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Simplified Composite Micromechanics Equations for Hygral, Thermal and Mechanical Properties

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ABSTRACT

A unified set of composite micromechanics equations of simple form is summarized and described. This unified set can be used to predict unidirectional composite (ply) geometric, mechanical, thermal and hygral properties using constituent material (fiber/matrix) properties. This unified set also includes approximate equations for predicting (1) moisture absorption; (2) glass transition temperature of wet resins; and (3) hygrothermal degradation effects. Several numerical examples are worked-out to illustrate ease of use and versatility of these equations. These numerical examples also demonstrate the interrelationship of the various factors (geometric to environmental) and help provide insight into composite behavior at the micromechanistic level.

1.0 INTRODUCTION

Hygral, thermal and mechanical properties of unidirectional composites are fundamental to analysis/design of fiber composite structures. Though some of these properties are determined by physical experiments, several of them are not readily amenable to direct measurement by testing. In addition, testing is usually time consuming, costly, and composites with specific configuration must have been made prior to testing. Furthermore, parametric studies of the effect of fiber volume ratio on properties such as impact resistance, and fracture toughness can be made only by conducting an extensive series of tests. Another approach is the use of composite micromechanics to derive equations for predicting composite properties based on constituent (fiber and matrix) properties. Over the last twenty years, composite micromechanics has been used to derive equations for predicting selected composite properties. However, many of these equations are not of simple form and are not readily available since equations for different properties are scattered throughout the literature (refs. 1, 2, and 3). In addition, recent developments on hygrothermal effects (ref. 4, 5, and 6) provide simple equations for estimating moisture absorption, glass transition temperature of wet resins and hygrothermal degradation. It is timely, therefore, to provide a summary of the various micromechanics equations.

Herein, a unified set of composite micromechanics equations is summarized and described. The set includes simple equations for predicting ply geometric, mechanical, thermal, and hygral properties using constituent material properties. In addition, the set includes simple equations for predicting the moisture absorption, the glass transition temperature of the wet resin and the degradation effects due to hygrothermal environments. The description consists of the significance of the participating variables in the equations and several numerical examples.

The equations for each subset (geometry, mechanical properties, thermal, moisture absorption and hygrothermal degradation effects) are summarized in chart form (labeled figures). This allows the equations for each subset to be on one page for convenience of use and identification of interrelationships. Constituent material properties used in the numerical examples are tabulated and identified with the same symbol used in the equations. The numerical examples are described in narrative form, rather than tabular, in order to conserve space. The symbols used are summarized in the Appendix for convenience of reference.

Many of the equations included in this composite micromechanics unified set appear in their present simplified form for the first time. Also, this is the first unified set which provides a quantified description of composite behavior at the micromechanistic level.

2.0 COMPOSITE MECHANICS - DEFINITIONS AND CONSTITUENT MATERIALS

The branch of composite mechanics which provides the formal structure to relate ply properties to constituent properties is called composite micromechanics. Composite micromechanics is identified concisely in the schematic in figure 1. The schematic in this figure defines the inputs to composite micromechanics and the outputs. The inputs consist of constituent material properties (fiber/matrix), geometric configuration, and the fabrication process. The outputs consist of ply thermal properties, ply mechanical properties and ply hygral properties.

The formal structure of composite micromechanics (concepts, math-models and equations) is developed based on certain assumptions (consistent with the physical situation) and the principles of solid mechanics. The two main assumptions made in deriving the equations described herein are: (1) the ply resists in-plane loads as depicted schematically in figure 2, and (2) the ply and its constituents behave linearly elastic to fracture as is illustrated in figure 3. Though the principle of solid mechanics can be used with various levels of mathematical sophistication, the mechanics of materials was used in deriving the equations summarized herein because it leads to explicit equations of simple form for each property.

Properties along the fiber direction (1-axis, fig. 2) are conventionally called longitudinal; those transverse (2-axis, fig. 2) to the fiber direction are called transverse; the in-plane shear is also called intralaminar shear (1-2 plane, fig. 2). All ply properties are defined with respect to the ply material axes denoted by 1, 2, and 3 in figure 2 for description/analysis purposes. Most ply properties are denoted by a letter with suitable subscripts. The subscripts are selected to identify type of property (ply, fiber, matrix), plane, direction, and sense in the case of strengths. For example, S_{l11T} denotes ply longitudinal tensile strength while S_{fT} denotes fiber tensile strength. Though this notation may seem cumbersome, it is necessary to properly differentiate among the multitude of ply and constituent properties.

A variety of fibers have been used to make composites. Some of these are summarized in Table 1 with their respective properties needed for composite micromechanics. Similarly, some typically matrix resins are summarized in Table 2.

3.0 GEOMETRIC RELATIONSHIPS

Several geometric relationships are important in composite micromechanics. These range from constituent material volume ratios to interfiber spacing. A schematic of a ply that can be used to derive equations for various geometric relationships is illustrated in figure 4. Micromechanics equations for some geometric relationships including density (ρ) are summarized in figure 5. Note that the fiber diameter and the fiber volume ratio are important parameters (variables). Oftentimes the amount of fiber or resin in the composite is given in terms of weight percent. The weight percent can be used to determine the fiber or resin volume ratio from the equations. These equations are expressed in terms of weight ratio (λ_m or λ_f) and constituent densities ρ_m and ρ_f which are normally known.

The equations for the geometric relationships summarized in figure 5 can be used in a number of ways.

