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ACOUSTIC MODAL ANALYSIS OF A FULL-SCALE ANNULAR COMBUSTOR

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Abstract

An acoustic modal decomposition of the measured pressure field in a full-scale annular combustor installed in a ducted test rig is described. The modal analysis, utilizing a least squares optimization routine, is facilitated by the assumption of randomly occurring pressure disturbances which generate equal amplitude clockwise and counter-clockwise pressure waves, and the assumption of statistical independence between modes. These assumptions are fully justified by the measured cross-spectral phases between the various measurement points. The resultant modal decomposition indicates that higher order modes compose the dominant portion of the combustor pressure spectrum in the range of frequencies of interest in core noise studies. A second major finding is that, over the frequency range of interest, each individual mode which is present exists in virtual isolation over a significant portion of the spectrum.

Finally, a comparison between the present results and a limited amount of data obtained in an operating turbofan engine with the same combustor is made. The comparison is sufficiently favorable to warrant the conclusion that the structure of the combustor pressure field is preserved between the component facility and the engine.

Introduction

As part of a program to investigate the nature and significance of turbofan engine core-noise sources, a comprehensive measurement program of the pressure field at a variety of locations within the core of a YF-102 turbofan engine, simultaneously with the far-field acoustic pressure, has been underway for several years at the Lewis Research Center. Extensive analyses of this data using a variety of advanced signal diagnostic methods such as correlation and ordinary coherence techniques, conditioned spectral analysis (i.e., partial and multiple coherence analysis) and a recently developed triple signal coherence analysis, have been conducted[1-4].

In Ref. 1 the combustor was unambiguously identified as the major source of core noise at frequencies below about 200 Hz. This was shown by rigorous interpretation of the geometric properties of certain two-point pressure correlation functions within the core and between core pressures and far-field acoustic pressures. Using coherent output power computations, this contribution was quantified in Ref. 2. In Ref. 3, using a novel triple signal coherence technique, Krejsa was able to quantify the core noise contribution from this engine very accurately at frequencies up to about 1 kHz, although the origin within the core of this noise was not identifiable. In Ref. 4, using conditioned spectral analysis as a diagnostic technique, evidence was presented which strongly suggested that the core noise measured between about 200 Hz and 1 kHz was

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also, in fact, combustor originated.

Having identified the combustor as the primary source of core noise, and having available accurate techniques to accurately quantify its contribution to overall engine noise from a given turbofan, a logical next step is to develop techniques to enable the ultimate prediction of core noise without having to make further measurements. This, in turn, requires a description of two elements: the source characteristics (i.e., the structure of the fluctuating pressure field within the combustor) and the propagation path characteristics within the engine core. This paper will deal with the pressure field within the combustor, specifically the modal content, as determined experimentally.

If the acoustic field generated in the combustor is to be tracked through the engine and to the far-field, its modal content must be known. Furthermore, extensive testing of many combustor designs on full-scale operating turbofan engines is extremely cumbersome and prohibitively expensive. Consequently, it is desirable to conduct such studies on less expensive, more flexible apparatus specifically designed for such purposes, such as ducted combustor component rigs. This paper, therefore, will deal with the measured pressure field - specifically, the modal content - in the combustor of a component test facility. A corollary question then arises as to the relationship between combustor internal pressure measurements obtained in a component test facility and those obtained in an engine. Comparisons, therefore, will also be presented with some limited data obtained earlier from an operating YF-102 turbofan.

Test Facility, Instrumentation, and

Data Processing

Component Test Facility and Combustor

The combustor component test facility was operated for the NASA Lewis Research Center, under contract, by the Lycoming Division of the AVCO Corporation at their manufacturing and research facilities in Stratford, Connecticut. The facility, shown schematically in Fig. 1, consists of: an inlet section to distribute the flow; the combustor containing the liner, fuel nozzles (Fig. 2), and igniters; a water cooled exhaust diffuser and plenum; an exhaust valve; and an exhaust stack to the atmosphere. Pressurized air was supplied to the facility by a series of compressors, and the desired inlet temperatures were obtained by electric heaters. Combustor inlet pressure, temperature, and mass flow rate were set to simulate engine operating conditions at the desired test points. A more detailed description of the test facility and apparatus can be found in Ref. 5.

The YF-102 combustor is a reverse flow, annular type, with a nominal outside diameter of 55.4 cm and an annulus height of 6.0 cm at the measurement plane (Fig. 2). Data were obtained in the test facility at five combustor conditions, which approximately

corresponded to those which would prevail at engine operating points between 25 and 60 percent of maximum design speed. At these conditions the combustor exit temperature varied between 1000 and 1100 K.

Internal Probes

The data and analysis reported in this paper were obtained from six dynamic pressure probes placed in the outer annular wall of the combustor liner at a single axial location in the combustion zone. The circumferential placement of the measuring stations corresponded to a reference location (0°) and five others at angular displacements of 30°, 60°, 90°, 135°, and 180° from the reference location (Fig. 1). The probes used to measure these fluctuating pressures in the combustor were cooled "semi-infinite" waveguide tubes using conventional condenser microphones as the actual transducers. A detailed description of their design and response characteristics may be found in Refs. 1 and 2.

Data Acquisition and Processing

All signals from the pressure probes were simultaneously FM-recorded on magnetic tape for later processing. The record lengths at each operating condition were a minimum of 2 minutes in duration to provide adequate statistical confidence in the results. The microphones were calibrated each day with a standard pistonphone prior to recording data. The pressure auto- and cross-spectra required for the modal analysis were obtained by off-line processing of the tape-recorded data on a two-channel fast Fourier transform digital signal processor with built-in analog to digital converters and 120 dB/octave anti-aliasing filters. The processor was capable of direct computation of up to a 4096 member ensemble average of a 1024 point forward or inverse Fourier transform to yield either frequency domain (coherence, amplitude, and phase auto- and cross-spectra, and transfer function) or time domain (correlation) information. The resulting digital spectra were then transmitted from the processor to a central computer with which it was interfaced to perform the computations required for the modal decomposition described later.

Analysis

Measured Cross-Spectra

The approach implemented here to perform the modal analysis entails the simultaneous use of six cross-spectra obtainable from the six combustor pressure signals, taken in pairs. It is essentially the same method reported in Ref. 6, and called by the authors a "Time Averaged Mode Separation Technique." In the present application, using the 0° location (see Fig. 1) as an arbitrary, but common, reference, six pressure cross-spectra were generated.

The cross-spectrum, in general a complex quantity, is defined, for a given pair of random pressure signals, at a given frequency f, as

$$\overline{G_{0\theta}(f)} = \overline{P_0(f)P_\theta^*(f)} \qquad (1)$$

The overbar indicates an ensemble average, and $P_0(f)$ and $P_\theta(f)$ are the (digitally computed)

Fourier transforms of the measured pressure signals at the reference position and at the angle θ, respectively. The *, as usual, designates the complex conjugate.

In general, $G_{0\theta}(f)$ is a complex quantity containing both amplitude and phase information. The measured results, for a single operating condition, are shown in Figs. 3 and 4 for the five angles of 30°, 60°, 90°, 135°, and 180° with the 0° position as the common reference. (The sixth "pair" is, of course, simply the auto-spectrum at the reference position, and therefore contains no phase information.)

While the amplitudes (Fig. 3) of the cross-spectra are not especially remarkable, the cross-spectral phases (Fig. 4) are quite notable in that the phase differences are predominantly either 0° or 180°, with essentially no measured phase angles lying between these limits. This feature of the phase spectra, characteristic of standing waves, is typical of all tested operating conditions of the combustor. Furthermore, it suggests a relatively simple model for the pressure field in the annulus and considerably facilitates the modal decomposition.

