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FLUTTER: A FINITE ELEMENT PROGRAM FOR AERODYNAMIC INSTABILITY ANALYSIS OF GENERAL SHELLS OF REVOLUTION WITH THERMAL PRESTRESS

By

Dennis J. Fallon

and

Earl A. Thornton, Principal Investigator

Final Report
For the period ending September 30, 1982

Prepared for the
National Aeronautics and Space Administration
Langley Research Center
Hampton, Virginia

Under
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FLUTTER: A FINITE ELEMENT PROGRAM FOR AERODYNAMIC INSTABILITY ANALYSIS OF GENERAL SHELLS OF REVOLUTION WITH THERMAL PRESTRESS

By

Dennis J. Fallon[1] and Earl A. Thornton[2]

SUMMARY

Documentation for the computer program FLUTTER is presented. A general discussion of the theory of aerodynamic instability with thermal prestress is given. Theoretical aspects of the finite element matrices required in the aerodynamic instability analysis are also discussed. General organization of the computer program is explained, and instructions are then presented for the execution of the program.

INTRODUCTION

The computer program FLUTTER was written in the course of research aimed at evaluating the effects of thermal prestressing on aerodynamic instability (flutter) characteristics of general shells of revolution. Specifically, the objective of the research was to compare the conventional finite element technique to the integrated finite element technique. Interested readers should refer to reference 1 for further details of the approach.

The main body of this report presents: (1) concept of aerodynamic instability, (2) finite element formulations, (3) program organization and (4) input instruction.

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CONCEPT OF AERODYNAMIC INSTABILITY

The free vibration equations of motion for a finite element analysis of shell of revolution subjected to the effect of prestressing forces and aerodynamic pressure is expressed as (2):

$$[K_e]\ \{q\} + [K_g]\ \{q\} + \lambda\ [A_e]\ \{q\} + [M]\ \{\ddot{q}\} = \{0\} \tag{1}$$

where the matrices $[K_e]$, $[K_g]$, $[A_e]$ and $[M]$ represents the first order stiffness, initial stress (geometric), aerodynamic and mass matrices, respectively. The vectors $\{q\}$ and $\{\ddot{q}\}$ represent the nodal displacements and accelerations as a function of time. The term λ represents an aerodynamic coefficient which is a function of the stagnation pressure and the Mach number. The derivation of all matrices in equation (1) is given in the following sections. It should be noted that prestressing of the shell is incorporated through the initial stress matrix.

Now assuming that the displacement varies as harmonic function of time,

$$\{q\} = \{\bar{q}\}\ e^{i\omega t} \tag{2}$$

where $\{\bar{q}\}$ is a vector of nodal displacement independent of time and ω is the natural frequency of the system, equation (1) reduces to the classical dynamic equation:

$$\left([K_e] + [K_g] + \lambda[A_e] - \omega^2\ [M]\right)\ \{\bar{q}\}\ e^{i\omega t} = \{0\} \tag{3}$$

For a solution to equation (3) to exist the determinant of the equation in the parenthesis must vanish. That is

$$\left|[K_e] + [K_g] + \lambda[A_e] - \omega^2\ [M]\right| = 0 \tag{4}$$

The objective of an aerodynamic instability analysis is to seek a set of vibration modes that are unbounded in the time domain. This criteria is achieved when the natural frequencies, ω, defined in equation (2) are complex quantities. When $\lambda=0$ in equation (4) the problem degenerates into the calculation of the in-vacuo natural frequencies of the free vibration case. The matrices $[K_e]$, $[K_g]$ and $[M]$ are symmetric, and the eigenvalues are real.

As λ is increased from zero, two of the eigenvalues approach each other and coalesce at a critical value of λ designated λ_{cr}. As the value of λ is increased beyond λ_{cr} the eiganvalues become complex conjugates. A typical plot of the natural frequencies (square root of the eigenvalues) versus the aerodynamic constant is illustrated in figure 1(a). Therefore, the value of λ_{cr} represents the onset of flutter of the shell.

