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SHAPE CONTROL OF LARGE SPACE STRUCTURES

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ABSTRACT

The development of the Space Transportation System now makes feasible the erection of large structures in space. Some of these structures will combine large size with very rigorous surface figure error requirements. The most stringent requirements will be made by large optical systems and antennas operating at very high frequencies.

The shape control of these large structures is made difficult because of their flexibility and their distributed nature. Their vibrational modes are numerous, densely packed, and low frequency. In addition, the characteristics of these systems cannot be accurately predicted before flight. The control problem is further complicated by the need to design a controller which has low enough order so that it can be implemented on the onboard computer and yet of high enough order to accurately control the structure.

A survey has been conducted to determine the types of control strategies which have been proposed for controlling the vibrations in large space structures. From this survey several representative control strategies were singled out for detailed analyses. The application of these strategies to a simplified model of a large space structure has been simulated. These simulations demonstrate the implementation of the control algorithms and provide a basis for a preliminary comparison of their suitability for large space structure control.

SECTION 1

INTRODUCTION

The purpose of this project has been to examine procedures for controlling vibrations in large space structures (LSS). This effort has consisted of three major parts:

- A survey of the literature related to LSS control, and control theory in general, to identify candidate techniques for LSS control.
- An analysis of the candidate techniques in order to categorize them and to determine those which have the most promise for successful LSS control.
- Simulation studies of representative techniques from several relevant categories.

This report will present the results of this project. Section 2 outlines the LSS control problem and describes some relevant LSS models. Section 3 discusses the various categories of solution which have been proposed for LSS control. Sections 4 through 8 describe in detail five representative control techniques which have been suggested for use in LSS control and illustrate their application to a common test problem. Section 9 summarizes the results and suggests areas for future investigation.

The scope of this project and of this report has been purposely limited because of time considerations. The control of LSS is a large field, and this report does not attempt to cover it completely. In particular, this report is limited to vibration control and will not consider pointing problems. In addition, the report discusses only continuous time control algorithms and does not treat problems of digital control, sampling rates, etc. Finally, there is no discussion of the hardware implementation of any of the control algorithms in terms of specific actuators, sensors, etc.

SECTION 2

THE MODELS

2.1 The Distributed Parameter Model

The most complete model of a LSS would be a system of partial differential equations of the form

$$m(x)u_{tt}(x,t) + D_0 u_t(x,t) + A_0 u(x,t) = F(x,t) \quad (2.1)$$

where $u(x,t)$ represents the displacement of the structure from its equilibrium position, $F(x,t)$ is the force distribution, $m(x)$ is the mass distribution, D_0 is a differential operator representing the damping of the LSS, and A_0 is a differential operator representing the stiffness of the LSS.

2.2 The Finite Dimensional Model

There are some control procedures which attempt to deal directly with the infinite dimensional distributed parameter model (eg. [1],[2]). However most procedures, including all those discussed in this report, use a finite dimensional approximation of the form

$$M\ddot{q} + D\dot{q} + Kq = f \quad (2.2)$$

where q is an n -dimensional vector representing the displacements of the structure in some generalized coordinate system, f is a vector of generalized forces, M is a real symmetric positive definite mass matrix, K is a real symmetric positive semi-definite stiffness matrix, and D is the damping matrix. Because the damping of the LSS is expected to be very low (~ 0.005), the D matrix will be set to zero for the purposes of the following discussion. The finite dimensional model of (2.2) is normally developed by the finite element method using a computer program such as NASTRAN.

The forces are applied through m actuators in a manner described by

$$f = B_a u \quad (2.3)$$

where u is an m -dimensional vector of inputs to the actuators, and B_a is a matrix which specifies the effects of the actuators on the generalized displacements. There are also k sensors which measure displacements and velocities in a manner described by

$$y = C_d q + C_v \dot{q} \quad (2.4)$$

where y is a k -dimensional vector of measurements, C_d is a

matrix which specifies the position measurements, C_v is a matrix which specifies the velocity measurements.

2.3 The Modal Form

Because of the form of the M and K matrices there exists a unitary matrix U such that

$$U^T M U = I \quad \text{and} \quad U^T K U = W \quad (2.5)$$

where

$$W = \begin{bmatrix} w_1^2 & 0 & \cdots & 0 \\ 0 & w_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & w_n^2 \end{bmatrix}$$

The values w_1, \dots, w_n are the natural frequencies of vibration of the structure and the columns of U are the corresponding mode shapes. It is now possible to represent the displacements of the structure in terms of the modal coordinates. The relationship between the generalized coordinates and the modal coordinates is given by

$$q = Uv \quad (2.6)$$

where v is an n-dimensional vector of modal coordinates. The model of the LSS can now be written in terms of the modal coordinates

$$\ddot{v} + Wv = U^T B_a u \quad (2.7)$$

$$y = C_d Uv + C_v U\dot{v} \quad (2.8)$$

2.4 The State Space Model

It will be convenient for later discussions to have the finite dimensional model of the LSS in state space form. One possible state space representation would be

$$\dot{x} = Ax + Bu \quad (2.9)$$

$$y = Cx \quad (2.10)$$

where

$$x = \begin{bmatrix} v \\ \dot{v} \end{bmatrix} \quad A = \begin{bmatrix} 0 & I \\ -W & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ U^T B_a \end{bmatrix} \quad C = \begin{bmatrix} C_d U & C_v U \end{bmatrix} \quad (2.11)$$

