ACTIVE STABILIZATION OF A FLEXIBLE ANTENNA FEED TOWER

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ABSTRACT

Active stabilization logic is synthesized to hold a feed at the focus of a spacecraft antenna dish. The feed support structure is modeled as a tetrahedron made up of flexible bars and connected to the dish by six short legs containing force actuators. Using the symmetry of the structure, the model can be decomposed into four uncoupled subsystems: (1) pitch/forward motions with four degrees of freedom (DOF) and two controls, (2) roll/lateral motions with four DOF and two controls, (3) vertical motions with three DOF and one control, and (4) yaw motion with one DOF and one control. This greatly simplifies the synthesis of control logic.

INTRODUCTION

A spacecraft consists of a massive central body with a large antenna dish at one end; the feed for this antenna is mounted to the dish with a flexible support structure consisting of twelve bar-like members. (See fig. 1.) Six of the bars form a regular tetrahedron, with the feed at the apex. Two legs connect each of the three joints at the base of the tetrahedron to the antenna-dish/spacecraft, which we shall approximate as an inertial frame of reference due to its large mass. The mass of the structure will be lumped at the four joints of the tetrahedron, and the bars will be approximated as springs with axial deformation only.

The design objective is to control the four lowest frequency vibration modes that involve lateral motions of the feed so that they are at least 10 percent critically damped.

SEPARATION INTO SYMMETRIC AND ANTISYMMETRIC MOTIONS

Motions symmetric with respect to y-z plane involve seven degrees of freedom:

$$y_1, \ z_1, \ x_2 = -x_3, \ \ y_2 = y_3, \ \ z_2 = z_3, \ y_4, \ z_4$$

Motions antisymmetric with respect to y-z plane involve five degrees of freedom:

$$x_1, \ x_2 = x_3, \ \ y_2 = -y_3, \ \ z_2 = z_3, \ x_4$$

Three of the symmetric modes involve only vertical (z_1) motions of the apex, and one antisymmetric mode is symmetric about the z axis (a yaw mode), producing zero motion of the apex. The remaining eight modes consist of two sets of four modes that have identical frequencies, but one set involves symmetric motions and the other set involves antisymmetric motions.

The actuator forces can be arranged into six sets, one of which controls only the yaw mode, another that controls only z_1 motions, and two sets of two that control the remaining symmetric and antisymmetric modes, respectively. Thus the stabilization problem may be reduced to two almost identical problems of controlling four modes with two controls.

EQUATIONS OF MOTION

Let $\vec{r}_i$ be displacement vector of ith joint, $\vec{m}_{ij}$ be position vector from ith joint to jth joint, and k_1, k_2, k_ℓ be equal to EA/mL^3 for base members, vertical members, and legs, respectively. Then

$$\ddot{\vec{r}}_1 = -k_2\vec{m}_{1,2}\vec{m}_{1,2}\cdot(\vec{r}_1 - \vec{r}_2) - k_2\vec{m}_{1,3}\vec{m}_{1,3}\cdot(\vec{r}_1 - \vec{r}_3) - k_2\vec{m}_{1,4}\vec{m}_{1,4}\cdot(\vec{r}_1 - \vec{r}_4)$$

$$\ddot{\vec{r}}_2 = -k_2\vec{m}_{1,2}\vec{m}_{1,2}\cdot(\vec{r}_2 - \vec{r}_1) - k_1\vec{m}_{2,3}\vec{m}_{2,3}\cdot(\vec{r}_2 - \vec{r}_3) - k_1\vec{m}_{2,4}\vec{m}_{2,4}\cdot(\vec{r}_2 - \vec{r}_4)$$

$$- k_\ell\vec{m}_{2,5}\vec{m}_{2,5}\cdot\vec{r}_2 - k_\ell\vec{m}_{2,6}\vec{m}_{2,6}\cdot\vec{r}_2 + \vec{m}_{2,5}f_{2,5} + \vec{m}_{2,6}f_{2,6}$$

$$\ddot{\vec{r}}_3 = -k_2\vec{m}_{1,3}\vec{m}_{1,3}\cdot(\vec{r}_3 - \vec{r}_1) - k_1\vec{m}_{2,3}\vec{m}_{2,3}\cdot(\vec{r}_3 - \vec{r}_2) - k_1\vec{m}_{3,4}\vec{m}_{3,4}\cdot(\vec{r}_3 - \vec{r}_4)$$

$$- k_\ell\vec{m}_{3,7}\vec{m}_{3,7}\cdot\vec{r}_{3,7} - k_\ell\vec{m}_{3,8}\vec{m}_{3,8}\cdot\vec{r}_3 + \vec{m}_{3,7}f_{3,7} + \vec{m}_{3,8}f_{3,8}$$

$$\ddot{\vec{r}}_4 = -k_2\vec{m}_{1,4}\vec{m}_{1,4}\cdot(\vec{r}_4 - \vec{r}_1) - k_1\vec{m}_{2,4}\vec{m}_{2,4}\cdot(\vec{r}_4 - \vec{r}_2) - k_1\vec{m}_{3,4}\vec{m}_{3,4}\cdot(\vec{r}_4 - \vec{r}_3)$$

$$- k_\ell\vec{m}_{4,9}\vec{m}_{4,9}\cdot\vec{r}_4 - k_\ell\vec{m}_{4,10}\vec{m}_{4,10}\cdot\vec{r}_4 + \vec{m}_{4,9}f_{4,9} + \vec{m}_{4,10}f_{4,10}$$

For nominal configuration,

$$k_1 = \frac{(1)(1000)}{(2)(10)^3} = 0.5$$

$$k_2 = \frac{(1)(100)}{(2)(10)^3} = 0.05$$

$$k_\ell = \frac{(1)(100)}{(2)(2\sqrt{2})^3} = 2.2097$$

COMPUTER CODE "TETRA"

Calculates 12×12 K matrix, where

$$\frac{d^2}{dt^2}\begin{bmatrix} r_1 \\ \cdots \\ r_2 \\ \cdots \\ r_3 \\ \cdots \\ r_4 \end{bmatrix} = -K\begin{bmatrix} r_1 \\ \cdots \\ r_2 \\ \cdots \\ r_3 \\ \cdots \\ r_4 \end{bmatrix} + Gf$$

Calculates 7×7 K_S matrix and 5×5 K_A matrix, where

$$\ddot{d}_S = -K_S d_S + G_S f_S$$

$$\ddot{d}_A = -K_A d_A + G_A f_A$$

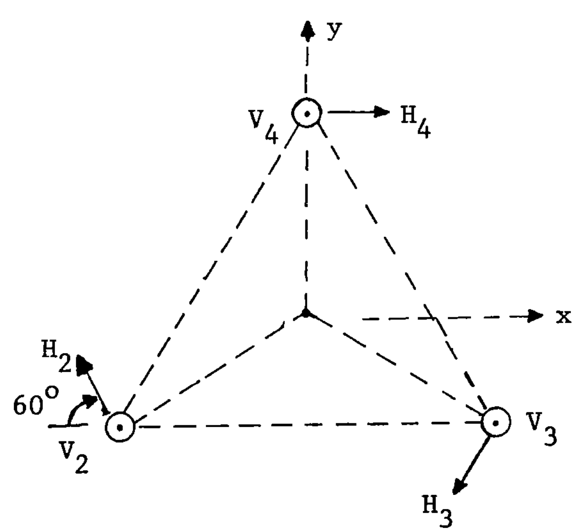
$$d_S \triangleq \left[y_1,\ z_1,\ \frac{x_2 - x_3}{2},\ \frac{y_2 + y_3}{2},\ \frac{z_2 + z_3}{2},\ y_4,\ z_4\right]^T$$

$$d_A \triangleq \left[x_1,\ \frac{x_2 + x_3}{2},\ \frac{y_2 - y_3}{2},\ \frac{z_2 - z_3}{2},\ x_4\right]^T$$

$$f_S \triangleq \left[\frac{H_2 - H_3}{2},\ \frac{V_2 + V_3}{2},\ V_4\right]^T$$

$$f_A \triangleq \left[\frac{H_2 + H_3}{2},\ \frac{V_2 - V_3}{2},\ H_4\right]^T$$

