Geared Power Transmission Technology

John J. Coy*

Gears are the means by which power is transferred from source to application. Gearing and geared transmissions drive the many machines of modern industry. Gears move the wheels and propellers to move us through the sea, land, and air. A very sizeable section of industry and commerce in today's world depends on gearing for its economy, production, and, indeed, livelihood.

The art and science of gearing has roots in pre-Christian times, and yet many scientists and researchers continue to delve into the regions where improvement is necessary, seeking to quantify, establish, and codify the methods to make gears meet the ever widening needs of mankind.

The art and science of geared power transmission has changed over the years from the simplest of beginnings in wooden toothed gears for pumping water and turning grist mills to advanced applications in helicopters and automobiles and ocean-going vessels. Gears, which were originally conceived by men of genius and invention, now command the attention of men of no less skill. The gearing art has become interdisciplinary, spanning areas of endeavor which are too varied for any one man to master, no matter how talented he may be. Gearing art, science, and applications span the fields of kinematics, chemistry, metallurgy, heat transfer, fluid dynamics, stress analysis, vibration, and acoustics. The gear practitioner draws on a staggering array of scientific disciplines to assemble a good gear design.

It is the purpose of this paper to briefly review the historical path of the science and art of gearing. The present state of gearing technology is discussed along with examples of some of the NASA-sponsored contributions to gearing technology. Future requirements in gearing are summarized.

History of Gearing

The earliest written descriptions of gears are said to be made by Aristotle in the fourth century B.C. (ref. 1), but the oldest surviving relic containing gears is the Antikythera mechanism (fig. 1), so named because of the Greek island of that name near which the mechanism was discovered in a sunken ship in 1900. Professor Price (ref. 2) of Yale University has written an authoritative account of this mechanism. The mechanism is not only the earliest relic of gearing, but also it is an extremely complex arrangement of epicyclic differential gearing. The mechanism is identified as a calendrical sun and moon computing mechanism and dated to about 87 B.C. Because of the many difficulties surrounding attempts to authoritatively document the earliest forms of gearing, there is sometimes disagreement among authorities. It has been pointed out that (refs. 2 and 3) the passage attributed to Aristotle by some (ref. 1) was actually from the writings of his school, in *Mechanical Problems* of Aristotle (ca. 280 B.C.). In the passage in question there was no mention of gear teeth on the parallel wheels, and they may just as well have been smooth wheels in frictional contact. Therefore, the attribution of gearing to Aristotle is, most likely, false. There is more agreement that the real beginning of gearing was with Archimedes who in about 250 B.C. invented the endless screw turning a toothed wheel, which was used in engines of war. Archimedes also used gears to simulate astronomical ratios. The Archimedian spiral was continued in the hodometer and dioptra, which were early forms of wagon mileage indicators (odometer) and surveying instruments. These devices were probably "thought" experiments of Heron of Alexandria (ca. 60 A.D.), who wrote on the subjects of theoretical mechanics and the basic elements of mechanism.

For such a complex device as the Antikythera mechanism to suddenly appear gives rise to speculation regarding the origins of the gearing art. Perhaps some unknown genius invented this complex mechanism in one shot, and spin-offs from the major invention gave manifold applications in the more mundane areas such as for grist mills and raising water. Or perhaps the art of gearing as
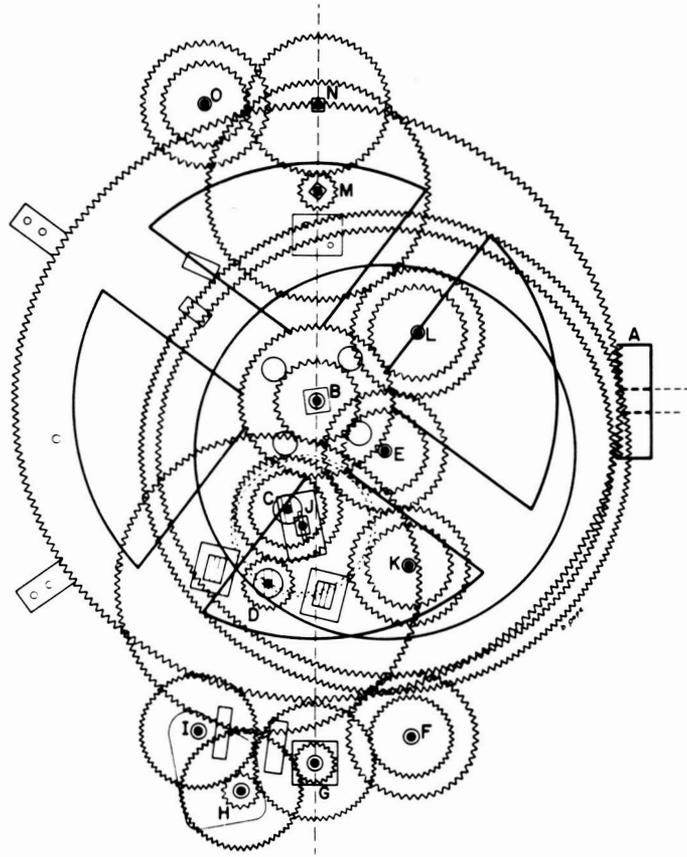
(a) General plan of all gearing, composite diagram.
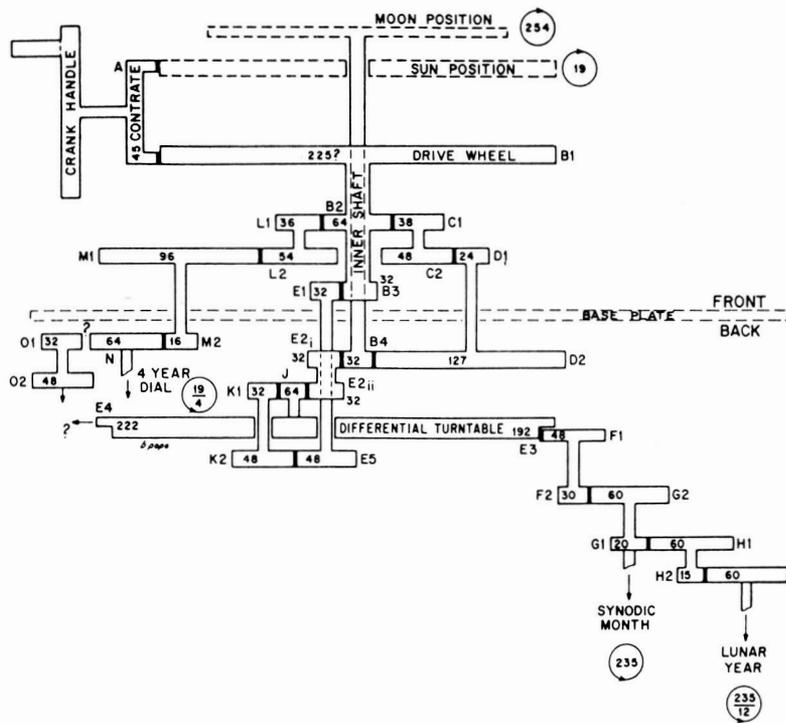
(b) Sectional diagram of complete gearing system.

Figure 1. - Antikythera mechanism (ca. 87 B.C.). (Reproduced with the permission of The American Philosophical Society.).

seen in the astronomical device and clockworks are the culmination of much evolutionary trial and error and represent the paragon of synthesis in design.

Notwithstanding, the art of gearing was carried through the European Dark Ages after the fall of Rome, appearing in Islamic instruments such as the geared astrolabes (fig. 2), which were used to calculate the positions of the celestial bodies. Perhaps the art was relearned by the clock and instrument making artisans of 14th century Europe, or perhaps some crystallizing ideas and mechanisms were imported from the East after the crusades of the 11th, 12th, and 13th centuries.

It appears that the English Abbot of St. Alban's monastery, born Richard of Wallingford, in A.D. 1330, reinvented the epicyclic gearing concept. He applied it to an astronomical clock, which he began to build at that time and which was completed after his death. Translated manuscripts clearly tell of the problems of finance, management, and skeptic's ridicule that Richard suffered as his clock was being built. Indeed, this 14th century technological project was not so different from the research and technical projects of today!

A mechanical clock of a slightly later period was conceived by Giovanni de Dondi (1348-1364), a model of which is in the Smithsonian Institution (fig. 3). Diagrams of this clock, which did not use differential gearing (ref. 2), appear in the sketchbooks of Leonardo da Vinci, who designed geared mechanisms himself (refs. 2 and 5). In 1967 two of Leonardo da Vinci's manuscripts, lost in the National Library in Madrid since 1830 (ref. 6), were rediscovered. One of the manuscripts, written between 1493 and 1497 and known as "Codex Madrid I" (ref. 7), contains 382 pages with some 1600 sketches. Included among this display of Leonardo's artistic skill and engineering ability are his studies of gearing. Among these are tooth profile designs and gearing arrangements that were centuries ahead of their "invention."

A common interest of the period was perpetual motion machines (fig. 4), many of which used gearing arrangements. Leonardo, who studied friction and understood the implications this had for negating perpetual motion, categorized the seekers of such with the alchemists who vainly sought to synthesize gold from the more basic elements. The first page of Leonardo's "Codex Madrid I" contains these observations: "Among the superfluous and impossible delusions of man there is the search for continuous motion . . . always the same thing happened to them as to the alchemists . . . because of the impossible things they promised to sovereigns and heads of state. I remember that many people, from different countries, went to Venice with great expectation of gain to make mills in dead (still) water, and after much expense and effort, unable to set the machine in motion, they were obliged to escape."

Figure 2. - Geared astrolabe of A.D. 1221-22 by Muhammed B. Abi Bakr of Isfahan. Interior mechanism is pictured. (Courtesy of The Smithsonian Institution.)
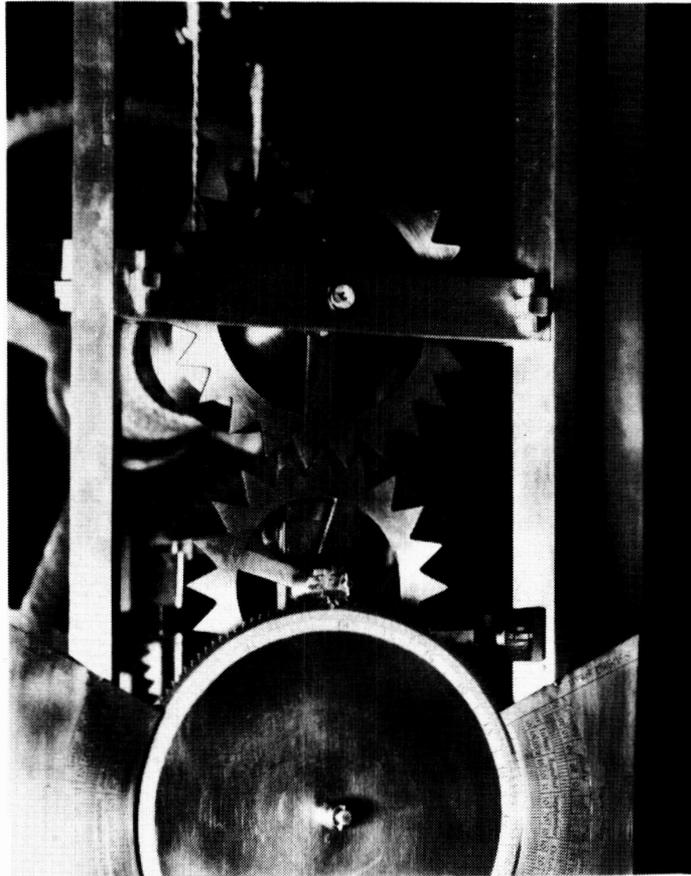
Figure 3. - Clock of Giovanni de Dondi (ca. 1364). Drive gears with triangular teeth are pictured. A modern reconstruction. (Courtesy of The Smithsonian Institution.)

In short, the gearing art and practice was certainly begun at least 2000 years ago and was preserved in the practical machines for doing work, measuring distance, and keeping time. Clearly, from the Antikythera machine onwards there was a high technology line of sophisticated gear work preserved in the craft of clockmakers all the way through history to the Industrial Revolution. Following another line from antiquity onwards in the tradition of the wheelwright and millwright, there were low technology gear pairs used in power transmission. These were mostly wooden gears with lantern pinions. There were therefore two traditions which only came together in the late 18th and early 19th centuries: clockwork gearing used for its ratio properties and rudimentary wooden gears used to transmit power.

Progress in Gearing

In the period 1450 to 1750, the mathematics of gear-tooth profiles and theories of geared mechanisms became established. Albrecht Dürer is credited with discovering the epicycloidal shape (ca. 1525). Philip de la Hire is said to have worked out the analysis of epicycloids and recommended the involute curve for gear teeth (ca. 1694). Leonard Euler worked out the law of conjugate action (ca. 1754; ref. 5). Gears, designed according to this law, have a steady speed ratio.

Gears were used in the early machines that powered the industrial revolution. Later, they were used in machines to make even more gears. Figure 5 shows a hand-powered production machine used to make clockwork gearing. This machine from the early 18th century is attributed to Christopher Polhem of Sweden.

