IDENTIFICATION AND CONTROL OF SPACECRAFT

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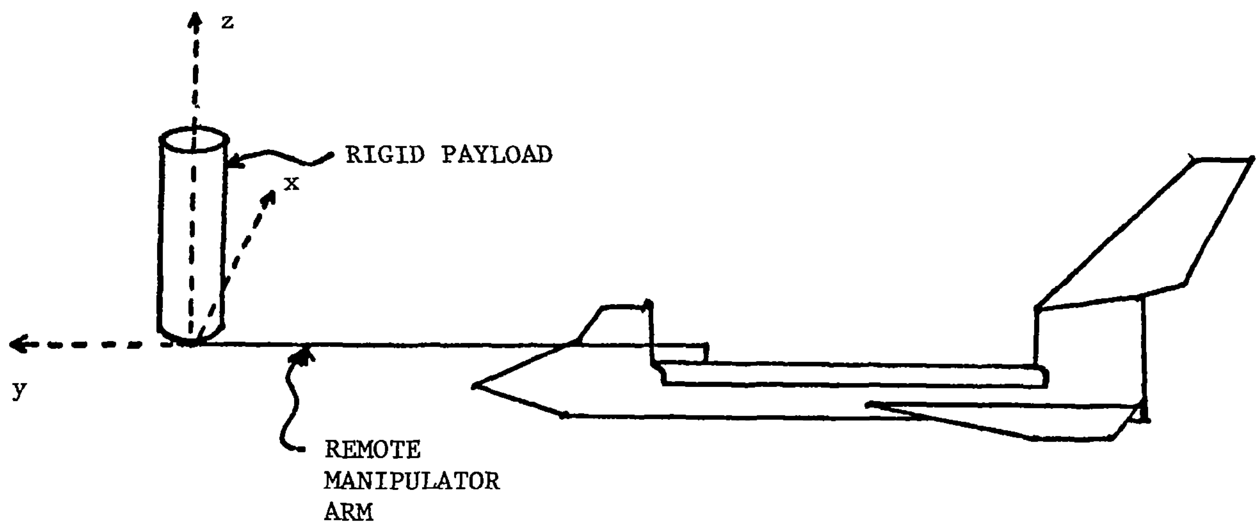
IDENTIFICATION & CONTROL OF SPACECRAFT