Note actuator forces resolved into vertical and horizontal components (V_i, H_i) at joints $i = 2,3,4$, which makes the determination of G_S, G_A quite simple.



```
10 REM ***** TETRA **** 1/19/82 *****
20 REM * FINDS STIFFNESS MATRIX GIVEN
30 REM * JOINT COORDINATES & MEMBER
40 REM * STIFFNESSES FOR TETRAHEDRON
50 REM * ON LEGS; THEN FINDS STIFFNESS
60 REM * MATRICES FOR SYMMETRIC &
70 REM * ANTI-SYMMETRIC MOTIONS.
80 REM ******************************
90 DIMX(10,3),M(4,10,3),K(12,12):FORI=1TO10:FORJ=1TO3:READX(I,J):NEXTJ,I
100 READK1,K2,KL:DEFFNR(S)=INT(10000*S+.5)/10000
110 FORI=1TO4:FORJ=1TO10:FORK=1TO3:M(I,J,K)=X(I,K)-X(J,K):NEXTK,J,I
120 FORI=1TO3:FORJ=4TO6:K(I,J)=M(1,2,I)*M(1,2,J-3)*K2:NEXTJ,I
130 FORI=1TO3:FORJ=7TO9:K(I,J)=M(1,3,I)*M(1,3,J-6)*K2:NEXTJ,I
140 FORI=1TO3:FORJ=10TO12:K(I,J)=M(1,4,I)*M(1,4,J-9)*K2:NEXTJ,I
150 FORI=4TO6:FORJ=7TO9:K(I,J)=M(2,3,I-3)*M(2,3,J-6)*K1:NEXTJ,I
160 FORI=4TO6:FORJ=10TO12:K(I,J)=M(2,4,I-3)*M(2,4,J-9)*K1:NEXTJ,I
170 FORI=7TO9:FORJ=10TO12:K(I,J)=M(3,4,I-6)*M(3,4,J-9)*K1:NEXTJ,I
180 FORI=1TO3:FORJ=4TO12:K(J,I)=K(I,J):NEXTJ,I
190 FORI=4TO6:FORJ=7TO12:K(J,I)=K(I,J):NEXTJ,I
200 FORI=7TO9:FORJ=10TO12:K(J,I)=K(I,J):NEXTJ,I
210 FORI=1TO3:FORJ=1TO3:K(I,J)=-K(I,J+3)-K(I,J+6)-K(I,J+9):NEXTJ,I
220 FORI=4TO6:FORJ=4TO6:K(I,J)=-K(I,J-3)-K(I,J+3)-K(I,J+6):NEXTJ,I
230 FORI=4TO6:FORJ=4TO6:K(I,J)=K(I,J)-M(2,5,I-3)*M(2,5,J-3)*KL:NEXTJ,I
240 FORI=4TO6:FORJ=4TO6:K(I,J)=K(I,J)-M(2,6,I-3)*M(2,6,J-3)*KL:NEXTJ,I
250 FORI=7TO9:FORJ=7TO9:K(I,J)=-K(I,J-6)-K(I,J-3)-K(I,J+3):NEXTJ,I
260 FORI=7TO9:FORJ=7TO9:K(I,J)=K(I,J)-M(3,7,I-6)*M(3,7,J-6)*KL:NEXTJ,I
270 FORI=7TO9:FORJ=7TO9:K(I,J)=K(I,J)-M(3,8,I-6)*M(3,8,J-6)*KL:NEXTJ,I
280 FORI=10TO12:FORJ=10TO12:K(I,J)=-K(I,J-9)-K(I,J-6)-K(I,J-3):NEXTJ,I
290 FORI=10TO12:FORJ=10TO12:K(I,J)=K(I,J)-M(4,9,I-9)*M(4,9,J-9)*KL:NEXTJ,I
300 FORI=10TO12:FORJ=10TO12:K(I,J)=K(I,J)-M(4,10,I-9)*M(4,10,J-9)*KL:NEXTJ,I
310 PRINTTAB(10);"STIFFNESS/MASS MATRIX, UPPER LEFT QUADRANT"
320 FORI=1TO6:PRINTTAB(10);:FORJ=1TO6:PRINTFNR(K(I,J));:NEXTJ:PRINT:NEXTI
330 PRINTTAB(10);"UPPER RIGHT QUADRANT":FORI=1TO6:PRINTTAB(10);:FORJ=7TO12
335 PRINTFNR(K(I,J));:NEXTJ:PRINT:NEXTI
340 PRINTTAB(10);"LOWER RIGHT QUADRANT"
350 FORI=7TO12:PRINTTAB(10);:FORJ=7TO12:PRINTFNR(K(I,J));:NEXTJ:PRINT:NEXTI
360 REM *** CALCULATES ANTI-SYMMETRIC, SYMMETRIC STIFFNESS MATRICES: ***
370 DIMT(12,12),TI(12,12),L(12,12),L1(12,12):C=.5:T(1,1)=1:T(2,6)=1
380 T(3,7)=1:T(4,2)=1:T(4,8)=1:T(5,3)=1:T(5,9)=1:T(6,4)=1:T(6,10)=1:T(7,2)=1
390 T(7,8)=-1:T(8,9)=1:T(8,3)=-1:T(9,10)=1:T(9,4)=-1:T(10,5)=1:T(11,11)=1
400 T(12,12)=1:TI(1,1)=1:TI(2,4)=C:TI(2,7)=C:TI(3,5)=C:TI(3,8)=-C:TI(4,6)=C
410 TI(4,9)=-C:TI(5,10)=1:TI(6,2)=1:TI(7,3)=1:TI(8,4)=C:TI(8,7)=-C:TI(9,5)=C
420 TI(9,8)=C:TI(10,6)=C:TI(10,9)=C:TI(11,11)=1:TI(12,12)=1
430 FORI=1TO12:FORJ=1TO12:FORK=1TO12:L1(I,J)=L1(I,J)+K(I,K)*T(K,J):NEXTK,J,I
440 FORI=1TO12:FORJ=1TO12:FORK=1TO12:L(I,J)=L(I,J)+TI(I,K)*L1(K,J):NEXTK,J,I
450 PRINTTAB(10);"ANTI-SYMMETRIC STIFFNESS/MASS MATRIX:"
460 FORI=1TO5:PRINTTAB(10);:FORJ=1TO5:PRINTFNR(L(I,J));:NEXTJ:PRINT:NEXTI
470 PRINTTAB(10);"CROSS-COUPLING MATRIX:"
480 FORI=1TO5:PRINTTAB(10);:FORJ=6TO12:PRINTFNR(L(I,J));:NEXTJ:PRINT:NEXTI
490 PRINTTAB(10);"SYMMETRIC STIFFNESS/MASS MATRIX:"
500 FORI=6TO12:PRINTTAB(10);:FORJ=6TO12:PRINTFNR(L(I,J));:NEXTJ:PRINT:NEXTI
510 PRINTTAB(10);"CROSS-COUPLING MATRIX:"
520 FORI=6TO12:PRINTTAB(10);:FORJ=1TO5:PRINTFNR(L(I,J));:NEXTJ:PRINT:NEXTI:END
530 REM *** ENTER X,Y,Z COORDINATES OF JOINTS 1 THRU 10: ***
540 DATA 0,0,10.165,-5,-2.887,2,5,-2.887,2,0,5.7735,2,-6,-1.1547,0
550 DATA -4,-4.6188,0,4,-4.6188,0,6,-1.1547,0,2,5.7735,0,-2,5.7735,0
560 REM *** ENTER STIFFNESS/MASS FOR HEAVY MEMBERS, LIGHT MEMBERS, & LEGS: ***
570 DATA .5,.05,2.2097
READY.
```

