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A PRELIMINARY LOOK AT CONTROL AUGMENTED DYNAMIC
RESPONSE OF STRUCTURES

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16. ABSTRACT This report deals with the augmentation of structural characteristics, mass, damping, and stiffness through the use of control theory in lieu of structural redesign or augmentation. Treated first is the standard single-degree-of-freedom system followed by a treatment of the same system using control augmentation. The system is extended to elastic structures using single- and multi-sensor approaches and concludes with a brief discussion of potential application to large orbiting space structures.			
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TECHNICAL MEMORANDUM

A PRELIMINARY LOOK AT CONTROL AUGMENTED DYNAMIC RESPONSE OF STRUCTURES

I. INTRODUCTION

Classically, structural dynamicists and structural engineers have looked at structural redesign as the technique for solving structural response and loads problems. Generally, control engineers have used their discipline as a means of controlling some system state, such as space vehicle attitudes, automobile pollution control, etc. In recent years, the two fields have started merging when control has been used as a means of reducing structural weight in the aeronautics, aerospace, and automobile industries. The Space Shuttle is a good example of the use of vehicle load relief control system approaches to reduce the overall vehicle aerodynamic loading and thus save structural weight. This system used pitch and yaw acceleration feedback to reduce aerodynamic loading by reducing angle of attack, side slip, and rolling of the vehicle in such a manner as to load the Orbiter wings in the most favorable direction. In addition, elevon load relief was employed to reduce elevon loading during ascent. This system, however, only treated vehicle rigid body response. Improved approaches are being developed which move beyond rigid body response dealing with elastic body response leading to control configured, optimized design configurations. Active control configured flutter suppression, aeroelastic tailoring, modal suppression, and optimal design techniques fall under this general heading. These techniques have evolved as structural dynamicists and control engineers have recognized the potential of using control systems and control logic as means of altering structural dynamic responses, thus replacing structural redesign or structural weight with more efficient use of already existing control systems. Optimized design of new configurations naturally follows through the use of more complex control systems.

The evolution of the multidiscipline, structural control interaction has not developed as fast as it should. Several reasons for this slow development are clear: (1) normal protection of one's own discipline, (2) the use of different transformations, terms, nomenclature, etc., for solving the same type differential equations, and (3) lack of proper systems engineering and organizations to force the cross fertilization and system trades across these major disciplines.

Classically, if a structural dynamicist wants to change response or reduce loads, he changes stiffness by (1) adding or subtracting materials, (2) passive isolation of components, etc., (3) addition of passive dampers, and (4) detuning the system from the forcing function. This report will treat these same concepts first from classical vibration and elasticity theory and then show how control logic can accomplish the same goals while still preserving the nomenclature and form structural dynamicists are familiar with. A brief look will also be made in the control engineer's field showing how, with minimum effort, one can transform the knowledge and insight from structures to control and vice versa. This is accomplished by first repeating the basic vibration characteristics of a single degree-of-freedom mass, spring, damped, forced system. The basic equation is then recast with a control system feedback logic, then put in the same form as the original equation, thus preserving the response characteristics a structural engineer is used to working with. In the reformulation with control logic, the basic parameters of inertia (mass), damping, and stiffness are augmented with control parameters. Next a single bending mode is treated to show the transference of the single degree-of-freedom to include structural gains, central control, and distributive control concepts. Finally, these same concepts are briefly looked at for two- and multimode-structural systems.

II. SINGLE DEGREE-OF-FREEDOM VIBRATION SYSTEM

The linear single degree-of-freedom forced vibration system has been analyzed, discussed, and published probably more than any other dynamic system. The reason for this is obvious; it is the basis for dynamics and vibration in general. The same is true for linear proportional gain control theory. Since the purpose of this report is to put fundamental control theory in the vernacular of the structural dynamicist, a brief review of the basic results of vibration theory will be stated. Additional information can be found in any vibration, dynamics, or control theory text book. Figure 1 depicts the classical single degree-of-freedom linear forced system. Constant properties are assumed.

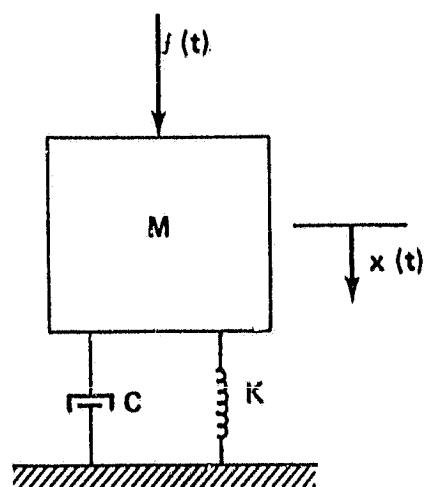


Figure 1. Single degree-of-freedom system.

The describing differential equation is:

$$m\ddot{x} + c\dot{x} + Kx = f(t) \quad (1)$$

Solutions to linear differential equations in this form break out into two parts, homogeneous and nonhomogenous. Out of these have grown the classical stability criteria, usually expressed in structural dynamics as frequency and damping and in control theory as complex roots, the real part depicting a product of damping and frequency and the imaginary part as the damped frequency. Without repeating all the derivations, equation (1) can be rewritten in the form:

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = \bar{f}(t) \quad (2)$$

where,

$$\bar{f}(t) = \frac{f(t)}{m} \quad \text{and} \quad 2\zeta\omega_n = \frac{c}{m} \quad \omega_n^2 = \frac{K}{m} \quad , \quad (3)$$

and in the homogeneous form where $f(t) = 0$, the classical roots are $\delta + i\omega$, where,

$$\delta = -\zeta\omega$$

$$\omega_d = \omega \sqrt{1 - \zeta^2} \quad (4)$$

Solutions to this equation for various type forcing functions are well known and are familiar to the control and structural dynamics community.

The solution for forced oscillation can be obtained in several forms. The results are presented as classical solution curves parameterized in terms of damping and frequency. In general, these solutions are expressed in terms of forcing functions that have the form of (1) impulses, (2) steps, (3) sinusoidal, (4) ramps, etc. The response, for example, to a step function, subcritically damped, is well known and takes the form:

$$X(t) = X_{st} \left[1 - \frac{e^{-\zeta\omega t}}{\sqrt{1 - \zeta^2}} \cos \left(\omega \sqrt{1 - \zeta^2} t - \alpha \right) \right] \quad (5)$$

where X_{st} is the new equilibrium position and

$$\tan \alpha = \frac{\zeta}{\sqrt{1 - \zeta^2}} \quad (6)$$

The maximum displacement is then

$$X_{max} = X_{st} \left[1 + \frac{e^{\frac{-\zeta\pi}{\sqrt{1 - \zeta^2}}}}{\sqrt{1 - \zeta^2}} \cos \alpha \right] \quad (7)$$

Plotting the solution for various damping values is shown on Figure 2.

It can be seen for increasing damping, the subcritical damping $\zeta < 1$, the extreme values shift to the right (frequency decreases), and the amplitude decreases. If $f(t)$ is a harmonic forcing function, the homogenous solution takes the form

$$X(t) = \sqrt{X^0 + \frac{(V_0 + \zeta\omega X_0)^2}{\omega^2 (1 - \zeta^2)}} e^{-\zeta\omega t} \cos \left(\omega \sqrt{1 - \zeta^2} t + \alpha \right), \quad (8)$$

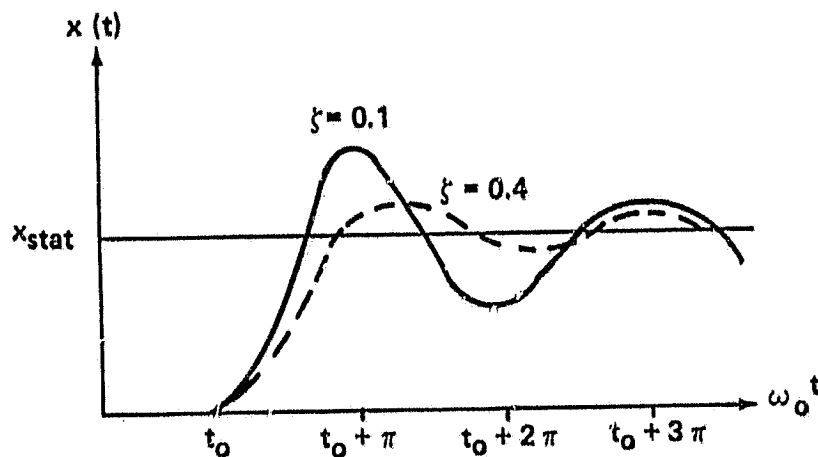


Figure 2. Displacement of a subcritically damped vibration system to step input.

with

$$\tan \alpha = \frac{-(V_0 + X_0 \zeta \omega)}{\lambda_0 \omega C \sqrt{1 - \zeta^2}} \quad (9)$$

The solution in graphical form is shown in Figure 3.

