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SUMMARY

The technique of tracing out a mean flow streamline with a three-dimensional laser-Doppler anemometer (LDA) is discussed with respect to cumulative, systematic errors that are inherent when the motion of the LDA test point is in the direction of the local measured velocity. Using simple potential flows that have variable curvature and inflection points to simulate an LDA experiment, a streamline-tracing algorithm is developed that minimizes these errors. Also, the test-point path remains close to the correct streamline even when simulated statistical measurement variations are included.

INTRODUCTION

Complex, highly turbulent fluid flows are difficult to measure or visualize experimentally. In general, flow surveys are conducted along straight-line paths to create a grid, or mesh, of locations where the velocity has been measured. Using these data, vector plots or arrow diagrams on the grid can provide a pictorial representation of the flow patterns from which the scientist can, it is hoped, gain insight into the fluid mechanics of the flow.

Where streamline patterns are desired, an alternative approach to vector plots is to measure the mean values of the three components of the velocity at a given location, compute the flow direction, move the measurement probe a short distance in that direction, and repeat the procedure. In this manner, the motion of the probe should trace out the mean streamline. However, in order to accomplish these measurements, the velocity measuring device must be capable of accurately measuring all three velocity components, even in regions of stagnation and reversed or recirculating flow. Whereas hot-wire anemometers and pressure-based devices are not acceptable in this kind of flow, recent advances (refs. 1-3) in three-dimensional, laser-Doppler anemometry (LDA) now make such measurements feasible.

Orloff and Snyder (ref. 4) and Snyder et al. (ref. 5) have reported a three-dimensional LDA technique based on statistics and sampling theory that allows the mean flow direction to be measured within a prescribed accuracy. Having applied the technique, they have shown it to be accurate for constructing vector plots of the mean-flow direction along a survey line in a swirling flow (with recirculation and local turbulence intensities in excess of 50%), and they have attempted to follow a mean streamline in this flow using the stepping method described above. Their conclusion is that the resulting path of the LDA test point is not a true fluid streamline, and they suggest that the discrepancy is not a result of measurement inaccuracies associated with the LDA. They claim instead that the error derives from the fact that the step segments are always straight lines <u>tangent</u> to the local streamline. Hence, for a flow with curvature, the cumulative errors result in a systematic crossing of the streamlines by the test point.

The purpose of this memorandum is to (1) verify that the authors of references 4 and 5 are correct in their explanation of the discrepancy between the true fluid path taken by the LDA focal point; (2) develop an improved algorithm for streamline tracing that reduces the systematic "drift" away from the correct streamline; and (3) test the new algorithm with a numerical simulation using some simple potential flows.

NUMERICAL SIMULATION

A streamline tracing scheme can be simulated numerically using a flow field generated by the superposition of simple potential flows. The simulated motion of the LDA test point is accomplished by starting at a given location, computing the velocity at that location, moving a computed distance along the streamline according to a motion algorithm, and repeating the procedure at each subsequent location.

In the experiment reported by Snyder et al. (ref. 5), the step length was calculated by scaling between a prespecified minimum and maximum value according to an estimate of the local path radius of curvature at the last measured point. A local quadratic fit through this point and through the previous four points was used to determine the radius of curvature. As the radius of curvature decreased, the step size was reduced so that the test point did not deviate excessively from the desired streamline; as the radius of curvature increased, the step size was increased without incurring large positioning errors. The test point was always moved, according to this step size, in the direction of the measured velocity vector.

To most accurately simulate the conditions of the swirling flow measurements reported in reference 5, a numerical flow was constructed from a potential vortex of strength Γ and a potential sink of strength Q. Because this flow has a sustained, but variable, radius of curvature, the cumulative effect of the stepping errors can be evaluated. Also, by adjusting Γ and Q, the velocities and scale can be closely matched to those reported in reference 5.