PRINT-OUT FROM "TETRA"

```
STIFFNESS/MASS MATRIX, UPPER LEFT QUADRANT
-2.5  0  0  1.25  .7218  2.0413
 0 -2.5001 -2E-04  .7218  .4167  1.1786
 0 -2E-04 -10.0001  2.0413  1.1786  3.3334
 1.25  .7218  2.0413 -68.1694 -14.7184 -2.0413
 .7218  .4167  1.1786 -14.7184 -51.1771 -1.1764
 2.0413  1.1786  3.3334 -2.0413 -1.1764 -21.011
UPPER RIGHT QUADRANT
 1.25 -.7218 -2.0413  0  0  0
-.7218  .4167  1.1786  0  1.6667 -2.357
-2.0413  1.1786  3.3334  0 -2.357  3.3334
 50  0  0  12.5  21.6513  0
 0  0  0  21.6513  37.5021  0
 0  0  0  0  0  0
LOWER RIGHT QUADRANT
-68.1694  14.7184  2.0413  12.5 -21.6513  0
 14.7184 -51.1771 -1.1764 -21.6513  37.5021  0
 2.0413 -1.1764 -21.011  0  0  0
 12.5 -21.6513  0 -42.6776  0  0
-21.6513  37.5021  0  0 -76.6709  2.357
 0  0  0  0  2.357 -21.011

ANTI-SYMMETRIC STIFFNESS/MASS MATRIX:
-2.5  2.5  1.4435  4.0825  0
 1.25 -18.1694 -14.7184 -2.0413  12.5
 .7218 -14.7184 -51.1771 -1.1764  21.6513
 2.0413 -2.0413 -1.1764 -21.011  0
 0  25  43.3025  0 -42.6776
CROSS-COUPLING MATRIX:
 0  0  0  0  0  0  0
 0  0  0  0  0  0  0
 0  0  0  0  0  0  0
 0  0  0  0  0  0  0
 0  0  0  0  0  0  0
SYMMETRIC STIFFNESS/MASS MATRIX:
-2.5001 -2E-04  1.4435  .8335  2.3572  1.6667 -2.357
-2E-04 -10.0001  4.0825  2.3572  6.6667 -2.357  3.3334
 .7218  2.0413 -118.1694 -14.7184 -2.0413  21.6513  0
 .4167  1.1786 -14.7184 -51.1771 -1.1764  37.5021  0
 1.1786  3.3334 -2.0413 -1.1764 -21.011  0  0
 1.6667 -2.357  43.3025  75.0043  0 -76.6709  2.357
-2.357  3.3334  0  0  0  2.357 -21.011
CROSS-COUPLING MATRIX:
 0  0  0  0  0
 0  0  0  0  0
 0  0  0  0  0
 0  0  0  0  0
 0  0  0  0  0
 0  0  0  0  0
 0  0  0  0  0
```

EQUATIONS OF MOTION IN MODAL FORM

Using computer code "MODALSYS", the symmetric and antisymmetric equations of motion were put into modal form. Sketches of these mode shapes are given in figures 2 through 4. Only four modes involve fore-aft (y_1) motions of the apex (fig. 2). Another four modes involve only lateral (x_1) motions of the apex (fig. 3). Another three modes involve only vertical (z_1) motions of the apex (fig. 4). One mode involves no motion of the apex (fig. 5).

SEPARATION INTO FOUR SUBSYSTEMS

Only two linear combinations of actuator forces enter into the y_1 apex motions. (See first example of modal controllability matrix.) We shall call them f_{pitch} and f_{fwd}. Two different linear combinations of actuator forces enter into the x_1 apex motions. (See second example of modal controllability matrix.) They will be referred to as f_{roll} and f_{flat}. One different linear combination of actuator forces enters into the z_1 apex motions. It is called f_{vert}. One different linear combination of actuator forces involves no apex motion, and is called f_{yaw}. The equations of motion for these four subsystems are given elsewhere in this paper.

ANALYSIS OF TETRAHEDRON WITH CONSTRAINED MOTION

Symmetric Tetrahedron

Constraints

$$x_1 = x_4 = 0,\ x_3 = -x_2,\ y_3 = y_2,\ z_3 = z_2$$

$$h_3 = -h_2,\ v_3 = v_2,\ h_4 = 0$$

System equations

$$x = (y_1,\ z_1,\ x_2,\ y_2,\ z_2,\ y_4,\ z_4)$$

$$u = (H_2,\ v_2,\ v_4)$$

$$y = (y_1,\ y_2)$$

$$x = Fx + Gu + G_A v$$

$$y = Hx$$

where

```
Units m ,sec

Dynamics Matrix F, is:
- 2.500 -   .000 + 1.443 +   .833 + 2.357 + 1.666 - 2.357
-   .000 -10.000 + 4.082 + 2.357 + 6.666 - 2.357 + 3.333
+   .721 + 2.041 -**.*** -14.718 - 2.041 +21.651 +   .000  (-118.1694)
+   .416 + 1.178 -14.718 -51.177 - 1.176 +37.502 +   .000
+ 1.178 + 3.333 - 2.041 - 1.176 -21.011 +   .000 +   .000
+ 1.666 - 2.357 +43.302 +75.004 +   .000 -76.670 + 2.357
- 2.357 + 3.333 +   .000 +   .000 +   .000 + 2.357 -21.011
Control Distribution Matrix, G, is:
+   .000 +   .000 +   .000
+   .000 +   .000 +   .000
-   .500 +   .000 +   .000
+   .866 +   .000 +   .000
+   .000 + 1.000 +   .000
+   .000 +   .000 +   .000
+   .000 +   .000 + 1.000
Feedback Gain Matrix C, is:
+   .000 +   .000 +   .000 +   .000 +   .000 +   .000 +   .000
+   .000 +   .000 +   .000 +   .000 +   .000 +   .000 +   .000
+   .000 +   .000 +   .000 +   .000 +   .000 +   .000 +   .000
Output Distribution Matrix, H, is:
+ 1.000 +   .000 +   .000 +   .000 +   .000 +   .000 +   .000
+   .000 + 1.000 +   .000 +   .000 +   .000 +   .000 +   .000
Disturbance Distribution Matrix GA, is:
+ 1.000 +   .000
+   .000 + 1.000
+   .000 +   .000
+   .000 +   .000
+   .000 +   .000
+   .000 +   .000
+   .000 +   .000
```

Modal Analysis

Eigenvalues are:

Real	Imaginary	Mode no.
-151.8372 +	.0000	12
- 85.5752 +	.0000	10
- 23.3724 +	.0000	9
- 21.7350 +	.0000	7
- 8.7456 +	.0000	4
- 7.4730 +	.0000	3
- 1.8009 +	.0000	1

Eigenvector Matrix, T, is:

y_1	+ .0000	- .0317	+ .0000	+ .1979	- .2629	- .0000	+1.0000	m_{12}
z_1	+ .0512	- .0000	- .7307	- .0001	- .0008	+1.0000	- .0000	m_{10}
x_2	- .8659	+ .9673	- .0275	+ .0057	+ .0832	+ .0107	+ .0165	m_9
y_2	- .5000	- .6753	- .0160	- .1190	+ .8557	+ .0064	+ .0997	m_7
z_2	- .0193	+ .0188	+ .9994	- .4997	- .1214	+ .2440	+ .0534	m_4
y_4	+1.0000	+1.0000	+ .0316	- .1090	+1.0000	- .0120	+ .1283	m_3
z_4	- .0193	- .0376	+1.0000	+1.0000	+ .2424	+ .2441	- .1069	m_1

Inverse of the Eigenvector Matrix is:

```
+ .0000 + .0170 - .5766 - .3329 - .0128 + .3329 - .0064
- .0083 + .0000 + .5109 - .3566 + .0099 + .2640 - .0099
- .0000 - .2067 - .0155 - .0090 + .5656 + .0089 + .2826
+ .1253 + .0000 + .0072 - .1507 - .6334 - .0690 + .6331
- .0997 - .0003 + .0631 + .6493 - .0921 + .3793 + .0920
- .0000 + .8480 + .0182 + .0109 + .4138 - .0102 + .2070
+ .9486 - .0000 + .0313 + .1892 + .1014 + .1218 - .1014
```