2.5 The Reduced Order Model

The LSS model of (2.9) and (2.10) is normally of very high order and therefore it must be simplified in some way before applying any control procedures. The standard method of simplifying the LSS system is to subdivide x into three parts:

- x_p will designate the primary states. These consist of those modes of the system which are most critical to the performance of the LSS (eg. line of sight errors).
- x_s will designate the secondary states. These states consist of those modes which can be accurately modeled but which are not as critical to system performance as x_p . These states can be used in evaluating the control system design.
- x_r will designate the residual states. These states consist of those modes which cannot be accurately modeled.

$$x_p = \begin{bmatrix} v_p \\ \dot{v}_p \end{bmatrix} \quad x_s = \begin{bmatrix} v_s \\ \dot{v}_s \end{bmatrix} \quad x_r = \begin{bmatrix} v_r \\ \dot{v}_r \end{bmatrix}$$

The state space representation of (2.9) and (2.10) can now be written

$$\begin{bmatrix} \dot{x}_p \\ \dot{x}_s \\ \dot{x}_r \end{bmatrix} = \begin{bmatrix} A_p & 0 & 0 \\ 0 & A_s & 0 \\ 0 & 0 & A_r \end{bmatrix} \begin{bmatrix} x_p \\ x_s \\ x_r \end{bmatrix} + \begin{bmatrix} B_p \\ B_s \\ B_r \end{bmatrix} u \quad (2.12)$$

$$y = \begin{bmatrix} C_p & C_s & C_r \end{bmatrix} \begin{bmatrix} x_p \\ x_s \\ x_r \end{bmatrix} \quad (2.13)$$

where A_p , A_s and A_r are defined in a manner analogous to A in (2.11).^p (Appendix A gives an example of such a model for a simplified representation of a LSS.)

Once the model has been partitioned as in (2.12) and (2.13) the problem is simplified by ignoring all secondary and residual modes to obtain the reduced order equations

$$\dot{x}_p = A_p x_p + B_p u \quad (2.14)$$

$$y = C_p x_p \quad (2.15)$$

It is clear from (2.12) and (2.13) that this reduced order model has two potential sources of error. First, states other than x_p influence the output y , and yet the terms $C_s x_s$ and $C_r x_r$ have not been included in the observation equation (2.15). These terms have been called "observation spillover" [3]. Secondly, the input u affects not only the primary states x_p but also x_s and x_r . The terms $B_s u$ and $B_r u$ are called "control spillover". If control and observation spillover are ignored it is possible that a controller could be designed which is stable for the reduced order model of (2.14) and (2.15) but unstable for the model described in (2.12) and (2.13).

To summarize this section, we have divided the modes of the LSS into the following categories

- Primary modes - These are the modes which have the largest effect on system performance.
- Secondary modes - These modes are not specifically controlled but may be used in evaluating the control design.
- Residual modes - These modes are included in the finite element model, but their accuracy is suspect and they would not be included in the design process.
- Unmodeled modes - The distributed parameter system (2.1) is truly infinite dimensional. These are the modes which are not included in the finite element model.

SECTION 3

METHODS OF CONTROL

Now that the LSS control problem has been outlined and the models have been described, this section will provide a brief overview of the categories of control methodology which have been proposed for active LSS vibration control. Later sections of this report will examine several specific control procedures in some detail.

3.1 Direct Output Feedback

The simplest feedback control is direct output feedback (DOFB) which is characterized by the following control law:

$$u = Gy \tag{3.1}$$

where G is a constant matrix of feedback gains. With this control law in force the closed loop equations for the controlled states - ignoring spillover - would be

$$\dot{x}_p = \begin{bmatrix} A_p & - B_p G C_p \end{bmatrix} x_p \tag{3.2}$$

The various methods which use DOFB have the same on-board computational requirements since they use the same control law (3.1). They are distinguished by the criteria by which they choose G . Some methods ([4],[5]) choose G in order to obtain a desired level of damping in the controlled modes. These methods are by far the easiest methods to implement and are the most robust methods in terms of closed loop stability. Their main problem stems from the fact that they were designed to obtain relatively small amounts of damping (up to approximately .2). This may not be a practical limitation, however, because of the large power requirements which would be needed to obtain large amounts of damping with any method.

Other types of DOFB choose G in such a way as to place the poles of the closed loop system (3.2) in some desired locations. There are a number of procedures for pole placement ([6]-[8]). Their advantage over the damping augmentation methods would appear to be the increased flexibility in placing the closed loop poles. However, the number of poles which can be positioned is limited by the number of sensors and actuators which are used, and there is no guarantee that those poles which are not placed will remain stable. Furthermore, even if the reduced order model of (2.14) and (2.15) is stable there is no guarantee that the total closed loop system will be stable.

A third type of DOFB chooses G so as to minimize a performance index of the form

$$J = \int_0^{\infty} [x_p^T R_1 x_p + u^T R_2 u] dt \quad (3.3)$$

where R_1 is a symmetric positive semi-definite matrix, and R_2 is a symmetric positive definite matrix. Procedures of this type are described in [9] and [10], and a stochastic version is given in [11]. The main advantage of these optimal output feedback methods lies in the performance index (3.3). The weighting matrices R_1 and R_2 can be chosen by the designer, and this allows great flexibility in choosing the design criteria. Each mode can be weighted according to its relative effect on the performance of the LSS. The main disadvantage of optimal output feedback is the computational burden. Although, as with all DOFB methods, the on-board computation is minimal, the computational requirements for determining the optimal gains can be prohibitive for problems of the magnitude of most LSS systems.