Figure 1 demonstrates the cumulative error incurred when straight-line motion in the direction of the measured velocity is used. The numerical results are compared with the experimental results of reference 5. Because the experimental flow was only quasi-cylindrical for a short axial distance, only the first 180° of rotation about the center can be realistically compared with the two-dimensional numerical simulation. Notice the tendency of the numerical trace to cross streamlines (fig. 1(a)) and its similarity to the experimental results (fig. 1(b)). More importantly, the numerical reverse-path segments (moving opposite to the velocity direction starting from a point on the forward path) show precisely the same tendency to diverge to the outside of the forward trace as is evidenced in the experimental reverse-path traces. This agreement supports the explanation given by Snyder et al. It follows, therefore, that an improved motion algorithm should produce a more faithful streamline trace.

In addition to radius of curvature errors, the ability to remain on a streamline is strongly influenced by the presence of inflection points along the streamline path. To evaluate this effect, the potential flow around a circular cylinder with circulation will be studied.

Finally, to match the numerical simulation to the LDA measurements, statistical variations in the measured flow direction must be simulated. This is accomplished by using a pseudo-Gaussian probability distribution for selecting direction perturbations between ±10° (the accuracy criterion used for the experimental trace shown in fig. 1). The angular perturbation $\Delta\theta$ is generated using

$$\Delta\theta = 0.245(10°)[1 - e^{-(RND-0.5)^2/3.38}]SGN(RND - 0.5) \tag{1}$$

where RND is a pseudorandom number between 0 and 1, and SGN( ) is a function that returns the sign of the argument. Figure 2 shows a typical probability distribution for 400 consecutive calculations.

STEP DISTANCE

Minimum and maximum stepping distances, SMIN and SMAX, respectively, are chosen, based on experience, appropriate to the flow being studied. The value actually used is calculated within this interval based on the following guidelines.

1. Radius of curvature— For the first five points along a streamline trace, SMIN is used, and the motion is in the direction of the measured velocity. For subsequent points, the radius of curvature R is computed using a local parametric quadratic fit through the current test-point location and the previous four points. The step size is then computed from

$$STEP = SMIN + (SMAX - SMIN)(R/RINF)^{1/2} \tag{2}$$

where RINF is a constant radius that influences the relative rate of variation of the step size with changing radius of curvature. RINF is usually chosen to be a radius at which the curvature is effectively "flat" relative to the scale of the flow.

2. Inflection— In regions of local streamline inflection, the step size must not abruptly increase to SMAX or else large deviations from the streamline may occur. To prevent this, the radius of curvature at the previous location, ROLD, is compared with the radius of curvature at the current location, R, and the ratio ROLD/R is computed. As the test point moves into a region of inflection, the ratio will become less than one, and the step size is decreased to prevent significant deviation from the streamline. This is accomplished by modifying equation (2),

$$STEP = SMIN + (SMAX - SMIN)(R/RINF)^{1/2}(RAT) \tag{3}$$

where RAT is a quantity defined by

$$RAT = (ROLD/R)^2 \quad \text{if} \quad ROLD/R < 1$$

$$RAT = 1 \quad \text{if} \quad ROLD/R \geq 1$$

STEP DIRECTION

The measured flow direction (assumed accurate) will always be tangent to the local streamline; the associated cumulative error that is generated by moving the test point in that direction has been graphically demonstrated in figure 1. In fact, this error will always be present if the flow has any amount of curvature at all. To correct for this error, we note that because the measured velocity is tangent to the local curvature, the test point should instead be moved along a circular arc corresponding to the local flow curvature based on the five-point curve fit.