Since the Industrial Revolution in the mid-19th century, the art of gearing blossomed, and gear designs steadily became based on more scientific principles. In 1893 (ref. 8) Wilfred Lewis published
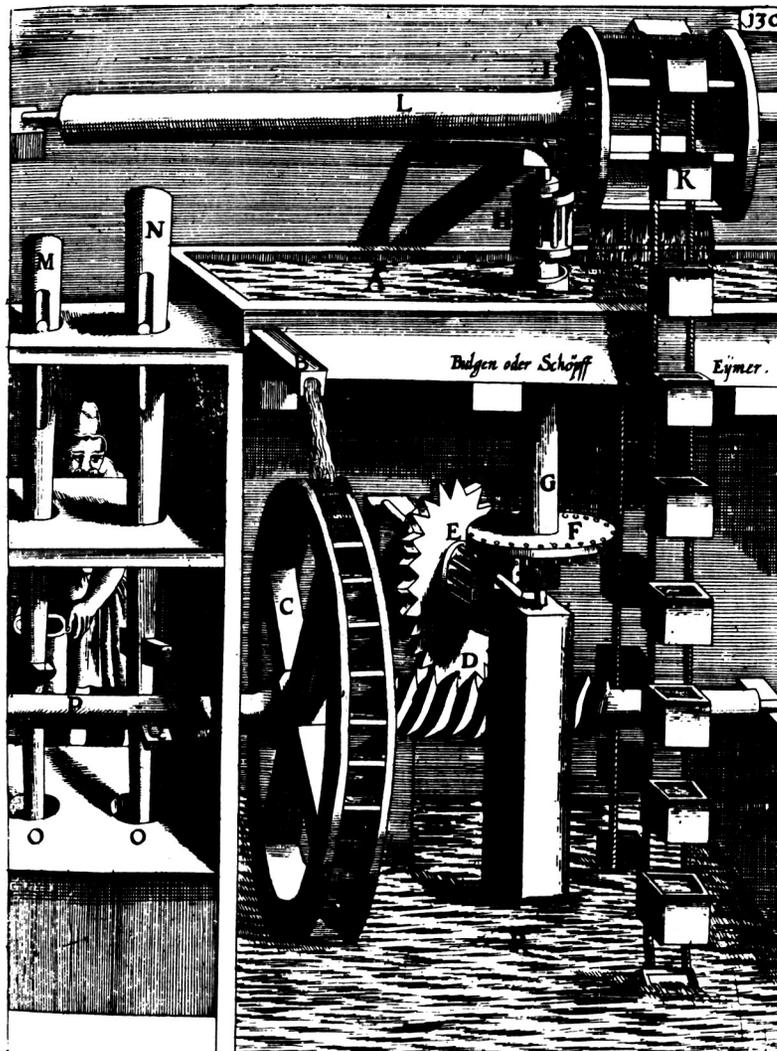
Figure 4. - Medieval perpetual motion machine driven by water wheel. Lantern
pinions drove peg-toothed wheels (E, F, H, I). The Spiral of Archimedes
(ca. 280 B.C.) was implemented as a worm drive (D, E). Cams (Q) on the
shaft lifted and dropped pounding posts (M, N) which pulverized material in
the container. Figure extracted from a book on machinery by George Andream
Bocklern published in Nuremberg in 1661. (Courtesy of The Smithsonian
Institution.)

a formula for computing stress in gear teeth. This formula is in wide use today in gear design. In 1899 George B. Grant, the founder of five gear manufacturing companies, published "A Treatise on Gear Wheels" (ref. 9).

New inventions lead to new applications for gearing. For example, in the early part of this century (1910), parallel shaft gears were introduced to reduce the speed of the newly developed reaction steam turbine enough to turn the driving screws of oceangoing vessels. This application achieved an overall increase in efficiency of 25 percent in sea travel (ref. 1).

By 1916 the age of specialized machine tools for producing gears had definitely arrived. It was in 1916 that the American Gear Manufacturers Association was founded. Figure 6 shows the staff of one of today's leading gear companies as it was in 1902, 10 years after its founding by George Grant. Figure 7 shows one of the machine tools that may have been inside this gear workshop. In today's gear shops it is common to find gears cut with machinery kept in temperature-controlled environments. Figure 8 shows a gear grinder finishing a pinion shaft such as would be found in a turbine-driven oceangoing vessel or in large rolling mill gearing.
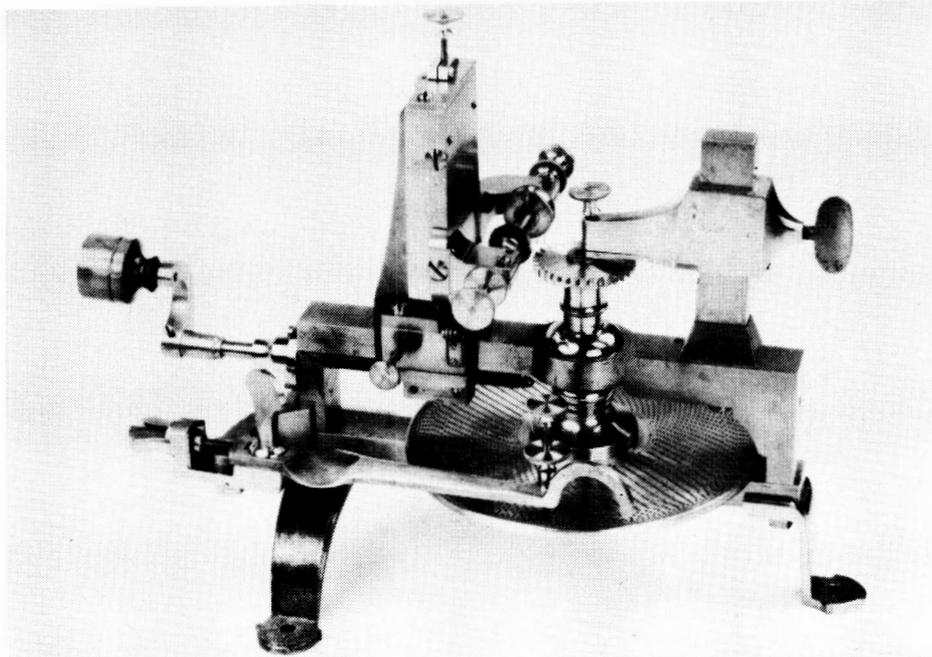
Figure 5. - Early hand-powered production machine for making clockwork gears by Chrisotopher Polhem of Sweden (ca. 1729). Index plate for spacing teeth. Courtesy of The Smithsonian Institution.)

Figure 6. - Philadelphia Gear Works in 1902, 10 years after its founding by George B. Grant. Today Philadelphia Gear Corp. is one of the leading gear producers, employing over 2000 persons. (Courtesy of the Philadelphia Gear Corp.)

The need for more accurate and quiet-running gears became obvious with the advent of the automobile. Although the hypoid gear was within our manufacturing capabilities by 1916, it was not used practically until 1926 when it was used in the Packard automobile. The hypoid gear made it possible to lower the drive shaft and gain more usable floor space. By 1937 almost all cars used hypoid geared rear axles. Special lubricant antiwear additives were formulated in the 1920's which
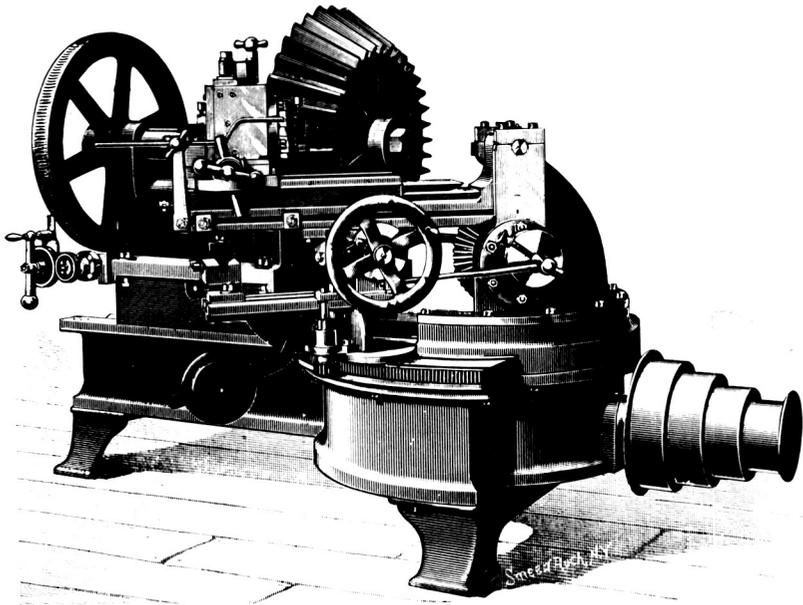
Figure 7. - Gleason bevel gear planing machine. This first commercially practical bevel gear planer was designed and built by William Gleason in 1874. (Courtesy of The Smithsonian Institution.)
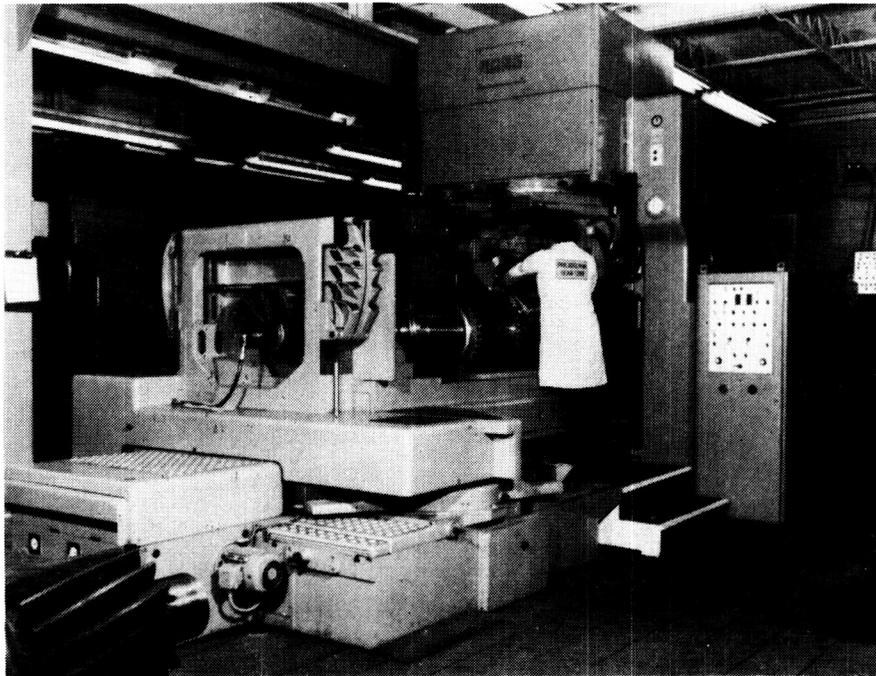

Figure 8. - Maag precision gear grinding machine, custom built to Philadelphia Gear Corp. specifications. Such machines are located in temperature and humidity controlled environments to maintain precision control of manufacture. (Courtesy of Philadelphia Gear Corp.)

made it practical to use hypoid gearing. In 1931 Earle Buckingham, a professor at the Massachusetts Institute of Technology and chairman of an ASME research committee on gearing, published a milestone report on gear-tooth dynamic loading (ref. 10). This led to a better understanding of why faster running gears sometimes could not carry as much load as slower running gears.

In the early 1900's wear was the major problem in gearing, but, with the more severe service seen in aircraft applications in the 1930's, scoring problems began to occur. Scoring is caused by a combination of high sliding speed and high contact stress. These factors cause the lubricating oil film to break down in a sudden and unstable manner. This mode of failure emphasized that, in addition to cutting down friction, a prime contribution of a lubricating fluid is to provide cooling to the gears. In 1949 (ref. 11) the first real breakthrough in the theory of lubrication of concentrated contacts, such as in gearing, occurred with Grubin's theory of elastohydrodynamic lubrication. Although lubrication of gears had been investigated since the early 1900's, Grubin connected the interaction of elastic deformations in the gears with the increase in lubricant viscosity with pressure.

High-strength alloy steels for gearing were developed during the 1920's and 1930's. Nitriding and case-hardening techniques to increase the surface strength of gearing were introduced in the 1930's. Induction hardening was introduced in 1950. With new capability to predict strength and lubricating conditions for gears, wear and scoring became somewhat less difficult to deal with. This left surface pitting as the life limiting failure mode for gears. In this mode of failure small cracks develop on and under the surface as a result of metal fatigue. Eventually, pieces of the surface are lost, and the gears become rough and noisy in their operation and may even fail by secondary tooth breakage. Extremely clean steels produced by vacuum melting processes introduced in 1960 have proven effective in prolonging gear life.

Current State of the Gearing Art and Science

Gearing problems continue to occur as the ever increasing demands of power transmission and transportation systems require. One of the major ways of identifying and solving these problems is through the free exchange of ideas at national and international forums.

The American Gear Manufacturers Association (AGMA) membership represents approximately 80 percent of the gearmaking capacity in the United States and Canada, or approximately two-billion dollars worth of business annually. The AGMA holds two technical meetings each year, publishes the proceedings of these meetings, and maintains the standards used by the American gear industry.