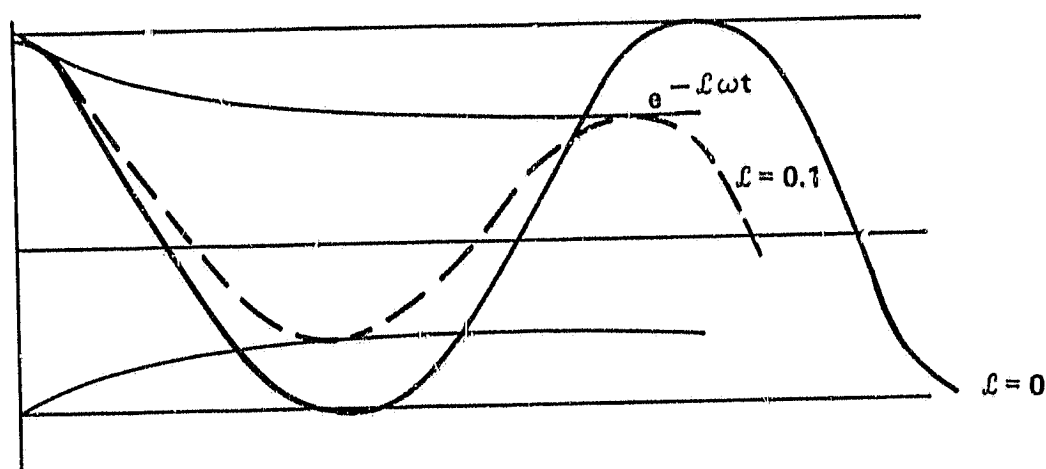


Figure 3. Subcritically damped vibration.

The nonhomogenous solution is better approached from the vectorial form if harmonic forcing functions are assumed.

$$M \ddot{\bar{X}} + C \dot{\bar{X}} + K \bar{X} = F_0 e^{i\Omega t} \quad (10)$$

Thus, the particular solution is of the form

$$\bar{X} = \bar{X} e^{i\Omega t} \quad (11)$$

giving

$$\bar{X} [-\Omega^2 + i\Omega C + K] = \bar{F}_0 \quad (12)$$

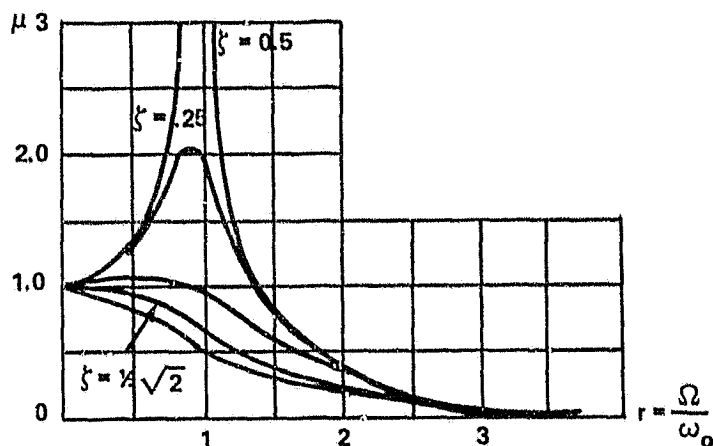
Equation (13) shows the equilibrium of the various forces, inertial, damping, restoring, and driving, with the \bar{F}_0 and \bar{X} being no longer collinear, as in undamped oscillations, but forming a phase angle, α . Letting $r = \Omega/\omega$ be the ratio of the forcing frequency to the undamped natural frequency ω , the equation becomes

$$\bar{X} = \frac{\bar{F}_0/K}{\sqrt{(1-r^2)^2 + 4\zeta^2 r^2}} = \frac{\bar{F}_0}{K} \mu \quad (13)$$

and

$$\tan \alpha = \frac{C \Omega}{K - \mu \Omega^2}$$

Plotting μ , the classical magnification factor gives the value as shown in Figure 4 and the phase angle as shown in Figure 5. The peak magnification factor μ_{\max} and μ versus damping ratio is shown in Figure 6. From these curves, one can see the classical changes in response familiar to all structural dynamicists and control engineers.



$$\left(\frac{\partial \mu}{\partial r} = r \frac{2(1-r^2) - 4\zeta^2}{[(1-r^2)^2 + 4\zeta^2 r^2]^{3/2}} \right)$$

Figure 4. Magnification factor.

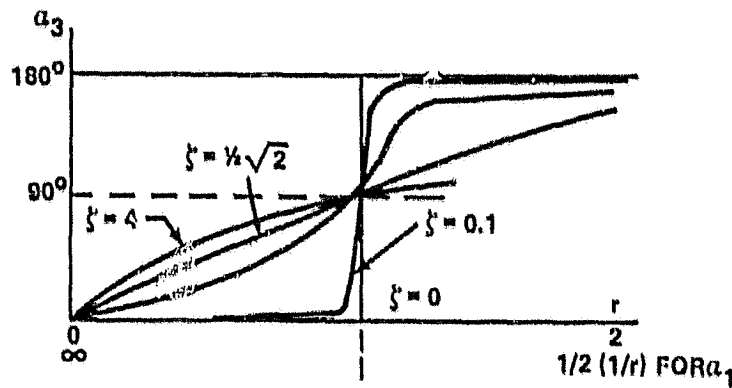


Figure 5. Phase angle α_3 .

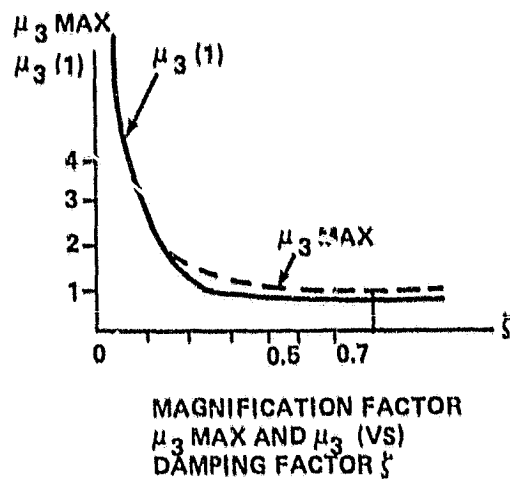


Figure 6. Damping factor ζ .

The effects of damping, inertia, and stiffness upon the response as well as how they interact with the forcing function are well known. For example, more damping reduces the response but increases the period of the oscillations, more mass reduces the static responses but increases the period, increasing the spring increases the frequency, and decreasing the period shortens the decay time. Finally, tuning the frequency of dynamic systems to the forcing function increases several fold the response amplitude. The structural engineer by changing mass, damping, and stiffness alters the response of the system. Isolation systems are derived using these concepts. In summary, the response can be changed by (1) adding mass, (2) adding structural damping, (3) changing stiffness through material changes, structural configuration changes, etc., or (4) altering the forcing function or detuning the structural response from it.

Section III shows how the system response alteration can be achieved by augmenting it with control concepts instead of structural changes.