As will be shown later the interpolation function used in the finite formulation will be a Fourier series. Hence, the analysis of a shell reduces to seeking the eigenvalue solution of equation (4) for each harmonic. To obtain a complete solution to a given shell, all harmonics must be searched to determine the lowest value for λ. This is illustrated in figure 1(b). Interested readers are referred to reference 3 for an expedient technique to determine the critical harmonic.

FINITE ELEMENT FORMULATION

General Remarks

The classical finite element technique involves the modeling of a large complex system by the assemblage of smaller elements (4). For the structural analyses in this report, a geometrically exact shell element was employed. Figure 2 illustrates a typical element where R_1 and R_2 are principal radii of curvature. This element has been shown to produce excellent results in the computation of natural frequencies and mode shapes for general shells of revolution (5). The interpolation functions for this element are expressed as a Fourier series in the circumferential direction and simple polynomials in the meridional direction as follows:

$$u = \sum_{n=o} U_n(s) \cos n\theta$$

$$v = \sum_{n=0} V_n(s) \sin n\theta$$

$$w = \sum_{n=0} W_n(s) \cos n\theta \qquad (5)$$

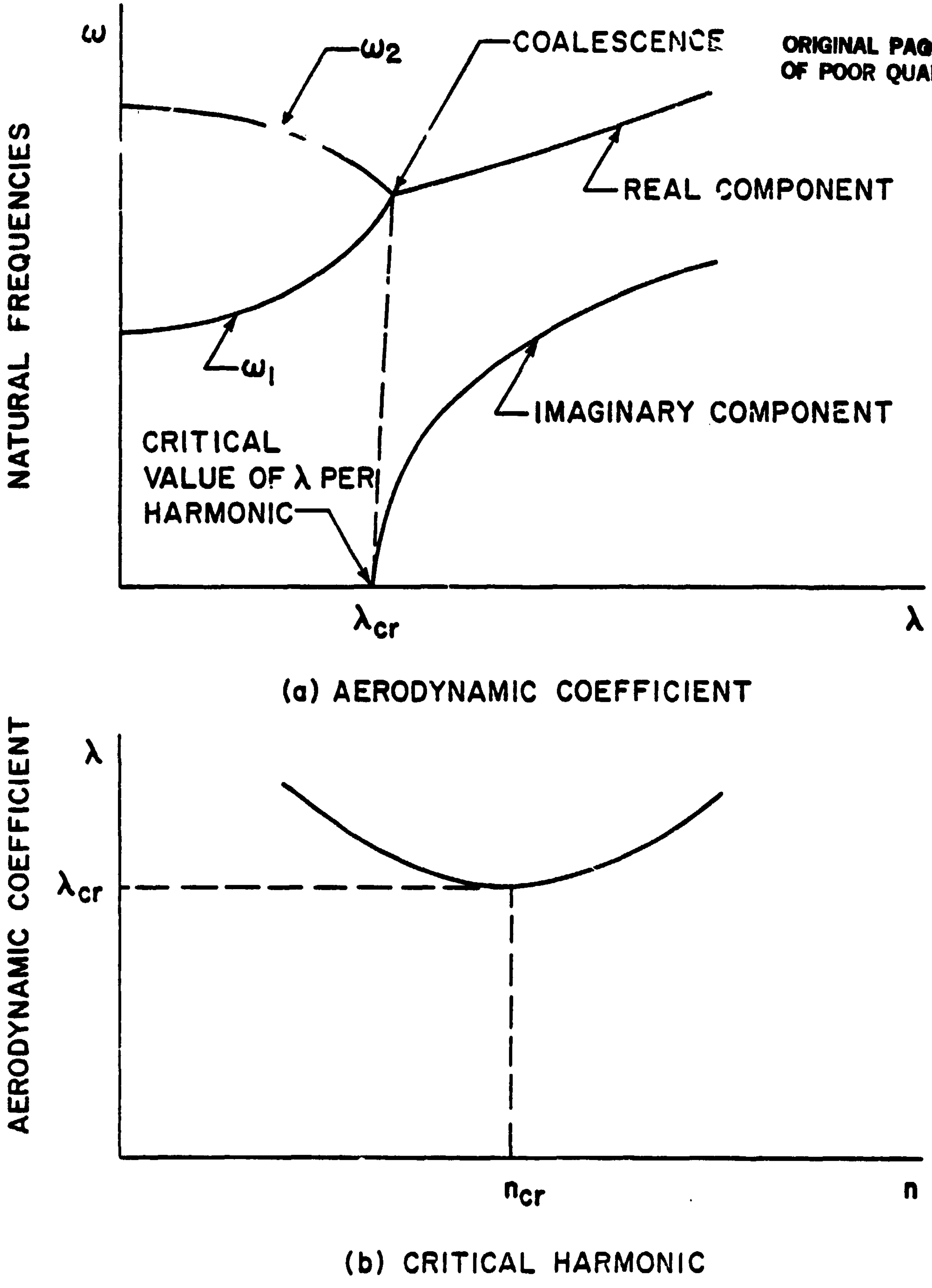

Figure 1. Evaluation of critical aerodynamic coefficient.

