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ATMOSPHERIC TURBULENCE PARAMETERS FOR MODELING WIND TURBINE DYNAMICS

W.E. Holley, Associate Professor
R.W. Thresher, Professor
Department of Mechanical Engineering
Oregon State University
Corvallis, Oregon 97331

ABSTRACT

This paper presents a model which can be used to predict the response of wind turbines to atmospheric turbulence. The model was developed using linearized aerodynamics for a three-bladed rotor and accounts for three turbulent velocity components as well as velocity gradients across the rotor disk. Typical response power spectral densities are shown. The system response depends critically on three wind and turbulence parameters, and models are presented to predict desired response statistics. An equation error method, which can be used to estimate the required parameters from field data, is also presented.

WIND TURBINE SYSTEM MODEL

Before embarking on a discussion of the detailed characteristics of atmospheric turbulence parameters, it is necessary to present the modeling framework in which the parameters will be used to predict system responses. The primary purpose of the model is to provide a tool by which designers can estimate the effects of fluctuating turbulence inputs on the wind turbine, structural and power system responses.

For an n degree of freedom system, the basic principles of Newtonian mechanics [1] give equations of motion of the form

$$[M]\{\ddot{z}\} + [C_s]\{\dot{z}\} + [K_s]\{z\} = \{f_a\} \qquad (1)$$

where $\{z\}$ = the nx1 vector of generalized displacement coordinates.
$[M]$ = the nxn inertia matrix.
$[C_s]$ = the nxn gyroscopic and structural and power train damping matrix.

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$[K_s]$ = the nxn structural and power train stiffness matrix.
$\{f_a^s\}$ = the nxl vector of aerodynamic forces and moments generated by the turbine rotor.

The aerodynamic forcing term of Eq. (1) depends upon the motion of the turbine rotor with respect to the ground as well as the motion of the air. If the aerodynamic forces and moments are linearized about a steady operating condition, the following equation results

$$\{f_a\} = \{f_n\} + [F]\{u\} - [C_a]\{\dot{z}\} - [K_a]\{z\} \qquad (2)$$

where $\{f_n\}$ = the nxl vector of steady, nominal aerodynamic forces and moments.
$\{u\}$ = the mxl vector of fluctuating turbulence inputs.
$[F]$ = the nxm matrix of aerodynamic influence coefficients.
$[C_a]$ = the nxn aerodynamic damping matrix.
$[K_a]$ = the nxn aerodynamic stiffness matrix.

In this particular model, the turbulence input vector {u} consists of three velocity components which are uniform over the turbine rotor disk and six additional gradient terms which account for variations in turbulent velocity over the rotor disk. Table 1 gives a verbal description of the nine turbulence input terms appropriate for a rigid, three-bladed wind turbine rotor.

TABLE 1. DESCRIPTION OF TURBULENCE INPUT TERMS

Component	Description
V_x	uniform lateral or side component (in rotor plane)
V_y	uniform longitudinal component along steady wind direction
V_z	uniform vertical component (in plane)
$V_{y,x}$	lateral gradient of longitudinal velocity
$V_{y,z}$	vertical gradient of longitudinal velocity
γ_{xz}	swirl about steady wind axis (in plane)
ε_r, $\bar{\gamma}_r$	shear strain rates (in plane) expressed in a reference frame rotating at three times the rotor rate
$\bar{\varepsilon}_{xz}$	in-plane dilation

Assuming that the atmospheric turbulence is adequately described by the homogeneous, isotropic Von Karman model [2], the turbulence input vector can be approximated by the following set of stochastic differential equations [3]

$$\{\dot{u}\} = [A_w]\{u\} + [B_w]\{w\} \tag{3}$$

where $\{w\}$ = an mx1 vector of white noise excitations with flat power spectral density, $S_w = \sigma^2 L/V_w^3$.

$[A_w]$ = the mxm dynamics matrix for the turbulence inputs.

$[B_w]$ = the mxm distribution matrix for the white noise excitations.

The matrices $[A_w]$ and $[B_w]$ are diagonal, except for two off diagonal terms in $[A_w]$, which account for the three rotations per rotor revolution effect in the ε_r and $\bar{\gamma}_r$ terms caused by the three blades moving through the in-plane turbulence gradients.

The motion Eqs. (1), the aerodynamic force Eqs. (2), and the wind turbulence inputs Eqs. (3) can be combined into a set of system equations of the form

$$\begin{aligned} \{\dot{x}\} &= [A]\{x\} + [B]\{w\} \\ \{y\} &= [C]\{x\} + \{y_n\} \end{aligned} \tag{4}$$

where

$\{x\} = \begin{Bmatrix} \delta z \\ \delta \dot{z} \\ u \end{Bmatrix}$ the Nx1 system state vector (N = 2n+m)

$\{w\}$ = the mx1 white noise turbulence excitation vector.

$\{y\}$ = the ℓx1 vector of system response variables.

$\{y_n\}$ = the ℓx1 vector of steady nominal system responses.

$[A] = \begin{bmatrix} 0 & I & 0 \\ -M^{-1}(K_s+K_a) & -M^{-1}(C_s+C_a) & M^{-1}F \\ 0 & 0 & A_w \end{bmatrix}$ = the NXN system matrix

$[B] = \begin{bmatrix} 0 \\ 0 \\ B_w \end{bmatrix}$ = the Nxm white noise excitation distribution matrix.

$[C]$ = the ℓxN response distribution matrix.

Note that δz and $\delta \dot{z}$ are deviations from the steady, generalized displacement and velocity components. The outputs $\{y\}$ and the corresponding matrix $[C]$ depend upon the particular set of displacements, velocities or load response variables of interest to the designer.