SECTION III. SINGLE DEGREE-OF-FREEDOM VIBRATION SYSTEM WITH CONTROL

From control theory, forces can be generated which are linearly proportional to the displacement, velocity, and acceleration. Using the familiar SDOF, the application of control theory to alter system response is illustrated in Figure 7 where

$$\delta(t) = [a_0 x(t) + a_1 \dot{x}(t) + a_2 \ddot{x}(t)] F^* \quad (14)$$

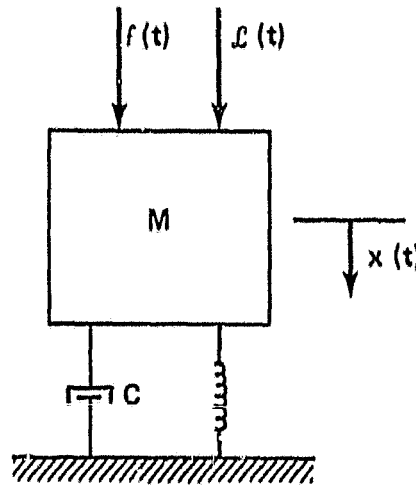


Figure 7. Mass spring system with control feedback.

A control force is generated linearly proportional to displacement, velocity, and acceleration, and $f(t)$ is a completely independent forcing function (non-feedback) of the system. For this formulation, the assumption is made that the control force is ideal; i.e., no phase lags exist in the control mechanism. The introduction of phase lag or lead can be added later to further illustrate control augmentation of dynamic response of structures. Introducing the augmented control force into the equation of motion of dynamical systems gives:

$$m\ddot{x} + c\dot{x} + Kx = f(t) + \delta(t) = m\ddot{x} + c\dot{x} + Kx = f(t) + (a_0 x + a_1 \dot{x} + a_2 \ddot{x}) F^* \quad (15)$$

Recombining or collecting terms,

$$(m - a_2 F^*) \ddot{x} + (c - a_1 F^*) \dot{x} + (K - a_0 F^*) x = f(t) \quad (16)$$

It is clear that equation (16) is identical in form to equation (2). What has changed is the definition of the parameters. No longer is the mass term true mass, but it is an apparent mass due to the control force augmentation. The same is true for damping and stiffness terms. In addition, the control parameters (gains) can take on positive or negative values, hence they must be carefully chosen in order to keep the system stable since it is possible to drive the augmented terms to negative values. Defining the augmented terms as:

$$m^* = m - a_2 F^*$$

$$c^* = c - a_1 F^* \quad (17)$$

$$K^* = K - a_0 F^*$$

equation (16) becomes:

$$m^* \ddot{x} + c^* \dot{x} + K^* x = f(t) \quad (18)$$

where,

$$\sigma = -2\zeta\omega = \frac{c^*}{m^*} = \frac{c - a_1 F^*}{m - a_2 F^*} \quad (19)$$

$$\omega^2 = \frac{K^*}{m^*} = \frac{K - a_0 F^*}{m - a_2 F^*}$$

The ability of dynamic engineers, through application of control augmentation, to alter the behavior of dynamical systems while preserving the structural design and geometry is clearly illustrated.

In general, control theory and control systems are thought of as tools for achieving a desired response of a system already in design, verification, or operational phases. These uses of control employ both active feedback and open loop command control techniques. The past few years, systems engineers have recognized the power of control to achieve the response goals and supplant in a more optimum way structural weight, stiffness, and damping. This allows for tighter design tolerances and higher performance systems at lower cost and risks. This is accomplished through the use of the extra variables introduced by control, including the choice of locating forces optimally for desired response. This will be clearer in the sections dealing with space vehicle elastic modes and non-ideal control. This approach has been exemplified extensively in the active flutter suppression techniques employed on modern aircraft and the new field commonly called "control configured design." What is being done essentially is using control theory prudently to augment structural design parameters, such as stiffness, damping, and mass in both a static and dynamic sense and detuning the system from the forcing function.

The example given used only one sensor of each type and one control force; however, for a multi-mass system, one can go to multisensor, multicontrol force systems and extend substantially the number of variables for optimizing the system. In the past, structural response has been augmented using a control system designed for another function; orientation of space vehicle along a desired flight path. Using the concepts presented, two control systems or an augmented single control system can be made more optimum to perform the two separate tasks, orienting a vehicle's attitude and changing the structural dynamic response to achieve a more optimum structural design.

The substitution of equation (10) is not made back into the equations, since it is obvious how it is accomplished without destroying the validity of the solutions or graphical solutions shown in Section II.

At this point, it is clear that a control system can be used to act as structural elements and alter the system responses. Classically, many ways are available for dealing with and understanding the system response and determining the best approach for arriving at a solution. These approaches, in general, deal with the stability, response time, and amplitude determined by solution of differential equations as discussed previously. These solutions fall into the categories classically called homogenous and nonhomogenous and typically are called stability and response. Determination of these characteristics in the stability area can be accomplished using (1) Routh criterion, (2) root locus, (3) Nyquist, and (4) Nichols techniques. Complex variable theory of functions is the underlying theory used in these techniques, such as Laplace transforms. Closed form solutions to the equations can be used in special cases, but are not a generally available technique. Numerical integration of the equations is generally applicable and with modern computers is efficient for large systems. Analog or hybrid computers also are an efficient approach, particularly for nonlinear systems.

Recognizing that all dynamic models of structural systems and control systems are usually cast in differential equation form, the techniques just described are applicable. The major differences between the two disciplines are in the way these equations are formulated.

Control engineers are concerned with several aspects of the structural dynamic interaction problem. First, control feedback logic is used as a technique for placing a dynamic system at a given state, normally a displacement or rotation. This is accomplished using a response command that drives the system to this position by actuating a control force. If the system is stable, then the dynamic system arrives at the desired position. This involves a position command about some static or normal position. Secondly, in achieving this position command, the engineer is concerned with the rate of reaching this position and the overshoot errors and recovery time produced in changing the dynamic system state. Thirdly, closely related to this is the stability of the system due to introducing control. Finally, he is concerned with the response of dynamic systems to any external environments that result in perturbations to the response position he is trying to achieve. These four areas exist whether one is concerned with using control to optimize structural design or the control of a system to a desired state which results in dynamic responses that are unwanted and must be contained or reduced (stability augmented systems, etc.). When these areas are considered, the basic vibration response characteristics discussed earlier result; however, it can be advantageous to formulate the equations in terms of a transformed or different coordinate system. This coordinate system is formulated in terms of an error coordinate which is the difference between the commanded state and the response state, normally called $X_{in} - X_{out} = e$. As a result, special analysis tools have been developed around these redefined equations which allows quick insights into the control system parameters in terms of the system stability and response. Obviously, one does not have to go to this special formulation, but can use the basic structural dynamics formulation given previously where control is an input force proportional to state. The latter formulation results, in many cases, in a less efficient analysis approach. It conserves, however, all the structural dynamics engineer's built-in understanding and intuition. Therefore, the control engineer needs awareness of this data base. At the same time, the structural dynamic engineer must strive to reinterpret his firm data base in terms of the transformed equations approach used by control so important to understanding and implementing control concepts. Extensive literature exists on these techniques from both disciplines and is not a focus of this paper; however, the following example depicts the general concept.

Using the original system with some different control concepts will allow reformulation of the equations. A differentiator will operate on a position signal to obtain a rate in such a manner that the resulting equation has the form:

$$\dot{e} = -C_e \dot{e} = -C_e (\dot{X}_{in} - \dot{X}_O) \quad (20)$$

Introducing an electrical amplifier as a means of changing signal, then control force becomes for a control amplifier of gain μ

$$\mu (a_0 e + a_1 C_e \dot{e}) F^* \quad (21)$$

Then,

$$m \ddot{X}_O + C \dot{X}_O + K X_O = f(t) + \mu F^* (a_0 e + a_1 C_e \dot{e}) \quad (22)$$

which becomes

$$M \ddot{X}_O + (C + C_e \mu F^* a_1) \dot{X}_O + (K + \mu F^* a_0) X_O = C_e \mu F^* a_1 \dot{X}_O + \mu F^* a_0 X_1 + f(t) \quad (23)$$

Setting $f(t) = 0$ and putting this equation in transform form, the response can be written as:

$$X_O = \frac{(C_e a_1 \delta + a_0) \mu F^* X_1}{M \delta^2 + (C + C_e \mu F^* a_1) \delta + (K + \mu F^* a_0)} \quad (24)$$

Or in conventional vibration form:

$$X_O = \frac{(C_e a_1 \delta + a_0) \mu F^* / m X_1}{\delta^2 + 2 \zeta \omega \delta + \omega^2} \quad (25)$$

where

$$2 \zeta \omega = \frac{C + C_e \mu F^* a_1}{m} \quad (26)$$

and

$$\omega^2 = \frac{K + \mu F^* a_0}{m} \quad (27)$$

which is the same as equation (20) when the acceleration term is set to 0 and $a_0 F^*$ and $a_1 F^*$ is redefined to have an amplifier gain. This system is shown in block diagram form in Figure 8.