Figure 3(a) depicts a local streamline section in the Y-Z plane that is concave upward ($d^2z/dy^2 > 0$). The measured velocity is assumed tangent to the streamline at the current measurement location (point 5). Because the flow can be in either direction along the path, two opposing velocity vectors are shown at point 5, and we denote the forward vector at an angle θ_f and the reverse vector at an angle θ_r. The angle δ is the change in the step direction to correct for the tangential motion error. The correction angle δ is given by

$$\delta = \sin^{-1} \frac{\text{STEP}}{2R}$$

Alternatively, the correction may be expressed in Cartesian coordinates $\Delta y'$ and $\Delta z'$ as shown in figure 3(b) and given by

$$\left.\begin{aligned} \Delta y' &= -2(\text{STEP})\sin \frac{\delta}{2} \sin\left(\frac{\delta}{2} + \theta_f\right) \\ \Delta z' &= 2(\text{STEP})\sin \frac{\delta}{2} \cos\left(\frac{\delta}{2} + \theta_f\right) \end{aligned}\right\} \text{forward direction}$$

$$\left.\begin{aligned} \Delta y' &= -2(\text{STEP})\sin \frac{\delta}{2} \sin\left(\frac{\delta}{2} - \theta_r\right) \\ \Delta z' &= -2(\text{STEP})\sin \frac{\delta}{2} \cos\left(\frac{\delta}{2} - \theta_r\right) \end{aligned}\right\} \text{reverse direction}$$

The corrected increments for motion along the path are

$$\Delta y = (\text{STEP})\cos \theta + \Delta y'$$

$$\Delta z = (\text{STEP})\sin \theta + \Delta z'$$

where θ may be either θ_f or θ_r.

In figure 3(a), the slope of the path at point 5 is positive; θ_f lies in the first quadrant, and θ_r is in the third quadrant. If, however, the path is such that point 5 is located in the concave upward region where the slope is negative, then θ_f will be in the fourth quadrant, and θ_r will be in the second quadrant. It can be shown that the general expressions for $\Delta y'$ and $\Delta z'$ that describe all possible combinations of concave-upward or concave-downward flow, and positive or negative slope locations are

$$\begin{aligned} \Delta y' &= -2(\text{STEP})\sin \frac{\delta}{2} \sin\left(\frac{\delta}{2} + \alpha\beta\theta\right) \\ \Delta z' &= 2\alpha\beta(\text{STEP})\sin \frac{\delta}{2} \cos\left(\frac{\delta}{2} + \alpha\beta\theta\right) \end{aligned} \tag{4}$$

where

$$\alpha = \text{SGN}(\cos \theta)$$

$$\beta = \text{SGN}\left(\frac{\dot{y}\ddot{z} - \dot{z}\ddot{y}}{\dot{y}^3}\right)$$

The derivatives shown in the above expression for β denote a parametric differentiation with respect to the point number $n = 1, \ldots, 5$, as indicated in figure 3(a). The parametric description must be used to prevent the curve-fitting routine from encountering double-valued functions in the calculation of the radius of curvature; this can occur as the streamline transitions between concave upward and concave downward.

It should be noted that $\vec{V}$ can only remain precisely tangent to the predicted curvature where the streamlines are circular. Generally, as the curvature varies, $\vec{V}$ will tend to follow the streamline, and the correction δ will cause the trace to lag slightly behind, thereby keeping the test-point motion from reacting too quickly to changes in curvature. Moreover, for the experimental measurements, statistical variations in the velocity direction will also be somewhat damped by the δ correction.

RESULTS

The ability of the radius-of-curvature-correction algorithm to compensate for the error inherent in tangential stepping is shown in figure 4 for the potential flow presented earlier in figure 1. The correction is quite good, and the only noticeable error occurs at the beginning of the trace where the first five steps are tangential before the quadratic curve fit can be used to compute the local radius of curvature. After that, the streamline is followed without any obvious error.

To model more clearly the numerical simulation to experimental (LDA) procedure, statistical variation can be added to the velocity computation at each location according to the probability distribution shown in figure 2. Figure 5 presents the results of three consecutive traces that begin at the same location in the flow. The effects of statistical variation are random, and the "average" path appears to follow the streamline faithfully. Additionally, the figure suggests that the deviation of the trace from the original streamline lies within a "probability band" that increases with increasing trace length. Nevertheless, the path is free of systematic, cumulative errors and is a reasonable representation of the true streamline.

