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ENHANCED MANUAL CONTROLLABILITY
VIA ACTIVE CONTROL OF
AEROELASTIC VEHICLES

Semi-Annual Status Report
For the Period Aug. 1, 1982 to Jan. 31, 1983

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Contents

1. Introduction

This constitutes the semi-annual progress report for the period 7/31/82-1/31/83 on the research being performed by the School of Aeronautics and Astronautics, Purdue University, for NASA's Langley Research Center under grant number NAG-1-254. The subject area of the research includes flexible aircraft dynamics, handling qualities of flexible aircraft, and active control for good handling characteristics for such vehicles.

2. Technical Progress

During this first year of grant activity, a modal analysis technique has been developed for evaluating the effects of elastic modes on aircraft dynamic response, and the handling qualities implication of these effects. This technique will be discussed, along with results of application of the technique in a technical paper entitled

A Modal Analysis of Flexible Aircraft Dynamics With Handling Qualities Implications.

This paper will be presented at the 1983 AIAA Atmospheric Flight Mechanics Conference, to be held in Gatlinburg, Tenn. in August. The extended abstract for this paper is included as Appendix A of this status report.

As discussed in the paper's abstract, the analytical technique for the analysis of flexible flight vehicle dynamics includes the analysis of the vehicle's dynamic modes, and specifically includes the dynamic model of the atmospheric disturbance and pilot lags in the analysis. Fundamental results obtained in such an analysis are not only the eigenvalues and eigenvectors for the system modes, but also the degree of controllability (disturbability), observability, and impulse response residues for each system mode. By obtaining these important measures of response to meaningful system inputs, such as those from the pilot and turbulence, the significance of the participation of each mode may be assessed. The analytical methodology is briefly summarized in Table I included here.

As of the date of this report, the longitudinal dynamics of a family of eight vehicle math models have been evaluated, and the lateral directional dynamics are under investigation presently. These eight configurations were evaluated by Yen in a fixed-base laboratory simulation study. The geometry of all of these configurations is similar to the B-1 aircraft, but the structural stiffness of each configuration is varied parametrically in such a way that the elastic mode shapes remain unchanged, but the in-vacuo frequencies are reduced. These eight configurations then are identified in terms of their eigenvalues in Table II. Shown in Figs. 1 and 2 are the attitude tracking results and subjective ratings obtained in Yen's simulations.

Note, for example, that in terms of eigenvalues alone, it is not at all clear why the results for configuration 3 were so much worse than for configuration 4. Both had unstable phugoid roots, and one rather low-frequency aeroelastic mode. In the case of configuration 3, this low-frequency mode included a large amount of fuselage bending, while for configuration 4, the eigenvectors indicated that the low-frequency mode included a good deal of wing bending. This fact partially explains why attitude tracking would be more difficult for configuration 3.

TABLE I

SUMMARY OF MATH METHOD

Have models

vehicle $\dot{x}_v = Ax_v + Bx_p + Dx_g$

gust $\dot{x}_g = A_g x_g + G_g \omega_g$

white noise

pilot lags $\dot{x}_p = A_p x_p + G_p \omega_p$

- where $x_v = \begin{bmatrix} x_{rigid} \\ \hdashline x_{elastic} \end{bmatrix}$ vehicle states

x_p pilot states

x_g gust states

Form

$$\begin{bmatrix} \dot{x}_p \\ \dot{x}_g \\ \dot{x}_v \end{bmatrix} = \begin{bmatrix} A_p & 0 & 0 \\ 0 & A_g & 0 \\ B & D & A_v \end{bmatrix} \begin{bmatrix} x_p \\ x_g \\ x_v \end{bmatrix} + \begin{bmatrix} G_p \\ 0 \\ 0 \end{bmatrix} \omega_p + \begin{bmatrix} 0 \\ G_g \\ 0 \end{bmatrix} \omega_g$$

and outputs (responses)

$$y = Cx_v + Ex_p + Fx_g$$

$$= [E \quad F \quad C] \begin{bmatrix} x_p \\ x_g \\ x_v \end{bmatrix}$$

so collecting we have

$$\dot{x} = A^*x + G_p^*\omega_p + G_g^*\omega_g$$

$$y = C^*x$$

Now Define

$$T \triangleq \text{modal matrix (matrix of eigenvectors) of } A^*$$

Then in Terms of Vehicle Modal States

$$\dot{q} = T^{-1}A^*Tq + T^{-1}G_p^*\omega_p + T^{-1}G_g^*\omega_g$$

$$y = C^*Tq$$

With

$$\tilde{B} = T^{-1}G_p^* \qquad \text{controllability matrix}$$

$$\tilde{D} = T^{-1}G_g^* \qquad \text{disturbability matrix}$$

$$\tilde{C} = C^*T \qquad \text{observability matrix}$$

$$\Lambda = T^{-1}A^*T \qquad \text{(diagonal)}$$

or

$$\dot{q} = \Lambda q + \tilde{B}\omega_p + \tilde{D}\omega_g$$

$$y = \tilde{C}q$$

so, after Laplace Transforming -

$$y = \tilde{C}[sI-\Lambda]^{-1}\tilde{B}\,\omega_p + \tilde{C}[sI-\Lambda]^{-1}\,\tilde{D}\omega_g$$

$$[sI-\Lambda]^{-1} = \begin{bmatrix} \frac{1}{s-\lambda_1} & & 0 \\ & \ddots & \\ 0 & & \frac{1}{s-\lambda_n} \end{bmatrix}$$

Table I (cont'd)

let

$$\tilde{C} = \begin{bmatrix} \bar{c}_1 \\ \hline \bar{c}_2 \\ \hline \vdots \\ \hline \bar{c}_n \end{bmatrix} \quad ; \quad \tilde{B} = [\bar{b}_1 \mid \bar{b}_2 \mid \cdots \mid \bar{b}_m] \; ; \quad \tilde{D} = [d]$$

Note for each output y_j and each input u_k we would have

$$\frac{y_j(s)}{u_k(s)} = \sum_{i}^{n} \frac{R_i}{s-\lambda_i} \quad ; \quad R_i = \text{Residue for } \lambda_i$$

So the Residues Are For Each y_i and ω_{p_j} or ω_g are

$$R_{ij} = [c_{i1}b_{j1} \, , \, c_{i2}b_{j2}, \cdots \, c_{in}b_{jn}] = \langle \bar{c}_i b_j \rangle$$

and

$$R_{ig} = \langle c_i d \rangle$$

where ω_{p_j} and ω_g are taken as impulses.

Table 2 Natural Frequencies and Damping Ratios of Eight Cases

Case #	ω_1 rad/sec	ω_2 rad/sec	ζ_{sp}	ω_{sp} rad/sec	ζ_{ph}	ω_{ph} rad/sec	ζ_{1e}	ω_{1e} rad/sec	ζ_{2e}	ω_{2e} rad/sec
1	13.59	21.18	0.5339	2.806	0.0197	0.0708	0.0494	13.312	0.0215	21.354
2	9.17	21.18	0.5235	2.5724	-0.00060267	0.0573	0.08769	8.7891	0.0213	21.356
3	6.16	21.18	0.5217	1.7691	Real Roots +0.090978 -0.076723		0.1999	5.8669	0.0213	21.357
4	13.59	4.79	0.6872	1.5745	Real Roots +0.14654 -0.13167		0.05284	13.270	0.1137	5.9702
5	11.66	11.66	0.5436	2.5819	-0.0001122	0.0537	0.0773	11.801	0.0162	11.574
6	6.93	6.93	0.7028	1.3665	Real Roots +0.17581 -0.15307		0.1919	7.3305	0.007599	6.9178
7	10.25	9.75	0.5517	2.3937	-0.0483	0.0282	0.1129	10.234	-0.0004277	9.8978
8	10.68	9.27	0.5549	2.3893	-0.0541	0.0256	0.11021	10.347	0.0005306	9.7781

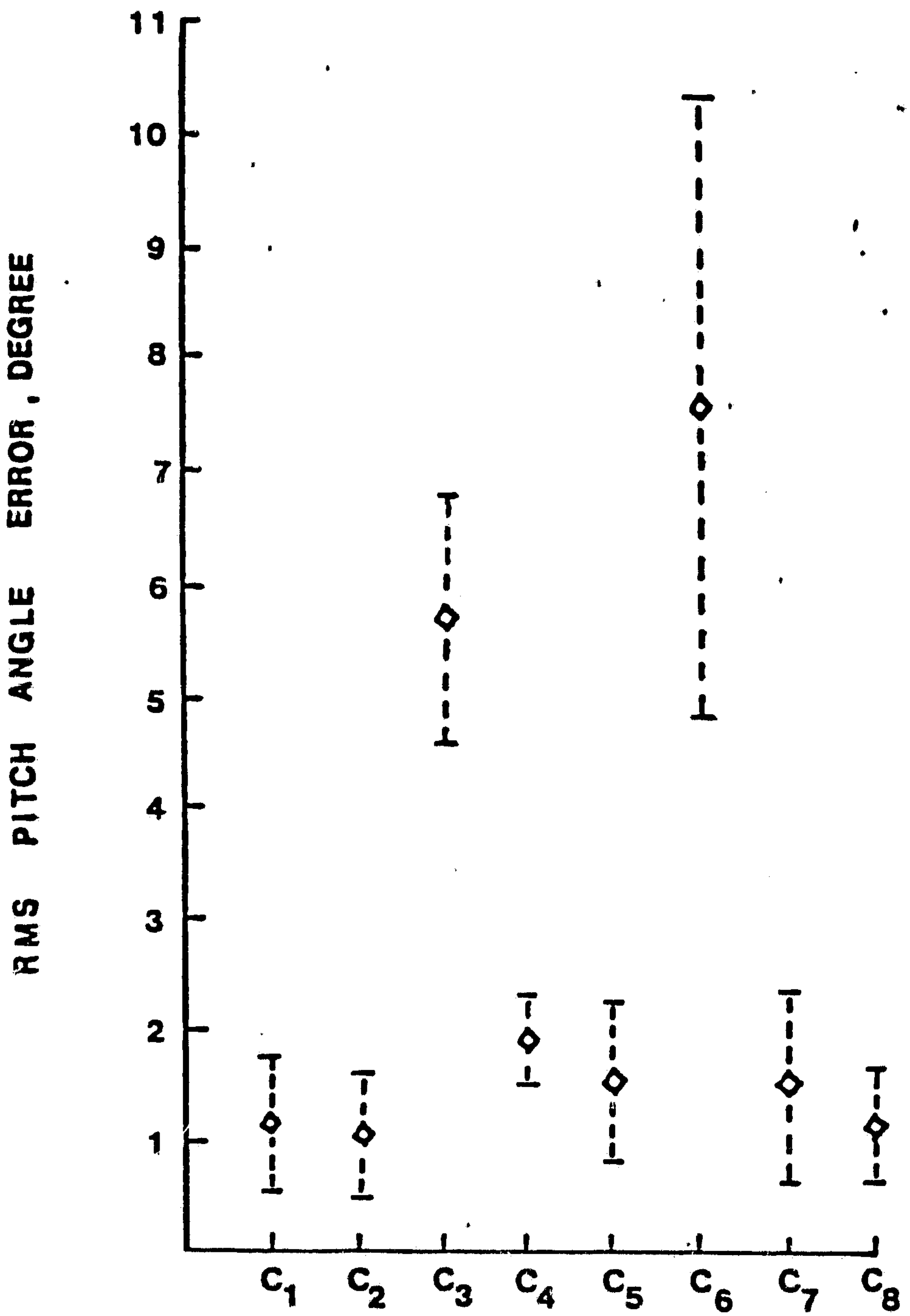


Figure 1 Tracking Performance Versus Cases

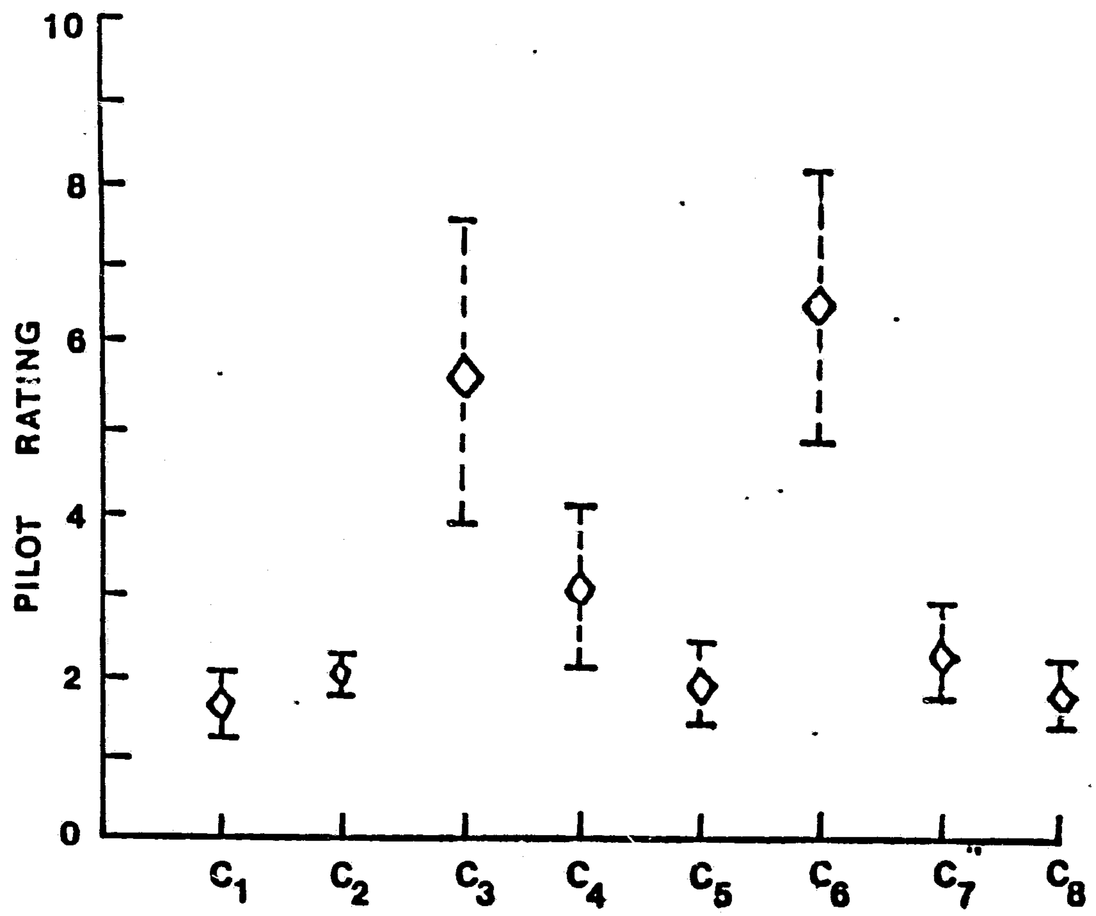


Figure 2 Cooper-Harper Ratings

PILOT RATING	PILOT COMPENSATION REQUIRED
8	UNACCEPTABLE
7	MAXIMUM
6	EXTENSIVE
5	CONSIDERABLE
4	MODERATE
3	MINIMAL
2	UNNEEDED

The remaining piece of information about these configurations relates to the magnitude of the participation of the modes in the attitude response of the vehicle. This is measured by the relative magnitudes of the residues of the modes.