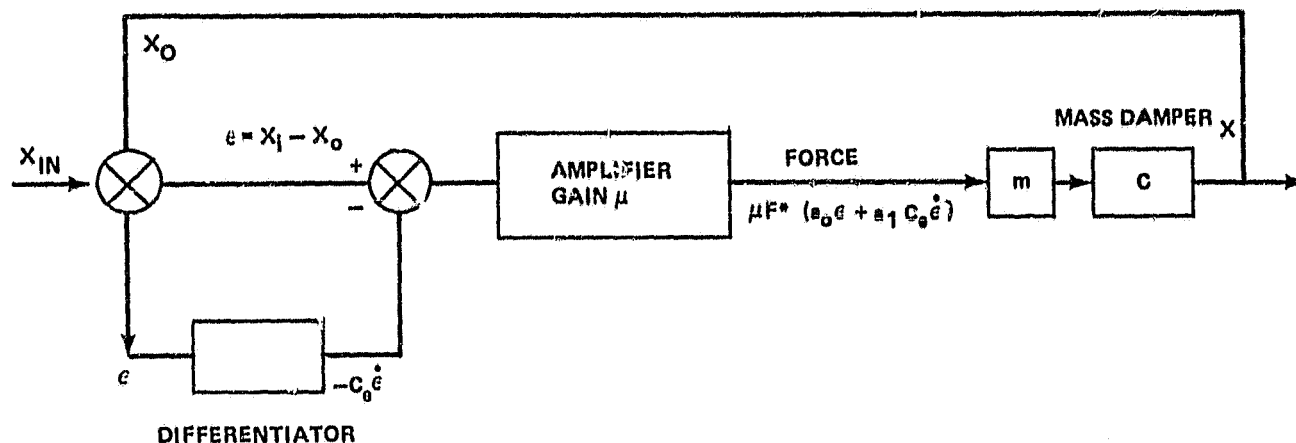


Figure 8. Closed loop control augmentation diagram.

Equation (25) is now in a form that is easily amenable to various control commands, response augmentation, and analysis techniques, achieved through the introduction of a command signal and feedback error signal while maintaining the basic physical system presented earlier. As discussed previously, what one is dealing with is the transformation between the time domain and frequency domain or vice versa using a transformed parameter ϵ . This formulation, therefore, allows the use of all the techniques developed in the field of operational mathematics as well as control theory. It is assumed that the reader is knowledgeable of these techniques for (1) generating time responses from equations in operator forms using inverse transforms, etc., (2) generation and understanding of transfer functions $Y(\delta)$ or $Y(i\omega)$, and (3) numerical integration of differential equations. The area where, in general, the engineers lack understanding or insight into control analysis techniques is the stability analysis techniques discussed earlier. Whether an engineer comes up through the control or structural dynamics side, he knows that the stability of a dynamic system is obtained from the homogenous solution through the roots of the characteristics equation. The loss of intuition arises because of the special formulation of the equations used in control theory.

The structural dynamics engineer basically thinks in terms of frequency and damping which is preserved only with the root locus technique. Other techniques for stability describe stability margins in terms of phase and gain which are meaningful terms but at the expense of the concrete damping expression. As was shown in equations (25), (26), and (27), the resulting transformed equation has the form for the transfer function:

$$\frac{X_O}{X_I} = Y(P) = \frac{b_m P^m + \dots + b_0}{a_n P^n + \dots + a_0}, \quad (28)$$

and the closed-loop differential equation is

$$(a_n P^n + \dots + a_0) b_0 = (b_m P^m + \dots + b_0) \theta_i.$$

Therefore, the homogeneous solution that determines stability is arrived at by setting the right-hand side equal to zero and solving for the complex roots δ_i 's where

$$\delta = \sigma \pm i\omega \quad (29)$$

The problem is not writing the characteristics equation but finding its roots. The characteristics equation can be written in the form of a polynomial, or can be expressed in state form, or as coefficients of second order differential equations in matrix form. The larger the number of degrees of freedom or the order of the polynomial, the more difficult the solution for the roots are. This results in the use of transformed equations in conjunction with the many techniques based on complex variable theory that use the open-loop transfer function as the basic equation formulation. These techniques arrive at the answers graphically or numerically in the frequency domain instead of using polynomial solvers or matrix root iteration techniques. The power of these techniques such as Routh criterion, Nyquist stability criteria, is that the stability boundaries (positive real roots) can be found without obtaining roots; however, as stated previously, the absolute level of stability is not known. The Routh criterion is based on the mathematical principle that the coefficients of the characteristic equation are the sums and products of the roots of the equation. When carried through to completion, it is easily shown that for stability all the coefficients must be positive. For simple characteristics equations (low order, few parameters), the coefficients can be written in terms of key parameters and thus the stability boundaries can be plotted versus parameter variations. Using equation (24) as an example, the coefficients are:

$$\begin{aligned} a_2 &= m \\ a_1 &= C + C_0 \mu F^* a_1 \\ a_0 &= K + \mu F^* a_0 \end{aligned} \quad (30)$$

which says that m must be positive which physically is correct and that $C + C_0 \mu F^* a_1$ and $K + \mu F^* a_0$ must be greater than zero. This gives two equations in two unknowns which can be solved as a function of the other parameters. Since there are no cross couplings, the solutions are independent which means the damping and frequency and thus the stability can be determined independently. In most cases, the system is coupled containing many parameters which must be balanced to achieve the most optimum stability characteristics. Literature is full of the application of Routh criterion.

Nyquist stability approaches are also well documented. It is based on complex variable theory which says that if you plot an envelop of points in the complex root plane (closed contour) and calculate the value of the complex function (transfer function), a closed contour will also be developed in the $f(\delta)$ plane. This closed contour about the origin will circle the origin a total of the number of poles minus the number of zeroes where the poles are the roots of the denominator and the zeroes are the roots of the numerator of the transfer function. The stability can be determined therefore by graphic or numerical techniques and does not require finding the roots of the characteristics equation. In general, the equations are transformed such that the transfer function has the form:

$$X_o = \frac{Y_o(s)}{1 + Y_o(s)} X_{in} \quad (31)$$

In this case, the number of contours encircled minus one instead of the origin. Root locus techniques using present day computer technology has not placed as much emphasis on graphic techniques as when the engineer had to determine system stability, sensitivity, and margins without high powered computer technology and was forced to use graphic techniques. Root locus techniques plot the position of poles and zeroes of the closed loop transfer function as a function of some control parameter such as the amplifier gain μ , which is a gain in the open loop transfer function. In other words, the root locus procedure starts with the open-loop transfer function and ends with the poles and zeroes of the closed-loop system. Before proceeding it is important to remember that the closed-loop transfer function using inverse transforms describes the transient response characteristics. In general form:

$$X_o(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t} + \dots C_n e^{s_n t} \quad (32)$$

where the poles $S_n = \sigma_n + i \omega_n$ determines the stability and frequency of response and the zeroes fix the size of the transient term for a particular input, i.e., the constants C_1 , C_2 , and C_n . In this case, equation (34) is rewritten:

$$\frac{X_o}{X_i} = \frac{X_o}{1 + X_o} = \frac{K_G}{1 + K_G} \quad (33)$$

where K is an arbitrary gain of the control amplifier. The characteristic equation becomes

$$(P - S_1)(P - S_2) \dots (P - S_n) = 1 + X_o(S) = 1 + G(S)K = 0 \quad (34)$$

Graphically, then one plots the points where

$$G(S) = -\frac{1}{K} \quad (35)$$

Using complex variable theory, the solution to equation (35) is found where the angle of $G(S)$

$$\text{Angle of } G(S) = \text{angle of } -\frac{1}{K} = \pi + K 2\pi \quad (36)$$

$$K = \text{integer and the magnitude of } G(P) = \frac{1}{K} \quad (37)$$

Thus, the angle of $G(S)$

$$\text{Angle } G(S) = \angle A + \sum^2 \text{ zero vectors} - \sum^P \text{ pole vectors} \quad (38)$$

$$\text{Magnitude } G(S) = |A| \frac{\text{Product of } Z \text{ zero vector magnitudes}}{\text{Product of } P \text{ pole vector magnitudes}},$$

where these equations are derived from the factoring of

$$G(S) = \frac{A (S - Z_1) \dots (S - Z_n)}{(S - P_1) \dots (S - P_n)} \quad (39)$$

Therefore, using complex variable theory, one can treat these complex members as vectors and graphically evaluate $G(S)$. It is not the purpose of this memo to go further into the details of root locus, since it is well documented. Modern computers by-pass this graphics approach and solve for the roots numerically which is a more efficient way. It is clear the solutions obtained in this manner are equal to and consistent with those normally dealt with in structural dynamics. As stated previously, modern day approaches do not depend on these graphic techniques; however, it is imperative that the underlying principles inherent in these approaches be understood as well as staying abreast of the data base developed in conventional structural dynamics analysis.

