N ·A Technical Memorandum 83040

(NASA-TM-83040) INTO MESH LUBRICATION OF SPUR GEARS WITH ARBITRARY OFFSET OIL JET. I: FOR JET VELOCITY LESS THAN OR EQUAL TO GEAR VELOCITY (NASA) 22 p HC A02/MF A01 CSCL 13I G3/37

N83-19092

Unclas
02952

Into Mesh Lubrication of Spur Gears With Arbitrary Offset Oil Jet
I—For Jet Velocity Less Than or Equal to Gear Velocity

STIF

L. S. Akin
California State University
Long Bec·h, California

and

D. P. Townsend
Lewis Research Center
Cleveland, Ohio

Prepared for the
Winter Annual Meeting of the
American Society of Mechanical Engineers
Phoenix, Arizona, November 15-19, 1982

NASA

INTO MESH LUBRICATION OF SPUR GEARS WITH ARBITRARY OFFSET OIL JET

I - FOR JET VELOCITY LESS THAN OR EQUAL TO GEAR VELOCITY

L. S. Akin*
Mechanical Engineering Department
California State University
Long Beach, California

and

D. P. Townsend**
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio

E-1389

ABSTRACT

An analysis was conducted for into mesh oil jet lubrication with an arbitrary offset and inclination angle from the pitch point for the case where the oil jet velocity is equal to or less than pitch line velocity. The analysis includes the case for the oil jet offset from the pitch point in the direction of the pinion and where the oil jet is inclined to intersect the common pitch point. Equations were developed for the minimum oil jet velocity required to impinge on the pinion or gear and the optimum oil jet velocity to obtain the maximum impingment depth.

The optimum operating condition for best lubrication and cooling is provided when the oil jet velocity is equal to the gear pitch line velocity with both sides of the gear tooth cooled. When the jet velocity is reduced from pitch line velocity the drive side of the pinion and the unloaded side of the gear is cooled. When the jet velocity is much lower than the pitch line velocity the impingement depth is very small and may completely miss the pinion.

---

*Fellow, ASME.

**Member, ASME.

INTRODUCTION

Several methods of oil jet lubrication of gears is practiced by the gear industry. These include the oil jet directed into the mesh, out of the mesh and radially directed into the gear teeth. In most cases an exact analysis is not used to determine the optimum condition for best cooling such as jet nozzle location, direction and oil jet velocity. As a result many gear sets are operating without optimum oil jet lubrication and cooling.

The reason for developing more precise gear lubrication and cooling analysis is to be able to provide more design control over the scoring mode of gear tooth failure (1-4). As a result, considerably more power capacity can be provided in smaller gear drives. This is especially crucial in high performance aircraft or aerospace and light weight marine gears where the gears are casehardened and ground. The first step in performing such an analysis is in the determination of the oil jet impingement depth down the tooth profile being cooled by the jet. The radial and out of mesh jet nozzle orientation analyses have already been published (2, 3). The analysis discussed in this paper is confined exclusively to the directly into mesh jet nozzle orientation where oil jet velocity is less than or equal to the gear pitch line velocity ($V_j \leq \omega R \sec \beta_p$).

Primary impingement only is considered here because it can be accurately controlled. If it is properly applied nearly all of the necessary cooling can be provided by primary impingement, only. Post impingement flow down the tooth profile is not considered in this analysis. This nozzle orientation is the least desirable from a cooling viewpoint in that the fling-off angle and fling-off time (5) are severely curtailed due to the immediate tooth engagement drastically reducing the lubricant residence time and the effective heat transfer.

The time when this orientation is desirable is

(a) When the speed is so slow as to provide inadequate EHD lubricant film thickness in the mesh conjunction zone, increasing the coefficient of friction and heat generation

(b) When the speed is so high that radial or out-of-mesh impingement allows the fling-off angle and time to be so small as to provide inlet or EHD entrance zone film starvation.

Thus, in this case the problem is one of lubricant supply in the conjunction zone not cooling. In such cases, multiple nozzle orientation may be desirable: one very small into mesh jet for lubrication and one or more larger out of mesh jets for cooling.

The objectives of the work reported herein was to develop the analytical methods for gear lubrication with the oil jet directed into the engaging side of the gear mesh. The analysis is considerably different when the oil jet velocity is less than pitch line velocity as opposed to the case where the oil jet is greater than pitch line velocity. Therefore, this paper deals only with the case of oil jet velocity equal to or less than pitch line velocity. At gear ratios larger than one, the oil jet should be offset from the pitch line and rotated to a precise location to get optimum oil jet impingement on both the gear and pinion.

ANALYSIS

The analyses presented herein used an arbitrary offset and angle for jet location. For best results the offset should be constrained within $0 \leq S \leq S_o$, where:

$$S_o = a \frac{M_g - 1}{M_g + 1}$$

and the impingement angle β should be constrained within $0 \leq \beta \leq \beta_{pp}$, where:

ORIGINAL PAGE IS OF POOR QUALITY

$$\beta_{pp} = \tan^{-1} \frac{S_o}{(R_o^2 - R_s^2)^{1/2}}$$

In this analysis β will be further constrained so that:

$$\beta_p = \tan^{-1}[(S/(R_o^2 - R_s^2)^{1/2}]$$

so that S is an independent variable and β_p is a dependent variable.