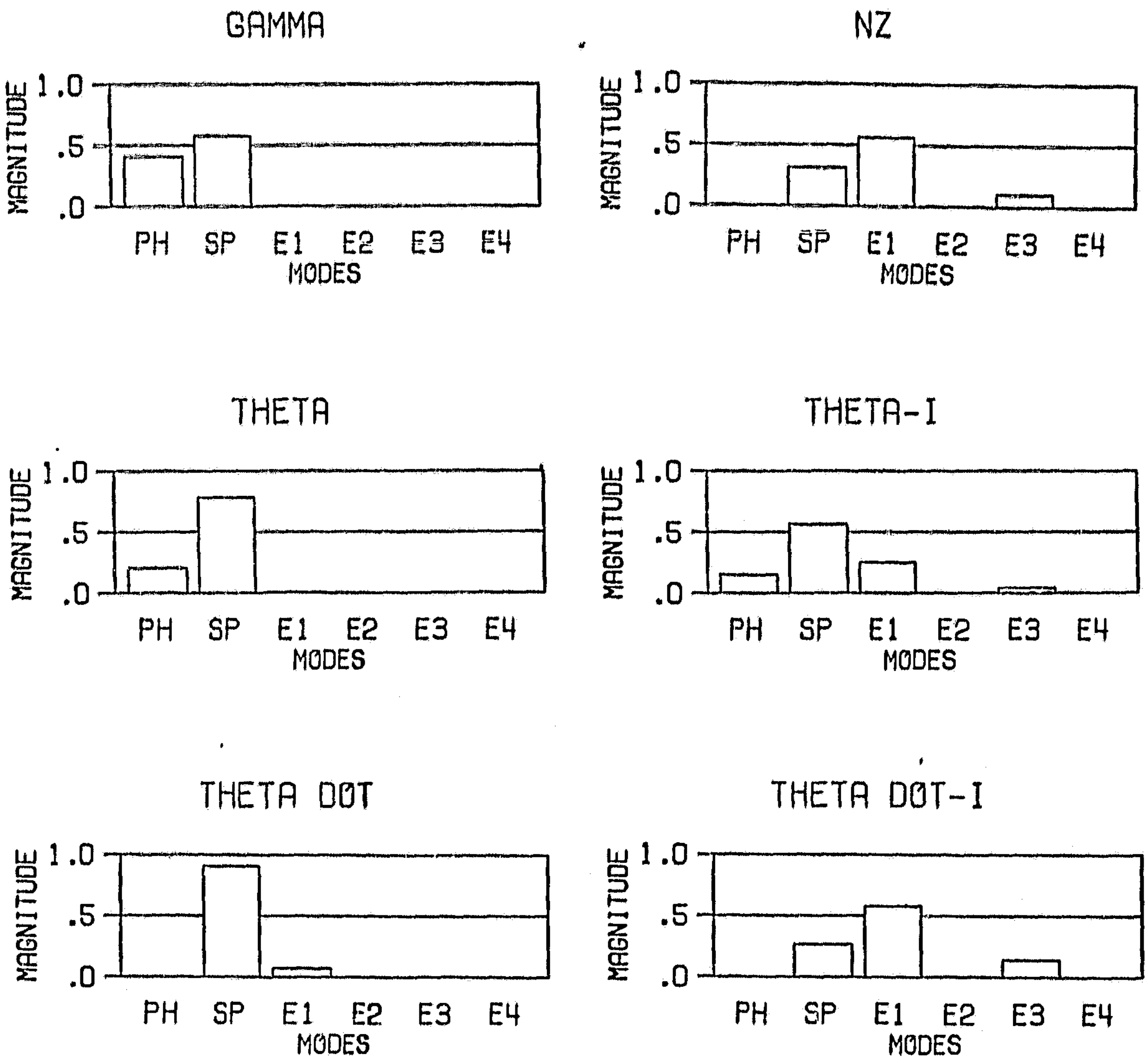
Now four vehicle responses known to be significant in longitudinal dynamics are flight path angle γ, vertical acceleration at the cockpit n_z, rigid-body pitch attitude Θ, and pitch rate $\dot{\Theta}$. In addition, the local indicated pitch attitude and pitch rate $(\Theta_I, \dot{\Theta}_I)$ are also important for flexible vehicles. Local indicated attitude differs from rigid attitude in that the former includes the local elastic slope of the structure relative to the rigid-body axis. This local attitude angle is referred to as "indicated" since this is the quantity measured by a sensor placed at the fuselage station in question.

Shown in Figs. 3-6 are the residues for the two rigid-body modes (Phugoid and short-period) and as well as four aeroelastic vehicle modes (E1-E4) for four of Yen's eight configurations. Two of these modes (E1 and E3) are the two modes included in the configurations simulated by Yen (who did not include the two less important modes E2 and E4 in his math model). All these modes are identified from their eigenvectors. Now the problem with configuration 3 is clear. Unlike the other three configurations, the rigid-body attitude response, Θ, is dominated by elastic mode E1 rather than by the rigid-body short-period mode! This is evident from the relative magnitudes of the modal residues in the theta responses. Note here that the input is an impulse passed through the pilot's lag model, or the actual input if the pilot attempted to input an impulse. From this result, it is clear that any attempts by the pilot to track a commanded attitude would be amost impossible. Also, characterizing the configuration handling qualities in terms of the rigid-body eigenvalues is meaningless.

RESIDUE MAGNITUDES

(DUE TO PILOT INPUTS)

CASE 1 -



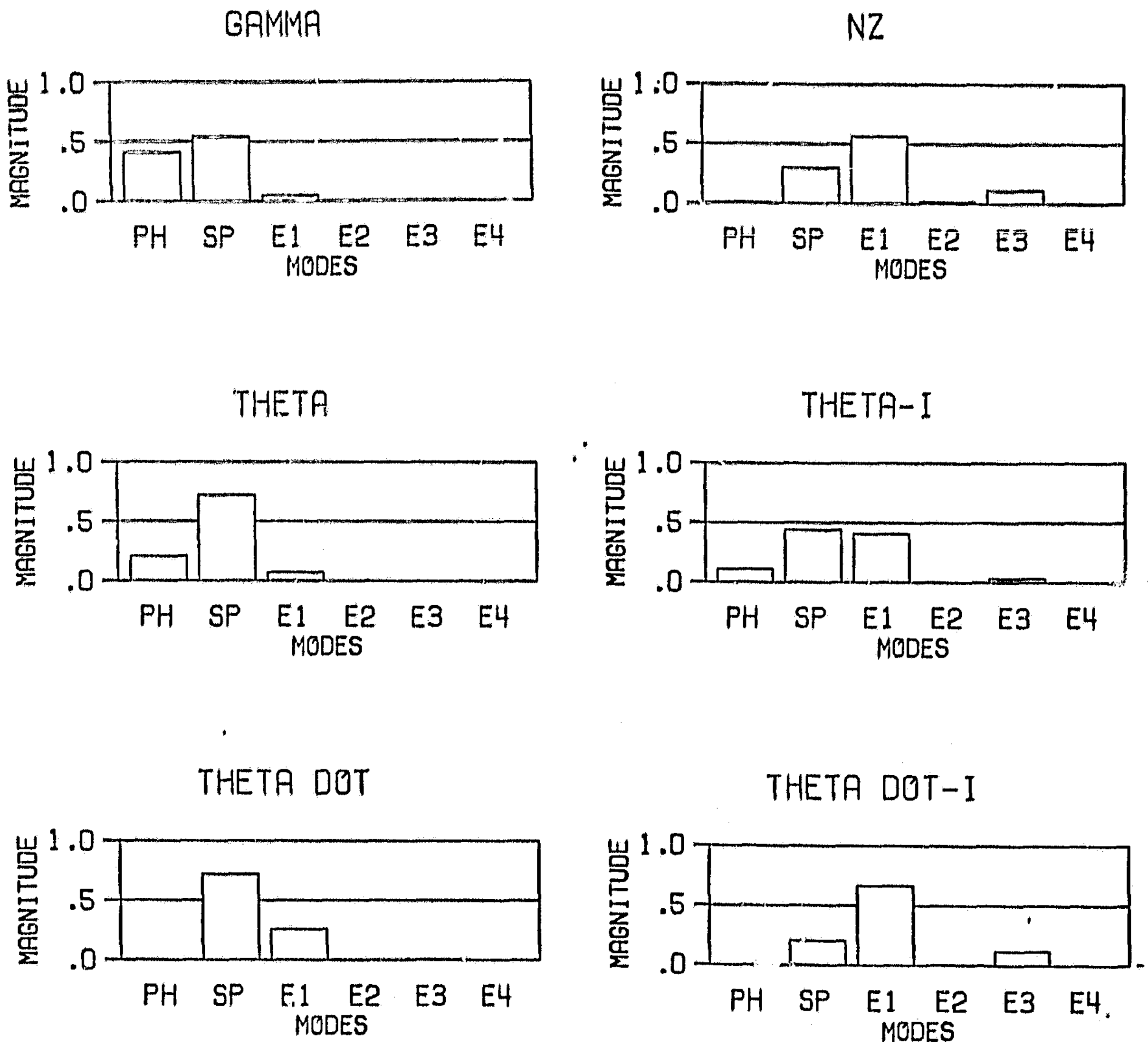
RESIDUES FOR EACH OUTPUT ARE NORMALIZED SO
THAT THEIR SUM IS 1.0

Figure 3

RESIDUE MAGNITUDES

(DUE TO PILOT INPUTS)

CASE 2 -



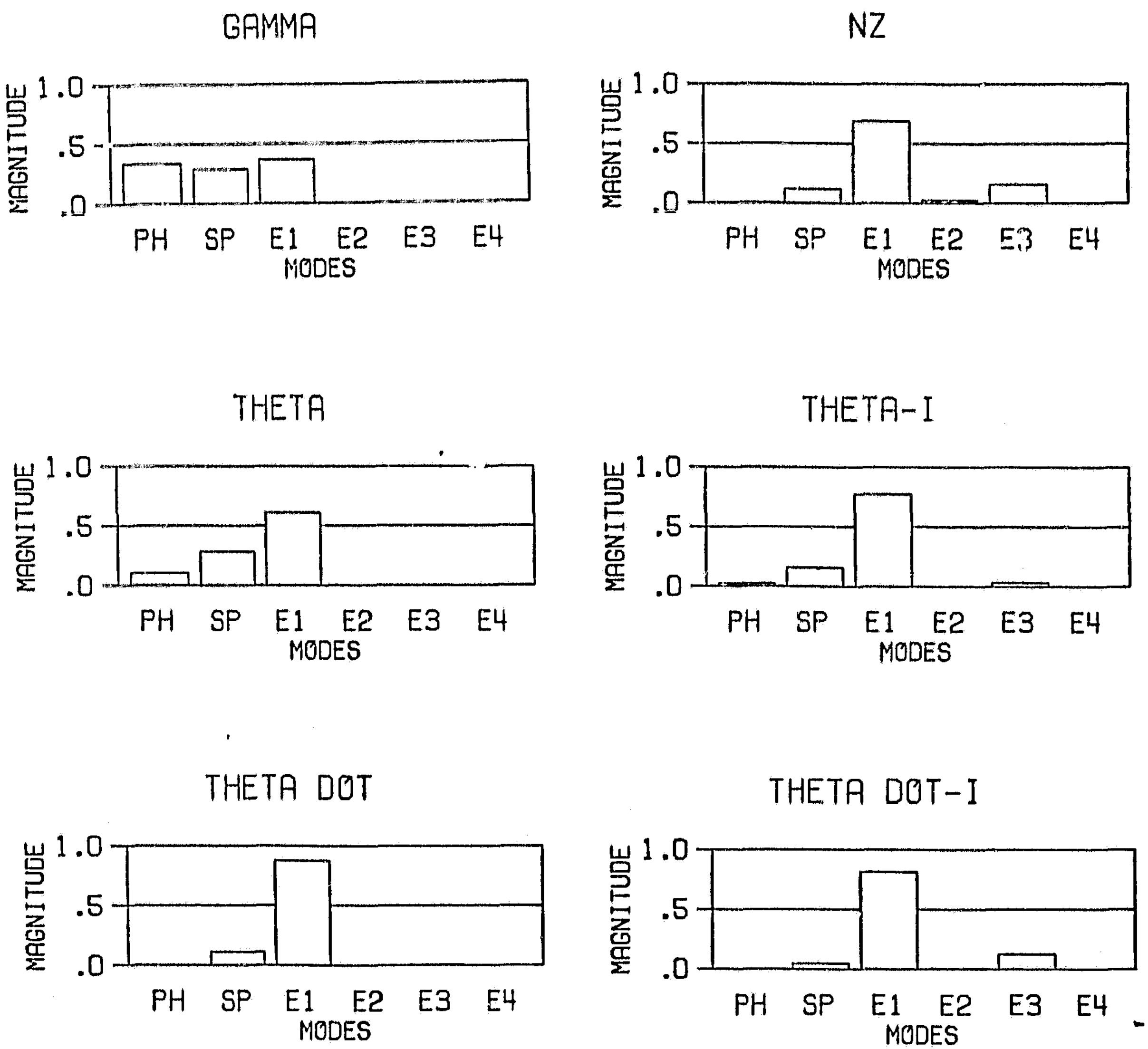
RESIDUES FOR EACH OUTPUT ARE NORMALIZED SO
THAT THEIR SUM IS 1.0

Figure

RESIDUE MAGNITUDES

(DUE TO PILOT INPUTS)

CASE 3 -



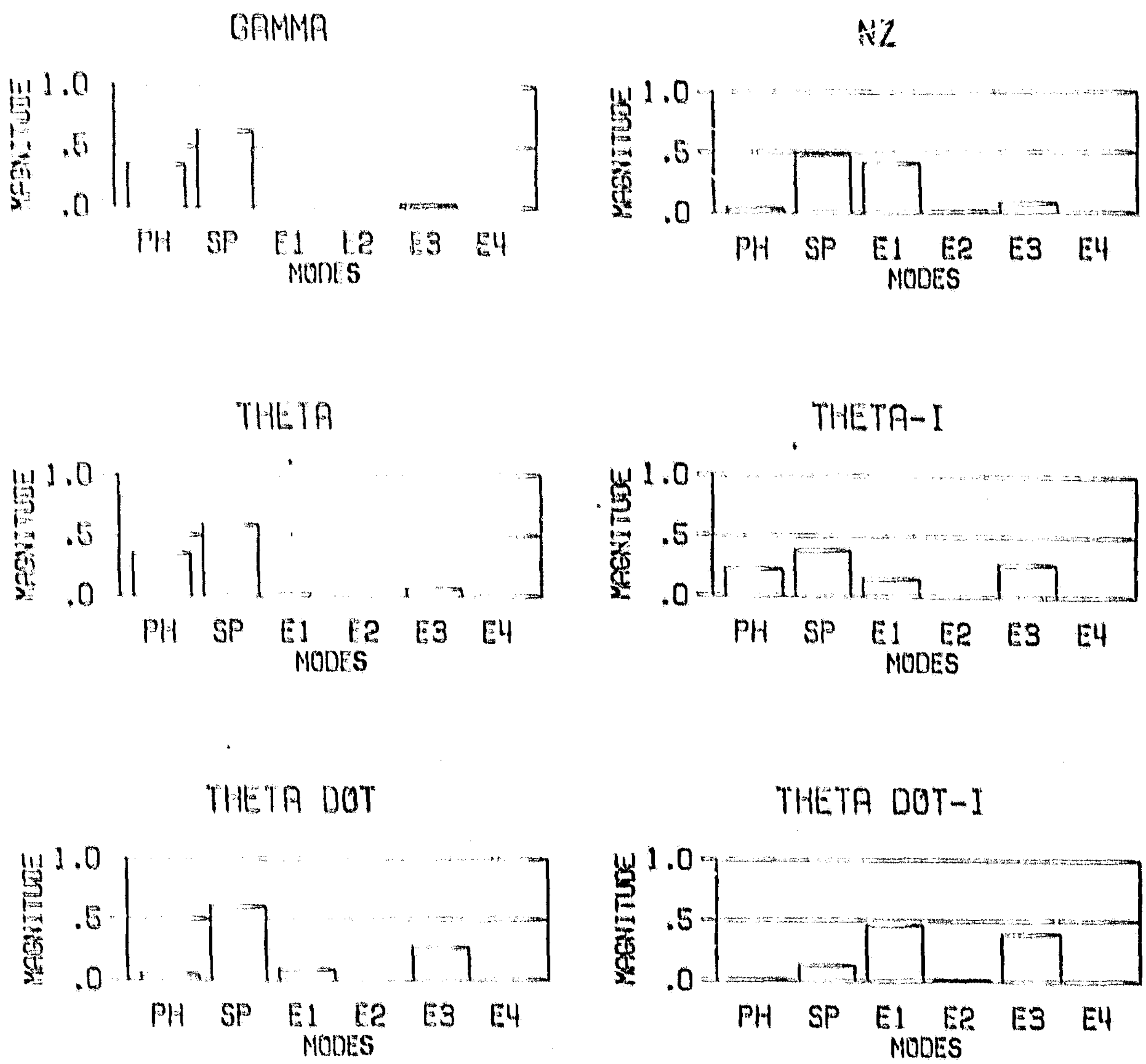
RESIDUES FOR EACH OUTPUT ARE NORMALIZED SO THAT THEIR SUM IS 1.0

Figure 5

RESIDUE MAGNITUDES

(DUE TO PILOT INPUTS)

CASE 4 -



RESIDUES FOR EACH OUTPUT ARE NORMALIZED SO THAT THEIR SUM IS 1.0

Figure 6

Included as Appendix B are the residues for all eight configurations for all six responses to pilot-filtered impulses. Also given in Appendix B are the residues for the same vehicle responses to atmospheric turbulence input. This input consists of an impulse passed through a Dryden gust model, and thereby provides indications of modal contributions to the vehicle's gust response. Upon comparing the "gust response residues" to the "pilot response residues" we note for example that the situation regarding θ residues on configuration 3 and 4 is very different. In the case of "gust response" the attitude θ response is dominated by mode E3 for configuration 4, while that was not the case for the "pilot response" for this configuration. This would indicate that a task involving attitude response in turbulence would be very difficult for configuration 4, while this configuration was much better than configuration 3 in attitude tracking, which doesn't involve vehicle gust response. Finally note the large participation of the elastic modes in the acceleration response (n_z) of all configurations for both types of inputs.