Example 3.1. The number of fibers through a ply thickness can be determined by solving the ply thickness (t_l) equation for N_f. Using representative values for a graphite fiber/resin composite: $t_l = 0.005$ in, $d_f = 0.0003$ in, and $k_f = 0.57$, N_f equals 14 fiber through the ply thickness which is relatively large considering the small ply thickness. The interfiber spacing (δ_l) is determined from its respective equation, figure 5. The interply (matrix layer) thickness can be determined from the interfiber spacing (δ_l) equation.

Example 3.2. The interply thickness of a graphite fiber/resin composite is needed. This composite has 40 percent resin by weight and 2 percent voids. The fiber volume ratio and the fiber diameter are needed to determine the interply thickness. Assume the fiber density to be 0.063 lb/in^3 and hat for the resin to be 0.044 lb/in^3. In addition, the fiber diameter (d_f) is 0.0003 in and the resin weight ratio (λ_m) is 0.4 and $\lambda_f = 0.6$ ($\lambda_m + \lambda_f = 1.0$). Using these values the fiber volume ratio is 0.5 and the interply thickness is 0.000076 in. This thickness is about 25 percent of the fiber diameter and about 1.5 percent of the ply thickness (0.005 in). Obviously the interply thickness is very thin. Use of the equations in figure 5 with other examples is instructive and provides insight/appreciation of the interrelationships of the various composite micromechanics geometric variables.

4.0 MECHANICAL PROPERTIES - ELASTIC CONSTANTS

The simple equations for relating ply elastic constants (moduli and Poisson's ratios) are summarized in figure 6. Note that the properties in the third direction are the same as those in the second because they were obtained by assuming that the ply is transversly isotropic in the 2-3 plane. Therefore, a total of five independent elastic constants are required (E_{l11}, E_{l22}, G_{l12}, G_{l23}, and ν_{l12}). In order to use the equations in figure 6, the fiber volume ratio (k_f), five fiber properties (E_{f11}, E_{f22}, G_{f12}, G_{f23} and ν_{f12}) and two matrix properties (E_m, and ν_m) are needed. The matrix shear modulus is related to E_m and ν_m by $G_m = E_m/2\,(1 + \nu_m)$ since the matrix is assumed to be isotropic.

The void effects are incorporated into these equations by using the fiber and matrix volume ratios (k_f and k_m) predicted by the appropriate equations in figure 5. This is illustrated by the following example:

Example 4.1. Determine the longitudinal transverse and shear moduli of a plymade from AS-graphite fiber/epoxy-resin composite with 70 percent fibers by weight and one (1) percent void volume ratio. The fiber volume ratio for this case is 0.624 (k_f eq., fig. 5) 0.624. Using this value for k_f and respective values for AS fibers from Table 1 and for matrix (IMLS) from Table 2, yields $E_{\ell 11} = 19.3 \times 10^6$ psi, $E_{\ell 22} = 1.23 \times 10^6$ psi, and $G_{\ell 12} = 0.63 \times 10^6$ psi. The corresponding values for zero voids are: $k_f = 0.630$, $E_{\ell 11} = 19.7 \times 10^6$ psi, $E_{\ell 22} = 1.24 \times 10^6$ psi and $G_{\ell 12} = 0.64 \times 10^6$ psi. This example illustrates the interrelationships of the various equations as well as the negligible effect of 1 percent voids on these properties.

Example 4:2 determine the $G_{\ell 23}$ shear modulus $\nu_{\ell 12}$ and $\nu_{\ell 23}$ for the same composite and "zero" voids. Again, with $k_f = 0.63$ and respective values from Table 1 for the fiber and Table 2 for the matrix, $G_{\ell 23} = 0.46 \times 10^6$ psi and $\nu_{\ell 12} = 0.278$. The $\nu_{\ell 23}$ value is determined from its respective equation in figure 6. Using the previously calculated values $E_{\ell 22} = 1.24 \times 10^6$ psi and $G_{\ell 23} = 0.46 \times 10^6$ psi yield $\nu_{\ell 23} = 0.348$. This value is greater than the value for $\nu_{\ell 12} = 0.278$. The examples just described illustrate the ease with which the various properties can be determined. This is more significant for $G_{\ell 23}$ and $\nu_{\ell 23}$ which are needed for any three dimensional analysis including finite element.

5.0 THERMAL PROPERTIES

The simple equations for predicting the ply thermal properties from constituent properties are summarized in figure 7. The thermal properties in this figure include heat capacity (C_ℓ), heat conductivity (K_ℓ) and thermal expansion coefficient (α_ℓ). All these thermal properties are expressed in terms of the respective constituent properties, the fiber volume ratio (k_f), the matrix volume ratio (k_m), the void volume ratio (k_v) and the heat conductivity of the air (K_v). The thermal expansion coefficients are also related to ply properties ($E_{\ell 11}$ and $E_{\ell 22}$).

The following examples illustrate the use of these equations.

Example 5.1. Calculate the transverse heat conductivity of S-Glass fiber/epoxy composite with four percent voids and 75 percent fiber by weight. The properties of the S-Glass fiber are obtained from Table 1 and those for the matrix (IMLS) from Table 2. Using densities from these tables and the appropriate equations in figure 5, $k_f = 0.58$ and $k_m = 0.38$. Using the appropriate equations in figure 7 with $K_v = 0.225$ Btu/hr/ft^2/$^\circ$F/in, and $K_m = 1.130$ Btu/hr/ft^2/$^\circ$F/in, $K_{\ell 22} = 3.35$ Btu/hr/ft^2/$^\circ$F/in. The corresponding ply transverse conductivity without voids is ($k_f = 0.605$ for this case) $K_{\ell 22} = 3.90$ Btu/hr/ft^2/$^\circ$F/in. It is interesting to note that a 4 percent void Fraction decreased the transverse conductivity by about 14 percent indicating that this conductivity is very sensitive to void content.

