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NASA TECHNICAL MEMORANDUM NASA TM-77013

SOME CHARACTERISTIC QUANTITIES OF KARMAN-TREFFTZ PROFILES

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Translation of "Enige Karateristieke Grootheden Van Von Karman-Trefftz Profielen", Technische Hogeschool, Delft (Netherlands). Department of Aerospace Engineering, Report No. VTH-LR-323, June 1981, 40 pp.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 JANUARY 1983

STANDARD TITLE PAGE

1. Report No. NASA TM-77013	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle SOME CHARACTERISTIC QUANTITIES OF KARMAN-TREFFTZ PROFILES		5. Report Date January 1983
		6. Performing Organization Code
7. Author(s) J. J. H. Blom		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address SCITRAN Box 5456 Santa Barbara, CA 93108		11. Contract or Grant No. NASw- 3542
		13. Type of Report and Period Covered Translation
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes
Translation of "Enige Karateristieke Grootheden Van Von Karman-Trefftz Profielen", Technische Hogeschool, Delft (Netherlands). Department of Aerospace Engineering, Report No. VTH-LR-323, June 1981, 40 pp. (N82-19189)

16. Abstract
For von Karman-Trefftz profiles, the characteristics which determine profile shape (profile nose dimensions, maximum thickness and position; tail slope and curvature) are stated as a function of transformation variables using the Timman method. The profile is obtained by iterative deformation of a von Karman profile with known transformation, corresponding as well as possible to the desired profile. The figures and relations which enable a good choice of the required profile are given.

17. Key Words (Selected by Author(s))		18. Distribution Statement Unclassified - Unlimited	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 33	22. Price

Summary /i

The conform transformation for a given random profile can be determined by means of the method designed by Timman (7). Here an auxiliary or starting profile is used of which the transformation is known and which is as consistent as possible with the given profile.

This starting profile is then deformed by iteration until the desired profile shape is achieved.

A Karman-Trefftz profile is used for the starting profile.

For these profiles the characteristic quantities which determine the profile shape, such as the size of the radius of the nose, the maximum thickness and position of the thickness as well as the slope of the tail and arching are determined as a function of the transformation variables. With the figures and drawings presented in this article it is possible to choose well the required starting profile.

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1. Introduction

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An important help in calculating the potential theoretical pressure distribution around a wing profile is the technique of conform image formation.

By this means a circular or almost circular contour in one surface (ζ) is converted into a wing profile into the other surface (z). The flow around the profile (z-plane) then starts from the known flow around the circle (ζ-plane) and the image function is totally determined.

In the years before and after the First World War a large number of transformation formulae were derived.

The best known of these image functions is that of Joukowsky (1) in which a circle in the ζ-plane is converted into a wing profile with an angle of slope $\delta = 0$ in the z-plane.

The image function required for this purpose has two zero points, that is, points for which the derivative of the image function is equal to zero. One zero point lies on the circular contour and is transferred in the image to the shaft rear edge of the profile.

The other zero point is surrounded by the circular contour and affects the form of the profile.

The image method of Joukowsky offers only two possibilities of varying the profile shape, specifically, thickness and arch.

The more general transformation formula of Karmann and Trefftz (2) has besides the already mentioned possibilities of variation, thickness and arch, the possibility of choosing the angle of slope. The image function also has two zero points. The zero point located on the circular contour is transferred in the image into the rear edge of the profile with an angle of slope δ different from 0.

For the special case of the angle of slope $\delta = 0$ the image function can be simplified to that of the Joukowsky profile.

For more practical applications sometimes an S-bend in the profile shape is desired; that is, the profile nose is bent downwards and the profile slope bent somewhat upwards, to form the S-shaped frame.

The von Karman-Trefftz transformation is not able to generate this type of profile. The skeleton of the von Karman-Trefftz profile is specifically sickle-shaped and consists of two circular arcs. Mises (3) made this S-shaped profile /2 possible by allowing more than one zero point to be formed inside the circular contour. The lower position of these zero points is regulated by the requirements that the image function must be unique, specifically that there should be no logarithmic turn in the image function.

For the derivative this means no term with $1/\zeta$. The necessary prerequisite is that the center of gravity of all the zero points; that is, the zero point placed on the circular contour as well as the ones enclosed by the contour coincide with the origin of the surface of the circle, $\zeta = 0$.

If the number of zero points inside the circular contour is limited to one, then the image function is simplified to that of the Joukowsky profile.

A detailed discussion of the Joukowsky, von Karman-Trefftz and Mises profiles is given in (4), (5) and (6).

In (4) and (5) attention is also paid to the two profile families designed by Mueller. These are closely related to the von Karman-Trefftz family and have the same possibilities for profile variation.

It is also worth mentioning that really random profile shapes cannot be described with the indicated transformation formula. According to the image position of Riemann, the conform image of the circle to a given random profile is always possible, but cannot be given in the form of a formula.

An elegant method by which the transformation of a random profile form can be achieved numerically by iteration was given by Tiemann (7).

Here, the conform transformation of an arbitrary profile is found by starting from a so-called initial profile of which the transformation is known.

The starting profile has the same angle of slope as the desired profile shape and must be as consistent as possible with it (the same nose radius and/or thickness and degree of arching). The starting profile (a von Karman-Trefftz profile) is then deformed by iteration until the desired profile shape is achieved (8), (9).

For a quick and responsible choice of the starting profile, it is desirable to have a survey in which: the nose radius, the maximum thickness, the place of the /3
maximum thickness and the slope of the tail are given as a function of the variables from the transformation formula.

This report will first give a short review of the von Karman-Trefftz profile family.

Subsequently, relations are derived by which the nose radius, the maximum thickness and the site of the maximum thickness can be determined. Finally, the indicated quantities are shown in a number of figures as a function of the transformation variables.

2. Von Karman-Trefftz profiles /4

The image function given by von Karman and Trefftz (2) is:

$$\frac{z + kb}{z - kb} = \left(\frac{\zeta + b}{\zeta - b}\right)^k \qquad (2.1)$$

in which b is a real constant.

With the choice of the angle of slope $k = 2 - \frac{\delta}{\pi}$.

The image function can also be written as:

$$z = kb\,\frac{(\zeta + b)^k + (\zeta - b)^k}{(\zeta + b)^k - (\zeta - b)^k} \tag{2.2}$$

The derivative over ζ of this image function is then:

$$\frac{dz}{d\zeta} = 4k^2b^2\,\frac{(\zeta + b)^{k-1}\,(\zeta - b)^{k-1}}{[(\zeta + b)^k - (\zeta - b)^k]^2} \tag{2.3}$$

The points $\zeta = \pm b$ as may be seen from equation (2.3) are points in which $\frac{dz}{d\zeta} = 0$.

If in infinity the field of flow in the z-plane must be identical to that in the ζ-plane than the conditions must be satisfied: $\frac{dz}{d\zeta} = 1$ for $\zeta \to \infty$.

The expansion into a series of functions (2.2) gives:

$$z = \zeta + \frac{k^2 - 1}{3}\frac{b^2}{\zeta} - \frac{(k^2 - 1)\,(k^2 - 4)}{45}\frac{b^4}{\zeta^3} + \frac{(k^2 - 1)\,(k^2 - 4)\,(2k^2 - 11)}{945}\frac{b^6}{\zeta^5} - \ldots\ldots \tag{2.4}$$

From equation (2.4) we obtain for the derivative:

$$\frac{dz}{d\zeta} = 1 - \frac{k^2 - 1}{3}\left(\frac{b}{\zeta}\right)^2 + \frac{(k^2 - 1)\,(k^2 - 4)}{15}\left(\frac{b}{\zeta}\right)^4 - \frac{(k^2 - 1)\,(k^2 - 4)\,(2k^2 - 11)}{189}\left(\frac{b}{\zeta}\right)^6 + \ldots\ldots \tag{2.5}$$

That is, the condition imposed is satisfied.

The circle C1 is shaped as a profile in which the zero point lies on a contour of the circle, $\zeta = + b$ is shown as a sharp rear edge of the profile, $z = + kb$ (see figure 2.1). /5

The skeleton of this type of profile is formed by two circular arcs; the latter lie between the images of the two zero points, $z = \pm kb$, and ėnclosed between them an angle δ (see figure 2.2). The skeleton arises from the circle C0 which touches in $\zeta = +b$ the circle C1 and passes through the other zero point, $\zeta = -b$.

It is easy to see that for the angle of slope $\delta = 0$, that is, $k = 2$ the image function (2.4) and the derivative (2.5) are transformed into the Joukowsky profile:

$$z = \zeta + \frac{b^2}{\zeta} \tag{2.6}$$

$$\frac{dz}{d\zeta} = 1 - \left(\frac{b}{\zeta}\right)^2 \tag{2.7}$$

The skeleton is reduced to a twice traveled circular arc placed between the points $z = \pm 2$ b, the images in the z-plane of the two zero points $\zeta = \pm$ b (see figures 2.3 and 2.4)

3. The Slope of the Tail and the Arch /6

In the previous chapter it was already stated that the circle CO is formed as a sickle-shaped contour consisting of two circular arcs placed between the points $z = \pm$ kb (figure 2.2).

The lines of contact at the points z = +kb on both circular arcs form with the real axis an angle (6) of formula:

$$\gamma_1 = \beta\left(2 - \frac{\delta}{\pi}\right) + \frac{\delta}{2} \tag{3.1}$$

$$\gamma_2 = \beta\left(2 - \frac{\delta}{\pi}\right) - \frac{\delta}{2} = \gamma_1 - \delta \tag{3.2}$$

respectively in which β, the angle between the "first profile" and the real axis (see figure 2.2) is a measure of the arching of the profile.

