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Lubrication Background

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ABS: Surface topography, including the various physical methods of measuring surfaces, and the various lubrication regimes (hydrodynamic, elastohydrodynamic, boundary, and mixed) are discussed. The historical development of elastohydrodynamic lubrication is outlined. The major accomplishments in four periods, the pre-1950's, the 1950's, the 1960's, and the 1970's are presented.
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A lubricant is any substance that is used to reduce friction and wear and to provide smooth running and a satisfactory life for machine components. Most lubricants are liquids (like mineral oils, the synthetic esters and silicone fluids, and water), but they may be solids (such as PTFE) for use in dry bearings or gases (such as air) for us in gas bearings. An understanding of the physical and chemical interaction between the lubricant and the tribological surfaces is necessary if the machine elements are to be provided with a satisfactory life.

A valid representation of the surface topography is required to understand the different regimes of lubrication. It is known (Dowson, 1979) that most tribological surfaces are covered by asperities having slopes within the range 0° to 25° (0 to 0.44 rad), with the vast majority in the narrow band between 5° and 10° (0.09 and 0.17 rad). Indeed, many well-produced engineering surfaces have asperities with slopes of about 1° or 2°. Various physical methods have been developed to yield information on surface topography, including oblique sectioning, optical interferometry, electron microscopy, and profilometry. Of all these physical methods profilometry, in which a fine diamond stylus is drawn across the surface and its vertical excursions magnified and recorded, has found the widest application. The departure of the profile from the centerline can be identified by the rms (root mean square) and cla (center-line average), or the arithmetical mean deviation Ra as it is now known internationally, both of which indicate the roughness of the surface. These parameters can be obtained by applying the voltage signal from the profilometer to a meter, directly for the rms value and after full-wave rectification for the Ra value. Surface topography is discussed in more detail in Section 4.1.

The film parameter Λ, defined in equation (3.125), is a ratio of the film thickness to composite rms and is used to define the lubrication regimes:

(1) Hydrodynamic lubrication - or fluid-film lubrication - occurs when the lubricant film is sufficiently thick to prevent the opposing solids from coming into contact. The behavior is governed by physical properties of the bulk lubricant, notably the viscosity. The film parameter is generally greater than 10 ($\Lambda > 10$).

(2) Elastohydrodynamic lubrication is a form of fluid-film lubrication where elastic deformation of the bearing surfaces becomes significant. Film thickness is smaller than in hydrodynamic lubrication. However, no gross asperity contact occurs

between the surfaces. Both viscous and elastic properties predominate. The film parameter is generally between 3 and 10 ($3 \leq \Lambda < 10$).

(3) Boundary lubrication is the surface interaction between one or more molecular layers of boundary lubricants. The solids dominate the operation of the contact, fluid-film effects are negligible, and there is considerable asperity contact. The friction behavior is similar to that in a dry contact and the film parameter is less than unity ($\Lambda < 1$).

(4) Mixed lubrication is governed by a mixture of "boundary" and "fluid film" effects. Some asperity contact may occur, and interaction takes place between one or more molecular layers of boundary lubricating films. Partial fluid lubrication action develops in the bulk of the space between the solids. The film parameter is generally between 1 and 4 ($1 \leq \Lambda < 4$).

A more detailed discussion of the four lubrication regimes, along with a discussion of the transition between them, is given in Section 4.2. The historical development of elastohydrodynamic lubrication is outlined in Section 4.3. The major accomplishments in four periods are presented, namely, the pre-1950's, the 1950's, the 1960's, and the 1970's. The account of experimental and theoretical research covered in Section 4.3 does not attempt to include all the work ever done on elastohydrodynamic lubrication, but only to cover the major achievements in the field.

4.1 Surface Topography

The first step in gaining insight into the lubrication of solid surfaces is to examine the surface profile or topography. Smooth surfaces are not flat on an atomic scale. The roughnesses of manufactured surfaces used in lubrication are between $1x10^{-8}$ and $20x10^{-8}$ m; whereas typical atomic diameters are between $1x10^{-10}$ and $10x10^{-10}$ m. Even a highly polished surface, when examined microscopically or with a profilometer, has an irregular nature. The surface consists of high and low spots. The high spots or protuberances are also called asperities.

A typical metallic surface might appear somewhat like that shown in Figure 4.1, as described by Halling (1976), who defines the various layers in the structure of metallic surfaces as follows:

> On top of the normal crystalline structure lies a layer of deformed material created by the processes used in the manufacture of the surface. This layer is often overlaid by a microcrystalline layer which is also produced in the manufacturing process. In such processes the outermost molecular layers are melted and smeared over

the underlying material. The sudden cooling of this molten layer produces a structure of very fine crystals which is harder than the underlying material. This hard thin layer can be very important in producing high resistance of the surface to wear. There is also an outermost layer produced by chemical reaction of the surface with its environment. With steels in air this will be an oxide layer (rust) and it is of considerable significance in tribology. In a sense it acts as a barrier between metallic surfaces that are apparently in contact, and so helps to reduce the friction between them. Finally, the surface will usually be covered with dust, wear debris and possibly lubricant, and as Figure 2.1 (Fig. 4.1) shows, such particles are of a similar size to the intrinsic roughnesses of the surface boundary.

The geometric characteristics or surface texture of irregular surfaces fall into the three categories shown in Figure 4.2:

(1) Error of form, in which the surface deviates from a well-defined pattern because of errors inherent in the manufacturing process

(2) Waviness, which takes the form of relatively long-wavelength variations in surface profile and is often associated with the unwanted vibrations that always occur in machine tool systems

(3) Roughness, irregularities that exclude waviness and errors of form and are inherent in the actual cutting and/or polishing process during production. In the study of lubricated contacts this is the geometric variation that is generally of greatest interest.

A wide range of instruments is used to study these geometric characteristics, but the one most widely adopted is the profilometer. In a profilometer a very fine diamond stylus (tip radius, $(2 \pm 0.5) \times 10^{-6}$ m) is drawn over the surface. The vertical movement of the stylus as it traverses the profile is measured and amplified, usually electronically, so that the recorded output provides a picture of the actual surface. The static load of the stylus on the surface is normally less than 0.0007 N (0.00256 oz).

Profilometry was introduced by Abbott and Firestone in 1933 at the University of Michigan. One of the best known instruments of this kind is the Talysurf. One of the most attractive features of this instrument is its flexibility in controlling the horizontal and vertical magnification independently. The horizontal magnification is controlled by the speed of traversing and the speed of the paper on which the profile record is produced. The vertical magnification is controlled electronically. The vertical magnification is normally between 100 and 100,000, and the horizontal magnification between 10 and 5000. A typical ratio of vertical-to-horizontal magnification is 50. The difference in the vertical and horizontal magnifications, although useful in giving greater emphasis to the height characteristics of the surface, does mean that the resulting record is distorted. The surface asperities are undulations rather than the sharp peaks that are shown in the distorted profiles. The finite size of the stylus is the ultimate limit on the resolution.