The analyses that follow are divided into two cases (or operational conditions) with subcases. The two cases are established by whether or not the resolved pitch line velocity in the direction of the cooling oil jet ($V_p = \omega_p r \sec \beta_p = \omega_g R \sec \beta_p$) is equal to or less than the oil jet velocity. The two cases only are considered in this paper. Also, starting with a unit gear ratio, $m_g = 1$, as a refernce, consider that the ratio is increased, $m_g > 1$, until the jet is gear tooth controlled, severely narrowing the operating jet velocity range that will wet the pinion tooth (that is avoid missing it entirely). This analysis has been broadened in scope from previous published analysis (6), by allowing an arbitrary offset distance S and a constrained inclination angle β_p. It is assumed that optimum conditions occur if $S = S_o$ and $\beta_p = \beta_{pp}$ removing the $V_j(\max)$ and $V_j(\min)$ limitations to be described later in the paper.

It becomes overwhelmingly clear as the analysis proceeds that it is very desirable to provide the optimum engaging jet velocity exactly equal to or slightly larger than the resolved pitch line velocity, $V_j = \omega_p r \sec \beta_p = \omega_g R \sec \beta_p$, to provide maximum addendum impingement surface area down to the pitch circle on both gear and pinion as pointed out by Fujita et al. in 1965 (7, 8).

Development of Primary Formulii - for the Generalized Pinion Cases ($0 \leq S \leq S_o$)

Assume that we begin to view the sequence of events for the case where (the jet stream velocity) $V_j < \omega_p r \sec \beta_p$ (where: $\omega_p r$ is the pitch line velocity) as shown in Figures 1 to 3. As can be seen the jet stream head initiated by gear tooth (1) at "A" in Figure 1 will impinge on the leading side of the pinion tooth (2) trailing gear tooth (1) in Figure 2 and the so-called impingement depth will be determined by the final impingement of the jet stream head as shown in Figure 3 at "A". The starting positions of the mating teeth (1) and (2) as shown in Figure 1 are calculated for the gear:

$$\theta_{g1} = \cos^{-1}(R_s/R_o) - \text{inv}\ \varphi_{og} + \text{inv}\ \varphi \tag{1}$$

where:

$\text{inv}\ \varphi_{og} = \tan \varphi_{og} - \varphi_{og}$ and

$$\varphi_{og} = \cos^{-1}(R_b/R_o) = \cos^{-1}[(N_g \cos \varphi)/(N_g + 2 \pm \Delta N)] \tag{2}$$

$\text{inv}\ \varphi = \tan \varphi - \varphi$

φ = Pressure angle of gear (or pinion) at pitch circle

$R_s = R + S$ and $r_s = r - S_p$ (See Figure 1 or 4)

S = Jet Offset, $0 \leq S \leq S_o$ and $S = S_p$ only when $\beta_p = \beta_{pp}$

$S_p = [(r_o^2 - r^2 \cos^2 \beta_p)^{1/2} + r \sin \beta_p] \sin \beta_p$, $0 < S_p < S$

S_o = Offset that places jet stream at intersection of O.D.'s (crotch)

R = Pitch radius of gear = $N_g/(2\ P_d)$

R_o = Outside radius of gear = $(N_g + 2 \pm \Delta N)/(2\ P_d)$

$R_b = R \cos \varphi$ = base radius of gear

$(R_s/R_o) = N_g/(N_g + 2 \pm \Delta N) = m_g\ N_p/(m_g N_p + 2 \pm \Delta N)$

m_g = Gear ratio = $N_g/N_p = R/r = \omega_p/\omega_g$

N_g = Number of teeth in gear circle

ΔN = Delta (or differential) number of teeth when long and short addendums are used (ΔN is usually set to zero)

P_d = Diametral pitch (usually cancels out)

ORIGINAL PAGE IS
OF POOR QUALITY

Assume $0 \leq S \leq S_o$ in this paper, unless otherwise stated.

For the Pinion:

$$\theta_{p1} = m_g \theta_{g1} + \pi/N_p - \text{inv}\ \varphi + 2\ B_p/N_p \tag{3}$$

where:

N_p = Number of teeth in the pinion circle

B_p - Blacklash at $P_d = 1$, for pinion only

$b_p = B_p/P_d$ = Blacklash at diametral pitch used

(b_p and b_g are usually set equal to zero in these calculations)

This locates the pinion tooth, at time equal to zero ($t = 0$) in Figure 1, to be impinged upon by the short jet stream, shown in Figure 2, when its initial end (called "head") reaches the final impingement point "A", shown in Figure 3 as "impingement length, L_p", and impingement depth, "d_p". The position of the pinion tooth (2) (in Figure 3) when (at $t = t_\omega$) the oil jet head makes this final point "A" is calculated from:

$$\theta_{p2} = \tan^{-1}\left(\frac{L_p \cos \beta_p}{r - L_p \sin \beta_p}\right) - \text{inv}\ \varphi_{p2} \tag{4}$$

where:

$$\text{inv}\ \varphi_{p2} = \tan \varphi_{p2} - \varphi_{p2}$$

$$\varphi_{p2} = \cos^{-1}\left(\frac{r_b}{[(r - L_p \sin \beta_p)^2 + (L_p \cos \beta_p)^2]^{1/2}}\right) \tag{5}$$