FINITE ELEMENT MODEL

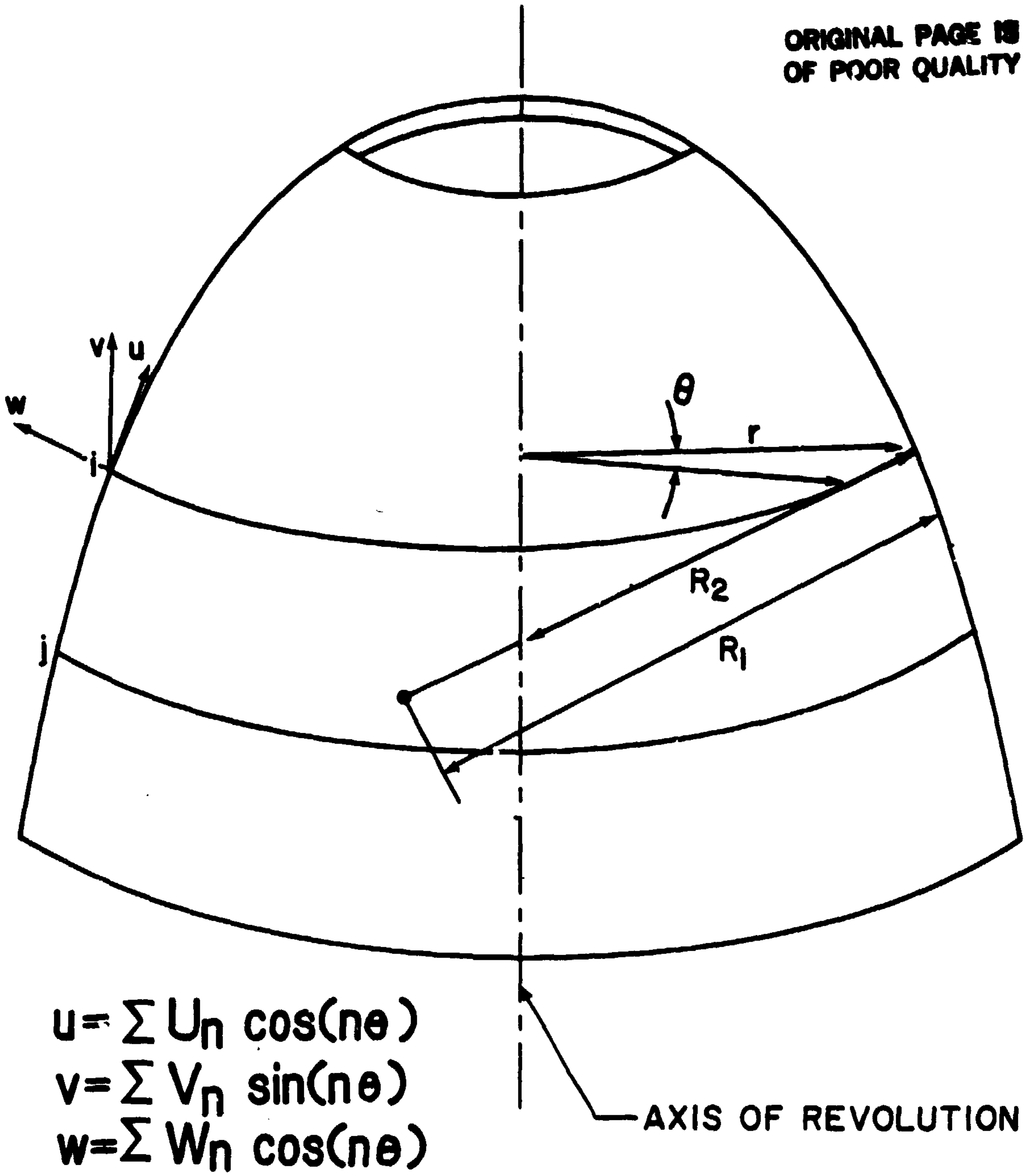

Figure 2. Typical shell element.

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where u, v, and w represent displacements in the meridional, circumferential and normal directions, respectively; U_n, V_n, W_n are simple polynomials expressed as:

$$U_n = a_{1,n} + a_{2,n}s + a_{3,n}s^2 + a_{4,n}s^3$$

$$V_n = a_{5,n} + a_{6,n}s + a_{7,n}s^2 + a_{8,n}s^3$$

$$W_n = a_{9,n} + a_{10,n}s + a_{11,n}s^2 + a_{12,n}s^3 + a_{13,n}s^4 + a_{14,n}s^5 \qquad (6)$$

where $a_{i,n}$ represents the ith generalized coordinate for the nth harmonic; s is the meridional coordinate and θ is the circumferential coordinate.

At each nodal circle designated by i and j in figure 2, there are seven degrees of freedom: w, u, v, $\left(w' - \frac{u'}{R_1}\right)$, u', v', $\left(w'' - \frac{u'}{R_1}\right)$. The prime denotes the differentiation with respect to s.

First Order Strain Energy

The first order stiffness matrix defined in equation (1) can be derived from the first variation of the first order potential energy. The potential energy for a general shell of revolution is:

$$V = \frac{1}{2} \iint (C_{11} \varepsilon_1^2 + 2C_{12} \varepsilon_1 \varepsilon_2 + C_{22} \varepsilon_2^2 + C_{66} \varepsilon_{12}^2) \, r \, d\theta ds$$