3.2 Modern Modal Control

The methods of the previous subsection were characterized by the minimal on-board computational requirements of (3.1). This subsection will discuss methods which require more on-board computation but which hold the potential for increased flexibility in affecting LSS performance. These methods, instead of using direct output feedback, estimate the state of the system and use these estimates for feedback. For the reduced order model of (2.14) and (2.15) the estimator (filter/observer) would take the form

$$\dot{\hat{x}}_p = A_p \hat{x}_p + B_p u + K_f [y - C_p \hat{x}_p] \quad (3.4)$$

The control law would take the form

$$u = K_c \hat{x}_p \quad (3.5)$$

As with DOFB, the various modern modal control (MMC) design methods [3] are distinguished by the manner in which they determine the gains K_f and K_c . There are two basic methods. The first method calculates the gains so as to place closed loop system poles in some desired locations. In contrast with DOFB, in which there are constraints on the number of poles which can be placed, there is no theoretical limit on the number of poles of the reduced order system which can be placed using MMC (assuming that the system is controllable). In addition, the procedures for pole positioning using MMC are more straightforward and present a smaller computational burden than those for DOFB. Of course,

just because the poles of the reduced order model are stable this does not guarantee that the total distributed parameter system will be stable.

The second approach to MMC is to choose the gains in order to minimize a performance index in the form of (3.3). This optimal MMC is different from optimal DOFB in that the computational burden in calculating the gains is decreased considerably, although the on-board computation is increased. Both methods have the advantage of choosing the gains based on a performance index which is very flexible and can be used, for example, to mitigate the effects of modeling error. Optimal MMC will be discussed in detail in a later section.

3.3 Suppression of Spillover

In all of the methods discussed so far the controller has been designed based on the reduced order model, ignoring all secondary and residual modes. As was mentioned earlier the effect of spillover from these modes can degrade the performance of the system and in some cases may cause instability. This subsection will provide an overview of some of the methods which have been designed to reduce the effects of spillover.

The first method is a variation of optimal MMC. It is called Model Error Sensitivity Suppression (MESS) [12]. In this procedure the control spillover to the secondary modes is weighted in the performance index (3.3). In this way the control gain is chosen so as to minimize spillover to the secondary modes. Of course there will still be spillover to the residual and unmodeled modes.

A second type of technique for spillover suppression is to include filters in the controller in order to reduce the controller bandwidth. This is done in order to limit spillover to the residual modes, which are of higher frequency than the primary and secondary modes. There are several approaches to this type of control (eg. [13]-[15]). The value of this method is that the high frequency modes cannot be accurately modeled and therefore it is not possible to weight these modes in the performance index, as is done in MESS. Filtering enables the designer to suppress spillover to the residual modes without having accurate knowledge of their characteristics. The disadvantage of these methods is that they require more on-board computation. It is necessary to implement the filter as well as the observer (3.4) (if an observer is used).

Another method of control system design which can be used to reduce the effect of spillover is multivariable

frequency domain design [16]. Because the design is done in the frequency domain it is possible to limit the effect of the controller on high frequency modes. The disadvantage of this procedure is that it is difficult to apply to very high order systems. The design procedure is not as well defined as those discussed previously, and the design is based to a larger extent on subjective decisions of the designer. This process becomes much more difficult as the dimension of the problem increases.

A further discussion of the spillover problem and combinations of methods which can be used to control it is given in [17].

3.4 Adaptive Control

The finite dimensional model of the LSS (2.7), which is normally developed using the finite element method, often contains substantial errors in the modal frequencies and mode shapes. In addition, these system parameters can change during flight due to changes in temperature, vehicle orientation, etc. For these reasons it may be necessary to design LSS control systems which will tune in on the true system parameters ([18],[19]).

These adaptive control techniques can be divided into two basic categories. The first is indirect adaptive control. The indirect methods first identify the parameters of the system and then use the parameter estimates to adjust the control law. The second is direct adaptive control. The direct methods do not explicitly identify the system parameters but rather directly adjust certain parameters in the control law.

Adaptive control has several disadvantages:

- It requires much more on-board computation than other control methods.
- The stability properties for adaptive control of distributed parameter systems are not well understood.
- Very little work has been done on the development of adaptive control procedures for LSS control.

3.5 Conclusions

This section has provided a brief overview of some of the methods which have been proposed for vibration control of LSS. It has suggested that there are two, often conflicting, requirements which these methods must satisfy:

- The computational requirements of the algorithm must not exceed the capacity of the on-board computer.
- The procedure must be robust. The controller which is designed for the reduced order model must perform adequately when applied to the distributed parameter LSS system whose characteristics cannot be accurately estimated.

The methods discussed in this section have ranged from simple output feedback controllers with small on-board computational requirements, to adaptive control systems with considerable computational requirements but with a potentially increased robustness. At this point in time the LSS control problem has not been sufficiently analyzed to significantly narrow the list of potential LSS control systems.