With regard to the lateral-directional dynamics, two pilot inputs, aileron and rudder, are involved of course, and the significant vehicle response variables must be selected before residues may be found. Important aircraft responses in this axis include sideslip (β) and bank (ϕ) angles, roll (p) and yaw (r) rates, and lateral acceleration at the cockpit n_y. In addition to these, experimental evidence points to the fact that the rate of onset of acceleration may be more important than the magnitude of lateral acceleration in ride and flying qualities. Therefore, an additional response is included in the analysis, and it is defined as "lateral jerk," or dn_y/dt. Finally, both the rigid-body portion of these response variables as well as the total, or indicated responses, which include rigid plus elastic contributions are included in the response variables considered. The "total" responses are again referred to as indicated since these variables would be measured by the appropriate sensor, and include total (rigid plus elastic) response. Consequently, the vector of responses considered in the lateral directional analysis is taken as

$$Y^T_{L.D.} = [\phi_{rig}, p_{rig}, r_{rig}, n_{Y_{rig}}, \dot{n}_{Y_{rig}}, \phi_I, p_I, r_I, n_{Y_I}, \dot{n}_{Y_I}]$$

The vector of control inputs is $\bar{u}^T = [\delta_A, \delta_R]$. And finally, pilot lags on rudder and aileron are included in the dynamic model, along with a Dryden gust model appropriate for this axis.

For the baseline B-1 type vehicle, the eigenvalues are listed below, while the residues for pilot and gust responses are shown in Figs. 7-9. Figs. 7 and 8 include the response residues for an impulse input through the pilot (lag) aileron and rudder, respectively. Fig. 9 includes the gust response residues. The significance of the elastic modes in many of these aircraft responses is quite evident, even for the baseline configuration.

Attention is now turning to the parametric variation of in-vacuo frequencies for the lateral-directional model, and thereby generate a family of "more flexible" configurations, as in the longitudinal case. Analysis will proceed then on these configurations, along with planning a fixed-base simulation.

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OF POOR QUALITY

RESIDUE MAGNITUDES
(DUE TO PILOT INPUTS)
CASE 1 - DA

RESIDUES FOR EACH OUTPUT ARE NORMALIZED SO
THAT THEIR SUM IS 1.0

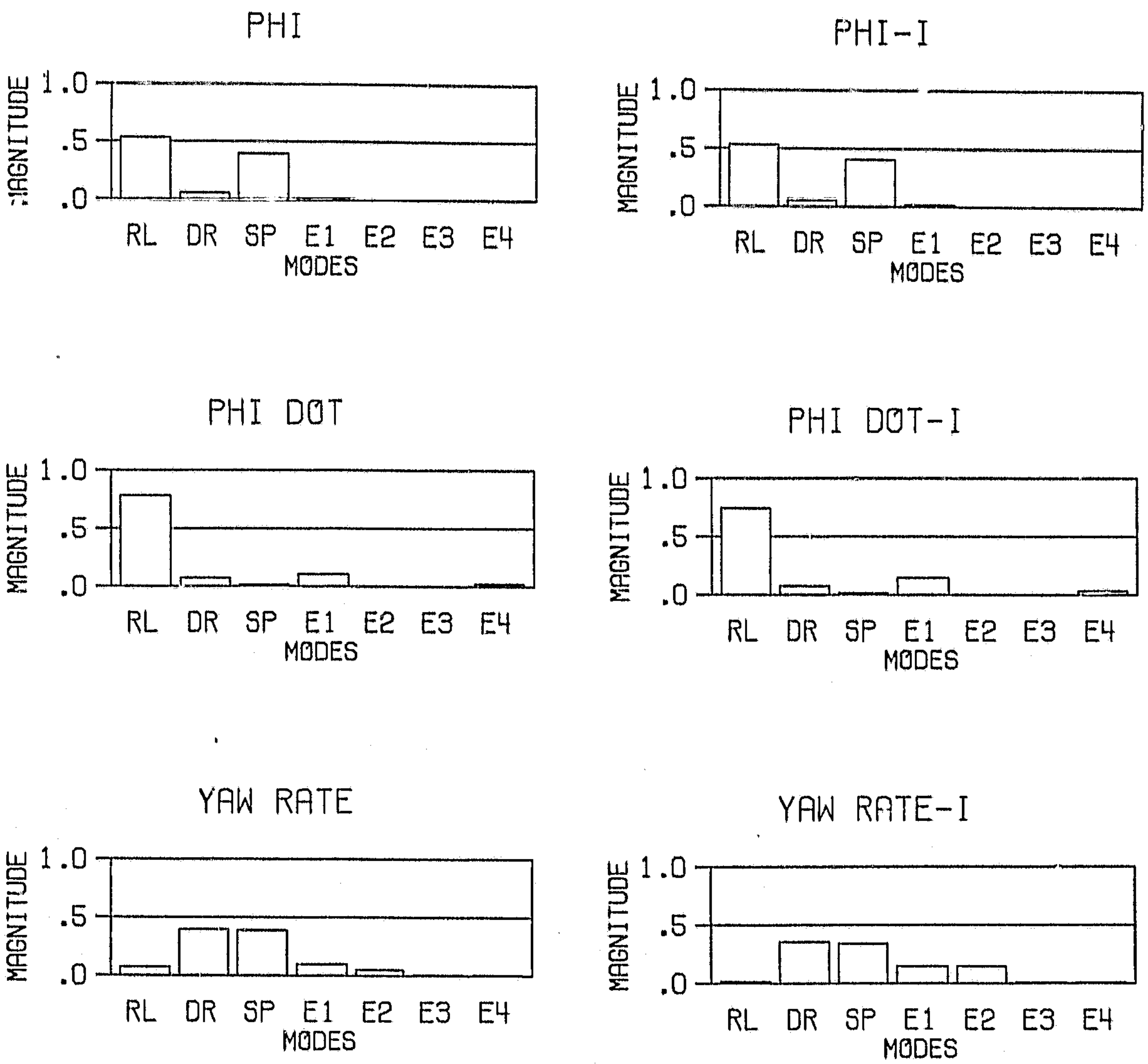

Figure 7

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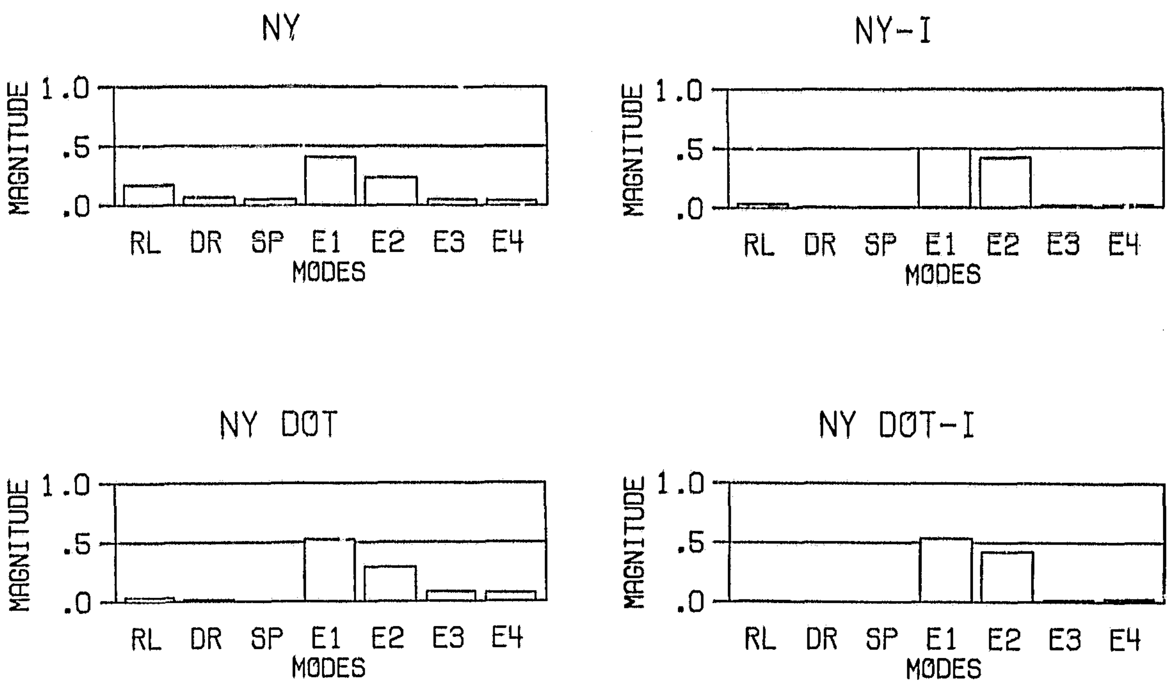

Figure 7, cont'd.

RESIDUE MAGNITUDES
(DUE TO PILOT INPUTS)
CASE 1 - DR

RESIDUES FOR EACH OUTPUT ARE NORMALIZED SO THAT THEIR SUM IS 1.0

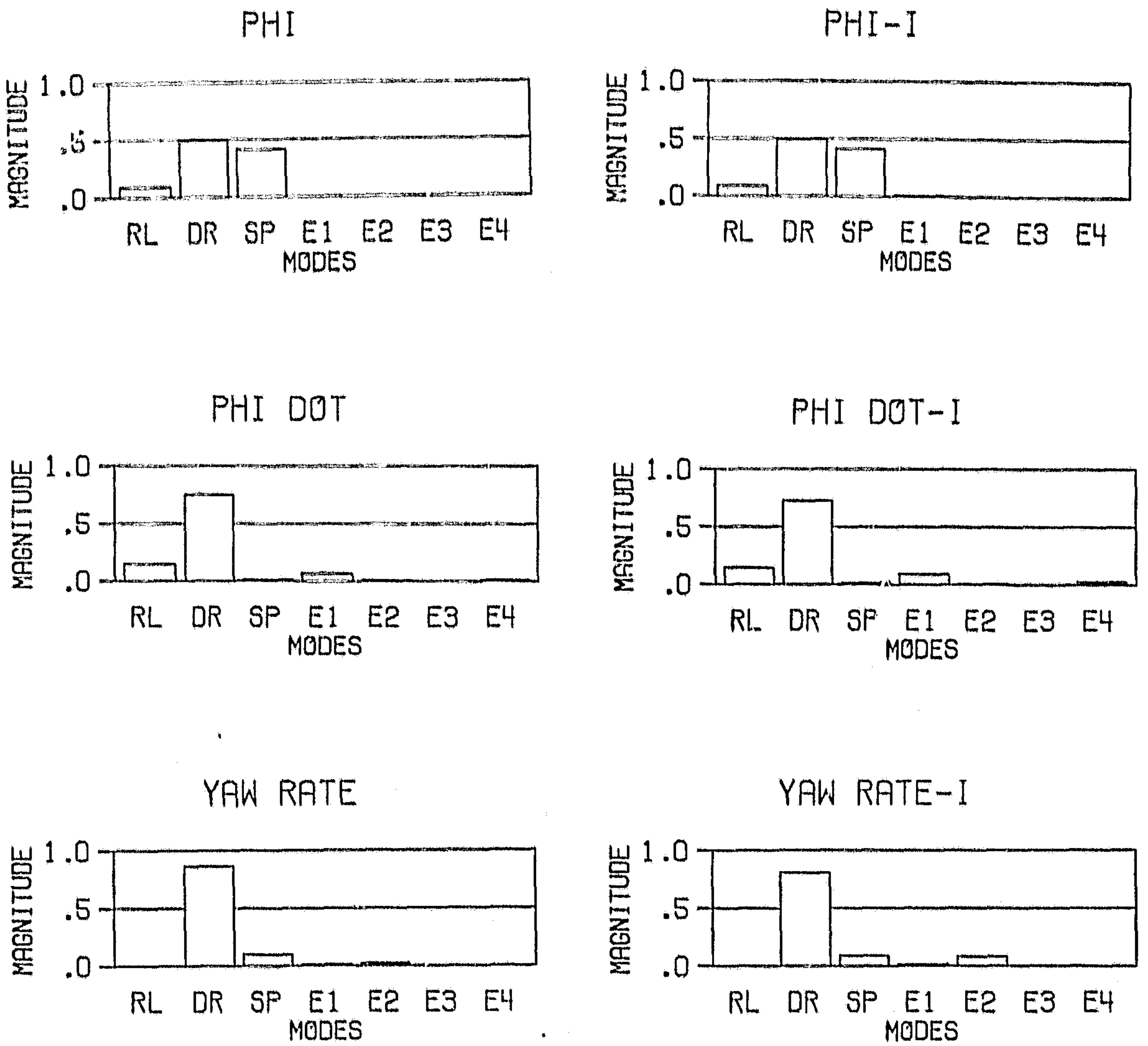

Figure 8

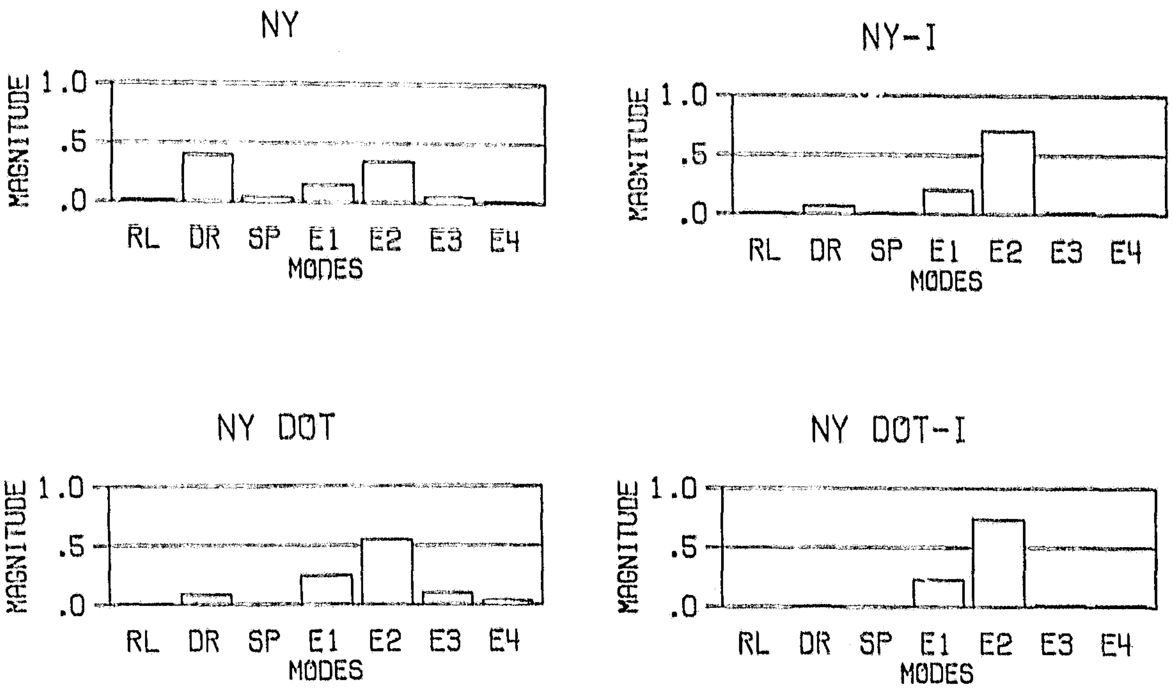

Figure 8, cont'd.