Modal Decomposition Technique

The analysis proceeds by assuming that a randomly occurring instantaneous pressure disturbance, produced by the combustion turbulence, sends a clockwise and counter-clockwise pressure wave traveling around the annulus. At any angle θ the two waves add to produce the resultant pressure:

$$P_\theta = P_\theta^+ + P_\theta^- = e^{j\omega t} \sum_{m=0}^{M-1} \left[A_m e^{j(\varphi_m - m\theta)} + B_m e^{j(\varphi_m + m\theta)} \right]$$

$$\left(j = \sqrt{-1} \right) \qquad (2)$$

Here, A_m and B_m are the amplitudes of the clockwise and counter-clockwise waves, respectively, associated with the lowest radial order of the mth circumferential mode, φ_m is the phase of the mth mode, and M is the total number of modes present. As will be justified later, higher order radial modes are neglected.

At the reference angle (θ = 0) we simply have

$$P_0 = e^{j\omega t} \sum_{m=0}^{M-1} (A_m + B_m) e^{j\varphi_m}$$

Hence

$$P_0 P_\theta^* = \sum_{m=0}^{M-1} (A_m + B_m) e^{j\varphi_m} \sum_{n=0}^{N-1} \left[A_n e^{-j(\varphi_n - n\theta)} \right.$$

$$\left. + B_n e^{-j(\varphi_n + n\theta)} \right]$$

or

$$P_0 P_\theta^* = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} (A_m A_n + B_m A_n) e^{j(\varphi_m - \varphi_n + n\theta)}$$

$$+ \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} (B_m B_n + A_n B_m) e^{j(\varphi_m - \varphi_n - n\theta)} \tag{3}$$

Upon performing the averaging process indicated by Eq. (1), the individual products in Eq. (3) become correlations. An important assumption is now made that the modes are independent and, therefore, their cross-correlations vanish. Consequently, all terms in the averaged version of Eq. (3) for which $m \neq n$ vanish, and the double sums which remain for $m = n$ may be contracted to single sums:

$$\overline{P_0 P_\theta^*} = \sum_{m=0}^{M-1} \left(\overline{A_m^2} + \overline{B_m A_m} \right) e^{jm\theta} + \sum_{m=0}^{M-1} \left(\overline{B_m^2} + \overline{A_m B_m} \right) e^{-jm\theta} \tag{4}$$

Finally, after some minor manipulation, Eq. (4) can be re-written as:

$$\overline{P_0 P_\theta^*} = \sum_{m=0}^{M-1} \overline{\left(A_m + B_m \right)^2} \cos m\theta$$

$$+ j \sum_{m=0}^{M-1} \left(\overline{A_m^2} - \overline{B_m^2} \right) \sin m\theta = \overline{G_{0\theta}} \tag{5}$$

Equation (5) is the general expression for the complex cross-spectrum between the pressure measured at the reference position and the pressure measured at a given angle θ relative to the reference position. The phase of this cross-spectrum, then, is:

$$\Phi_{0\theta}(f) = \tan^{-1} \frac{\sum_{m=0}^{M-1} \overline{\left(A_m^2 - B_m^2 \right)} \sin m\theta}{\sum_{m=0}^{M-1} \overline{\left(A_m + B_m \right)^2} \cos m\theta} \tag{6}$$

If the characteristics of the acoustic sources are such that an instantaneous pressure disturbance created in the combustor sends equal amplitude clockwise and counter-clockwise pressure waves traveling around the annulus, then $A_m = B_m$. From Eq. (5), the imaginary part of the cross-spectrum vanishes, and from Eq. (6) it is seen that the phase must then be either 0° or 180°, depending upon the sign of the real part. From the data shown in Fig. 4, this is essentially what is observed.

The equation for the complex cross-spectrum then becomes, when it is assumed that $A_m = B_m$:

$$\overline{G_{0\theta}(f)} = \overline{P_0 P_\theta^*} = 4 \sum_{m=0}^{M-1} \overline{A_m^2} \cos m\theta \tag{7}$$

The phase angle, then, is given by:

$$\Phi_{0\theta}(f) = 0° \quad \text{for} \quad \overline{G_{0\theta}(f)} \geq 0$$
$$= 180° \quad \text{for} \quad \overline{G_{0\theta}(f)} < 0 \tag{8}$$

Equations (7) and (8), together with the measured pressure cross-spectra from the combustor, form the basis for the modal analysis conducted in this investigation.

Equation (7) can be written for each of the six measured cross-spectra. It is further assumed that, at most, only the first six modes made significant contributions to the pressure field in the combustor over the frequency range of interest in core noise studies. This latter assumption will be shown to be amply justified by the results of the modal decomposition. The resulting six equations in six unknowns can then, in principle, be solved on a frequency by frequency basis to obtain the modal amplitudes.

As noted above, the assumption of equal amplitude clockwise and counter-clockwise spinning modes gives rise to the requirement that the cross-spectral phase angles be either 0° or 180°. It is clear from the phase spectra shown in Fig. 4, however, that while the measured data are qualitatively highly consistent with this assumption, the assumption is, nevertheless, not perfectly valid, even allowing for normal experimental error and statistical variance. Hence, the basic model being proposed for the pressure field, as given by Eqs. (7) and (8) is, at best, a good approximation. Furthermore, the six cross-spectra required as input to the six versions of Eq. (7) are themselves experimentally determined variables, also subject to nominal experimental error and statistical uncertainties.

For these reasons there is obviously no assurance that the set of six equations in six unknowns will have, at any given frequency, a solution in the sense of yielding six unique modal coefficients, or amplitudes. The result will depend upon the particular combination of six probes chosen. The system of equations will, however, have an optimum solution in, for example, a least squares sense. That is, a single set of coefficients for Eq. (7) may be found which minimizes the sum of the squares of the differences between the six measured cross-spectra and the corresponding cross-spectra computed via Eq. (7). Quantitatively, then a search is conducted to find a set of coefficients A_m to minimize the function S:

$$S = \sum_{i=1}^{6} \left[\left| G_{0\theta_i} \right|_e - 4 \left| \sum_{m=0}^{5} A_m^2 \cos m\theta_i \right| \right]^2 \tag{9}$$

Here, $\left| G_{0\theta_i} \right|_e$ is the magnitude of the measured cross-spectrum between the pressure at the reference position and the pressure at the angle θ_i. There are numerous existing optimization routines which can be used for such a minimization problem. The search technique used in this study is described in Ref. 7, and the computer program used was adapted from Ref. 8.

Before proceeding to the results, one additional point requires some discussion. At some

angles the measured cross-spectrum can be inherently small over a broad range of frequencies because of the distribution of the modes which may dominate the spectrum at those frequencies. For example, at $\theta = 90°$, if it should occur that one or more odd order modes dominate the spectrum over some frequency range, then the magnitude of the cross-spectrum will be very small in that range. If the cross-spectrum is sufficiently small then the resulting signal-to-noise ratio in the measurement is inadequate to produce a satisfactory set of modal coefficients from the optimization routine. That is, the difference between the measured cross-spectrum and that which would be reconstructed by Eq. (7) would be unacceptably large. Furthermore, the reconstructed phase would randomly jump between $0°$ or $180°$, in contrast to the systematic behavior exhibited by the measured phase and shown in Fig. 4.

In general, as can be seen from Figs. 3 and 4, the measured phase is much better behaved than the cross-spectrum magnitude. This observation was the basis for slightly altering the actual function which was minimized. The function given by the expression in Eq. (9) was modified to include a weighting factor:

$$S = \sum_{i=1}^{6} \left[\left| G_{0\theta_i} \right|_e - 4 \left| \sum_{m=0}^{5} A_m^2 \cos m\theta_i \right| \right]^2 W_i$$

The weighting factor, W_i, was set equal to unity (i.e., no weighting) when the sign of the reconstructed cross-spectrum (which is pure real) was the same as the sign of the real part of the measured cross-spectrum. If the signs were different, the weighting was set equal to a factor of five. This weighting, or penalty factor, effectively requires the optimization routine to search for a set of coefficients which accurately reconstructs both the amplitude and phase data, simultaneously. (Various values for W_i were tried. A value of five, however, appeared to provide an optimum sensitivity for both amplitude and phase.)

