end{bmatrix},$$

ω_i is the frequency of ith mode in rad/sec,

ζ_i is the damping of ith mode,

s_i is the ith modal slope at the control effector,

t_i is the ith modal slope at the attitude sensor,

and

v_i is the ith modal slope at the attitude rate sensor.

The system model used for this study is a two mode case and the A, B, and C system matricies are shown in table 1. The simplification to the two mode case allows insight to be gained into the pole migration without the burden of undue complications that a higher order system possesses. The pole migrations for the two mode case are shown in tables 1-3. As the rigid body poles are placed on the 45° axis lines in the complex plane, the bending mode poles migrate in a circular arc until they reach the real axis, then they move along the real axis. How and why these poles migrate as they do is a function of the modal data and this functional relationship is developed in the next section.

ALGEBRAIC CONSTRAINT RELATIONSHIPS

Given the system

$$\dot{x} = Ax + Bu \tag{6}$$

$$y = Cx \tag{7}$$

$$u = Fy \tag{8}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -.394 & -.00628 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 0 & a & 0 & b \end{bmatrix},$$

$$C = \begin{bmatrix} a & 0 & b & 0 \\ o & a & 0 & b \end{bmatrix} \text{, and}$$

$$F = \begin{bmatrix} p & q \end{bmatrix} \text{,}$$

the open loop characteristic polynominal is

$$| \lambda I_4 - A | = \lambda^2 \Delta$$

where

$$\Delta = (\lambda^2 + .00628\lambda + .394).$$

The control equation constraint (see equation 5) for this system is

$$Q_1 = \lambda^4 + A_1\lambda^3 + B_1\lambda^2 + C_1\lambda + D_1 \tag{9}$$

where

$$A_1 = .00628 + b^2q + a^2p, \tag{10}$$

$$B_1 = a^2p + a^2(.00628)q - b^2p + .394, \tag{11}$$

$$C_1 = (.394)a^2q + (.00628)pa^2, \text{ and} \tag{12}$$

$$D_1 = (.394)a^2p. \tag{13}$$

Now if the roots of (9) are to be

$$r_{1,2} = \alpha \pm i\alpha \text{ and} \tag{14}$$

$$r_{3,4} = \beta \pm i\gamma \tag{15}$$

where (α, $\beta < 0$),

then the characteristic polynominal is

$$Q_z = \lambda^4 + \lambda^3(-2\alpha - 2\beta) + \lambda^2(2\alpha^2 + \gamma^2 + \beta^2 + 4\alpha\beta)$$

$$+ \lambda(-2\alpha\gamma^2 - 2\alpha\beta^2 - 4\beta\alpha^2) + 2(\alpha\gamma)^2 + 2(\alpha\beta)^2 = 0 \tag{16}$$

Equating the coefficients of Q_1 and Q_2 leads to the equations

$$-2(\alpha + \beta) = A_1,\tag{17}$$

$$\beta^2 + 2\alpha^2 + \gamma^2 + 4\alpha\beta = B_1,\tag{18}$$

$$-2\alpha(\beta^2 + \gamma^2 + 2\alpha\beta) = C_1, \text{ and}\tag{19}$$

$$2\alpha^2(\gamma^2 + \beta^2) = D_1.\tag{20}$$

Equations (17-20) yield 3 unknowns (α, β, γ) on the left hand side and 4 on the right hand side (A_1, B_1, C_1, D_1), so that placing a pair of roots on the 45° line gives a relationship between A_1, B_1, C_1, and D_1 - but it will be fourth order. This is not very tractable, so we impose a constraint that we want simple relationships. This can be done by fixing α and asking what a, b, p, q can produce this α, and, if so, are they a feasible set.

So let

$$C_1 = -2\alpha\, C_2,\tag{21}$$

$$D_1 = 2\alpha\, D_2,\tag{22}$$

$$A_1 = -2A_2, \text{ and}\tag{23}$$

$$B_1 = B_2,\tag{24}$$

and we arrive at the relationships

$$\alpha + \beta = A_2,\tag{25}$$

$$\beta^2 + \gamma^2 + 2\alpha^2 + 4\alpha\beta = B_2,\tag{26}$$

$$\beta^2 + \gamma^2 + 2\alpha\beta = C_2, \text{ and}\tag{27}$$

$$\gamma^2 + \beta^2 = D_2.\tag{28}$$

which eliminate the third and fourth degree equations. So for a pair of roots to lie on the 45° axis in the left half side of the complex plane and for the absolete value of that root to equal $|\alpha|$, the set (A_2, B_2, C_2, D_2) must satisfy

$$2\alpha^2 = D_2 - 2C_2 + B_2 \text{ and} \tag{29}$$

$$B_2 - D_2 = 2a^2. \tag{30}$$

If (a, b, p, q) exists so that (29) and (30) are satisfied and if they also belong to an attainable set, then we have conditions for placing the root at a desired location. We can treat α as a parameter and get a set of solutions in terms of α, and observe what effect increasing or decreasing α has upon the set (a, b, p, q).

Substituting for (a, b, p, q) in (29) and (30) when the damping is zero yields generic equations of the form

$$-b^2 p = f_1(\alpha, a, p, q) \text{ and} \tag{31}$$

$$2a^2 = f_2(\alpha, a, p, q) - b^2 p. \tag{32}$$

Equations (31) and (32) show that

$$2a^2 = f_3(\alpha, a, p, q) \tag{33}$$

which decouples α from the flexible mode's shape.

CONCLUSIONS AND RECOMMENDATIONS

Algebraic constraint relationships are developed which can be used to predict the unplaced pole migrations of a fourth order system in terms of the structural characteristics. These constraint relationships add insight in the control synthesis technique used by the CFPPS. These relationships will be implemented on a computer and the results of the study will be published.

It is recommended that the system be generalized to n modes to see if tractable results can be obtained. If so, the results would be of great value in the synthesis of control systems for Large Space Structures.

REFERENCES

1. Waites, H. B., "Control Pole Placement Methods," NASA Memorandum ED12-81-1, dated March 2, 1981.

2. Davison, E. J. and Wang, S. H., "On Pole Assignment in Linear Multivariable Systems Using Output Feedback," IEEE Transactions and Automatic Control, Vol. AC-20, No. 4, August 1975, pp 516-518.

LEGEND:
NFDRIVER a b c
a implies multiplicity of complex
root b±jc.
Outputs after NFDRIVER are the
closed loop characteristic roots.

TABLE 1. MODEL DESCRIPTION AND POLE PLACEMENT

TABLE 2.   POLE PLACEMENT FOR TWO MODE MODEL

```
NFDRIVER 1   100 100
-3.11E-5  -6.25E-2
-3.11E-5  -6.25E-2
-1.00E2   -1.00E2
-1.00E2   -1.00E2

NFDRIVER 1   200 200
-3.11E-5  -6.25E-2
-3.11E-5  -6.25E-2
-2.00E2   -2.00E2
-2.00E2   -2.00E2

NFDRIVER 1   .4  .4
0.023     0
-0.0349   0
0        -.4
0        -.4

NFDRIVER 1   .42 .42
0.0146    0
0.025     0
0         .42
-0.42    -0.42

NFDRIVER 1   .44 .44
0.000727  0
-0.00816  0
0         .44
0.44     -0.44

NFDRIVER 1   .45 .45
0.00415  -0.0126
-0.00415 -0.0126
-0.45    -0.45
-0.45    -0.45

NFDRIVER 1   .5  .5
-0.00303  0.0295
-0.00303  0.0295
0.5      -0.5
0.5      -0.5
```

TABLE 3.   POLE PLACEMENT FOR TWO MODE MODEL