The Modal Dynamics Matrix F_q, Inv(T)*F*T, is:

```
-*.**** + .0000 + .0000 + .0000 + .0005 + .0000 + .0000   (-151.8)
+ .0000 -*.**** - .0000 - .0000 + .0005 + .0000 + .0000   (-85.57)
+ .0000 + .0000 -*.**** - .0003 + .0000 - .0002 + .0000   (-23.37)
+ .0000 + .0000 + .0003 -*.**** + .0002 + .0000 - .0000   (-21.73)
+ .0000 + .0000 + .0000 + .0000 -8.7466 + .0000 + .0000
+ .0000 + .0000 - .0002 - .0002 - .0000 -7.4721 + .0000
+ .0000 + .0000 - .0000 - .0000 - .0002 + .0000 -1.8009
```

The Modal Controllability Matrix G_q, Inv(T)*G, is:

```
- .0000 - .0128 - .0064
- .5643 + .0099 - .0099
- .0000 + .5656 + .2826
- .1341 - .6334 + .6331
+ .5307 - .0921 + .0920
+ .0004 + .4138 + .2070
+ .1482 + .1014 - .1014
```

The Modal Observability Matrix H_q, H*T, is:

```
+ .0000 - .0317 + .0000 + .1979 - .2629 - .0000 +1.0000
+ .0512 - .0000 - .7307 - .0001 - .0008 +1.0000 - .0000
```

The Modal Disturbability Matrix G_{Aq}, Inv(T)*(GA), is:

```
+ .0000 + .0170
- .0083 + .0000
- .0000 - .2067
+ .1253 + .0000
- .0997 - .0003
- .0000 + .8480
+ .9486 - .0000
```

Antisymmetric Tetrahedron

Constraints

$x_3 = x_2$, $y_3 = -y_2$, $z_3 = -z_2$, $z_1 = 0$, $y_1 = 0$, $y_4 = 0$, $z_4 = 0$,

$h_3 = h_2$, $v_3 = -v_2$, $v_4 = 0$

System equations

$$x = (x_1, x_2, y_2, z_2, x_4)$$

$$u = (h_2, v_2, h_4)$$

Modal amplitudes

$$q = [m_{12},\ m_{10},\ m_9,\ m_7,\ m_4,\ m_3,\ m_1]$$

$$\ddot{q} = F_q q + G_q u + G_{Aq} v$$

$$y = H_q q$$

where

```
Units m ,sec

Dynamics Matrix F, is:
- 2.500 + 2.500 + 1.443 + 4.082 +  .000
+ 1.250 -18.169 -14.718 - 2.041 +12.500
+  .721 -14.718 -51.177 - 1.176 +21.651
+ 2.041 - 2.041 - 1.176 -21.011 +  .000
+  .000 +25.000 +43.302 +  .000 -42.677
Control Distribution Matrix, G, is:
+  .000 +  .000 +  .000
-  .500 +  .000 +  .000
+  .866 +  .000 +  .000
+  .000 + 1.000 +  .000
+  .000 +  .000 + 1.000
Feedback Gain Matrix C, is:
+  .000 +  .000 +  .000 +  .000 +  .000
+  .000 +  .000 +  .000 +  .000 +  .000
+  .000 +  .000 +  .000 +  .000 +  .000
Output Distribution Matrix, H, is:
+ 1.000 +  .000 +  .000 +  .000 +  .000
Disturbance Distribution Matrix GA, is:
+ 1.000
+  .000
+  .000
+  .000
+  .000

Eigenvalues are:
   Real          Imaginary      Mode no.
- 85.5753 +    .0000              11
- 21.7350 +    .0000               8
- 17.6775 +    .0000               6
-  8.7462 +    .0000               5
-  1.8009 +    .0000               2
Eigenvector Matrix, T, is:
x1  + .0256 - .2286 - .0001 - .2761 +1.0000   m11
x2  - .3579 + .1300 - .4998 +1.0000 + .1188   m8
y2  - .7840 - .0068 + .8659 + .0874 + .0165   m6
z2  - .0264 +1.0000 + .0004 - .2207 + .0926   m5
z4  +1.0000 + .1409 +1.0000 + .8483 + .0901   m2
Inverse of the Eigenvector Matrix is:
+ .0103 - .2877 - .6303 - .0212 + .4019
- .1085 + .1234 - .0065 + .9496 + .0669
```

```
- .0000 - .3333 + .5774 + .0002 + .3334
- .0949 + .6875 + .0601 - .1518 + .2916
+ .9487 + .2254 + .0313 + .1757 + .0855
The Modal Dynamics Matrix, Inv(T)*F*T, is:
-*.**** + .0000 + .0000 + .0000 + .0000     (-85.58)
+ .0000 -*.**** + .0000 + .0000 + .0000     (-21.73)
+ .0000 + .0000 -*.**** + .0000 + .0000     (-17.68)
+ .0000 + .0000 + .0000 -8.7462 + .0000
+ .0000 + .0000 + .0000 + .0000 -1.8009
The Modal Controllability Matrix, Inv(T)*G, is:
- .4020 - .0212 + .4019
- .0673 + .9496 + .0669
+ .6667 + .0002 + .3334
- .2917 - .1518 + .2916
- .0855 + .1757 + .0855
The Modal Observability Matrix, H*T, is:
+ .0256 - .2286 - .0001 - .2761 +1.0000
The Modal Disturbability Matrix Inv(T)*(GA), is:
+ .0103
- .1085
- .0000
- .0949
+ .9487
Residues;All Outputs, Ctrl 1,Then Ctrl 2, etc.
- .0103 + .0154 - .0000 + .0805 - .0855
- .0005 - .2171 + .0000 + .0419 + .1757
+ .0103 - .0153 - .0000 - .0805 + .0855
Residues;All Outputs, Dist. 1, Then Dist. 2, etc.
+ .0002 + .0248 + .0000 + .0262 + .9487
```

PITCH/FORWARD TRANSLATION SUBSYSTEM

$$\ddot{m}_1 = -(1.342)^2 m_1 + 0.1907 f_p$$

$$\ddot{m}_4 = -(2.957)^2 m_4 + 0.6652 f_F$$

$$\ddot{m}_7 = -(4.662)^2 m_7 - 0.9848 f_p + 0.7914 f_F$$

$$\ddot{m}_{10} = -(9.251)^2 m_{10} - 0.1320 f_p - 0.5788 f_F$$

where

$$\begin{bmatrix} y_1 \\ z_1 \\ x_2 \\ y_2 \\ z_2 \\ y_4 \\ z_4 \end{bmatrix} = \begin{bmatrix} 1 & -0.2629 & 0.1979 & -0.0317 \\ 0 & 0 & 0 & 0 \\ 0.0165 & 0.0832 & 0.0057 & 0.9673 \\ 0.0997 & 0.8557 & -0.1190 & -0.6753 \\ 0.0534 & -0.1214 & -0.5000 & 0.0188 \\ 0.1283 & 1 & -0.1090 & 1 \\ -0.1069 & 0.02424 & 1 & -0.0376 \end{bmatrix} \begin{bmatrix} m_1 \\ m_4 \\ m_7 \\ m_{10} \end{bmatrix}$$

and

$$\begin{bmatrix} H_z \\ V_2 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0.02602 & 1 \\ 0.5000 & -0.4872 \\ -1 & 0.9744 \end{bmatrix} \begin{bmatrix} f_p \\ f_F \end{bmatrix} \qquad \begin{matrix} H_3 = -Hz \\ V_3 = V_2 \\ H_4 = 0 \end{matrix}$$