Results

Reconstruction of Measured Data

An indicator of the degree to which the numerical optimization procedure discussed above yields an acceptable set of modal amplitudes is how well the coefficients can be used to reconstruct the original measured spectra, using Eq. (7). Examples of the precision with which the procedure was in fact capable of reconstructing the measured data are shown in Figs. 5 and 6. In Fig. 5 are shown the spectral amplitudes for several cases, with the solid line being the measured spectrum and the plotted circles being the reconstructed values. (The measured spectra contain 350 points with a 4-Hz bandwidth. Although the coefficients were computed at all 350 frequencies, the reconstructed points are plotted only at every third frequency for clarity.) The effectiveness in reconstructing the phase is shown in Fig. 6. (Here, it is the circles which are the measured data, and the solid lines which are the reconstructed phases.) It is quite clear from both Figs. 5 and 6 that the search routine utilized is extremely effective in determining a set of coefficients which accurately reconstruct the measured amplitude and phase spectra.

Estimation of Resonant Frequencies

Data for this study were obtained for five different operating conditions of the combustor, with the pressure ranging from about 255 to 550 kPa and the temperatures from 950 to 1100 K. The Mach number was approximately 0.07 for all conditions. These conditions roughly correspond to equivalent operating conditions of between 30 and 60 percent of maximum speed.

Neglecting the Mach number, and considering the transverse modes only, the characteristic, or cut-on, frequencies of the normal modes are obtained from the usual expression:

$$f_{m,n} = \frac{\alpha_{m,n} C}{2\pi R_0}$$

Here, C is the speed of sound in the duct, R_0 is the outer radius, and $\alpha_{m,n}$ is the eigenvalue associated with the mth circumferential and n^{th} radial modes. The eigenvalues are obtained as the zeroes of the characteristic function associated with the particular geometry and may be found in numerical tables. The estimated values for the resonant frequencies of the first five circumferential modes and the first radial mode for an annular duct with an inner to outer radius ratio of 0.785 (the same as that of the combustor studied, at the measurement plane) are shown in Table 1. They were obtained by interpolating from tabulated eigenvalues in Refs. 10 and 11.

TABLE 1. - ESTIMATED RESONANT FREQUENCIES

Con-di-tion	Tem-pera-ture, K	Zeroth order radial					First order rad-ial
		$f_{1,0}$, Hz	$f_{2,0}$, Hz	$f_{3,0}$, Hz	$f_{4,0}$, Hz	$f_{5,0}$, Hz	$f_{0,1}$, Hz
1	947	396	794	1194	1595	1995	4142
2	955	397	796	1197	1599	2001	4154
3	982	403	807	1214	1622	2029	4213
4	989	404	810	1218	1627	2036	4228
5	1100	427	854	1285	1716	2147	4459

A number of aspects concerning the estimated values shown in the table are worth noting. First, the lowest frequency at which a radial mode (other than the plane wave) is expected to be resonant in the combustor is approximately 4100 to 4400 Hz, depending upon operating condition. The basic assumption of the analysis that the modal decomposition can be conducted over the frequency range of interest in core noise studies by considering only the circumferential modes seems, therefore, to be justified. Furthermore, the table indicate that if we are concerned only with frequencies up to about 1400 or 1500 Hz, we can probably expect significant contributions only from the first three or four circumferential modes, depending upon how rapidly a particular mode diminishes away from its resonant frequency. Finally, it is noted that with the exception of condition No. 5, there should be insignificant changes in the observed modal peaks as a function of operating conditions. This is a consequence of the essentially constant thermodynamic conditions in the combustor over the range of conditions tested.

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Modal Content in Component Combustor

The actual modal content is represented by the individual coefficients as a function of frequency. The results for the entire range of combustor operating conditions in this study are shown in Fig. 7. The symbols represent the calculated modal amplitudes, as obtained from the optimization routine. Also shown, as solid lines, are the measured autospectra. Before proceeding to further discussion, a brief explanation of the plots shown in Fig. 7 is required.

First, as in Figs. 5 and 6, the bandwidth for the measured data is 4 Hz, with the solid line containing 350 measured points. Again, for clarity, although modal coefficients were computed at all 350 frequencies, only every third point is plotted. Additionally, on Fig. 7 those computed modal amplitudes which were more than 10 dB below the measured auto-spectrum were not plotted. When the contributions of one or more particular modes at a given frequency to the overall spectral amplitude were very small, the optimization routine would yield values for the dominant coefficients which were, not unexpectedly, insensitive to the precise numerical values of the small coefficients. Hence, the small coefficients would essentially vary randomly resulting in quite "noisy" spectra for their associated modes in those frequency ranges where they made negligible contributions.

The results shown in Fig. 7 are significant on a number of levels. First, there is direct confirmation that a very major portion of the source region contains higher order acoustic modes. In fact, those modes dominate the spectrum beyond about 400 Hz and even make significant contributions to the spectrum at frequencies as low as about 250 Hz. Knowledge of how the energy contained in these transverse acoustic modes is distributed is essential to enable the combustor acoustic source information to be tracked through the engine.

It is also worth noting that for all of the combustor operating conditions investigated, there are distinct frequency bands over which the individual modes, including the plane wave, exist nearly in isolation in the sense that there is virtually no overlapping of the individual modes. This is in sharp contrast to the situation normally found in fan ducts. There, because of the higher frequencies of interest, there are many more modes to contend with, including radial modes, and individual modes do not dominate the spectrum over any given frequency range. The modal analysis, therefore, is significantly more difficult, usually requiring far more measurement locations than in the present study, normally at a number of positions which is larger than the number of modes thought to be present in the duct.

A Comparison of Fig. 7 with Table 1 clearly shows that the individual modal peaks, as obtained from the experimental decomposition, correspond very closely to the estimated resonant frequencies. Each local peak in the measured spectrum may be identified or associated with a single mode. Furthermore, other than the plane wave, no peak occurs in the measured spectrum at any frequency other than the resonant frequency of one of the circumferential modes. This suggests that the basic source generating mechanism itself has a relatively smooth, somewhat featureless, spectral shape but the annular

geometry of the combustor is extremely effective in "selecting" those frequencies to be resonant in the combustor.

Comparison with Engine Data

Earlier it was mentioned that one of the primary motivations for conducting core noise studies in ducted component apparatus is because of the inherent experimental flexibility of a component rig compared to an engine, and the significantly lower cost of testing new combustor designs in such a facility. A natural question then arises as to the relationship between the pressure field measured in a component test facility and that obtained in an engine.

This issue is at least partially addressed in Fig. 8, which contains data from pressure measurements taken in the combustor of an operating YF-102 turbofan engine. The data for this one operating condition are typical for all conditions and were first reported, in part, in Ref. 9. For the engine tests conducted in conjunction with Ref. 9, only two pressure measurement stations were available in the combustor. These corresponded to the reference and 90° circumferential positions, and were at the same axial location, as in the present study. Only a limited comparison with the present data can be made, therefore. The engine data shown in Fig. 8 are for the operating condition of the engine which correspond to combustor component operating condition 3.

In part (a) of Fig. 8 is shown the measured cross-spectral phase between the pressure at the reference position and at the 90° position in the engine combustor. This corresponds to the measured cross-spectral phase for the component combustor in this investigation shown in Fig. 4(c). It is clear from Fig. 8(a) that, at least for the one angular displacement of 90°, the cross-spectral phase in the engine combustor exhibits the same characteristic behavior as in the component combustor: the phase is either 0° or 180°, with essentially no measured phase angles lying between these limits. A comparison of these data with those in Fig. 4(c) for the component combustor, furthermore, indicates that the frequency at which the phase switches from 0° to 180° is also the same as the component combustor. (The random scatter in the phase data of the engine combustor, which occurs beyond about 1100 Hz, results from the relatively low coherence between the two pressures due to a weak signal-to-noise ratio.) For this one angle, at least, the phase spectra between engine and component combustor compare favorably.