RESIDUE MAGNITUDES
(DUE TO DISTURBANCE)
CASE 1 - GUST

RESIDUES FOR EACH OUTPUT ARE NORMALIZED SO THAT THEIR SUM IS 1.0

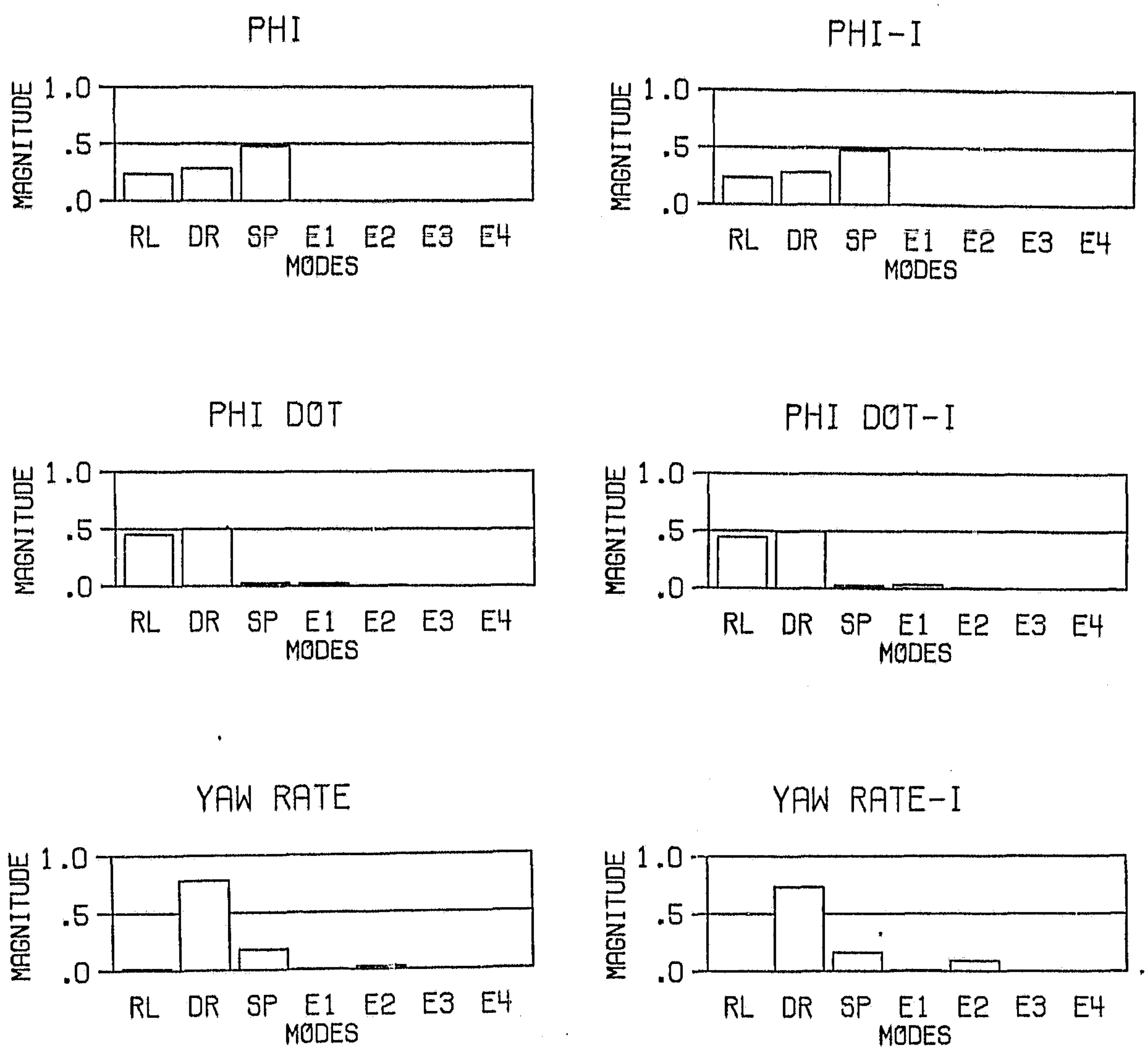

Figure 9

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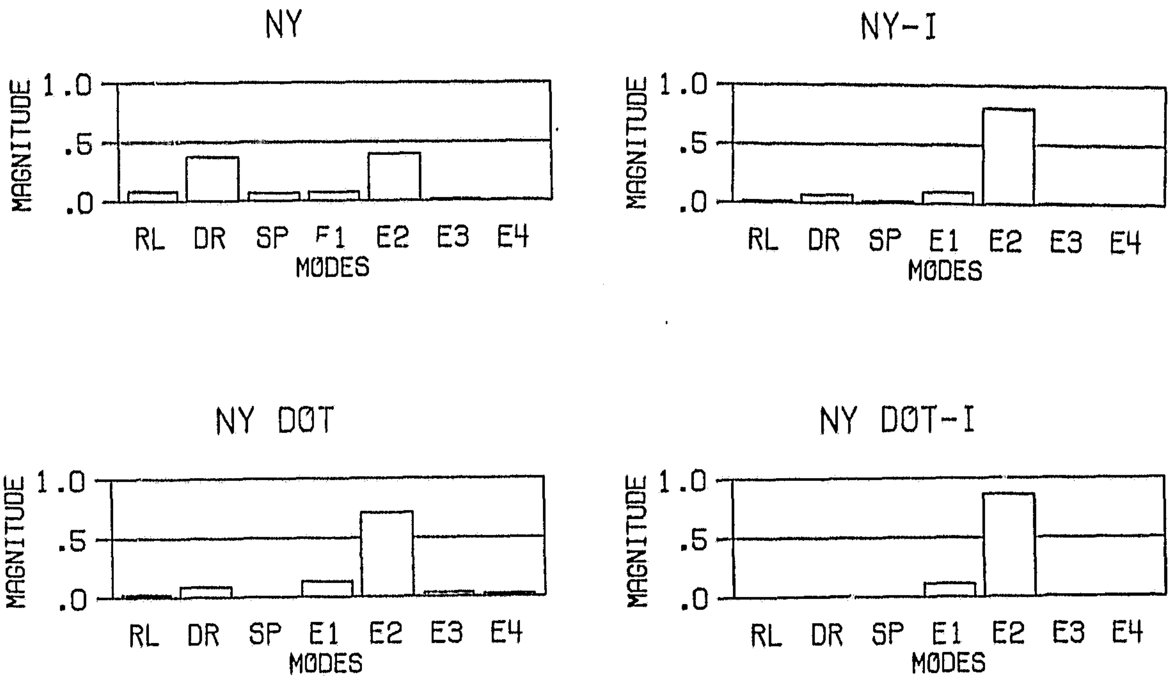

Figure 9, cont'd.

Table III

Lateral-Directional Eigenvalues (Baseline)

Spiral, - .050	Elastic 1, -1.959 ± 16.389j
Roll, -2.191	Elastic 2, -0.570 ± 15.599j
Dutch roll, -0.159±2.002j	Elastic 3, -0.286 ± 20.288j
	Elastic 4, -3.753 ± 23.849j

3. Personnel and Plans

Three graduate students are currently supported on this grant project. They are Mr. Marty Wasak, an M.S. student currently responsible for the dynamic analysis of the longitudinal axis. Mr. Wasak will also perform a Neal-Smith analysis of these configurations via an optimal control pilot model, as developed in research projects here at Purdue. This work will constitute Mr. Wasak's M.S. thesis to be completed in Aug. 1983. He has just indicated that he plans to remain after August to pursue doctoral studies, and will therefore continue to be supported by this grant.

Mr. Frank Leban is the second M.S. student supported by the grant, and is currently completing the computer software for the lateral-directional dynamic analysis. This software should be completed by May, 1983, and results from this analysis will thereby be available for the next status report.

The third student recently starting on the project is Mr. John Davidson, also an M.S. student. Mr. Davidson began investigating a modal control technique considered applicable to structural mode control. This control synthesis method is based on a method described by Moore, and extended by Cunningham of Honneywell. As yet, it had not been applied to flexible vehicle control, but it appears to be a prime candidate for shaping the response of such vehicles. In application of this method, optimum handling characteristics, will be the goal.

A fourth student, Mr. William Smith, will also contribute to this research project, though as an Air Force officer he will not be financially supported by the grant. Mr. Smith's responsibilities are expected to focus on the fixed-base simulation of the lateral-directional dynamics of a family of flexible vehicle configurations analogous to the eight configurations on which the longitudinal analysis has been performed. It is hoped that these simulations will begin this summer. Roll tracking will be the task investigated, and it is to be noted that the Purdue's simulations will all be fixed base.

Appendix A

Extended Abstract

of

A Modal Analysis of Flexible Aircraft Dynamics With Handling Qualities Implications

by

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to be presented at the
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A Modal Analysis of Flexible Aircraft Dynamics with Handling Qualities Implications

by

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Extended Abstract

Introduction

The "flying qualities" or "handling qualities" or a flight vehicle refer to the ease or difficulty with which the pilot is able to control the vehicle in performing the various missions for which the system was designed. In this definition we see three important factors. They include

The manual controllability of a dynamic system

The use of subjective measures ("ease...difficulties")

The multiplicity of missions and tasks.

Inherent in the first two is the all important fact that a human is involved, hence psychological and physiological factors strongly come into play. Through extensive training of pilots, the attempt is made to minimize the deviation among these variables and to minimize inherent human limitations. However, biases of opinion, likes, dislikes, and so forth, are still present. This fact, coupled with the use of subjective measures make the evaluation of flying qualities, not to mention the design problem, quite challenging. However, with a careful and systematic approach, both can be accomplished.

Historically, the subjective opinions from highly trained test pilots on the performance and workload involved in performing the missions were correlated with the damping and frequency of the vehicle's rigid body modes. These parameters (damping and frequency) were then used to infer the handling qualities of future vehicles, or for handling qualities specifications.[1]

This approach was very successful over the years, but in a very real way caused many of the problems (primarily conceptual) experienced recently with modern, more complex vehicles. This damping and frequency became synonymous with the vehicle's handling qualities. That is, they became thought of as the handling qualities rather than two parameters that affect the handling qualities!

Here is the key point - the handling qualities of a vehicle are not determined by the damping and frequency in a transfer function, but by the dynamic behavior, the time response, of the vehicle system. This simple fact explains why the classic approach was successful for rigid, unaugmented aircraft, while high-order dynamic effects (aeroelastic or control system dynamics) have been found to so significantly affect the pilot opinion as to render the approach useless.

The purpose of the work to be presented in this paper is to address the question, "Can dynamic aeroelastic effects significantly affect aircraft handling qualities, and if so, how?" By answering this question we are then able to consider the enhancement of the handling qualities, if required, through active control or structural design.

To move toward answering this question, we will discuss an analytical approach for evaluating a subject vehicle's dynamics, and apply the method to a spectrum of generic vehicles for which experimental data has been obtained. The complete methodology includes not only open loop analyses of the aircraft dynamics, but also an analytical pilot/vehicle analysis to be presented.

Modal Analysis Concepts

In the transfer function for the short-period approximation for longitudinal aircraft dynamics, the numerator depends on the same dimensional stability derivatives as those in the characteristic equation.[2] So the transfer function does not involve four independent parameters (two poles, a zero, and gain) but rather more like three. Therefore, once the stick gain is fixed, and short period damping and frequency are specified, very little variability remains. As a result, note that the short period time response is essentially specified as well (assuming the linear model to be valid). The point is that in this case, specifying the rigid body _modal_ characteristics nearly completely _specifies the time response_ as well.

Clearly, for systems for which a simple, rigid-body transfer function is a poor model, the parameters in that transfer function cannot guarantee good or bad handling qualities, for they cannot alone determine the system's time response. This is our first important point - _parameters in a vehicle dynamic model are important if they significantly affect the system's time response._ Note that this is an operational definition for a systems dominant modes, but is more general.

Let us now turn to how parameters in a dynamical model affect time response. Again, assuming a linear model to be valid (or valid enough to gain insight), the impulse response may be found from the partial fraction expansion of the transfer function. Taking the important pitch attitude response, for example,

$$\dot{\theta}(s) = \sum_{i=1}^{n} \frac{R_i}{s+\lambda_i}$$

where

n = order of the system

R_i = ith residue λ_i

λ_i = ith system pole or eigenvalue

Then the time response is

$$\dot{\theta}(t) = \sum_{i=1}^{n} R_i e^{-\lambda_i t}$$

Now for a classical rigid vehicle a good model was obtained with n simply equal to two (2), or a short period approximation. Now n may have to be larger than two for a good model.

Here rather than the time response expressed in terms of eigenvalues, it will convenient to the use the system's modes. So we may express the same time response as

$$\dot{\theta}(t) = \sum_{i=1}^{N} |R_i| e^{-\sigma_i t} \cos(\omega_i t + \psi_i)$$

where N = number of system <u>modes</u>

$-\sigma_i \pm j\omega_i = \lambda_i, \lambda_i^*$ (conjugate pair of eigenvalues)

$R_i = |R_i|\, e^{\psi_i j}$ (residue associated with root λ_i)

So the time response is completely determined by the real and imaginary parts of the eigenvalues (σ_i and ω_i, or by ζ_i and ω_{n_i}). And the magnitude and phase of the residues ($|R_i|$ and ψ_i) associated with each mode. Or not only the modal damping and frequencies are involved in defining the time response, but the residues as well. Furthermore, in general these are the minimum number of variables required - 4N.

Returning for a moment to the classic, rigid body case, the use of only the short period damping and frequency in specifying the handling qualities is justifiable since the residues depend on the interaction between the poles and zeros of the transfer function, and as cited previously, the numerator characteristics are not independent of the denominator in a rigid aircraft transfer function. Therefore, the variation in residues was limited. Note also, that the rigid-body assumption implies the residues for the infinity of elastic modes actually present in all vehicles are small compared to those of the rigid-body modes.

This is certainly not so with the introduction of significant aeroelastic effects and control system dynamics, for example. The modal characteristics are still fundamental in their affect on time response, but the "numerator dynamics" or the interaction between the poles and zeros are significant in terms of <u>their effect on the residues</u>. The zero's locations in themselves are not important, but rather how the position affects the residues.

Let's consider a simple example involving the short-period plus a single elastic mode, for which the transfer function might be

$$\frac{\dot{\theta}(s)}{\delta(s)} = \frac{K(s+z)\ (s^2+as+b)}{(s^2 + 2\zeta_{sp}\ \omega_{sp}\ s + \omega_{sp}^2)(s^2 + 2\zeta_1\ \omega_1\ s + \omega_1^2)}$$

The impulse (time) response of this vehicle may be written as

$$\dot{\theta}(t) = |R_{sp}|e^{-\sigma_{sp}t}\cos(\bar{\omega}_{sp}\ t + \Psi_{sp}) + |R_1|e^{-\sigma_1 t}\ \cos(\bar{\omega}_1 t + \Psi_1\ )$$

or

$$\dot{\theta}(t) = \dot{\theta}_{Rigid} + \dot{\theta}_{Flex}$$

with

$$\sigma_{sp} = \zeta\omega_{sp};\quad \bar{\omega}_{sp} = \omega_{sp}\sqrt{1 - \zeta_{sp}^2}$$

$$\sigma_1 = \zeta_1\omega_1;\quad \bar{\omega}_1 = \omega_1\sqrt{1 - \zeta_1^2}$$

Now, the affect of the flexible mode on the time response consists of three components.

The coupling affect on the short period modal characteristics (changing σ_{sp} and ω_{sp})

The coupling affect on the short period residues (changing $|R_{sp}|$ and ψ_{sp})

The time response of the mode itself (θ_{Flex})

This leads to a second important point. The effects of higher-order modes may be quantified in terms of these effects on the dominant (usually rigid body) modal characteristics and residues, as well as the time response of the higher modes themselves.