Example 5.2. Determine the longitudinal and transverse thermal expansion coefficients for the same S-glass/epoxy composite without voids. To determine these coefficients from the equations in figure 7, the ply longitudinal modulus is needed This is calculated from the appropriate equation in figure 6 using properties from Table 1 for the fiber and Table 2 for the matrix, and using the previously determined value for $k_f = 0.605$. The longitudinal modulus $E_{\ell 11} = 7.70 \times 10^6$ psi. The longitudinal thermal expansion coefficient $\alpha_{\ell 11} = 4.19 \times 10^{-6}$ in/in/$^\circ$F. The transverse thermal expansion coefficient $\alpha_{\ell 22} = 19.9 \times 10^{-6}$in/in/$^\circ$F.

6.0 HYGRAL PROPERTIES

The simple micromechanics equations for determining ply hygral properties from constituent properties are summarized in figure 8. The ply hygral properties summarized include: diffusivity (D_l) and moisture expansion coefficients. To determine ply hygral properties summarized in figure 8, the respective properties of the matrix and the fiber volume ratio are needed. The longitudinal moisture expansion coefficient (β_{l11}) depends also on the ply longitudinal modulus (E_{l11}) while the transverse (β_{l22}) depends on the transverse modulus (E_{l22}). The following examples will illustrate the use of these equations.

Example 6.1. Calculate the ply longitudinal and transverse diffusivities for an AS-graphite-fiber/epoxy composite with 35 percent epoxy (resin) by weight and "zero" voids. Using the equation in figure 5, the fiber volume ratio $k_f = 0.58$. Using the matrix diffusivity from Table 2 of $6\text{x}10^{-11}\text{in}^2/\text{sec}$, the ply longitudinal diffusivity $D_{l11} = 2.52\text{x}10^{-11}\text{in}^2/\text{sec}$, and the transverse diffusivity $D_{l22} = 1.43\text{x}10^{-11}\ \text{in}^2/\text{sec}$. The ply transverse diffusivity is about 60 percent of the longitudinal. This implies that exposed fiber ends enhance moisture absorption/desorption.

Example 6.2. Determine the ply longitudinal moisture expansion coefficient for the composite in the previous example. First the ply longitudinal modulus needs to be determined from the appropriate equation in figure 6. Using respective values for the constituents from Table 1 for the AS-graphite fiber, from Table 2 for the matrix (IMHS) and $k_f = 0.58$ (determined previously) the ply longitudinal modulus $E_{l11} = 18.19\text{x}10^6$ psi. The ply longitudinal moisture expansion coefficient $\beta_{l11} = 0.0038$ in/in/% M (percent moisture by weight).

Example 6.3. Determine the corresponding ply transverse moisture expansion coefficient. First the transverse ply modulus is needed. Using respective properties for the constituents and $k_f = 0.58$ in the appropriate equation in figure 6, the ply transverse modulus $E_{l22} = 1.17\text{x}10^6$ psi. Using the equation for the ply transverse moisture coefficient (β_{l22}) in figure 8, the respective resin properties for matrix (IMHS) from Table 2, and the above values for $E_{l22} = 1.17\text{x}10^6$ and $k_f = 0.58$, $\beta_{l22} = 0.086$ in/in/% M (percent moisture by weight).

Example 6.4. Determine β_{l22} in the above example assuming incompressible matrix. The ply density (ρ_l) is needed to perform this calculation using the equation in figure 8. Using respective constituent material densities in the ply density equation in figure 5, $\rho_l = 0.056\ \text{lb/in}^3$ and from the equation in figure 8, $\beta_{l22} = 0.201$ in/in/% M (percent moisture by weight). Thus, the ply moisture expansion coefficient of a composite with incompressible matrix is about three times greater than that with a compressible matrix.

7.0 MOISTURE ABSORPTION

The micromechanics equations for estimating moisture in the resin and composite as a function of relative humidity ratio (RHR = 1.0 for 100 percent relative humidity) are summarized in figure 9. The equations in this figure are for the moisture in the matrix (M_m) and the moisture in the ply (M_l.) To use these equations, the lineal moisture expansion coefficient of the matrix

β_m and the saturation moisture of the matrix at 100 percent relative humidity are needed. If β_m is not known, it can be estimated from the equation in figure 9 by using the wet and dry density of the matrix. The saturation moisture M_∞ at 100 percent relative humidity for the particular resin is also needed. If it is not known, $M_\infty \sim 7$ percent by weight is a reasonable approximation. The following examples illustrate use of the equations in figure 9.

Example 7.1. Determine the matrix moisture for 70 percent relative humidity exposure. Using $M_\infty = 7$ percent (assuming not known) and RHR = 0.7 in the matrix moisture equation (fig. 9) M_m = 4.9 percent by weight.