* THE PROBLEM

* CONTROL
  - CLASSICAL
  - MODERN

* IDENTIFICATION



CLASSICAL CONTROL

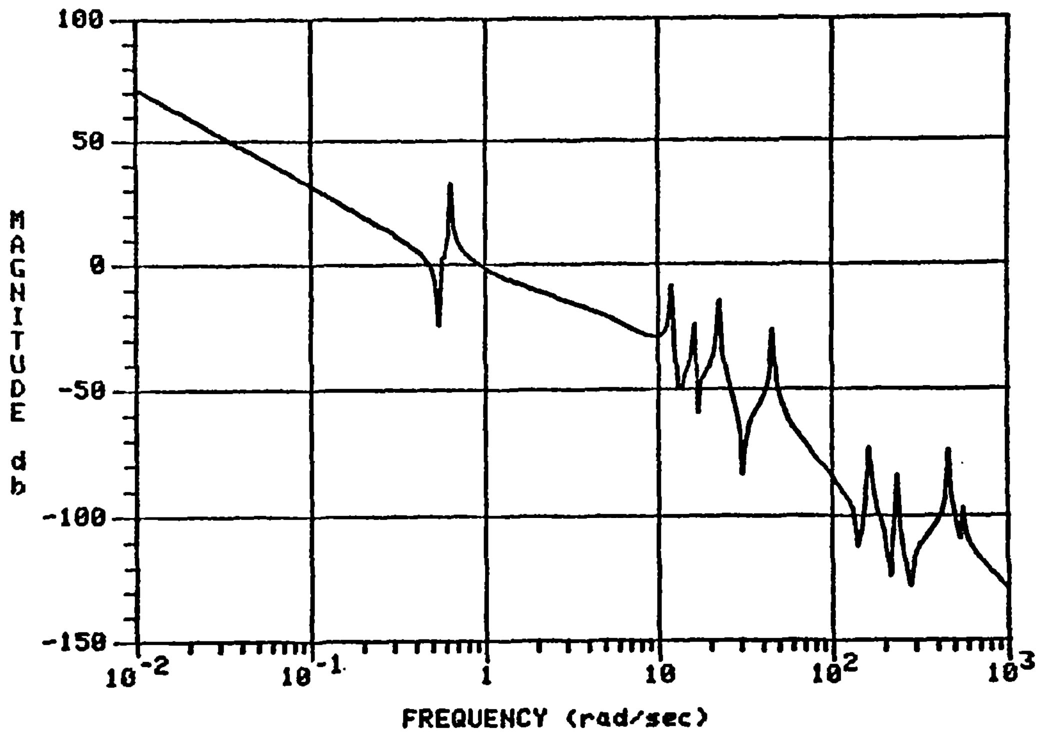
* SCALAR FEEDBACK DESIGN

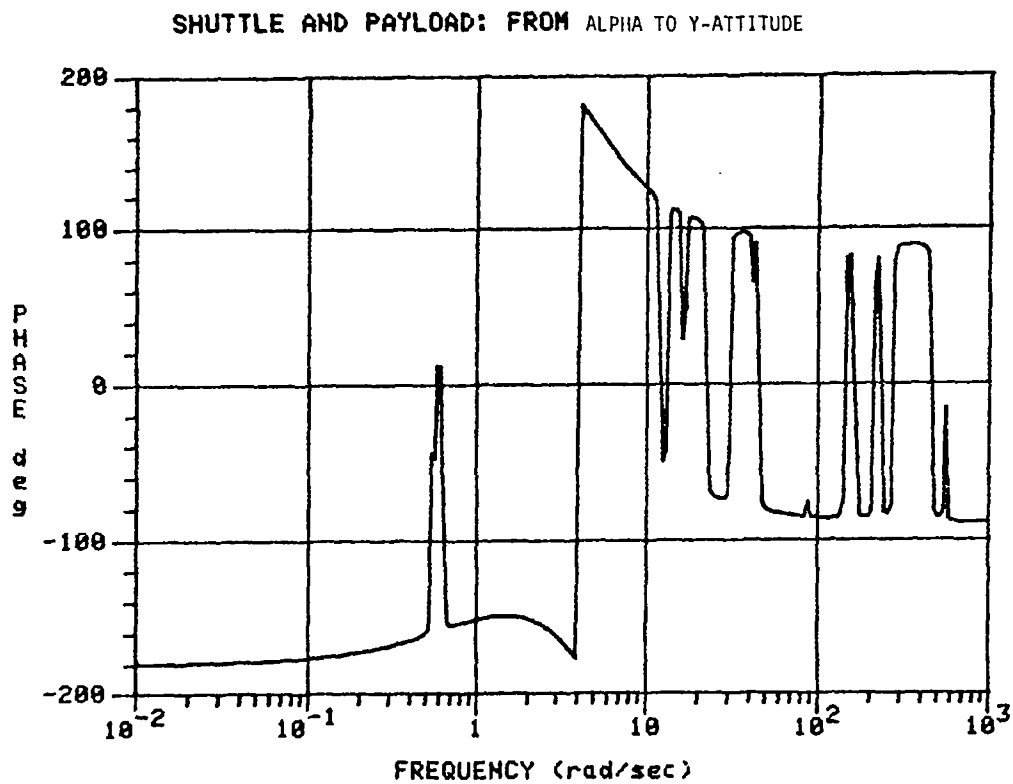
* LEAD-LAG CONTROLLER

$$K(s) = \frac{k(s+a)}{(s^2+2\zeta b+b^2)}$$

* BANDWIDTH AROUND 1 RAD/SEC

SHUTTLE AND PAYLOAD: FROM ALPHA TO Y-ATTITUDE