There are a number of other methods of studying surface geometry besides profilometry. Some of these other alternative techniques are

(1) Oblique sectioning

(2) Optical interferometry

(3) Electron microscopy

(4) Light microscopy

A discussion of these approaches can be found, for example, in Halling (1976).

A horizontal line passing through the center of the area of the distribution curve obtained from a profilometry trace is defined as the centerline of the profile. The areas generated by the surface profile above and below this centerline are equal. The departure of the profile from this centerline may be identified by the parameters rms (root mean square) and cla (centerline average) or Ra (roughness average). The Ra value is defined as the arithmetic average value of the vertical deviation of the profile from the centerline, and the rms value as the square root of the arithmetic mean of the square of this derivation. In mathematical form they can be written as

$$\text{cla or Ra} = \frac{1}{n}\sum_{i=1}^{n} |z_i| \tag{4.1}$$

$$\text{rms} = \left[\frac{1}{n}\sum_{i=1}^{n} (z_i)^2\right]^{0.5} \tag{4.2}$$

where n is the number of points on the centerline at which the profile deviation z_i is measured. For most surfaces the Ra and rms values are very similar. Both parameters can be obtained by applying the voltage signal from the profilometer to a meter, directly for the rms value and after full-wave rectification for the Ra value. For this reason these parameters are extensively used in defining surfaces.

4.2 Lubrication Regimes

By the middle of the twentieth century two distinct regimes of lubrication were generally recognized. The first of these was fluid-film or hydrodynamic lubrication. The development of the understanding of this lubrication regime began with classical experiments by Tower (1883) in which the existence of a film was detected from measurements of pressure within the lubricant and by Petrov (1883), who reached the same conclusion from friction measurements. This work was closely followed by Reynolds' (1886) celebrated analytical paper in which he used a reduced form of the Navier-Stokes equations in association with the continuity equation to generate a second-order differential equation for the pressure in the narrow converging gap of a bearing contact. Such a pressure enables a load to be transmitted between the surfaces with very low friction, since the surfaces are completely separated by a film of fluid. In such a situation it is the physical properties of the lubricant, notably the dynamic or absolute viscosity, that dictate the behavior of the contact.

The second lubrication regime clearly recognized by 1950 was boundary lubrication. The understanding of this lubrication regime is normally attributed to Hardy and Doubleday (1922a and b), who found that very thin films adhering to surfaces were often sufficient to assist relative sliding. They concluded that under such circumstances the chemical composition of the fluid was important, and they introduced the term "boundary lubrication." Boundary lubrication is at the opposite end of the lubrication spectrum from fluid-film or hydrodynamic lubrication. In boundary lubrication it is the physical and chemical properties of thin films of molecular proportions and the surfaces to which they are attached that determine contact behavior, the lubricant viscosity not being an influential parameter.

In the last 30 years research has been devoted to a better understanding and more precise definition of lubrication regimes between these extremes. One such lubrication regime occurs in nonconformal contacts, where the pressures are high and the bearing surfaces deform elastically. In this situation the viscosity of the lubricant rises considerably, and this further assists the formation of an effective fluid film. A lubricated contact in which such effects are to be found is said to be operating elastohydrodynamically. Significant progress has been made in understanding the mechanism of elastohydrodynamic lubrication, and it is generally viewed as reaching maturity.

Since 1970 it has been recognized that between elastohydrodynamic and boundary lubrication some combined mode of action can occur. This mode is generally termed "mixed lubrication." To date, most of the scientific unknowns lie in this lubrication regime. An interdisciplinary approach will be needed to gain an understanding of this important lubrication mechanism.

4.2.1 Hydrodynamic or Fluid-Film Lubrication

Fluid-film lubrication occurs when the lubricant film is sufficiently thick to prevent the opposing solids from coming into contact. This condition is often referred to as the ideal form of lubrication since it provides low friction and a high resistance to wear. The behavior of the contact is governed by the bulk physical properties of the lubricant, notably viscosity, and the frictional characteristics arise purely from the shearing of the viscous lubricant.

The lubricant films are normally many times thicker than the surface roughness. The physical properties of the lubricant dictate contact behavior, and surface effects are negligible. The film thickness normally exceeds 10^{-6} m, and the film parameter Λ is in excess of 10 and may even rise to 100. Films of this thickness are clearly insensitive to chemical action in surface layers of molecular proportions. This lubrication mechanism normally occurs in self-acting, squeeze-film, and externally pressurized bearings.

4.2.2 Elastohydrodynamic Lubrication

Elastohydrodynamic lubrication is a form of fluid-film lubrication where elastic deformation of the bearing surfaces becomes significant. It is usually associated with highly stressed machine components of low conformity, like gears and rolling-element bearings. This lubrication mechanism is also encountered with soft bearing materials, like elastomeric seals and rubber tires. The common factors in these applications are that local elastic deformation of the solids provides coherent fluid films and that asperity interaction is largely prevented. Elastohydrodynamic lubrication normally occurs in contacts where the film thicknesses is in the range 10^{-7} m $\leq h \leq 10^{-6}$ m and the film parameter Λ is in the range $3 \leq \Lambda < 10$.

4.2.3 Boundary Lubrication

In boundary lubrication the solids are not separated by the lubricant, so fluid-film effects are negligible and there is considerable asperity contact. The contact lubrication mechanism is governed by the physical and chemical properties of thin surface films of molecular proportions. The properties of the bulk lubricant are of minor importance, and the coefficient of friction is essentially independent of the viscosity of the fluid. The frictional behavior is similar to that encountered in dry friction between solids. The surface films vary in thickness from 5×10^{-9} m to 10^{-8} m and the film parameter Λ is less than unity ($\Lambda < 1$).

4.2.4 Mixed Lubrication

The behavior of the conjunction in a mixed lubrication regime is governed by a combination of boundary and fluid-film effects. Some asperity contact may occur. Interaction takes place between one or more molecular layers of boundary lubricating films. A partial fluid-film lubrication action develops in the bulk of the space between the solids. The film thickness in a mixed lubrication contact is less than 10^{-8} m and greater than 10^{-6} m. The film parameter Λ is normally between 1 and 4.