The system equations of motion given by Eq. (4) are derived assuming a rigid, three-bladed turbine rotor. It is possible to derive system equations for two-bladed rotors similar to these equations, except that several of the terms in the [A] matrix will have periodic terms instead of being constant as in Eq. (4).

At this point, we will describe briefly how the wind parameters enter the various coefficients of the overall system model. First, the steady wind speed, V_w, affects the nominal aerodynamic forces and the linearized aerodynamic coefficients in the matrices $[C_a]$, $[K_a]$ and $[F]$. Second, both the steady wind speed, V_w, and the turbulence integral scale, L, affect the matrices $[A_w]$ and $[B_w]$. Finally, the turbulence component variance, σ^2, as well as V_w and L, affect the power spectral density, S_w, for each of the white noise excitation components. Thus, three atmospheric turbulence parameters, V_w, σ, and L, must be known in order to utilize the model given by Eq. (4).

Once the appropriate turbulence parameters are specified, the response, power spectral densities can be computed using the model given by Eq. (4). Since the white noise inputs are uncorrelated, the following equation results

$$\{S_y(\omega)\} = [T(\omega)]\{S_w\} \tag{5}$$

where $\{S_y(\omega)\}$ = the ℓx1 vector of response power spectral densities.
$\{S_w\}$ = the mx1 vector of white noise excitation power spectral densities.
$[T(\omega)]$ = the ℓxm matrix of squared, complex magnitudes of the system frequency response matrix elements.
ω = the radian frequency.

If $T_{jk}(\omega)$ is one element of $[T(\omega)]$, then

$$T_{jk}(\omega) = |H_{jk}(i\omega)|^2 \tag{6}$$

where $H_{jk}(i\omega)$ = the corresponding element of the complex frequency response matrix.
i = $\sqrt{-1}$.

Assuming the eigenvalues of the system dynamics matrix [A] are distinct, the complex frequency response matrix is given by

$$[H(i\omega)] = [C][M][i\omega[I] - [\Lambda]]^{-1}[M]^{-1}[B] \tag{7}$$

where [M] = complex modal matrix consisting of columns of eigenvectors of [A].
$[\Lambda]$ = diagonal complex matrix of eigenvalues of [A].
[I] = identity matrix.

TYPICAL WIND TURBINE RESPONSE CHARACTERISTICS

A simplified five-degree-of-freedom model for a three-bladed, horizontal-axis wind turbine was developed by Thresher, et al. [4]. The five generalized displacement degrees of freedom are given by

$$\{z\}^T = (U, V, \phi, \chi, \psi) \tag{8}$$

where
- U = lateral displacement of the nacelle in x direction.
- V = fore-aft displacement of the nacelle in y direction.
- ϕ = yaw angle.
- χ = pitch angle.
- ψ = rotor angular displacement about spin axis.

Figure 1 shows the coordinate system used for this model. The configuration shown in the figure is appropriate for a down-wind rotor design.

Thresher and Holley [5] utilized the model for two typical wind turbines of widely differing size. The first, designated the Mod-M, is an 8 kW free yaw system with a down-wind rotor. The second, the Mod-G, is a large 2.5 MW machine with a fixed yaw, up-wind rotor. The system characteristics for these two machines are given in Tables 2 and 3.

TABLE 2. MOD-M CHARACTERISTICS

Rotor Characteristics:		
Rotor Radius	5.081 m	(16.67 ft)
Hub Height	16.8 m	(55 ft)
Blade Chord (constant)	.457 m	(1.5 ft)
Coning Angle	.061 rad	(3.5°)
Blade Twist	0 rad	(0.0°)
Pitch Setting (to ZLL)	.052 rad	(3.0°)
Steady Operating Conditions:		
Rotor Speed, Ω	7.681 rad/s	(73.35 RPM)
Wind Speed, V_w	7.434 m/s	(16.63 MPH)
Approximate Output	6 kW	
Aerodynamic Properties:		
Lift Curve Slope	5.7	
Drag Coefficient C_{D0}	.02	
Turbulence Parameters:		
Standard Deviation, σ	.619 m/s	(2.03 ft/s)
Integral Length Scale, L	91.44 m	(300 ft)
System Frequencies (Tower Motion):		
1st Bending (fore-aft)	15.1 rad/s	(2.0 Ω)
2nd Bending (fore-aft)	53.1 rad/s	(7.0 Ω)
1st Bending (side-to-side)	15.9 rad/s	(2.1 Ω)
1st Torsion	0.0 rad/s	(Free Yaw)