β = Inclination angle of jet stream ($0 < \beta < \beta_{pp}$)

β_p = Inclination angle pointing at pitch point (PP), $0 < S < S_o$

$\beta_p = \tan^{-1}[S/(R_o^2 - R_s^2)^{1/2}]$

β_{pp} = Inclination angle pointing at pitch point when $S = S_o$ only

$\beta_{pp} = \tan^{-1}[S_o/R_o^2 - R_s^2)^{1/2}]$

r = Pitch radius of pinion = $N_p/(2\ P_d)$

r_b = $r \cos \varphi$ = Base radius of pinion

The design solution to the problem of pinion cooling is to set the desired impingement depth "d_p" for the needed cooling surface and solve explicitly for the needed jet velocity, "V_j". It is assumed that $\beta = \beta_p$ in this paper to simplify the mathematics. Noting that the time of jet stream flight "t_f" must be equal to the time of rotation "t_ω", it can be shown that the required jet velocity may be calculated from

$$V_j = \omega_p \frac{[(R_o^2 - R_s^2)^{1/2} \sec \beta - L_p]}{(\theta_{p1} - \theta_{p2})} = V_{jp} \tag{6}$$

$$V_j = \omega_p [(R_o^2 - R_s^2)^{1/2} \sec \beta - L_p]/(\theta_{p1} - \theta_{p2}) = V_{jp}$$

where:

$$L_p = [(r_o - d_p)^2 - r^2 \cos^2 \beta_p]^{1/2} + r \sin \beta_p \tag{7}$$

r_o = Outside radius of pinion = $(N_p + 2 \pm \Delta N)/2\ P_d$

The analysis solution to the problem, when V_j is specified and the resulting impingement depth d_p is desired for the pinion, can be calculated implicitly by solving iteratively for "L_p" from (see Figures 1 and 3):

$$\omega_p[(R_o^2 - R_s^2)^{1/2} \sec \beta_p - L_p] = (\theta_{p1} - \theta_{p2})V_j \tag{8}$$

then

$$d_p = r_o - [(r - L_p \sin \beta_p)^2 + (L_p \cos \beta_p)^2]^{1/2} \tag{9}$$

for $0 < V_j < V_j(\text{Opt}, L)_p$

and

ORIGINAL PAGE IS
OF POOR QUALITY

$$d_p = a = \frac{1 \pm \Delta N/2}{P_d} \tag{10}$$

for $V_j(\mathrm{Opt}, L)_p \leq V_j$

where

a = tooth addendum

$V_j(\mathrm{Opt}, L)_p$ = Lowest jet velocity to obtain maximum impingement depth

A minimum jet velocity condition exists for the pinion when $m_g > 1.0$ or $S \neq S_o$ and $\beta \neq \beta_{pp}$. This happens when V_j is less than that required to place the jet head at position "A" in Figure 2 as the leading edge of the top land of the pinion crosses the jet stream line. This position of the pinion can be calculated from (see Figures 1 and 2):

$$\theta_{p3} = \cos^{-1}(r_s/r_o) - \mathrm{inv}\ \varphi_{op} + \mathrm{inv}\ \varphi \tag{11}$$

where

$$\mathrm{inv}\ \varphi_{op} = \tan \varphi_{op} - \varphi_{op}$$

$$\varphi_{op} = \cos^{-1}(r_b/r_o) \qquad \text{and}$$

$$r_s = r - S_p = r - [(r_o^2 + r^2 \cos^2 \beta_p)^{1/2} + r \sin \beta_p] \sin \beta_p \tag{12}$$

Now the minimum jet velocity V_j that will wet the top land of the pinion (when $m_g > 1.0$) can be calculated from:

$$V_j(\min)_p = \frac{[(R_o^2 - R_s^2)^{1/2} - (r_o^2 - r_s^2)^{1/2}] \sec \beta_p\ \omega_p}{\varphi_{pi} - \varphi_{p3} + \mathrm{inv}\ \varphi} \tag{13}$$

Note that when $m_g = 1$ or when $S = S_o$ or both, $V_j(\min)_p = 0$. The impingement depth at $V_j(\min)_p$ is $d_p(\min, L) = 0$.

Equations (6) and (13) are shown in Table 1 with the other equations being developed herein and provide a form of graphical visibility based on the oil jet velocity scale (larger velocity at top of table).

Moving up the velocity scale to the next point of interest, there is a "constant impingement depth range" of oil jet velocity centered around $V_j = V_g = \omega_g R \sec \beta_p = \omega_p r \sec \beta_p$. The lower limit of this range of $V_j < \omega_p r \sec \beta_p$ can be calculated from:

$$V_j(\text{Opt, L}) = \frac{\omega_g (R_o^2 - R_s^2)^{1/2} \sec \beta_p}{\theta_{g1} + (\pi + 2B_g)/N_g} \tag{14}$$

This is the jet velocity that will barely get the stream head to a depth $d_p = a$. When $V_j = V_p = V_g = \omega_g r \sec \beta_p$, exactly, the pinion tooth is wetted on both the leading and trailing profiles down to the pitch line ($d_p = a$).

A selection or specification for V_j must be kept within the bounds of Equation (13) and $\omega_p r \sec \beta_p$ if impingement on the leading side of the tooth profile is required.