Example 7.2 Determine the ply moisture for the previous example for AS-graphite-fiber/epoxy matrix (IMLS) with 35 percent resin by weight and zero voids. Using the matrix volume ratio (k_m) equation in figure 5 and respective constituent material densities from Tables 1 and 2, $k_m = 0.42$. The corresponding ply density $\rho_l = 0.056$ lb/in^3. Using these values, M_m = 4.9 percent and respective values for the other variables in the ply moisture (M_l) equation in figure 9, M_l = 1.7 percent by weight which is about 1/3 of the moisture in the resin in the previous example.

8.0 HYGROTHERMAL EFFECTS

The equations for predicting, hygrothermal degradation effects in composites using micromechanics are summarized in figure 10. The equations in this figure are for the glass transition temperature of the wet matrix (T_{GW}), and the degraded mechanical property (P_{HTM}), and the degraded thermal property (P_{HTT}). Note that two equations are given for the glass transition temperature of the wet resin (T_{GW}). One equation is in terms of the matrix moisture content (M_m) and the other in terms of a hygrothermally degraded mechanical property (P_{HTM}). A hygrothermally degraded property can be used with the equation for P_{HTT} as well. Note also that the effects on the thermal properties are the reciprocal of those on the mechanical properties.

It is worth noting that the equations in figure 10 were obtained by curve fitting experimental data. Consequently, they should be used judiciously and cross checked with available data for a specific case. The following examples illustrate use of the equations in figure 10.

Example 8.1. Determine the ply transverse modulus of an AS-graphite-fiber/epoxy matrix (IMLS) composite with fiber volume ratio $k_f = 0.60$, "zero" voids and in a hygrothermal environment of 80 percent relative humidity and 250° F.

The moisture in the matrix M_m = 5.6 percent (eq., fig. 9 and $M_\infty = 7$ percent). The ply density $\rho_l = 0.056$ lb/in^3 (eq., fig. 5, Tables 1 and 2). The ply moisture M_l = 1.82 percent (eq., fig. 9). The glass transition temperature of the wet matrix in the ply is $T_{GW} = 353.3°$ F (eq., fig. 10 and T_{GD} from Table 2). The ratio of the degraded mechanical in situ matrix property $(P_{HTM}/P_O) = 0.543$, ($T_{GW} = 353.3°$ F, $T_{GD} = 420°$ F, $T_O = 70°$ F and $T = 250°$ F). Note that the corresponding matrix thermal property ratio $(P_{HTT}/P_O) = 1.842$, the reciprocal of 0.543. Using the degraded ratio 0.543, the corresponding degraded matrix modulus $E_m = 0.272\text{x}10^6$ psi (degradation ratio times IMLS matrix modulus, Table 2). Using this value for E_m, $k_f = 0.6$ and $E_{f22} = 2.0\text{x}10^6$ psi in the appropriate equation in figure 10, the degraded ply transverse modulus $E_{l22} = 0.822\text{x}10^6$ psi. This is a substantial reduction (about 31 percent) compared to the undegraded $E_{\ell 22} = 1.19\times10^6$ psi indicating that matrix - dominated ply properties are very sensitive to hygrothermal environments.

Example 8.2. Determine the ply transverse thermal expansion coefficient for the composite and environmental conditions of the previous example, using the equation for $\alpha_{\ell 22}$ in figure 7.

First, the changes in α_m are determined from the equation in figure 10. Using the thermal property degradation ratio $P_{HTT}/P_0 = 1.842$ (determined previously) and the reference $\alpha_m = 57\times10^{-6}$ in/in/$^\circ$F from Table 2, the degraded value of $\alpha_m = (P_{HTT}/P_0)\ \alpha_m$ (reference) or $\alpha_m = 1.84 \times (57 \times 10^{-6})$ in /in/$^\circ$F = 105×10^{-6} in/in/$^\circ$F. Next, the degraded ply longitudinal modulus is determined from the equation in figure 6 with $E_{f11} = 31\times10^6$ psi, $E_m = 0.272 \times10^6$ psi and $k_f = 0.6$, or $E_{\ell 11} = 18.71\times10^6$ psi lb/in^2 compared to 18.8×10^6 psi for the dry conditions (insignificant change as would be expected). The matrix Poisson's ratio (ν_m) is not degraded by the hygrothermal environment (ref. 6). Using the values just determined and respective constituent material properties from Tables 1 and 2 in the equation for $\alpha_{\ell 22}$ in figure 7, $\alpha_{\ell 22} = 37.7\times10^{-6}$ in/in/$^\circ$F. This is a significant change (about 68 percent) compared to $\alpha_{\ell 22} = 22.4\times10^{-6}$ in/in/$^\circ$F for the dry room temperature condition. The numerical values from these examples show that fiber dominated properties are not sensitive to hygrothermal environments while the matrix dependent properties are very sensitive.

Example 8.3. Determine the matrix glass transition temperature and moisture in a composite made from S-Glass-fiber/IMHS-epoxy with a ply transverse modulus of $E_{\ell 22} = 1.6\times10^6$ psi at 200° F and with a fiber volume ratio $k_f = 0.65$ The corresponding ply transverse modulus $E_{\ell 22} = 2.2\times10^6$ psi at room temperature. The mechanical property degradation ratio $(P_{HTM}/P_0) = (E_{\ell 22}\text{ (wet)} / E_{\ell 22}\text{ (dry)}) = (1.6/2.2) = 0.72$. Using this value together with $T = 200^\circ$ F, $T_{GD} = 420^\circ$ F, $T_0 = 70^\circ$ F and the appropriate equation in figure 10, $T_{GW} = 385^\circ$ F which is the glass transition temperature of the wet resin. Using the ratio $T_{GW}/T_{GD} = 385^\circ\text{ F}/420^\circ\text{ F} = 0.917$ in the first equation in figure 10 yields $M_\ell = 0.87$ percent by weight. The corresponding moisture in the resin only can be determined by solving the M_ℓ equation for M_m in figure 9. The desired result is $M_m = 4.23$ percent by weight. The relative humidity corresponding to this moisture, assuming $M_\infty = 7$ percent, is 60 percent. The interesting conclusion from the previous example is that considerable information about the composite behavior may be obtained with relatively little measured or known data.