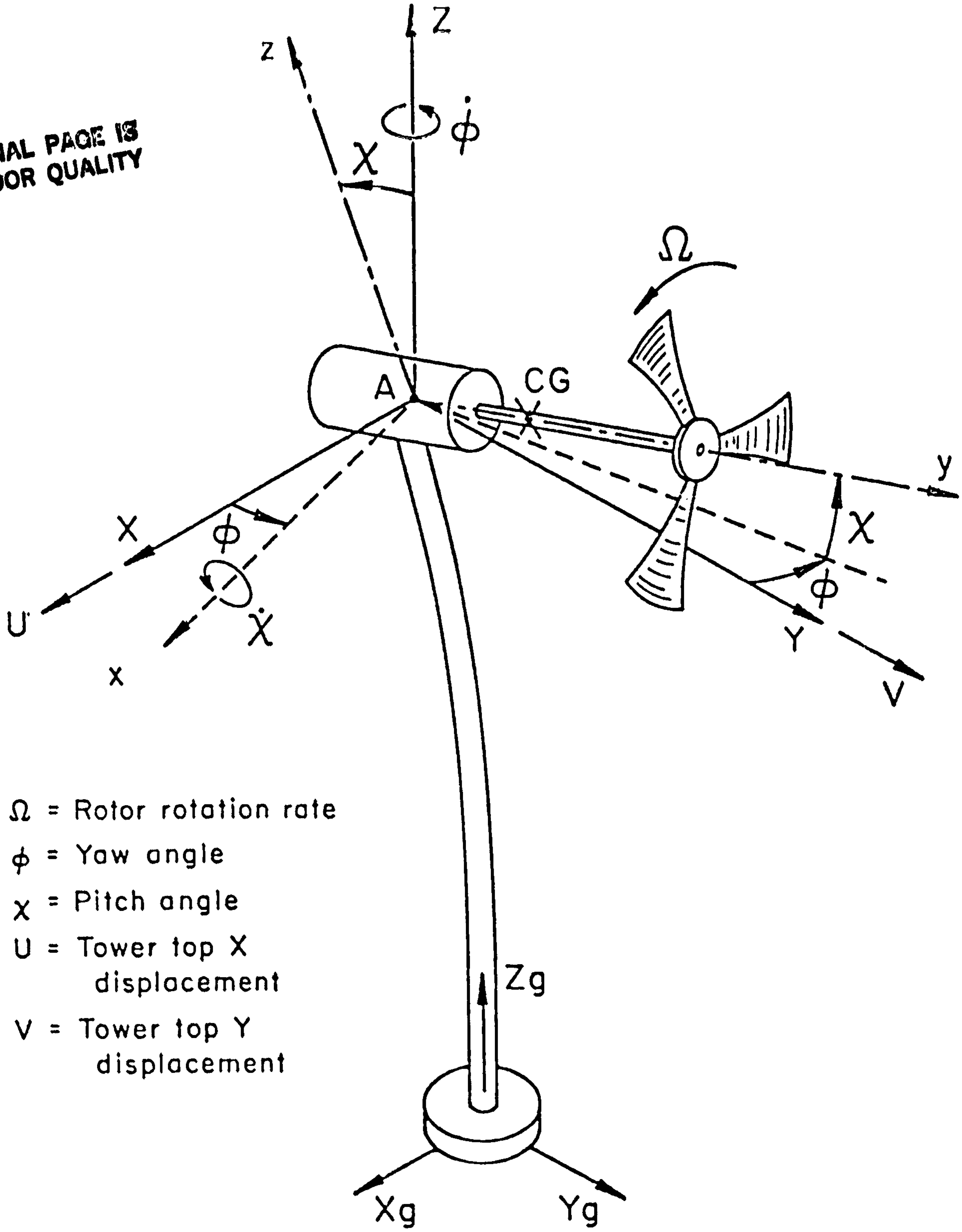

Figure 1. Coordinate Definitions for the Wind Turbine Model.

TABLE 3. MOD-G CHARACTERISTICS

Rotor Characteristics:		
Rotor Radius	45.7 m	(150 ft)
Hub Height	61.0 m	(200 ft)
Blade Chord (linear taper)	2.36 m to .96 m	(7.74 ft to 3.15 ft)
Coning Angle	.070 rad	(4.0°)
Blade Twist (linear)	.140 rad	(8.0°)
Pitch Setting at Tip (to ZLL)	.108 rad	(-6.2°)
Steady Operating Conditions:		
Rotor Speed, Ω	1.833 rad/s	(17.5 RPM)
Wind Speed, V_w	8.940 m/s	(20.0 MPH)
Approximate Output	1.1 MW	
Aerodynamic Properties:		
Lift Curve Slope	5.73	
Drag Coefficient, C_{D0}	.008	
Turbulence Parameters:		
Standard Deviation, σ	.744 m/s	(2.44 ft/s)
Integral Length Scale, L	152.4 m	(500 ft)
System Frequencies (Tower Motion):		
1st Bending (fore-aft)	2.75 rad/s	(1.5 Ω)
2nd Bending (fore-aft)	12.8 rad/s	(7.0 Ω)
1st Bending (side-to-side)	2.9 rad/s	(1.6 Ω)
1st Torsion	9.5 rad/s	(5.2 Ω)

Two aerodynamic wake models were used for each system to compute the coefficients in the aerodynamic system matrices $[C_a]$, $[K_a]$, and [F]. In the first, the steady conditions are used with standard momentum theory to compute the steady distribution of induced velocity across the rotor disk. This induced velocity is then assumed constant for the given conditions. This model is called the "Frozen Wake." In the second model, the induced velocity which results from a slowly varying velocity field is computed using a quasi-steady momentum balance. In this model, the turbine thrust is always in equilibrium with the driving turbulent velocity, and is called the "Equilibrium Wake." Aerodynamic stall is not modeled in either case.

Figures 2 and 3 show the power spectral densities of the thrust load and the yaw angle for the Mod-M machine. In the low frequency portion of Figure 2, the thrust load response closely follows the power spectrum of the V_y turbulence input. At higher frequencies the resonance effects of the tower bending modes are observed. In Figure 3, the yaw response is dominated by the $V_{y,x}$ turbulence input. This turbulence input term can be interpreted as the rate of change of the direction in the horizontal turbulent velocity component. A smaller additional effect is due to the uniform side velocity, V_x, turbulence term.