4.2.5 Transition Between Regimes

The transition from boundary to fluid-film lubrication does not take place instantaneously as the severity of loading is decreased, but an increasing proportion of the load is carried by pressures within the fluid that fills most of the space between the opposing solids. Indeed, it is often difficult to eliminate fluid-film lubrication effects to enable true boundary lubrication to occur, and there is evidence to suggest that micro-fluid-film lubrication associated with a number of miniature bearings formed by surface irregularities is an important effect in some processes.

The variation of coefficient of friction μ with the film parameter Λ is shown in Figure 4.3. In this figure the approximate locations of the various lubrication regimes already discussed are shown. This figure shows that as the film parameter Λ increases there is initially a decrease in coefficient of friction in the elastohydrodynamic regime followed by an increase in coefficient of friction in the hydrodynamic regime.

In explaining this phenomenon let us assume that the surface roughness is the same in both lubrication regimes. The coefficient of friction is defined as

$$\mu = \frac{\tilde{T}}{F} \tag{4.3}$$

where $\tilde{T}$ is the tangential (friction) force and F is the normal force.

In hydrodynamic lubrication of conformal contacts as found in journal and thrust bearings $F \propto 1/h^2$. In elastohydrodynamic lubrication the normal force has little effect on the film thickness. In the film thickness equation (8.23) the exponent on F is -0.073, so we can say that F is essentially proportional to a constant. In hydrodynamic and elastohydrodynamic lubrication $\tilde{T} \propto 1/h$. Making use of this we can write

$$(\mu)_{HL} \propto \frac{\frac{1}{h}}{\left(\frac{1}{h}\right)^2} \propto h \tag{4.4}$$

$$(\mu)_{EHL} \propto \frac{\frac{1}{h}}{\text{Constant}} \propto \frac{1}{h} \tag{4.5}$$

This then explains the general form of Figure 4.3.

4.3 Elastohydrodynamic Lubrication History

Historically, a recognition and understanding of elastohydrodynamic lubrication may be viewed as one of the major developments in the field of tribology in the twentieth century. It not only revealed the existence of a previously unsuspected regime of lubrication in highly stressed and nonconformal machine elements like gears and rolling-element bearings, but it brought order to the understanding of the complete spectrum of lubrication, ranging from boundary to hydrodynamic, as was shown in the preceding section. The present section attempts to trace the development of understanding of elastohydrodynamic lubrication by pointing out landmarks in the short history of this subject.

4.3.1 Pre-1950's - Martin to Grubin

Much of the early interest in the subject now known as elastohydrodynamic lubrication was generated by the need to understand the mechanism of gear lubrication. A suggestion that a full hydrodynamic film separated the opposing teeth was examined theoretically by Martin in 1916. Martin considered rigid solids and an incompressible, isoviscous lubricant. His solution of the Reynolds equation for a lubricating film between two rigid circular cylinders presented a useful beginning to theoretical studies.

Martin's theoretical work discouraged the view that spur gears could be lubricated by hydrodynamic action since his results indicated that film thicknesses of the order of 3×10^{-8} m would be required to support the applied load under typical conditions. This separation is, of course, considerably smaller than the known surface irregularities of gear teeth. This discouraging result probably accounts for the long-time interval of about 20 years before the next significant flurry of theoretical work on gear lubrication. Peppler (1936) and Meldahl (1941) considered the effect of local elastic distortion on the predictions of hydrodynamic theory. Although their investigations failed to demonstrate the full significance of elastic effects, they did point the way for future investigations.

A very important step in the development of experimental investigations was recorded in 1935 when Merritt built a disc machine to simulate gear tooth contact conditions. The two discs were designed to represent the contact geometry at a selected position in the meshing cycle, and the coefficient of friction was recorded over a wide speed range. Disc machines in various forms have provided the basic apparatus for most elastohydrodynamic lubrication experiments in recent years and are discussed more fully in Chapter 10.

The first satisfactory solution to take account of the effect of elastic distortion and viscosity-pressure action was reported by Grubin (1949). Grubin adopted a set of very plausible assumptions to enabled him to obtain an approximate analytical solution to a complex theoretical problem.

The essence of Grubin's analysis was that he assumed that the shape of the elastically deformed solids in a highly loaded lubricated contact was the same as the shape produced in a dry contact. This assumption facilitated the solution of the Reynolds equation in the inlet region to the conjunction and enabled the separation of the solids in the central region to be determined with commendable accuracy. The most valuable result of this analysis was a film thickness equation for highly loaded elastic contacts. The equation predicted film thicknesses one or two orders of magnitude greater than the corresponding rigid-cylinder, isoviscous fluid values, and the possibility of satisfactory fluid-film action was established theoretically for the first time. Grubin's analysis produced an excellent account of the physical mechanism of the lubrication process in highly loaded line contacts, and it marked a very important development in the history of elastohydrodynamic lubrication. Although this solution is commonly known as the Grubin theory, it has been suggested that Ertel's name should also be associated with the theory (Cameron, 1966).

4.3.2 1950's - Blok to Dowson and Higginson

The influence of pressure-viscosity effects on theoretical solutions to rigid-cylinder lubrication was examined by McEwen (1952) and Blok (1952). McEwen (1952) considered a relationship of the form $\eta = \eta_0(1 + p/k)^n$, and Blok (1952) introduced the now familiar exponential expression $\eta = \eta_0 e^{\alpha p}$.

The outstanding feature of these investigations was that they predicted a minimum film thickness increase of about 150 percent over the constant-viscosity (Martin) solution. Blok also noted that the exponential relationship introduced a mathematical limitation on minimum film thickness since the pressure reached an infinite value when the minimum film thickness fell to a certain value. The load capacity under these limiting conditions was found to be about 2.3 times the corresponding Martin theory.

More complete solutions to the elastohydrodynamic lubrication problem that simultaneously satisfied the governing elastic and hydrodynamic equations were presented by Petrusevich (1951). The three main features of these solutions are now recognized as general characteristics of many elastohydrodynamic contacts.