Figure 6 presents the results of simulated streamline tracing in a potential flow where the path goes through moderate curvature and two points of inflection. Although the corrected path is not exact, it is significantly better than the path that results from tangential stepping. Most noticeable is the deviation that occurs for the uncorrected path along the second half of the trace. This is a result of the convergence of the streamlines as the flow accelerates over the top of the cylinder. In this region, any small spatial error in the stepping motion is amplified because it represents a large percentage of the distance between streamlines; subsequently, as the trace proceeds downstream (where the streamlines are farther apart) the spatial extent of the error becomes more pronounced. In view of this, the accuracy of the corrected stepping is quite reasonable.

The major effect of adding statistical variation in the computed velocity to the circular cylinder flow (fig. 7) is the rapid broadening (owing to expanding streamlines) of the "probability band." The fact that inflection is present does not appear to affect the accuracy of the trace or to create any systematic errors.

CONCLUDING REMARKS

The streamline-tracing algorithm developed in this memorandum is promising as a workable scheme for using a three-dimensional LDA instrument to follow mean-flow streamlines. Although not exact, the inherent inaccuracies appear to be masked by the presence of statistical variations in the measurement of the flow direction; hence, the algorithm should be adequate for experimental purposes.

Final verification of the algorithm will come when the technique is applied to a real flow using the three-dimensional LDA. The degree to which the streamline traces are found to be statistically similar to those presented in figures 5 and 7, as well as the reversibility of the trace without systematic drift, will be a measure of the success of the algorithm.

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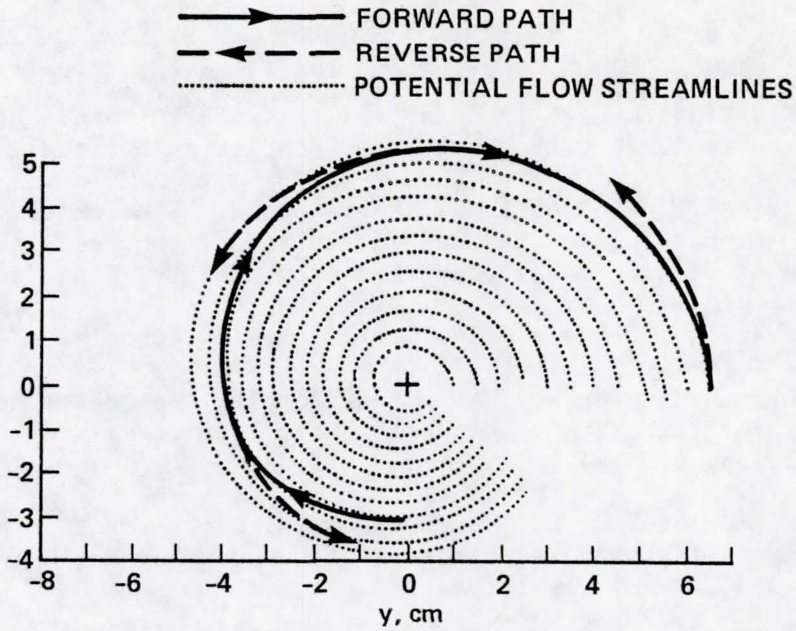
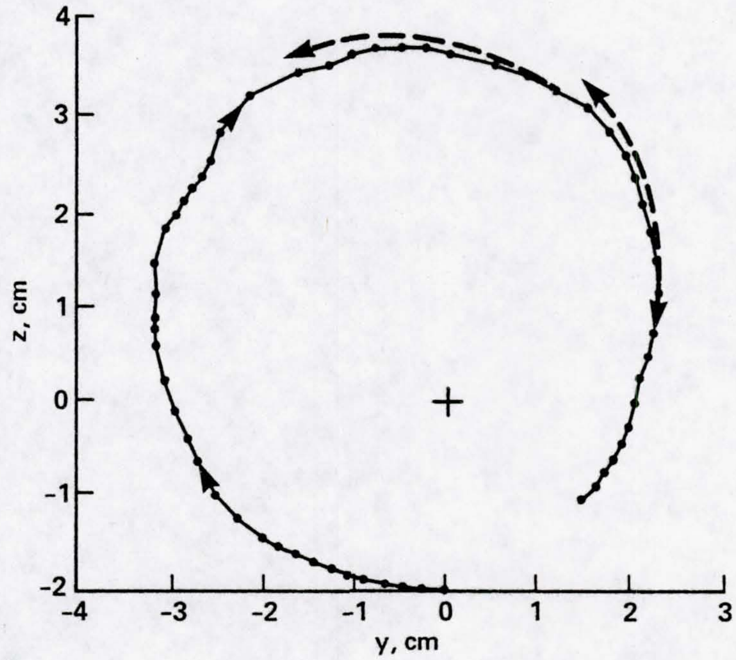