In 1977 at the American Society of Mechanical Engineers (ASME) International Power Transmission and Gearing Conference held in Chicago, 82 technical papers were presented (fig. 9). At the 1980 conference in San Francisco 112 technical papers were given, and a 4-hour panel session on the gear standards of AGMA and the International Standards Organization (ISO) was presented. At both of these conferences there was worldwide contribution and attendance. The Fifth World Congress of the International Federation for the Theory of Machines and Mechanisms (IFToMM) held in Montreal in 1979, drew 350 contributing technical papers. Approximately 35 of those papers were devoted to subjects in power transmission and gearing (ref. 12). These conferences present a very good collection of the problems, solutions, and achievements of today's gearing community. State of the art reviews on various aspects of gearing are periodically published in the ASME Transactions, Journal of Mechanical Design.

A few of the developments, originally reported in the previously mentioned forums and regarded as advances in the state of the gear art, are described next.

Applications and Capabilities

In the past 20 years there has been increased use of industrial gas turbines for electric power generation. In the range of 1000 to 14 000 hp epicyclic gear systems have been used successfully (fig. 10). Pitch-line velocities are from 50 to 100 m/sec (10 000 to 20 000 ft/min). These gear sets must work reliably for 10 000 to 30 000 hr between overhauls.

Bevel gears produced to drive a compressor test stand ran successfully for 235 hr at 2984 kW (4000 hp) and 200 m/sec (40 000 ft/min) (ref. 13). From all indications these gears could be used in an industrial application if needed. A reasonable maximum pitch-line velocity for commercial spiral-bevel gears with curved teeth is 60 m/sec (12 000 ft/min)(ref. 14).

Gear system development methods have been advanced in which lightweight, high-speed, highly loaded gears are used in aircraft applications. The problems of strength and dynamic loads, as well as resonant frequencies for such gearing, are now treatable with techniques such as finite-element analysis, siren and impulse testing for mode shapes, and application of damping treatments where required (ref. 15).
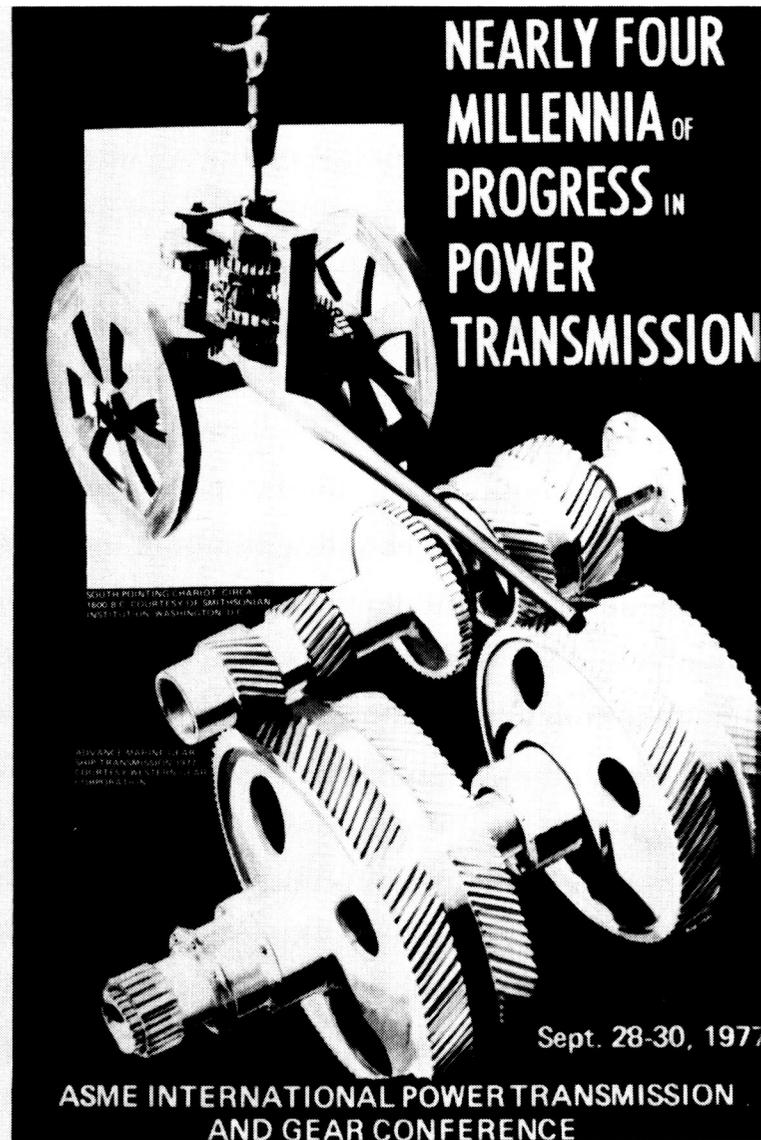
Figure 9. - Publicity poster used for 1977 ASME International Gear Conference portraying conference theme. Encyclopedia Brittannica (ref. 1) dates the Chinese South Pointing Chariot to the 27th century B.C. The figure on the chariot points south and is driven by differential gearing as elaborated by George Lanchester in a lecture before the China Society. (Dates and gearing pertaining to the legendary South Pointing Chariot are conjectural.) Reference 4 contains a balanced summary of scholarly evidence connected with this interesting chariot.

Gear Materials

Gear materials and treatments are continuing to advance the capabilities of gears in hardness retention at elevated temperatures, in higher strength, in scoring resistance, and in durability (refs. 16 to 19).

Plastic gearing and other alternative materials are being studied, and their performances quantified to establish the boundaries on their usefulness. This endeavor is mandated by reasons of economy and the shortage of critical raw materials. Molybdenum disulphide-filled cast nylon gears are finding use today in a wide range of industries. If allowances for the special problems of thermal
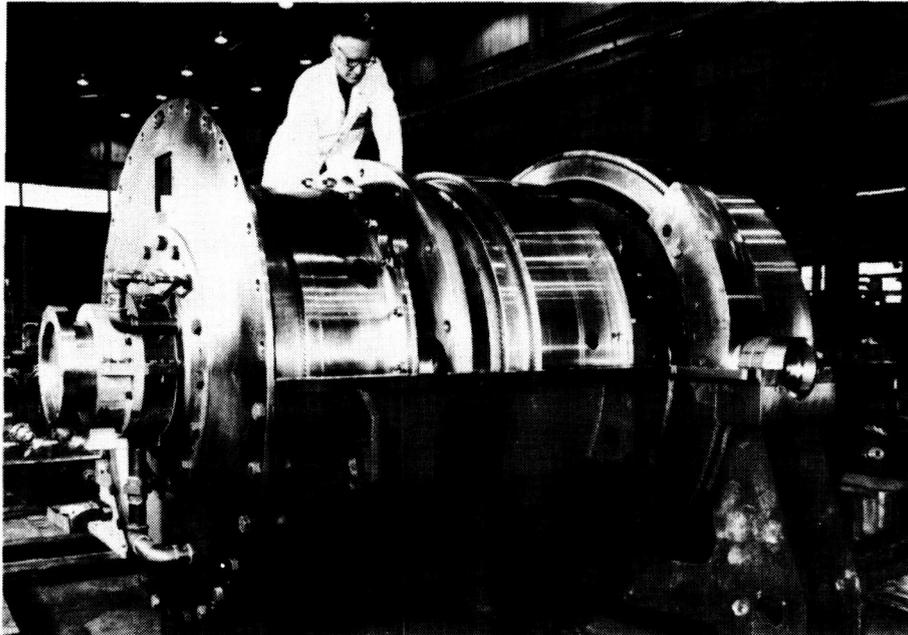
Figure 10. - Gearbox with epicyclic gearing capable of handling 3700 hp used in a huge, 2500-kW wind turbine. Gear drive is a three-stage epicyclic speed increaser utilizing the Stoekicht principle of flexible annulus coupling rings for even load sharing. Design is smaller and lighter than parallel shaft configuration. (Courtesy of Philadelphia Gear Corp.)

expansion and creep are made, these gears were found to fail eventually by a fatigue fracture after many millions of stress cycles (ref. 20). Studies on the temperature rise in plastic gears during operation have also been conducted (ref. 21).

Gear Manufacture

Demands for high-power, high-strength bevel gears are made for steel rolling mills, bow thrusters for off-shore drilling rigs, and high-performance marine craft such as surface-effect ships. Spiral bevel gears for these applications have resulted in the development of production machinery for generating 203-cm (80-in.) diameter, 30-cm (12-in.) face-width gears (fig. 11) (ref. 14). The carbide toothed skiving hob for finishing hardened gear teeth was developed in Japan in 1963 and continues to be the object of research (ref. 22). Honing the skived surface can produce a finish of 0.5 to 0.8 μm rms roughness using a newly developed screw-shaped hone made of an elastic polymer embedded with abrasives. There is also a new shaped shaving hob for finishing gears of large pitch. While shaving to improve surface finish is not new, conventional gear-shaped shavers are expensive to maintain. The new screw-shaped shaver is economical to make and resharpen in a lathe (ref. 23).

Spiral bevel gears may now be finish-machined in the hardened condition on the same machine as was used to rough-cut the gear (fig. 12). The gears are brought to a surface hardness of 60 Rockwell C by carburizing and quenching, while holding the gear in a press to minimize distortions. The gears are then finished in the hard condition using carbide cutters to obtain surface finishes in the range of 0.5 to 0.8 μm (20 to 30 μin.) and precision comparable to AGMA quality 12.

Numerically controlled, three-axis dressing of finish grinding wheels has been developed. The system is capable of dressing complex shapes in the grinding wheel used to finish grind complicated worm-gear profiles (ref. 24).

In addition to this use of computers to control the gear manufacturing process, a more comprehensive application of computerized methods has been applied in Switzerland. The CAD/CAM manufacture of spiral-bevel gears is used to do design layout, calculate machine settings for manufacture, analyze the meshing conditions, and do the stress analysis (ref. 25).
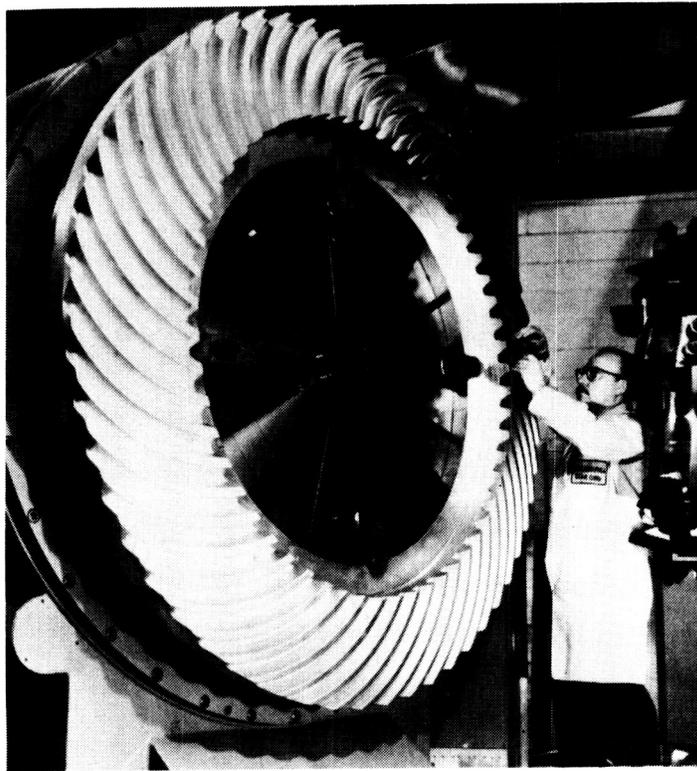
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Figure 11. - Large spiral-bevel gear is manufactured using rotary cutter
method. Gears can be cut in the 50- to 80-in.-diam range with up to 12-in.
face and 0.75-in.-diam pitch. Such gears can transmit 5000 to 6000 hp.
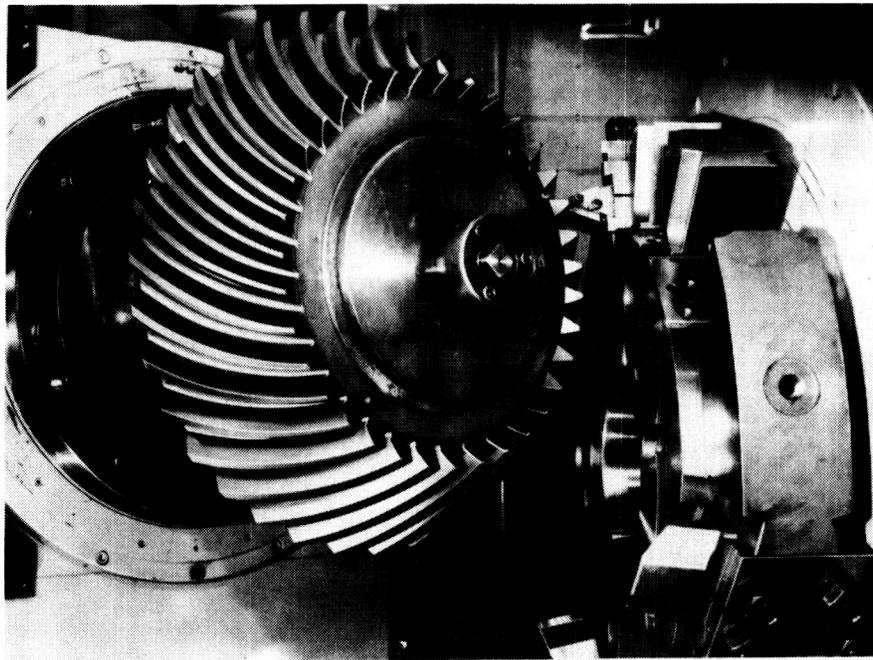(Courtesy of Philadelphia Gear Corp.)