The remainder of this report will investigate in more detail some representative LSS control systems and will illustrate their application to a simplified model of a LSS. The control systems which are covered have been chosen to have a range of complexity from simple output feedback to MMC with filters for spillover suppression.

SECTION 4

MODAL DASHPOTS

This section describes a method for adding damping to specific modes of a LSS through output feedback [4]. This is one of the simplest methods for LSS control and requires a minimal amount of computation - both for calculation of feedback gains and for on-board algorithm implementation.

Consider again the finite dimensional LSS model of (2.7) and (2.8)

$$\ddot{v} + Wv = U^T B_a u \quad (4.1)$$

$$y = C_d U v + C_v U \dot{v} \quad (4.2)$$

If we now consider only the primary modes and ignore all secondary and residual modes we obtain the reduced order model

$$\ddot{v}_p + W_p v_p = U_p^T B_a u \quad (4.3)$$

$$y = C_d U_p v_p + C_v U_p \dot{v}_p \quad (4.4)$$

where the primary mode shapes make up the columns of U_p . The method of modal dashpots uses velocity feedback only so that the closed loop model can be written

$$u = Gy \quad (4.5)$$

$$\ddot{v}_p + W_p v_p = U_p^T B_a G C_v U_p \dot{v}_p \quad (4.6)$$

or

$$\ddot{v}_p - U_p^T B_a G C_v U_p \dot{v}_p + W_p v_p = 0 \quad (4.7)$$

The principle of modal dashpots is to provide a specified amount of damping to the primary modes. This could be done by restricting the damping matrix of the closed loop system to be

$$-U_p^T B_a G C_v U_p = \begin{bmatrix} 2\zeta_1 \omega_1 & 0 & \cdots & 0 \\ 0 & 2\zeta_2 \omega_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 2\zeta_p \omega_p \end{bmatrix} = \Lambda \quad (4.8)$$

The following decoupled controller is chosen

$$G = -[U_p^T B_a]^T [U_p^T B_a (U_p^T B_a)^T]^{-1} \Lambda [(C_v U_p)^T C_v U_p]^{-1} (C_v U_p)^T \quad (4.9)$$

It can be shown [4] that if the feedback gain matrix is symmetric then under minor restrictions the closed loop system will always be stable. This can be assured by requiring that the sensors and actuators be colocated, ie.

$$B_a = C_v^T \quad (4.10)$$

Another restriction is that the number of sensor/actuator pairs be greater than the number of primary modes. This is necessary to ensure that the inverse matrices in (4.9) exist.

It should be emphasized that this discussion has ignored all secondary and residual modes. The spillover from these modes will cause the damping of the system to be different than the designed damping in Λ . However, the system will remain stable under minor restrictions [4].

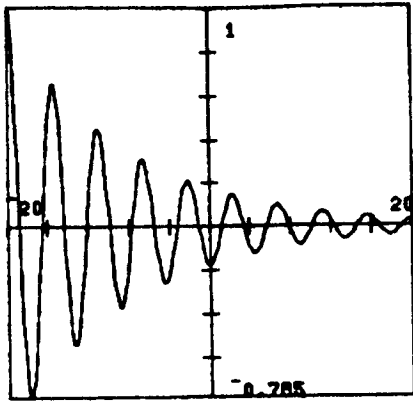
The modal dashpot method has been applied to the CSDL tetrahedral model which is described in [21]. The feedback gains were computed for three values of damping in the primary modes - .1, .2 and .3. As an example, for .1 damping the desired damping matrix would be

$$\Lambda = \begin{bmatrix} 0.27 & 0 & 0 & 0 \\ 0 & 0.33 & 0 & 0 \\ 0 & 0 & 0.51 & 0 \\ 0 & 0 & 0 & 0.68 \end{bmatrix} \quad (4.11)$$

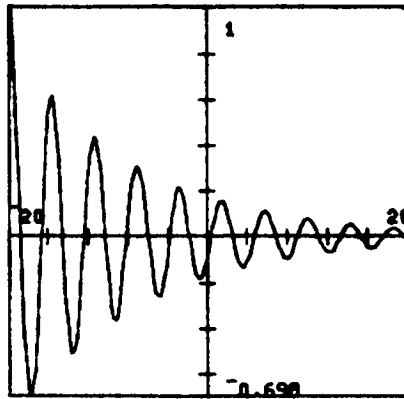
and the gain matrix was calculated from (4.9) to be

$$G = \begin{bmatrix} 3 & 0.35 & -1.0 & -2 & -0.61 & -2.1 \\ -0.35 & 3 & 2.1 & -0.61 & -2 & -1.0 \\ -1.0 & 2.1 & 7.4 & 4.6 & -6 & -7.2 \\ -2 & -0.61 & -4.6 & -0.0 & -0.49 & -6 \\ -0.61 & -2 & -6 & -0.49 & 0.0 & 4.6 \\ 2.1 & -1.0 & -7.2 & 6 & 4.6 & 7.4 \end{bmatrix}$$

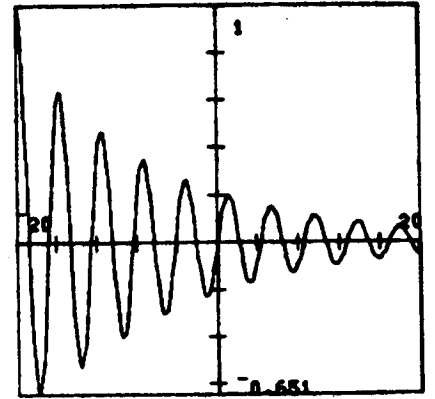
For each of the three values of damping the response of the closed loop system to an initial disturbance in the first mode was simulated. The response of the first mode for each case is shown in Figure 4.1. Notice that the actual maximum damping occurs for the system designed for .1 damping. This occurs because of increased spillover. The secondary and residual modes were ignored during the design process, and as the design attempts to obtain more damping in the primary modes spillover from the other modes degrades system performance.