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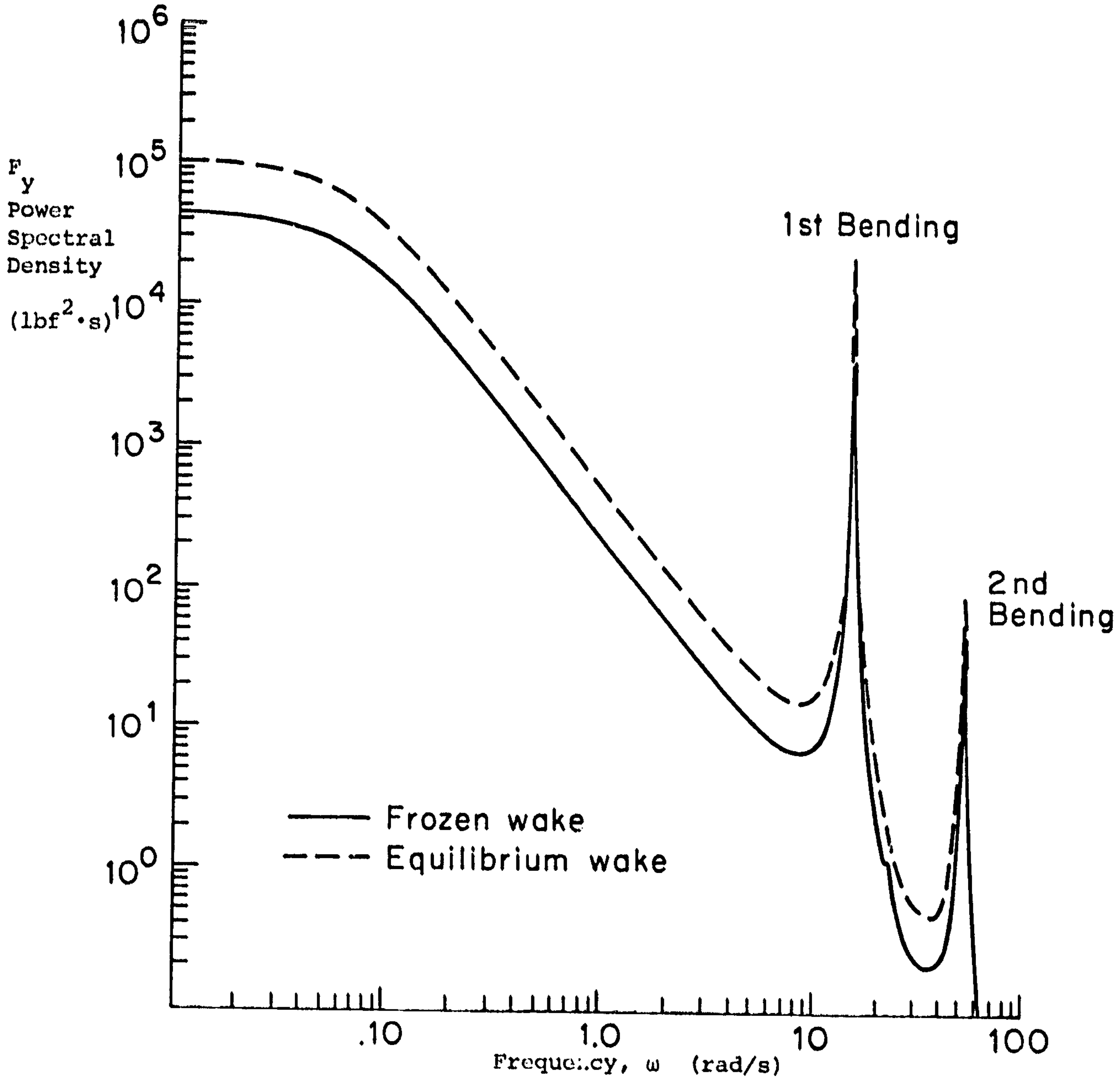

Figure 2. Thrust Load, F_y, for Mod-M.