Figure 12. - Precision hard-cut process for generated spiral-bevel gears
Gears are cut in soft condition, deep carburized to Rockwell C 60, press
quenched to minimize distortion, then precision finished in hard condition
by special carbide cutters on the same machine as the original cut.
(Courtesy of Philadelphia Gear Corp.)

NASA Contributions to Gearing

Beginning in 1969 the NASA Lewis Research Center embarked on a comprehensive gear technology research program. This work is being continued under the NASA Helicopter Transmission System Technology Program. Some of the important results and continuing programs are presented in this section.

Metallurgical Effects

The metallurgical processing imposed on a gear steel from its elemental plate to the finished component can significantly affect its ultimate performance. Even the type of ore from which the various elements are extracted can exercise some influence over later component life. Theoretically then, a large number of variables could be considered in determining the rolling-element (surface pitting) life of a potential gear material. This becomes a nearly impossible task.

There is only a small body of published data on material effects on gear pitting life. Many of the gear alloy improvement programs have been evaluated by mechanical tests rather than by rolling-element component or full-scale gear surface pitting fatigue tests. Since rolling-element fatigue is a unique property, it is not, as such, necessarily possible to correlate it with more standard mechanical tests (refs. 26 and 27).

An extensive program is being conducted by NASA to evaluate potential gear materials. Tests have been conducted on NASA Lewis' gear fatigue apparatus (fig. 13; see refs. 17 and 28 to 33) and General Electric's rolling-contact (RC) tester (fig. 14; see refs. 34 and 35). Figure 15 shows the spur test gears used at Lewis in the gear fatigue test rig. These gears are 8.9 cm (3.5 in.) in pitch diameter and have 28 teeth. Four such rigs (fig. 13) are used to perform long-term endurance tests. The effects of materials, lubrication, and design on gear life are being studied.

Several different types of gear failure occur, as shown in figure 15. Scoring is caused by poor lubrication and high-temperature operation. Tooth fracture is caused by high tooth bending stresses and is aggravated by poor heat treatment. Fracture usually begins at the root of the tooth, but may also originate at a surface fatigue pit. Surface fatigue pitting is caused by repeated application of contact stress. The higher the contact stress, the more rapidly failure occurs.

Gear Materials

Scoring and fracture failures may be controlled by proper design, but fatigue pitting is an intrinsic problem in gear applications. Pitting is an unavoidable event that eventually ends the useful life of a gear. Figure 16 shows data for test gear failures due to pitting fatigue. The failure points are plotted on Weibull coordinates, which give the cumulative percentage of failed specimens as a function of running time. The notable feature of presenting the data this way is that it emphasizes the relation between life and reliability. For instance, the life at which 10 percent of the gear specimens have failed is denoted the L_{10} life. This is conceptually identical to the B_{10} life for bearings. The L_{10} life corresponds to 90-percent reliability; the L_{50} life corresponds to 50-percent reliability. Other life numbers may be defined as the degree of reliability required changes.

Figure 17 shows the results of surface fatigue tests of six gear steels. AISI 9310, the baseline for comparison, is assigned a relative life of 100 percent. In the tests only pitting fatigue failures occurred in this material. The CBS 600 gears had an L_{10} life 7.5 times that of the AISI 9310 gears. The failure mechanism was primarily pitting. In some cases the cracks near the pitting failure propagated through the tooth to cause fracture. The forged AISI M-50 material behaved similarly, with a life six times that of the baseline AISI 9310. The AISI M-50 and CBS 1000 gear steels have good high-temperature hardness retention, which results in longer life at elevated operating temperatures.

Gear Forging

Gears made from through-hardened materials or case-carburized materials (having a high percentage of alloying elements) have a tendency for gear tooth fracture due to bending fatigue as a result of extended running after a surface fatigue spall. Figure 15 shows a typical tooth fracture emanating from a surface fatigue spall. One fabrication method that has the potential to improve the strength and life of gear teeth is ausforging. Ausforging is a thermomechanical metalworking process
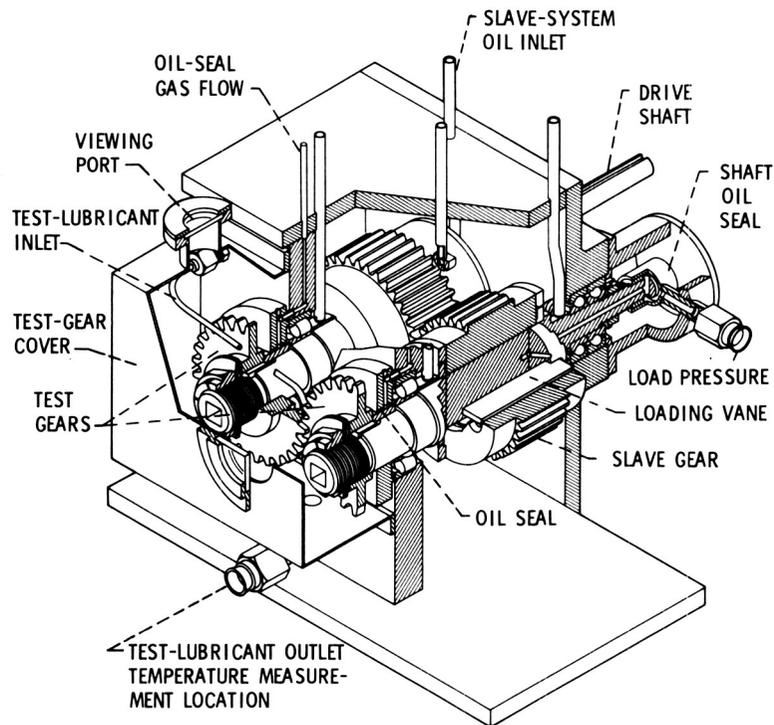
(a) CUTAWAY VIEW.
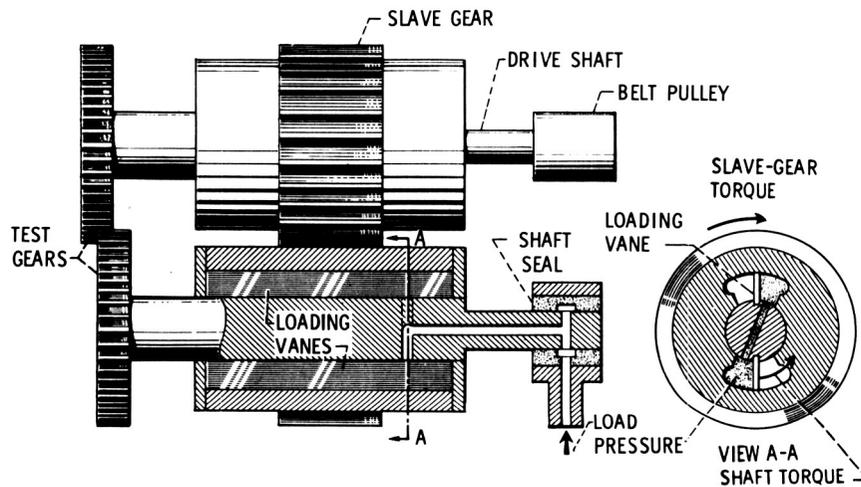
(b) SCHEMATIC DIAGRAM.

Figure 13. - NASA Lewis Research Center's gear fatigue test apparatus.

whereby a steel is forged or otherwise worked while it is in the metastable austentic condition (ref. 36). The application of ausforging to machine elements such as rolling-element bearing was first reported (ref. 37).

Tests were conducted (refs. 31 and 32) at 350 K (170° F) with three groups of 8.9-cm (3.5-in.) pitch diameter spur gears made of double-vacuum melted (VIM–VAR) AISI M–50 steel and one group of CVM AISI 9310 steel. The pitting fatigue life of the standard forged and ausforged gears was approximately 5 times that of the CVM AISI 9310 gears and 10 times that of the bending fatigue life of the standard machined VIM–VAR AISI M–50 gears run under identical conditions. There was a slight decrease in the 10-percent life of the ausforged gears from that for the standard forged gears. However, the difference is not statistically significant.
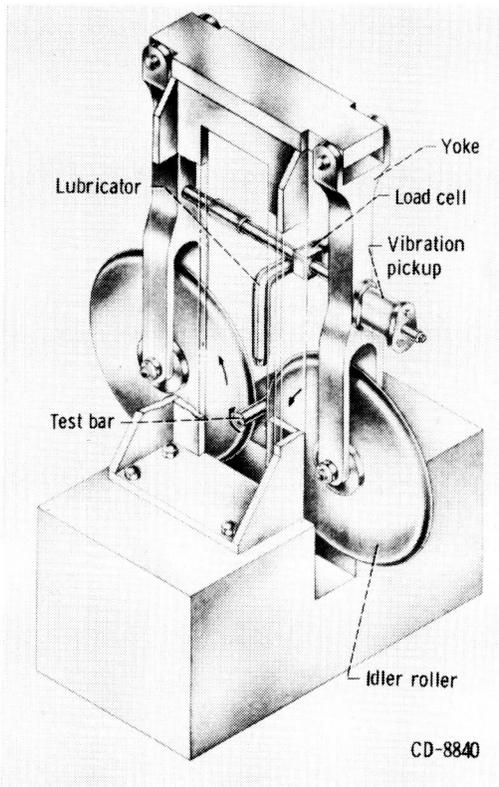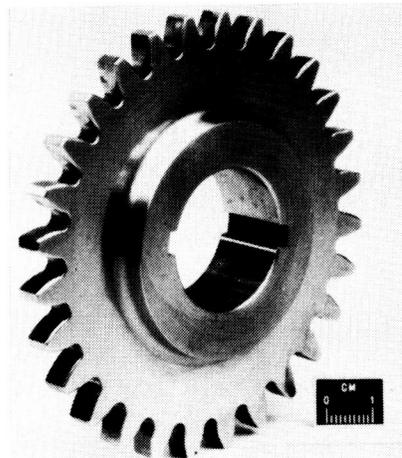
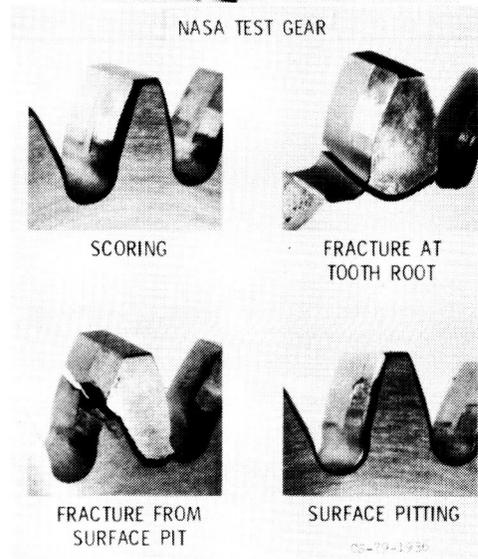Figure 14. - Rolling-contact fatigue apparatus.

NASA TEST GEAR

SCORING

FRACTURE AT TOOTH ROOT

FRACTURE FROM SURFACE PIT

SURFACE PITTING

Figure 15. - Typical gear-tooth failure modes.

STATISTICAL % OF GEAR SPECIMENS FAILED

$L_{10} = 45 \times 10^6$ REV
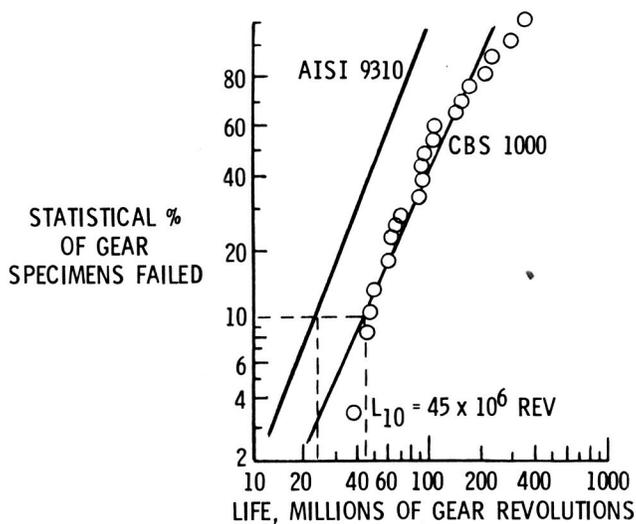
LIFE, MILLIONS OF GEAR REVOLUTIONS

Figure 16. - Weibull plot for pitting fatigue failure of CBS 1000 specimen gears compared with baseline dispersion in life for AISI 9310 gears. Tests conducted on the NASA gear-fatigue test apparatus.