The above presentation was a review of known facts, but demonstrates concepts upon which we have developed our analytical approach. For more realistic dynamics (i.e. higher-order), numerical analysis using multi-variable systems techniques is appropriate. We assume a linear state-variable representation for the vehicle is available, and although the multi-input, multi-output case (e.g. θ and n_z) will be treated, consider for now the case above, or θ/δ_E. We have

$$\dot{\bar{x}} = A\bar{x} + B\delta_E$$

$$\dot{\theta}(t) = C\bar{x}$$

Now diagonalizing the system via the model matrix T consisting of the system eigenvectors $\bar{\nu}_i$, or

$$T = [\bar{\nu}_1 \bar{\nu}_2 \quad \ldots \quad \bar{\nu}_n]$$

one obtains

$$\dot{\bar{q}} = \Lambda\bar{q} + T^{-1} B\delta_E$$

the vehicle dynamics expressed in modal coordinates [3]. With the pitch rate expressed in terms of the <u>system modes</u> as

$$\dot{\theta}(t) = CT\bar{q}$$

where Λ = diagonal matrix of eigenvalues, $\lambda_i = -\sigma_i \pm j\bar{\omega}$;

T^{-1} B = Modal controllability matrix, $\begin{vmatrix} b_1 \\ \vdots \\ b_n \end{vmatrix}$

CT = Modal observability matrix, $[c_1 \ldots c_n]$

And the partial fraction expansion of the system's <u>impulse</u> response is simply

$$\dot{\theta}(s) = \sum_{i=1}^{n} \frac{c_i\, b_i}{s-\lambda_i}$$

obtained directly from Laplace transforming the above equations. So we have the residues R_i simply as the product $\hat{c}_i \hat{b}_i$, available by <u>inspection from the system's modal representation</u>. The three effects of a higher-order mode discussed previously may therefore be found numerically.

<u>The Analysis Approach - Overview</u>

Although all the above ideas are available from linear systems theory, blind use of these ideas will lead to incorrect answers to our fundamental questions cited at the outset. The straightforward modal analysis of this linearized math model of the aircraft dynamics alone ignores the salient characteristics of the <u>actual</u> disturbances exciting these dynamics in <u>actual</u> flight conditions. Two such disturbances of interest here are the pilot's control input and atmosphere turbulence. Neither of these may be treated as "white" random processes, for example. The significant characteristics of these "exciting subsystems" must be taken into account. How this is accomplished will thoroughly explained in the complete paper.

The counterpart to this "open-loop" analysis is the pilot/vehicle analysis, performed using an optimal-control-theoretic model of the pilot's control loop closures. We will show, unlike the reduced-order pilot model of Swaim [4], how our full-order pilot model, consistent with that of Ref. [5], may be used to obtain good correlation between tracking error statistics obtained analytically and those obtained from fixed base simulation. Through this pilot modeling approach we are able to infer a different piloting strategy rather than a different pilot "internal model" [4] in the control of flexible aircraft dynamics, and this supported by both simulation results and the open-loop aircraft analysis presented previously.

Sample Results

Shown in Figure I are the rms attitude tracking errors obtained from previous fixed base simulation of several generic flexible vehicles [6]. The only difference between these vehicles is their structural stiffness, quantified in terms of the in-vacuo vibration modal frequencies. The effects of flexibility in the vehicle dynamics are clearly evident in the increased tracking error, particularly in configuration C_3. What is not clear is why configuration C_4, an equally flexible configuration to C_3 in terms of elastic modal frequencies, resulted in significantly better tracking. The modal analysis technique outlined above and to be described completely in the paper will reveal the answer to this paradox. In addition, how the full-order pilot model is applied to achieve the correlation between analytical and experimental results shown in the figure will also be discussed. These are just four of eight vehicle configurations that will be presented, with analytical results compared to experimental. In all cases, excellent correlation is obtained. Finally, the modal analysis method is used to demonstrate the affects of flexibility on a variety of vehicle responses, in addition to pitch attitude, and these results will be presented as well.

References

1. Military Specification - Flying Qualities of Piloted Airplanes, MIL-F-8785B (ASG), 1969.

2. Etkin, B., Dynamics of Flight, Stability and Control, Wiley, New York, 1982.

3. DeRusso, P., Roy, R., and Close, C., State Variables for Engineers, Wiley, New York, 1965.

4 Swaim, R.L. and Poopaka, S., "An Analytical Pilot Rating Method for Highly Elastic Aircraft", AIAA Journ. of Guid. Cont., and Dyn., Vol. 5, No. 6, Nov.-Dec., 1982, pp. 578-582.

5. Kleinman, D., Baron, S., and Levison, W., "An Optimal Control Model of Human Response", Parts I and II, *Automatica*, Vol. 6, 1970, pp. 357-383.

6. Swaim, R.L., and Yen, W.Y., "Effects of Dynamic Aeroelasticity on Handling Qualities and Pilot Rating", AIAA Paper 78-1365, Atm. Flight Mech. Conf., Palo Alto, CA, Aug. 1978.

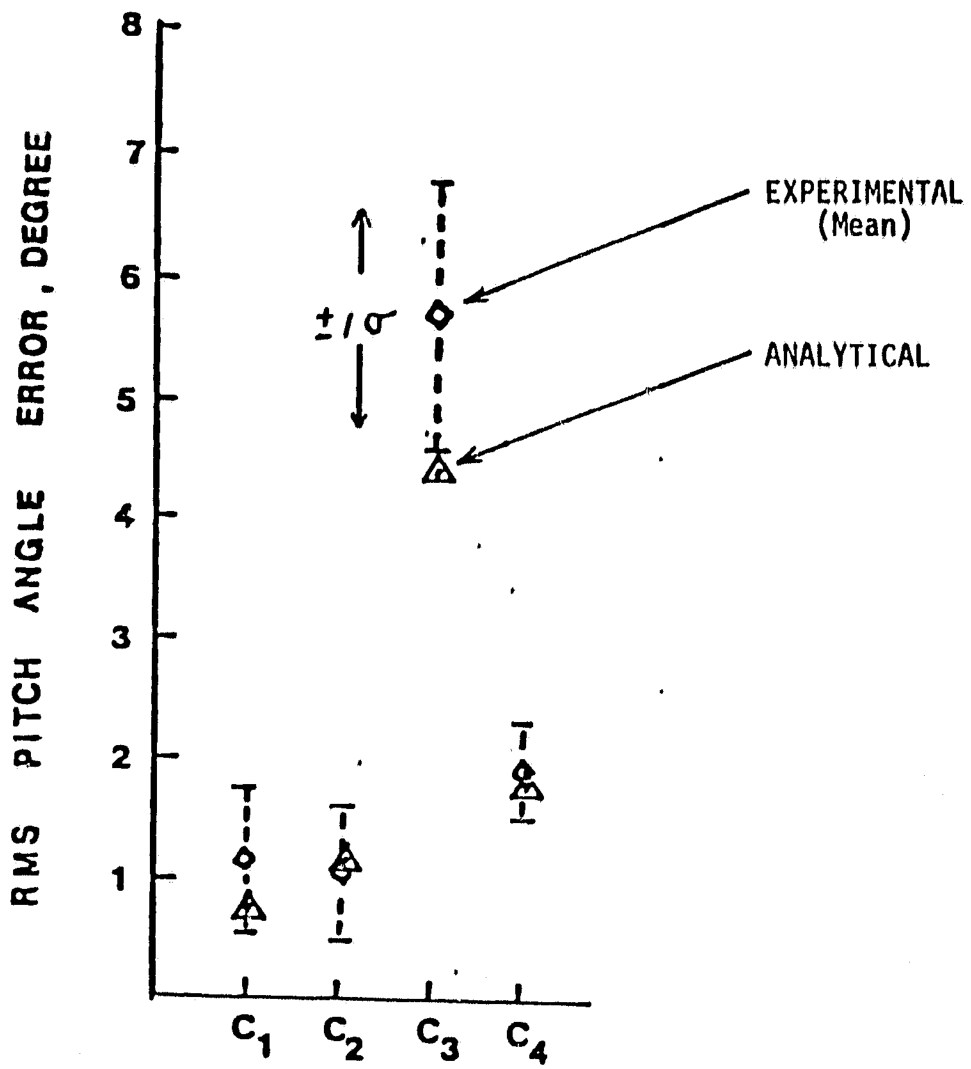


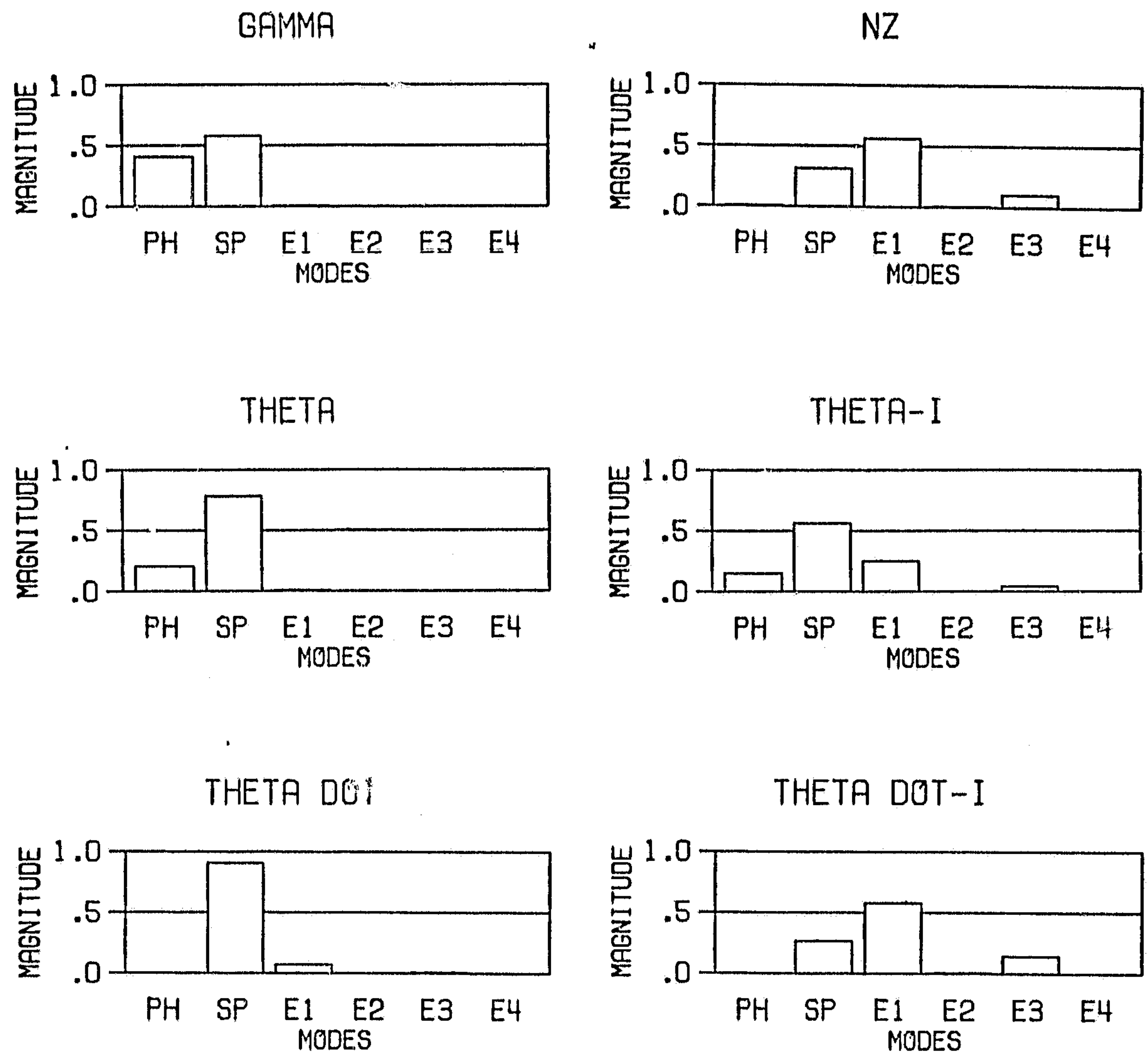
Figure 1 Tracking Performance

Appendix B

RESIDUE MAGNITUDES

(DUE TO PILOT INPUTS)

CASE 1 -

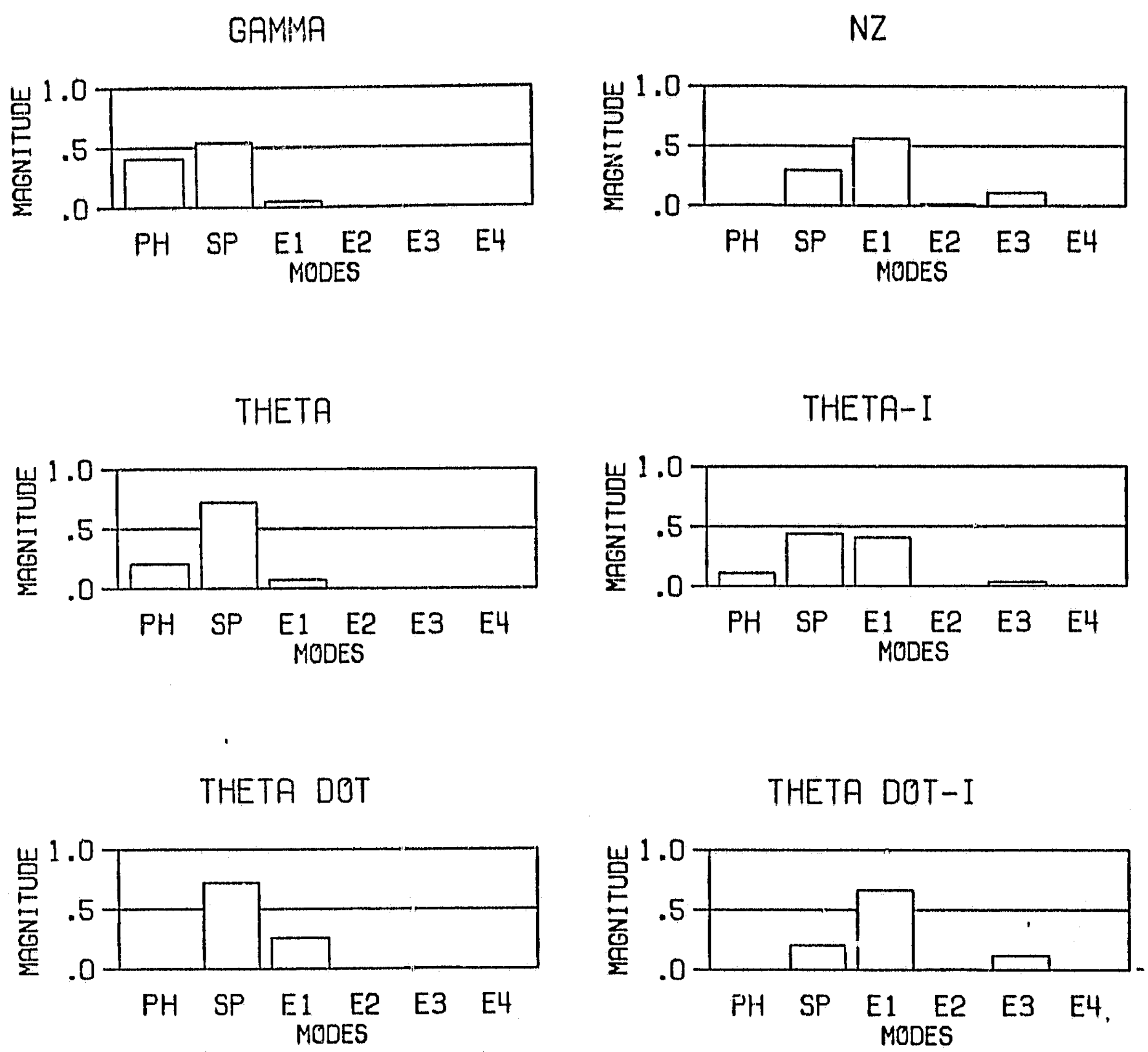


RESIDUES FOR EACH OUTPUT ARE NORMALIZED SO
THAT THEIR SUM IS 1.0

RESIDUE MAGNITUDES

(DUE TO PILOT INPUTS)