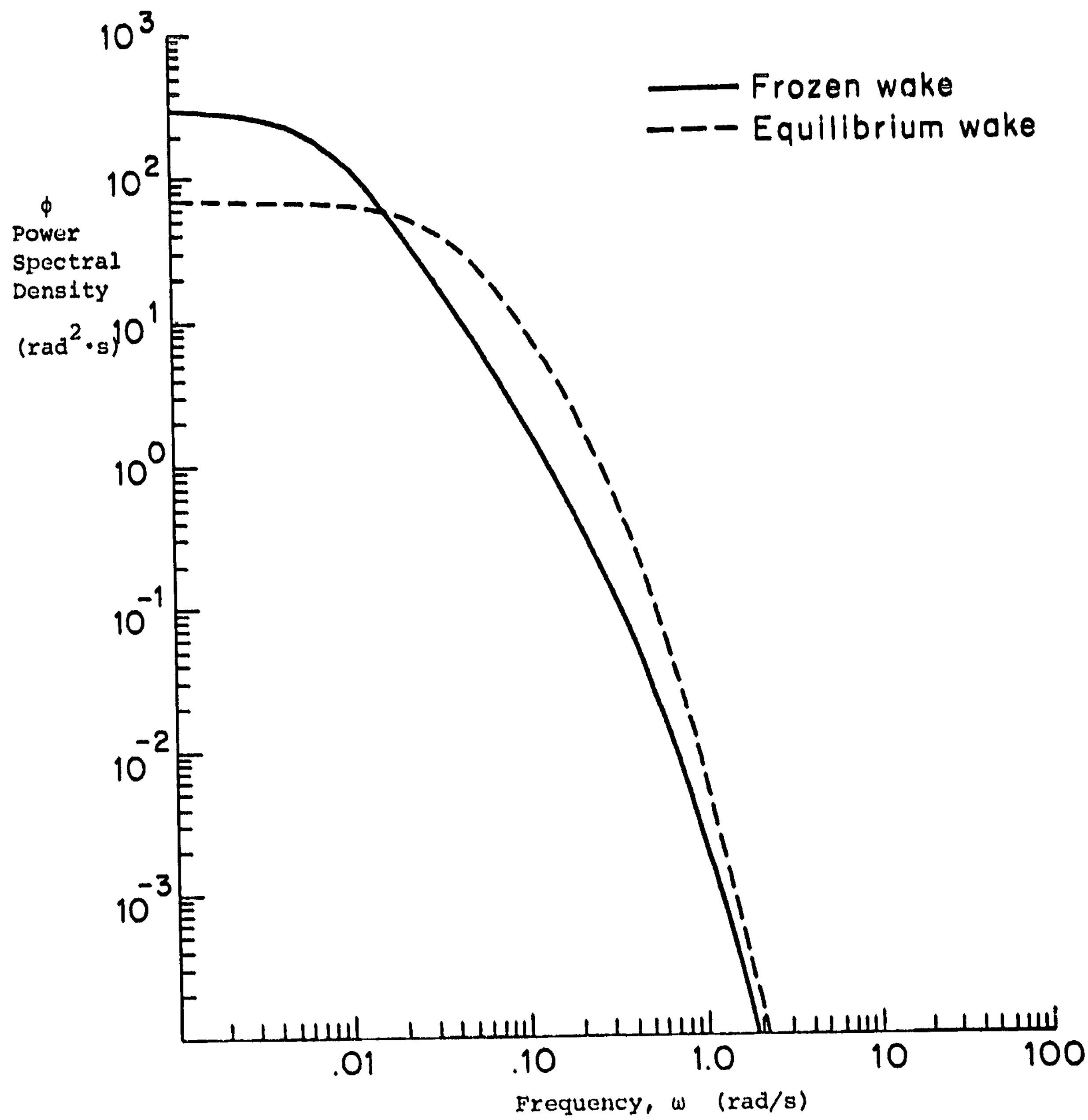

Figure 3. Yaw Response, ϕ, for Mod-M.

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Figures 4 and 5 show similar results for the Mod-G machine except that the tower torsion load is shown instead of the yaw angle for this fixed yaw machine. The Mod-G machine shows a greater sensitivity to the 3Ω effects of the ε_r and γ_r turbulence terms.

METHODOLOGY FOR COMPUTATION OF RESPONSE STATISTICS

This section gives a brief discussion of the techniques by which the model given by Eq. (4) can be used to compute desired response statistics. Assuming that the fluctuating components of the atmospheric turbulence are adequately described by Gaussian statistics [6,7], the model will give the conditional probability density function of the response given the steady wind speed V_w, and the turbulence parameters σ and L. Thus, considering only a single, scalar response variable

$$p(y|V_w,\sigma,L) = \frac{1}{\sigma_y\sqrt{2\pi}} e^{-1/2(y-\mu_y/\sigma_y)^2} \tag{9}$$

where $\mu_y = y_n(V_w)$ = the steady response for given V_w

$\sigma_y = \sigma_y(V_w\,\sigma,L)$ = the rms response for given V_w, σ, and L.

This function can be recognized as the standard Gaussian density function. The conditional mean, μ_y, is a nonlinear function of V_w, and the conditional rms response, σ_y, depends nonlinearly on V_w and L and is proportional to σ. The rms response, σ_y, can be computed from the response power spectral density by the relation

$$\sigma_y^2 = \frac{1}{\pi}\int_o^\infty S_y(\omega)d\omega \tag{10}$$

The response variance σ_y^2 can also be calculated directly using the relation [8]

$$\sigma_y^2 = [C][M][P][M]^{*T}[C]^T \tag{11}$$

where [C] = the row matrix relating the response to the system state vector.

[M] = modal matrix with column eigenvectors.

* = complex conjugate of the matrix.

The Hermitian matrix, [P], satisfies the linear relation

$$[\Lambda][P] + [P][\Lambda]^* + [M]^{-1}[B][B]^T[M]^{-*T}\left(\frac{\sigma^2 L}{V_w^3}\right) = 0 \tag{12}$$

where [Λ] = the diagonal matrix of complex eigenvalues.

[B] = the white noise input distribution matrix.