The amplitudes of the combustor pressure spectra measured in each facility are compared directly in Fig. 8(b). There are significant differences between the two especially in level, with the engine combustor spectrum (solid line) approximately 3 to 8 dB higher than the component combustor spectrum (dashed line) at most frequencies. Furthermore, in the region of the spectra previously identified with the plane wave in the component combustor (up to about 300 Hz) the two are radically different. Beyond this frequency, however, except for the level difference, there is an important similarity. Though not as distinct, there are clearly local peaks in the engine combustor pressure spectrum which correspond to the individual modal peaks previously identified in the component combustor spectrum.

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This observation, combined with the similarities in the characteristics of the cross-spectral phases, previously discussed, strongly suggest that the basic structure of the pressure field in the combustor is preserved between the component facility and the engine. The differences in detail in the spectral content are likely due to inherent differences in the spectra of the source generating mechanisms.

Conclusions

An acoustic modal decomposition of the measured pressure field in a full-scale annular combustor installed in a ducted test rig was performed. The modal analysis was facilitated by the assumption of randomly occurring pressure disturbances which generate equal amplitude clockwise and counter-clockwise pressure waves, and an assumption of statistical independence between modes. These assumptions were consistent with the measured cross-spectral phases between the various measurement points. A least squares optimization routine was then used to decompose the measured data into a distribution of modes consistent with the assumed model.

The resultant modal decomposition indicated that higher order modes compose the dominant portion of the combustor pressure spectrum in the range of frequencies of interest in core noise studies. It was further found that the individual modes present existed in virtual isolation in the sense of a particular mode dominating the spectrum over a very broad frequency range. That is, individual regions of the combustor spectrum could be identified uniquely with a single individual mode.

A comparison with some limited data obtained previously from an operating turbofan engine was also made. The comparison indicated sufficient similarities, especially with respect to the phase spectra, to warrant the conclusion that although there are differences in the combustor pressure spectral content, the structure of the pressure field between the component and engine combustors is duplicated. The use of ducted component apparatus, therefore, in place of fully operating turbofan engines can be a valuable technique for conducting certain types of combustion noise source diagnostics.

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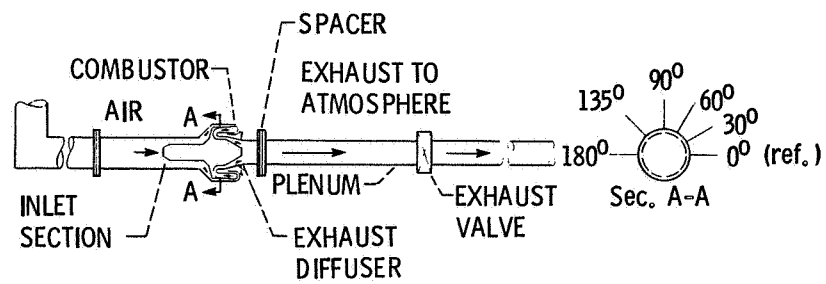
Figure 1. - Combustor component test facility.

Figure 2. - Schematic of flowpath and probe location.
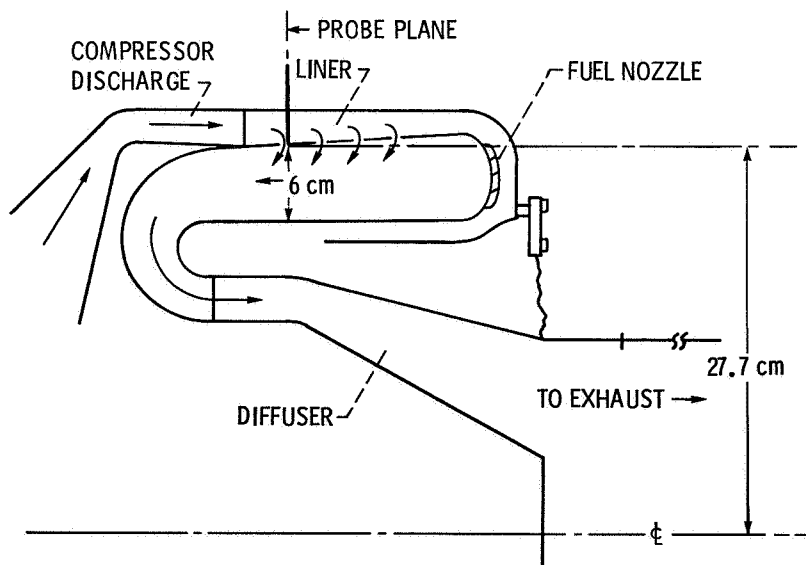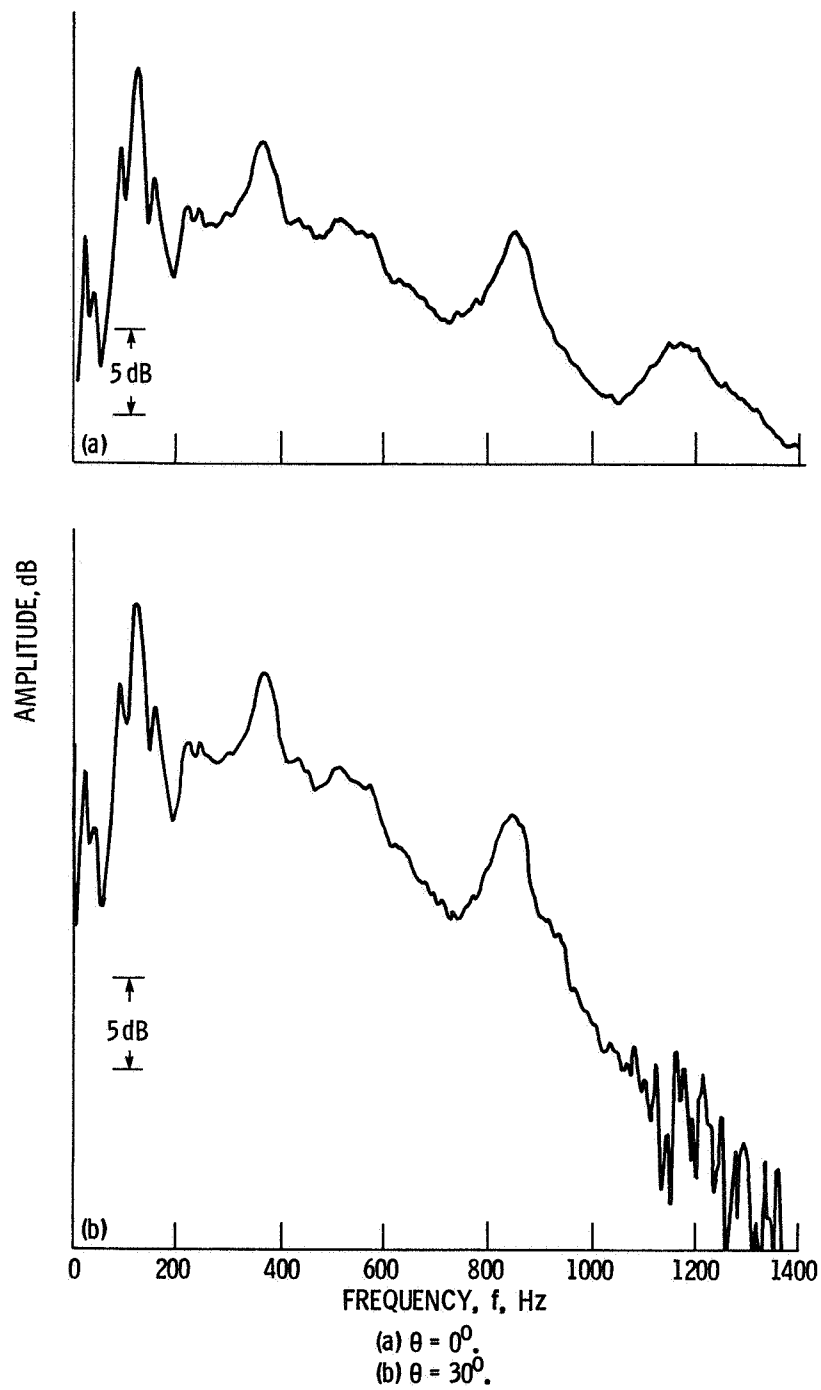
Figure 3. - Measured cross-spectral amplitude, $G_{0\theta}$ (f); condition 3, P = 373 kPa.