$$+ \frac{1}{2} \iint (D_{11} \kappa_1^2 + 2D_{12} \kappa_1 \kappa_2 + D_{22} \kappa_2^2 + D_{66} \kappa_{12}^2) \, r \, d\theta ds \qquad (7)$$

where ε_1, ε_2, ε_3, are first order meridional, circumferential and shearing strains according to Novozhilov shell theory (6), κ_1, κ_2, κ_{12} are first order meridional, circumferential and cross curvature according to Novozhilov shell theory; C_{kl} and D_{kl} are elastic stiffness coefficients. By use of the definitions (see Appendix A) of the strains and curvature which are explicit functions of the displacements and, with the additional use of the orthogonality properties of the sine and cosine, i.e.:

$$\int_0^{2\pi} \sin n\theta \sin m\theta d\theta = \begin{cases} 0 & m\neq n;\ m=n=0 \\ \pi & m=n \end{cases}$$

$$\int_0^{2\pi} \cos n\theta \cos m\theta d\theta = \begin{cases} 0 & m\neq 0 \\ 2\pi & m=n=0 \\ \pi & m=n\neq 0 \end{cases} \tag{8}$$

the first order stiffness matrix in terms of the nth harmonic is obtained. Exact expression for this matrix can be found in reference 5. Note that integration with respect to the meridional coordinate s was performed using a ten point Gauss scheme.