0.1 Damping



0.2 Damping



0.3 Damping

Figure 4.1

Response to Initial Disturbance - Modal Dashpots

SECTION 5

LOW AUTHORITY CONTROL

Low authority control (LAC) [5] is a method for adding a moderate amount of damping to the primary modes of the system. It is similar in concept to modal dashpots.

Consider again the reduced order state space model (2.14) and (2.15)

$$\dot{x}_p = A_p x_p + B_p u \quad (5.1)$$

$$y = C_p x_p \quad (5.2)$$

where

$$x_p = \begin{bmatrix} v_p \\ \dot{v}_p \end{bmatrix} \quad A_p = \begin{bmatrix} 0 & I \\ -W_p & 0 \end{bmatrix} \quad B_p = \begin{bmatrix} 0 \\ U_p^T B_a \end{bmatrix} \quad (5.3)$$

$$C_p = [0 \quad C_v U_p]$$

with output feedback

$$u = Gy = GC_p x_p \quad (5.4)$$

Combining equations (5.1), (5.2) and (5.4) we obtain

$$\dot{x}_p = A_p x_p + B_p GC_p x_p \quad (5.6)$$

or

$$\dot{x}_p = A_p^* x_p \quad (5.6)$$

where

$$A_p^* = \begin{bmatrix} 0 & I \\ -W_p & U_p^T B_a GC_v U_p \end{bmatrix} \quad (5.7)$$

LAC considers the changes in the eigenvalues of A_p^* from those of A_p when the feedback gain G is small. The eigenvalues of A_p are p

$$\lambda_n = \pm j\omega_n \quad n = 1, \dots, p$$

Aubrun [5] has shown that the changes in the eigenvalues for small gains will be approximately

$$d\lambda_n \approx \frac{1}{2} [U_p^T B_a G C_v U_p]_{nn} \quad (5.8)$$

where $[]_{nn}$ denotes the n, n element of the matrix. Because the matrices in (5.8) are all real, it follows that the new root $\lambda_n + d\lambda_n$ now has a real part. The definition of the damping ratio now gives

$$2\zeta_n \omega_n \approx -[U_p^T B_a G C_v U_p]_{nn} \quad (5.9)$$

To illustrate the method for computing G , let

$$B^* = U_p^T B_a \quad C^* = C_v U_p \quad (5.10)$$

Then (5.9) can be written

$$\sum_j \sum_k B_{nj}^* C_{kn}^* G_{jk} = -2\zeta_n \omega_n \quad (5.11)$$

There is in general more than one solution for G which satisfies (5.11). By using a pseudo-inverse it is possible to compute G in such a way as to minimize

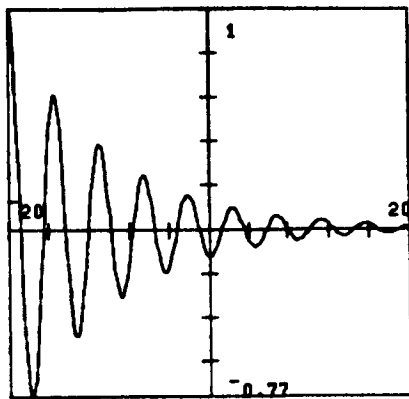
$$\sum_i \sum_j G_{ij}^2 \quad (5.12)$$

The gains were calculated in this manner for the CSDL tetrahedral model. Three values of damping in the primary modes - .1, .2 and .3 were used. The gain matrix for .1 damping is

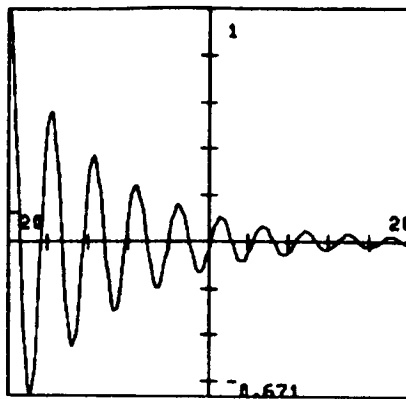
$$G = \begin{bmatrix} -4.0 & -0.49 & -3.9 & -3.1 & -0.22 & -4.3 \\ -0.49 & 4.0 & 4.3 & -0.22 & -3.1 & -3.9 \\ -3.9 & 4.3 & 6.4 & 1.6 & -2.7 & -6.2 \\ -3.1 & -0.22 & 1.6 & 4.2 & 2.0 & -2.7 \\ -0.22 & -3.1 & -2.7 & -2.0 & 4.2 & 1.6 \\ 4.3 & -3.9 & -6.2 & -2.7 & 1.6 & 6.4 \end{bmatrix} \quad (5.13)$$

Notice that this matrix is significantly different than the modal dashpot .1 damping gain matrix (4.12).