From equations (3.1) and (3.2) it now follows:

$$\beta = \frac{\gamma_1 + \gamma_2}{2\left(2 - \frac{\delta}{\pi}\right)} \tag{3.3}$$

or also:

$$\beta = \frac{\gamma}{2 - \frac{\delta}{\pi}} \tag{3.4}$$

in which γ is the slope of the bisectrix of the angle of slope δ. The relation between the slope γ and the angle β is given in figure 3.1.

For the case when the lower of the two circular arcs, which form together the skeleton is reduced to a straight line ($\gamma_2 = 0$) expression (3.3) becomes:

$$\beta = \frac{\delta}{2\left(2 - \frac{\delta}{\pi}\right)} \tag{3.5}$$

The variation of the angle of slope δ with the arch β, according to the above expression is given in figure 3.2.

Figure 3.3. shows a profile with flat lower portion of the skeleton.

4. The Chord of the Profile

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For standardizing the profile coordinates the chord c is introduced.

The first point of the chord is considered as coming from the real point N (see figure 4.1).

For the point N we have:

$$\zeta = b - 2 \cos \beta \tag{4.1}$$

assuming that the circle C1 has a radius R = 1.

The substitution in the image function (2.2) gives:

$$z = -kb \frac{1 + \left(1 - \frac{b}{\cos \beta}\right)^k}{1 - \left(1 - \frac{b}{\cos \beta}\right)^k} \tag{4.2}$$

The rear point of the chord arises from the point T for which we have:

$$\zeta = + b \tag{4.3}$$

With the image function (2.2) this gives:

$$z = + kb \tag{4.4}$$

The size of the chord is then:

$$c = kb \frac{1 + \left(1 - \frac{b}{\cos \beta}\right)^k}{1 - \left(1 - \frac{b}{\cos \beta}\right)^k} + kb \tag{4.5}$$

After some conversion we find:

$$c = \frac{2kb}{1 - \left(1 - \frac{b}{\cos \beta}\right)^k} \tag{4.6}$$

We can speak of a symmetrical profile ($\beta = 0$) and the expression (4.6) is simplified to:

$$c_{symm} = \frac{2kb}{1 - (1 - b)^k} \tag{4.7}$$ /8

For a Joukowsky profile, the angle of slope $\delta = 0$ ($k = 2$) relations (4.6) and (4.7) become respectively:

$$c_{Joukowsky} = \frac{4\cos^2\beta}{2\cos\beta - b} \tag{4.8}$$

and:

$$c_{symm.Joukowsky} = \frac{4}{2 - b} \tag{4.9}$$

5. The Radius of the Profile Nose /9

To establish an expression for the nose radius, we use a method given by Timman (7). If two real functions σ and τ are defined in such a way that:

$$\frac{dz}{d\zeta} = e^{\sigma + i\tau} \tag{5.1}$$

then the radius of the curve of the profile section may be written as:

$$r = \frac{e^{\sigma}}{1 + \frac{d\tau}{d\theta}} \tag{5.2}$$

in which θ gives the corresponding place on the circle contour. To further simplify the expression of the derivative (2.3), the following relations are introduced:

$$\zeta - b = r_1 e^{i\varphi_1} \tag{5.3}$$

$$\zeta + b = r_2 e^{i\varphi_2} \tag{5.4}$$

$$(\zeta + b)^k - (\zeta - b)^k = r_3 e^{i\varphi_3} \tag{5.5}$$

The geometrical meaning of r1, r2, φ_1 and φ_2 follows from figure 5.1.

The substitution of: (5.3), (5.4) and (5.5) in (2.3) gives:

$$\frac{dz}{d\zeta} = 4k^2b^2 \frac{(r_2 e^{i\varphi_2})^{k-1} (r_1 e^{i\varphi_1})^{k-1}}{(r_3 e^{i\varphi_3})^2} \tag{5.6}$$

From this we find for:

$$e^{\sigma} = \left|\frac{dz}{d\zeta}\right| = 4k^2b^2 \frac{(r_2 r_1)^{k-1}}{r_3^2} \tag{5.7}$$

$$\tau = \arg(dz) - \arg(d\zeta) = (k - 1)\ (\varphi_1 + \varphi_2) - 2\varphi_3 \qquad (5.8)$$

and:

$$\frac{d\tau}{d\theta} = (k - 1)\left(\frac{d\varphi_1}{d\theta} + \frac{d\varphi_2}{d\theta}\right) - 2\,\frac{d\varphi_3}{d\theta} \qquad (5.9)$$

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From figure 5.1 follows:

$$r_1 = 2 \sin\left(\frac{\theta}{2}\right) \qquad (5.10)$$

$$\varphi_1 = \frac{1}{2}(\pi + \theta) - \beta \qquad (5.11)$$

$$r_2 = \sqrt{4b^2 + 2 - 2\cos\theta + 4b(\cos(\theta - \beta) - \cos\beta)} \qquad (5.12)$$

$$\varphi_2 = \arctan\left(\frac{\sin\beta + \sin(\theta - \beta)}{2b - \cos\beta + \cos(\theta - \beta)}\right) \qquad (5.13)$$

The substitution of (5.3) and (5.4) in (5.5) gives:

$$r_3 = \sqrt{r_2^{2k} + r_1^{2k} - 2r_1^k r_2^k \cos(k(\varphi_2 - \varphi_1))} \qquad (5.14)$$

$$\varphi_3 = \arctan\left(\frac{r_2^k \sin k\varphi_2 - r_1^k \sin k\varphi_1}{r_2^k \cos k\varphi_2 - r_1^k \cos k\varphi_1}\right) \qquad (5.15)$$

The derivatives of r1, r2, r3, φ_1, φ_2 and φ_3 over θ are then:

$$\frac{d}{d\theta}(r_1) = \cos\left(\frac{\theta}{2}\right) \qquad (5.16)$$

$$\frac{d}{d\theta}(\varphi_1) = \frac{1}{2} \qquad (5.17)$$

$$\frac{d}{d\theta}(r_2) = \frac{1}{2r_2}(2\sin\theta - 4b\sin(\theta - \beta)) \qquad (5.18)$$

$$\frac{d}{d\theta}(\varphi_2) = \frac{1}{r_2^2}(2b\cos(\theta - \beta) - \cos\theta + 1) \qquad (5.19)$$

$$\frac{d}{d\theta}(r_3) = \frac{k}{r_3}\left(r_1^{2k-1}\frac{dr_1}{d\theta} + r_2^{2k-1}\frac{dr_2}{d\theta} - (r_1\ r_2)^{k-1}\right.$$
$$\left(r_1\frac{dr_2}{d\theta} + r_2\frac{dr_1}{d\theta}\right)\cos(k\varphi_2 - k\varphi_1) + (r_1\ r_2)^k$$
$$\left.\sin(k\varphi_2 - k\varphi_1)\left(\frac{d\varphi_2}{d\theta} - \frac{d\varphi_1}{d\theta}\right)\right) \qquad (5.20)$$

$$\frac{d}{d\theta}(\varphi_3) = \frac{k}{r_3^2}\left(r_1^{2k}\frac{d\varphi_1}{d\theta} + r_2^{2k}\frac{d\varphi_2}{d\theta} + r_2^k\, r_1^{k-1}\sin(k\varphi_2 - k\varphi_1)\frac{dr_1}{d\theta}\right.$$
$$- r_2^{k-1}\, r_1^k \sin(k\varphi_2 - k\varphi_1)\frac{dr_2}{d\theta} - r_1^k\, r_2^k\cos(k\varphi_2 - k\varphi_1)$$
$$\left.\left(\frac{d\varphi_1}{d\theta} + \frac{d\varphi_2}{d\theta}\right)\right) \qquad (5.21)$$

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By substitution of (5.10), (5.12) and (5.14) in the expression for e^{σ} (5.7) and of (5.17), (5.19) and (5.21) in the expression for $\frac{d\tau}{d\theta}$ (5.9), for each value of the variable θ in the ζ-plane the radius of the curve of the profile circumference in the z-plane can be determined.

5.1 Symmetrical Profiles

For a symmetrical profile ($\beta = 0$) the nose of the profile arises from the point $\theta = \pi$ of the circular contour C1 in the ζ-plane. The expressions for r1, r2, r3 and the derivatives over θ of φ_1, φ_2 and φ_3 are then respectively:

$$r_1 = 2 \tag{5.22}$$

$$r_2 = 2(1 - b) \tag{5.23}$$

$$r_3^2 = 4^k\left((1 - b)^{2k} + 1 - 2(1 - b)^k\right) \tag{5.24}$$

$$\frac{d}{d\theta}(\varphi_1) = \frac{1}{2} \tag{5.25}$$

$$\frac{d}{d\theta}(\varphi_2) = \frac{1}{2(1 - b)} \tag{5.26}$$

$$\frac{d}{d\theta}(\varphi_3) = \frac{k}{2}\,\frac{(1 - b)^{2k-1} + 1 - (2 - b)(1 - b)^{k-1}}{(1 - b)^{2k} + 1 - 2(1 - b)^k} \tag{5.27}$$

For e^{σ} and $\frac{d\tau}{d\theta}$ we may write:

$$e^{\sigma} = k^2 b^2 \frac{(1 - b)^{k-1}}{1 + (1 - b)^{2k} - 2(1 - b)^k} \tag{5.28}$$

$$\frac{d\tau}{d\theta} = (k - 1)\frac{(2 - b)}{2(1 - b)} - k\,\frac{1 + (1 - b)^{2k-1} - (2 - b)(1 - b)^{k-1}}{1 + (1 - b)^{2k} - 2(1 - b)^k} \tag{5.29}$$ /12

and therefore for the radius of the profile nose:

$$r_{symm} = \frac{2k^2 b(1 - b)^k}{(k - 1) - (k + 1)(1 - b)^{2k} + 2(1 - b)^k} \tag{5.30}$$

or rendered dimensionless with the expression for the chord (4.7):

$$\left(\frac{r}{c}\right)_{symm} = \frac{k(1 - b)^k}{(k - 1) + (k + 1)(1 - b)^k} \tag{5.31}$$

For a Joukowsky profile, the angle of slope $\delta = 0$ (k = 2) simplifies the expression (5.31) to:

$$\left(\frac{r}{c}\right)_{\text{symm. Joukowsky}} = \frac{2(1-b)^2}{1+3(1-b)^2} \tag{5.32}$$

In figure 5.2 for a series of angles of slope, the relation between the radius of the nose of the symmetrical von Karman-Trefftz profile and the transformation variable b is given.