Control theory is not ideal in application. Forces cannot be generated without introducing phase lags. Also, through the use of electrical networks, and a digital control theory, the control engineer can generate a control force or control signal that changes in amplitude and phase as a function of frequency given additional flexibilities and power to design engineers. These effects do not change the basic concepts presented. They do increase significantly the complexity of obtaining the solutions.

Section IV looks at these concepts in terms of the response of an elastic space vehicle or aircraft response to environmental excitation.

SECTION IV. SPACE VEHICLE ELASTIC BODY RESPONSE CONCEPTS

Before going to elastic body response, it should be mentioned that the rigid body response of a space vehicle in rotation and translation are mere extensions of the concepts just presented where the major stiffness and damping are due to the control system with some augmentation from aerodynamics. The major mass moment of inertia portion comes from the structural configuration with little or no augmentation from control. This subject, rigid body response, has been treated extensively for many years [1] and is not repeated here. This section will discuss only the elastic body effects because of their

strong interaction with control and performance issues of large space structures, where not only control of response but control of curvature or shape is important. In these cases, requirements on ferrying weight and overall size result in a very light, large, and low frequency structural system. These constraints place strong emphasis upon a coupled system optimized design approach. Also, during ascent flight and aircraft flight, large aerodynamic and gust loads and uncomfortable ride qualities result from the elastic body transient response to these disturbances. Use of control to reduce these responses is a key design area.

As all structural dynamicists are aware, the solution of the structural dynamic equation with no external forces leads to a set of normal modes and frequencies commonly called eigenvectors and eigenvalues. The modes are orthogonal and thus uncoupled. Since each mode can be characterized as an effective mass and stiffness, called generalized, one can then couple them with external forces, including aerodynamic and control system using energy approaches such as LaGrange's equations. Using these approaches, including generalized coordinates, the equations are in the same general form used in vibration theory of both the single degree-of-freedom forced response and multidegree-of-freedom forced response systems. Extensive documentation is available depicting these approaches and solutions used.

To understand elastic body responses (loads and stability), the assumption will be made (later removed) that one elastic body mode is uncoupled from the other and that the rigid body angle of attack and engine deflection act as known (time wise) forcing functions to this model (Fig. 9).

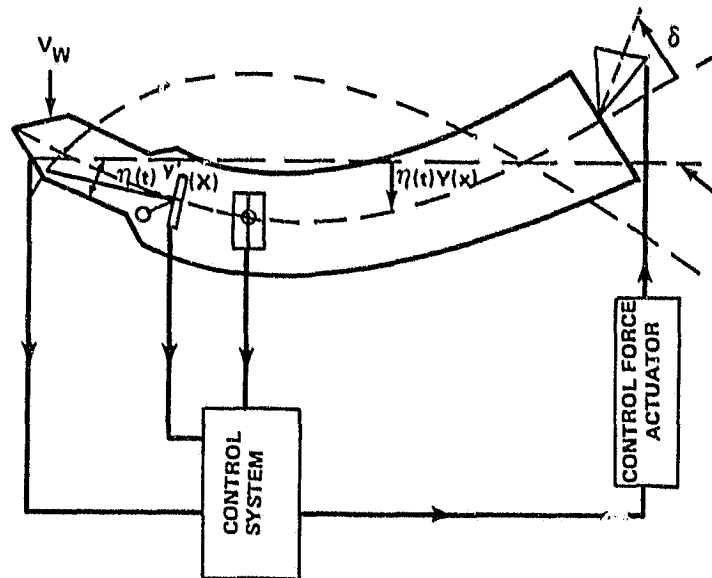


Figure 9. Controlled elastic body response.

In order to write the equations under these assumptions, the gimbal engine generated force is split into two parts; the rigid body generated control induced force is a known function of time, and the elastic mode introduced control force is a function of the elastic body modal state (deflection, rate, and acceleration). Phasing between the rigid body generated engine and aerodynamic forces is neglected for simplicity. When phasing is neglected, the equation for a bending mode is written as follows:

$$\ddot{\eta}_{\mu}(t) + 2 \zeta_{B\mu} \omega_{B\mu} \dot{\eta}_{\mu}(t) + \omega_{B\mu}^2 \eta(t) = \frac{F_s Y_E}{M_B} \delta_{\text{elastic}}(t) + \frac{B_{\mu}}{M_B} \eta_{\mu}(t) + \frac{C_{\mu}}{M_B} \dot{\eta}_{\mu}(t) + \frac{D_{\mu}}{M_B} \alpha_{\text{rigid}}(t) + \frac{F_s Y_E}{M_B} \delta_{\text{rigid}}(t) , \quad (40)$$

or

$$\ddot{\eta}_{\mu}(t) + \left(2 \zeta_{B\mu} \omega_{B\mu} - \frac{C_{\mu}}{M_B} \right) \dot{\eta}_{\mu}(t) + \left(\omega_{B\mu}^2 - \frac{B_{\mu}}{M_B} \right) \eta(t) = \frac{F_s Y_E}{M_B} \delta_{\text{elastic}}(t) + \frac{Q(t)}{M_B} \quad (41)$$

(rigid body forcing
function plus wind gust) ,

where

$\eta_{\mu}(t)$ = bending mode generalized coordinate,

$\omega_{B\mu}$ = bending mode natural frequency,

M_B = bending mode generalized mass,

$\zeta_{B\mu}$ = structural damping,

B_{μ} = local angle of attack aerodynamic term,

C_{μ} = local angle of attack aerodynamic term,

D_{μ} = rigid body aerodynamic force term,

F_s = vehicle thrust,

Y_E = mode deflection at engine.

To the uninitiated, the engine nozzle is gimbaled to produce a lateral control force proportional to the thrust times the sine of the deflected angles. For small angles, this force is equal to the thrust times the angle in radians. Assuming that δ_{elastic} results from signals arising from body-fixed accelerometers, rate gyros, and position gyros, the control equation becomes:

$$\delta_{\text{elastic}} = a_0 \eta_{\mu}(t) Y'(X_g) + a_1 \dot{\eta}_{\mu}(t) Y'(X_R) + a_2 \ddot{\eta}(t) Y(X_A) , \quad (42)$$

where a_0 is position signal gain, a_1 rate signal gain, a_2 accelerometer signal gain, $Y'(X_g)$ the bending mode slope at the position sensor, $Y'(X_R)$ the bending mode slope at the rate sensor, and $Y(X_A)$ the bending mode deflection at the acceleration sensors.