(1) An almost parallel oil film in the central region of the contact with a restriction near the outlet

(2) A near-Hertzian pressure curve over most of the contact region

(3) A very local second pressure maximum of considerable height near the outlet end of the contact region

Several numerical solutions to the isothermal elastohydrodynamic problem for cylindrical contacts were presented in the 1950's. Weber and Saalfeld presented their findings in 1954 and a new approach to elastohydrodynamic lubrication theory was presented by Dowson and Higginson (1959). Even for the relatively simple case of a constant-viscosity fluid, the straightforward iterative processes employed by Meldahl (1941) are tedious and slowly convergent. By introducing a solution to the inverse hydrodynamic lubrication problem, Dowson and Higginson (1959) were able to overcome this difficulty and to obtain satisfactory solutions to the elastic and hydrodynamic equations after a small number of calculation cycles. Normally a solution of the Reynolds equation calls for the determination of a pressure distribution corresponding to a given film shape. In the inverse problem the film shape responsible for the generation of a given pressure distribution is determined. In the procedure adopted by Dowson and Higginson the computed film shape was compared with the shape of the elastically deformed solids, and the pressure curve was then modified to improve the agreement between the two shapes. Although the computational method produced an acceptable solution in a small number of cycles, the procedure was not fully automatic. Judgment was needed in modifying the pressure curve after each elastic and inverse hydrodynamic calculation. Although it was relatively easy to provide this judgment when calculations were performed on hand-operated desk calculating machines, the procedure was not well suited to high-speed digital computers.

These advances in the theoretical work in the 1950's were matched by reliable experimental work. Interest was focused on the all-important film thickness, and disc machines were widely employed for the investigations. The capacitance method was the most reliable measuring system to emerge during the 1950's. Lewicki (1955) reported interdisc capacitance measurements on a two-disk machine that seemed to indicate a film thickness of about 1 μm (40 μin.). The method was refined and related to a more accurate shape of deformed cylinders by Crook. At an early stage of his experiments with disc machines, Crook (1957) noted that the lubricating oil formed films on the surfaces and was only slowly replaced by fresh oil from the supply. He later found (Crook, 1958) that if the oil supply was cut off, the film of retained oil continued to lubricate the disc satisfactorily for at least 30 minutes. During this period the film thickness fell by about 30 percent, and it was amply demonstrated that this reduction was not caused by a loss of oil but was attributable to a rise in the temperature of the disc and the associated reduction in the viscosity of the oil. It was therefore clear that the surface temperature of the discs had a marked influence on their lubrication.

4.3.3 1960's - Crook to Archard and Cowking

Crook's experimental studies continued in the 1960's. Crook (1961) confirmed the order of magnitude of film thickness deduced by Lewicki (1955), and he was able to provide direct evidence of the influence of load and speed on film thickness.

Load was found to have an almost negligible effect on film thickness, but speed was found to play a very important role. The method used by Crook was refined and used convincingly by Dyson, et al. (1965-66) in a study that covered a wide range of lubricants.

An interesting paper by Archard and Kirk (1961) demonstrated that fluid-film lubrication could also occur in some highly loaded point contacts. Before the publication of this paper it had been considered that only boundary lubrication could occur under such extreme conditions. Archard and Kirk's experiments with crossed cylinders also showed that the values of film thickness at a point contact with a circular Hertzian region differed less than might have been expected from those at a line contact. A comparison of the values of film thickness for a line contact (Crook, 1961) with those for a point contact (Archard and Kirk, 1961) under otherwise similar conditions shows that the two values differ by roughly a factor of 2.

An alternative experimental approach to the capacitance method was described by Sibley and Orcutt (1961) when they presented film thickness measurements based on an X-ray transmission technique. Results were obtained for a range of speeds and loads and for three lubricants having quite different viscosities. In addition, the transverse profiles of the elastically deformed solids in the vicinity of the contact were recorded.

The film thicknesses recorded by Sibley and Orcutt were in overall agreement with Crook's (1961) measurements.

The latter half of the 1960's saw a flurry of activity in optical elastohydrodynamics. The first significant publication on this topic was by Cameron and Gohar (1966). They loaded a lubricated, rotating steel ball against a stationary plate of high-refractive-index glass and obtained interference patterns that were the first to show the now classic horseshoe constriction of elastohydrodynamic point contact. Their use of sliding contact and special glass imposed a severe restriction on loads, speeds, and fringe quality. Gohar and Cameron (1966) reduced these limitations by using sapphire and diamond as the transparent member. A great improvement in fringe quality was obtained by Foord, et al. (1968) who, instead of relying on differences in refractive index, used a 20-percent-reflectivity layer of chromium. This allowed the transparent material to be selected for its mechanical properties and enabled them to use pure rolling and high speeds. It is clear that the experimental work of the 1960's provided valuable data on the lubrication of highly loaded contacts, and we now turn to the development of elastohydrodynamic theory during this era.

The Dowson and Higginson theory developed in 1959 was further automated in their 1961 paper. The digital computer was used to great advantage in developing numerical solutions for particular values of the independent variables. These solutions have enabled the roles of the speed, load, and materials properties to be clearly ascertained. In particular the influence of these parameters on minimum film thickness became clear. Dowson and Higginson (1961) then produced an empirical formula for this important feature of highly loaded, lubricated contacts on the basis of their theoretical solutions for line contacts. The importance of adequate minimum film thickness in elastohydrodynamic contacts cannot be overemphasized. It represents a necessary condition for successful fluid-film lubrication, and the machine designer must consider this point whenever he is concerned with the lubrication of highly stressed contacts. An interesting historical point is that it took 45 years to develop from the Martin (1916) solution to the full isothermal elastohydrodynamic solution for minimum film thickness of line contacts (Dowson and Higginson, 1961). Close agreement was found between the theoretical minimum-film-thickness predictions of Dowson and Higginson (1961) and the experimental results of Crook (1961) and Sibley and Orcutt (1961). This close agreement demonstrated that the gap between theory and experiment that was present at the time of Martin's 1916 work had been largely closed for highly stressed line contacts by the early 1960's.

Once agreement had been obtained, after nearly half a century, between theory and experiment in relation to film thickness in highly stressed line contacts, attention turned to other aspects of elastohydrodynamic lubrication. Analyses of thermal effects in line contacts exhibiting mixed rolling and sliding was performed by Cheng and Sternlicht in 1964 and Dowson and Whitaker in 1966. These authors considered the difficult mathematical problem presented by the requirement for a solution of the Reynolds, elasticity, energy, and heat-transfer equations for a line contact. The results indicated that the basic features of elastohydrodynamic contacts indicated by isothermal theory were also evident in solutions for sliding or thermal conditions. In particular, the sharp secondary pressure peak predicted for a Newtonian fluid in pure rolling conditions was found to persist, and even grow, when some sliding was introduced.

Another feature of the Cheng and Sternlicht (1964) solutions that was of considerable practical importance was that the calculated film thickness was not appreciably influenced by the oil film temperatures generated in sliding contacts. The film thickness formula for isothermal line contacts thus gives good approximations to the measured film thickness if the lubricant viscosity corresponding to the temperature at the film inlet is considered.