MATERIAL

AISI 9310   (PITTING)
SUPER NITRALLOY   (PITTING /SCORING)
AISI M-50   (PITTING /FRACTURE)
FORGED M-50   (PITTING → FRACTURE)
CBS 1000   (PITTING → FRACTURE)
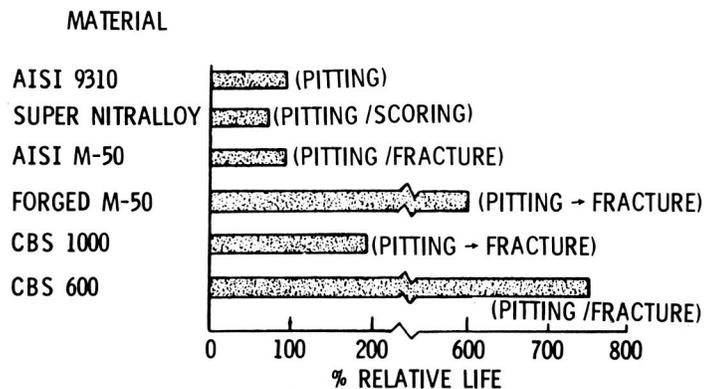CBS 600   (PITTING /FRACTURE)

% RELATIVE LIFE

Figure 17. - Summary of gear-fatigue test results for six gear steels.

The standard machined gears failed primarily by gear-tooth fracture, while the forged and ausforged VIM–VAR M–50 and CVM AISI 9310 gears failed primarily by surface pitting fatigue. The ausforged gears had a slightly greater tendency to fail by tooth fracture than the standard forged gears.

While gear forging offers the potential for long-lived, reliable gearing, especially for the high alloy steels, both the cost and the availability of forging facilities may outweigh its advantages at the present time.

Gear Life Predictions

The fatigue-life model proposed in 1947 by Lundberg and Palmgren (ref. 38) is the commonly accepted theory to determine the fatigue life of rolling-element bearings. The probability of survival is expressed as follows:

$$\log \frac{1}{s} \propto \frac{\tau^c \eta^e v}{z^h} \tag{1}$$

where

e Weibull slope

h,c material dependent exponents

s probability of survival

v volume representative of the stress concentration or "stressed volume"

z depth of the critical stress

η millions of stress cycles

τ critical stress

Much of the work by Lundberg and Palmgren was concerned with connecting the basic equation to common bearing geometry and operating parameters (ref. 39). In order for the theory to be directly useful for gears, the NASA research used the same approach (refs. 40 and 41.)

The Weibull slope, e, and the load-life exponent, p, were directly determined by conducting life tests under several load conditions for a group of gears. The results of these tests conducted for three gear loads with three groups of AISI 9310 gears (ref. 42) are shown in figures 18 and 19. Life was found to vary inversely with load to the 4.3 and 5.1 power at the 10-percent and 50-percent life levels, respectively. The average Weibull slope was 2.5.

Using the exponent values determined experimentally from the gear tests, the life distributions for the three groups of AISI 9310 gears were calculated. These distributions are plotted for comparison with the experimental data in figure 18.

The AGMA has published two standards for tooth surface fatigue (refs. 43 and 44). These standards are AGMA 210.02 and 411.02. AGMA 210.02 provides for an endurance limit for surface fatigue below which it is implied that no failure should occur. In practice, there is a finite surface fatigue life at all loads. AGMA 411.02 recognizes this finite life condition. Therefore, it does not contain an endurance limit in the load-life curve but does show a continuous decrease in life with increasing load. Both standards are illustrated in figure 20. The AGMA load-life curves shown are for a 99-percent probability of survival or the L_1 life (ref. 45). The experimental L_1, L_{10}, and L_{50} lives are plotted for comparison.

It is evident that the load-life relation used by AGMA is different from the experimental results reported herein. The difference between the AGMA life prediction and the experimental lives could be the result of differences in stressed volume. The AGMA standard does not consider the effects of stressed volume, which for many gears would be considerably different from that of the test gears used herein. The larger the volume of material stressed, the greater the probability of failure or the lower the life of a particular gearset. Therefore, changing the size or contact radius of a gearset, even though the same contact stress is maintained, would have an effect on gear life.

(a) TANGENTIAL LOAD W_t/f 463x10³ N/M (2645 lb/in.).

(b) TANGENTIAL LOAD W_t/f 578x10³ N/M (3305 lb/in.).

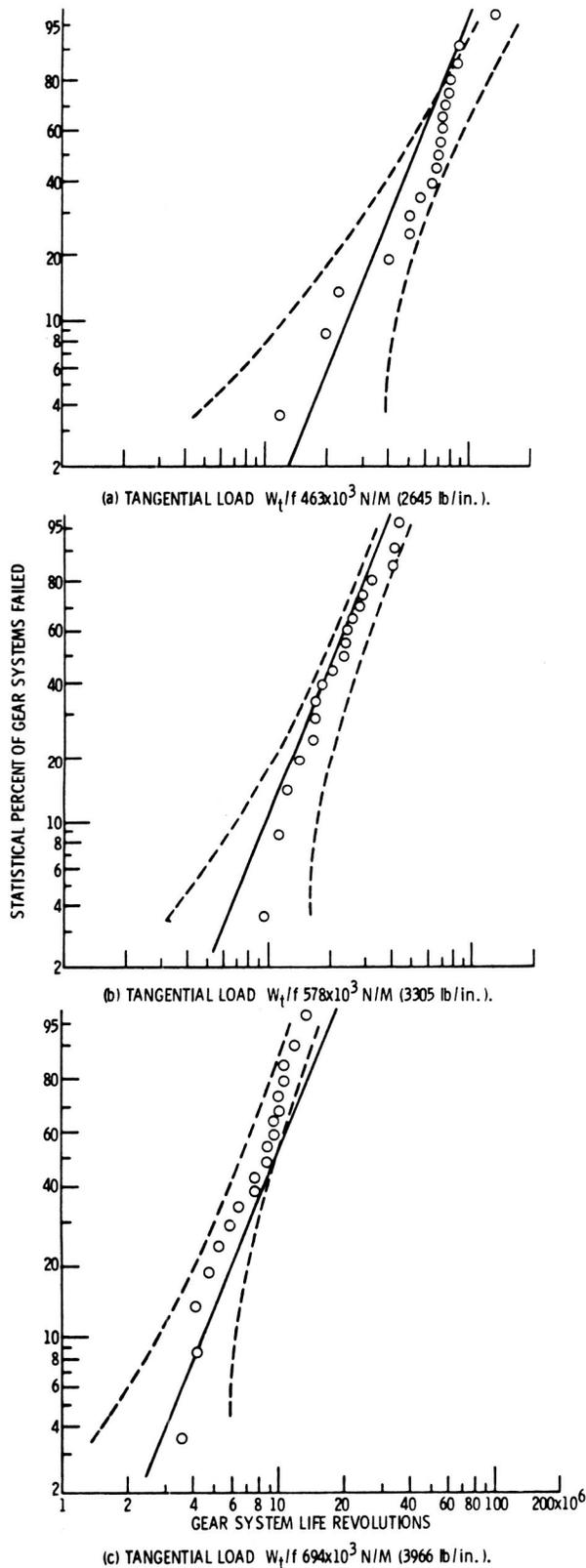(c) TANGENTIAL LOAD W_t/f 694x10³ N/M (3966 lb/in.).

Figure 18. - Comparison of life prediction theory with experimental results for VAR AISI 9310 steel spur gears. Speed, 10 000 rpm; lubricant, superrefined naphthenic mineral oil with additive package.
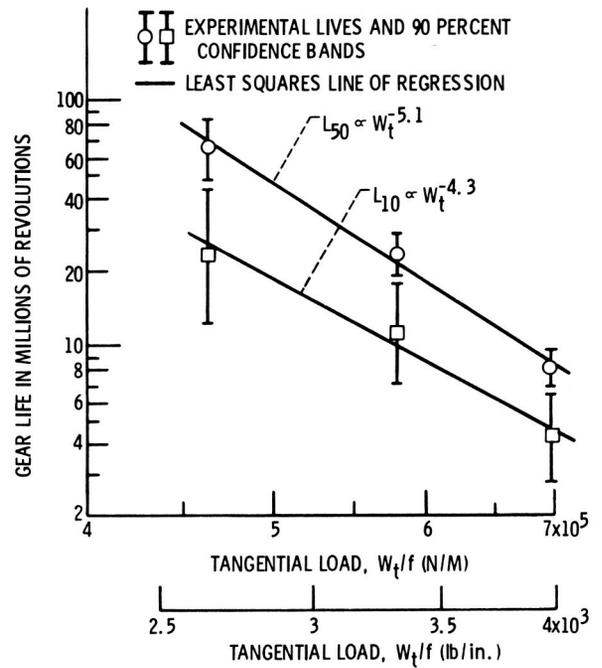

Figure 19. - Load life relationship for VAR AISI 9310 steel spur gears. Speed, 10 000 rpm; lubricant, superrefined naphthenic mineral oil with additive package.
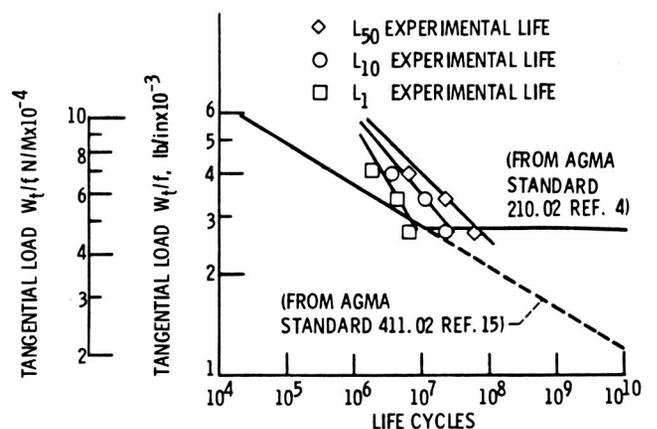

Figure 20. - Comparison of experimental life for (CVM) AISI 9310 spur gears with AGMA life prediction. Speed, 10 000 rpm; lubricant, superrefined naphthenic mineral oil with additive package.

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Tooth Profile and Pressure Angle

A majority of current aircraft and helicopter transmissions have spur-gear contact ratios (average number of teeth in contact) of less than 2. The contact ratios are usually from 1.3 to 1.8, so the number of teeth in engagement is either one or two. Many gear designs use a pressure angle of 25° for improved tooth strength, giving a contact ratio of approximately 1.3. This low contact ratio increases dynamic loading of the gear teeth, increases noise, and may sometimes cause lower pitting fatigue life.

High-contact-ratio gears (contact ratio greater than 2) have been in existence for many years but have not been widely used. These gears have load sharing between two or three teeth during engagement; there is, therefore, usually less load per tooth, and the gears should operate with lower dynamic loads and thus less noise.

High contact ratios can be obtained in several ways: (a) by smaller teeth (large pitch), (b) by smaller pressure angle, and (c) by increased addendum. Any of these methods produce gears that tend to have lower bending strength and increased tooth sliding. Increased sliding may cause the gears to run hotter and have a greater tendency for surface-distress-related failures such as micropitting and scoring.

Profile modification (changing the involute profile at the addendum or dedendum or both) is normally done on all gears to reduce tip loading and scoring (ref. 46). However, if done improperly, the dynamic load could be increased (ref. 30). Several profile modifications have been produced that would reduce scoring and improve the performance of high-contact-ratio gears. One such proposal is the so-called new-tooth-form (NTF) gear, which has a large profile modification at both the addendum and dedendum. The profile radius of curvature of this proposal is also reduced at the addendum and increased at the dedendum in an attempt to lessen sliding and thereby reduce scoring of high-contact-ratio gears. However, a gear geometry analysis (ref. 47) indicates that sliding is independent of the profile radius of curvature.

Under NASA contract, the Boeing Vertol Co. designed and manufactured two sets of NTF gears and two sets of standard gears for the purpose of evaluating and comparing them. Scoring, surface-fatigue, and single-tooth-bending-fatigue tests were conducted using four sets of spur gears of standard design and three sets of spur gears of NTF design (ref. 48; fig. 21). The scoring tests were conducted on a Wright Air Development Division (WADD) gear test rig at a speed of 10 000 rpm at the Southwest Research Institute. The surface fatigue tests were conducted on the same test rig at a speed of 10 000 rpm and at maximum Hertz stresses of 173×10^7 and 148×10^7 Pa (250 000 and 214 000 psi). The single-tooth bending fatigue tests were conducted on both the standard and NTF gears starting at a bending stress of 104×10^7 Pa (150 000 psi). The stress was increased until failure occurred at 3×10^6 cycles or less.