ROLL/LATERAL TRANSLATION SUBSYSTEM

$$\ddot{m}_2 = -(1.342)^2 m_2 + 0.2202 f_R$$
$$\ddot{m}_5 = -(2.957)^2 m_5 + 0.5482 f_L$$
$$\ddot{m}_8 = -(4.662)^2 m_8 + 0.9845 f_R - 0.5925 f_L$$
$$\ddot{m}_{11} = -(9.251)^2 m_{11} + 0.1880 f_R + 0.6184 f_L$$

where

$$\begin{bmatrix} x_1 \\ x_2 \\ y_2 \\ z_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 & -0.2761 & -0.2286 & 0.0256 \\ 0.1188 & 1 & 0.1300 & -0.3579 \\ 0.0165 & 0.0874 & -0.0068 & -0.7840 \\ 0.0926 & -0.2207 & 1 & -0.0264 \\ 0.0901 & 0.8483 & 0.1409 & 1 \end{bmatrix} \begin{bmatrix} m_2 \\ m_5 \\ m_8 \\ m_{11} \end{bmatrix} \qquad \begin{matrix} x_3 = x_2 \\ y_3 = -yz \\ z_3 = z_2 \end{matrix}$$

and

$$\begin{bmatrix} H_2 \\ V_2 \\ H_4 \end{bmatrix} = \begin{bmatrix} -0.1735 & -0.5000 \\ 1 & -0.7299 \\ 0.3470 & 1 \end{bmatrix} \begin{bmatrix} f_R \\ f_L \end{bmatrix} \qquad \begin{matrix} H_3 = Hz \\ V_3 = -V_z \\ V_4 = 0 \end{matrix}$$

VERTICAL SUBSYSTEM

$$\ddot{m}_3 = -(2.734)^2 m_3 + 0.6208 f_V$$
$$\ddot{m}_9 = -(4.835)^2 m_9 + 0.8476 f_V$$
$$\ddot{m}_{12} = -(12.322)^2 m_{12} + 0.0192 f_V$$

where

$$
\begin{bmatrix} y_1 \\ z_1 \\ x_2 \\ y_2 \\ z_2 \\ y_4 \\ z_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & -0.7307 & 0.0512 \\ 0.0107 & -0.0275 & -0.8659 \\ 0.0064 & -0.0160 & -0.5000 \\ 0.2440 & 0.9994 & -0.0193 \\ -0.0120 & 0.0316 & 1 \\ 0.2440 & 1 & -0.0193 \end{bmatrix} \begin{bmatrix} m_3 \\ m_9 \\ m_{12} \end{bmatrix} \qquad \begin{aligned} x_3 &= -x_2 \\ y_3 &= y_2 \\ z_3 &= z_2 \end{aligned}
$$

and

$$
\begin{bmatrix} V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} f_V \end{bmatrix}
$$

YAW SUBSYSTEM

$$
\ddot{m}_6 = -(4.204)^2 m_6 + 1.000 f_Y
$$

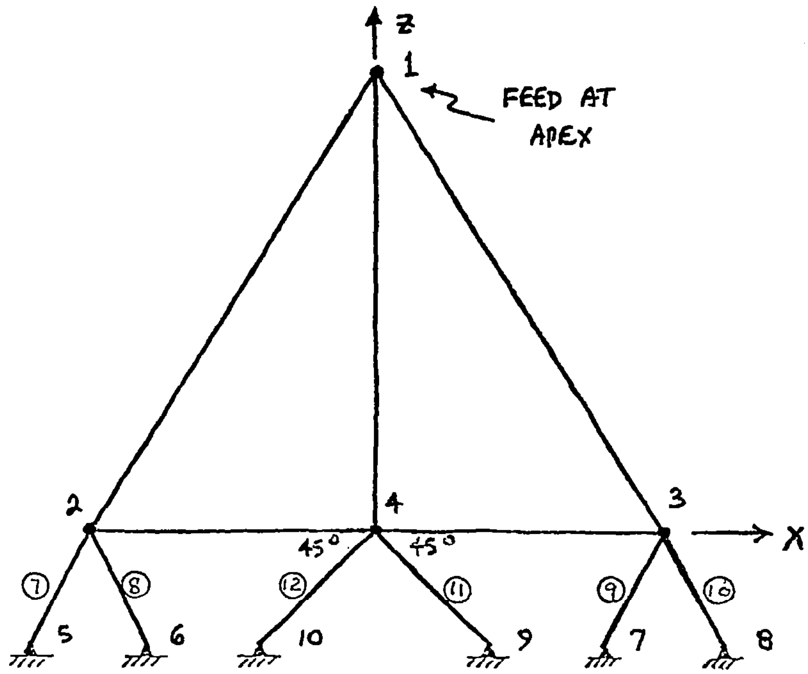
where

$$
\begin{bmatrix} x_1 \\ x_2 \\ y_2 \\ z_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.5000 \\ 0.8660 \\ 0 \\ 1.000 \end{bmatrix} \begin{bmatrix} m_6 \end{bmatrix} \qquad \begin{aligned} x_3 &= x_2 \\ y_3 &= -yz \\ z_3 &= z_2 \end{aligned}
$$

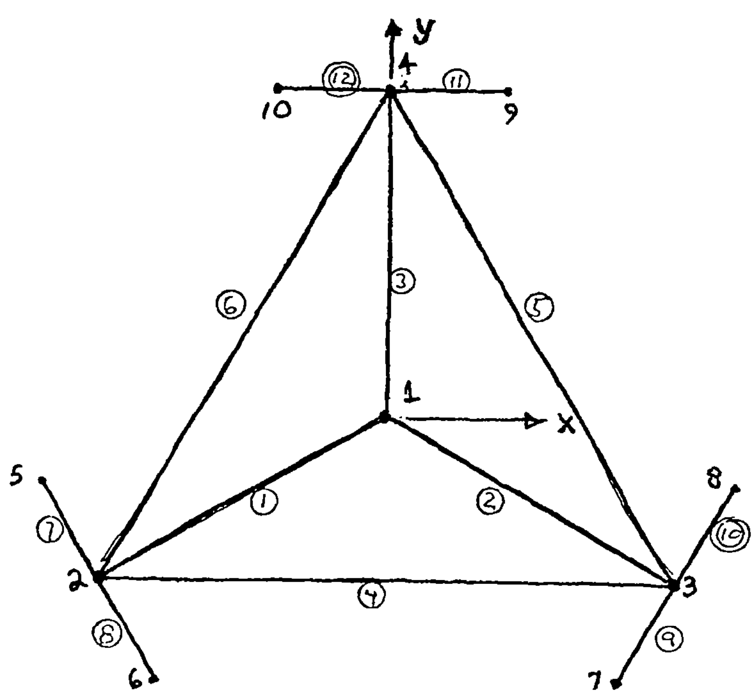
and

$$
\begin{bmatrix} H_2 \\ H_3 \\ H_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} f_Y \end{bmatrix} \qquad \begin{aligned} V_2 &= 0 \\ V_3 &= 0 \\ V_4 &= 0 \end{aligned}
$$

Figure 6 shows the combinations of controls that control only modes 1 and 4 (and also modes 7 and 10). Figure 7 shows the combinations of controls that control modes 2 and 5 (and also 8 and 11). Figure 8 shows the combinations of controls that control the vertical and yaw modes.



Rear view



Top view

Figure 1.- Antenna feed tower.