5.2 Arch Profiles

If the profile is arched (β different from 0) then it is not clear directly which point θ of the circular contour is transfered in the ζ-plane into the nose of the profile in the z-plane (10).

Betz and Keune (11) and Ginzel (12) give for this the point E located on the circular contour (figure 5.1).

In the ζ-plane is the shortest distance from the circular contour to the zero point $\zeta = -b$ (11).

Now the θ of the point E is known, so the relations for e^{σ} (5.7) and $\frac{d\tau}{d\theta}$ (5.9) can be calculated and the radius of the nose of the arched profile is also known.

In figure 5.3 to figure 5.7 for a series of angles of slope the relation is given between the radius of the nose of the arched von Karman-Trefftz profile in the transformation variable b.

If the figures 5.2 to 5.7 are placed one over the other, then each following figure can be produced from the previous one by accomplishing the displacement. /13

It is apparent that this can be done at least with one figure, specifically 5.2 and the following simple relation:

$$(b)_{\text{arched}} = (b)_{\text{symm}} - (1 - \cos\beta) \tag{5.33}$$

It can be noted that the curves in figure 5.3 to 5.7 all intersect at the point $b = \cos \beta$, $r/c = 0$.

In the case of the symmetrical profile $\beta = 0$, the point is: $b = 1$, $r/c = 0$ (figure 5.2).

The earlier mentioned shift is then: $1 = \cos \beta$.

If the radius of the nose, the angle of slope δ and the arched spine of the given profile are known, then it is possible to determine the transformation variable b with figure 5.2 and expression (5.33).

With the quantities δ, β and b, the starting profile of Timman (7) has been established.

6. The Profile Shape

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Expressions for the coordinates of a von Karman-Trefftz profile can be derived in a very simple manner.

The substition of (5.3), (5.4) and (5.5) in expression (2.2) gives:

$$z = kb \frac{(r_2 e^{i\varphi_2})^k + (r_1 e^{i\varphi_1})^k}{r_3 e^{i\varphi_3}} \tag{6.1}$$

After conversion we obtain:

$$z = \frac{kb}{r_3} (\cos \varphi_3 - i \sin \varphi_3) \, (r_2{}^k(\cos k\varphi_2 + i \sin k\varphi_2) + r_1{}^k(\cos k\varphi_1 + i \sin k\varphi_1)) \tag{6.2}$$

The splitting of z into a real and imaginary portion gives:

$$x = \mathrm{Re}(z) = \frac{kb}{r_3} (r_2{}^k \cos (k\varphi_2 - \varphi_3) + r_1{}^k \cos (k\varphi_1 - \varphi_3)) \tag{6.3}$$

$$y = \mathrm{Im}(z) = \frac{kb}{r_3} (r_2{}^k \sin (k\varphi_2 - \varphi_3) + r_1{}^k \sin (k\varphi_1 - \varphi_3)) \tag{6.4}$$

Using the relations (5.10) to (5.15) for each point θ of the circular contour in the ζ-plane, the coordinates x and y of the profile are determined in the z-plane.

The equations (4.4) and (4.6) are used for standardization of the chord c of the profile.

7. The Maximum Profile Thickness

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The maximum thickness of a profile can be defined as the distance between the lines of contact on the profile contour which are parallel to the real axis (see figure 7.1).

The determination of the profile thickness is then reduced basically to finding the extreme values of y; specifically, the solution of the equation $\frac{d}{d\theta}(y) = 0$.

The differentiation of the expression (6.4) over θ gives:

$$\frac{d}{d\theta}(y) = \frac{kb}{r_3^2}\Big\{r_3\Big(k\, r_2^{k-1}\frac{dr_2}{d\theta}\sin(k\varphi_2-\varphi_3) + r_2^k\cos(k\varphi_2-\varphi_3)\Big(k\frac{d\varphi_2}{d\theta}-\frac{d\varphi_3}{d\theta}\Big) + k\, r_1^{k-1}\frac{dr_1}{d\theta}\sin(k\varphi_1-\varphi_3) + r_1^k\cos(k\varphi_1-\varphi_3)\Big(k\frac{d\varphi_1}{d\theta}-\frac{d\varphi_3}{d\theta}\Big)\Big) - \Big(r_2^k\sin(k\varphi_2-\varphi_3) + r_1^k\sin(k\varphi_1-\varphi_3)\Big)\frac{dr_3}{d\theta}\Big\} \tag{7.1}$$

With expressions (5.10) to (5.21), it is now possible to determine for each value of the variable θ in the ζ-plane the $\frac{d}{d\theta}(y)$.

Generally there are three points θ on the profile contour for which y reaches an extreme value $\left(\frac{d}{d\theta}(y) = 0\right.$; see figure 7.1).

The above given definition is not suitable for arched profiles. From figure 7.1 with this definition it follows from 7.1 that the point of the maximum thickness cannot be indicated uniquely. Therefore in the following we were only considering symmetrical profiles ($\beta = 0$).

7.1 Symmetrical Joukowsky Profiles

For a symmetrical Joukowsky profile the expression (2.6) may be written as:

$$z = (b-1) + (\cos\theta + i\sin\theta) + \frac{b^2(b-1) + b^2(\cos\theta - i\sin\theta)}{(b-1)^2 + 2(b-1)\cos\theta + 1} \tag{7.2}$$

The splitting of z into a real and imaginary portion gives for the x and y coordinates: /16

$$x = \mathrm{Re}(z) = (b - 1) + \cos\theta + \frac{b^2(b - 1) + b^2 \cos\theta}{(b - 1)^2 + 2(b - 1)\cos\theta + 1} \qquad (7.3)$$

$$y = \mathrm{Im}(z) = \sin\theta - \frac{b^2 \sin\theta}{(b - 1)^2 + 2(b - 1)\cos\theta + 1} \qquad (7.4)$$

For the derivative of y over θ we may write:

$$\frac{d}{d\theta}(y) = \cos\theta - \frac{b^2 \cos\theta\,\{(b - 1)^2 + 1\} + 2b^2(b - 1)}{\{(b - 1)^2 + 2(b - 1)\cos\theta + 1\}^2} \qquad (7.5)$$

The solution of the equation $\frac{d}{d\theta}(y) = 0$ gives the values of θ for which y reaches an extreme value.

After the necessary conversion and the introduction of:

$$(b - 1)^2 + 1 = B \qquad (7.6)$$

a third degree equation in cos θ is obtained:

$$4(b - 1)^2 \cos^3\theta + 4B(b - 1)\cos^2\theta + B(B - b^2)\cos\theta - 2b^2(b - 1) = 0 \qquad (7.7)$$

To solve this equation, the method indicated in (13) is used.

The first root $(\cos\theta)_1$, gives two values for θ in which $\theta_2 = 2\pi - \theta_1$. The substitution of θ_1 or θ_2 in expression (7.4) gives half the maximum figures. The position of the thickness is obtained with (7.3).

The second root is: $(\cos\theta)_2 = 1$; that is, $\theta_3 = 0$ and $\theta_4 = 2\pi$, specifically giving the tail point of the profile.

The third root, $(\cos\theta)_3$ is more than 1 and has no other practical meaning.

7.2 Symmetrical von Karman-Trefftz Profiles

The maximum profile thickness (see figure 7.2) should occur when:

$$\theta + \frac{\pi}{2} + \tau = \pi \qquad (7.8)$$ /17

The substitution of the expression for τ (5.8) in the above relation gives:

$$\theta + \frac{\pi}{2} + (k - 1)(\varphi_2 + \varphi_1) - 2\varphi_3 = \pi \tag{7.9}$$

in which φ_1, φ_2 and φ_3 are functions of θ (see specifically the relations (5.11), (5.13) and (5.15)).

The value of θ which satisfies the requirements formulated in (7.9) is found by means of an iteration process.

The substitution of the θ calculated by this means in expression (6.4) gives half the maximum profile thickness. The position of the maximum thickness is obtained with equation (6.3).

Figures 7.3 and 7.4 give for a series of angle of slope the maximum profile thickness (in percentage c) and the position of the maximum thickness of the function of the transformation variable b.