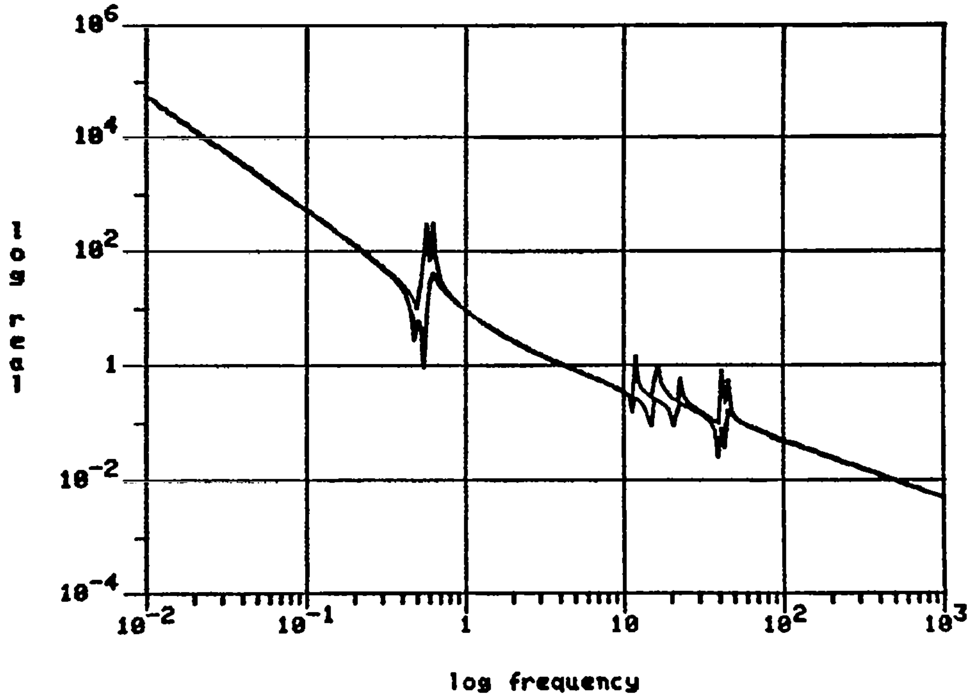
MODERN CONTROL

* START WITH FULL-STATE DESIGN
  - GOAL: MINIMIZE PAYLOAD ATTITUDE ERRORS
  - ITERATE ON CONTROL PENALTY TO ACHIEVE BW OF 5 R/S

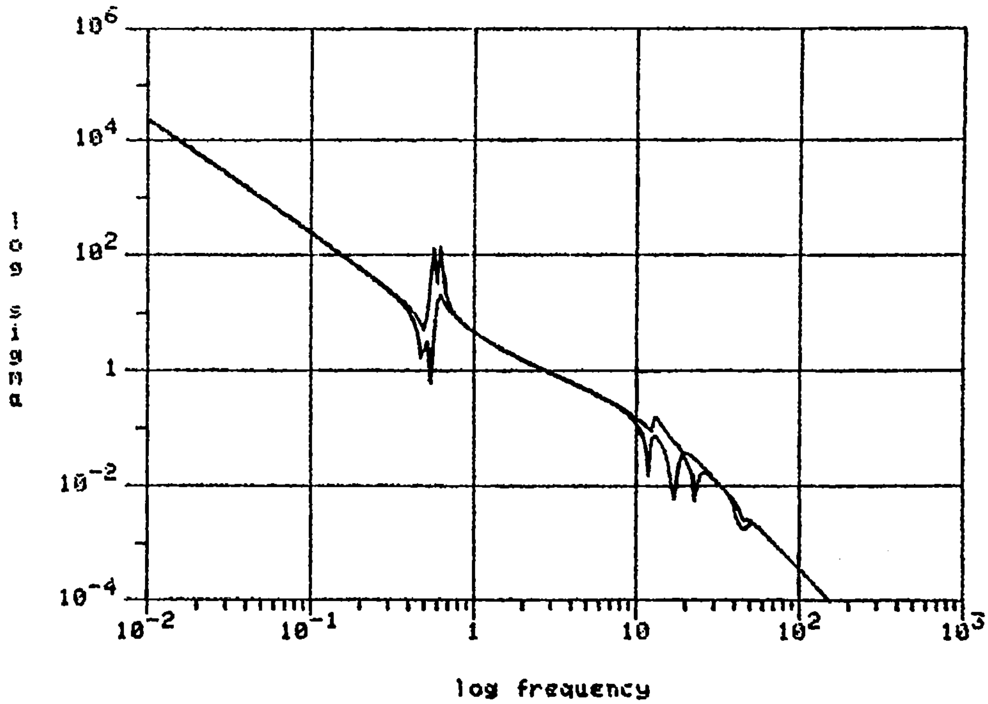
* DESIGN FILTER TO RECOVER LQ RESPONSE
  - USE STEIN/DOYLE ROBUSTNESS RECOVERY RESULTS