(a) $\theta = 0^{\circ}$.

(b) $\theta = 30^{\circ}$.
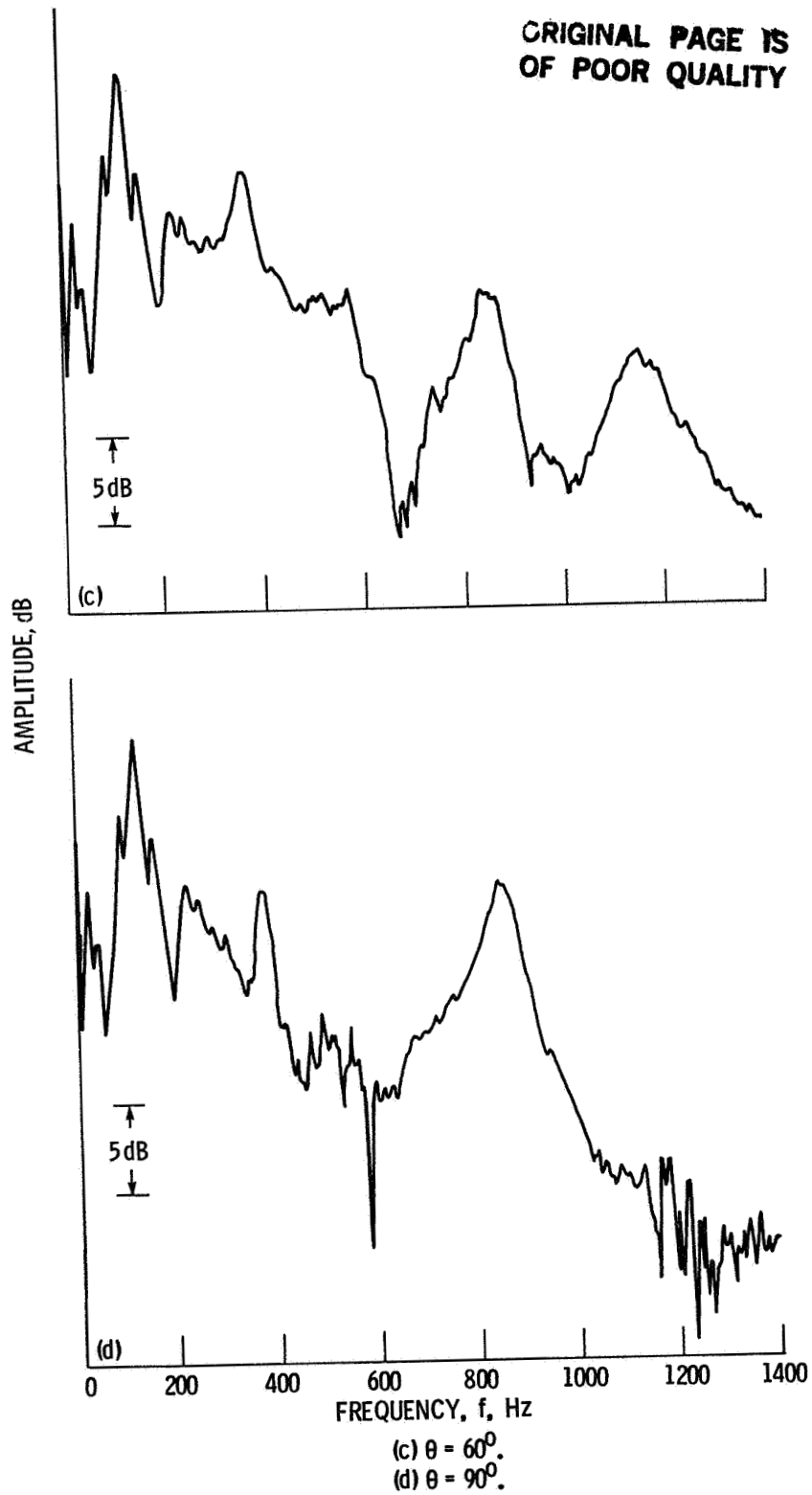
Figure 3. – Continued.

(c) θ = 60°.
(d) θ = 90°.

FREQUENCY, f, Hz

AMPLITUDE, dB

5 dB

5 dB

(c)

(d)

0   200   400   600   800   1000   1200   1400

(e) θ = 135°.
(f) θ = 180°.
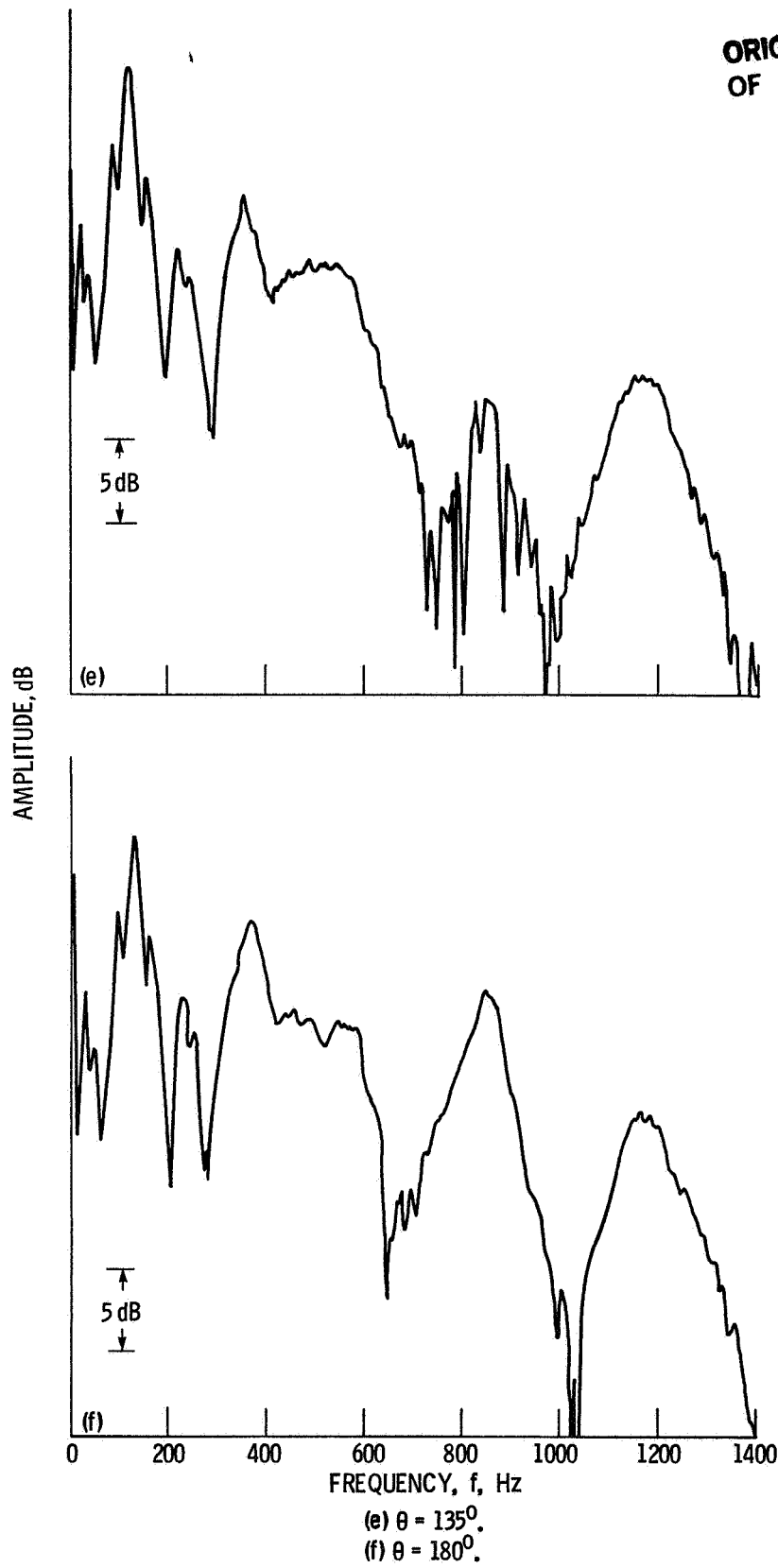
Figure 3. - Concluded.