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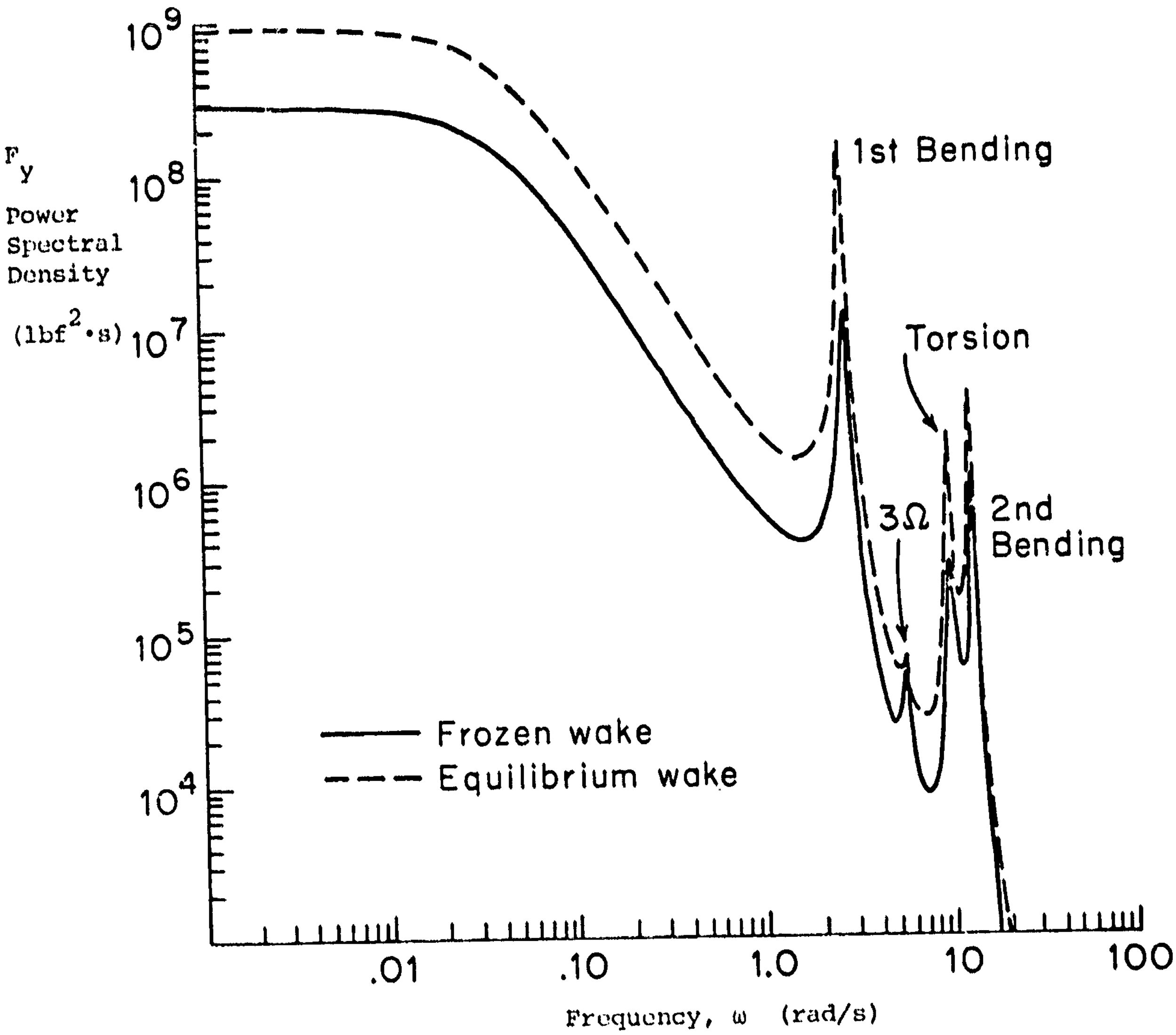

Figure 4. Thrust Load, F_y, for Mod-G.

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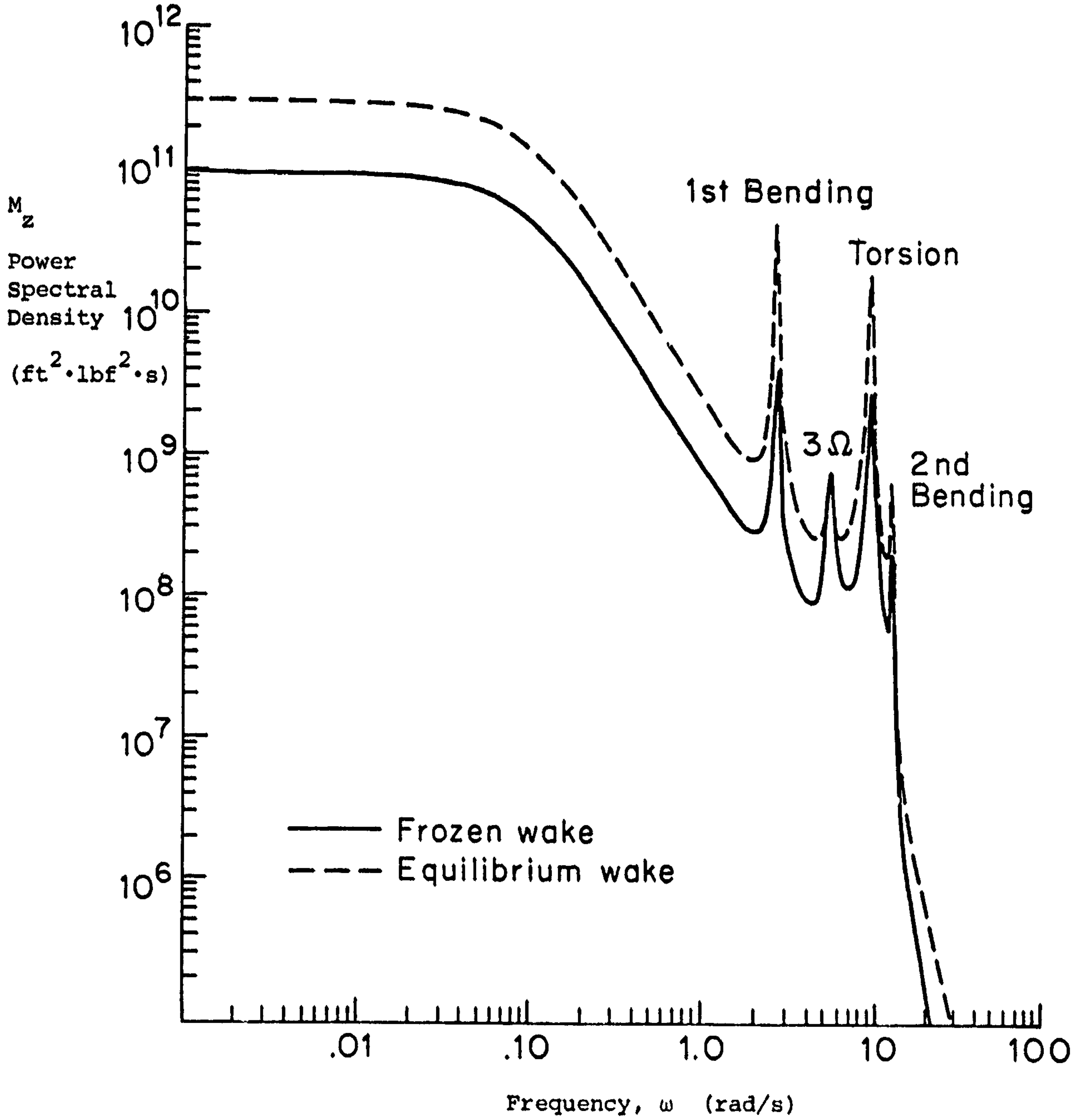

Figure 5. Tower Torsion Load, M_z, for Mod-G.