Substituting equations (41) and (42) and simplifying results gives:

$$\ddot{\eta}_\mu(t) + \frac{[2\zeta_{B\mu} \omega_{B\mu} M_B - C_\mu - a_1 F_s Y_E Y'(X_R)] \dot{\eta}_\mu(t)}{M_B - a_2 F_s Y_E Y(X_A)} + \frac{[\omega_{B\mu}^2 M_B - B_\mu - a_0 F_s Y_E Y'(X_g)] \eta_\mu(t)}{M_B - a_2 F_s Y_E Y(X_A)} = \frac{Q(t)}{[M_B - a_2 F_s Y_E Y(X_A)]} \quad (43)$$

It is clear that the above generalizations were made for one sensor of each type; however, the use of more than one sensor does not destroy the use of the analogy, since the total signal is the sum of the voltage coming from each control loop. The effects of multisensors on the roots, and therefore the cross-coupling between modes, etc., will be addressed later. How the response is altered by the control system on the roots is now discussed for the ideal case. The equation becomes:

$$\ddot{\eta}_\mu(t) + 2\bar{\alpha} \dot{\eta}_\mu(t) + (\bar{\alpha}^2 + \bar{\beta}^2) \eta_\mu(t) = R Q(t) \quad , \quad (44)$$

where

$$R^2 = \frac{1}{M_B - a_2 F_s Y_E Y(X_A)} \quad (\text{Apparent Mass}) \quad (45)$$

$$\bar{\alpha} = \frac{1}{2} [2\zeta_{B\mu} \omega_{B\mu} M_B - C_\mu - a_1 F_s Y_E Y'(X_R)] R^2 \quad (\text{Apparent Damping}) \quad (46)$$

$$(\bar{\alpha}^2 + \bar{\beta}^2) = [\omega_{B\mu}^2 M_B - B_\mu - a_0 F_s Y_E Y'(X_g)] R^2 = \omega_0^2 \quad (\text{Apparent Frequency}) \quad (47)$$

The roots to the equations are obviously $\alpha \pm \beta$ defined in equations (46) and (47). Using these roots, the solution to equation (44) gives insight into response-relieving mechanisms. Typical solutions can be found for constant coefficients and known $Q(t)$'s. Assuming that $Q(t)$ is some known (Laplace) transform, then

$$\eta(s) = \frac{R^2}{\beta} \left[\frac{\beta Q(s)}{(s-\alpha)^2 + \beta^2} \right] \quad , \quad (48)$$

for initial conditions equal to 0. Letting $Q(t)$ be a ramp input or $Q(t) = K_3 t$, then

$$\eta(t) = \frac{R^2 K_3}{\bar{\beta}} \left[\frac{\bar{\beta} t}{\bar{\alpha}^2 + \bar{\beta}^2} - \frac{2\bar{\alpha}\bar{\beta}}{(\bar{\alpha}^2 + \bar{\beta}^2)} + \frac{1}{\bar{\alpha}^2 + \bar{\beta}^2} \right] e^{-\bar{\alpha}t} \sin(\bar{\beta}t - \psi_4) , \quad (49)$$

where

$$\psi_4 = 2 \tan^{-1} \left(\frac{\bar{\beta}}{-\bar{\alpha}} \right) , \quad (50)$$

and

$$\ddot{\eta}(t) = \frac{R^2 K_4}{\bar{\beta}} \left[e^{-\bar{\alpha}t} \sin(\bar{\beta}t) \right] , \quad (51)$$

Finally, if $Q(t)$ is a sine function

$$Q(t) = K_4 \sin \Omega t , \quad (52)$$

then

$$\eta(t) = R^2 K_4 \frac{\Omega}{[(\bar{\alpha}^2 + \bar{\beta}^2 - \Omega^2) + 4 \bar{\alpha}^2 \Omega^2]^{1/2}} \left[\frac{1}{\Omega} \sin(\Omega t - \psi_6) + \frac{e^{-\bar{\alpha}t}}{\bar{\beta}} \sin(\bar{\beta}t + \psi_7) \right] , \quad (53)$$

where

$$\psi_6 = \tan^{-1} \frac{2\bar{\alpha}\Omega}{\bar{\alpha}^2 + \bar{\beta}^2 - \Omega^2} \quad (54)$$

$$\psi_7 = \tan^{-1} \frac{2\bar{\alpha}\bar{\beta}}{\bar{\alpha}^2 - \bar{\beta}^2 + \Omega^2} ,$$

and

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$$\eta(t) = R^2 K_4 \frac{\Omega}{[(\bar{\alpha}^2 + \bar{\beta}^2 - \Omega^2) + 4\bar{\alpha}^2 \Omega^2]^{1/2}} \left[-\Omega \sin(\Omega t + \psi_6) + \frac{\bar{\alpha}^2 + \bar{\beta}^2}{\alpha} e^{-\bar{\alpha}t} \sin(\bar{\beta}t + \psi_9) \right], \quad (55)$$

where

$$\psi_9 = \psi_7 - \tan^{-1} \frac{2 \bar{\alpha} \bar{\beta}}{\bar{\alpha}^2 - \bar{\beta}^2} \quad (56)$$

As expected, all cases show a difference between acceleration and displacement of the ω^2 factor on the transient part of the solution except for the sinusoidal forcing function which also contains a steady-state term with a factor of Ω^2 difference. Considering the solution to the ramp, step, or impulse, the magnitude of the constant can be changed by the term R^2 by use of accelerometers. These solutions — ramp, step, and impulse — can also be altered through each of the sensors as they alter the frequency or damping [equations (46) and (47)]. Rate gyros change the damping of the system either positively or negatively depending on the sign of the modal deflection Y_E and $Y'(X_R)$ and the rate gyro gain, a_1 . Choosing the sensor location or gain such that α increases results in greater damping and lower transient. Choosing an accelerometer location and gain such that R increases, increases both the damping and the frequency, thus allowing the accelerometer to be used as a modal suppressor from both the damping term and the frequency term. Position gyros can be used to alter the frequency by a proper choice of the sensor location or feedback gain, a_0 . The amplitude response (both steady-state and transient) in these cases is reduced if the frequency is increased; however, the accelerometer output is proportional to the frequency squared times the transient portion of the solution. Increasing the damping lowers the peak transient response. All three types of input forces are expected during flight since the wind contains some form of each type of input. The response (acceleration or amplitude) can be reduced by increasing the frequency or damping.

A more important type of force from the bending mode standpoint is the sinusoidal input. This represents the turbulence portion of the atmosphere, which can have frequency content in resonance with the bending mode. Also, not only is the transient term important but the steady-state term can be of a larger magnitude in both acceleration and amplitude. Again, increasing the damping decreases the amplitude and thus reduces the transient response. Increasing the frequency may not be feasible; however, since the resonance term contains $\bar{\beta}^2 + \bar{\alpha}^2$ and Ω^2 , and the amplitude of the frequency increases as $\bar{\beta}^2 + \bar{\alpha}^2$ and Ω^2 approach equal values. In this case, the frequency shift must be chosen to detune the system from the forcing frequency. Additionally, for this case, the accelerometer can be used to reduce the overall amplitude through R^2 which multiplies the solution (Fig. 10). Care must be exercised in using this term for reducing amplitude when at the same time it may increase the amplitude through either finer tuning (with forcing function), or decreased damping and frequency. The change in damping and frequency can be obtained by using the various sensors as discussed previously.

The discussion thus far has indicated that accelerometers, rate gyros, or position sensors, can be used for augmenting elastic system response. The results point out very clearly that one must have a very accurate description of both the vehicle modal characteristics and the input force (wind) to effectively design a control system for modal suppression using these types of sensors. Also, the coupling of these sensors in the rigid body control (used as input force for mode) is very important and cannot be neglected.