* TEST ROBUSTNESS

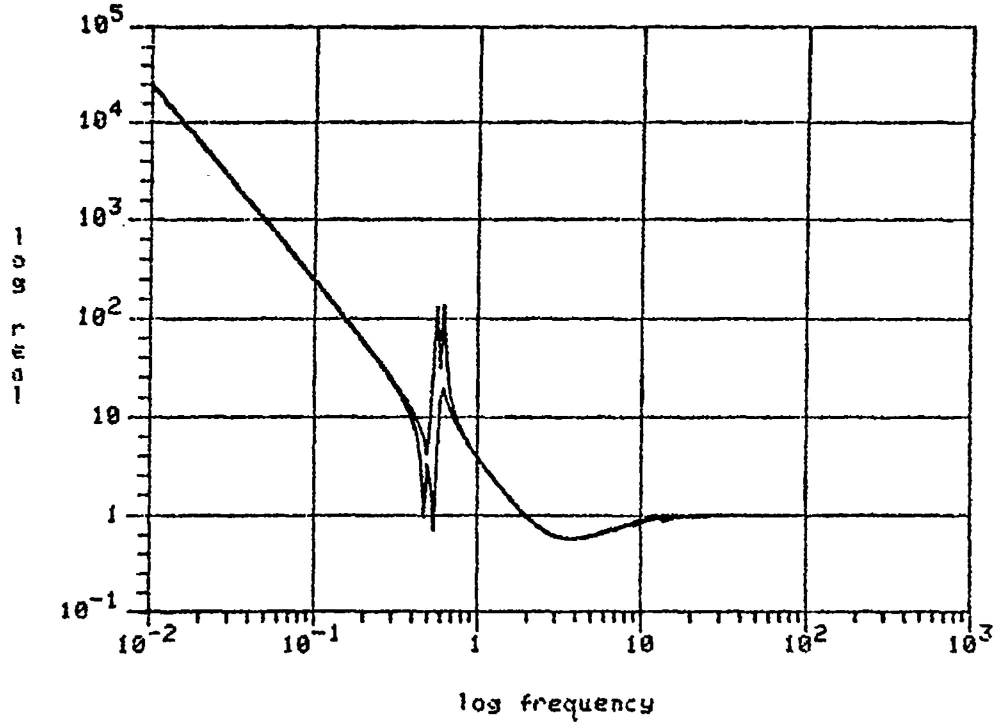
STATE FEEDBACK LQ



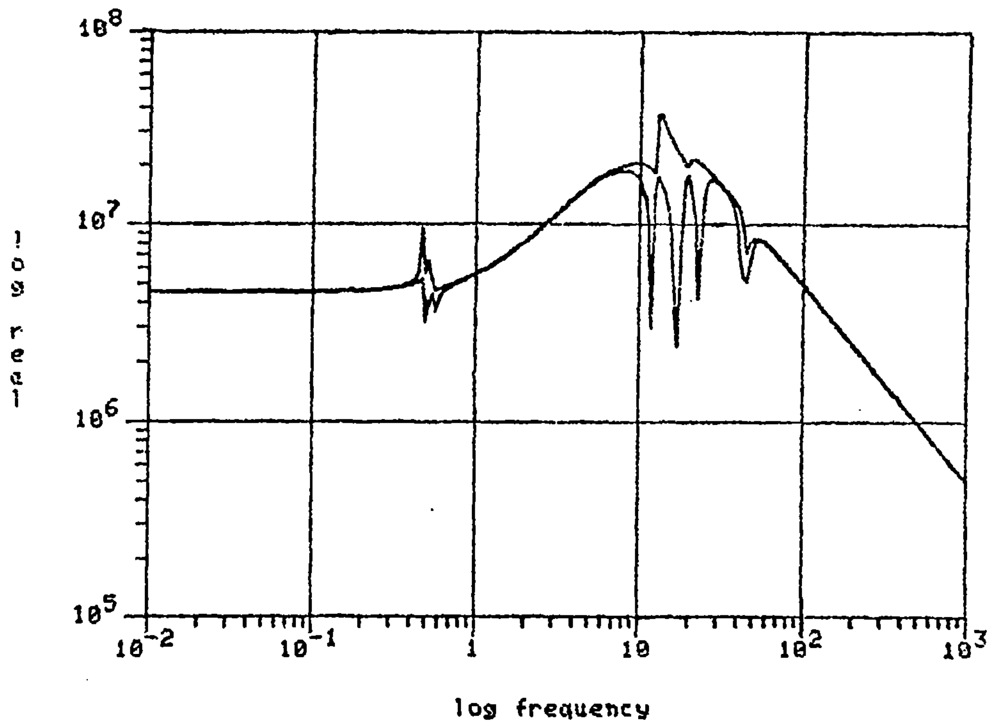
plant.r: Singular values of K(s)*G(s) q=1E8