CASE 2 -



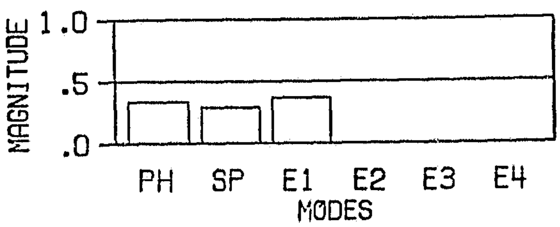
RESIDUES FOR EACH OUTPUT ARE NORMALIZED SO THAT THEIR SUM IS 1.0

RESIDUE MAGNITUDES

(DUE TO PILOT INPUTS)

CASE 3 -

GAMMA



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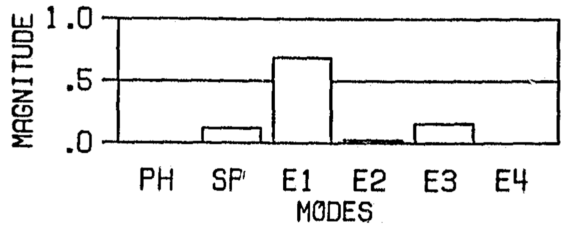

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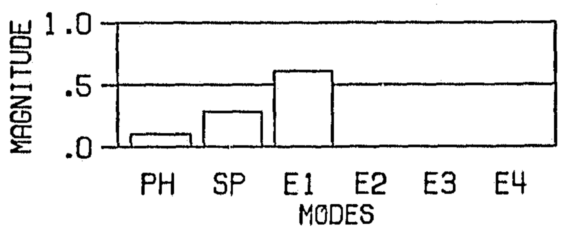

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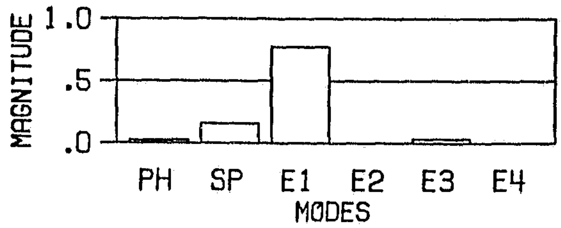

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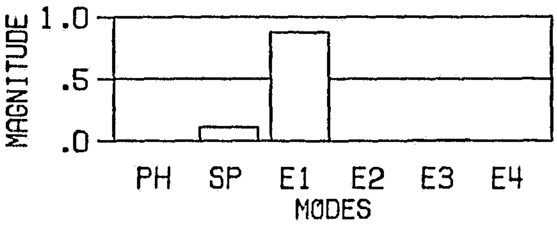

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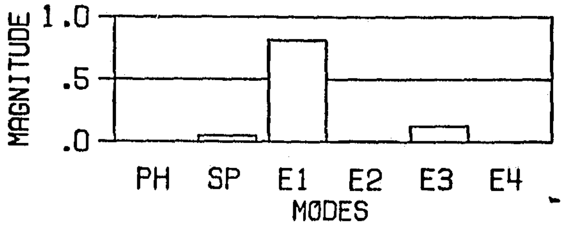
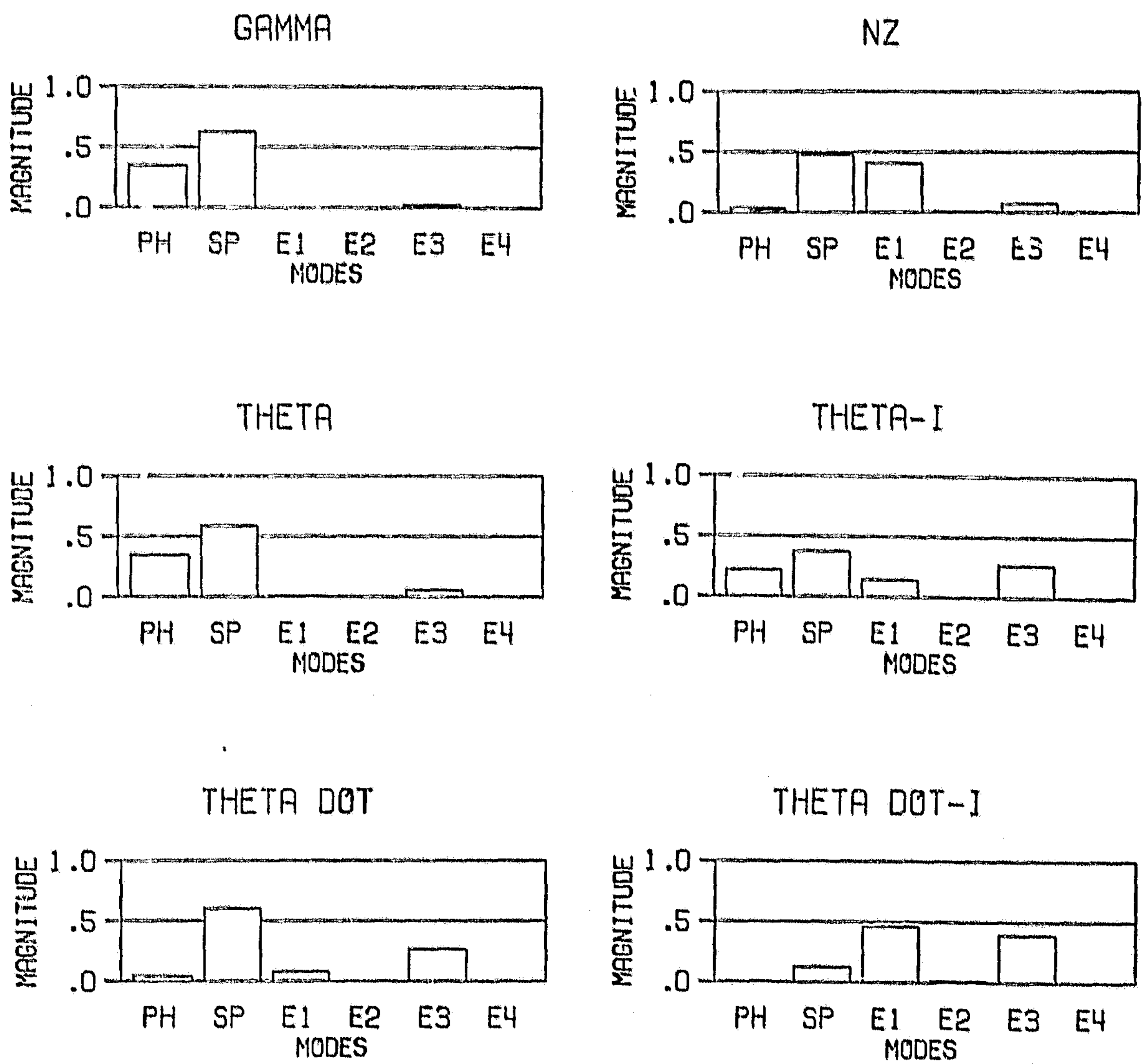


RESIDUES FOR EACH OUTPUT ARE NORMALIZED SO THAT THEIR SUM IS 1.0

RESIDUE MAGNITUDES

(DUE TO PILOT INPUTS)

CASE 4 -



RESIDUES FOR EACH OUTPUT ARE NORMALIZED SO
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RESIDUE MAGNITUDES

(DUE TO PILOT INPUTS)

CASE 5 -

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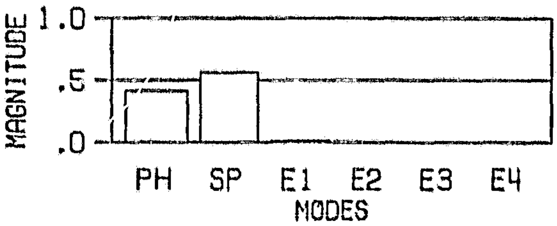

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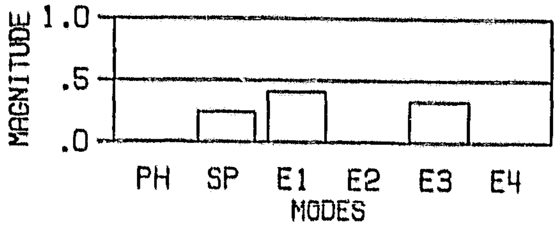

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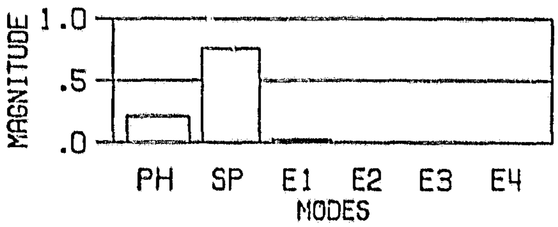

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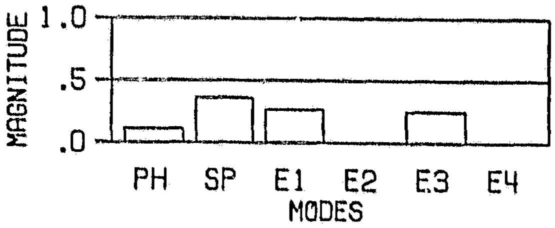

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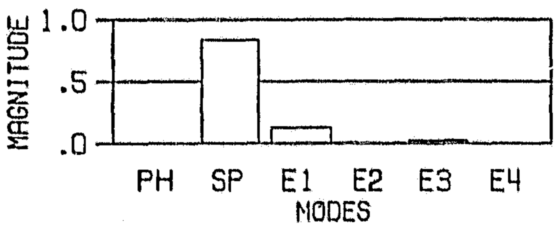

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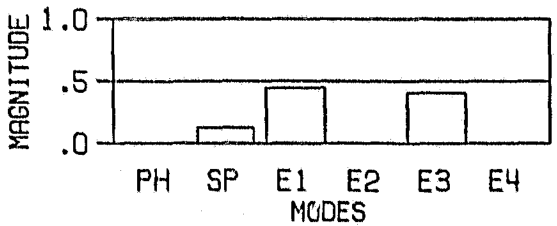

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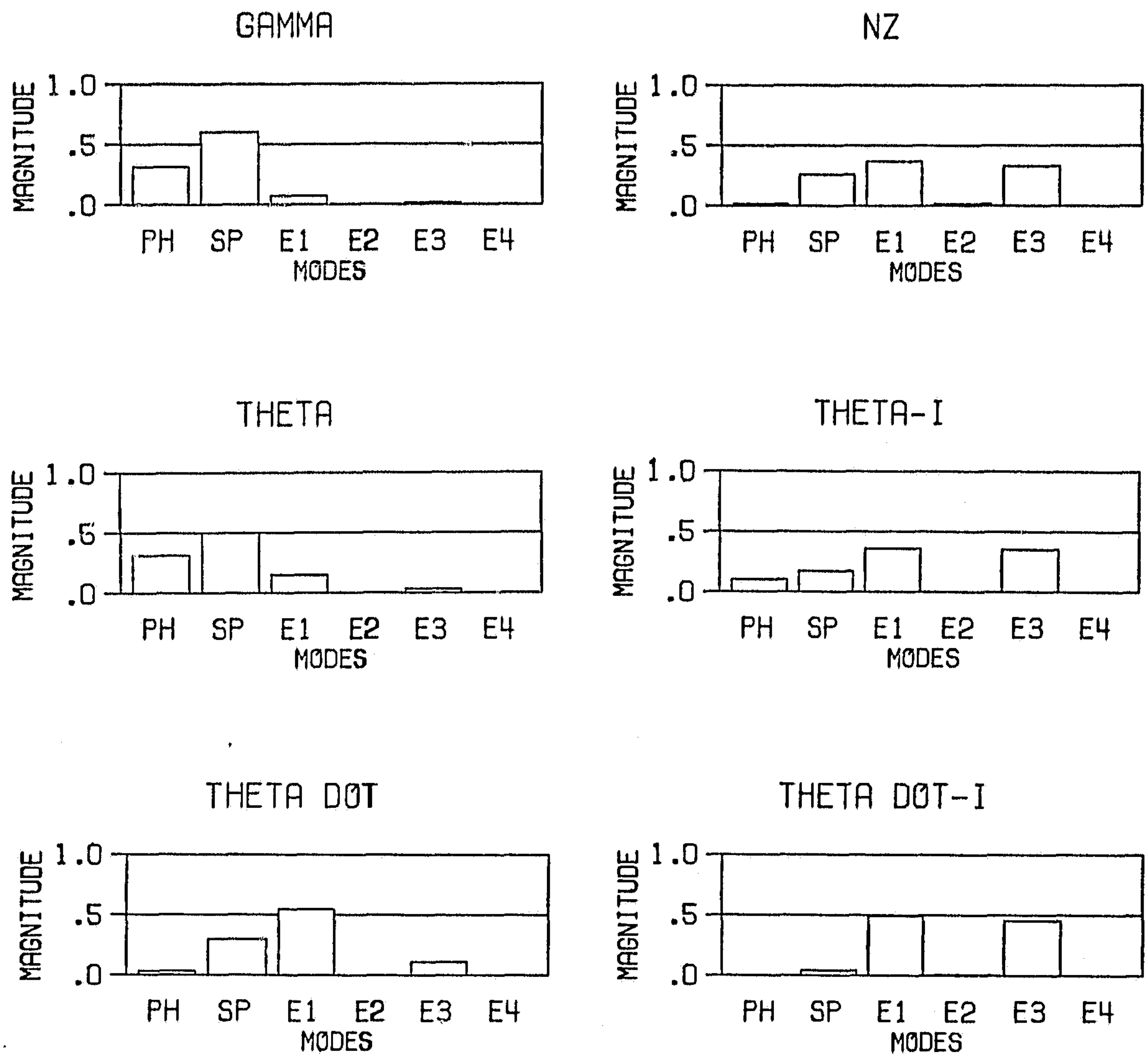
RESIDUE MAGNITUDES

(DUE TO PILOT INPUTS)

CASE 6 -



RESIDUES FOR EACH OUTPUT ARE NORMALIZED SO
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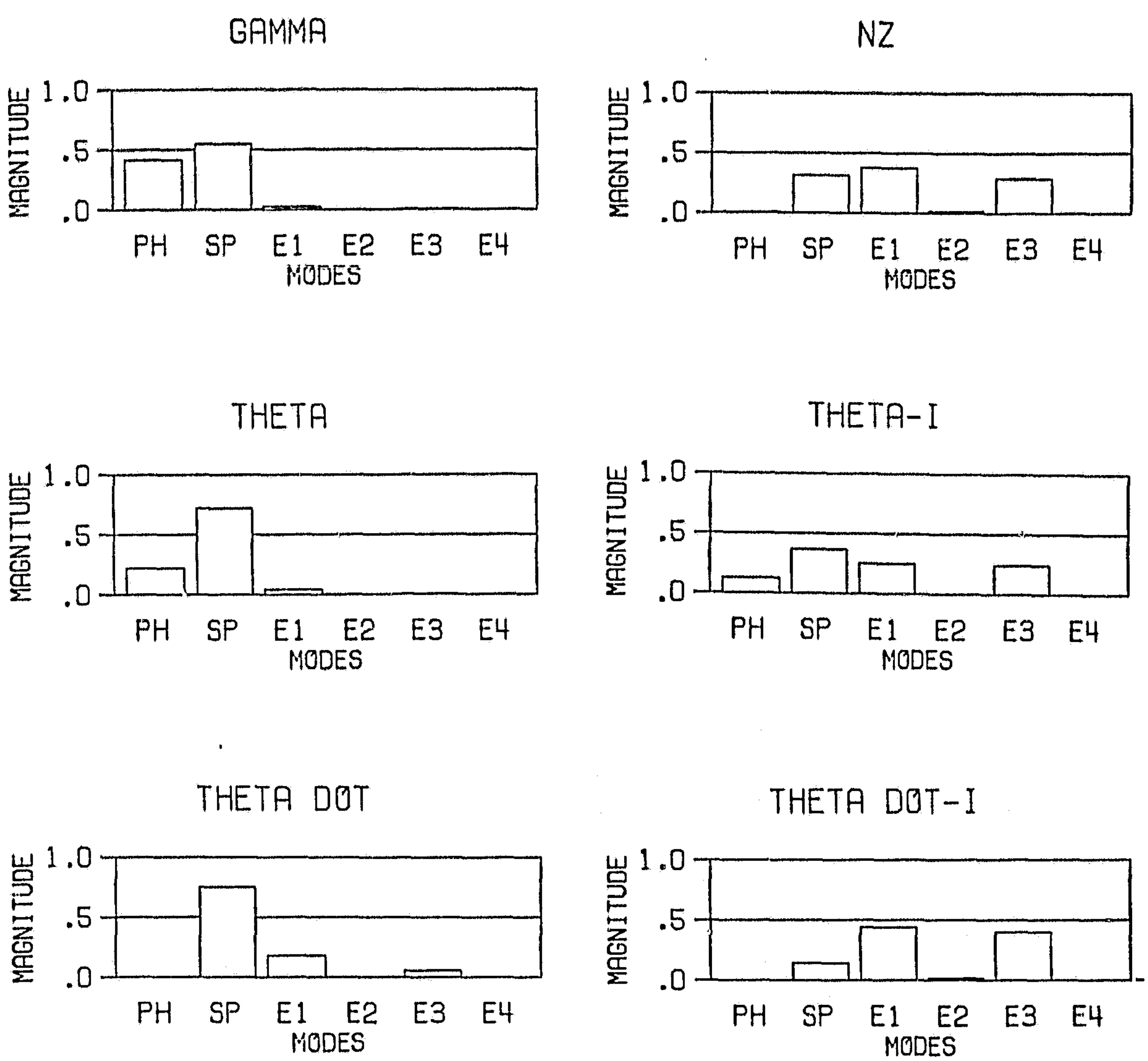
RESIDUE MAGNITUDES