Development of Primary Formulii - for the Generalized Gear Case ($0 \leq S \leq S_o$)

This sequence of events can be seen by looking at Figures 2, 4, and 5 for the gear when $V_j < \omega_g R \sec \beta_p$. The jet stream is initially chopped by the pinion tooth (2) at "A" in Figure 2, when the pinion is in the position θ_{p3}. At this time the gear tooth (3), where impingement will take place, is in position θ_{g3} which can be calculated from Figure 2:

$$\theta_{g3} = (\theta_{p3}/m_g) + \pi/N_g + 2\,B_g/N_g - \text{inv}\,\varphi \tag{15}$$

where:

N_g = Number of teeth in gear = 2 $P_d R$

B_g = Backlash for gear at P_d = 1 or $b_g\ P_d$

b_g = Backlash for gear (actual size), usually neglected.

As can be seen in Figure 4, the pinion tooth (2) goes off and leaves the jet stream head and stream is subsequently chopped again at its tail end by the gear leading tooth profile (3) at the top land, as shown at "A" in Figure 4. At this time the gear position can be calculated from:

$$\theta_{g4} = \cos^{-1}(R_s/R_o) - \text{inv}\ \varphi_{og} \tag{16}$$

The intermediate position of the jet head here is at

$$L_{ig} = (r_o^2 - R_s^2)^{1/2} \sec \beta_p = (\theta_{g3} - \theta_{g4})V_j/\omega_g \tag{17}$$

From here the jet head continues to fly toward the common line of centers at PP and the gear tooth (3) rotates until the leading profile intersects the head positions at "A" in Figure 5 at impingement distance L_g. At this time the gear has rotated to the angular position calculated from:

$$\theta_{g5} = \tan^{-1}\left(\frac{L_g \cos \beta_p}{R + L_g \sin \beta_p}\right) - \text{inv}\ \varphi_{g5} \tag{18}$$

where:

$$\text{inv}\ \varphi_{g5} = \tan^{-1} \varphi_{g5} - \varphi_{g5}$$

$$\varphi_{g5} = \cos^{-1} \frac{R_b}{[(R + L_g \sin \beta_p)^2 + (L_g \cos \beta_p)^2]^{1/2}} \tag{19}$$

The analysis solution for the gear is similar to but somewhat more complicated than for the pinion alone. Using Equation (6) first solve explicitly for the pinion jet velocity, V_{jp}, as a function of the selected pinion impingement depth d_p. Then, because a given "gear mesh" must have a common jet velocity, V_{jp}, the gear impingement depth d_g is solved for implicitly by first solving iteratively for the impingement distance "L_g" from Figures 2, 4, and 5.

$$[(r_o^2 - r_s^2)^{1/2} \sec \beta_p - L_g]\omega_g = (\theta_{g3} - \theta_{g5})V_{jp} \qquad \text{then} \tag{20}$$

$$d_g = R_o - [(R + L_g \sin \beta_p)^2 + (L_g \cos \beta_p)^2]^{1/2} \qquad (21)$$

for $V_j(\min)_p < V_j < V_j(\mathrm{Opt},\ L)_p$ and

$$d_g = a = \frac{1 \pm \Delta N/2}{P_d} \qquad (22)$$

for, $V_j(\mathrm{Opt},\ L)_g \leq V_j$

where $V_j(\mathrm{Opt},\ L)_g$ is defined mathematically later.

The design solution, for the gear, is similar to the pinion except more complex due to the boundary conditions imposed by the requirement to not allow jet velocities outside the range $V_j(\min)_g < V_j < V_j(\max)_g$ set by the pinion parameters. Noting again the equivalence of $t_f = t_\omega$, the velocities inside this range can be calculated from:

$$V_j = \frac{\omega_g[(r_o^2 - r_s^2)^{1/2} \sec \beta_p - ((R_o - d_g)^2 - R^2 \cos^2 \beta_p)^{1/2} + R \sin \beta_p]}{\theta_{g3} - \theta_{g5}} \qquad (23)$$

with the additional restriction that

$$V_j(\min)_p = V_j(\min)_g < V_j < V_j(\mathrm{Opt},\ L)_g$$

to be explained later. Equation (23) is shown in Table 2 on a graphical velocity scale to give a perspective for this and the other velocity formulas relative to each other.

A minimum jet velocity condition also exists for the gear when $m_g > 1$ as shown above and explained in detail for the pinion using Figure 2.

The initial position of the gear, to establish this minimum velocity cycle, is at θ_{g1} in Figure 1 and the final or pinion chopping position at "A" in Figure 2 for the gear may be calculated from:

$$\theta_{g2} = (\theta_{p3}/m_g) - (\pi/N_g) + 2\ B_g/N_g \qquad (24)$$

ORIGINAL PAGE IS OF POOR QUALITY

Therefore the minimum jet velocity V_j that can be allowed to avoid not wetting the pinion when $m_g > 1$, can be calculated, using gear parameters, from:

$$V_j(\text{min})_g = \frac{\omega_g[(R_o^2 - R_s^2)^{1/2} - (r_o^2 - r_s^2)^{1/2}] \sec \beta_p}{\theta_{g1} - \theta_{g2}} = 0 \quad \text{when} \quad S = S_o \tag{25}$$

and the associated minimum impingement depth, d_g, is equal to zero (0).