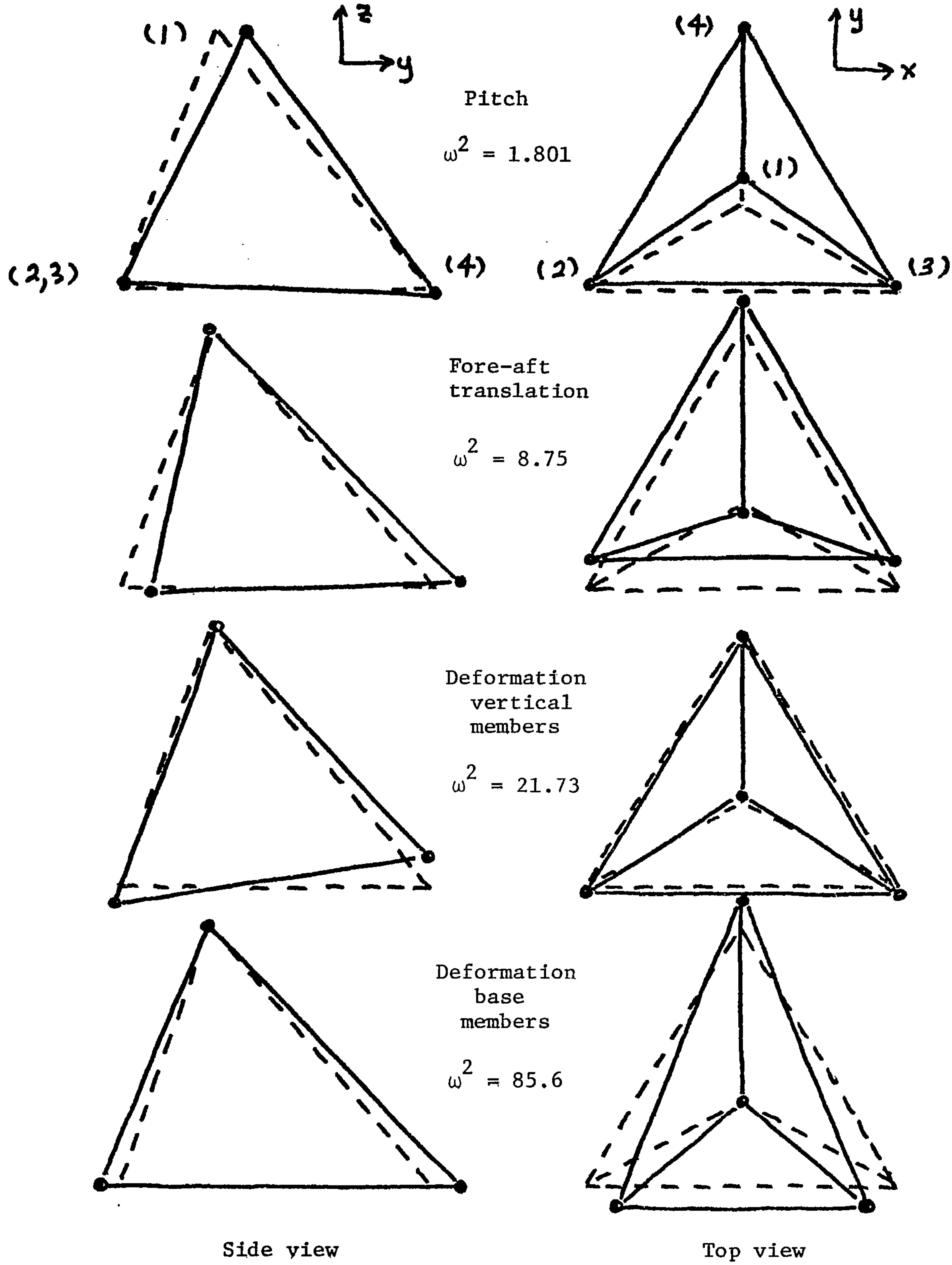


Figure 2.- Modes that involve only y motions of the apex.

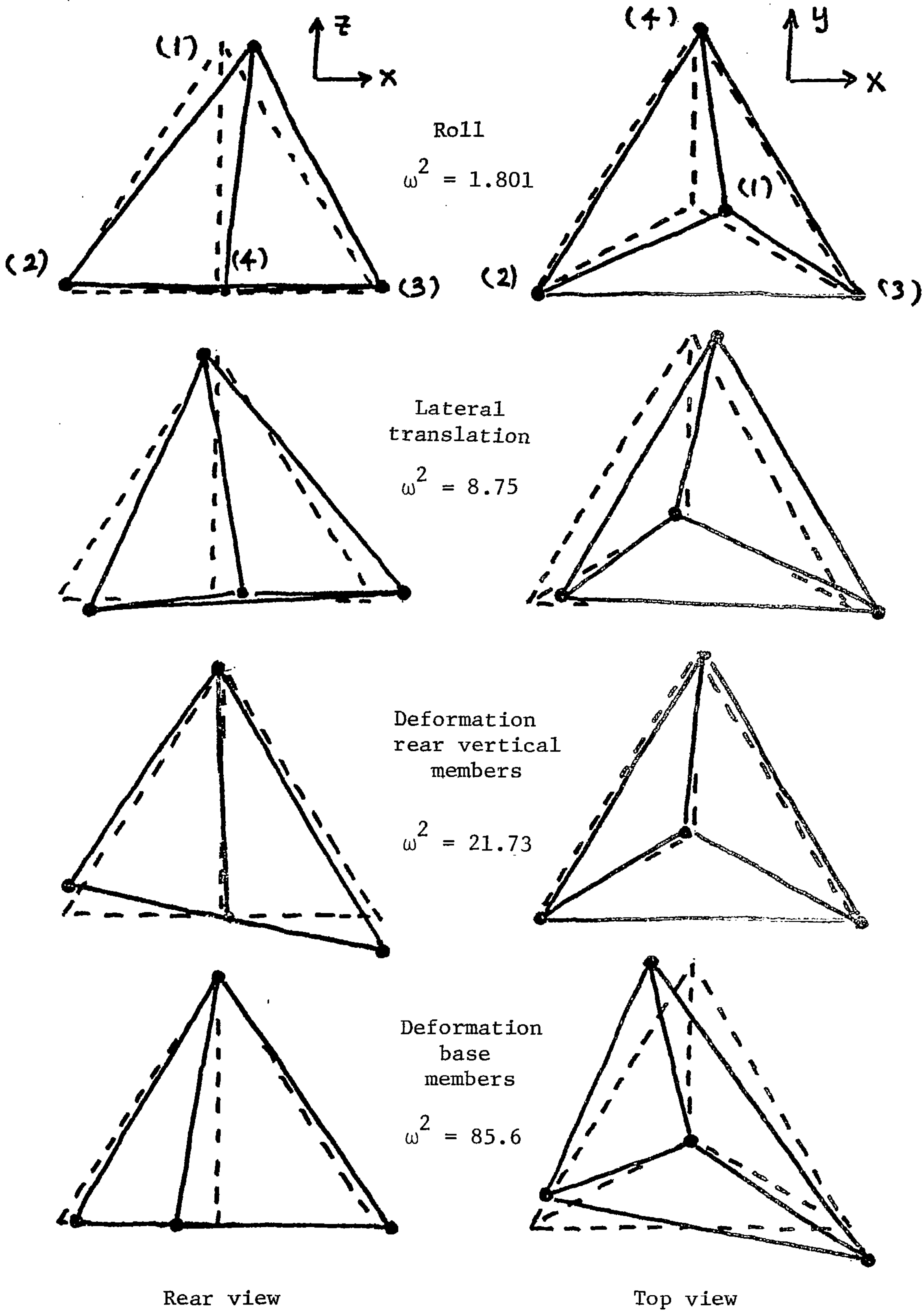


Figure 3.- Modes that involve only x motions of the apex.