8. Literature /18

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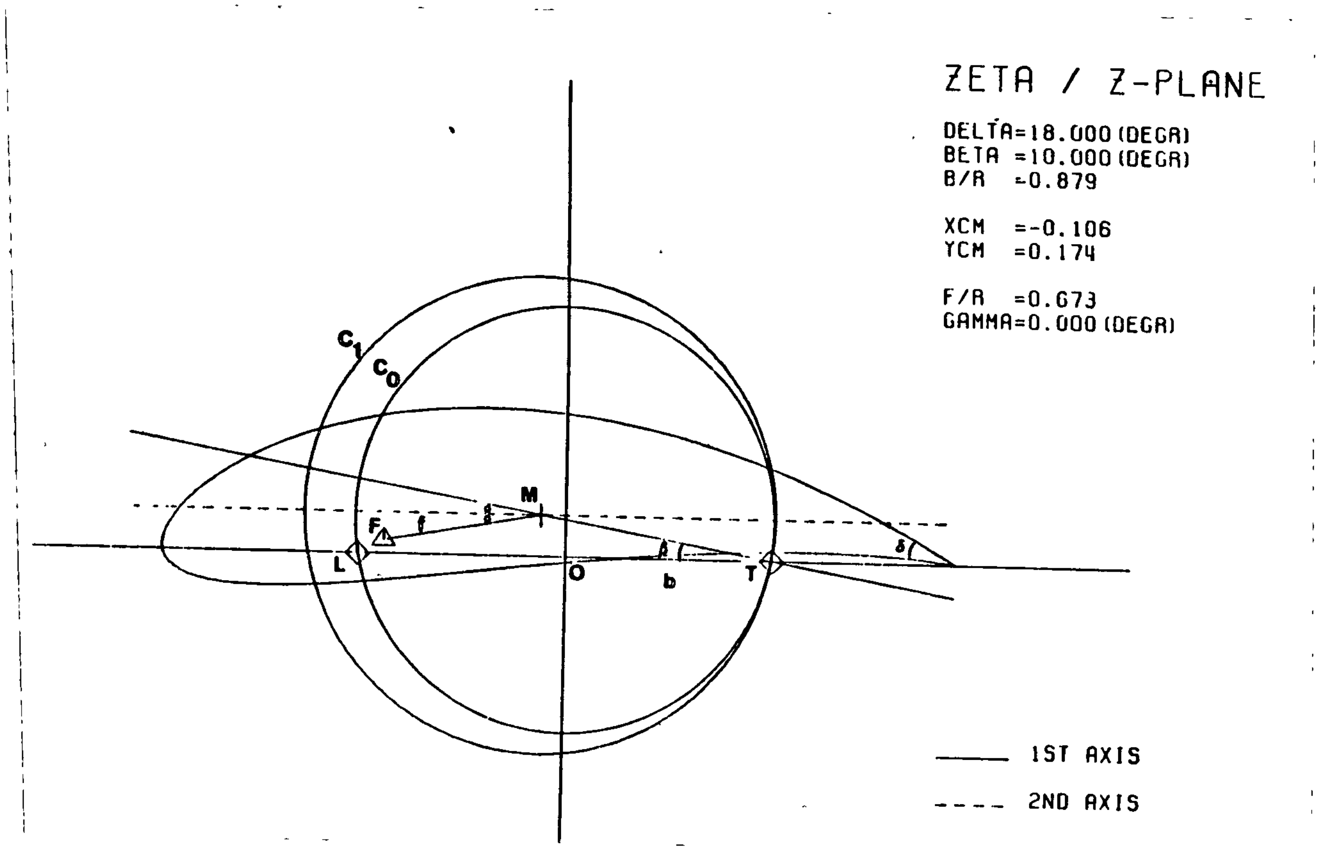