(a) Numerical simulation in a potential flow; minimum step size 0.3 cm; maximum step size 0.8 cm.



(b) Experimental 3-D LDA results from reference 5.

Figure 1.- Results of straight-line stepping in the direction of the velocity vector.

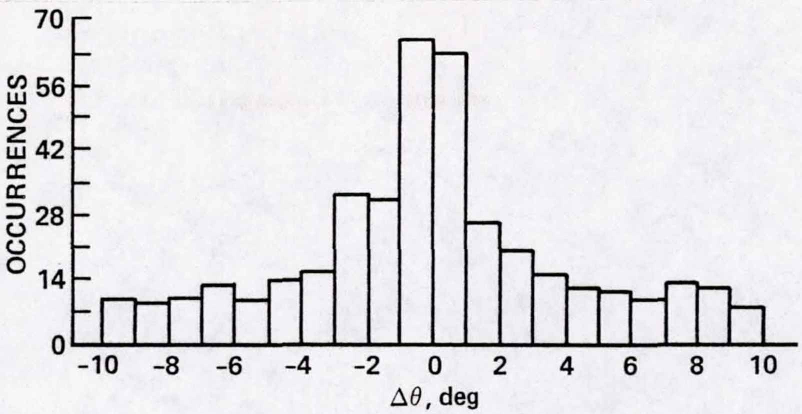
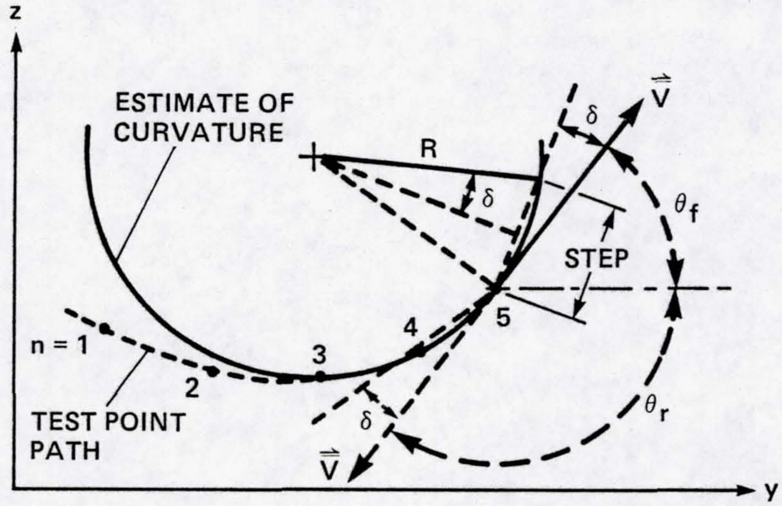
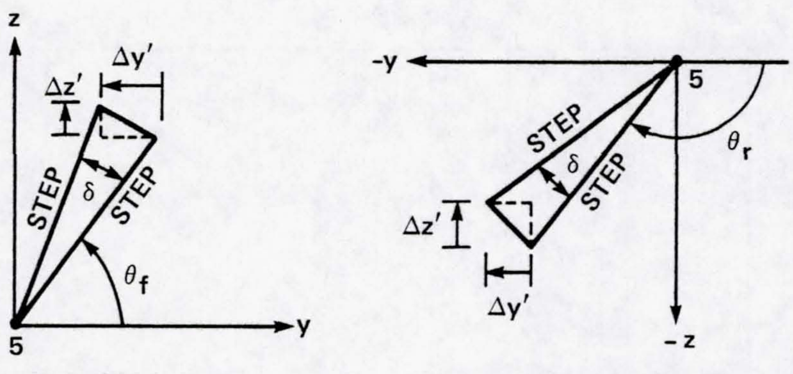