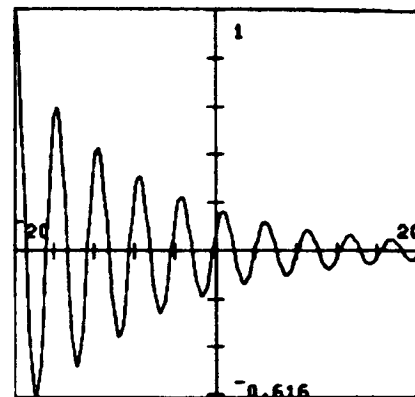
For each of the three values of damping the response of the closed loop system to an initial disturbance in the first mode was simulated. The response of the first mode in each case is shown in Figure 5.1. The results here are very similar to those for modal dashpots. Spillover causes a deterioration in performance if one tries to obtain large amounts of damping.



0.1 Damping



0.2 Damping



0.3 Damping

Figure 5.1

Response to initial disturbance - LAC

SECTION 6

OPTIMAL CONTROL

The theory of optimal control is well described in many textbooks (eg. [20]). This section will state the problem and the principle results in order to introduce the appropriate notation.

The theory of linear optimal control assumes that the system can be characterized by the differential equation

$$\dot{x} = Ax + Bu + w_1 \quad (6.1)$$

$$y = Cx + w_2 \quad (6.2)$$

where w_1 and w_2 are disturbance vectors containing white, Gaussian noise processes with intensities V_1 and V_2 respectively. With the exception of the disturbance vectors this model is the same as the LSS state variable model (2.9), (2.10).

It can be shown that the control

$$u^* = -K_c x \quad (6.3)$$

is optimal in the sense that it minimizes the performance index

$$J = E \left\{ \int_0^{\infty} [x^T R_1 x + u^T R_2 u] dt \right\} \quad (6.4)$$

where R_1 is positive semidefinite and R_2 is positive definite, if the control gain

$$K_c = R_2^{-1} B^T P \quad (6.5)$$

is computed from the algebraic Riccati equation

$$A^T P + PA - PBR_2^{-1} B^T P + R_1 = 0 \quad (6.6)$$

Because x is not measured directly it cannot be used in (6.3). For this reason an estimate of x is found from the filter/observer

$$\dot{\hat{x}} = A\hat{x} + Bu + K_f [y - C\hat{x}] \quad (6.7)$$

where the steady state filter/observer gain

$$K_f = QCV_2^{-1} \quad (6.8)$$

is computed from the algebraic Riccati equation

$$A^T Q + Q A - Q C^T V_2^{-1} C Q + V_1 = 0 \quad (6.9)$$

The estimate \hat{x} is optimal in the sense that it minimizes the mean square estimation error.

The optimal control law was found for the reduced order model (primary modes only) of the CSDL tetrahedral structure. The following weighting matrices were used

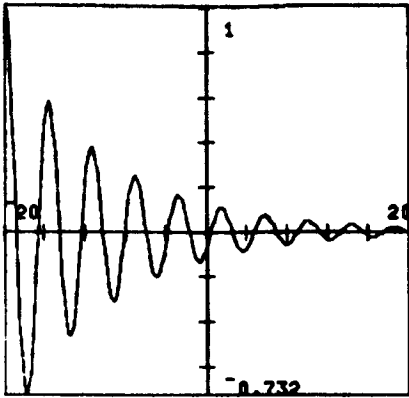
$$R_1 = I_8 \quad R_2 = \rho_c I_6 \quad V_1 = I_8 \quad V_2 = \rho_f I_6 \quad (6.10)$$

Where ρ_c is used to change the relative weighting, in the performance index, of control energy and regulation error. As ρ_c is decreased the system regulation error will decrease while the required control energy will increase. The ρ_f parameter serves an analogous role for the filter/observer.

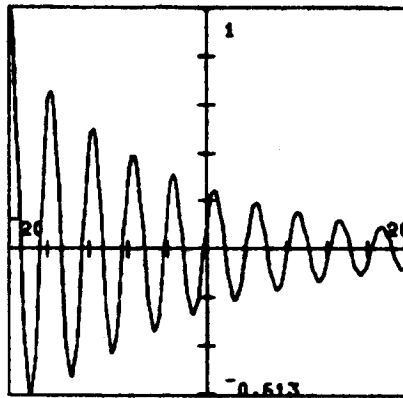
Optimal gains were computed for a number of values of ρ_c and ρ_f using the reduced order model. The response of the full order closed loop system was then simulated for various combinations of ρ_c and ρ_f . The results show the same major effect that was seen with the method of modal dashpots and LAC. As one attempts to achieve decreased regulation error (more damping) there comes a point at which the spillover from secondary and residual modes begins to cause a degradation in system performance and limits the actual damping which can be obtained.

Figure 6.1 shows the response of the system to an initial disturbance in the first mode for three combinations of ρ_c and ρ_f . It is clear that as the control energy is allowed to increase (decreasing ρ_c) the damping actually decreases after a certain point. Note that the initial peak does decrease with decreasing ρ_c , but as the secondary and residual modes are excited the spillover causes subsequent peaks to remain high.