Figure 2.1: Von Karman-Trefftz profile

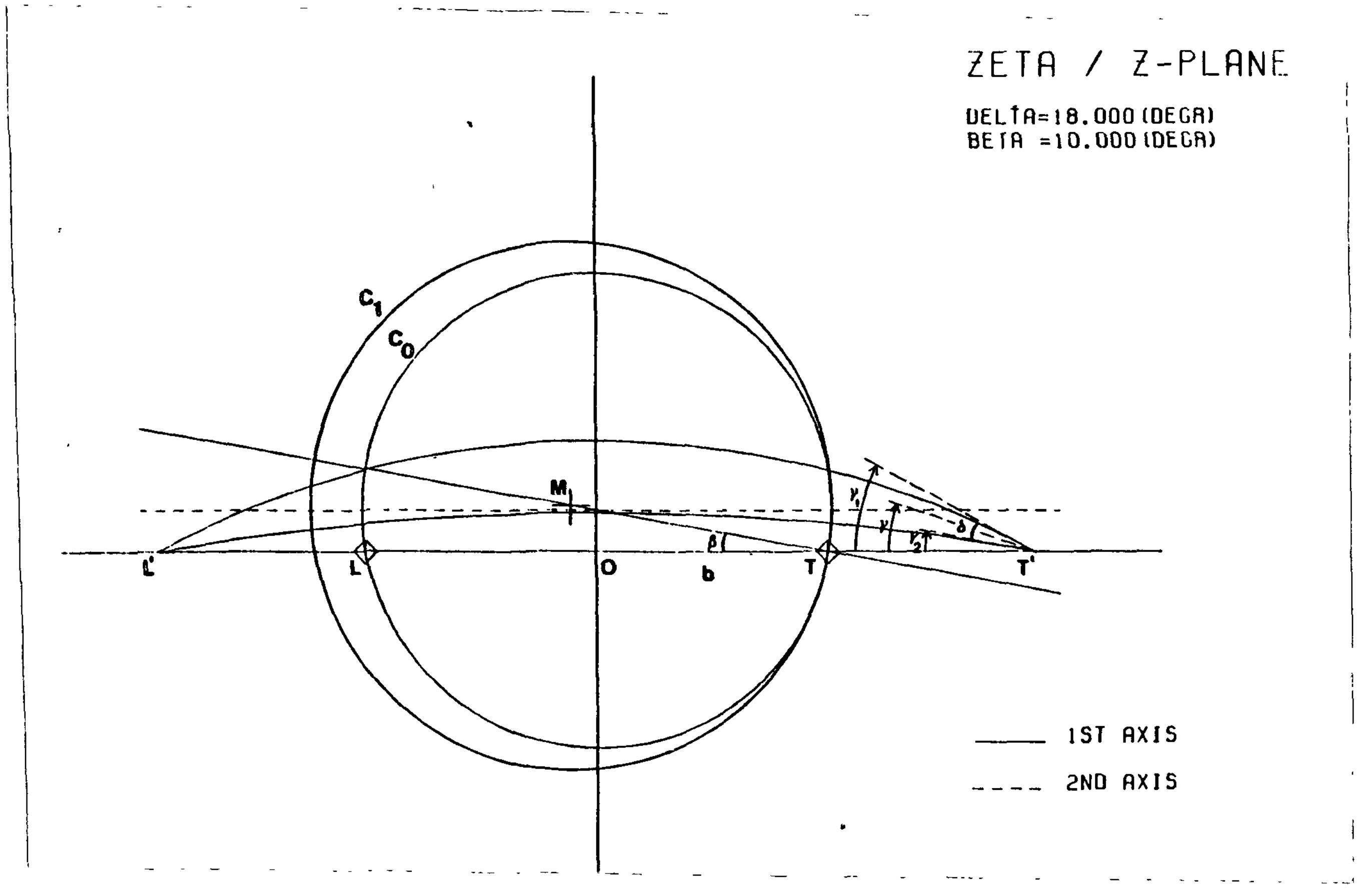

Figure 2.2: Von Karman-Trefftz skeletons

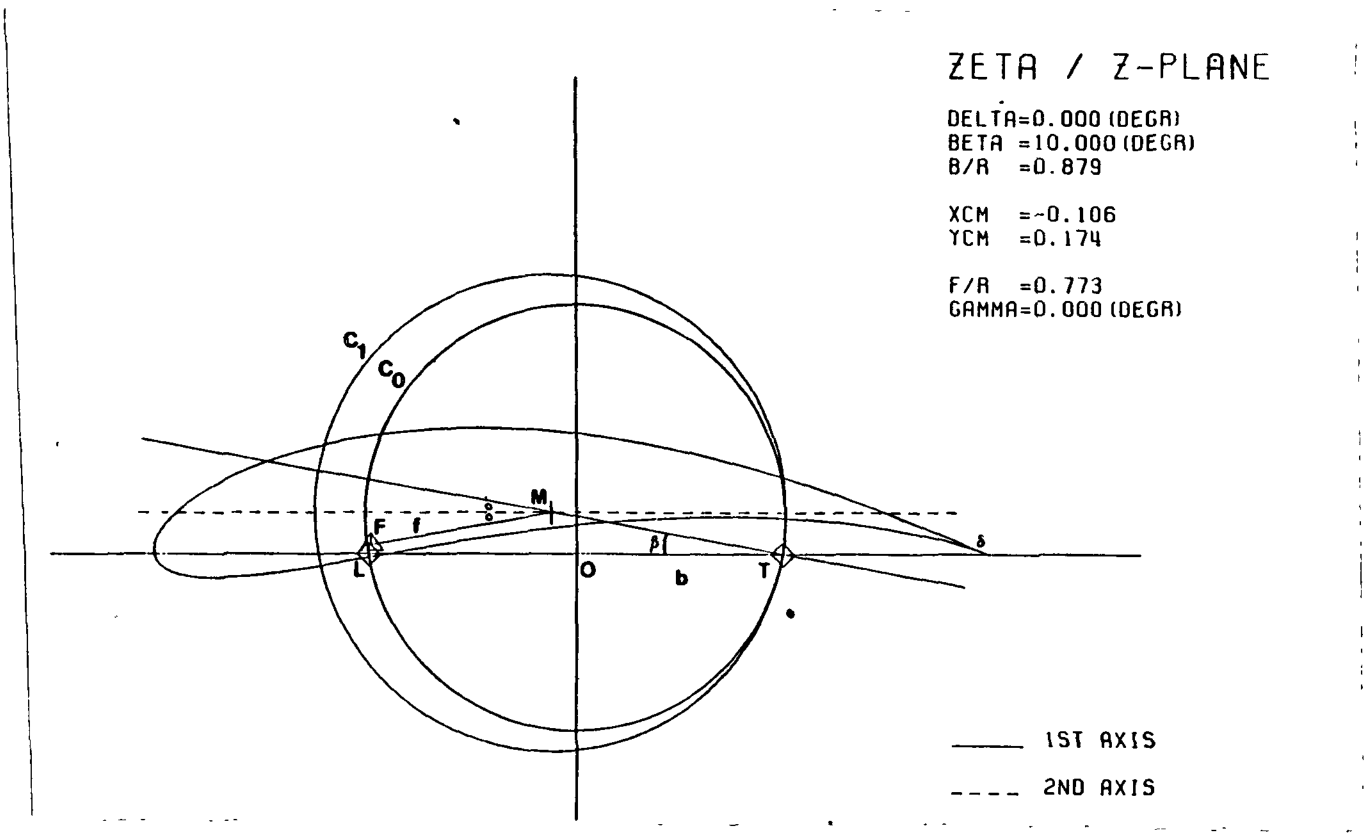

Figure 2.3: Joukowsky profile

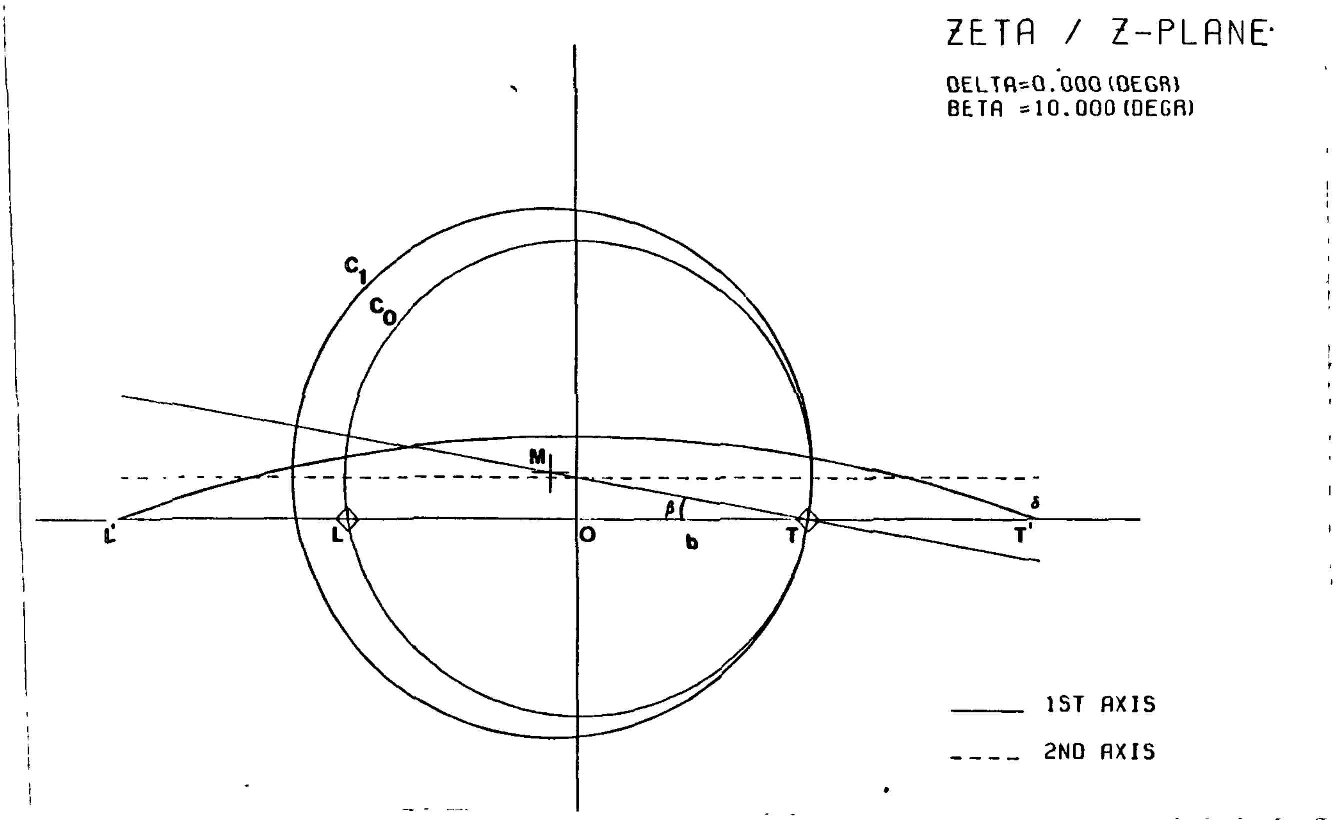

Figure 2.4: Joukowsky skeleton

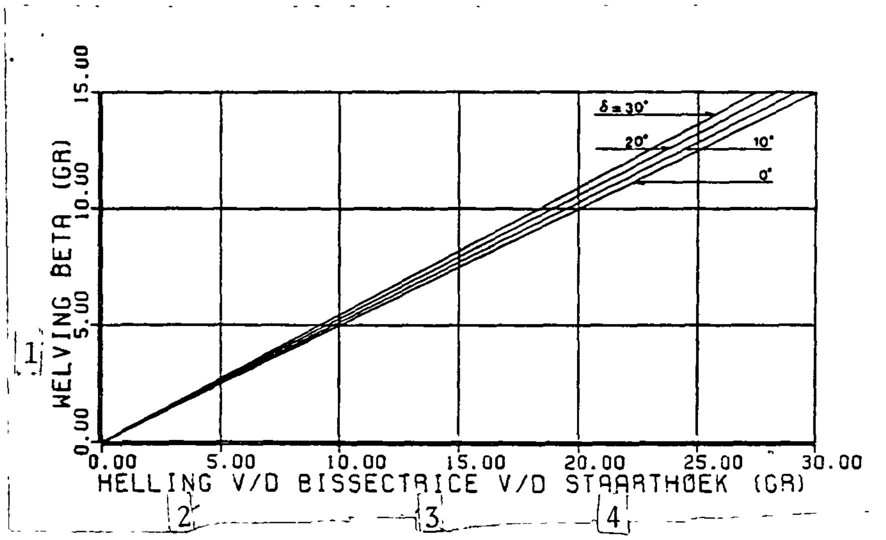

Figure 3.1: Arching β as a function of the slope γ for different δ.
Key: (1) arching; (2) slope; (3) bisectrix; (4) angle of slope

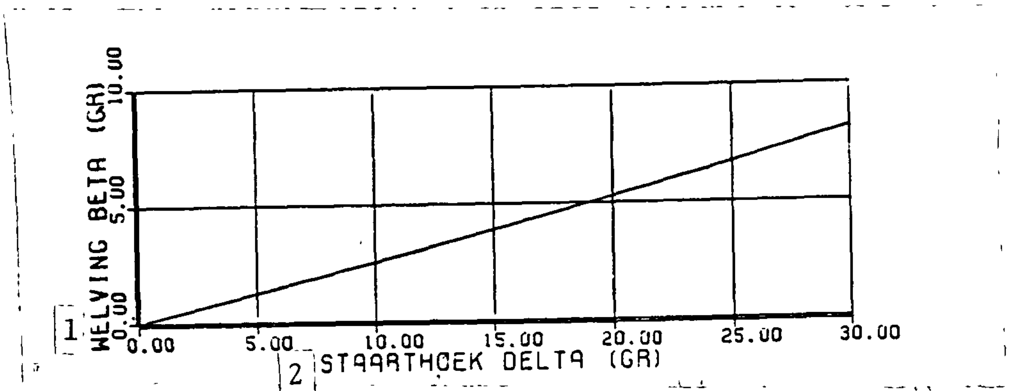

Figure 3.2: Arching β as a function of the angle of slope δ for a flat skeleton bottom ($\gamma_2 = 0$).
Key: (1) arching; (2) angle of slope.

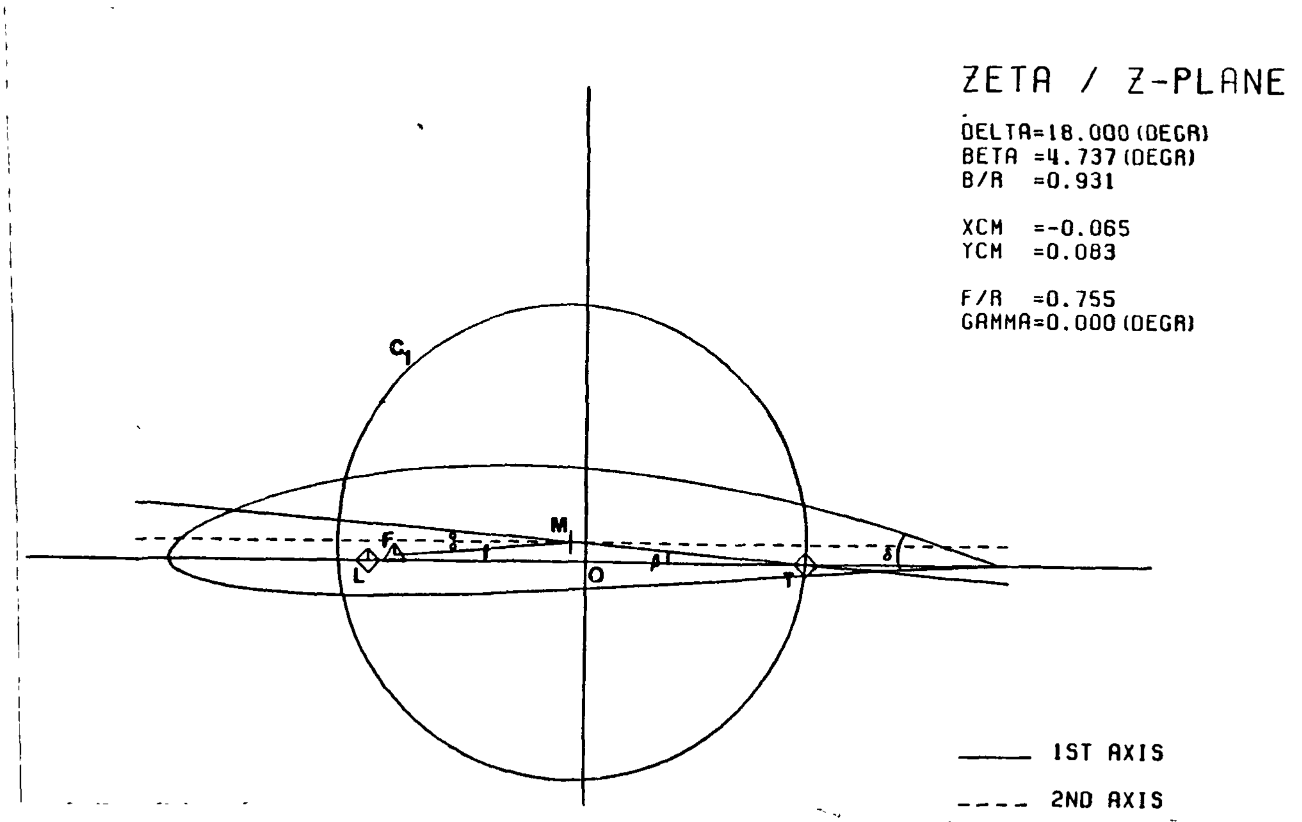

Figure 3.3: Von Karman-Trefftz profile with flat skeleton bottom ($\gamma_2 = 0$).