(a) θ = 30°.
(b) θ = 60°.
(c) θ = 90°.
(d) θ = 135°.
(e) θ = 180°.
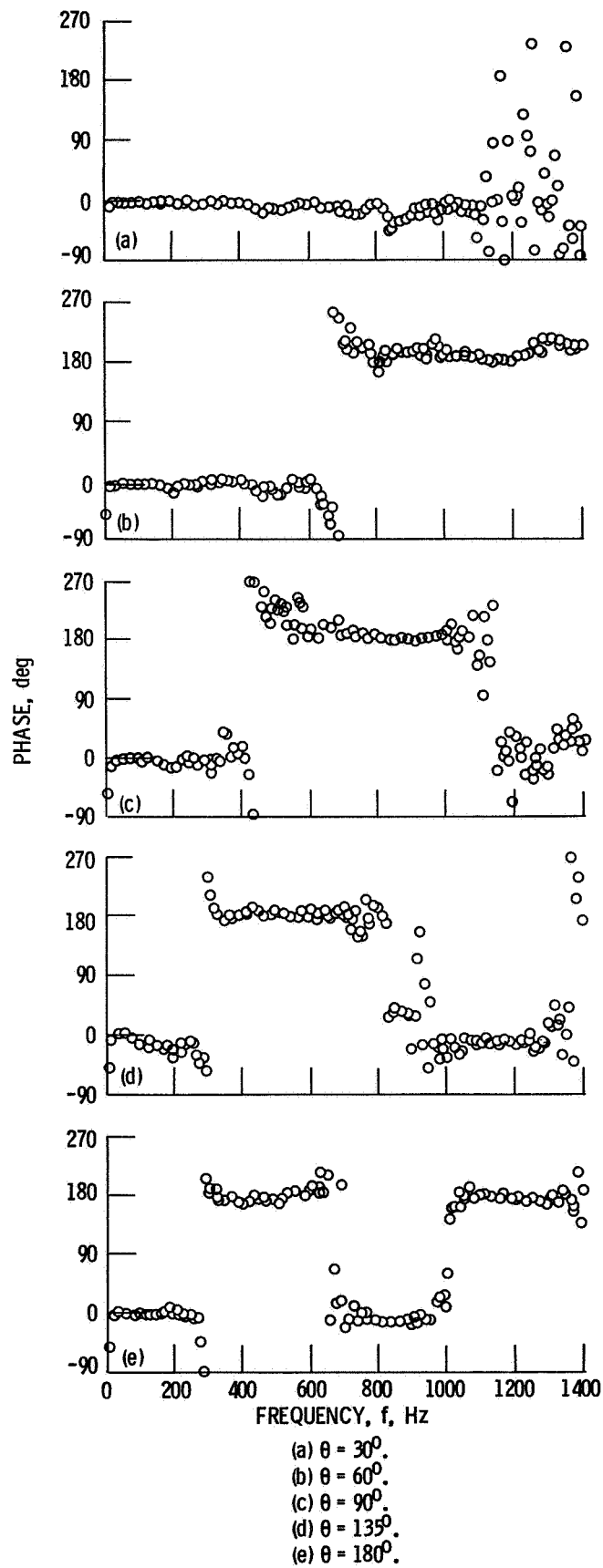
Figure 4. - Measured cross-spectral phase, $\varphi_{o\theta}$ (f);
condition 3, P = 373 kPa.
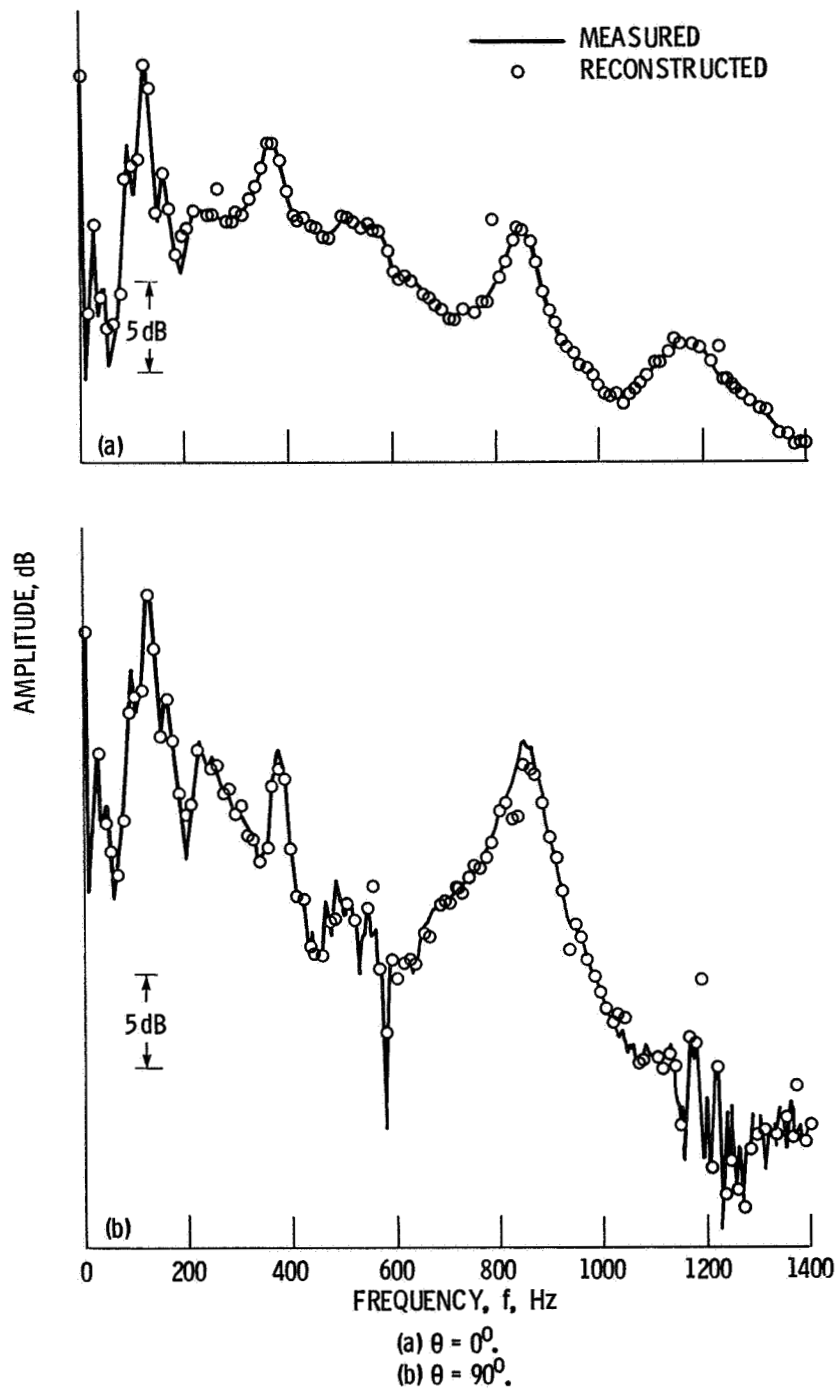
Figure 5. – Reconstructed cross-spectral amplitude; condition 3,
P = 373 kPa.