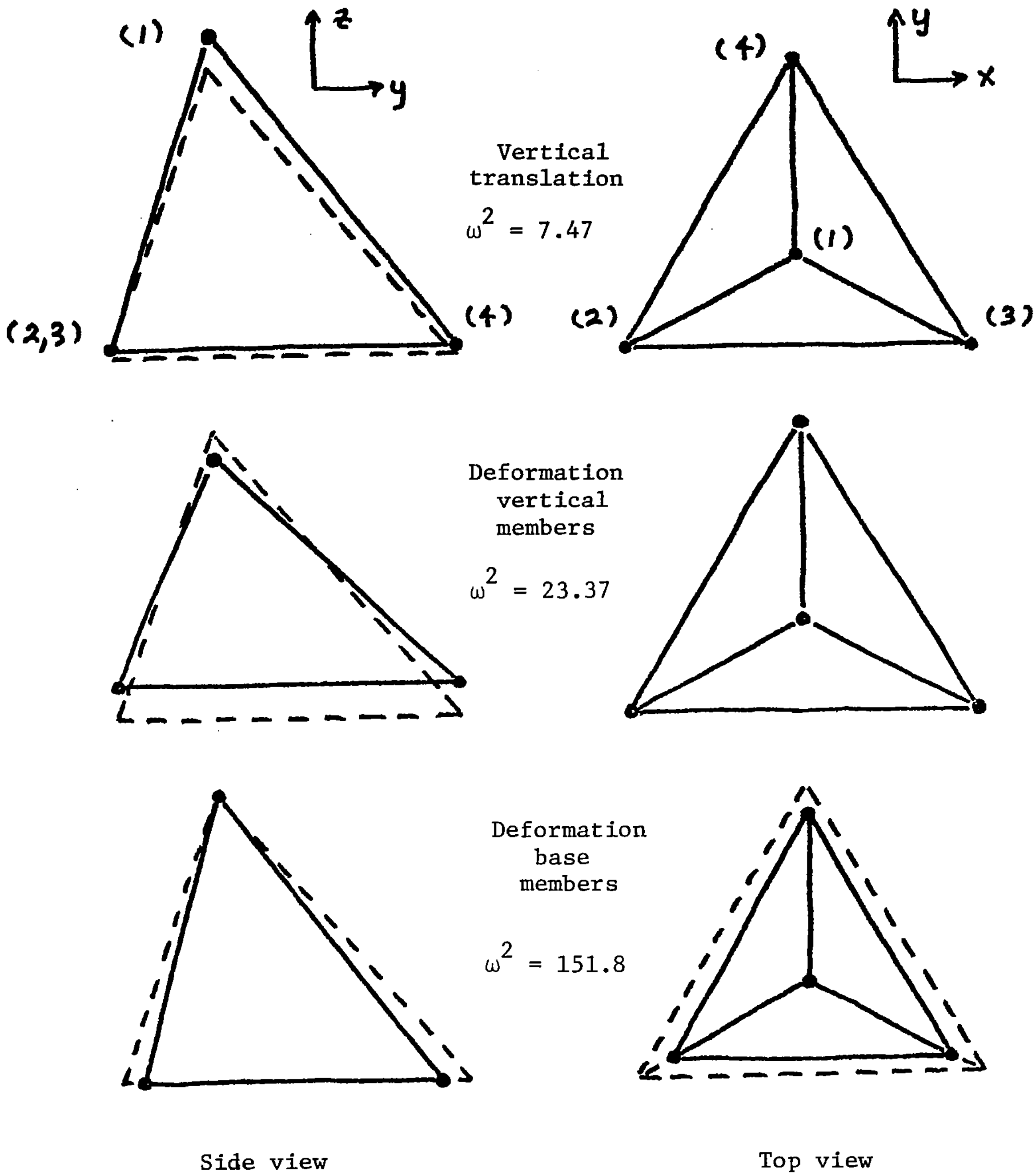


Figure 4.- Modes involving only z motions of the apex.

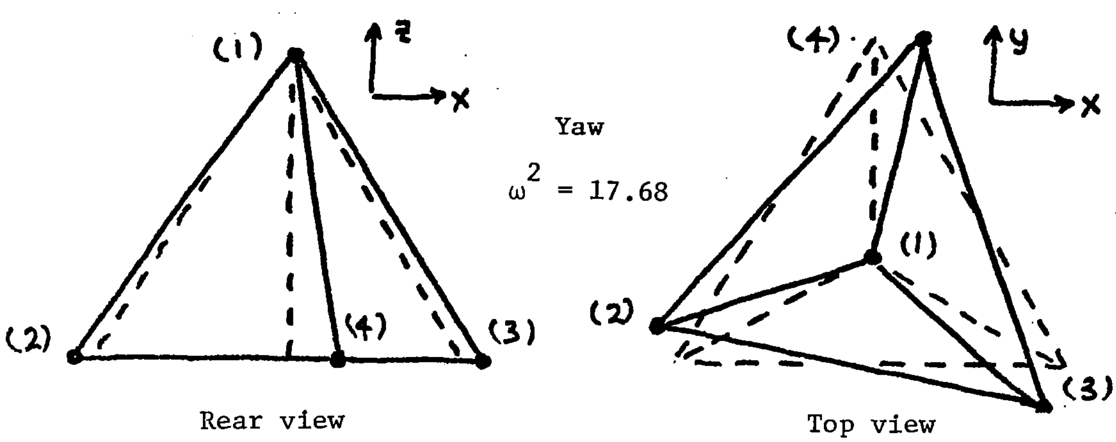
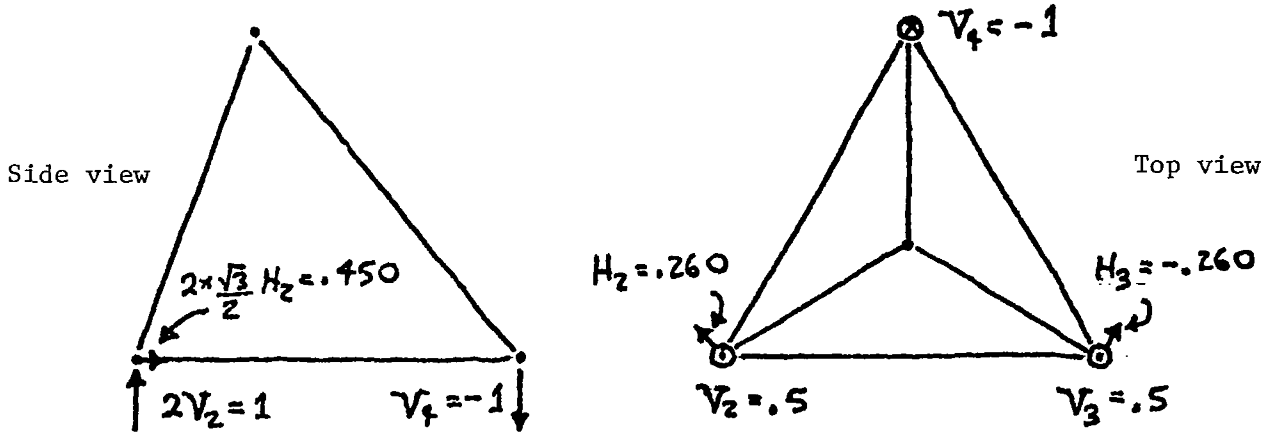
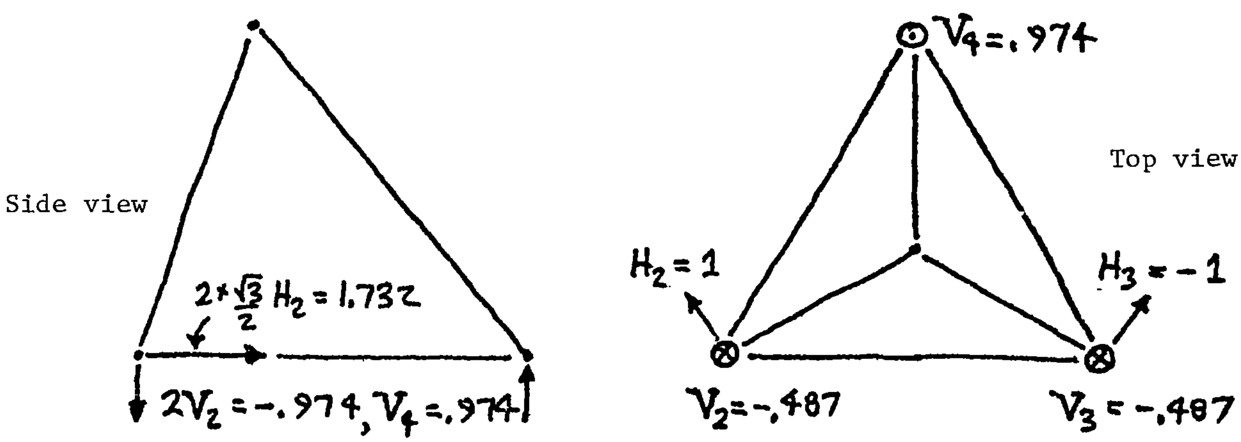


Figure 5.- Antisymmetric mode involving no motion of apex.