Archard and Cowking (1965-66) developed elastohydrodynamic theory for nominal point contacts of the form encountered between two spheres and thus extended the classical study of Kapitza (1955) as Grubin (1949) had extended that of Martin (1916). The Hertzian contact zone was assumed to form a parallel film region, and the generation of high pressure in the approaches to the Hertzian zone was considered. The results of the Archard and Cowking (1966) analysis led to the concept of a side-leakage or ellipticity factor λ_b, which represents the proportional reduction in pressure attributable to side leakage. In the isoviscous theory of an undeformed nominal point contact, λ_b is a constant equal to $\left[1 + (2R_x/3R_y)\right]^{-1}$ where R_x and R_y are the effective radii of curvature parallel to and perpendicular to the direction of motion, respectively. This concept was used by Archard and Cowking (1966) in deriving simple elastohydrodynamic theories for a nominal point contact. Their theories agree reasonably well with the film thickness experiments of Archard and Kirk (1961), who used a crossed-cylinders machine under conditions in which R_y/R_x was varied between approximately 0.3 and 12.0.

4.3.4 1970's - Cheng to the Present Day

By the beginning of 1970 experimentally determined film thicknesses were fairly well established. Experimentally the 1970's saw attention turned to other aspects of elastohydrodynamic lubrication such as pressure, temperature, rheology, and traction. Pressure distribution within a conjunction was revealed by the use of fine strips of manganin (an alloy of copper, nickel, and manganese, whose resistance is influenced by pressure) deposited on the insulated surfaces of discs. The technique originally developed by Kannel, et al. (1964) was used to reveal the pressure distribution in an impressive study by Hamilton and Moore (1971). They found that the pressures near the outlet that are predicted in theory are considerably attenuated in practice but do occur in the correct positions.

Hamilton and Moore (1971) also measured temperatures within elastohydrodynamic conjunctions by embedded and trailing thermocouples, by films of nickel on insulated discs, and by direct measurement of infrared radiation. Although the probes used were small, they accounted for 8 to 10 percent of the film thickness under typical operating conditions. The conditions were limited in severity because of the glass surface and the fragile nature of the gauges.

A noncontacting technique based on infrared measurements was developed by Turchina, et al. (1974) and further developed by Ausherman, et al. (1976) and Nagaraj, et al. (1977) for the measurement of film temperature. It does not interfere in any way with the operation of the elastohydrodynamic lubricated conjunction, and it can be used under conditions of contact severity comparable to those encountered in real engineering applications.

Two areas of research in the physics of viscous fluids in the 1970's cast valuable light onto the problem of the rheology of elastohydrodynamic lubricated films. The response of viscous liquids to high-frequency shear has shown that such "super-cooled" liquids have viscoelastic properties when the strain rates are comparable with the natural relaxation rates of the fluid. Studies by Alsaad, et al. (1978) have shown that such fluids exhibit a "glass transition" and become solidlike, when either the temperature is reduced or the pressure increased sufficiently for the liquid "free volume" to be virtually eliminated.

Direct observation of the viscoelastic behavior of elastohydrodynamic films in the linear region, where sliding speeds are extremely low, were made by Johnson and Roberts (1974) and Hirst and Moore (1974). At higher sliding speeds the relationship between shear stress (traction) and strain rate (slip) becomes very nonlinear. This nonlinearity in terms of the stress-strain behavior of the lubricant in its "glassy" state was considered by Johnson and Tevaarwerk (1977). They found that it was indeed better to describe the oil in a highly loaded elastohydrodynamic lubricated contact as an elastic-plastic solid rather as a viscous liquid. Based on this new understanding, they put forward a theory of elastohydrodynamic lubrication traction that is being applied to engineering components such as ball bearings and traction drives.

In the calculations of elastohydrodynamic oil film thickness discussed in this text the lubricant has been assumed to be a Newtonian fluid. That is, it is assumed to be perfectly viscous, with the rate of shear being linearly related to the shear stress. Jacobson (1970, 1972, and 1973) indicated that in nonconforming contacts, such as those that occur in rolling-element bearings, the lubricant will solidify because of the high pressures normally encountered. When the oil is solidified, the pressure buildup is a result of the shear strength and the compressibility of the oil, as well as the elastic properties of the solids. Jacobson described the basic approach used to determine the pressure distribution and the fluid-film thickness in such conjunctions in his 1970 paper. The basic requirement is that the condition of continuity of mass flow has to be satisfied at the boundaries between the liquid oil and the solidified oil regions, within the solidified oil region at the boundaries between sliding and nonsliding solidified oil, and on the cavitation boundary forming the downstream part of the liquid oil regime at the outlet. Jacobson applied his method to a circular contact in his 1972 paper and to a rectangular contact in his 1973 paper. The results presented in these papers are interesting, but at the present time they must be viewed as an initial attempt to incorporate some of the developing ideas of lubricant rheology into elastohydrodynamic theory for nominal line and point contacts.

A major change in the direction of theoretical studies of elastohydrodynamic lubrication in the 1970's arose from a switch from considerations of nominal line contacts to elliptical contacts. Other than the Archard and Cowking (1966) formulation of a side-leakage factor, the literature was essentially devoid of theoretical elliptical-contact film thickness formulations.

In 1970 Cheng developed a Grubin type of inlet analysis that was applicable to contacts whose Hertzian contacting area was elliptical. The surface deformation was assumed to be governed by classical Hertzian theory (Hertz, 1881) and the surface velocities u_a and u_b were assumed to be in the direction of the principal axes. Results were presented as side-leakage film reduction factors for an ellipticity parameter of $k \geq 5$ (approximates a line contct), for $k = 1$ (circular contact), and for $1 < k < 5$ (elliptical contact).

An interesting numerical solution of the elastohydrodynamic point-contact problem for a sphere near a plane was put forth by Ranger, et al. (1975). This solution was presented in dimensional terms, which thereby limited general usage. A puzzling feature of the Ranger, et al., work, however, was that their resulting equation for the minimum film thickness had a positive load exponent, which contradicts experiments (e.g., Cameron and Gohar, 1966, and Archard and Kirk, 1961).

Only in the 1970's did the complete numerical solution of the isothermal elastohydrodynamic lubrication of elliptical contacts successfully emerge. The analysis requires the simultaneous solution of the elasticity and Reynolds equations. In the years 1974 to 1979 Hamrock and Dowson published eight papers on elastohydrodynamic lubrication (1974, 1976a and b, 1977a and b, 1978, and 1979a and b). Their approach to the theoretical solution is presented in Hamrock and Dowson (1974 and 1976a). The first of these publications presents an elasticity model in which the conjunction is divided into equal rectangular regions with a uniform pressure applied over each region. This material is also summarized in Chapter 5. The second paper (Hamrock and Dowson, 1976a) gives a complete approach to the solution of the elastohydrodynamic lubrication problem for elliptical contacts.