The results of the surface fatigue tests are shown in figure 22. The pitting fatigue lives of the standard and NTF gears were statistically equal for the same maximum Hertz stress. The pitting fatigue life of the NTF gears was approximately 300 percent better than that of the standard gears at equal torque or load.
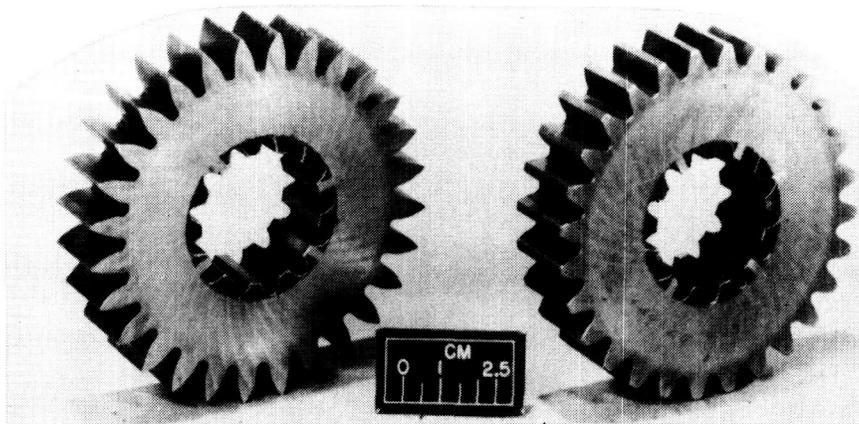

Figure 21. - New-tooth-form (high contact ratio) and standard low-contact-ratio gears. Test specimens for Wright Air Development Division gear test rig.
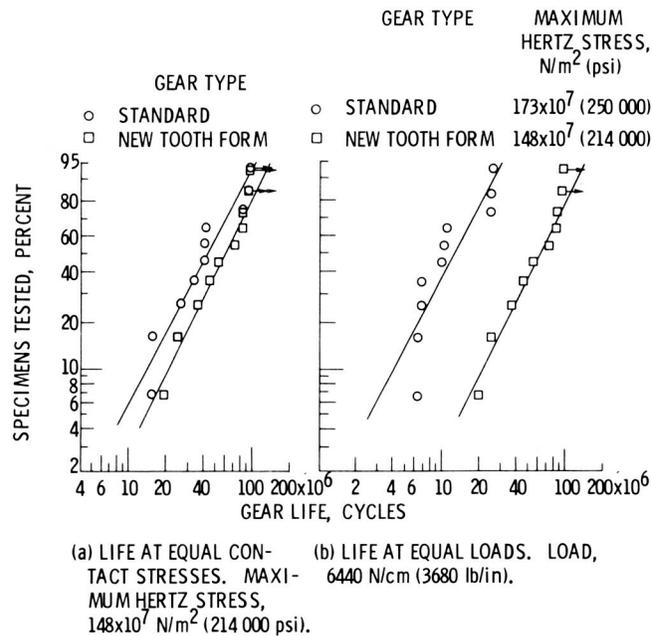
Figure 22. – Pitting fatigue lives of standard and new-tooth-form spur gears. Speed, 10 000 rpm; temperature, 370 K (207° F); lubricant, synthetic polyol ester.
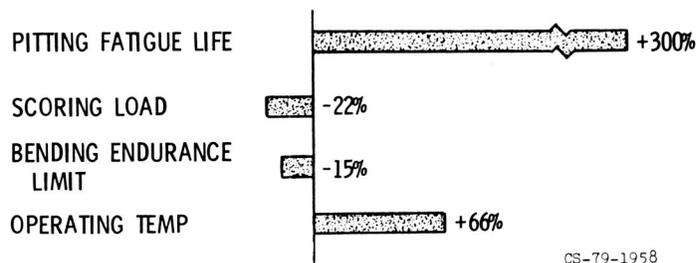

Figure 23. – Performance of new-tooth-form gears referenced to performance of standard low-contact-ratio gears. Testing conducted in Wright Air Development Division test rig.

The improvement in fatigue life, however, is not without a price (fig. 23). Additional tests showed that high-contact-ratio gears are more susceptible to scoring.

Both the standard and NTF gears scored at a gear bulk temperature of approximately 409 K (277° F). At this temperature the load on the NTF gears was 22 percent less than the load on the standard gears. The scoring failure was a function of gear bulk temperature, where for a given lubricant the temperature is a function of gear design, operating load, and speed.

The bending endurance fracture limit is 15 percent lower because the teeth of the high-contact-ratio gear are more slender. Also, these gears run hotter because of increased sliding of the longer teeth. With proper lubrication and design, though, the high-contact-ratio gear can be used to achieve longer life and more reliability.

Spiral Bevel Gears

NASA is conducting a fundamental study of spiral-bevel gear technology. From a study of the current state-of-the-art, significant advances can be achieved in reducing gear noise and vibration, better estimates of gear strength and life, as well as better lubrication techniques which will reduce wear and minimize temperature rise. Figure 24 shows a set of test specimen gears that are used on a spiral bevel gear fatigue tester.
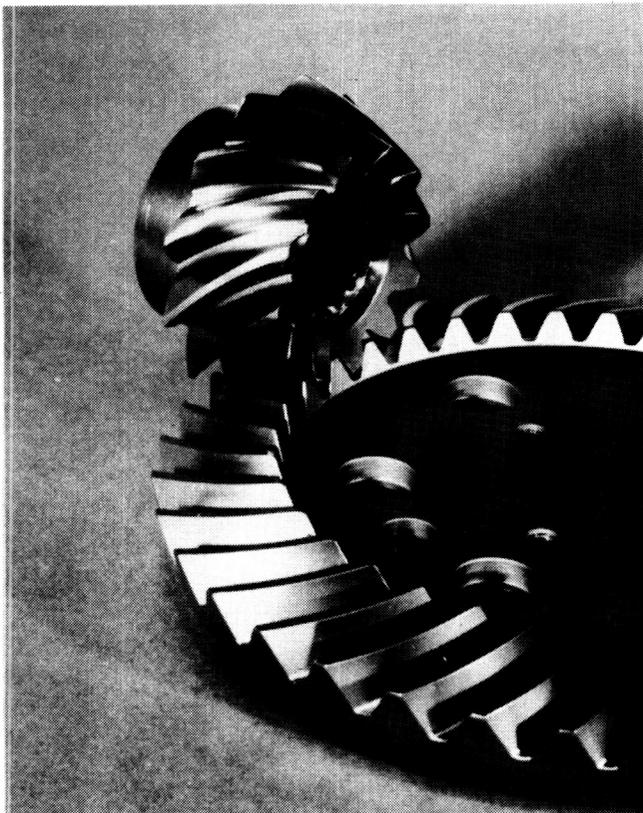
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Figure 24. - Spiral bevel test specimen gears for NASA spiral bevel gear-fatigue test rig. Spiral angle, 35°; face width, 1 in.; shaft angle, 90°; pressure angle, 22 1/2°; 12T pinion, 36T gear.
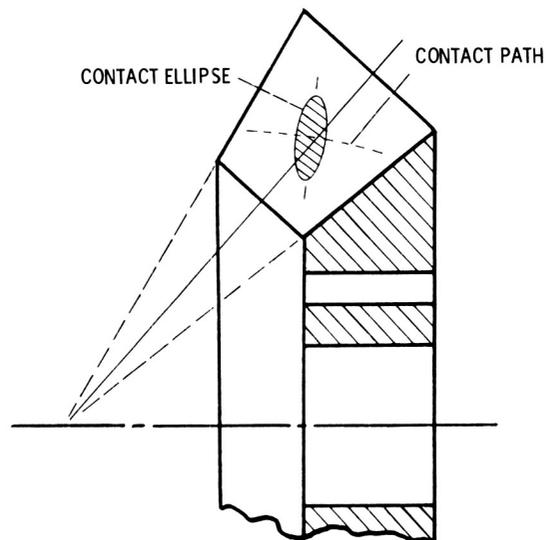

Figure 25. - Spiral bevel gear tooth showing features of tooth contact geometry.

Spiral-bevel gear noise is significantly higher than spur-gear noise. If gear noise in helicopter drives is to be significantly reduced, then this largest source of noise must be reduced. Also, stresses in the root of the spiral-bevel gears are significantly higher than handbook formulas would indicate.

To adequately understand the effects of spiral-bevel gear design parameters on noise, vibration, stress, temperatures, and lubrication, the gear-tooth surface geometry must be defined. Analysis is being performed to define the tooth surface geometry using differential geometry theory. Methods of optimizing the tooth surface contact are being studied (fig. 25).

A spiral-bevel gearset computer program has been developed that makes use of mesh stiffness calculations based on finite-element methods, gear shaft and bearing stiffnesses, current theories of tooth contact analysis (TCA), and elastohydrodynamic lubrication theory. The program will enable the calculation of such things as dynamic loads, bulk and flash temperatures, contact patterns, and lubricant film thickness. The NASA method of gear life prediction will be used as a subroutine of the gear program.

A method for calculating spur-gear efficiency has also been developed at NASA Lewis (ref. 49). This method algebraically accounts for gear sliding, rolling, and windage loss components and incorporates an approximate ball-bearing power-loss expression to estimate the loss of a ball-bearing support system. A theoretical breakdown of the total spur system loss into individual components was performed to show their respective contributions to the total system loss (fig. 26).

Full-Scale Transmission Experiments

When analysis and bench testing are successful in identifying those components that hold promise for improving helicopter transmission performance, they are designed into full-scale
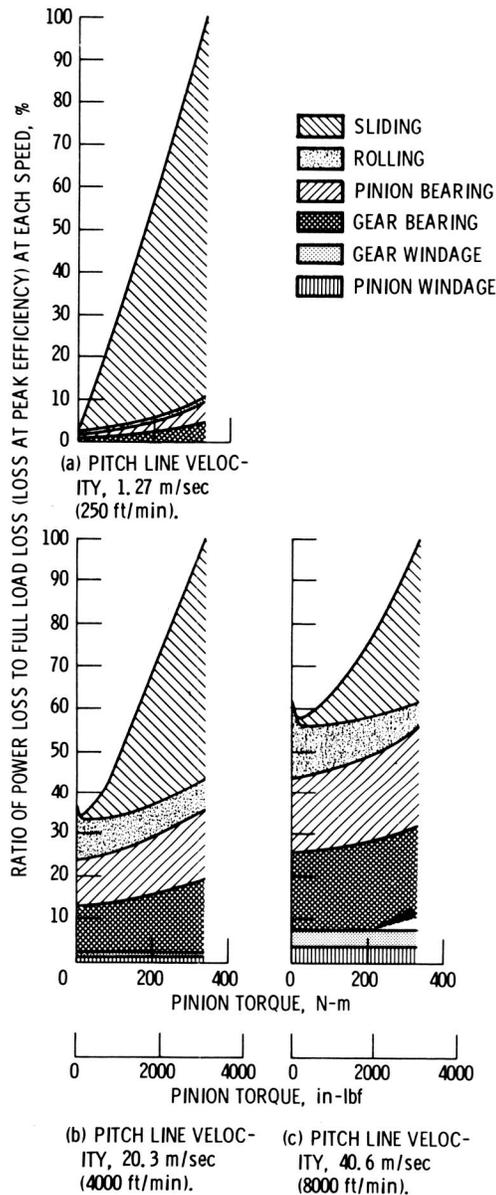
Figure 26. - Percentage breakdown of the sources of gearbox power loss as function of input torque for three speeds. Pitch diameter, 10 cm (4.0 in.); diametral pitch, 8; ratio, 1.0; pinion width to diameter ratio, 0.5; lubricant viscosity, 30 cP.

transmissions. The Lewis Research Center has two helicopter transmission test stands, the 500-hp test stand (figs. 27 and 28) and the 3000-hp test stand (figs. 29 and 30). Both stands operate on the torque regenerative principle, where power is recirculated through the closed loops of shafting. The 500-hp test stand has one loop of shafting, and the 3000-hp test stand has three. The 3000-hp test stand is capable of testing twin-engine input transmissions having a power takeoff to drive a tail rotor. Rotor loads are input by means of hydraulic actuators.