Note that Equations (13) and (25) are equal:

$V_j(\text{min})_p = V_j(\text{min})_g$, and also

when $m_g = 1$ $\quad V_j(\text{min})_g = d_g(\text{min}, L) = 0.$

Moving up the velocity scale of Table 2 to the next point of interest, there is a constant impingement depth range of oil jet velocity for the gear also centered around $V_j = V_g = \omega_g R \sec \beta_p$. The lower limit of this range can be calculated from (see Figure 2):

$$V_j(\text{Opt}, l)_g = \frac{\omega_g(r_o^2 - r_s^2)^{1/2} \sec \beta_p}{\theta_{p3}/m_g + (\pi/N_g + b_g/R)} = \frac{\omega_p(r_o^2 - r_s^2)^{1/2} \sec \beta_p}{\theta_{p3} + (\pi + 2\, B_p)/N_p} \tag{26}$$

Notice that this jet velocity will barely get the jet stream head initiated at "A" on tooth (2) to a depth $d_g = a$ or the gear tooth (3) at PP. Also when $V_j = V_g = \omega_j R \sec \beta_p$, exactly, the gear tooth is wetted on both sides of the tooth profiles down to the pitch line ($d_g = a$).

SUMMARY

An analysis was conducted for into mesh oil jet lubrication with an arbitrary offset and inclination angle from the pitch point for the case where the oil jet velocity is equal to or less than the pitch line velocity. The analysis includes the case for the oil jet nozzle offset from the pitch point in the direction of the pinion and where the oil jet is inclined to intersect the common pitch point. Equations were developed for the minimum oil jet velocity

required to impinge on the pinion or gear and the optimum oil jet velocity required to obtain the best lubrication condition of maximum impingement depth and gear tooth cooling. The following results were obtained.

1. The optimum operating condition for best lubrication and cooling is provided exactly when $V_j = V_g = \omega_p r \sec \beta_p = \omega_g R \sec \beta_p$ so that both sides of the pinion and gear will be wetted and, therefore, cooled.

2. When the jet velocity is slightly less than the gear velocity $(V_j \text{ Opt L}) \leq V_j < V_g)$ the loaded side of the driver is forward and receives the best cooling.

3. As the jet velocity is much less than the gear pitch line velocity, $V_j << (V_j(\text{Opt, L})) < V_g$, the impingement depth is considerably reduced. This may result in the pinion being completely missed with no primary pinion cooling provided.

NOMENCLATURE

a	$1/P_d$ or $(1 \pm \Delta N/2)/P_d$ = addendum
b_p, b_g	pinion and gear backlash respectively
B_p, B_g	total, pinion, gear backlash at $P_d = 1$
d_p, d_g	radial impingement depth
L_p, L_g	pinion, gear final impingement distance
L_{ig}	intermediate impingement distance
m_g	$N_g/N_p = R/r = \omega_p/\omega_G$ = gear ratio
N_p, N_g	number of teeth in pinion, gear
ΔN	differential number of teeth
P_d	diametral pitch
r, R	pinion and gear pitch radii
r_α, R_α	perpendicular distance from pinion, gear center to jet line
r_s, R_s	distance along line of centers to jet line origin

ORIGINAL PAGE IS
OF POOR QUALITY

r_x, R_x	distance along line of centers to jet line intersection at x
r_o, R_o	pinion and gear outside circle diameter
r_b, R_b	pinion and gear base radii
S, S_o, S_p	arbitrary, jet nozzle offset to intersect O.D.'s, offset for pinion only
t	time
t_f, t_ω	time of flight, rotation
$V_p = V_g$	linear velocity of pinion and gear at pitch line
V_j, V_{jp}	oil jet velocity, general, pinion controlled
x	distance from offset perpendicular to jet line intersection
$V_j(\max)_p$	maximum velocity at which $d_p = 0$
$V_j(\min)_p$	minimum velocity at which $d_p = 0$
β	arbitrary oil jet inclination angle
β_p	constrained inclination angle
β_{pp}	inclination angle for pitch point intersection
φ	pressure angle at pitch circles
φ_{pi}, φ_{gi}	pinion and gear pressure angle at points specified at i
ω_p, ω_g	pinion and gear angular velocities
inv φ	$\tan \varphi - \varphi$ = involute function at pitch point or operating pressure angle
$V_j(\mathrm{Opt}, L)_p$	lower limit jet velocity to impingement at pitch line

REFERENCES

1. Akin, L., Mross, J.; and Townsend, D.; "Theory for the Effect of Windage on the Lubricant Flow in the Tooth Spaces of Spur Gears", Trans. ASME, J. Engr. for Industry, Vol. 79, Ser. B. No. 4, 1975, pp. 1266-1283.

2. Akin, L., and Townsend, D.; "Study of Lubricant Jet Flow Phenomena in Spur Gears" Trans. ASME, J. Lubrication Tech., Vol. 97, Ser. F, No. 2, 1975, pp. 283-288.

3. Townsend, D. and Akin, L.; "Study of Lubricant Jet Flow Phenomena in Spur Gears - Out of Mesh Condition", Trans. ASME, J. Mech. Design, Vol. 100, No. 1, Jan. 1978, pp. 61-68.