(DUE TO PILOT INPUTS)

CASE 7 -

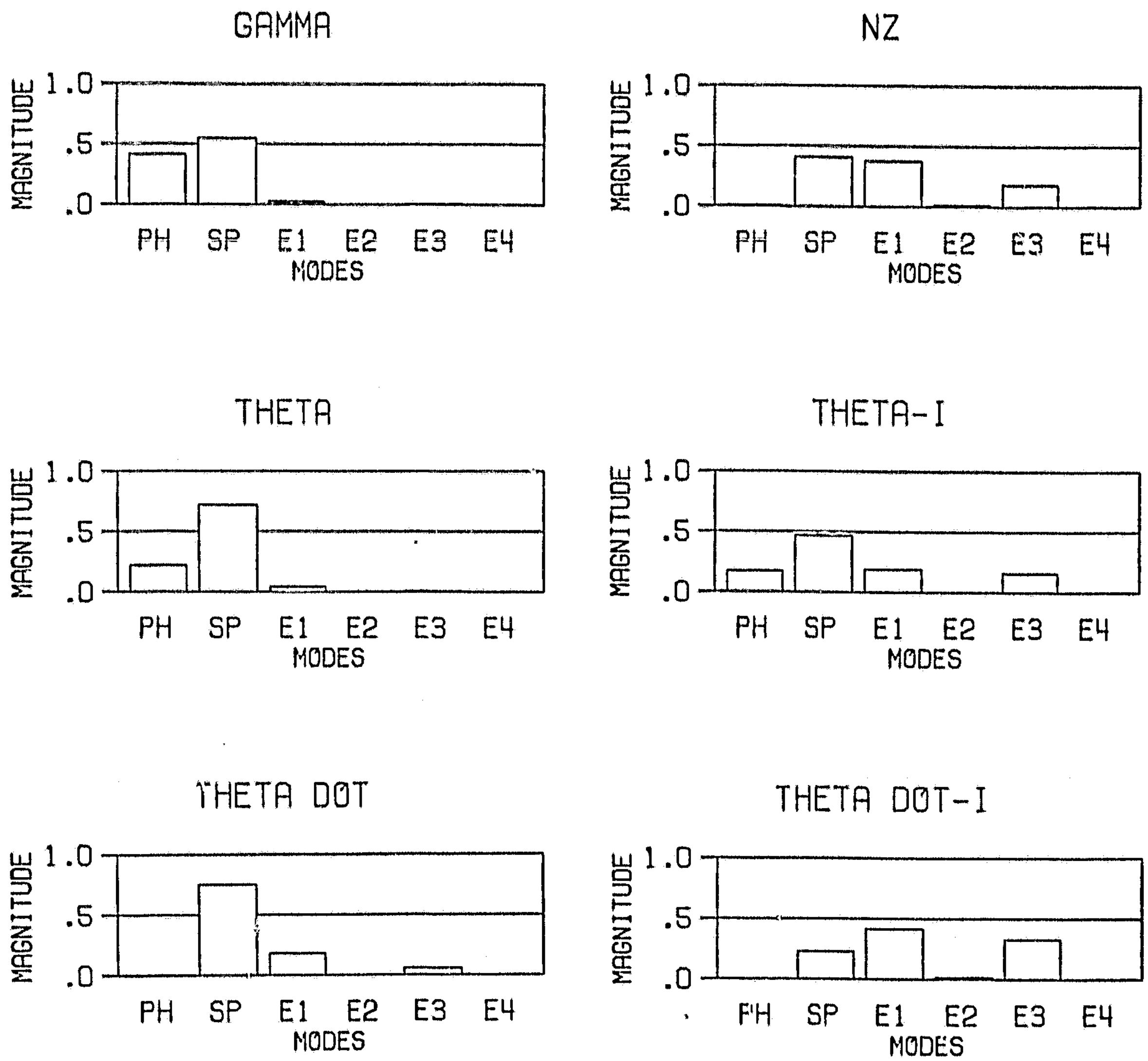


RESIDUES FOR EACH OUTPUT ARE NORMALIZED SO THAT THEIR SUM IS 1.0

RESIDUE MAGNITUDES

(DUE TO PILOT INPUTS)

CASE 8 -



RESIDUES FOR EACH OUTPUT ARE NORMALIZED SO
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RESIDUE MAGNITUDES

(DUE TO DISTURBANCE)

CASE 1 -

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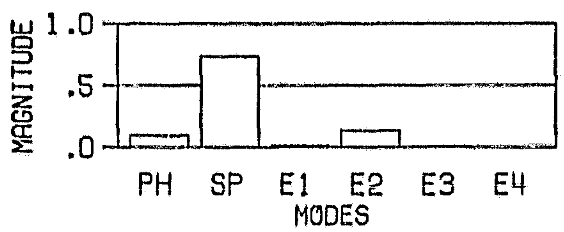

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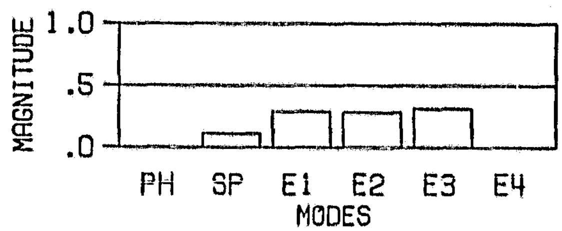

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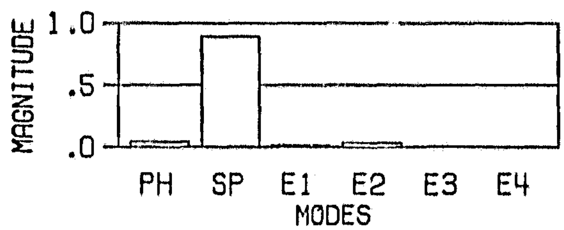

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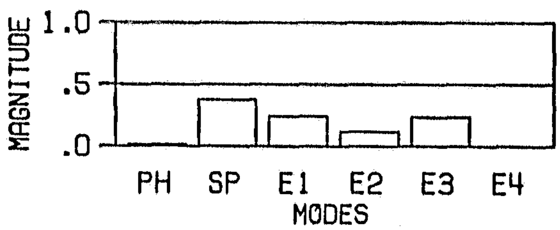

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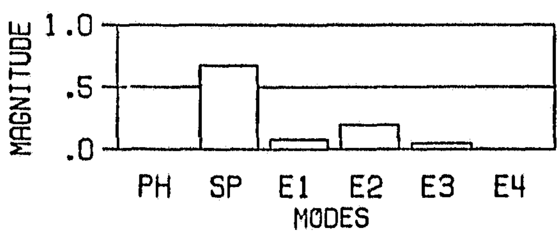

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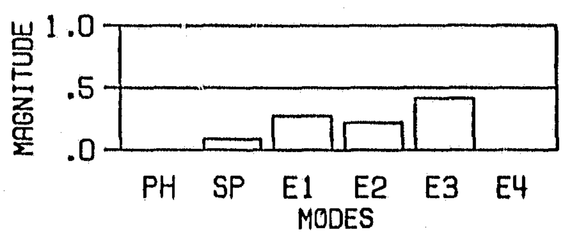

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RESIDUE MAGNITUDES

(DUE TO DISTURBANCE)

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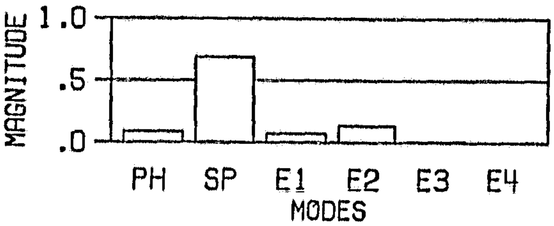

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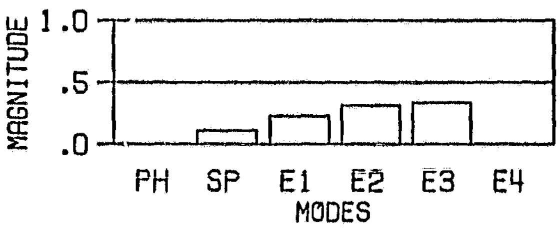

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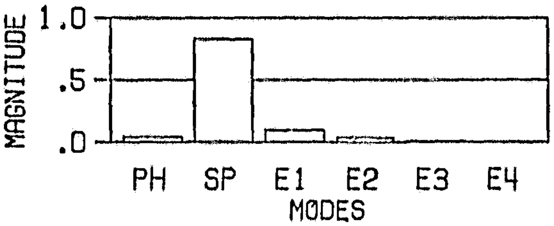

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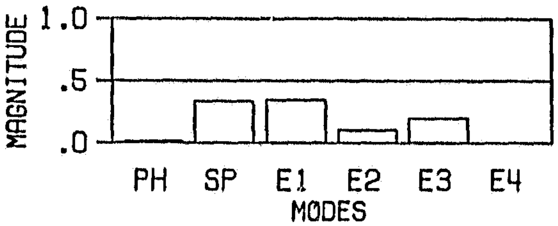

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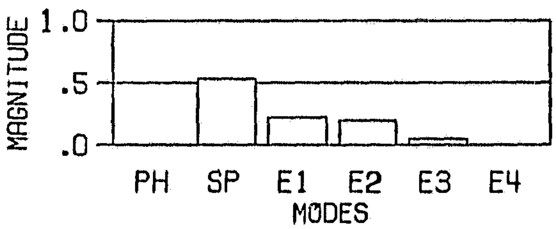

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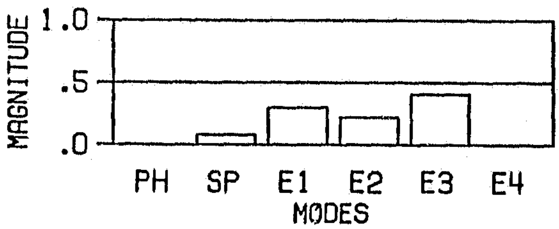
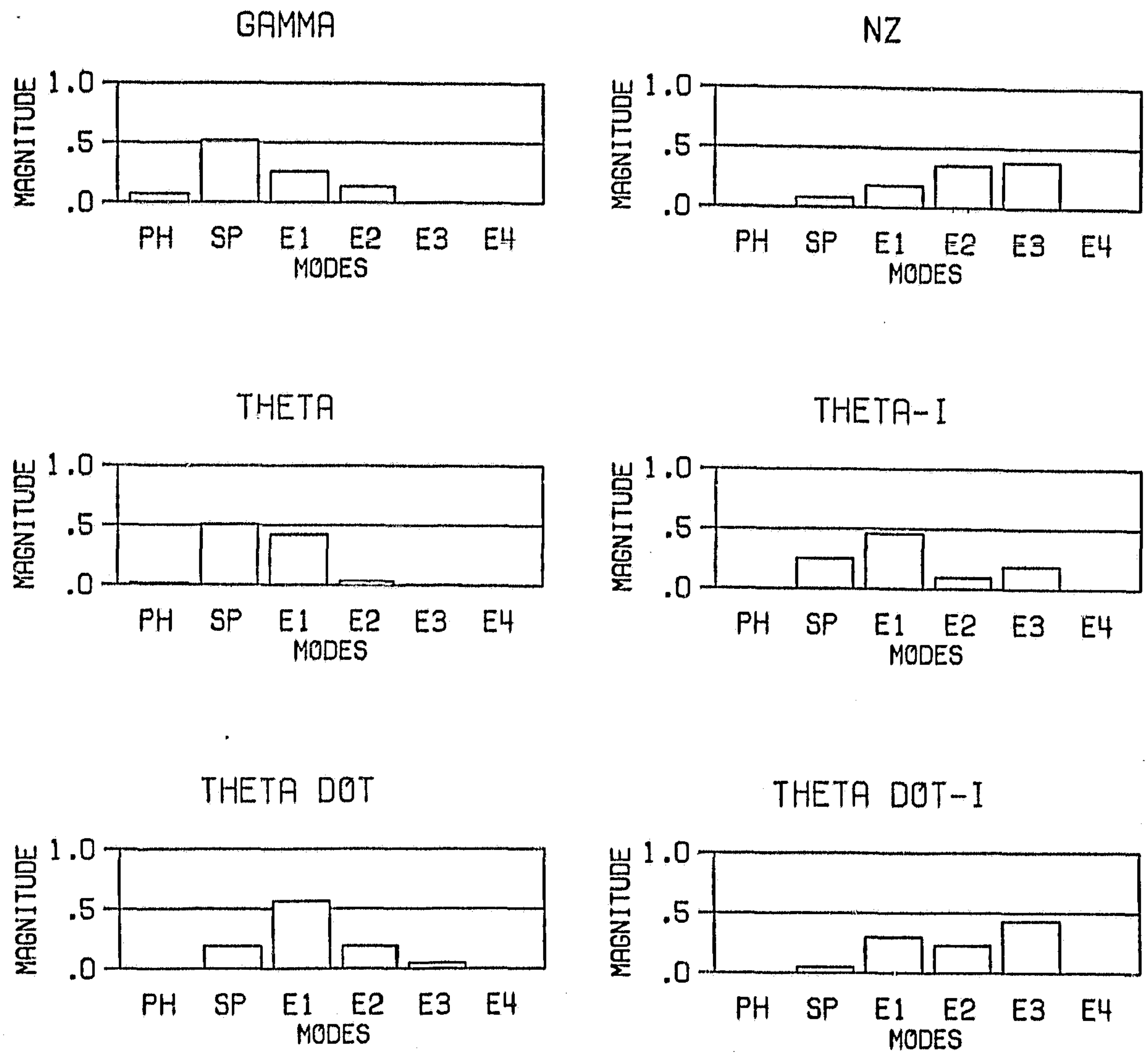


RESIDUES FOR EACH OUTPUT ARE NORMALIZED SO THAT THEIR SUM IS 1.0

RESIDUE MAGNITUDES

(DUE TO DISTURBANCE)

CASE 3 -



RESIDUES FOR EACH OUTPUT ARE NORMALIZED SO
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RESIDUE MAGNITUDES

(DUE TO DISTURBANCE)

CASE 4 -

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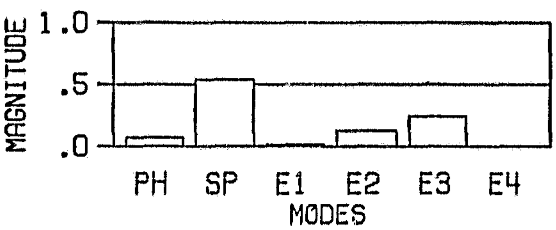

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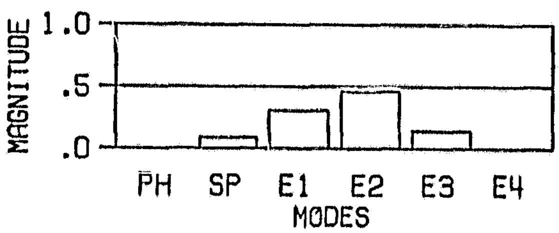