Second Order Strain Energy

A consistent initial stress (geometric) matrix which incorporates the effects of prestressing is formulated from the contribution of the second order strains in the strain energy expression. This energy is (7):

$$U_g = N_s^e \iint \epsilon_1^{(2)} r\, d\theta ds + N_{\theta\theta}^e \iint \epsilon_2^{(2)} r\, d\theta ds$$

$$+ 12 \frac{(M_{\theta\theta}^e - \nu M_{ss}^e)}{(1-\nu^2)} \iint \{[\epsilon_1 + \nu \epsilon_2]\ \beta_s + [1-\nu]\ \beta_\theta\ \epsilon_{12}\} \frac{r^2\, d\theta ds}{t^2 \cos\phi} \tag{9}$$

where N_{ss}^e, $N_{\theta\theta}^e$ are initial stresses and M_{ss}^e, $M_{\theta\theta}^e$ are initial moments due to prestressing in the meridional and circumferential directions, respectively; $\epsilon_1^{(2)}$, $\epsilon_2^{(2)}$ are second-order strains, ϕ is the slope of the shell surface in the meridional direction; and β_s, β_θ are perturbation rotations in the meridional and circumferential directions, respectively. The development of the initial stress matrix follows in a similar manner as the first order stiffness matrix. For definitions of the strains and rotations in terms of the displacements refer to Appendix A. Integration with respect to the meridional coordinate was also performed by a ten point Gauss scheme.

Kinetic Energy

The consistent mass matrix used in this study was derived from the kinetic energy of the system. Specifically, the kinetic energy is expressed as (5):

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$$E_k = \frac{\rho}{2} \iint \{\dot{u}^2 + \dot{v}^2 + \dot{w}^2\}\ t\ r\ d\theta ds \tag{10}$$

where ρ is the mass density; t is the shell thickness and a dot denotes differentiation with respect to time.

Aerodynamic Virtual Work

The aerodynamic matrix expressed in equation (1) was derived from the virtual work of the aerodynamic forces acting on the shell. These aerodynamic forces are formulated using a first order, high Mach number (>1.7) approximation to linear potential flow theory (8):

$$\delta W = - \iint \delta w \lambda \left[\frac{\partial w}{\partial s} + \frac{1}{U} \left(\frac{M_\infty^2 - 2}{M_\infty - 1}\right) \ddot{w} - \frac{w}{2\beta r}\right] r\ d\theta ds \tag{11}$$

where $\lambda = \frac{2q}{\beta}$; $\beta = (M_\infty^2 - 1)^{1/2}$; M_∞ is the freestream Mach number, q is the freestream dynamic pressure and U is the local flow velocity. For exact expression for the aerodynamic matrix refer to Appendix B.

PROGRAM ORGANIZATION

The computer program consists of four basic steps: (1) reading of input data, (2) the evaluation of element matrices, (3) the assembly of element matrix to form global matrices and (4) the evaluation of eigenvalues and eigenvectors. A flow chart of the program is illustrated in figure 3.

The input data which will be defined in more detail later in this report consists of material properties and geometry data of the particular shell. Such material properties as the modulus of elasticity and Poisson's ratio are input parameters for the stiffness calculation. The mass density is required for the evaluation of the mass matrix. These parameters are assumed to be constant throughout the shell. Some geometry data required on input is the length of each element and thickness of the shell (also assumed to be constant). At present the program is set up to do conical shells. To evaluate the flutter boundaries of other shells the subroutine Radius must be revised. Interested readers are referred to reference 5 for a detailed