4. Townsend, D, and Akin, L.; "Analytical and Experimental Spur Gear Tooth Temperature as Affected by Operating Variables", Trans. ASME, J. Mech. Design, Vol. 103, No. 4, Jan. 1981, pp. 219-226.

5. Winter, Arend de, and Blok, Harmen; "Fling Off Cooling of Gear Teeth, Trans. ASME, Engr. for Industry, Vol. 96, Ser. B, No. 1, 1974, pp. 60-70.

6. Akin, L, and Townsend, D.; "Cooling of Spur Gears with Oil Jet Directed into the Engaging Side of Mesh at Pitch Point", JSME - Proc. of Inter. Symposium on Gearing and Power Transmissions, 1981, Vol. 1, b-4, pp. 261-274.

7. Fujita, K., Obata, F., and Matsuo, K.; "Behavior of Lubricating Oil Onto the Meshing Parts of Spur Gears - The Case of Jet Lubrication onto the Engaging Side", J. of Japan Society of Lubrication Engineers, Vol. 19, No. 6, 1974, pp. 437-447.

8. Fujita, K., and Obata, F.; "The Influence of Lubricating Oil on the Scoring Resistance of Spur Gears", J. of Japan Society of Lubrication Engineers, Vol. 18, No. 5, 1973, pp. 383-393.

TABLE 1. - EQUATIONS FOR OIL JET VELOCITY AND PINION IMPINGEMENT DEPTH FOR PITCH LINE AND LOWER OIL JET VELOCITY

Relative velocity scale	Oil jet velocity	Pinion impingement depth
Pitch line and slightly lower	$V_j = \omega_p r \sec \beta_p = \omega_g R \sec \beta_p$ $V_j(\text{opt}, L)_p = \dfrac{\omega_g (R_o^2 - R_s^2)^{1/2} \sec \beta_p}{\theta_{g1} + \pi/N_g + 2B_g/N_g}$	$d_p = a = (1 \pm \Delta N_p/2)/P_d$, (both profiles) $d_p(\text{head}) = a$, (leading profile only)
Less than pitch line velocity down to where the oil jet starts to miss the pinion	$V_j = \dfrac{\omega_p \{(R_o^2 - R_s^2)^{1/2} \sec \beta_p - L_p\}}{\theta_{p1} - \theta_{p2}}$ V_j = given (when $0 \leq d_p \leq a$ <u>only</u>)	d_p = given (usual design solution) and $L_p = [(r_o - d_p)^2 - (r \cos \beta_p)^2]^{1/2} + r \sin \beta_p$ Iterate L_p from $[(R_o^2 - R_s^2)^{1/2} \sec \beta_p - L_p]\omega_p = (\theta_{p1} - \theta_{p2})V_j$ then, $d_p = r_o - [(r - L_p \sin \beta_p)^2 + (L_p \cos \beta_p)^2]^{1/2}$
Critical low velocity to miss the pinion	$V_j(\min)_p = \dfrac{\omega_p[(R_o^2 - R_s^2)^{1/2} \sec \beta_p - (r_o^2 - r_s^2)^{1/2} \sec \beta_p]}{\theta_{p1} - \theta_{p3} + \text{inv}\,\varphi}$ $V_j(\min)_p = 0$ when $s = s_o$ and $\beta_p = \beta_{pp}$	$d_p(\min, L) = 0$, when $m_g > m_g(\text{crit})$ note that: $0 < s < s_o$ and $0 < \beta_p \leq \beta_{pp}$ always

TABLE 2. - EQUATIONS FOR OIL JET VELOCITY AND GEAR IMPINGEMENT DEPTH FOR PITCH LINE AND LOWER OIL JET VELOCITIES

Relative velocity scale	Oil jet velocity	Pinion impingement depth
Pitch line and slightly lower	$V_j = \omega_g R \sec \beta$ (at pitch point) $V_j(\text{opt}, L) = \dfrac{\omega_p (r_o^2 - r_s^2)^{1/2} \sec \beta_p}{r_{p3} + (\pi + 2\beta_p)/N_p}$	$d_g = a = \dfrac{1}{P_d} \pm \dfrac{\Delta N}{2P_d}$
Less than pitch line velocity down to where oil jet starts to miss the gear	$V_j = \dfrac{\omega_g \{(r_o^2 - r_s^2)^{1/2} \sec \beta_p - [(R_o - d_g)^2 - (R \cos \beta_p)^2]^{1/2} - R \sin \beta_p\}}{\theta_{g3} - \theta_{g5}}$ V_j given	d_g given $L_g = [(R_o - d_g)^2 - R^2 \cos^2 \beta_p]^{1/2} - R \sin \beta_p$ Iterate L_g from: $[(r_o^2 - r_s^2)^{1/2} \sec \beta_p - L_g]\omega_g = (r_{g3} - r_{g5})V_J$ then $d_g = R_o - [(R + L_g \sin \beta_p)^2 + (L_g \cos \beta_p)^2]^{1/2}$
Critical low velocity to miss the gear	$V_j(\min)_g = \dfrac{\omega_g [(R_o^2 - R_s^2)^{1/2} - (r_o^2 - r_s^2)^{1/2}] \sec \beta_p}{r_{g1} - r_{g2}}$ $V_j(\min)_g = 0$ when $S = S_o$ and $\beta_p = \beta_{pp}$ or when $m_g = 1$	$d_g(\min, L) = R_o - \{[R + L_g(\min) \sin \beta_p]^2 + [L_g(\min) \cos \beta_p]^2\}^{1/2}$ $d_g(\min, L) = 0$ when $S = S_o$ and $\beta_p = \beta_{pp}$ or when $m_g = 1$