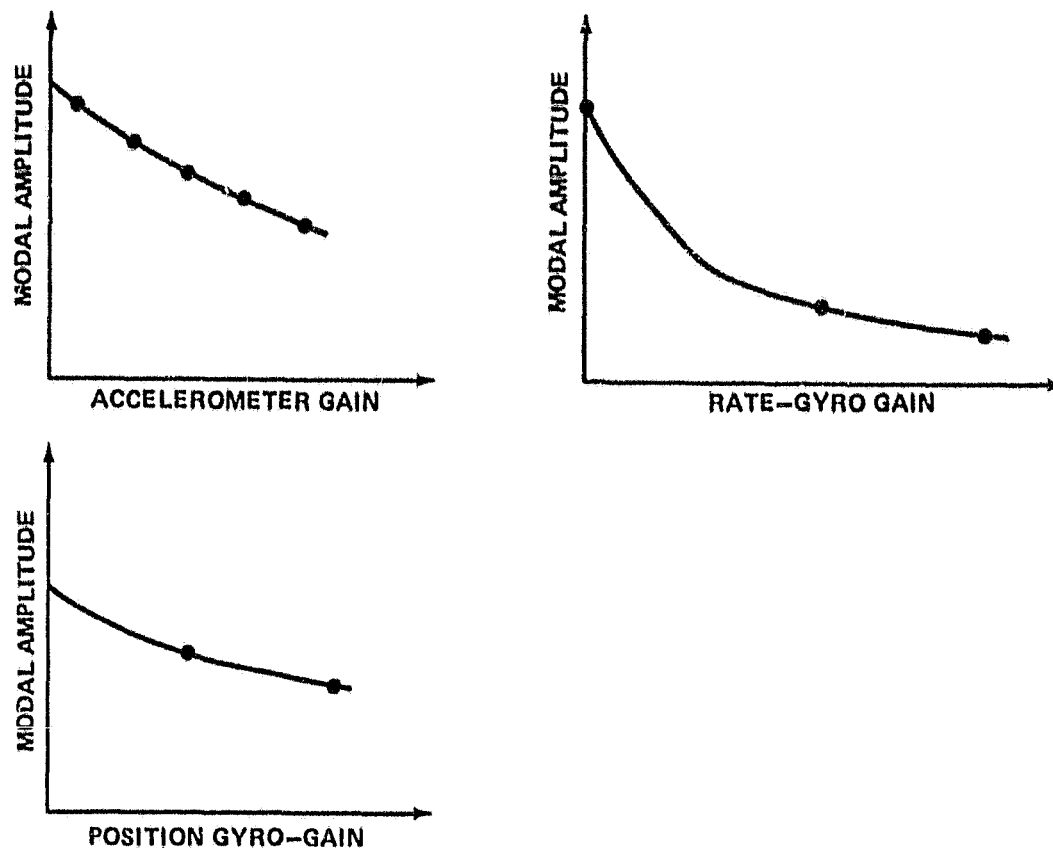


Figure 10. Modal suppression.

The previous interpretations can be stated in another way. The basic notion here is the freedom offered by a sensor complement in locating closed-loop eigenvalues as a possible source of quality measures. This is motivated by two considerations. First, classical experience with root loci and frequency domain design techniques provides tested insightful relationships between the performance capabilities of a controlled system and the closed-loop pole arrangements permitted by sensors. Such notions as stability, frequencies of oscillation, damping of individual modes of response, and dominance are all apparent from the pole constellation. Secondly, there is a fundamental connection between pole placement and the concept of controllability.

The previous discussion was based on the assumption that ideal control signals and response exist. Although this is not true, the principles remain the same as long as the gain and phase lag changes that take place in reality are considered. Also, the assumption is made that each mode is completely independent of the other, which is not true. To illustrate this, a two-sensor case will be presented first, then a two-mode case.

If two accelerometers are used instead of one, the denominator in equation (47) becomes:

$$\frac{1}{R^2} = M_B - a_{21}F_s Y_E Y(X_{A_1}) - a_{22}F_s Y_E Y(X_{A_2}) \quad (57)$$

This allows a choice of gains and sensor locations that would cancel the accelerometer effect or allow any mixture of effects (gains) between accelerometer locations. The other coefficients in equation (47) could be modified in the same manner by using two or more rate or position gyros. This not only illustrates the complexity of using many sensors but also the flexibility.

Extending the concept to two bending modes, but neglecting certain rigid body coupling, results in the following equations which are derived by assuming only one sensor of each type in the control equation. The control equation (elastic body feedback portion) is

$$\delta_{\text{elastic}} = a_0 [\eta_1 Y_1'(X_g) + \eta_2 Y_2'(X_g)] + a_1 [\dot{\eta}_1 Y_1'(X_R) + \dot{\eta}_2 Y_2'(X_R)] + g_2 [\ddot{\eta}_1 Y_1(X_A) + \ddot{\eta}_2 Y_2(X_A)] \quad (58)$$

The coupled bending dynamics equations given in matrix form, using this control law are as follows:

$$\begin{bmatrix} M_{B1} - g_2 F_s Y_{E1} Y_1(X_A) & - g_2 F_s Y_{E1} Y_2(X_A) \\ - g_2 F_s Y_{E2} Y_1(X_A) & M_{B2} - g_2 F_s Y_{E2} Y_2(X_A) \end{bmatrix} \begin{Bmatrix} \ddot{\eta}_1 \\ \ddot{\eta}_2 \end{Bmatrix}$$

$$\begin{bmatrix} 2r_{B1} M_{B1} - C_{11} - a_1 F_s Y_{E1} Y_1'(X_R) & - a_1 F_s Y_{E2} Y_2'(X_R) - C_{12} \\ - a_1 F_s Y_{E2} Y_1'(X_R) - C_{21} & 2r_{B2} \omega_{B2} M_{B2} - C_{22} - a_1 F_s Y_{E2}' X_R \end{bmatrix} \begin{Bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{Bmatrix}$$

$$\begin{bmatrix} M_{B1} - B_1^2 - B_{11} - a_0 F_s Y_{E1} Y_1'(X_g) & - B_{12} - a_0 F_s Y_{E1} Y_2'(X_g) \\ - B_{12} - a_0 F_s Y_{E2} Y_1'(X_g) & M_{B2} \omega_{B2}^2 - B_{22} - a_0 F_s Y_{E2} Y_2'(X_g) \end{bmatrix} \begin{Bmatrix} \eta_1 \\ \eta_2 \end{Bmatrix}$$

$$- \begin{Bmatrix} Q_1(t) \\ Q_2(t) \end{Bmatrix} = 0 \quad (59)$$

The coefficients show that what is done with one sensor for one mode can be offset by the redundant signal from the second mode. If the system is extended to include many sensor gains and force input locations, the tradeoffs are apparent but too difficult to illustrate. Although the concepts for one mode hold for this more general case of two modes, the design problem is increased many fold because of cross-coupling through the control system. Obviously, the things that help suppress one mode could easily aggravate another. With several modes and sensors, a procedure must be used that provides

insight into important characteristics and that gives first cuts at the gains and sensor values and locations. Numerical response techniques, optimal control techniques, sensitivity studies, and root locus techniques (eigenvectors and eigenvalue routines) allow the development of trades between control force locations, sensor locations, and control law logic that provide the desired response in each mode or the over response as a deflection and load.

Illustrated in this section has been the extension of the concepts discussed previously to elastic body response of a space vehicle. Additional concepts of modal gains and multisensor control logic as well as extension to multimodes systems were addressed, all prescribing the response in the general form a structural dynamicist is familiar with. Non-ideal control was not addressed; however, it is easily included by writing the differential equations for these effects and adding them into the matrices and generalized coordinates given in equation (30) [2].

SECTION V. NON-IDEAL CONTROL EFFECTS ON THE SOLUTION

As stated previously, the control system does not perform in this manner. Also the control engineer can alter the gain and phase electronically, producing greater flexibility to alter structural response. Therefore, the analog presented must be extended to cover these conditions. Assumptions must be made in this case also; namely, that these changes can be made without introducing excessive degrees of freedom and that those introduced do not create unstable modes at some other frequency, all of which can be accomplished with proper design attention. Also, it is assumed that the coupled frequency is fairly close to the uncoupled frequency of the spring mass system. This assumption can be removed later. Using these assumptions and going to the equations using the control equation in the form

$$\delta = F^* [a_0 \bar{K}_1(\omega) e^{i\phi_1(\omega)} X + a_1 K_1(\omega) e^{i\phi_2(\omega)} \dot{X} + a_2 \bar{K}_3(\omega) e^{i\phi_3(\omega)} \ddot{X}] , \quad (60)$$

where K_1 is the gain and ϕ_1 the phase of the control system near the natural frequency of mass spring system either due to inherent lags or is artificially introduced to achieve a desired response. For each of these terms, they can be rewritten in complex form as

$$\bar{K}_i (\cos \phi_i(\omega) + i \sin \phi_i(\omega)) . \quad (61)$$

It is desirable to remove the complex term (i). This can be accomplished under the assumption already states that the frequency will be near the natural frequency by dividing by ω and taking a time derivative of the variable. Performing these operations and assuming that the acceleration can be approximated as $-\omega^2 X$, the control equation becomes

$$\begin{aligned} \gamma = F^* a_0 K_1 [X \cos \phi_1(\omega) + X/\omega \sin \phi_1(\omega)] + F^* a_1 K_2 [X \cos \phi_2(\omega) + X/\omega \sin \phi_2(\omega)] \\ + F^* a_2 K_2 [X \cos \phi_3 - X/\omega \sin \phi_3(\omega)] . \end{aligned} \quad (62)$$