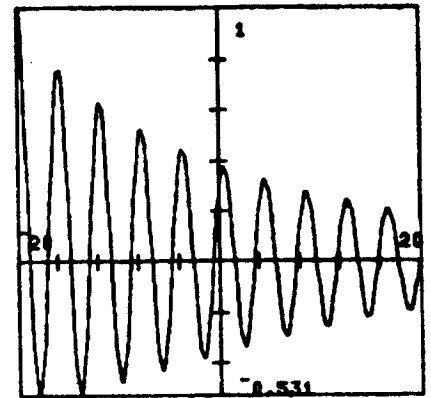
A comparison with Figures 4.1 and 5.1 show that optimal control, even though it requires significantly more on-board computation, does not significantly out-perform the simpler LAC and modal dashpots methods. Of course these results are of a preliminary nature, since the simulations performed were not exhaustive.



$\rho_C = 0.1$
 $\rho_f = 0.001$



$\rho_C = 0.01$
 $\rho_f = 0.001$



$\rho_C = 0.0001$
 $\rho_f = 0.0001$

Figure 6.1

Response to Initial Disturbance - Optimal Control

SECTION 7

MODEL ERROR SENSITIVITY SUPPRESSION

Model Error Sensitivity Suppression (MESS) is a method for reducing spillover from secondary modes. It is essentially the same as the optimal control method of the previous section except in the manner in which the weighting matrices R_2 and V_2 are determined.

Consider the primary, secondary and residual modes of (2.12) and (2.13) taken separately

$$\dot{x}_p = A_p x_p + B_p u \quad (7.1)$$

$$\dot{x}_s = A_s x_s + B_s u \quad (7.2)$$

$$\dot{x}_r = A_r x_r + B_r u \quad (7.3)$$

We want to include in the performance index some penalty on control spillover to the secondary modes - $B_s u$. Therefore the performance index (6.4) is changed to

$$J_2 = E \left\{ \int_0^{\infty} [x_p^T R_1 x_p + (B_s u)^T W_s (B_s u) + u^T R_2 u] dt \right\} \quad (7.4)$$

In terms of implementation this will require a change in the R_2 matrix to be used in (6.5) and (6.6)

$$R_2^* = R_2 + B_s^T W_s B_s \quad (7.5)$$

It is also possible by using a dual argument to develop a similar procedure for penalizing observation spillover by changing the V_2 matrix which is used in (6.7) and (6.8).

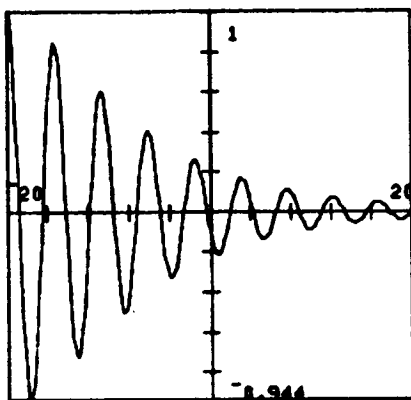
As with the other control methods MESS was simulated on the CSDL tetrahedral model. The spillover weighting matrix which was used was

$$W_s = I_6 \quad (7.6)$$

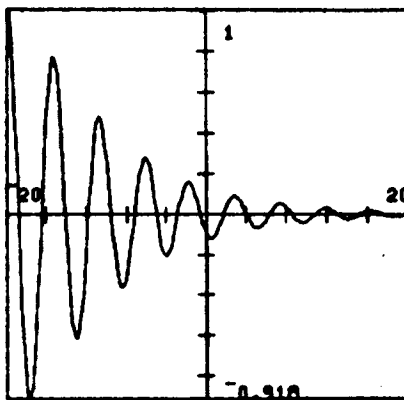
The R_1 , R_2 , V_1 and V_2 matrices were the same as those of (6.9).¹ Figure 7.1 Shows the response of the system to an initial disturbance in the first mode for three combinations of ρ_c and ρ_f . Notice that by comparison with Figure 6.1 the MESS^C technique has reduced the spillover problem significantly. It is now possible to achieve a greater level of damping. It should be emphasized that these are preliminary results, and that there are many possible combinations of weighting matrices and disturbance inputs which have not been investigated.

It should be noted that no attempt is made to reduce spillover to residual modes. This is because it has been assumed that knowledge of residual mode characteristics is

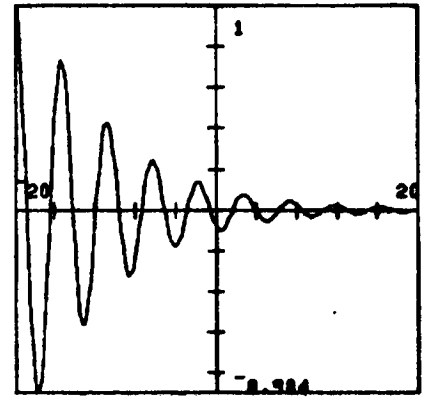
very inaccurate and therefore that B cannot be accurately computed. A method for reducing spillover to residual modes will be discussed in Section 8.