9.0 DISCUSSION

The several examples presented illustrate the usefulness and advantage of having a unified set of micromechanics equations summarized in figures 5 to 10 for the mechanical, thermal and hygral properties of composites. The examples also illustrate how the various properties are interrelated. In addition they provide detailed and quantitative insight into the micromechanistic behavior of composites. Furthermore, the various equations can be selectively used to conduct parametric studies as well as sensitivity analyses to assess acceptable ranges of various constituent material, geometric, and environmental factors.

No comparisons were provided between predicted values and available measured data for any of the numerical examples described herein. This was done intentionally. The primary purpose of this report is to describe a unified set of simple, working equations and illustrate its versatility with numerous numerical examples. These examples demonstrate computational effectiveness and illustrate interrelationships of various properties at the micromechanistic level. It is highly recommended that the reader use this unified set of micromechanics equations to predict various properties of interest to him and compare them with measured data or with known values. This provides a direct approach to assess the application and limitations of these equations as well as guidelines on how to modify them.

Another important aspect of having this unified set of micromechanics equations is that they can be used to plan and guide experimental programs for maximum benefit with minimum testing. These micromechanics equations can be advantageous in a number of other ways. Many of these other ways become "self evident" after some familiarity has been obtained.

The two tables summarizing constituent material properties illustrate the amount of data needed for effective use of a unified set of micromechanics equations. The data in these tables were compiled from many sources and many values are estimates which were inferred from predicted results and curve fits. The data are included for three main reasons: (1) to illustrate that the micromechanics equations need numerous properties; (2) to bring attention to the fact that many of these properties have not been measured and, hopefully, stimulate enough interest to develop experimental methods to measure them; and (3) to provide indicative ranges of properties of both fibers and matrices. It cannot be overemphasized that the data should be considered dynamic in the sense that they should be continuously modified if better values are known or become available.

Lastly, the unified set of micromechanics equations described herein, in conjunction with laminate theories (ref. 7), can be used to generate all the ply hygrothermomechanical properties needed to perform thermal and structural analysis of fiber composite structures.

10.0 SUMMARY

A unified set of micromechanics equation of simple form is summarized and described. These micromechanics equations are for predicting ply (unidirectional composite) geometric, mechanical, thermal and hygral properties using respective constituent material properties. Equations are also included for mositure absorption predictions, for the glass transition temperature of the wet resin and for hygrothermal degradation effects. Several numerical examples are worked out to illustrate ease of use of these equations. The examples were selected, in part, to demonstrate the interelationships of the various geometric and material parameters as well as to provide insight into composite micromechanistic behavior. Furthermore, the examples illustrate the effectiveness of the equations of this unified set in predicting several properties from minimal test data. This unified set of micromechanics equations makes it possible to generate all the ply material properties needed for inputs to laminate theories and to structural analysis of composite structures subjected to hygrothermomechanical environments.

APPENDIX - SYMBOLS

c	heat capacity
D	diffusivity
d	diameter
E	modulus of elasticity
G	shear modulus
K	heat conductivity
k	volume ratio
M	moisture - percent by weight
N_f	number of filaments per roving end
P	property
RHR	relative humidity ratio
S	strength
s	filament spacing, fig. 4
T	temperature
t	thickness
x,y,z	structural reference axes
1,2,3	ply material axes
α	thermal expansion coefficient
β	moisture expansion coefficient
δ	interfiber, interply spacing
$\mathcal{E}, \varepsilon$	fracture strain, strain
θ	ply orientation angle
λ	weight percent
ρ	density
δ	stress

Subscripts:

f	fiber property
C	compression property
D	dry property
G	glass-transition
l	ply property
m	matrix property
S	shear
T	tension
v	void
W	wet
0	reference property, temperature
∞	saturation
1,2,3	direction corresponding to 1,2,3 ply material axes

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TABLE I. - FIBER PROPERTIES

Name	Symbol	Units	Boron	Hms	AS	T300	KEV	S-G	E-G
Number of fibers/end	Nf	-	1	10000	10000	3000	580	204	204
Fiber diameter	df	in.	0.0056	0.0003	0.0003	0.0003	0.00046	0.00036	0.00036
Density	ρf	lb/in^3	0.095	0.070	0.063	0.064	0.053	0.09	0.090
Long. tensile modulus	Ef11	106psi	58	55.0	31.0	32.0	22	12.4	10.6
Transv. tensile moduls	Ef22	106psi	58	0.90	2.0	2.0	0.6	12.4	10.6
Long. shear modulus	Gf12	106psi	24.2	1.1	2.0	1.3	0.22	5.17	4.37
Transv. shear modulus	Gf23	106psi	24.2	0.7	0.8	0.7	0.22	5.17	4.37
Long. Ppisson's ratio	νf12	-	0.20	0.20	0.20	0.20	0.35	0.20	0.22
Transv. Poisson's ratio	νf23	-	0.20	0.25	0.25	0.25	0.35	0.20	0.22
Heat capacity	Cf	$Btu/lb/^oF$	0.31	0.20	0.20	0.22	0.25	0.17	0.17
Long. heat cond.	K_{f11}	$Btu/hr/ft^2/{}^oF/in$	22	580	580	580	1.7	21	7.5
Transv. heat cond.	K_{f22}	$Btu/hr/ft^2/{}^oF/in$	22	58	58	58	1.7	21	7.5
Long. th. exp. coef.	α_{f11}	$10^{-6} in/in/{}^oF$	2.8	-0.55	-0.55	-0.55	-2.2	2.8	2.8
Transv. th. exp. coef.	α_{f22}	$10^{-6} in/in/{}^oF$	2.8	5.6	5.6	5.6	30	2.8	2.8
Long tensile strength	S_{ft}	ksi	600	250	300	350	400	600	40
Long compression str.	S_{fc}	ksi	700	200	260	300	75	-	-
Shear strength	S_{fc}	ksi	100	-	-	-	-	-	-