These test stands with their accompanying data acquisition equipment are capable of gathering all bearing and gear temperatures, measuring the noise and vibration signatures, recording mechanical strain on the gears and housing components, and measuring the operating efficiency. Figure 31 shows an installation of 20 strain gages on a spiral-bevel gear which is on the input shaft of one of the experimental transmission models.
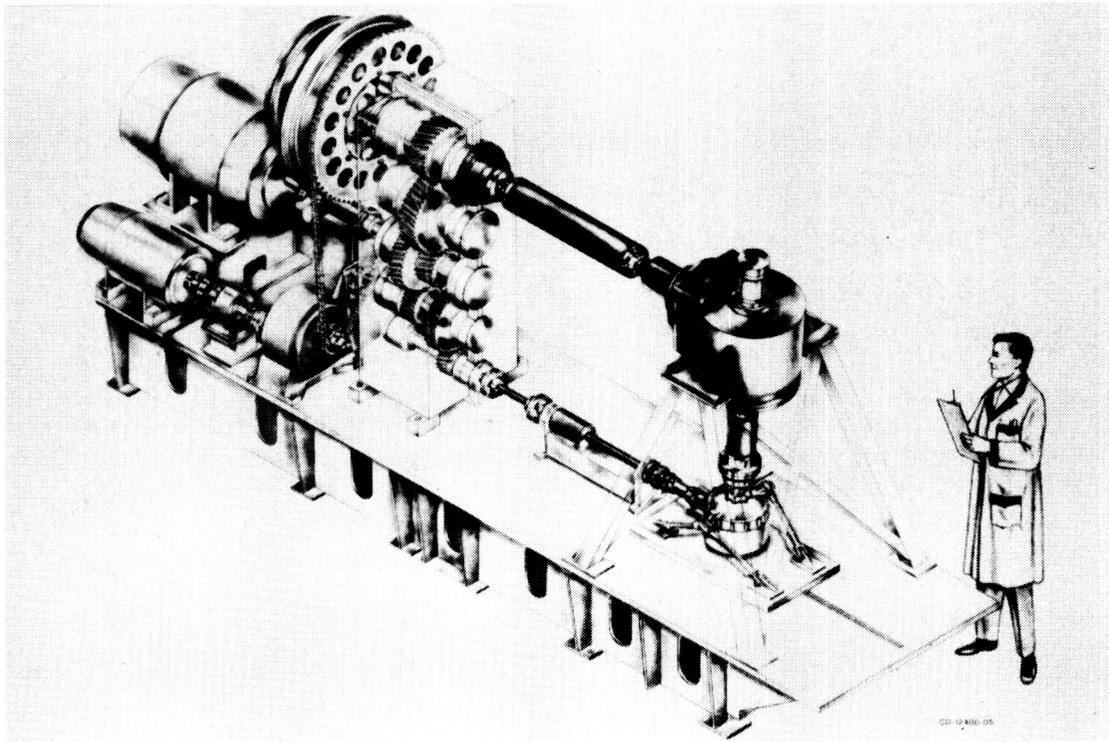
Figure 27. - NASA Lewis Research Center's 500-hp helicopter transmission test stand chain drive applies torque using the torque regenerative principle. Chain drive may turn continuously. Helicopter transmission mounted in pylon supports at lower right. A 200-hp motor provides driving power.
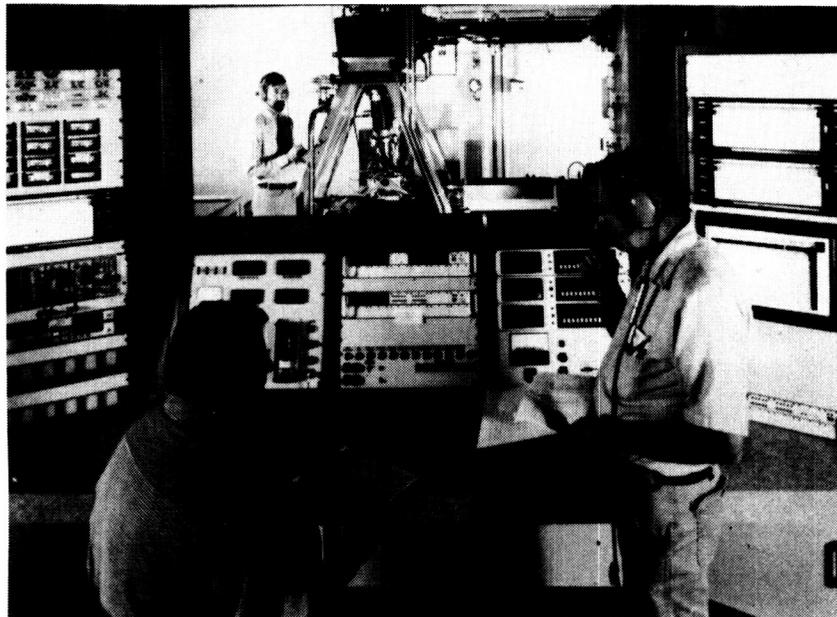

Figure 28. - View of NASA 500-hp test stand from the control room. Approximately 100 channels of static and dynamic data are recorded during experimental testing.

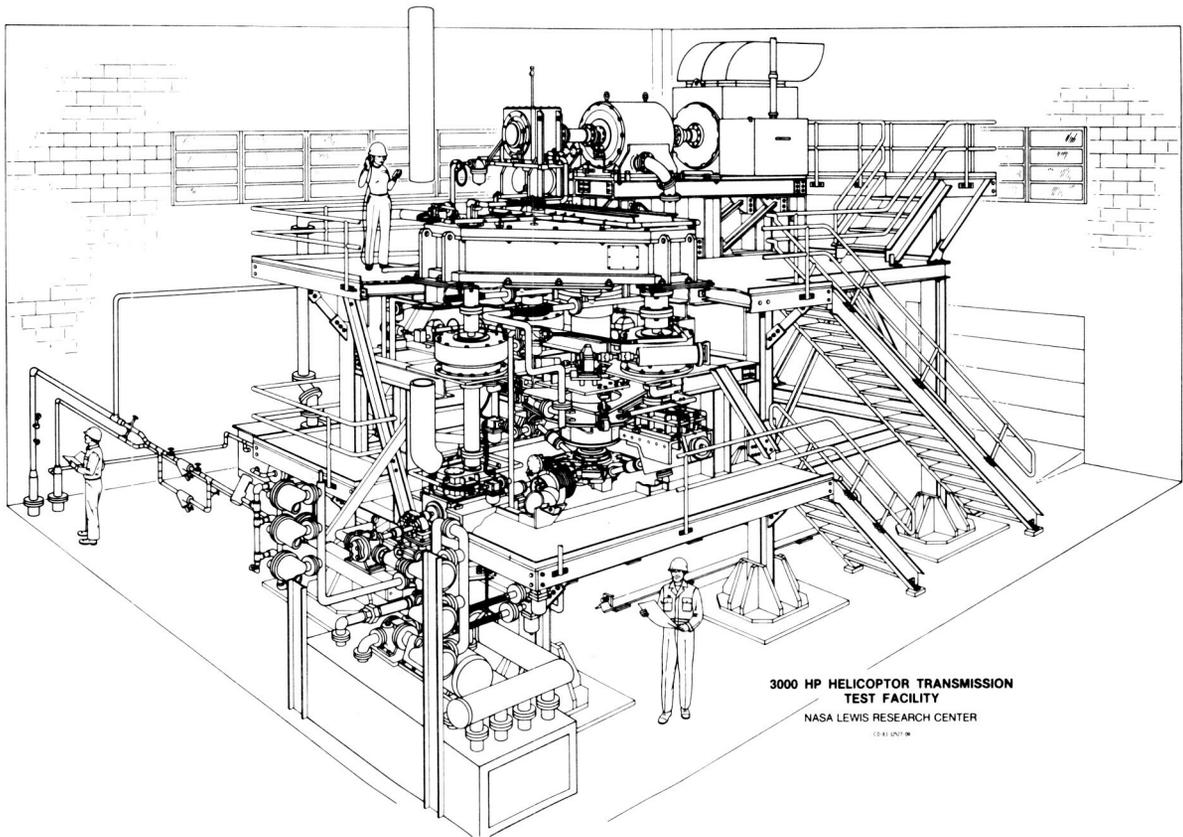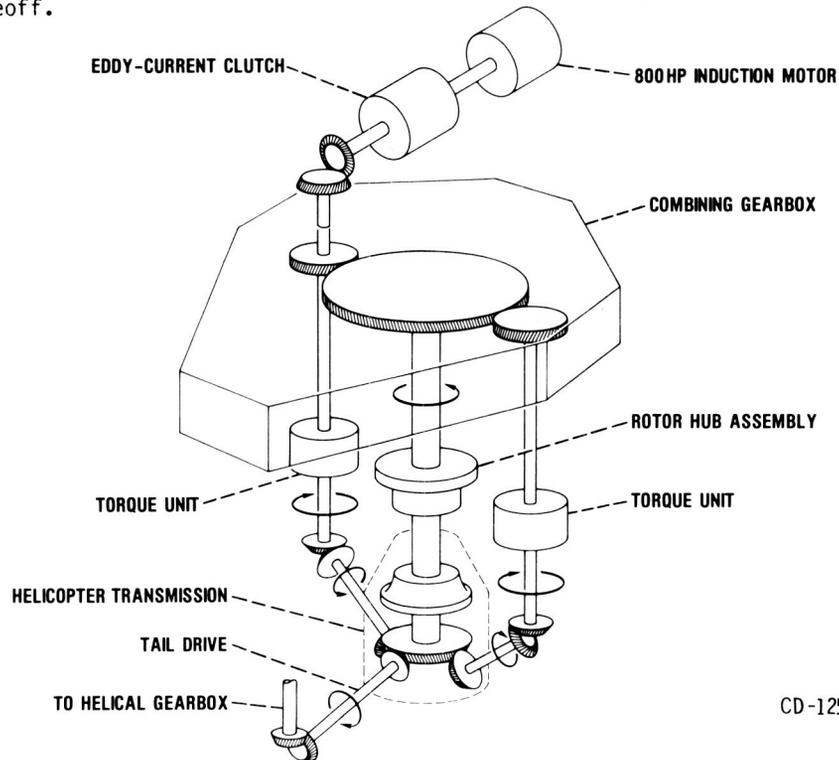Figure 29. - NASA Lewis Research Center's 3000-hp helicopter transmission test stand. Stand has been operated since March 1981. Consists of three loops of torque regenerative shafting. Capable of providing rotor lift and shear loads for transmissions with twin engine drive and a tail rotor shaft power takeoff.

CD-12550-09

Figure 30. - 3000-hp helicopter transmission test stand schematic arrangement.
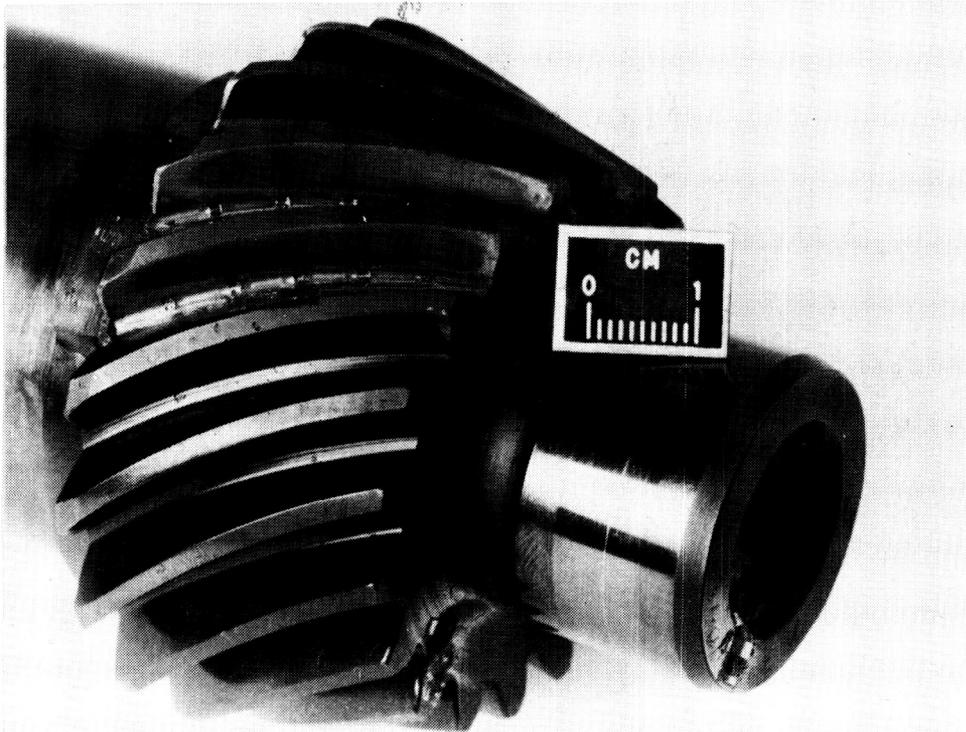
Figure 31. - Spiral bevel gear input shaft for OH-58 helicopter transmission. Twenty strain gages and nine thermocouples installed for research measurements.

Spur-Gear Dynamic Analysis

A gear dynamic analysis has been developed for standard and high-contact-ratio gears by Hamilton Standard Division of United Technologies under contract to NASA. The program will predict the gear dynamic loads for standard and high-contact-ratio gears with variations in gear-tooth profile modifications, in addition to variations in system mass and damping. The program is currently being extended to include rim effects, internal gears, and multiple gear meshes.

NASA is also conducting a spur-gear dynamic analysis program with Cleveland State University. This program is using a different approach for the dynamic analysis and has a finite-element program for gear-tooth deflections. Using this computer program for gear design will give a much better determination of the effects of various gear and system operating parameters on gear dynamic loads and life, and make it possible to improve the load-carrying capacity and life of gear systems.

Performance Prediction

A computer program was developed for calculating spur-gear performance characteristics (ref. 50). The computer program consists of an iterative solution of the bulk temperature, flash temperature, local traction, and the lubricant film thickness along the path of contact. The dynamic load is calculated from a torsional vibration analysis of the gear train. The bulk temperature is calculated from heat-transfer influence coefficients obtained from a finite-element analysis. This is solved iteratively with the elastohydrodynamic lubrication problem for the gears. It is assumed that the vibration problem is uncoupled from the thermal and EHD problems.