I + KG



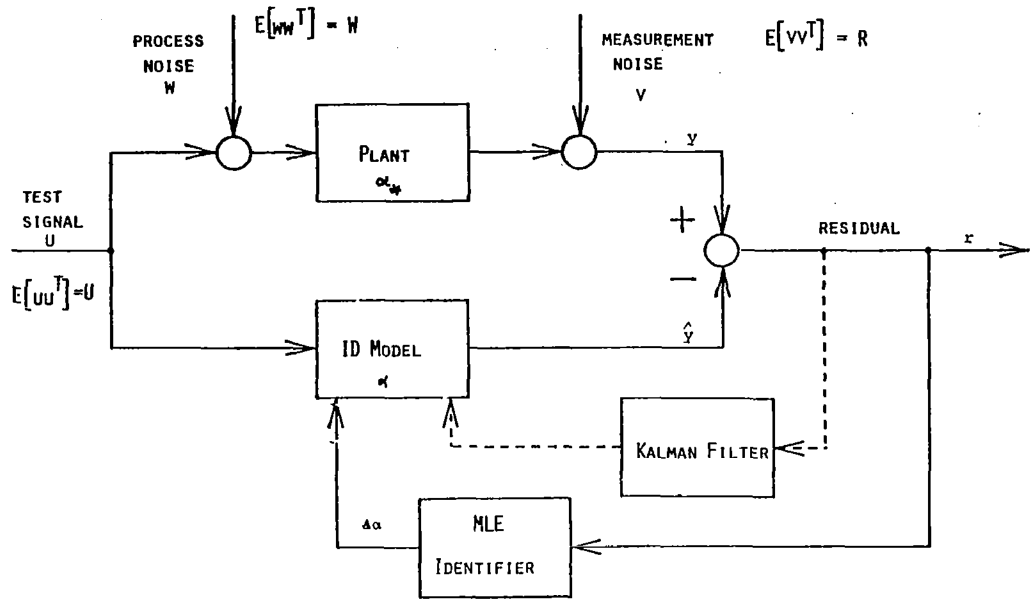
CONTROLLER



IDENTIFICATION

MAXIMUM LIKELIHOOD ESTIMATION (MLE)

IDENTIFICATION



MODEL STRUCTURE

- STATE SPACE

  n_* MODES (2x2 BLOCKS)

  m INPUTS

  p OUTPUTS

$$\begin{bmatrix}\vdots\\ \dot{x}_i\\ \ddot{x}_i\\ \vdots\end{bmatrix}(t)=\begin{bmatrix}\ddots & & 0\\ & \begin{matrix}0 & 1\\ -\omega_i^2 & -2\zeta_i\omega_i\end{matrix} & \\ 0 & & \ddots\end{bmatrix}\begin{bmatrix}\vdots\\ x_i\\ \dot{x}_i\\ \vdots\end{bmatrix}(t)+\begin{bmatrix}\vdots\\ 0^T\\ b_i^T\\ \vdots\end{bmatrix}[u(t)+w(t)]$$

$$\begin{bmatrix}y\end{bmatrix}(t)=\begin{bmatrix}\cdots & \vert & c_i \quad 0 & \vert & \cdots\end{bmatrix}\begin{bmatrix}\vdots\\ x_i\\ \dot{x}_i\\ \vdots\end{bmatrix}(t)+\begin{bmatrix}v\end{bmatrix}(t)$$

- FREQUENCY DOMAIN

$$y(s) = G_*(s)[u(s)+w(s)] + v(s)$$

$$G_*(s) = \sum_{i=1}^{n_*} \frac{c_i\, b_i^T}{s^2+2\zeta_i\omega_i s+\omega_i^2}$$

- PARAMETER VECTOR

$$\alpha_* = \{\omega_{*i}^2,\ 2\zeta_{*i}\omega_{*i},\ b_{*i},\ c_{*i};\ i = 1, \ldots, n_*\}$$