The most important practical aspect of the elastohydrodynamic lubrication elliptical-contact theory is the determination of the minimum film thickness within the conjunction. That is, the prediction of a film of adequate thickness is extremely important for the successful operation of machine elements in which these thin continuous fluid films occur. Hamrock and Dowson (1977a) presented results for fully flooded conjunctions. A fully flooded condition is said to exist when the extent to which the inlet region of the conjunction is filled with lubricant ceases to influence in any significant way the minimum film thickness. In Hamrock and Dowson (1977a) the influence of the ellipticity parameter and the dimensionless speed, load, and materials parameters on minimum film thickness was investigated. Thirty-four cases were used to generate a fully flooded minimum-film-thickness formula that can be used in the analysis of a wide range of highly stressed, lubricated machine elements.

In Hamrock and Dowson (1977b) the effect of lubricant starvation on the pressure and film thickness was studied. A simple expression for the critical dimensionless boundary distance at which lubrication starvation starts to become important was obtained. Fifteen cases, in addition to three presented in Hamrock and Dowson (1977a), were used to obtain simple expressions for the minimum and central film thicknesses in a starved conjunction.

The work presented in Hamrock and Dowson (1974, 1976a and b, and 1977a and b) related to materials of high elastic modulus (e.g., steel). The subsequent work reported in Hamrock and Dowson (1978 and 1979a) related to materials of low elastic modulus (e.g., rubber). For such materials the distortions are large, even with light loads. Hamrock and Dowson (1978) presented, to the best of the authors' knowledge, the first complete numerical solution of the problem of fully flooded, isothermal, elastohydrodynamic lubrication of elliptical contacts for low-elastic-modulus materials. That is, no assumptions were made as to pressure or film thickness within the contact, and compressibility and viscous effects were considered. Hamrock and Dowson (1979a) later extended the analysis to take account of lubricant starvation in such conjunctions.

4.4 Closure

In this chapter we have traced the development of understanding of the physical and chemical action of the lubricant within lubricated conjunctions. The first step is to develop a valid representation of the surface topography. Various physical methods have been mentioned that yield information on surface topography, including oblique sectioning, optical interferometry, electron microscopy, and profilometry. Of all these, profilometry, in which a fine diamond stylus is drawn across the surface and its vertical excursions magnified and recorded has found the widest application. The profilometer can be used to measure the root mean square (rms) and the centerline average (cla), or arithmetical mean deviation (Ra), which are well-known parameters used to define the roughness of surfaces.

The film parameter Λ was defined as a ratio of the film thickness to the composite surface roughness. The film parameter has been used to define four important lubrication regimes: fluid film or hydrodynamic, elastohydrodynamic, boundary, and mixed. <u>Hydrodynamic or fluid-film lubrication</u> occurs when the lubricating film is sufficiently thick to prevent the opposing solids from coming into contact. <u>Elastohydrodynamic lubrication</u> is a form of fluid-film lubrication where elastic deformation becomes significant. In <u>boundary lubrication</u> the surface interaction between one or more molecular layers of boundary lubricants and the solids dominates the operation of the contact. Mixed lubrication is governed by a mixture of boundary and fluid-film effects. Most of the scientific unknowns lie in this lubrication regime.

A brief history of elastohydrodynamic lubrication has also been presented in this chapter to introduce the major subject of this text. The major accomplishments in four periods have been presented, namely, the pre-1950's, the 1950's, the 1960's, and the 1970's. The approach to experimental and theoretical research covered in these periods has necessarily been selective rather than exhaustive, but we have endeavored to cover major accomplishments during these periods, which have witnessed most important and exciting developments in our understanding of the lubrication of highly stressed, lubricated machine elements.

SYMBOLS

A	constant used in equation (3.113)
$A^*, B^*, C^*, D^*, L^*, M^*$	relaxation coefficients
A_v	drag area of ball, m^2
a	semimajor axis of contact ellipse, m
$\bar{a}$	$a/2\bar{m}$
B	total conformity of bearing
b	semiminor axis of contact ellipse, m
$\bar{b}$	$b/2\bar{m}$
C	dynamic load capacity, N
C_v	drag coefficient
$C_1,\ldots,C_8$	constants
c	19,609 N/cm^2 (28,440 lbf/in^2)
$\bar{c}$	number of equal divisions of semimajor axis
D	distance between race curvature centers, m
$\tilde{D}$	material factor
$\bar{D}$	defined by equation (5.63)
De	Deborah number
d	ball diameter, m
$\bar{d}$	number of divisions in semiminor axis
d_a	overall diameter of bearing (Figure 2.13), m
d_b	bore diameter, m
d_e	pitch diameter, m
d_e'	pitch diameter after dynamic effects have acted on ball, m
d_i	inner-race diameter, m
d_o	outer-race diameter, m

E	modulus of elasticity, N/m^2
E'	effective elastic modulus, $2\Bigg/\left(\dfrac{1-\nu_a^2}{E_a}+\dfrac{1-\nu_b^2}{E_b}\right)$, N/m^2
E_a	internal energy, m^2/s^2
$\tilde{E}$	processing factor
E_1	$[(\tilde{H}_{min} - H_{min})/H_{min}] \times 100$
$\mathscr{E}$	elliptic integral of second kind with modulus $(1 - 1/k^2)^{1/2}$
$\bar{\mathscr{E}}$	approximate elliptic integral of second kind
e	dispersion exponent
F	normal applied load, N
F^*	normal applied load per unit length, N/m
$\tilde{F}$	lubrication factor
$\bar{F}$	integrated normal applied load, N
F_c	centrifugal force, N
F_{max}	maximum normal applied load (at $\psi = 0$), N
F_r	applied radial load, N
F_t	applied thrust load, N
F_ψ	normal applied load at angle ψ, N
$\mathscr{F}$	elliptic integral of first kind with modulus $(1 - 1/k^2)^{1/2}$
$\bar{\mathscr{F}}$	approximate elliptic integral of first kind
f	race conformity ratio
f_b	rms surface finish of ball, m
f_r	rms surface finish of race, m
G	dimensionless materials parameter, αE
G^*	fluid shear modulus, N/m^2
$\tilde{G}$	hardness factor
g	gravitational constant, m/s^2