[a]Transverse, Shear and Compression Properties are Estimates Inferred from Corresponding Composite Properties

TABLE 2. - MATRIX PROPERTIES

Name	Symbol	Units	LM	IMLS	IMHS	HM	Poly-imide	PMR
Density	ρ_m	lb/in^3	0.046	0.046	0.044	0.045	0.044	0.044
Tensile modulus	E_m	10^6psi	0.32	0.50	0.50	0.75	0.50	0.47
Shear modulus	G_m	10^6psi	-	-	-	-	-	-
Poissons's ratio	ν_m	-	0.43	0.41	0.35	0.35	0.35	0.36
Heat capacity	Cm	Btu/lb/^{0}F	0.25	0.25	0.25	0.25	0.25	0.25
Heat conductivity	K_m	Btu/hr/ft^2/^{0}F/in	1.25	1.25	1.25	1.25	1.25	1.25
Thermal Exp. Coef.	α_m	10^{-6}in/in/^{0}F	57	57	36	40	20	28
Diffusivity	D_m	10^{-10}in^2/sec	0.6	0.6	0.6	0.6	0.6	0.6
Moisture exp. coef.	β_m	in/in/% M	0.33	0.33	0.33	0.33	0.33	0.33
Tensile strength	S_{mt}	ksi	8	7	15	20	15	8
Compression strength	S_{mc}	ksi	15	21	35	50	30	16
Shear strength	S_{ms}	ksi	8	7	13	15	13	8
Tensile fracture strain	ε_{mt}	in/in (%)	8.1	1.4	2.0	2.0	2.0	2.0
Compr. fracture strain	ε_{mc}	in/in (%)	15	4.2	5.0	5.0	4.0	3.5
Shear fracture strain	ε_{ms}	in/in (%)	10	3.2	3.5	4.0	3.5	5.0
Air heat conductivity	K_v	Btu/hr/ft^2/^{0}F/in	0.225	0.225	0.225	0.225	0.225	0.225
Glass trans. temp. (Dry)	T_{GD}	^{0}F	350	420	420	420	700	700

Notes: LM = Low Modulus; IMLS = Intermediate Modulus Low Strength; IMHS = Intermediate Modulus High Strength; HM = High Modulus.
Thermal, Hygral, Compression and Shear Properties are estimates only;
$G_m = E_m/2\ (1 + \nu_m)$.

CONCEPTS, MATH-MODELS AND EQUATIONS USED TO PREDICT UNIDIRECTIONAL COMPOSITE (PLY) PROPERTIES FROM CONSTITUENT MATERIAL PROPERTIES, GEOMETRIC CONFIGURATION AND FABRICATION PROCESS VARIABLES

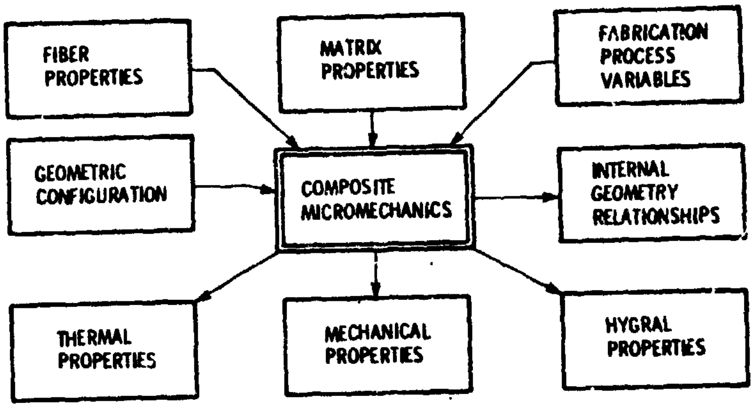

Figure 1. - Composite micromechanics definition.