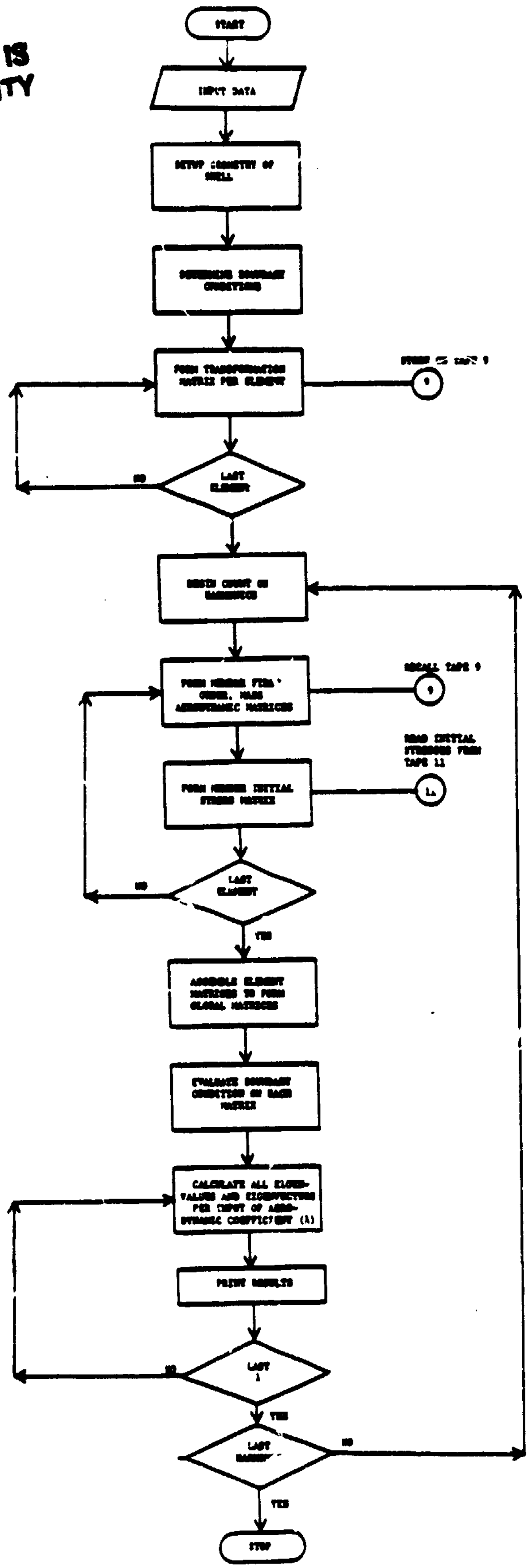

Figure 3. Flow chart FLUTTER.

procedure on how this may be done. Also an input parameter for the boundary conditions must be defined. This is done through the boundary condition code ICASE which has the interpretation shown in table 1. A free boundary condition means all displacements and rotations are released. A freely supported condition implies that only the normal displacements and the rotation are released, whereas in the clamped condition the shell is assumed to be completely fixed.

The calculation of the member element begins with the evaluation of the transformation matrix per each element. This matrix defines the relationship between the generalized coordinate (see equation) and the displacements and rotations at the ends of each element. Then per each harmonic the mass, first order stiffness, aerodynamic and initial stress matrices are calculated per element. Internal stresses are calculated at each integration point by a thermal stress computer program. These stresses are read off Tapell by FLUTTER.

After element matrices are formed they are superimposed to form the global matrices. Rows and columns are eliminated according to the applicable boundary condition specified by ICASE. Then using a NASA/Langley Computer Center subroutine complex eigenvalues and eigenvectors are computed per each specified value of aerodynamic coefficient. This procedure is repeated per each harmonic specified.

INPUT PROCEDURE

The following is the procedure per line of input data for the proper execution of the program. Note all input is free formatted.

First Line: IDEN

IDEN = any alphanumeric characters to define a particular run.

Second Line: K, NBEG, NLAST, ICASE

K = number of element;
NBEG = beginning harmonic;
NLAST = stopping harmonic;
ICASE = boundary condition code.

Third Line: SO

SO = origin of shell's coordinate system

Table 1. Boundary Conditions Codes

Value ICASE	Type of Boundary Conditon
1	Free - Free
2	Free - Freely Supported
3	Freely Supported - Free
4	Free - Simply Supported
5	Simply Supported - Free
6	Free - Clamped
7	Clamped - Free
8	Freely Supported - Freely Supported
9	Simply Supported - Simply Supported
10	Clamped - Clamped
11	Freely Supported - Simply Supported
12	Freely Supported - Clamped
13	Simply Supported - Freely Supported
14	Simply Supported - Clamped
15	Clamped - Freely Supported
16	Clamped - Simply Supported