Figure 2.- Histogram showing the probability distribution for angular perturbations used in numerical simulation for a streamline tracing accuracy of 10° (400 consecutive calculations).



(a) Concave-upward flow; δ is correction angle, R is estimated radius of curvature at point 5.



(b) Cartesian corrections $\Delta y'$ and $\Delta z'$ for forward- and reverse-flow directions at point 5.

Figure 3.- Correction scheme used to reduce error created by tangential stepping.

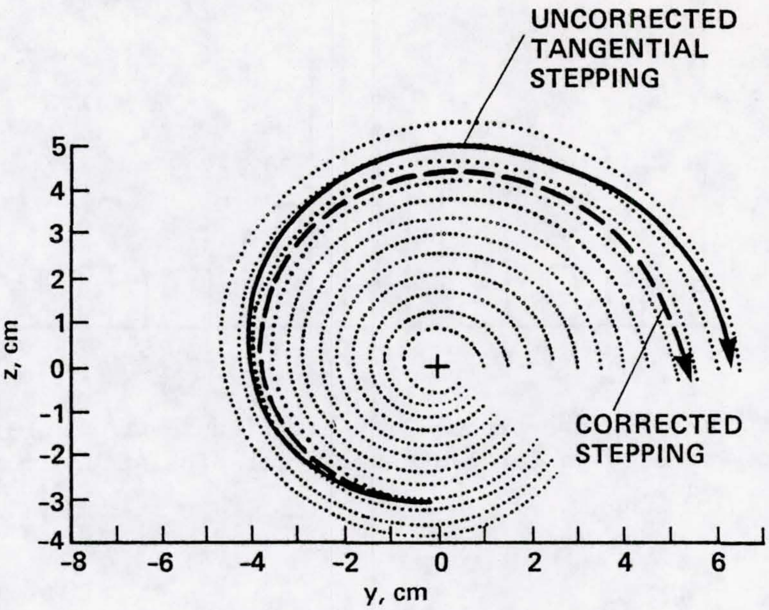


Figure 4.- Application of the radius-of-curvature correction algorithm (eq. (4)) to a potential flow; uncorrected result is shown for comparison: SMIN = 0.2 cm; SMAX = 0.6 cm.