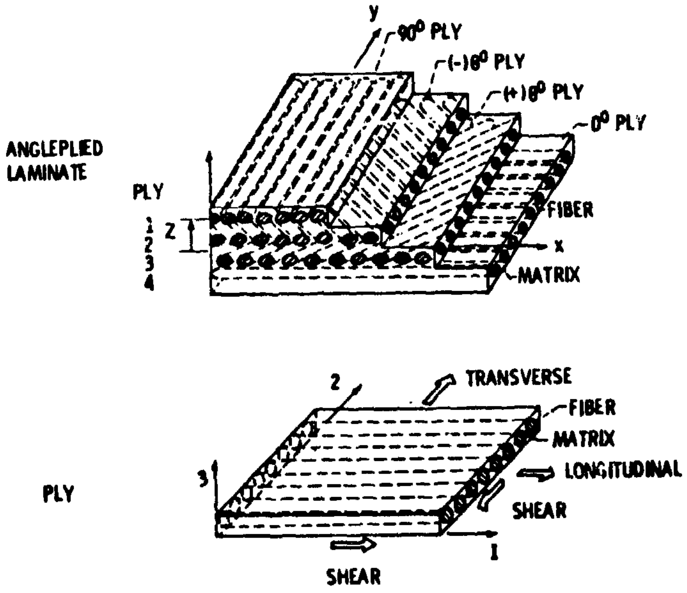

Figure 2. - Typical fiber composite geometry, plane ABCD is the reference plane.

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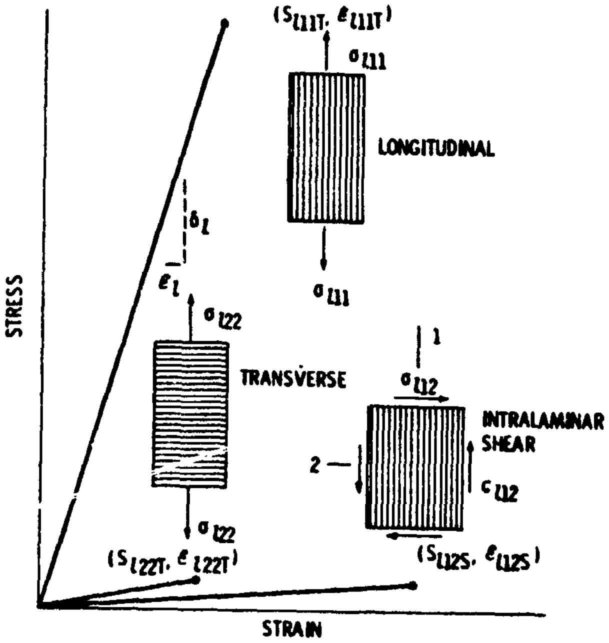


Figure 3. - Typical stress-strain behavior of unidirectional fiber composites.

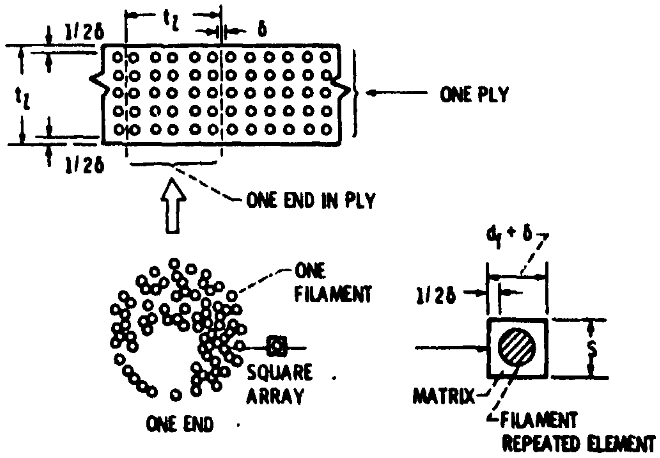


Figure 4. - Schematic of ply.

PARTIAL VOLUMES: $k_f + k_m + k_v = 1$

PLY DENSITY: $\rho_\ell = k_f \rho_f + k_m \rho_m$

RESIN VOLUME RATIO: $k_m = (1 - k_v) / [1 + (\rho_m/\rho_f)(1/\lambda_m - 1)]$

FIBER VOLUME RATIO: $k_f = (1 - k_v) / [1 + (\rho_f/\rho_m)(1/\lambda_f - 1)]$

WEIGHT RATIOS: $\lambda_f + \lambda_m = 1$

PLY THICKNESS (S. A.): $t_\ell = \left[\boxed{1/2} N_f d_f \sqrt{\pi/k_f}\right]$

INTERPLY THICKNESS: $\delta_\ell = \boxed{1/2}\left[\sqrt{\pi/k_f} - 2\right] d_f$

INTER FIBER SPACING (S. A.): $\delta_s = \delta_\ell$

CONTIGUOUS FIBERS (S. A.): $k_f = \pi/4 \sim 0.785$

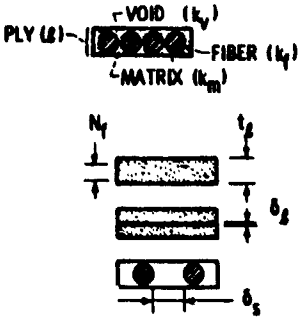


Figure 5. - Micromechanics: Geometric relationships.

LONGITUDINAL MODULUS: $E_{\ell 11} = k_f E_{f11} + k_m E_m$

TRANSVERSE MODULUS: $E_{\ell 22} = \dfrac{E_m}{1 - \sqrt{k_f}\,(1 - E_m/E_{f22})} = E_{\ell 33}$

SHEAR MODULUS: $G_{\ell 12} = \dfrac{G_m}{1 - \sqrt{k_f}\,(1 - G_m/G_{f12})} = G_{\ell 13}$

SHEAR MODULUS: $G_{\ell 23} = \dfrac{G_m}{1 - \sqrt{k_f}\,(1 - G_m/G_{f23})}$

POISSON'S RATIO: $\nu_{\ell 12} = k_f \nu_{f12} + k_m \nu_m = \nu_{\ell 13}$

POISSON'S RATIO: $\nu_{\ell 23} = \dfrac{E_{\ell 22}}{2G_{\ell 23}} - 1$

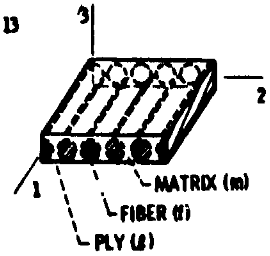
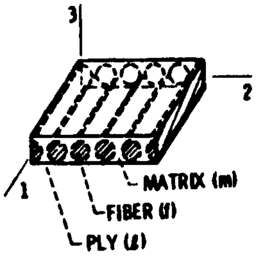


Figure 6. - Composite micromechanics, mechanical properties.