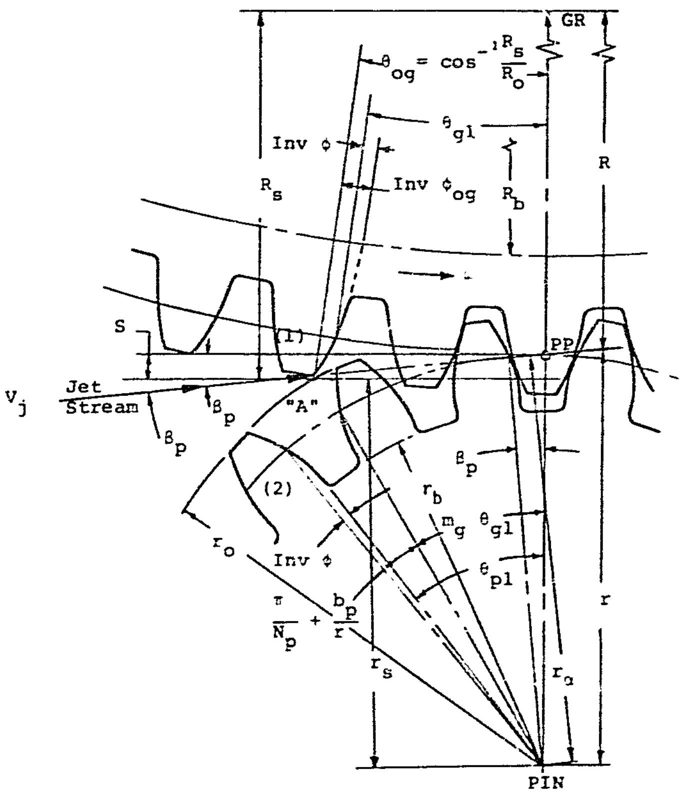

Figure 1.

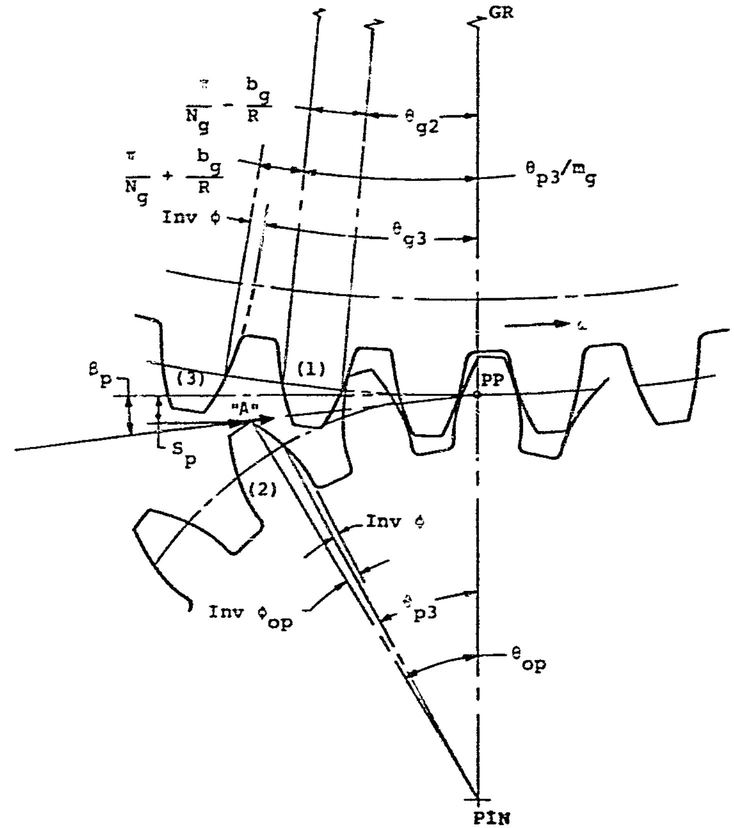

Figure 2.