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Note that the matrices [B], [M], and [A] depend nonlinearly on the parameters V_w and L.

Now, suppose it is desired to compute the probability that y exceeds a certain critical value y_c. The conditional probability is given by

$$\Pr\{y > y_c | V_w, \sigma, L\} = \int_{y_c}^{\infty} p(y|V_w, \sigma, L)\,dy \tag{13}$$

Substituting Eq. (9) into Eq. (13) yields

$$\Pr\{y > y_c | V_w, \sigma, L\} = \frac{1}{2} - \mathrm{erf}\left(\frac{y_c - \mu_y}{\sigma_y}\right) \tag{14}$$

where $\mathrm{erf}(\cdot) = \frac{1}{\sqrt{2\pi}} \int_0^{(\cdot)} e^{-y^2/2}\,dy$ = the error function.

The total probability is thus given by

$$\Pr\{y \geq y_c\} = \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} \left(\frac{1}{2} - \mathrm{erf}\left(\frac{y_c - \mu_y}{\sigma_y}\right)\right) p(V_w, \sigma, L)\,dV_w\,d\sigma\,dL \tag{15}$$

where $p(V_w, \sigma, L)$ = the joint probability density function of the positive wind and turbulence parameters.

For computational purposes, the integrals can be approximated by discrete summations, so that

$$\Pr\{y \geq y_c\} = \sum_{j,k,\ell} \left(\frac{1}{2} - \mathrm{erf}\left(\frac{y_c - \mu_y}{\sigma_y}\right)\right) p(V_{wj}, \sigma_k, L_\ell) \tag{16}$$

where the subscripts denote discrete values of the parameters associated with "counting bins." The probability required is the joint probability that V_w is in bin j, σ is in bin k, and L is in bin ℓ.

Unfortunately, complete data for determining the joint density function for the wind and turbulence parameters is generally lacking. However, several simplifying assumptions make an approximate model possible.

In an atmospheric boundary layer with neutral buoyant stability the logarithmic profile has been found to adequately model the variation of V_w with height [9]. This model is of the form

$$V_w = \frac{u_*}{0.4} \ln\left(\frac{z - z_n + z_o}{z_o}\right) \tag{17}$$

where u_* = friction velocity.
z = height above the ground.
z_n = nominal height where V_w = 0 (often zero).

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z_o = terrain roughness length.

Frost, et al. [10] recommend the Weibull probability distribution for the steady wind speed at the reference height of 10 m. Thus solving Eq. (17) for u_* when $z = z_r = 10$ m yields

$$V_w = V_r \frac{\ln\left(\frac{z-z_n+z_o}{z_o}\right)}{\ln\left(\frac{z_r-z_n+z_o}{z_o}\right)} \tag{18}$$

where V_r = V_w at the reference height.

z_r = reference height.

Since V_w and V_r are linearly related, V_w also satisfies the Weibull distribution which can be differentiated to give the density function of the form

$$p(V_w) = \frac{k}{V_o}\left(\frac{V_w}{V_o}\right)^{k-1} e^{-(V_w/V_o)^k} \tag{19}$$

where k = a site parameter (≈ 2).

$V_o = \frac{\bar{V}_w}{\Gamma(1+\frac{1}{k})}$

$\bar{V}_w$ = annual mean wind speed at the desired height.

$\Gamma(\cdot)$ = gamma function.

The annual mean wind speed at the desired height can be found from the value at the reference height by the use of Eq. (18).

The rms, turbulent component velocity, σ, is found to be highly correlated with the steady wind speed. Panofsky, et al. [11] give the relation

$$\sigma = 2.3\ u_* \tag{20}$$

so that when Eq. (17) is used for u_*,

$$\sigma = \frac{0.92}{\ln\left(\frac{z-z_r+z_o}{z_o}\right)} V_w \tag{21}$$

The turbulence integral scale, L, is much less understood. Most evidence indicates that it is independent from the steady wind speed, V_w, and the variance, σ^2. Several authors [12,13,14] recommend different power laws for the variation of integral scale with height. However, these relations are inconsistent and the experimental data exhibit wide scatter. It is highly recommended that an experimental program be undertaken to determine an appropriate height scaling law and to account statistically for the variation observed at a given height. In the interim, we will assume the integral scale is deterministic and satisfies the height relation

$$L = L_r \sqrt{\frac{z}{z_r}} \tag{22}$$

where L_r = a site parameter ($\approx$ 65 m).
z_r = reference height = 10 m.

Using these simplifying approximations for the parameter models, the statistical procedure given by Eq. (15) reduces to

$$Pr\{y \geq y_c\} = \int_0^\infty \left(\frac{1}{2} - erf\left(\frac{y_c - \mu_y}{\sigma_y}\right)\right) p(V_w) dV_w \tag{23}$$

where $p(V_w)$ is given by Eq. (19).

The quantities μ_y and σ_y will be complicated functions of V_w given by the model of the turbine response, with Eqs. (21) and (22) used for the parameters σ and L. Obviously, numerical procedures would be used to perform this computation.