HEAT CAPACITY: $C_\ell = \frac{1}{\rho_\ell}(k_f \rho_f C_f + k_m \rho_m C_m)$

LONGITUDINAL CONDUCTIVITY: $K_{\ell 11} = k_f K_{f11} + k_m K_m$

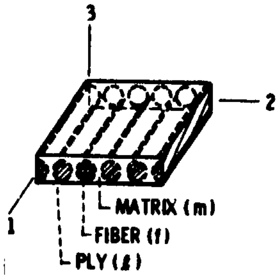
TRANSVERSE CONDUCTIVITY: $K_{\ell 22} = (1-\sqrt{k_f})K_m + \frac{K_m\sqrt{k_f}}{1-\sqrt{k_f}\,(1-K_m/K_{f22})} = K_{\ell 33}$

FOR VOIDS: $K_m = (1-\sqrt{k_v})K_m + \frac{K_m\sqrt{k_v}}{1-\sqrt{k_v}\,(1-K_m/K_v)}$

LONGITUDINAL THERMAL EXPANSION COEFFICIENT: $\alpha_{\ell 11} = \frac{k_f \alpha_{f11} E_{f11} + k_m \alpha_m E_m}{E_{\ell 11}}$

TRANSVERSE THERMAL EXPANSION COEFFICIENT: $\alpha_{\ell 22} = \alpha_{f22}\sqrt{k_f} + (1-\sqrt{k_f})(1 + k_f \nu_m E_{f11}/E_{\ell 11})\alpha_m$
$= \alpha_{\ell 33}$

Figure 7. - Composite micromechanics: Thermal properties.



LONGITUDINAL DIFFUSIVITY $D_{\ell 11} = (1-k_f)D_m$

TRANSVERSE DIFFUSIVITY $D_{\ell 22} = (1-\sqrt{k_f})D_m = D_{\ell 33}$

LONGITUDINAL MOISTURE EXP. COEF. $\beta_{\ell 11} = \beta_m (1-k_f) E_m / E_{\ell 11}$

TRANSVERSE MOISTURE EXP. COEF. $\beta_{\ell 22} = \beta_m (1-\sqrt{k_f}) \left[1 + \frac{\sqrt{k_f}(1-\sqrt{k_f})E_m}{\sqrt{k_f}E_{\ell 22} + (1-\sqrt{k_f})E_m}\right] = \beta_{\ell 33}$

FOR INCOMPRESSIBLE MATRIX

$\beta_{\ell 11} = 0$

$\beta_{\ell 22} = \beta_{mp}\, 1/2_{pm} = \beta_{\ell 33}$

Figure 8. - Composite micromechanics: Hygral properties.

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MOISTURE PICK UP IN MATRIX $M_m \approx M_\infty (RHR)$

MOISTURE PICK UP IN COMPOSITE $M_\ell = M_m (3\beta_m k_m + k_v)\rho_m/\rho_\ell$

MATRIX MOISTURE EXPANSION COEF. (LINEAL) $\beta_m \sim \left[(\rho_m/\rho_{mw}) - k_v\right]/3$

NOTATION

M_∞ = MATRIX SATURATION MOISTURE AT 100% RELATIVE HUMIDITY AND ROOM TEMP. (USE $M_\infty \approx 7\%$ BY WEIGHT IF UNKNOWN)

RHR = RELATIVE HUMIDITY RATIO (100% = 1.0)

SUBSCRIPTS

ℓ = COMPOSITE (PLY) PROPERTY

m = MATRIX

w = WET

v = VOID

Figure 9. - Composites micromechanics: Moisture absorption.

GLASS TRANSITION TEMP. OF WET RESIN

$$T_{GW} = (0.005M_m^2 - 0.10M_m + 1.0)\,T_{GD}$$

EFFECTS ON MECHANICAL PROPERTIES

$$\frac{P_{HTM}}{P_O} = \left[\frac{T_{GW} - T}{T_{GD} - T_O}\right]^{1/2}$$

EFFECTS ON THERMAL PROPERTIES

$$\frac{P_{HTT}}{P_O} = \left[\frac{T_{GD} - T_O}{T_{GW} - T}\right]^{1/2}$$

GLASS TRANSION TEMP. OF WET RESIN

$$T_{GW} = T + (T_{GD} - T_O)(P_{HTM}/P_O)^2$$

TEMPERATURE (T) ANY CONSISTENT UNITS

MOISTURE (M) WEIGHT PERCENT ($M \leq 10\%$)

SUBSCRIPTS: G = TRANSITION; D = DRY; W = WET; 0 = REFERENCE
HTM = HYGROTHERMAL MECHANICAL; HTT = HYGROTHERMAL THERMAL

Figure 10. - Governing equations: Micromechanics - Hygrothermal effects.