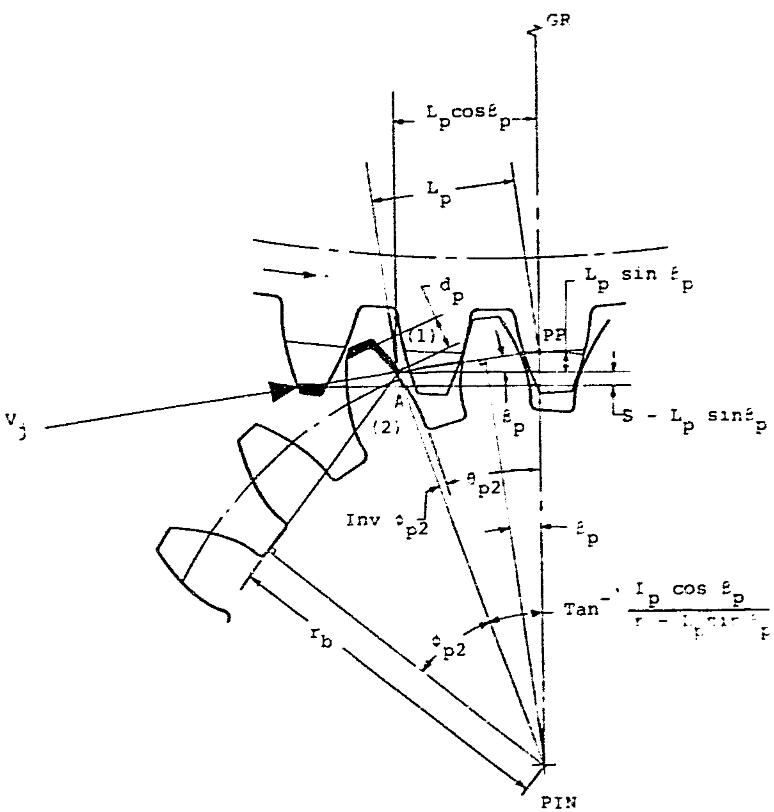

Figure 3.

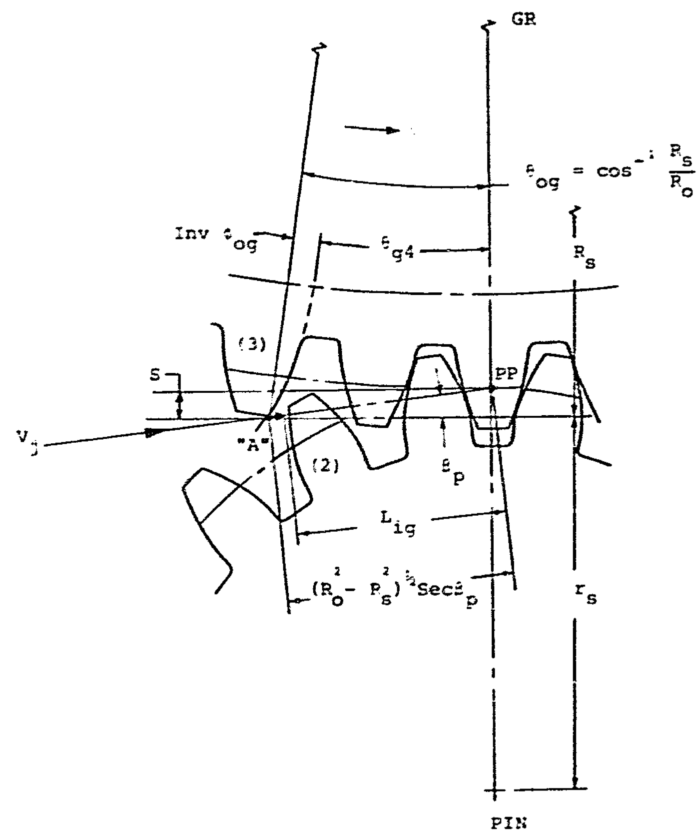

Figure 4.

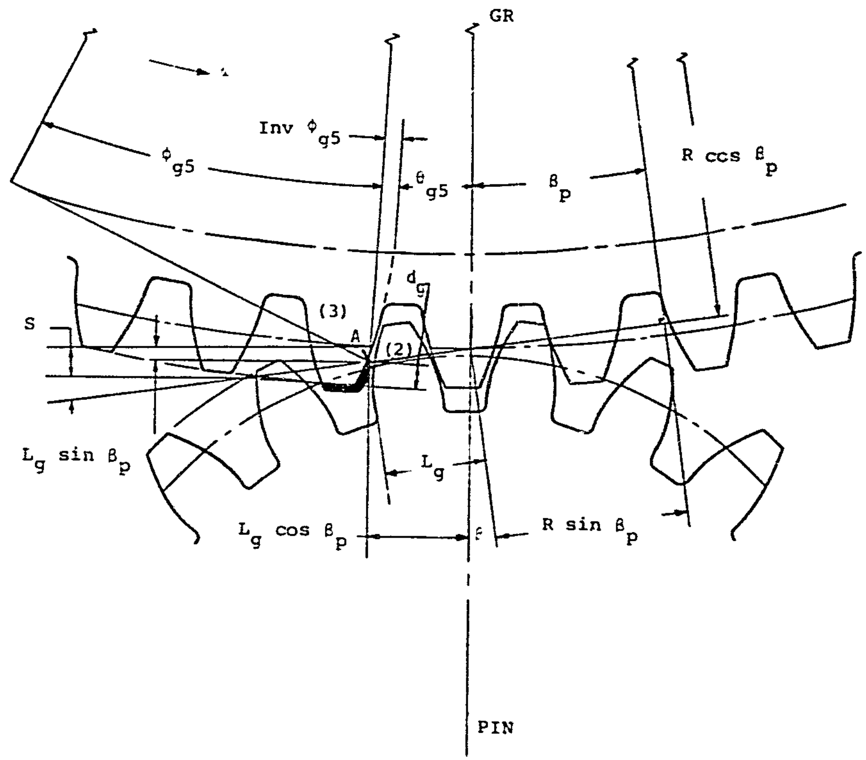
GR
Inv ϕ_{g5}
ϕ_{g5}
θ_{g5}
β_p
R ccs β_p
d_g
(3)
A
(2)
S
$L_g \sin \beta_p$
L_g
$L_g \cos \beta_p$
R sin β_p
PIN

Figure 5.