$$\rho_c = 0.1$$
$$\rho_f = 0.01$$



$$\rho_c = 0.01$$
$$\rho_f = 0.001$$



$$\rho_c = 0.0001$$
$$\rho_f = 0.0001$$

Figure 7.1

Response to Initial Disturbance - MESS

SECTION 8

FREQUENCY SHAPED COST FUNCTIONALS

The method of Frequency Shaped Cost Functionals (FSCF) [14] is an extension of the standard optimal control procedure of Section 6. The basic purpose is to minimize excitation of high frequency modes by the controller. To accomplish this consider the performance index written in the frequency domain

$$J = \frac{1}{2} \int_{-\infty}^{\infty} [x^*(j\omega)R_1x(j\omega) + u^*(j\omega)R_2u(j\omega)]d\omega \quad (8.1)$$

In its present form the weighting matrices are not a function of frequency, but we can generalize (8.1)

$$J = \frac{1}{2} \int_{-\infty}^{\infty} [x^*(j\omega)R_1(j\omega)x(j\omega) + u^*(j\omega)R_2(j\omega)u(j\omega)]d\omega \quad (8.2)$$

which will allow us to penalize high frequency activity of x and u . The problem is to translate this into an equivalent time domain problem which can be solved using the standard Riccati equation. Under certain restrictions on $R_1(j\omega)$ and $R_2(j\omega)$ this can be done [14]. Assume that $R_1(j\omega)$ and $R_2(j\omega)$ can be factored:

$$R_1(j\omega) = P_1^*(j\omega)P_1(j\omega) \quad (8.3)$$

$$R_2(j\omega) = P_2^*(j\omega)P_2(j\omega) \quad (8.4)$$

where P_1 and P_2 are rational matrices. Define

$$P_1(j\omega)x = x_+ \quad (8.5)$$

$$P_2(j\omega)u = u_+ \quad (8.6)$$

Equations (8.5) and (8.6) can also be written as differential equations. For example, let

$$R_2(j\omega) = I (w^2 + w_0^2)/w_0^2 \quad (8.7)$$

This would penalize controls at high frequencies. R_2 can be factored so that

$$P_2(j\omega) = I(j\omega + w_0)/w_0 \quad (8.8)$$

Equation 8.6 can now be written as a differential equation

$$\dot{u} + w_0u = w_0u_+ \quad (8.9)$$

We now augment our reduced order design model

$$\begin{bmatrix} \dot{x}_p \\ \dot{u} \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & -w_0I \end{bmatrix} \begin{bmatrix} x_p \\ u \end{bmatrix} + \begin{bmatrix} 0 \\ w_0I \end{bmatrix} u_+ \quad (8.10)$$

(To simplify the example we will assume that R_1 is a constant, although it can be handled in a manner similar to R_2 .) The performance index can now be written

$$J = \frac{1}{2} \int_{-\infty}^{\infty} [x^*(j\omega)R_1x(j\omega) + u_+^*(j\omega)Iu_+(j\omega)] d\omega \quad (8.11)$$

or in the time domain

$$J = \int_0^{\infty} [x^T R_1 x + u_+^T I u_+] dt \quad (8.12)$$

which is equivalent to

$$J = \int_0^{\infty} \{ [x^T \ u^T] \begin{bmatrix} R_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} + u_+^T I u_+ \} dt \quad (8.13)$$

Equations (8.10) and (8.13) can now be used to set up the Riccati equation which will be used to find the control law:

$$u_+ = -[K_1 \ K_2] \begin{bmatrix} x \\ u \end{bmatrix} \quad (8.14)$$

Combining (8.9) and (8.14)

$$\dot{u} = -w_0(I + K_2)u - w_0K_1x \quad (8.15)$$

which is the FSCF control law. Of course \hat{x} will replace x when implementing (8.15).

The FSCF method was applied to the CSDL tetrahedral model. The R_1 , R_2 , V_1 and V_2 matrices were chosen to be the same as those of (6.9). The filter parameter w_0 was set to .5. For all values of ρ_c and ρ_f which have been tested the closed loop full order system has been unstable. It appears that this procedure may be sensitive to filter bandwidth, and further testing needs to be done.

SECTION 9
CONCLUSIONS

9.1 Summary

The LSS vibration control problem can be stated:

How can one restore figure shape to an acceptable accuracy within a reasonable length of time when the structure is subjected to a large impulsive disturbance (or maintain figure shape at an acceptable accuracy in the presence of small continuous disturbances) when there exists limited computational capacity and limited knowledge of structural characteristics?

This report has discussed a variety of active control techniques that attempt to solve this problem - from simple damping augmentation techniques which use direct output feedback, to optimal control strategies with filtering to suppress spillover. Preliminary results seem to indicate that the simple damping augmentation methods provide performance which is comparable to the more sophisticated procedures with a fraction of the computational requirement. The MESS method does show promise, and when combined with some sort of filtering to suppress spillover to residual modes (as has been suggested in [13]) it may provide significantly better performance.

9.2 Future Work

Now that a test procedure has been organized and has been applied to several control algorithms, it would be useful to subject other methods to the same test procedure (eg. filter accommodated control, adaptive control, etc.). This would allow direct comparison of the methods and would make clear the relative advantages and disadvantages of each algorithm, and might suggest improvements. In addition, the test procedure should be expanded to test more aspects of the control systems. For example, it should test response to initial disturbances in secondary and residual modes, response to continuous type disturbances, effect of errors in system models on performance, effect of changes in system parameters on performance, etc. In this way a more complete analysis could be made and a more accurate comparison of their capabilities would result.

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