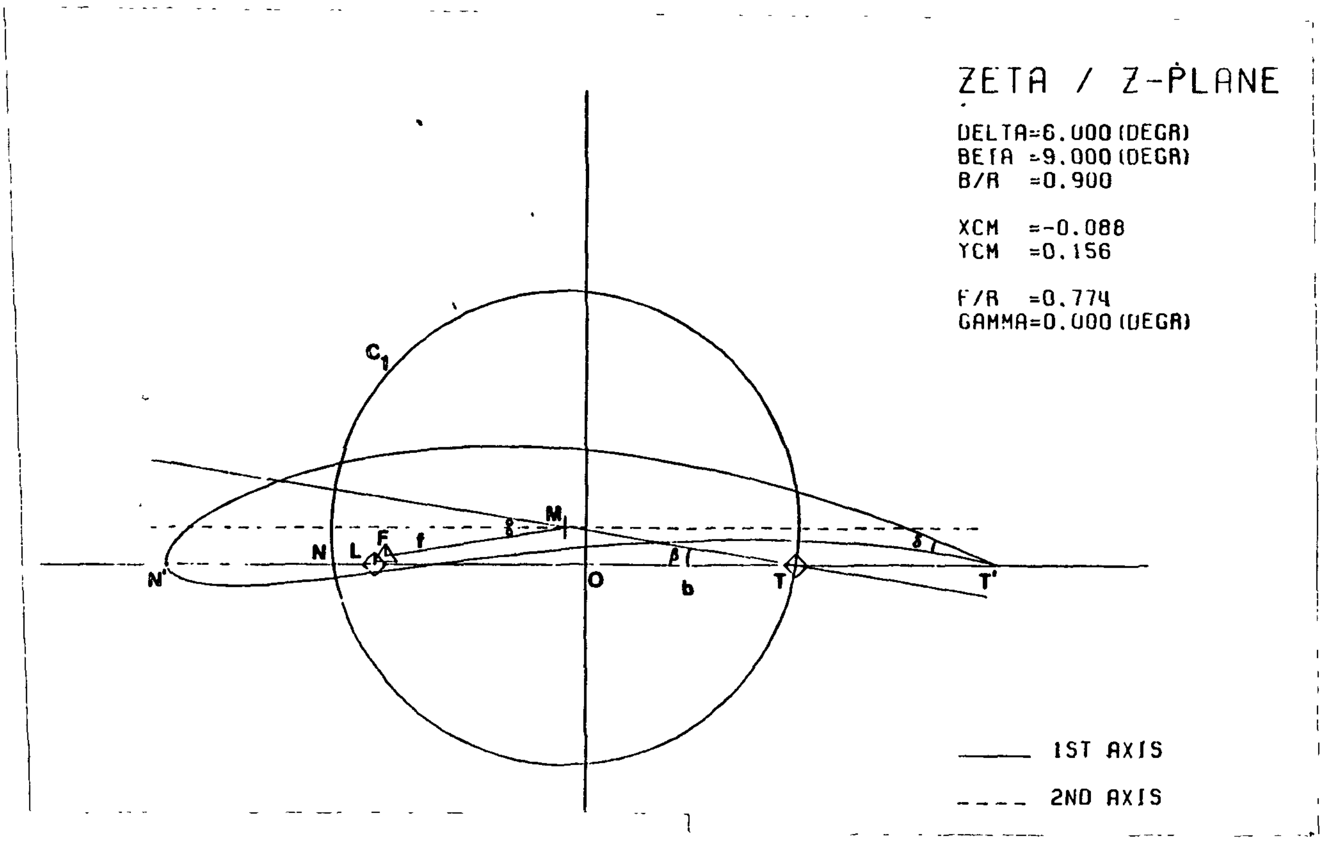

Figure 4.1: Definition of the profile chord

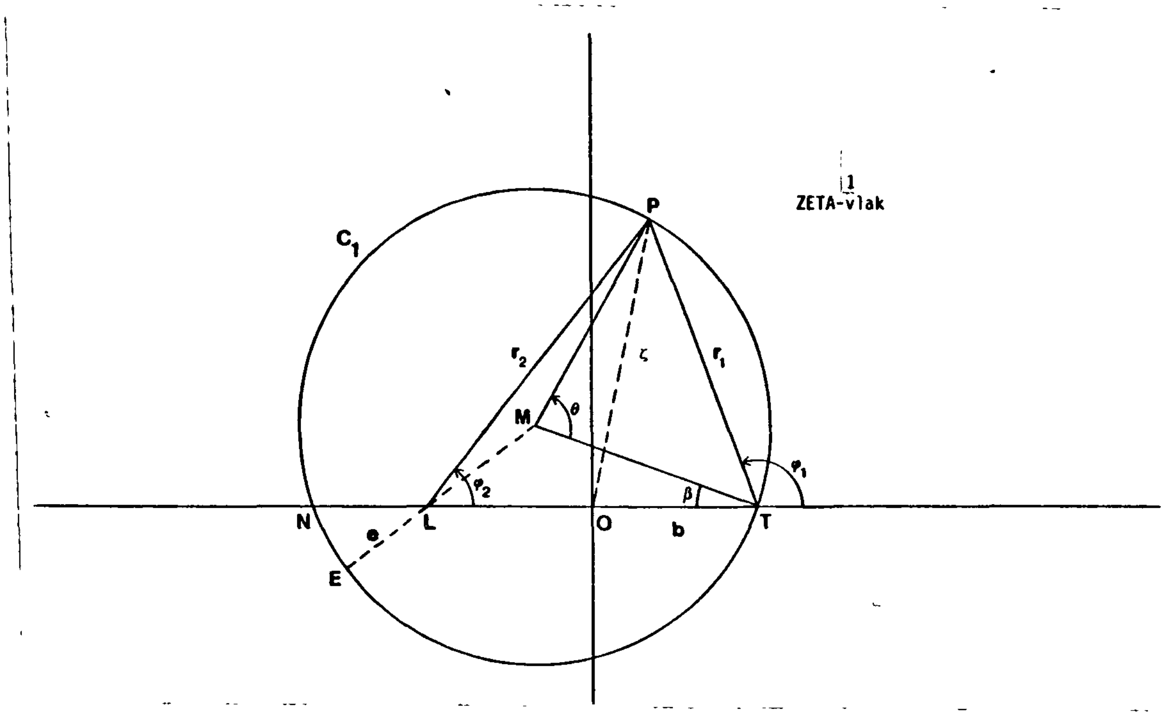

Figure 5.1: Geometrical meaning of r1, 42, φ_1. and φ_2.
Key: (1) ZETA-plane

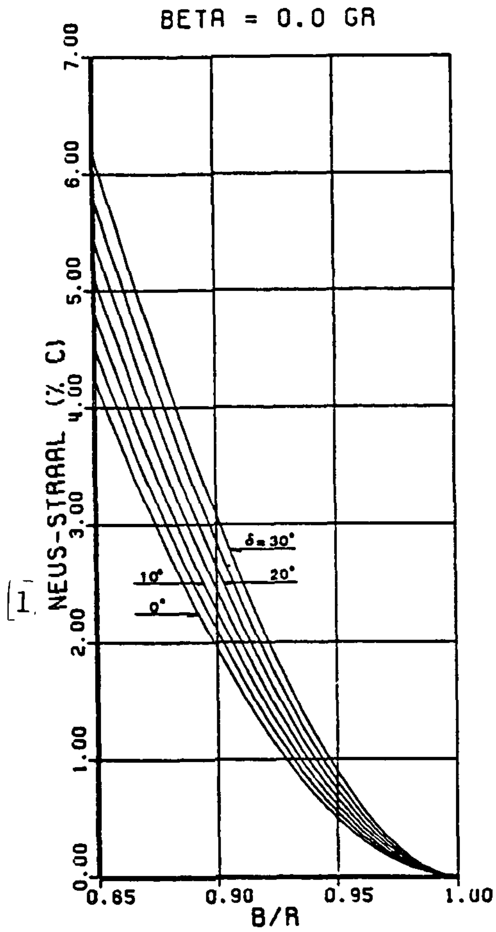

Figure 5.2: Radius of nose on (% c) as a function of the transformation variable b. (β = 0-degrees): Key: (1) radius of nose.