g_E	dimensionless elasticity parameter, $W^{8/3}/U^2$
g_V	dimensionless viscosity parameter, GW^3/U^2
H	dimensionless film thickness, h/R_x
$\hat{H}$	dimensionless film thickness, $H(W/U)^2 = F^2h/u^2\eta_0^2R_x^3$
H_c	dimensionless central film thickness, h_c/R_x
$H_{c,s}$	dimensionless central film thickness for starved lubrication condition
H_f	frictional heat, N m/s
H_{min}	dimensionless minimum film thickness obtained from EHL elliptical-contact theory
$H_{min,r}$	dimensionless minimum film thickness for a rectangular contact
$H_{min,s}$	dimensionless minimum film thickness for starved lubrication condition
$\tilde{H}_c$	dimensionless central film thickness obtained from least-squares fit of data
$\tilde{H}_{min}$	dimensionless minimum film thickness obtained from least-squares fit of data
$\overline{H}_c$	dimensionless central-film-thickness - speed parameter, $H_cU^{-0.5}$
$\overline{H}_{min}$	dimensionless minimum-film-thickness - speed parameter, $H_{min}U^{-0.5}$
$\overline{H}_0$	new estimate of constant in film thickness equation
h	film thickness, m
h_c	central film thickness, m
h_i	inlet film thickness, m

h_m	film thickness at point of maximum pressure, where $dp/dx = 0$, m
h_{min}	minimum film thickness, m
h_0	constant, m
I_d	diametral interference, m
I_p	ball mass moment of inertia, m N s^2
I_r	integral defined by equation (3.76)
I_t	integral defined by equation (3.75)
J	function of k defined by equation (3.8)
J^*	mechanical equivalent of heat
$\bar{J}$	polar moment of inertia, m N s^2
K	load-deflection constant
k	ellipticity parameter, a/b
$\bar{k}$	approximate ellipticity parameter
$\tilde{k}$	thermal conductivity, N/s °C
k_f	lubricant thermal conductivity, N/s °C
L	fatigue life
L_a	adjusted fatigue life
L_t	reduced hydrodynamic lift, from equation (6.21)
$L_1,\ldots,L_4$	lengths defined in Figure 3.11, m
L_{10}	fatigue life where 90 percent of bearing population will endure
L_{50}	fatigue life where 50 percent of bearing population will endure
ℓ	bearing length, m
$\bar{\ell}$	constant used to determine width of side-leakage region
M	moment, Nm

M_g	gyroscopic moment, Nm
M_p	dimensionless load-speed parameter, $WU^{-0.75}$
M_s	torque required to produce spin, N m
m	mass of ball, N s^2/m
m^*	dimensionless inlet distance at boundary between fully flooded and starved conditions
$\tilde{m}$	dimensionless inlet distance (Figures 7.1 and 9.1)
$\bar{m}$	number of divisions of semimajor or semiminor axis
m_W	dimensionless inlet distance boundary as obtained from Wedeven, et al. (1971)
N	rotational speed, rpm
n	number of balls
n^*	refractive index
$\bar{n}$	constant used to determine length of outlet region
P	dimensionless pressure
P_D	dimensionless pressure difference
P_d	diametral clearance, m
P_e	free endplay, m
P_{Hz}	dimensionless Hertzian pressure, N/m^2
p	pressure, N/m^2
p_{max}	maximum pressure within contact, $3F/2\pi ab$, N/m^2
$p_{iv,as}$	isoviscous asymptotic pressure, N/m^2
Q	solution to homogeneous Reynolds equation
Q_m	thermal loading parameter
$\bar{Q}$	dimensionless mass flow rate per unit width, $q\eta_0/\rho_0 E'R^2$
q_f	reduced pressure parameter
q_x	volume flow rate per unit width in x direction, m^2/s

q_y	volume flow rate per unit width in y direction, m^2/s
R	curvature sum, m
R_a	arithmetical mean deviation defined in equation (4.1), m
R_c	operational hardness of bearing material
R_x	effective radius in x direction, m
R_y	effective radius in y direction, m
r	race curvature radius, m
r_{ax}, r_{bx}, r_{ay}, r_{by}	radii of curvature, m
r_c, ϕ_c, z	cylindrical polar coordinates
r_s, θ_s, ϕ_s	spherical polar coordinates
$\bar{r}$	defined in Figure 5.4
S	geometric separation, m
S^*	geometric separation for line contact, m
S_0	empirical constant
s	shoulder height, m
T	τ_0/p_{max}
$\tilde{T}$	tangential (traction) force, N
T_m	temperature, °C
T_b^*	ball surface temperature, °C
T_f^*	average lubricant temperature, °C
ΔT^*	ball surface temperature rise, °C
T_1	$(\tau_0/p_{max})_{k=1}$
T_v	viscous drag force, N
t	time, s
t_a	auxiliary parameter
u_B	velocity of ball-race contact, m/s

u_c	velocity of ball center, m/s
U	dimensionless speed parameter, $\eta_0 u/E'R_x$
u	surface velocity in direction of motion, $(u_a + u_b)/2$, m/s
$\overline{u}$	number of stress cycles per revolution
Δu	sliding velocity, $u_a - u_b$, m/s
v	surface velocity in transverse direction, m/s
W	dimensionless load parameter, $F/E'R^2$
w	surface velocity in direction of film, m/s
X	dimensionless coordinate, x/R_x
Y	dimensionless coordinate, y/R_x
X_t, Y_t	dimensionless grouping from equation (6.14)
X_a, Y_a, Z_a	external forces, N
Z	constant defined by equation (3.48)
Z_1	viscosity pressure index, a dimensionless constant
$x, \tilde{x}, \overline{x}, \overline{x}_1$; $y, \tilde{y}, \overline{y}, \overline{y}_1$; $z, \tilde{z}, \overline{z}, \overline{z}_1$	coordinate system
α	pressure-viscosity coefficient of lubrication, m^2/N
α_a	radius ratio, R_y/R_x
β	contact angle, rad
β_f	free or initial contact angle, rad
β'	iterated value of contact angle, rad
Γ	curvature difference
γ	viscous dissipation, N/m^2 s
$\dot{\gamma}$	total strain rate, s^{-1}
$\dot{\gamma}_e$	elastic strain rate, s^{-1}
$\dot{\gamma}_v$	viscous strain rate, s^{-1}