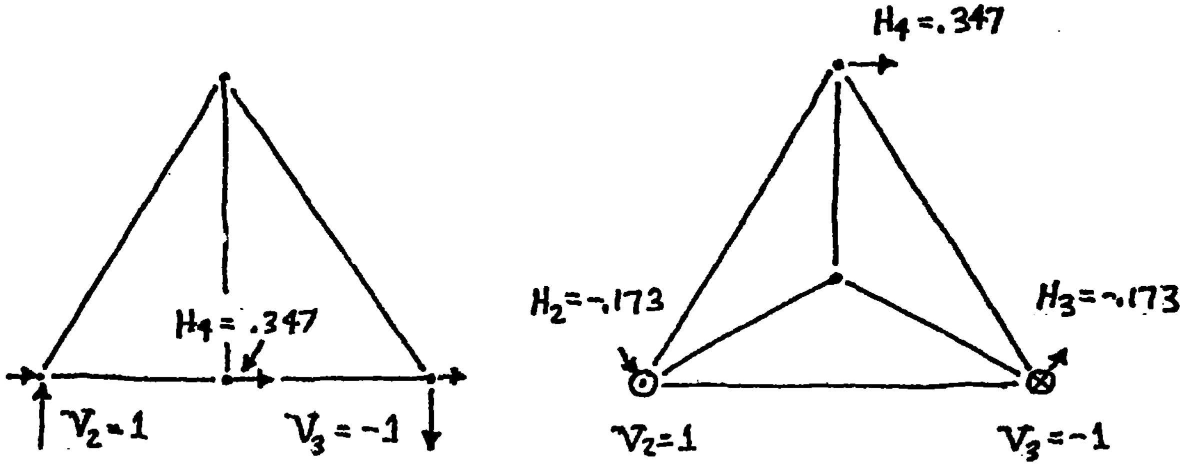


(a) Mode m_1 (pitch) controls (f_p).



(b) Mode m_4 (forward translation) controls (f_F).

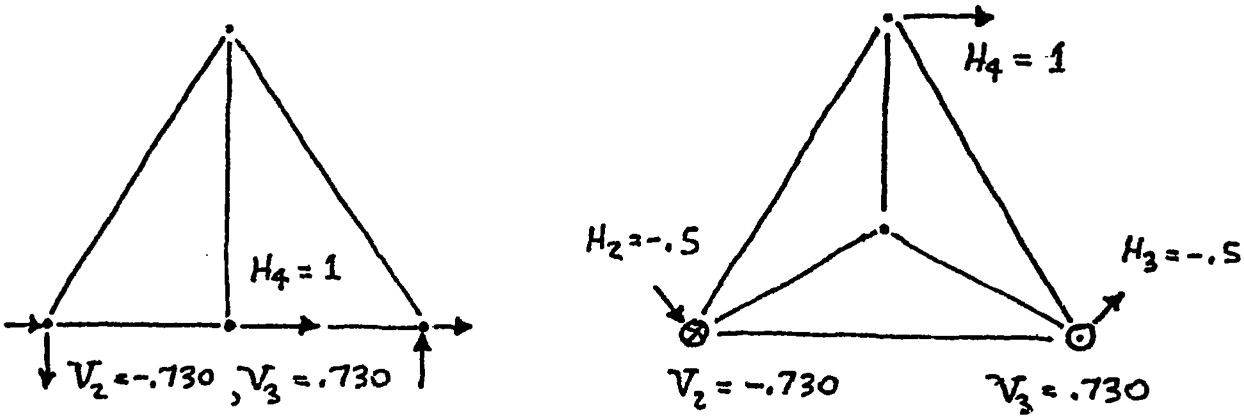
Figure 6.- Pitch and forward translation controls.



Rear view

Top view

(a) Mode m_2 (roll) controls (f_R).

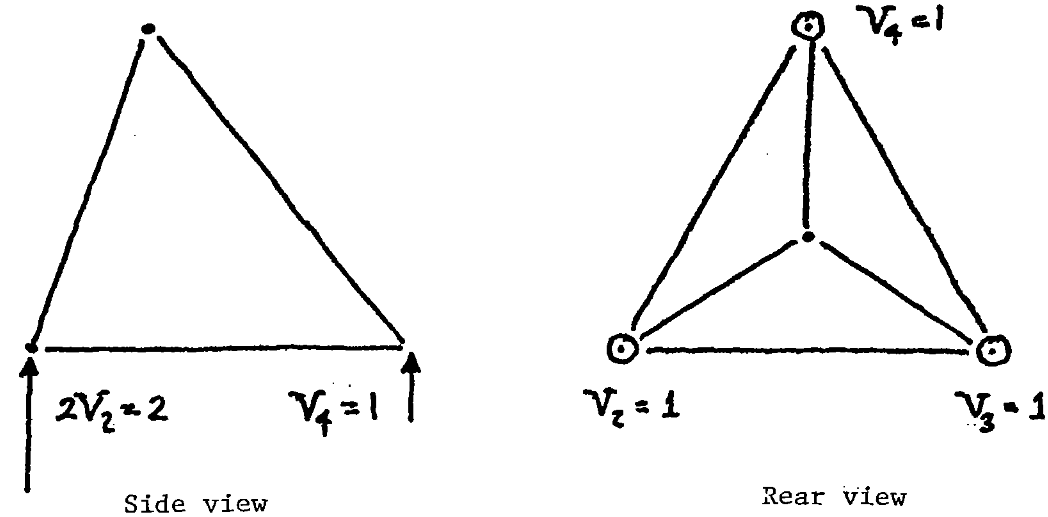


Rear view

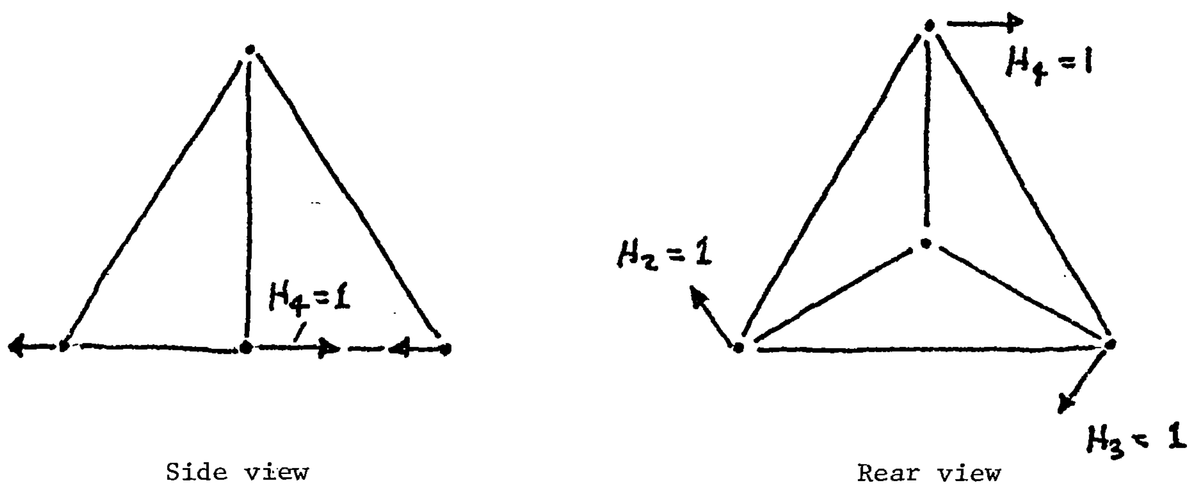
Top view

(b) Mode m_5 (lateral translation) controls (f_L).

Figure 7.- Roll and lateral translation controls.



(a) Vertical modes control (f_v).



(b) Yaw mode controls (f_y).

Figure 8.- Vertical and yaw controls.