MLE IDENTIFICATION SETUP

- RESIDUAL DEFINITION

$$r_k \triangleq y(kT) - \hat{y}(kT)$$

- LIKELIHOOD FUNCTION (NEGATIVE LOG)

$$L(\alpha) \triangleq \sum_{k=0}^{N} L_k(\alpha)$$

$$\triangleq \sum_{k=0}^{N} \frac{1}{2} \left[\text{LOG DET } S_k + r_k^T S_k r_k \right]$$

WHERE

$\alpha \triangleq \{\omega_i^2,\ 2\zeta_i\omega_i,\ b_i,\ c_i;\ i = 1, \ldots, n\}$ = UNKNOWN PARAMETERS

$S_k \triangleq E_\alpha\{r_k r_k^T\}$ = PREDICTED RESIDUAL COVARIANCE

MLE IDENTIFICATION SOLUTION

- PARAMETER ESTIMATE (THEORETICAL)

$$\hat{\alpha} \triangleq \text{ARG} \left\{ \underset{\alpha}{\text{MIN}}\ L(\alpha) \right\} = \text{PARAMETER ESTIMATE}$$

- ITERATIVE ALGORITHMS

GRADIENT: $\hat{\alpha}_{J+1} = \hat{\alpha}_J - \epsilon_J \nabla L(\hat{\alpha}_J)$

NEWTON-RHAPSON: $\hat{\alpha}_{J+1} = \hat{\alpha}_J - \left[\nabla^2 L(\hat{\alpha}_J) \right]^{-1} \nabla L(\hat{\alpha}_J)$

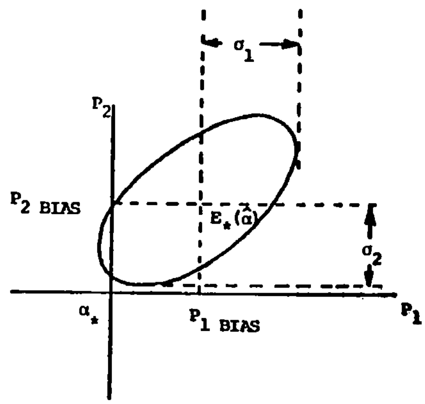
WHERE

$$\nabla L(\alpha) \triangleq \frac{\partial L}{\partial \alpha}(\alpha)$$

$$\nabla^2 L(\alpha) \triangleq \frac{\partial^2 L}{\partial \alpha^2}(\alpha)$$

IDENTIFICATION ACCURACY ISSUES

- SYSTEMATIC ERRORS: $E_*(\hat{\alpha}) - \alpha_*$
  - MODEL-ORDER MISMATCH
  - TEST SIGNAL AMPLITUDE AND SHAPING
  - SYSTEMATIC DISTURBANCES
  - SENSOR/ACTUATOR MODEL ERRORS



- STOCHASTIC ERRORS: $\sigma_\alpha \sim \frac{1}{\sqrt{TN}}$
  - RANDOM DISTURBANCES AND SENSOR NOISE
  - TEST SIGNAL AMPLITUDE AND SHAPING
  - IDENTIFICATION TIME INTERVAL

STEADY-STATE IDENTIFIABILITY ANALYSIS (YARED)

- EXPECTED LIKELIHOOD FUNCTION

$$
\begin{aligned}
I^*(\alpha) &= E_* \left\{ L_k(\alpha) \right\} \\
&= \frac{1}{2} \left[\text{LOG DET } S + \text{TR}(S^{-1} S_*) \right]
\end{aligned}
$$

- RESIDUAL COVARIANCES

$$S = E_\alpha \left\{ r_K r_K^T \right\} = \text{KALMAN FILTER PREDICTED RESIDUAL COVARIANCE MATRIX}$$

$$S_* = E_* \left\{ r_K r_K^T \right\} = \text{ACTUAL RESIDUAL COVARIANCE MATRIX}$$

NOTE: S_* AND $I^*(\alpha)$ CAN ONLY BE COMPUTED WHEN THE TRUE PLANT PARAMETERS ARE KNOWN.