γ_a	flow angle, deg
δ	total elastic deformation, m
δ^*	lubricant viscosity temperature coefficient, $^{o}C^{-1}$
δ_D	elastic deformation due to pressure difference, m
δ_r	radial displacement, m
δ_t	axial displacement, m
δ_x	displacement at some location x, m
$\bar{\delta}$	approximate elastic deformation, m
$\tilde{\delta}$	elastic deformation of rectangular area, m
ϵ	coefficient of determination
ϵ_1	strain in axial direction
ϵ_2	strain in transverse direction
ζ	angle between ball rotational axis and bearing centerline (Figure 3.10)
ζ_a	probability of survival
η	absolute viscosity at gauge pressure, N s/m^2
$\bar{\eta}$	dimensionless viscosity, η/η_0
η_0	viscosity at atmospheric pressure, N s/m^2
η_∞	6.31×10^{-5} N s/m^2(0.0631 cP)
θ	angle used to define shoulder height
Λ	film parameter (ratio of film thickness to composite surface roughness)
λ	equals 1 for outer-race control and 0 for inner-race control
λ_a	second coefficient of viscosity
λ_b	Archard-Cowking side-leakage factor, $(1 + 2/3\ \alpha_a)^{-1}$
λ_c	relaxation factor

μ coefficient of sliding friction

μ^* $\bar{\rho}/\bar{\eta}$

ν Poisson's ratio

ξ divergence of velocity vector, $(\partial u/\partial x) + (\partial v/\partial y) + (\partial w/\partial z)$, s^{-1}

ρ lubricant density, N s^2/m^4

$\bar{\rho}$ dimensionless density, ρ/ρ_0

ρ_0 density at atmospheric pressure, N s^2/m^4

σ normal stress, N/m^2

σ_1 stress in axial direction, N/m^2

τ shear stress, N/m^2

τ_0 maximum subsurface shear stress, N/m^2

$\tilde{\tau}$ shear stress, N/m^2

$\tilde{\tau}_e$ equivalent stress, N/m^2

$\tilde{\tau}_L$ limiting shear stress, N/m^2

Φ ratio of depth of maximum shear stress to semiminor axis of contact ellipse

Φ^* $P_H^{3/2}$

Φ_1 $(\Phi)_{k=1}$

ϕ auxiliary angle

ϕ_T thermal reduction factor

ψ angular location

ψ_ℓ limiting value of ψ

Ω_i absolute angular velocity of inner race, rad/s

Ω_o absolute angular velocity of outer race, rad/s

ω angular velocity, rad/s

ω_B angular velocity of ball-race contact, rad/s

ω_b angular velocity of ball about its own center, rad/s

ω_c	angular velocity of ball around shaft center, rad/s
ω_s	ball spin rotational velocity, rad/s

Subscripts:

a	solid a
b	solid b
c	central
bc	ball center
IE	isoviscous-elastic regime
IR	isoviscous-rigid regime
i	inner race
K	Kapitza
min	minimum
n	iteration
o	outer race
PVE	piezoviscous-elastic regime
PVR	piezoviscous-rigid regime
r	for rectangular area
s	for starved conditions
x,y,z	coordinate system

Superscript:

$(\bar{\ })$	approximate

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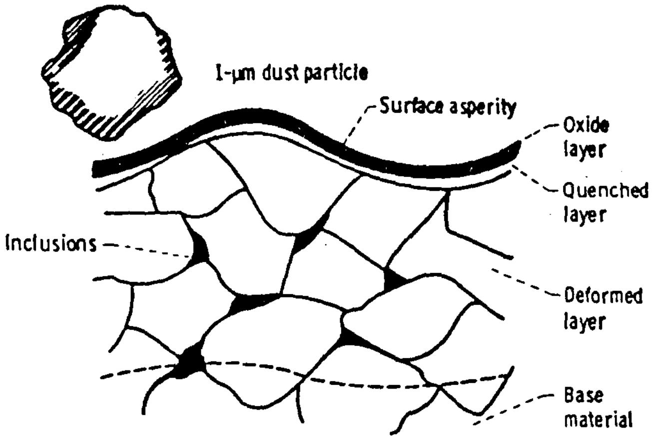

Figure 4.1. - Schematic diagram showing structure of metallic surfaces. (From Halling, 1976.)

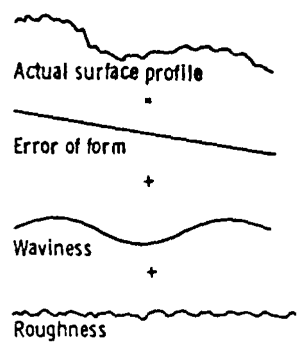

Figure 4.2. - Geometric characteristics of solid surfaces.

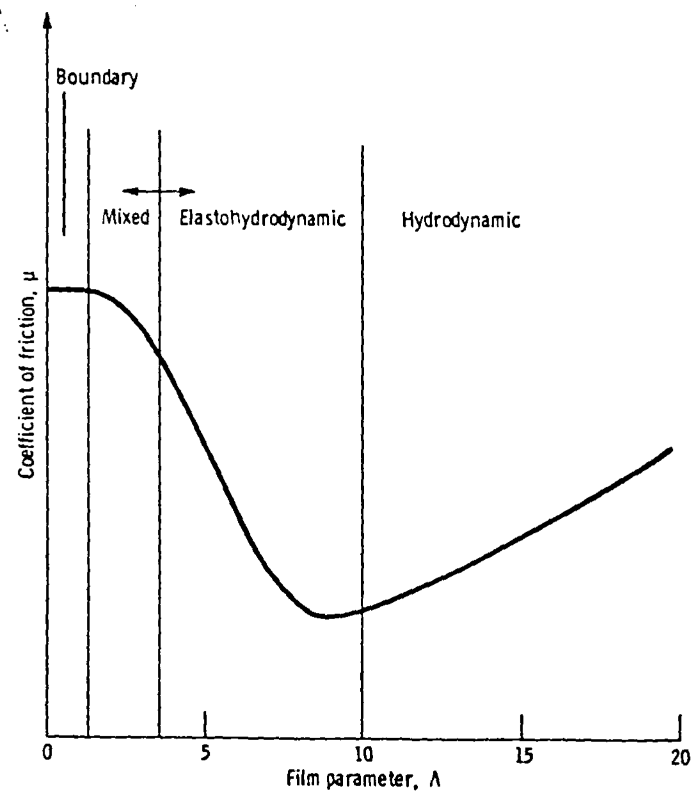

Figure 4.3. - Variation of coefficient of friction with film parameter.

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16. Abstract

Surface topography, including the various physical methods of measuring surfaces, and the various lubrication regimes (hydrodynamic, elastohydrodynamic, boundary, and mixed) are discussed. The historical development of elastohydrodynamic lubrication is outlined. The major accomplishments in four periods, the pre-1950's, the 1950's, the 1960's, and the 1970's are presented.

17. Key Words (Suggested by Author(s))	Surface topography; Hydrodynamic lubrication; Elastohydrodynamic lubrication; Mixed lubrication; Boundary lubrication; History of elastohydrodynamic lubrication
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