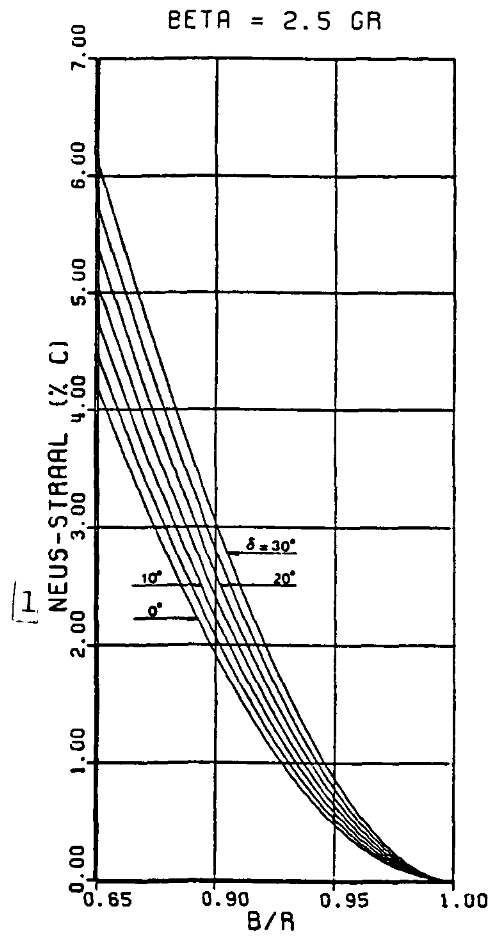

Figure 5.3: The radius of the nose r (% c) as a function of the transformation variable b (β = 2.5 degrees).
Key: (1) nose radius.

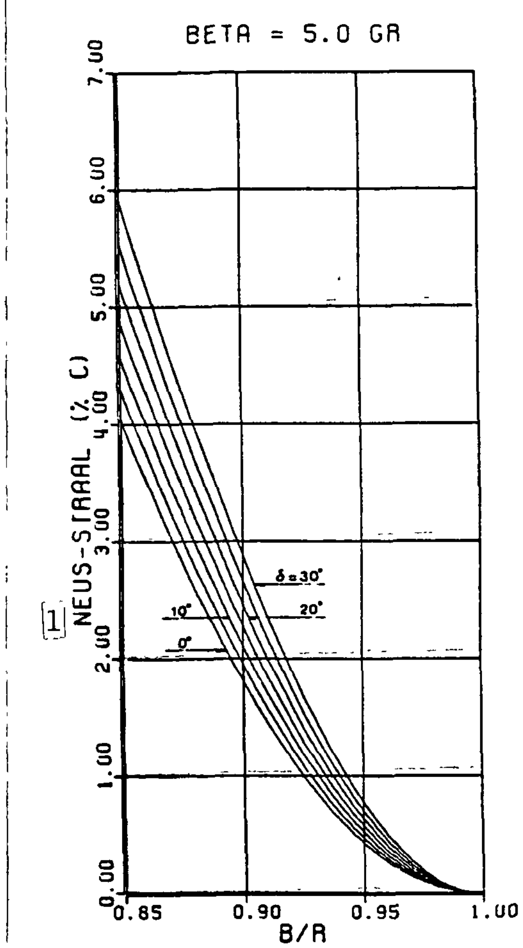

Figure 5.4: Radius of the nose r (% c) as a function of the transformation variable b. (β = 5.0 degrees). Key: (1) radius of the nose.

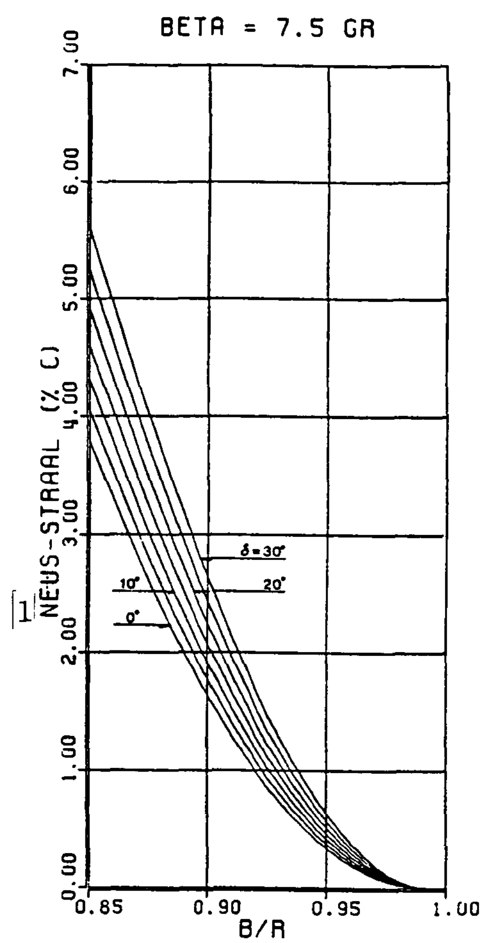

Fig..5:5:Radius of the nose r (% c) as a function of the transformation variable b. (β = 7.5 degrees). Key: (1) radius of the nose.

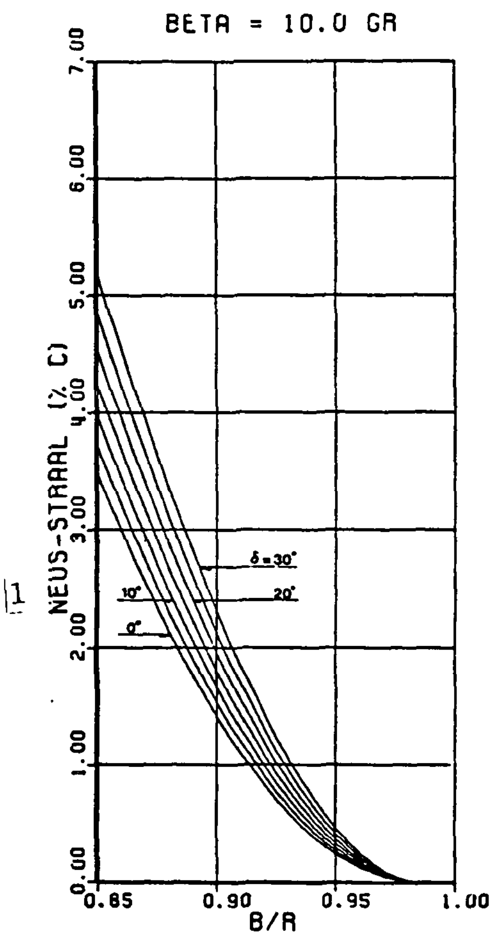

Figure 5.6: Radius of the nose r (% c) as a function of the transformation variable b (β = 10.0 degrees). Key: (1) radius of the nose.

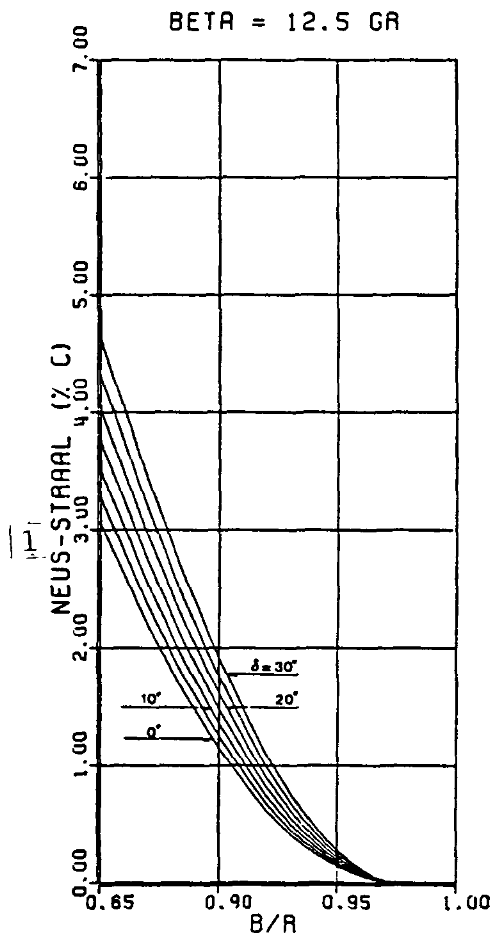

Figure 5.7: Radius of the nose r (% c) as a function of the transformation variable b (β = 12.5 degrees). Key: (1) radius of the nose.