EXPECTED MLE IDENTIFICATION SOLUTION

- Expected Parameter Estimate (Theoretical)

$$\hat{\alpha}_* \overset{\Delta}{=} E_* \left\{ \alpha \right\} = \text{Arg} \left\{ \min_{\alpha} \quad I^*(\alpha) \right\}$$

- Iterative Algorithms

Gradient: $\hat{\alpha}_{*_{J+1}} = \hat{\alpha}_{*_J} - \varepsilon_J \nabla I^*(\hat{\alpha}_{*_J})$

Newton-Rhapson: $\hat{\alpha}_{*_{J+1}} = \hat{\alpha}_{*_J} - \left[\nabla^2 I^*(\hat{\alpha}_{*_J}) \right]^{-1} \nabla I^*(\hat{\alpha}_{*_J})$

where $\nabla I^*(\alpha) = \dfrac{\partial^2 I^*}{\partial \alpha}(\alpha)$

$\nabla^2 I^*(\alpha) = \dfrac{\partial^2 I^*}{\partial \alpha^2}(\alpha)$

STEADY STATE IDENTIFICATION ACCURACY

- SYSTEMATIC ERRORS (BIASES)
  - PARAMETER ERRORS

    $\alpha_{BIAS} \overset{\Delta}{=} \hat{\alpha}_* - \alpha_* = 0$ WHEN NO MODEL MISMATCH

  - INFORMATION MEASURE (YARED)

    $I(\alpha_* \, ; \, \hat{\alpha}_*) \overset{\Delta}{=} I^*(\hat{\alpha}_*) - I^*(\alpha_*) \geq 0$

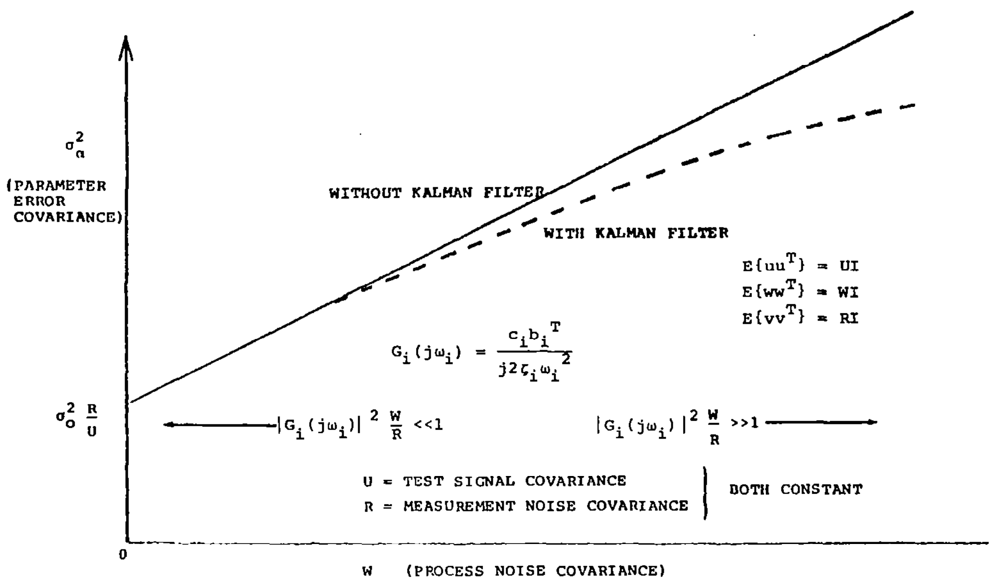
- STOCHASTIC ERRORS

$$C_{OV}\{\hat{a}\} \overset{\Delta}{=} \lim_{N \to \infty} E_* \left\{ (\hat{\alpha} - \hat{\alpha}_*)(\hat{\alpha} - \hat{\alpha}_*)^T \right\}$$

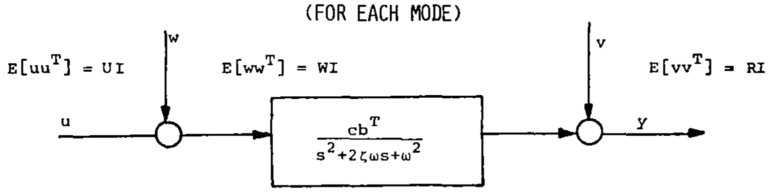
$$\overset{\Delta}{=} \left[\nabla^2 I^*(\hat{\alpha}_*) \right]^{-1} E_* \left\{ \left[\frac{\partial L}{\partial \alpha}(\hat{\alpha}_*) \right] \left[\frac{\partial L}{\partial \alpha}(\hat{\alpha}_*) \right]^T \right\} \left[\nabla^2 I^*(\hat{\alpha}_*) \right]^{-1} \frac{1}{(N+1)^2}$$

$$= \left[\nabla^2 I^*(\hat{\alpha}_*) \right]^{-1} \frac{1}{(N+1)} \quad \text{WHEN NO MODEL MISMATCH}$$