ESTIMATION OF MODEL PARAMETERS FROM FIELD DATA

Since the steady wind and turbulence parameters, V_w, σ, and L, critically affect the statistics of the response, it is highly desirable to have a reliable method for extracting the parameters from real field data. One such method is the equation error method [15]. Basically, the method determines a set of parameter values which minimize the difference between the data and predicted values based on the model equations. The resulting parameters will then serve to characterize the turbulence sample observed. A whole collection of such parameter values will then give the required statistical information discussed in the previous section.

Before proceeding to give the detailed procedure for estimating the mean wind and turbulence parameters, a brief description of the equation error method will be given. Suppose we have an accurate, noise-free measurement of a random process, u, modeled by the stochastic differential equation.

$$\dot{u} = au + bw \tag{24}$$

where w = white noise with flat PSD = S_w.
a,b = model parameters.

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The measurements will be a set of N values, u(i) taken at discrete times with a constant time interval, τ, between measurements. The continuous time model can be converted to the discrete time form

$$u(i+1) = e^{a\tau} u(i) + \xi(i) \tag{25}$$

where $\xi(i)$ = a random sequence of uncorrelated values.

The variance σ_ξ^2 of $\xi(i)$ is found by matching the stationary variance of u(i) and u(t). Thus, from Eq. (25)

$$E[u^2(i+1)] = e^{2a\tau} E[u^2(i)] + E[\xi^2(i)] \tag{26}$$

which when solved yields

$$\sigma_\xi^2 \triangleq E[\xi^2(i)] = (1 - e^{2a\tau})\sigma_u^2 \tag{27}$$

From Eq. (24) (assuming $a < 0$),

$$2a\sigma_u^2 + b^2 S_w = 0 \tag{28}$$

Using Eq. (28) in Eq. (27) yields

$$\sigma_\xi^2 = (1 - e^{2a\tau})(-\frac{b^2}{2a} S_w) \tag{29}$$

Now, since u(i+1) and u(i) are linearly related and the noise term is sequentially uncorrelated, standard regression methods [16] can be used to estimate $e^{a\tau}$ and σ_ξ^2 from the data sequence. Thus, we choose the parameter, a, to minimize the estimated variance

$$\hat{\sigma}_\xi^2 = \frac{1}{N-1} \sum_{i=1}^{N-1} (u(i+1) - e^{a\tau}u(i))^2 \tag{30}$$

The product, $b^2 S_w$, is determined from Eq. (29)

$$b^2 S_w = \frac{-2a\ \hat{\sigma}_\xi^2}{1 - e^{2a\tau}} \tag{31}$$

It is impossible to estimate b and S_w separately.

With the mathematical preliminaries out of the way, let us return to the turbulence parameter estimation problem. Suppose we have two propeller type anemometers set up to measure orthogonal horizontal components of the wind. Let $v_1(i)$ and $v_2(i)$ be sequences of measurements taken from the anemometers. The first step in the procedure is to find the steady wind speed and direction. Thus, determine

$$<v_1> = \frac{1}{N} \sum_{i=1}^{N} v_1(i)$$

$$<v_2> = \frac{1}{N} \sum_{i=1}^{N} v_2(i) \tag{32}$$

Now,

$$V_w = \sqrt{<v_1>^2 + <v_2>^2}$$

$$\phi = \tan^{-1} \frac{<v_2>}{<v_1>} \tag{33}$$

The lateral and longitudinal turbulence components are thus determined from

$$V_x(i) = v_2(i) \cos\phi - v_1(i) \sin\phi$$

$$V_y(i) = v_1(i) \cos\phi + v_2(i) \sin\phi - V_w \tag{34}$$

The next step is to determine the parameter, L, using the equation error regression procedure. According to the model developed by Holley [17], the lateral and longitudinal components of the turbulence satisfy the stochastic differential equations

$$\dot{V}_x = -\frac{2V_w}{L} V_x + \frac{2V_w^2}{L} w_1$$

$$\dot{V}_y = -\frac{V_w}{L} V_y + \frac{\sqrt{2}\, V_w^2}{L} w_2 \tag{35}$$

where w_1 and w_2 are independent white noise processes with equal power spectral densities, $S_w = \sigma^2 L/V_w^3$.

Applying the equation error regression technique of Eq. (30) and normalizing each of the equation errors by the variance gives the variance estimate

$$\hat{\sigma}^2 = \frac{1}{2} \left(\frac{\hat{\sigma}_1^2}{1-e^{-4V_w\tau/L}} + \frac{\hat{\sigma}_2^2}{1-e^{-2V_w\tau/L}} \right) \tag{36}$$

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where
$$\hat{\sigma}_1^2 = \frac{1}{N-1} \sum_{i=1}^{N-1} \left(V_x(i+1) - e^{-2V_w \tau/L} V_x(i)\right)^2$$

$$\hat{\sigma}_2^2 = \frac{1}{N-1} \sum_{i=1}^{N-1} \left(V_y(i+1) - e^{-V_w \tau/L} V_y(i)\right)^2$$

The value of L is chosen to minimize $\hat{\sigma}^2$ and σ is the resulting $\hat{\sigma}$ after the minimization.

The parameter values determined by this method will characterize the particular turbulence sample observed during a given sampling period. It is expected that the values will be different for different days and times at which the data is taken. This collection of parameter values can then be used to estimate the statistics discussed in the previous section.

CONCLUSIONS

The paper has presented a modeling technique which can be used to estimate wind turbine response statistics due to atmospheric turbulence. Up to this point all of the modeling results have been theoretical. Before these techniques can be put to use by designers, it is required that they be verified using atmospheric and wind turbine field data.

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