Reinserting these terms back into equation (42) gives

$$\left[m + F^* \left(a_1 \frac{K_2}{\omega} \sin \phi_2(\omega) + a_2 K_3 \cos \phi_3(\omega) \right) \right] \ddot{X} + \left[c + F^* \left(\frac{a_0 K_1}{\omega} \sin \phi_1(\omega) + a_1 K_0 \cos \phi_2(\omega) - a_2 K_3 \sin \phi_3(\omega) \right) \right] \dot{X} + [K + F^* (a_0 K_1 \cos \phi_2(\omega))] X = f(t). \quad (63)$$

The power of control becomes obvious when it is recognized that the K's and ϕ 's can be determined using state-of-art filter-design techniques. The ϕ_1 's can take on any value from 0 to 2π , the K's are usually values between 0 and 1. The choice of the signs and magnitudes on the control gains, a_0, a_1, a_2 , coupled with the choices of K's and ϕ 's allows altering the response of the basic equations in many ways: (1) by changing the structural dynamic design, (2) through basic control system gain choice, and (3) by introducing filters or networks that change the feedback as a function of frequency. In all cases, the basic understanding reached for the single degree of freedom system is preserved. Obviously, one cannot get these frequency dependent changes without altering the number of degrees of freedom; however, if proper attention is given, it will not destroy the analog given.

In summary, up to this point in the report, one sees that a control system in all its basic linear forms can be interpreted as pseudo structural elements, preserving the general response characteristics which a structural dynamics engineer is so familiar with. Ideally, then control can be interpreted as a means of adding or subtracting stiffness, damping, and facatis, all gained without changing the structural design. Adding the influence of filtering networks, any feedback term can be made to act as springs, dampers, or mass. This opens up a whole new vista of design approaches that includes substituting control for structure and is the basis for the relative new field of control configured vehicles. The penalty paid for these additional design parameters is the redundancy required in the control system for fail operational or fail safe requirements.

SECTION VI. ANALOGIES TO VARIOUS CONTROL CONCEPTS

So far, all the discussion has centered on the use of linear, proportional, central, control theory. The analogies shown do not preclude, however, nonlinear control or decentralized control. Here, the structural dynamicist would just move into nonlinear vibration theory or specialized analysis and develop the same kind of analogies.

Digital control theory can usually be put in the form of the continuous analog systems used so far in this paper if the sample rate is high enough relative to the structural frequencies of concern. This is always a good starting place. From that point on, new analogies need development but fall into the categories of sample data system analysis techniques currently employed and sample data control theory analysis techniques, which through transformations put the operations back in this same general form discussed previously. Certainly, one can think easily in terms of RMS, spectrums, etc., and how these are altered for dynamic system with control. The development of various analogies are left to the reader.

Force point sensing is an old concept; however, it has new emphasis in that by using distributed control (at the force inputs) the concept can be implemented for many modes. The idea is to sense and

control the response at the same place on the structure. Ideally, this system is always response reducing or stabilizing; however, control force actuators introduce phase lags that can change this effect. Introducing a filter or network that compensates for the actuator characteristics would put one back to the ideal case discussed.

The concept of distributed control is to put the logic, sensors, and force actuators in selected areas of the structure. This means the logic and desired response for each area must be well understood; otherwise, cross talk between each area creates real problems as they compete and couple with each other. This approach does not, however, destroy the analogies developed so far. This concept is very powerful for shape control. Using the bending mode equation (14) derived in the previous section requires only that terms are included for each sensor, control force, and logic accounting for the slopes and deflection where each element is located. The only complexity is the additional terms and ability to differentiate their influence. The problems get very messy when going to a multimode system; however, many more variables are available to get the optimum response solution.

Putting the centralized control and distributed control together and allowing the centralized control to be hierarchal gives additional flexibility but at the expense of complexity (Fig. 11). The reader can easily develop these analogies if desired.

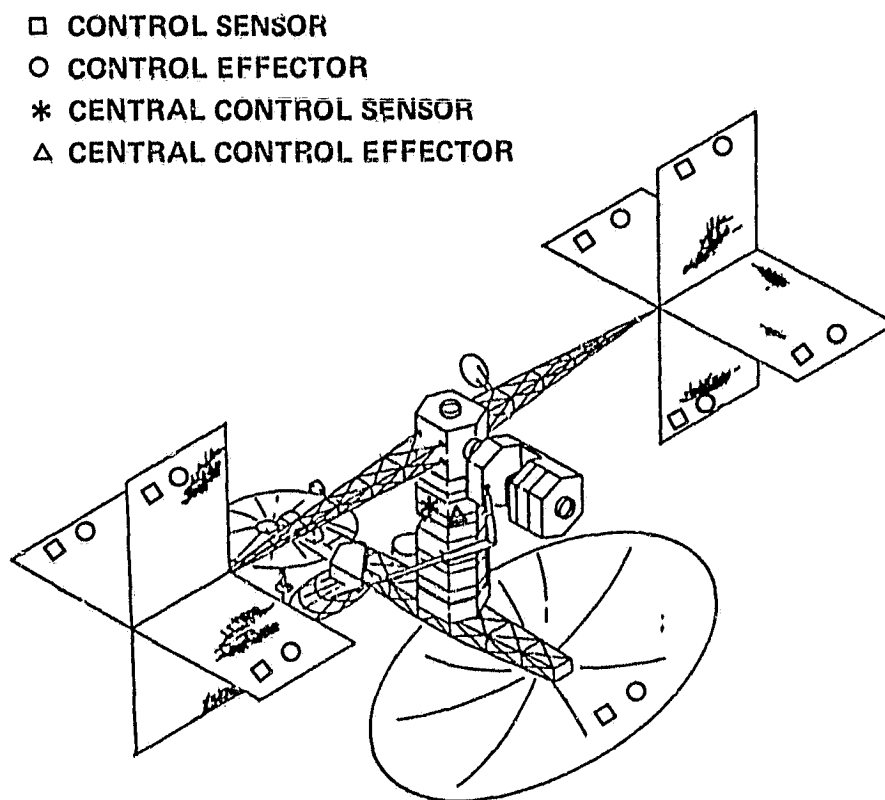


Figure 11. Distributed control system.

SECTION VII. PROBLEMS, CONCERNS, AND FUTURE CHALLENGES

Everything discussed has made the tacit assumption that the sensed state is ideally identified. This is not the case in the real world. Many times noise, other subsystem response frequencies, etc., obscure the desired signal. The whole area of state identification and pattern recognition must be extended. With better and better sensors, more complex control logic, and high structural modal density of very complex and highly coupled modes makes this a prime area to attack.

Optimal control approaches have basically dealt with the system response with no weighting factors for structural design changes. These areas need to be integrated further than has been accomplished to date and development of a means of doing total system optimized design in terms of structure, materials, thermal, and control.

A major problem facing this new discipline is the basic sensitivities of the responses, design, etc., to the uncertainties of both the structural model and the control system model. Here is needed learning control devices, desensitizing approaches, and sensitivity analysis techniques for large parameter systems.

Techniques for truncating or selecting critical structural modes and control system characteristics is one of the major problems facing this new discipline. If the problem solutions are reduced to a manageable size and complexity, major effort needs to be started in both criteria and reduction techniques developed.

Obviously, in the future not only space vehicles and large space systems but the whole field of energy and transportation must deal with this new discipline. The future challenge is to either create a new discipline or cause the separate disciplines to understand, communicate, and work together in a way not generally done in the past. It is hoped that this short introduction to some of the analogies will cause someone to look afresh at these areas.

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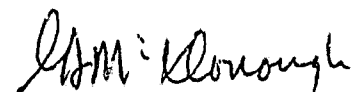
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APPROVAL

A PRELIMINARY LOOK AT CONTROL AUGMENTED DYNAMIC
RESPONSE OF STRUCTURES

By Robert S. Ryan and Ronald E. Jewell

The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.



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