STOCHASTIC ERROR WITH PROCESS NOISE



SIMPLIFIED IDENTIFICATION ACCURACY ANALYSIS (FOR EACH MODE)



$$\alpha \triangleq \{\omega^2,\ 2\zeta\omega,\ b_1, \ldots\ b_m,\ c_1, \ldots,\ c_p\} - \{b_L \text{ or } c_M\}$$

$$\frac{\sigma_{\omega^2}^2}{(\omega^2)^2} = \frac{8\zeta^2}{SNR}\ \frac{1}{(N+1)T} \quad ; \quad \rho \triangleq \max_{\ell,m} \left\{\frac{b_\ell^2}{|b|^2},\ \frac{c_m^2}{|c|^2}\right\} \le 1$$

$$\frac{\sigma_{2\zeta\omega}^2}{(2\zeta\omega)^2} = \frac{4}{SNR}\ \frac{1}{(N+1)T} \quad ; \quad SNR \triangleq \begin{cases} \frac{|c|^2|b|^2}{4\zeta\omega^3}\ \frac{U}{R} & \text{(MEAS. NOISE)} \\ \zeta\omega\ \frac{U}{W} & \text{(PROC. NOISE)} \end{cases}$$

$$\frac{\sigma_{b\ell}^2}{b_\ell^2} = \frac{1}{SNR}\left[\frac{|b|^2}{b_\ell^2} + \rho^{-1}\right] \frac{1}{(N+1)T}$$

$$\frac{\sigma_{cm}^2}{c_m^2} = \frac{1}{SNR}\left[\frac{|c|^2}{c_m^2} + \rho^{-1}\right] \frac{1}{(N+1)T}$$

NOTE: THIS ANALYSIS ASSUMES THAT $\omega T << 1$, $\zeta << 1$ AND APPLIES FOR EACH MODE

NUMERICAL RESULTS

- PROBLEM SIZE

  12 MODES (=22 - 8R.B. - 2 SMALL)

  2 INPUTS (α, β GIMBAL ANGLES)

  2 OUTPUTS (x, y ATTITUDES)

  60 PARAMETERS ($\approx$12 MODES x 5 PARAMETERS/MODE)

- TEST SIGNAL, NOISE STATISTICS

  SAMPLE TIME: $T = 0.1$ SEC

  TEST SIGNAL: $U = 4000(\text{IN-LB})^2$

  PROCESS NOISE: $W = 40(\text{IN-LB})^2$

  MEASUREMENT NOISE: $R = 4 \times 10^{-12}$ RAD^2

- WORST-CASE RELATIVE ERRORS AT TIME (MODE 9):

PARAMETER	1 SEC	14 SEC	0.39 HRS
ω^2	0.0265	0.007	0.0007
$2\zeta\omega$	3.75	1.0	0.1
b_1	13.6	3.6	0.36
c_1	2.7	0.7	0.07
c_2	11.7	3.1	0.31

SUMMARY

- CONTROL PROBLEM
  - MODERN LQ CONTROL DESIGN WITH ROBUSTNESS RECOVERY PRODUCES ROBUST CONTROLLERS FOR LSS
  - ACCURATE ID ALLOWS A FIVE-FOLD INCREASE IN LOOP BW
- IDENTIFICATION PROBLEM
  - STRUCTURAL MODES MAY BE IDENTIFIED ONE AT A TIME FOR SMALL DAMPING
  - LSS ID W/O KF

    --GREATLY REDUCES PARAMETER BIASES

    --GIVES ONLY MODEST INCREASE IN STOCHASTIC ERRORS
  - RELATIVE ERRORS IN PARAMETERS AFTER ID ARE SMALLER FOR FREQUENCY THAN FOR DAMPING OR MODE SHAPES
- OPEN ISSUE: HOW ACCURATE MUST ID BE FOR ROBUST CONTROL DESIGN?