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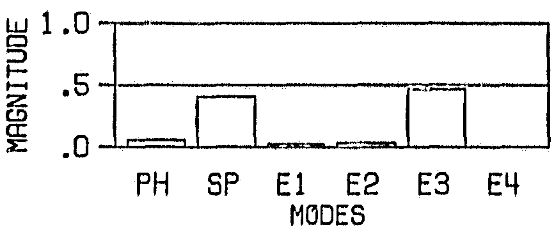

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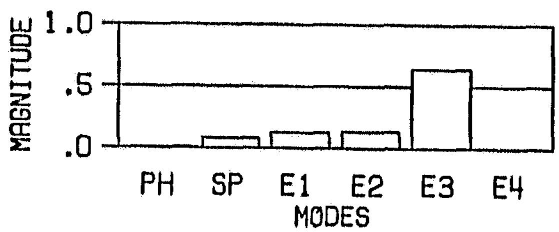

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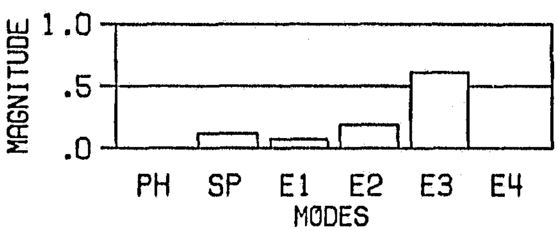

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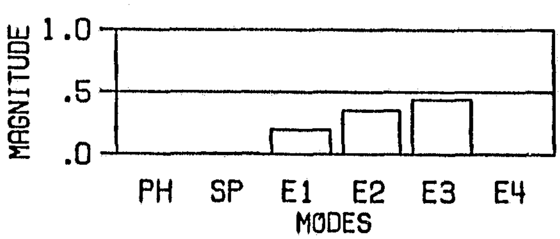

RESIDUES FOR EACH OUTPUT ARE NORMALIZED SO THAT THEIR SUM IS 1.0

RESIDUE MAGNITUDES

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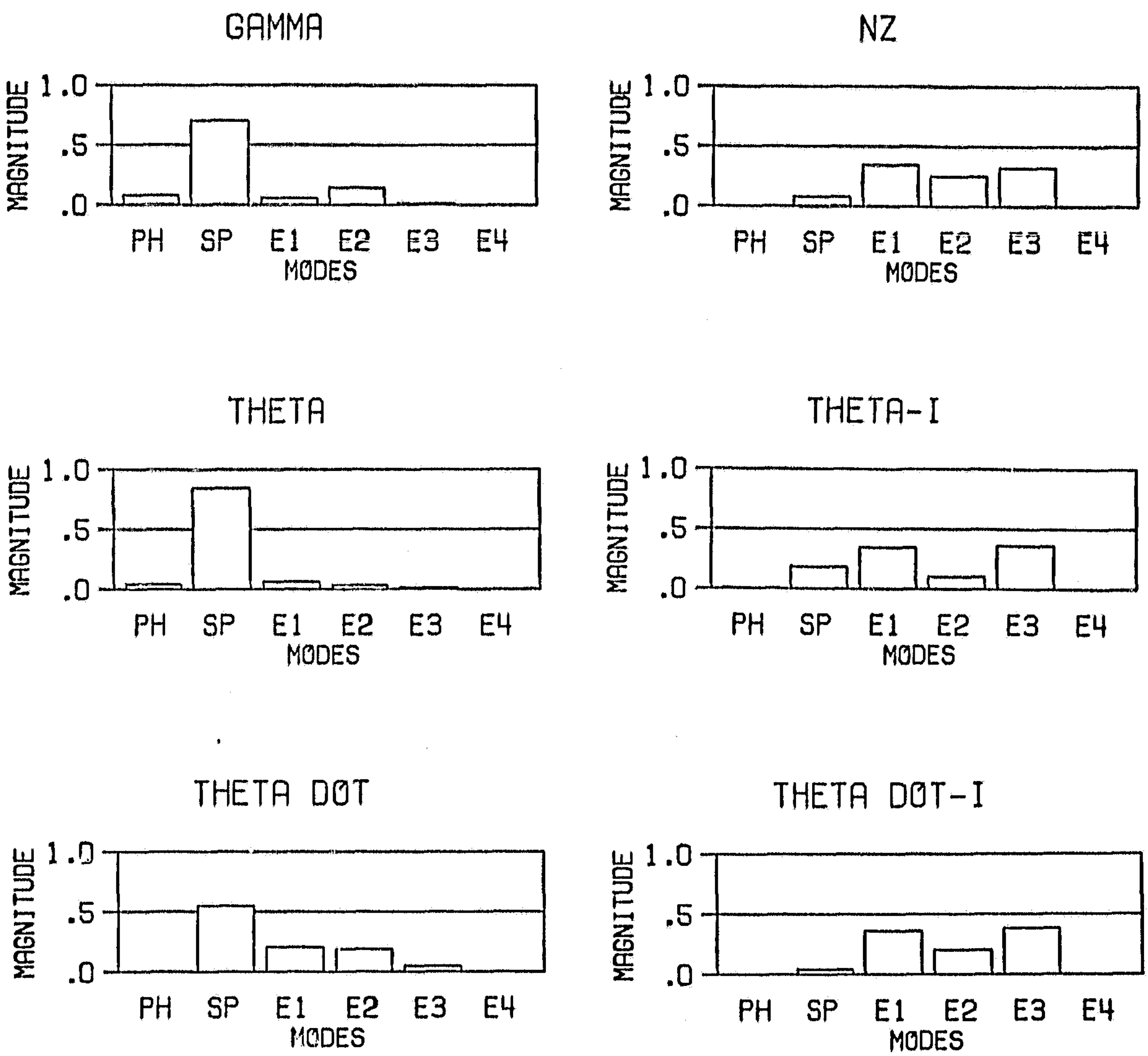

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RESIDUE MAGNITUDES

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CASE 6 -

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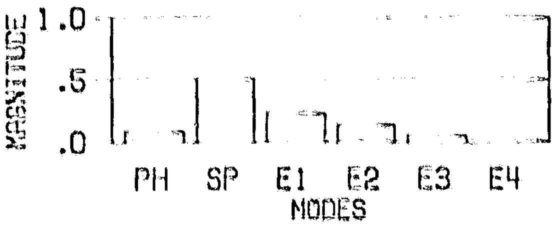

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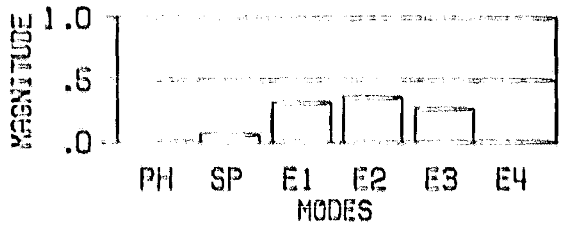

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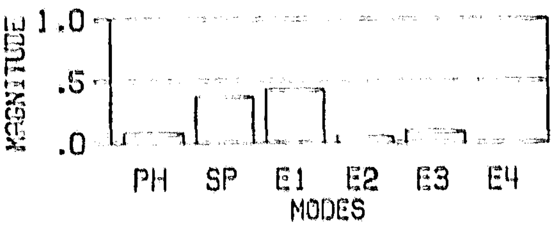

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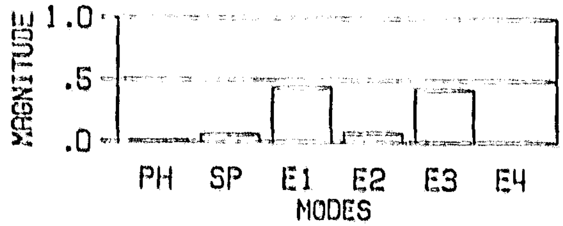

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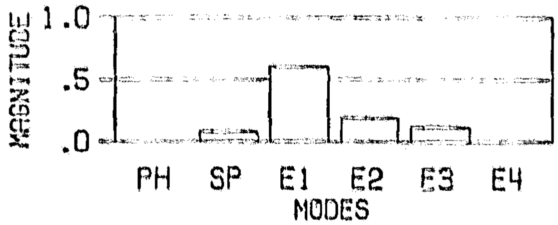

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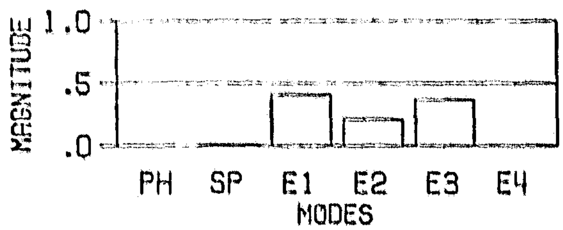

RESIDUES FOR EACH OUTPUT ARE NORMALIZED SO THAT THEIR SUM IS 1.0

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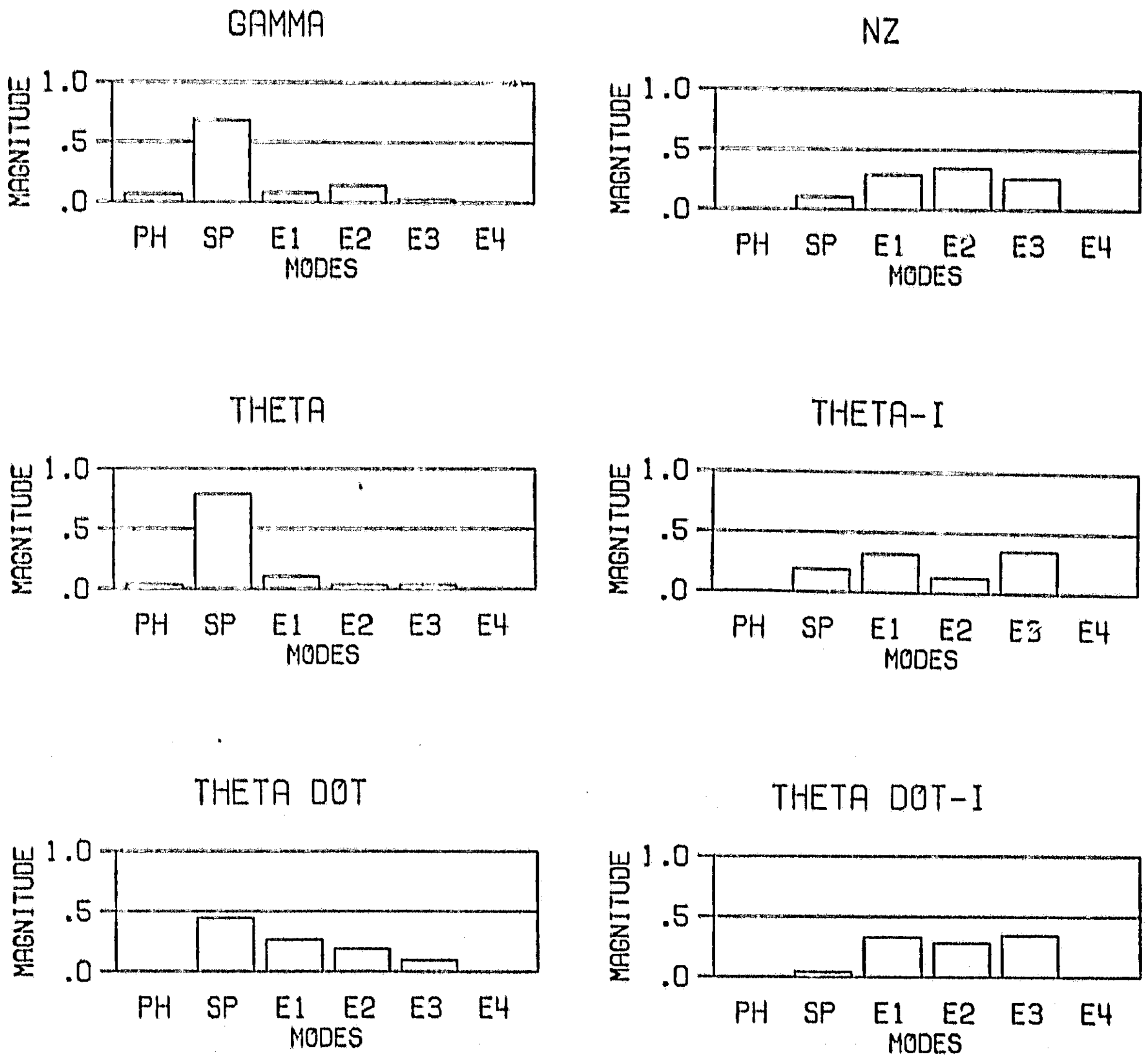
RESIDUE MAGNITUDES

(DUE TO DISTURBANCE)

CASE 7 -



RESIDUES FOR EACH OUTPUT ARE NORMALIZED SO
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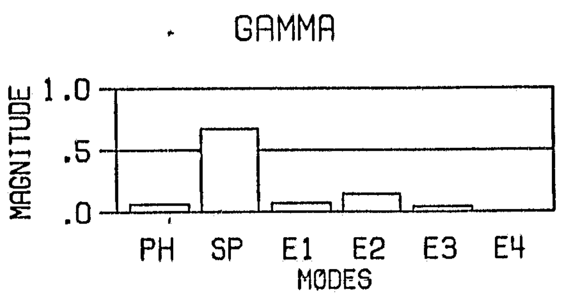
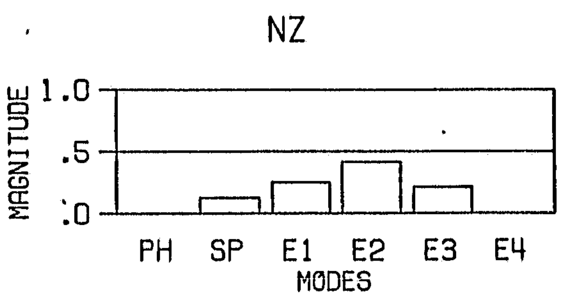
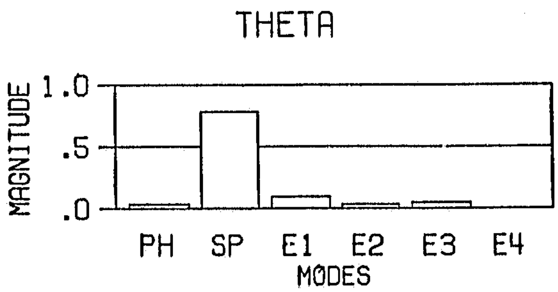
RESIDUE MAGNITUDES

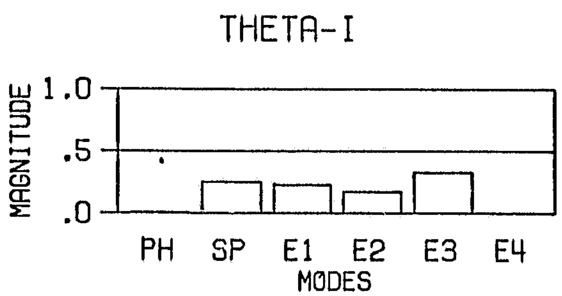
(DUE TO DISTURBANCE)

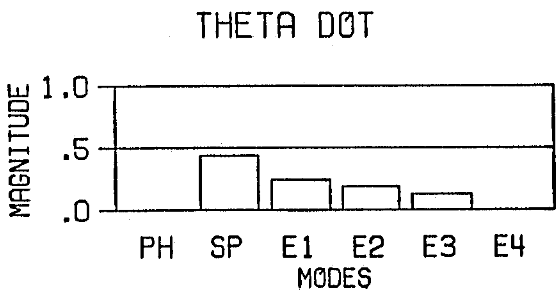
CASE 8 -

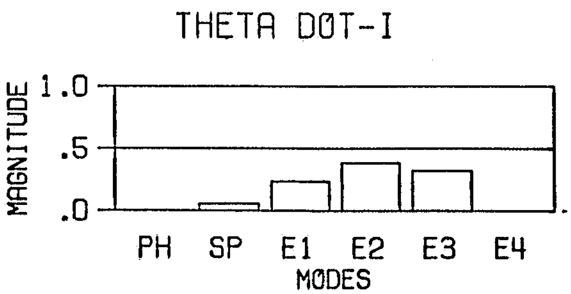












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