Fourth Line: E(I) I = 1, K

E(I) = length of the ith element; K values required on this line

Fifth Line: YOUNG1, YOUNG2, XMU1, XMU2, RHO, TH, G12

YOUNG1 = Young's Modulus in meridional direction;
YOUNG2 = Young's Modulus in circumferential direction;
XMU1 = Poisson's ratio in meridional direction;
XMU2 = Poisson's ratio in circumferential direction;
RHO = mass density of shell;
TH = thickness of shell;
G12 = shear modulus of shell

Sixth Line: MACH, NLAMB, IBUCK

MACH = reference Mach number;
NLAMB = number of aerodynamic coefficients;
IBUCK = 0 (aerodynamic stability problem)
= 1 (bifurcation buckling loads - eigenvalues represent buckling loads).

Seventh Line: LAMB

LAMB = aerodynamic coefficient (This line will be repeated NLAMB times).

CONCLUDING REMARKS

A finite element program for the computation of the aerodynamic instability of general shells of revolution with thermal prestress is described. The theoretical formulation of the finite element matrices is discussed and input instructions for execution of the program are given. Application of the program are presented in reference 1.

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APPENDIX A: EQUATION FOR STRAINS AND ROTATION

Definition of strains and rotations according to Novozhilov shell theory.

$$\varepsilon_1 = u' + \frac{w}{R_1}$$

$$\varepsilon_2 = \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{r'}{r}u + \frac{w}{R_2}$$

$$\varepsilon_{12} = \frac{1}{r}\frac{\partial u}{\partial \theta} + v' - \frac{r'}{r}v$$

$$\kappa_1 = -w'' + \frac{1}{R_1}u' - \frac{R_1}{R_1^2}u$$

$$\kappa_2 = -\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} + \frac{1}{r R_2}\frac{\partial v}{\partial \theta} - \frac{r' w'}{r} + \frac{r'}{r R_1}u$$

$$\kappa_{12} = -\frac{1}{r}\frac{\partial^2 w}{\partial s \partial \theta} + \frac{1}{r^2} r' \frac{\partial w}{\partial \theta} + \frac{1}{r R_1}\frac{\partial u}{\partial \theta} + \frac{v'}{R_2} - \frac{r' v}{r R_2}$$

$$\varepsilon_1^{(2)} = \frac{1}{2}\left[\frac{\partial w}{\partial s} - \frac{u}{R_1}\right]^2 + \frac{1}{8 r^2}\left[\frac{\partial u}{\partial \theta} - vr' - r\frac{\partial v}{\partial s}\right]^2$$

$$\varepsilon_2^{(2)} = \frac{1}{2 r^2}\left[\frac{\partial w}{\partial \theta} - \frac{vr}{R_2}\right]^2 + \frac{1}{8 r^2}\left[\frac{\partial u}{\partial \theta} - vr' - r\frac{\partial v}{\partial s}\right]^2$$

$$\beta_s = -\left[\frac{\partial w}{\partial s} - \frac{u}{R_1}\right]$$

$$\beta_s = -\frac{1}{r}\left[\frac{\partial w}{\partial \theta} - \frac{vr}{R_2}\right]$$

where

(') denotes differentation with respect to s.

APPENDIX B: AERODYNAMIC COEFFICIENTS

Aerodynamic coefficients in terms of generalized coordinates.

$$[A_e] = \int_{-E/2}^{E/2} r \begin{bmatrix} 0 & 1 & 2\,s & 3\,s^2 & 4\,s^3 & 5\,s^4 & \\ 0 & s & 2\,s^2 & 3\,s^3 & 4\,s^4 & 5\,s^5 & 0 \\ 0 & s^2 & 2\,s^3 & 3\,s^4 & 4\,s^5 & 5\,s^6 & \\ 0 & s^3 & 2\,s^4 & 3\,s^5 & 4\,s^6 & 5\,s^7 & \\ 0 & s^4 & 2\,s^5 & 3\,s^6 & 4\,s^7 & 5\,s^8 & \\ 0 & s^5 & 2\,s^6 & 3\,s^7 & 4\,s^8 & 5\,s^9 & \\ & & 0 & & & & 0 \end{bmatrix} ds$$