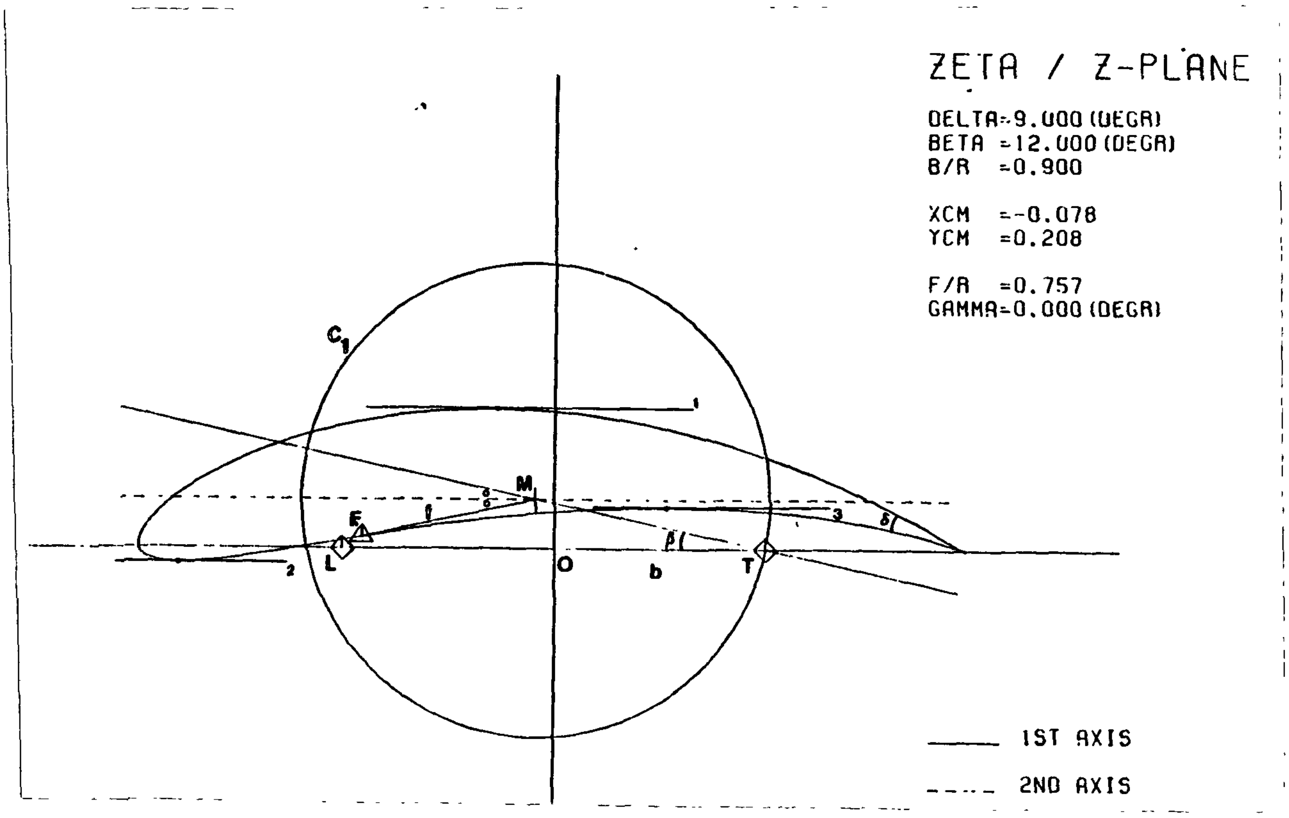

Figure 7.1: Lines of contact on the profile contour proportional to the rear axis.

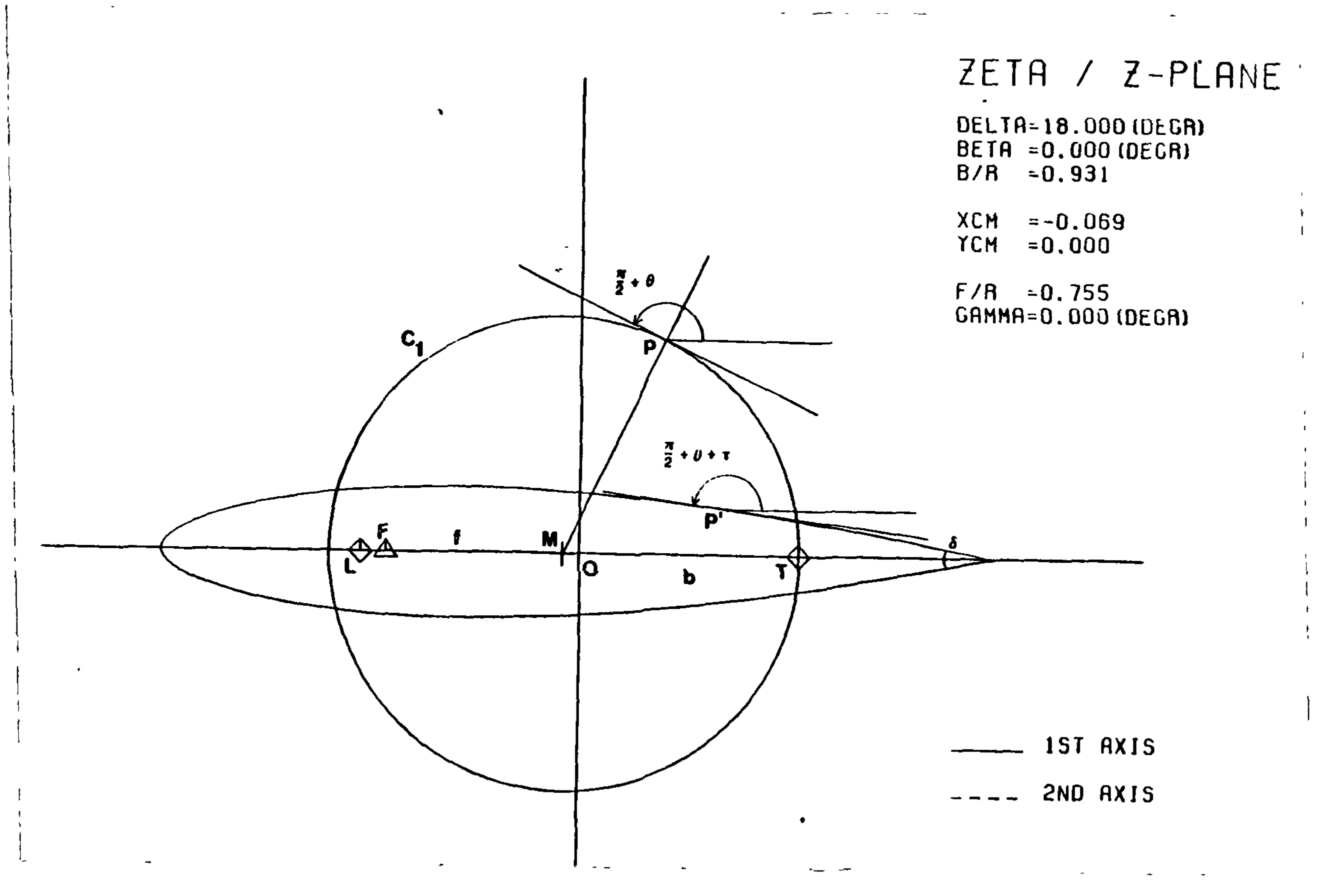

Figure 7.2: Determination of the maximum thickness for a symmetrical von Karman-Trefftz profile.

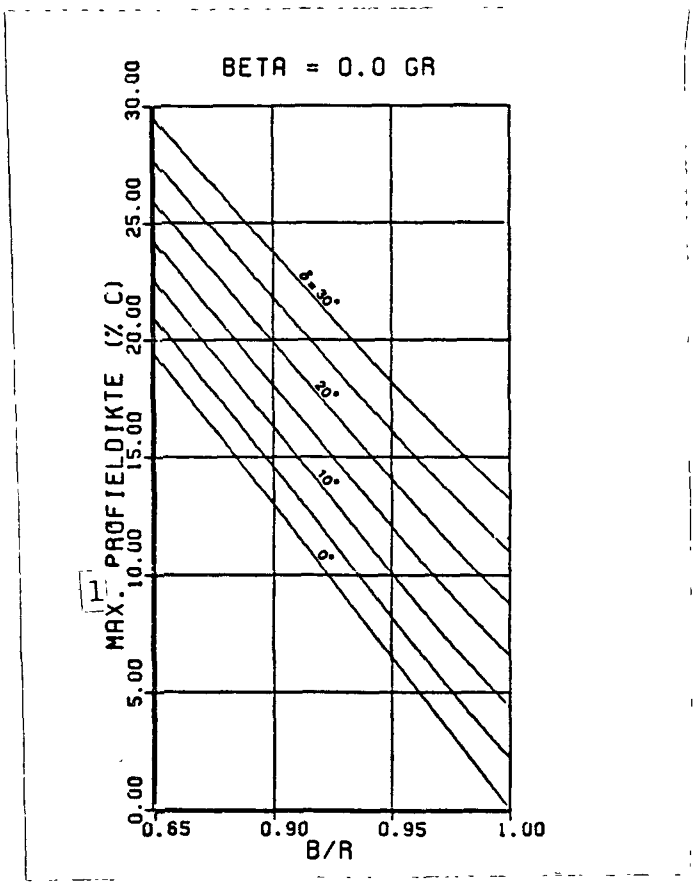

Figure 7.3: Maximum profile thickness (% c) as a function of the transformation variable b (β = 0 degrees).
Key: (1) maximum profile thickness.

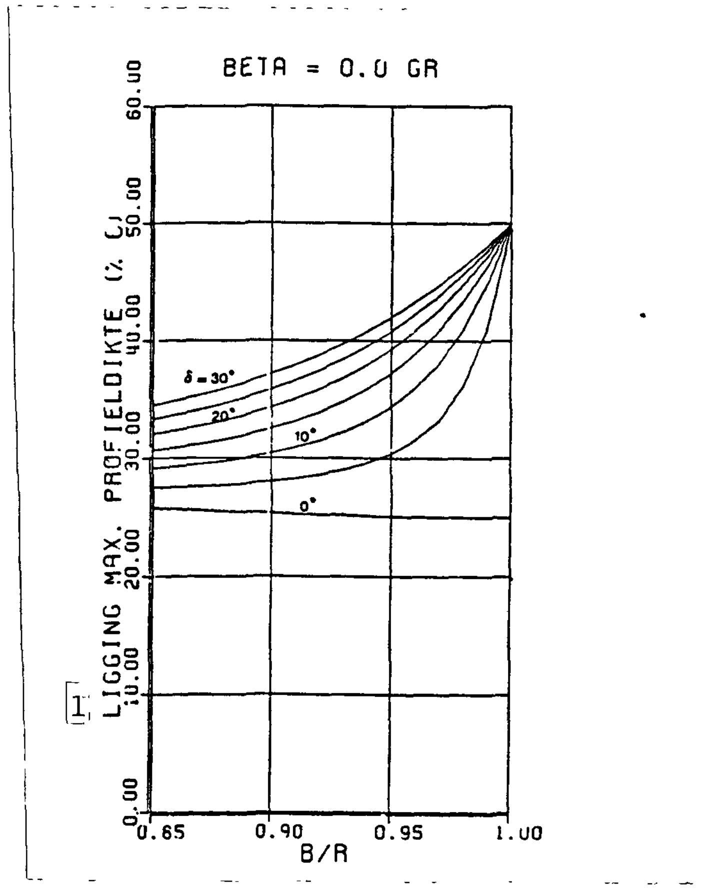

Figure 7.4: Position of the maximum profile thickness (% c) as a function of the transformation variable b (β = 0 degrees). Key: (1) position of the maximum profile thickness.