(c) θ = 180°
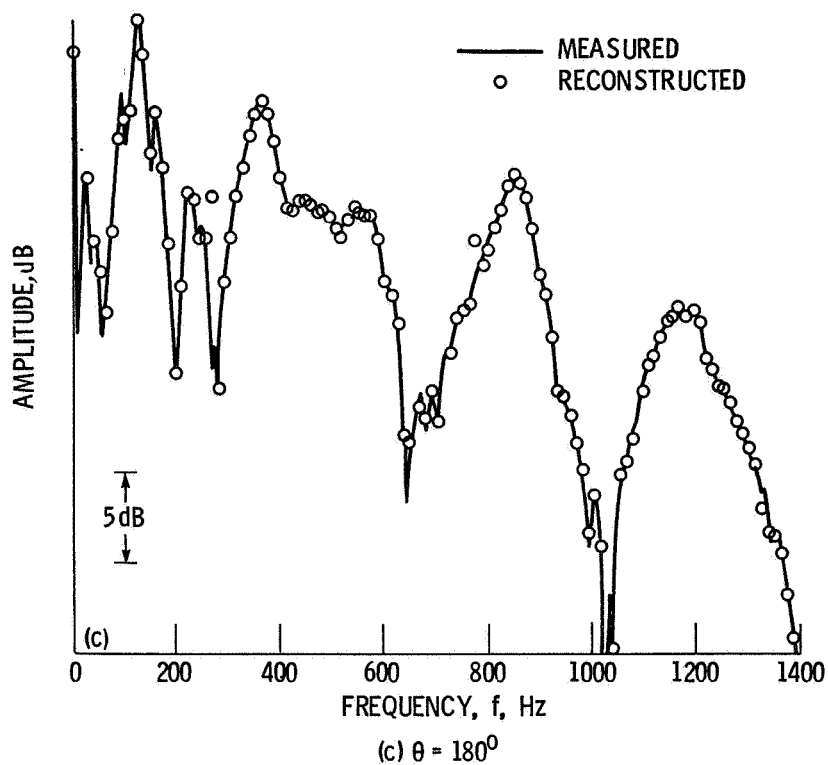
Figure 5. - Concluded.

(a) θ = 90°.

(b) θ = 180°.
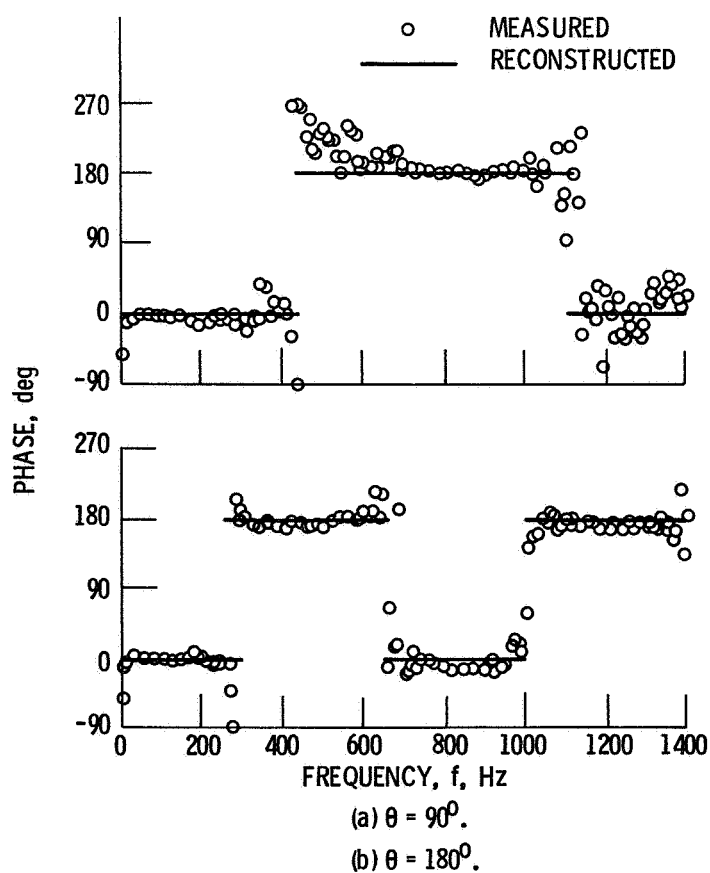
Figure 6. - Reconstructed cross spectral phase;
condition 3 P = 373 kPa.