Typical results for dynamic load, bulk temperture, flash temperature, and EHD film thickness are shown in figure 32. The calculations were done for the case of two 20° pressure angle, 36-tooth, gears running at 7800 rpm and transmitting 750 hp. Figure 32 is a computer-generated plot, giving the results as a function of distance along the path of contact. The origin is taken at the pitch point.
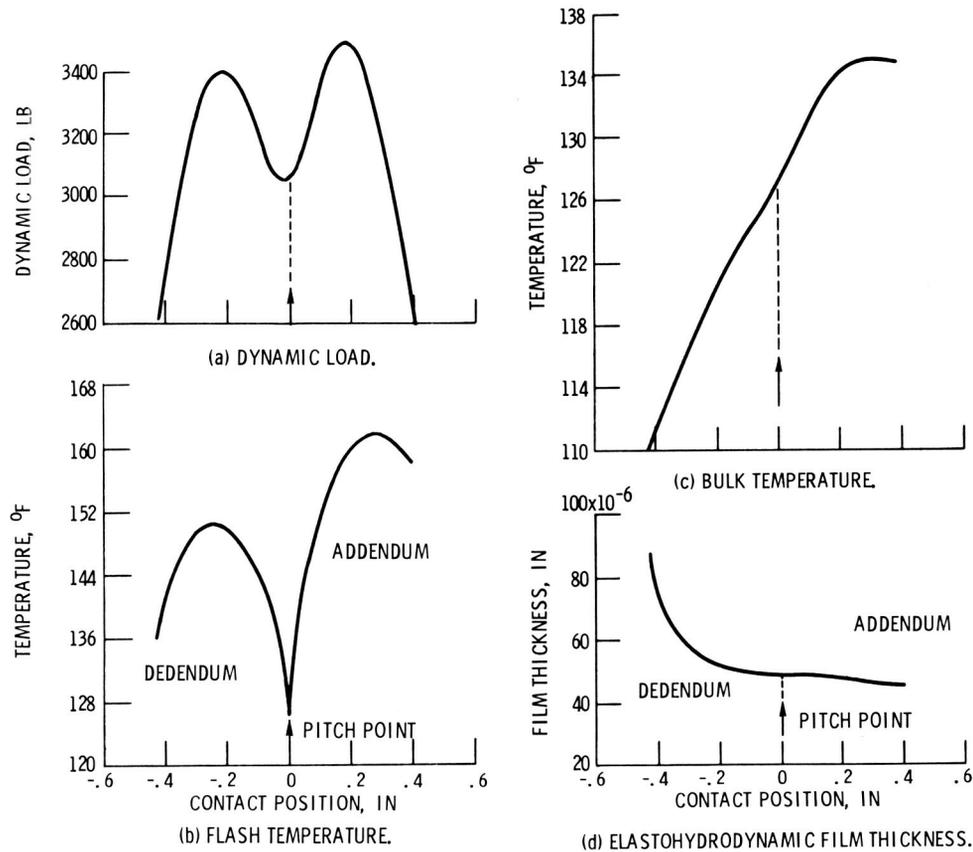
Figure 32. - Computer program results for a spur gear mesh. Ratio 1.1; pressure angle, 20°; 36 teeth; load, 2000 lb; speed, 7800 rpm.

Gear Lubrication and Cooling

The type of lubrication that prevents or minimizes surface asperity interaction is referred to as elastohydrodynamic lubrication. This lubrication mode is differentiated from boundary lubrication in which a chemical, usually oxide, film prevents gross wear or welding of asperities during metal-to-metal contact. In most gear applications a combination of elastohydrodynamic and boundary lubrication exists. Definition of those parameters that affect the lubrication of gears must be obtained and applied to gear design and operation. These parameters include surface finish, tooth design, lubricant type, and elastohydrodynamic lubrication principles (ref. 51).

As a first step in understanding the cooling phenomena in gears, it is important to understand how oil penetrates into the gear tooth spaces under dynamic conditions. Lubricant jet flow impingement and penetration depth into a gear tooth space were measured at 4920 and 2560 rpm using a 8.89-cm (3.5-in.) pitch diameter, 8-pitch spur gear at oil pressures from 7×10^4 to 41×10^4 Pa. (10 to 60 psi) (ref. 52). A high-speed motion picture camera with xenon and high-speed stroboscopic lights was used to slow down or "stop" the motion of the oil jet so that the impingement depth could be determined (fig. 33). An analytical model was developed for the vectorial impingement depth and for the impingement depth with tooth space windage effects included. A comparison of the calculated and experimental impingement depth versus oil-jet pressure is shown in figure 34. The windage effects on the oil jet were small for oil drop sizes greater than 0.0076 cm (0.003 in.). The analytical impingement depth compared favorably with experimental results for an oil-jet pressure greater than 7×10^4 Pa (10 psi). Some of the oil penetrates farther into the tooth space after impingement. Much of this postimpingement oil is thrown out of the tooth space without further contacting the gear teeth.

A computer analysis was conducted for oil-jet lubrication on the disengaging side of a gear mesh (ref. 53; fig. 35). The analysis was used to determine the oil-jet impingement depth for several gear
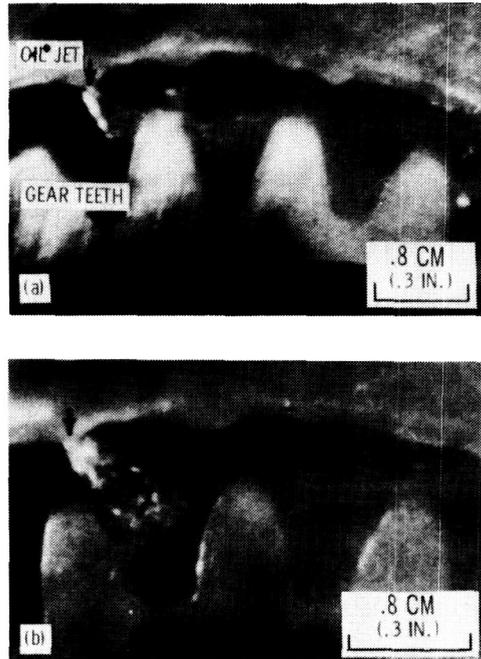
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Figure 33. - Oil-jet penetrating tooth space and impinging gear tooth. Speed, 4920 rpm; oil pressure, 10.5×10^4 Pa (15 psi); xenon and stroboscopic light.
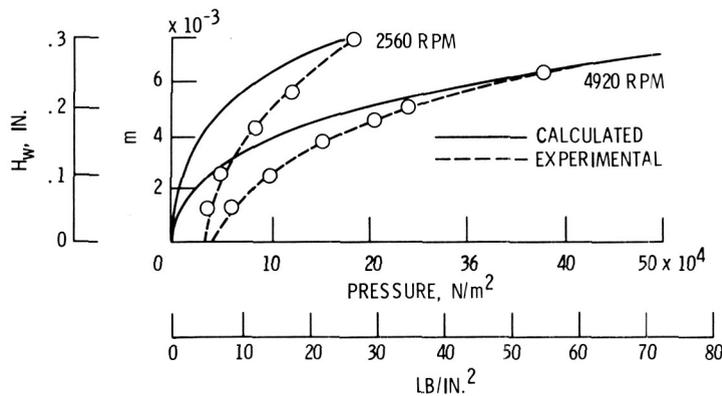

Figure 34. - Calculated and experimental impingement depth versus oil-jet pressure at 4920 and 2560 rpm.

ratios and oil-jet to pitch-line-velocity ratios. An experimental program was conducted on the Lewis gear test rig using high-speed photography to determine the oil-jet impingement depth on the disengaging side of mesh. Impingement depth reaches a maximum at gear ratios near 1.5 where chopping by the leading gear tooth limits the impingement depth. The pinion impingement depth is zero above a gear ratio of 1.172 for a jet velocity to pitch-line velocity ratio of 1.0 and is similar for other velocity ratios. The impingement depth for gear and pinion are equal and approximately one-half the maximum at a gear ratio of 1.0. Impingement depth on either the gear or pinion may be improved by relocating the jet away from the pitch line or by changing the jet angle. Results of the analysis were verified by experimental results using the high-speed camera and a well-lighted oil jet.

Gear Noise

Gear noise is being studied analytically by Lewis contractor Bolt, Beranek, and Newman, Inc. Since noise absorption material in helicopters adversely affects payload capacity, a method of
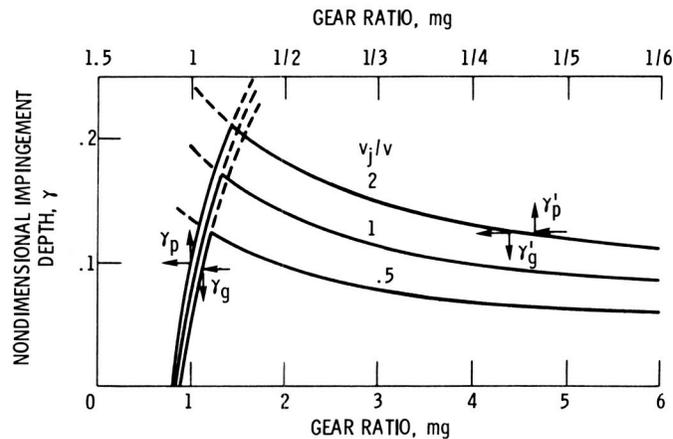
Figure 35. - Gear ratio versus nondimensional impingement depth. Speed, 3600 rpm; jet pressure, 17×10^4 Pa (25 psi); $N_p = 28$.

minimizing noise through better gear design is needed. This way gear noise can be minimized at the source, which is at the gear mesh. The approach is to develop a method for synthesizing optimum modifications of tooth surfaces that will minimize the dynamic loading and noise generated by gear teeth. The work includes developing computer programs for the tooth modification synthesis procedure and for predicting the Fourier series coefficients of vibratory excitation caused by elastic tooth deformations and deviations of tooth faces from perfect involute surfaces. A gear dynamic analysis program also will be written. These programs will be used to design an optimum gear pair to be tested at Lewis and to compute the expected dynamic improvement of the new design in comparison with conventional profile modifications for helicopter transmissions.

Effect of NASA Gear Research Programs

Current design methodology for transmission systems uses relatively standard stress calculations and methods derived from AGMA standards. These methods have proven satisfactory for current state-of-the-art applications. However, transmissions can be sized not only for stress, speed, and ratio, but also for life. Methods for transmission life predictions are inadequate and, for the most part, inconsistent from one design group to another. Assuming an infinite gear pitting life is not technically acceptable. The life-prediction analysis being developed at Lewis will allow for consistent and more accurate life-prediction methods.

Because of the requirement for higher temperature transmission applications, new gear materials, for which we have limited experience, are being used. In addition, heat treat specification and control can significantly affect the life and reliability of a gear system. Experimental definition of the relative life of gear materials and their heat treatment can aide in the selection of gear materials and in determining life-adjustment factors for life-prediction methods. Materials can be rationally selected for longer life application.

Proper gear lubrication and cooling has basically been an art. In general, choice of lubricant flow and pressure and nozzle position and type has been based on previous experience and trial and error methodology. Equations have been developed whereby the position of the oil jets and lubricant flow rate can be determined with reasonable certainty to obtain optimized oil volume and gear operating temperatures for a given transmission design.

The application of elastohydrodynamic (EHD) lubrication analysis to gear design and operation will enhance gear life and operation. The effect of EHD film thickness in determining transmission life and reliability has, for the most part, been ignored by the transmission designer. Furthermore, lubricant temperature affects film thickness and gear-tooth temperature. Hence, cooling analysis of the gear must be integrated with the EHD analysis. Selection of a lubricant affects both parameters and can affect the efficiency of a gearbox as well as its life.

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Analytical methods to accurately predict transmission noise is another requirement which becomes a potential design tool. While life and reliability as well as efficiency are prime design considerations, alternative designs of equal merit may result in significant reduction of noise amplitudes. Proven analytical methods for predicting noise would aid in the proper design, selection, or modification of gear systems that minimize noise and optimize mechanical performance.

Accurate predictions for bending stress and dynamic Hertz stress in the contact zone of gear teeth are very important factors in transmission design. Finite-element techniques for both spur and bevel gears will greatly improve current stress predictions. This would allow the design to more fully utilize the potential load capacity of advanced transmission systems and at the same time minimize transmission weight.

Tooth profile modifications and high-contact-ratio gearing offer the potential to increase the power-to-weight ratio for a given transmission application. Conversely, for a given transmission load, reliability and life should improve.

In essence, the effect of the NASA gear program will be to contribute technology towards more efficient transmission systems having higher power-to-weight ratios, longer lives, and higher reliabilities than current state-of-the-art systems. Furthermore, the use of improved gear materials and design methodology should improve transmission maintainability and mean time between removal (MTBR).

Future Requirements

Gearing requirements will continue to evolve. The condition of society and its needs as well as new discoveries, inventions, and new goals will keep the pressure on gear practitioners to meet the challenges. For example, 40 years ago, not many suspected that gears may be required to operate in the extreme environment of outer space, in cryogenic temperatures, in a vacuum, and without conventional lubrication. Now we can only imagine what may be required of the gearing technologists in the next 20 or 30 years.

From the standpoint of aircraft applications for gearing, continuing improvements are needed. There are several important topics that need further research for applications both now and in the next decade.

The requirements for advanced helicopter transmission and aircraft engine gearboxes include weight reduction and higher temperature operations than present-day aircraft, as well as increased reliability and service life. The gearing systems in these aircraft are expected to carry greater loads, operate at higher temperatures because of increased engine speeds, and provide improved system life, in addition to providing low maintenance rates and higher reliability. Very-high-temperature operation of gears is also required where the transmission must operate for short periods without lubrication and cooling.

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