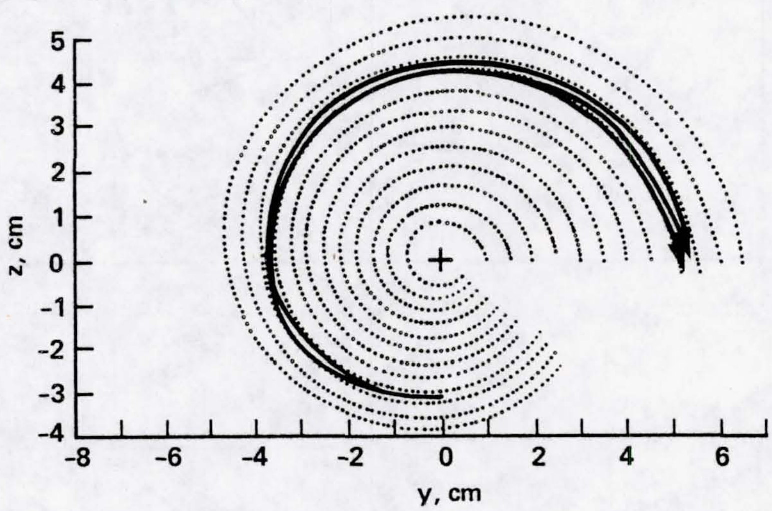


Figure 5.- Simulation of streamline tracing with radius-of-curvature correction algorithm and statistical variation in flow direction showing three traces beginning at same location: SMIN = 0.2 cm; SMAX = 0.6 cm.

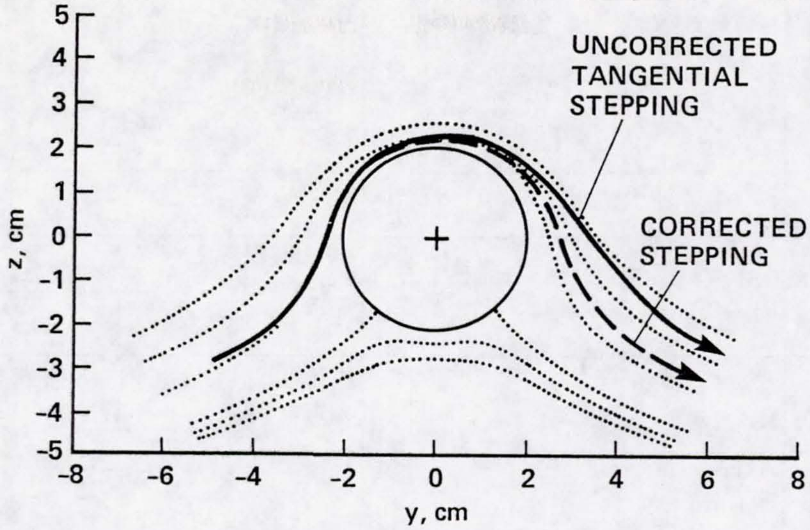


Figure 6.- Using radius-of-curvature algorithm to follow streamlines in a simulated potential flow with local inflection; uncorrected result is shown for comparison: SMIN = 0.2 cm; SMAX = 0.6 cm.

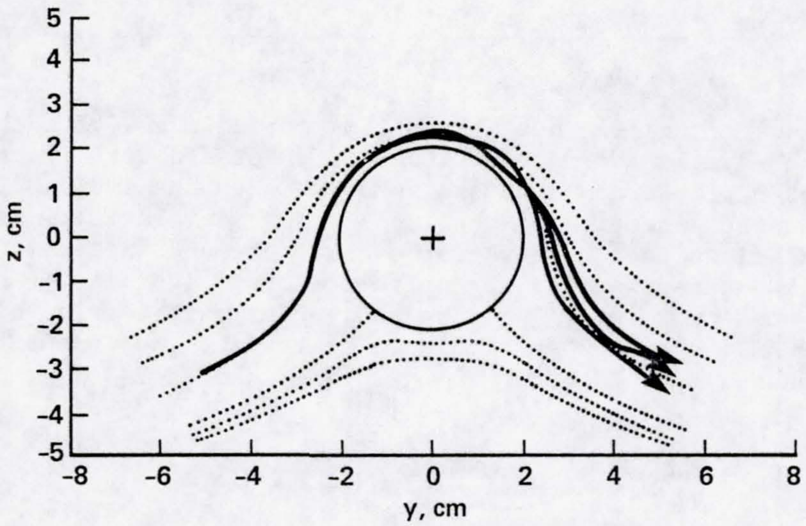


Fig. 7.- Effect of statistical variation in the flow direction showing three traces beginning at the same location in a flow with streamline inflection: SMIN = 0.2 cm; SMAX = 0.6 cm.

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16. Abstract

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