(a) Condition 1 (P = 257 kPa).
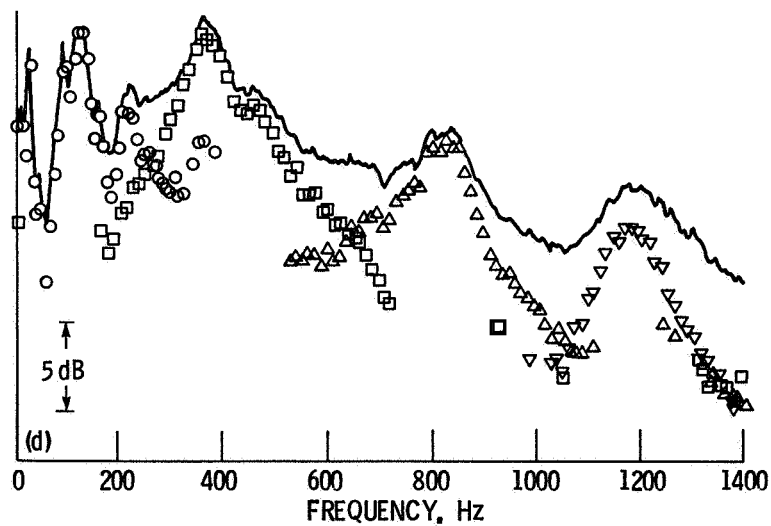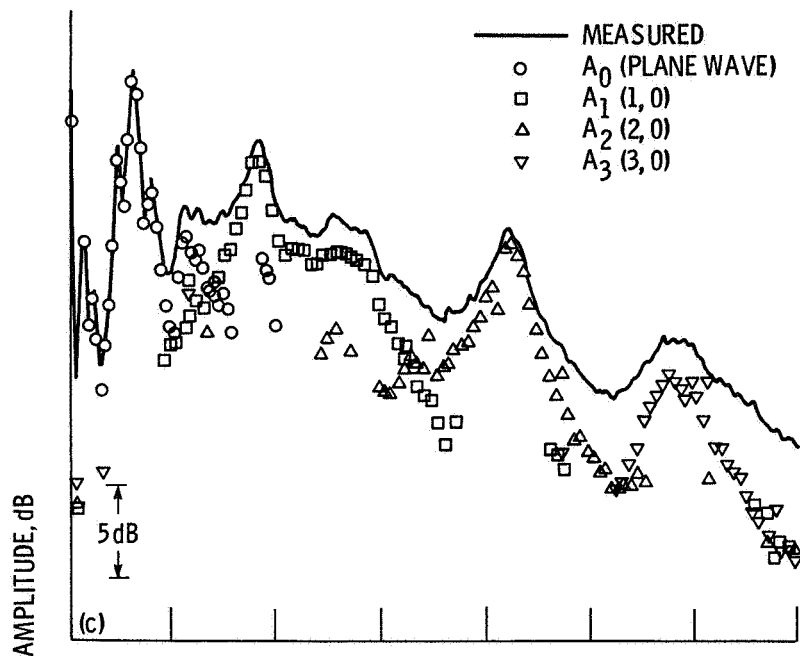(b) Condition 2 (P = 311 kPa).
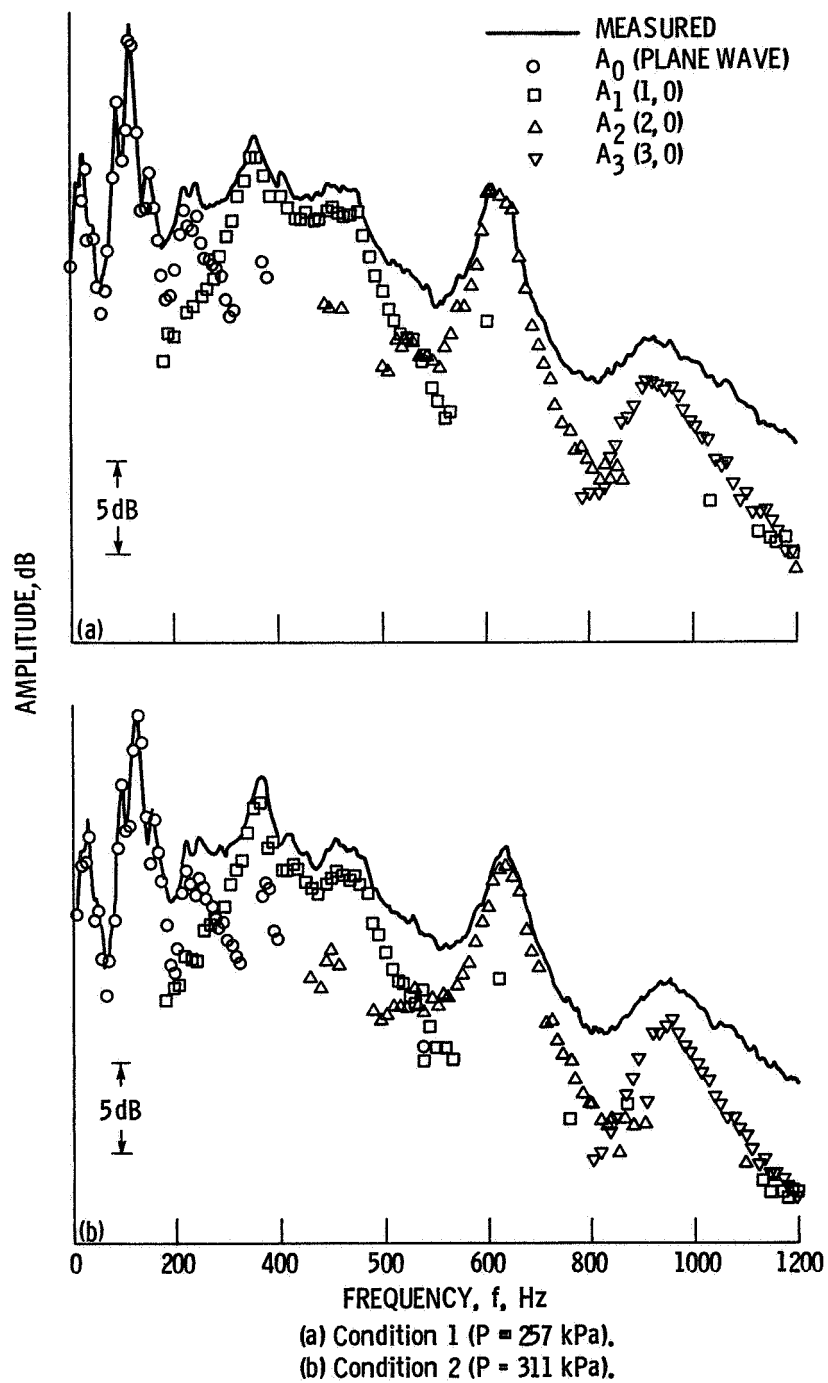
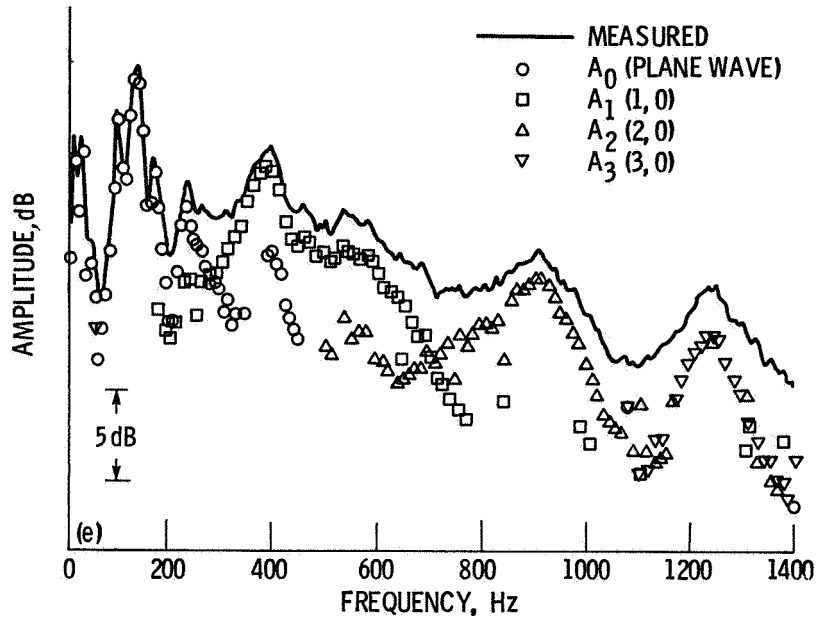Figure 7. - Modal decomposition of combustor pressure.
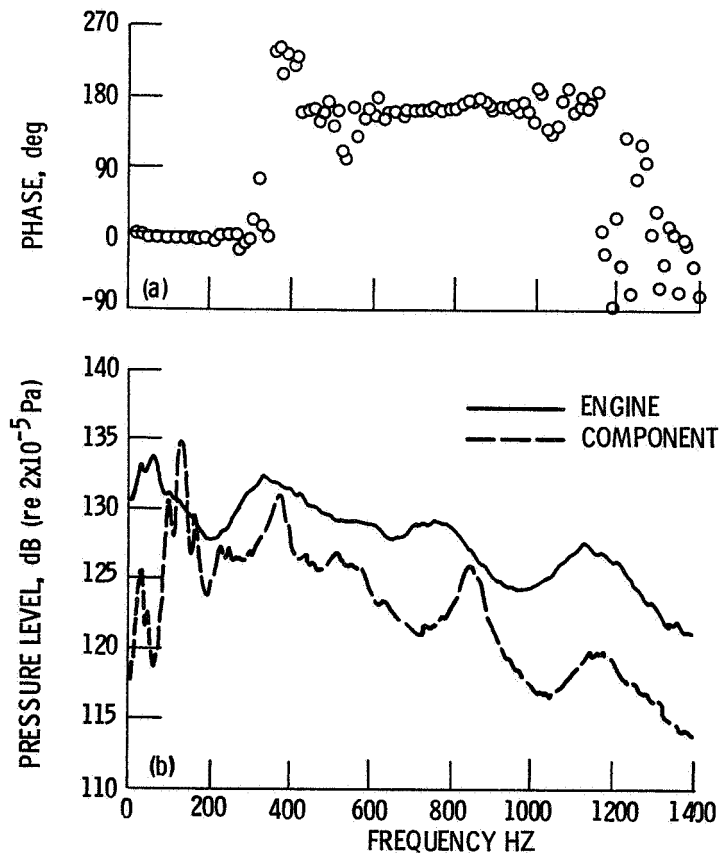
(c) Condition 3 (P = 373 kPa).
(d) Condition 4 (P = 447 kPa).

Figure 7. - Continued.

(e) Condition 5 (P = 557 kPa).

Figure 7. - Concluded.

(a) Cross-spectral phase in engine combustor
(θ = 90°).

(b) Comparison of engine and component combustor
auto-spectra, (P = 373 KPa).

Figure 8. - YF-102 engine combustor data.

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15. Supplementary Notes

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16. Abstract

An acoustic modal decomposition of the measured pressure field in a full-scale annular combustor installed in a ducted test rig is described. The modal analysis, utilizing a least squares optimization routine, is facilitated by the assumption of randomly occurring pressure disturbances which generate equal amplitude clockwise and counter-clockwise pressure waves, and the assumption of statistical independence between modes. These assumptions are fully justified by the measured cross-spectral phases between the various measurement points. The resultant modal decomposition indicates that higher order modes compose the dominant portion of the combustor pressure spectrum in the range of frequencies of interest in core noise studies. A second major finding is that, over the frequency range of interest, each individual mode which is present exists in virtual isolation over significant portions of the spectrum. Finally, a comparison between the present results and a limited amount of data obtained in an operating turbofan engine with the same combustor is made. The comparison is sufficiently favorable to warrant the conclusion that the structure of the combustor pressure field is preserved between the component facility and the engine.

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