CONTROL OF STRUCTURES IN SPACE

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EQUATION OF MOTION FOR DISTRIBUTED SYSTEMS

Differential equation: $Lu(P,t) + M(P)\partial^2u(P,t)/\partial t^2 = f(P,t)$, $P \in D$

Boundary conditions: $B_i u(P,t) = 0$, $i=1,2,\ldots,p$, $P \in S$

$u(P,t)$ = displacement at point P

L, B_i = differential operators (L is self-adjoint of order $2p$)

M = mass density

$f(P,t)$ = distributed control force

EIGENVALUE PROBLEM

Differential equation: $L\phi = \lambda M\phi$

Boundary conditions: $B_i\phi = 0$, $i = 1,2,\ldots,p$

Solution: eigenvalues $\lambda_r = \omega_r^2$, eigenfunctions ϕ_r $(r=1,2,\ldots)$

Because L is self-adjoint, eigenfunctions are orthogonal

L is generally positive semidefinite $\rightarrow \lambda_r$ are all nonnegative

$\lambda_r = 0$ for rigid-body modes

$\omega_r = \sqrt{\lambda_r}$ = natural frequencies

Orthonormality conditions: $\int_D M\phi_s\phi_r \, dD = \delta_{rs}$, $\int_D \phi_s L\phi_r \, dD = \lambda_r\delta_{rs}$

MODAL EQUATIONS

Expansion theorem: $u(P,t) = \sum_{r=1}^{\infty} \phi_r(P)u_r(t)$

$u_r(t)$ = modal coordinates

Modal equations: $\ddot{u}_r(t) + \omega_r^2 u_r(t) = f_r(t)$, $r=1,2,\ldots$

Modal controls: $f_r(t) = \int_D \phi_r(P)f(P,t)dD$, $r=1,2,\ldots$

Coupled controls: $f_r(t) = f_r(u_1, \dot{u}_1, u_2, \dot{u}_2,\ldots)$, $r=1,2,\ldots$

Independent modal-space control (IMSC): $f_r(t) = f_r(u_r,\dot{u}_r)$, $r=1,2,\ldots$

CONTROL IMPLEMENTATION

Distributed actuators: $f(P,t) = \sum_{r=1}^{\infty} M(P)\phi_r(P)f_r(t)$

Coupled controls: unable to design distributed controls

IMSC: design modal controls first, then use above formula
no control spillover

Discrete actuators: $f(P,t) = \sum_{j=1}^{m} F_j(t)\,\delta(P-P_j)$

$$f_r(t) = \sum_{j=1}^{m} \phi_r(P_j)F_j(t) = \sum_{j=1}^{m} B_{rj}F_j(t) \ , \quad r=1,2,\ldots,n$$

n = number of controlled modes

$$\underset{\sim}{F} = [F_1\ F_2\ \ldots\ F_m]^T, \quad \underset{\sim}{f} = [f_1\ f_2\ \ldots\ f_n]^T, \quad B = B_{rj}$$

Coupled controls: Design $\underset{\sim}{F}(t)$ so as to ensure controllability
IMSC: Design $\underset{\sim}{f}(t)$ so as to control a given number of modes
Then, $\underset{\sim}{F}(t) = B^{+}\underset{\sim}{f}(t)$, B^{+} = pseudo-inverse of B
To avoid pseudo-inverses, let m = n, or the number of actuators must equal the number of controlled modes.

CONTROL IMPLEMENTATION (CONT'D)

Distributed measurements:

Measurements: $u(P,t)$, $\dot{u}(P,t)$ for all P and at any t

Then, modal coordinates and velocities, $u_r(t)$ and $\dot{u}_r(t)$, are computed by using the modal filters

$$u_r(t) = \int_D M(P)\,\phi_r(P)u(P,t)dD,\quad \dot{u}_r(t) = \int_D M(P)\,\phi_r(P)\dot{u}(P,t)dD,\quad r=1,2,\ldots$$

Discrete measurements:

Measurements: $y_j(t) = u(P_j,t)$, $\dot{y}_j(t) = \dot{u}(P_j,t)$, $j = 1,2,\ldots,k$

k = number of sensors

Standard approach: Use Luenberger observer to estimate state

Discrete measurements treated as distributed:

Use interpolation functions to compute estimate $\hat{u}(P,t)$ of $u(P,t)$

Then, use modal filters to compute estimates $\hat{u}_r(t)$ of $u_r(t)$

Divide structure into s segments (elements)

Approximate displacement: $\hat{u}(P,t) = \sum_{j=1}^{s} \underset{\sim}{L}_j^T(P)\underset{\sim}{v}_j(t)$

$\underset{\sim}{v}_j$ = measurements at the boundaries of j'th interval

$\underset{\sim}{L}_j$ = vector of interpolation functions (from the finite element method)

Estimated modal coordinates:

$$\hat{u}_r(t) = \int_D M(P)\phi_r(P) \sum_{j=1}^{s} \underset{\sim}{L}_j^T\underset{\sim}{v}_j(t)dD = \sum_{j=1}^{s} \underset{\sim}{I}_{rj}\underset{\sim}{v}_j(t),\quad r = 1,2,\ldots,n$$

Similarly $\hat{\dot{u}}_r(t) = \sum_{j=1}^{s} \underset{\sim}{I}_{rj}\dot{\underset{\sim}{v}}_j(t)$

$$\underset{\sim}{I}_{rj} = \int_D M(P)\phi_r(P)\underset{\sim}{L}_j(P)dD = \text{const}$$

$\underset{\sim}{I}_{rj}$ are computed off-line, in advance.

CONTROL IMPLEMENTATION (CONT'D)

Rearrange $\underset{\sim}{I}_{rj}$ such that

$$\hat{u}_r(t) = \sum_{j=1}^{k} C_{rj} y_j(t) \;, \quad \hat{\dot{u}}_r(t) = \sum_{j=1}^{k} C_{rj} \dot{y}_j(t)$$

Let

$$\underset{\sim}{\hat{u}}(t) = [\hat{u}_1 \; \hat{u}_2 \ldots \; \hat{u}_n]^T, \quad \underset{\sim}{\hat{\dot{u}}}(t) = [\hat{\dot{u}}_1 \; \hat{\dot{u}}_2 \; \ldots \; \hat{\dot{u}}_n]^T$$

$$C = C_{rj} \;, \quad r=1,2,\ldots,n; \; j=1,2,\ldots,k$$

$$\underset{\sim}{y}(t) = [y_1 \; y_2 \; \ldots \; y_k]^T, \quad \underset{\sim}{\dot{y}}(t) = [\dot{y}_1 \; \dot{y}_2 \; \ldots \; \dot{y}_k]^T$$

$$\underset{\sim}{\hat{u}}(t) = C\underset{\sim}{y}(t) \quad , \quad \underset{\sim}{\hat{\dot{u}}}(t) = C\underset{\sim}{\dot{y}}(t)$$

The way C is assembled depends on the nature of the interpolation functions (see example later).

THE LANGLEY BEAM EXPERIMENT

Free-free beam controlled by using 4 actuators and 9 sensors

Use IMSC to control four modes, two rigid-body and two elastic modes

Actuator forces $F_j(t) = \sum_{r=1}^{4} (B^{-1})_{rj} f_r(t), \quad j = 1,2,3,4$

Modal forces:

1) For rigid body modes

   $\eta_r = |\hat{u}_r| + |\hat{\dot{u}}_r|/c_r \quad , \quad r=1,2$

   c_r = weighting factor

   if $\eta_r < d_r$, then $f_r = 0$

   if $\eta_r > d_r$ and

   i) $\hat{u}_r > 0$, $\hat{\dot{u}}_r > 0$, or $\hat{u}_r > 0 > \hat{\dot{u}}_r$ and $|\hat{\dot{u}}_r| < \varepsilon_r$, then $f_r = -k_r$

   ii) $\hat{u}_r < 0$, $\hat{\dot{u}}_r < 0$, or $\hat{u}_r < 0 < \hat{\dot{u}}_r$ and $|\hat{\dot{u}}_r| < \varepsilon_r$, then $f_r = k_r$

   d_r = magnitude of the deadband region

   ε_r = threshold velocity, k_r = modal control force

2) for elastic modes

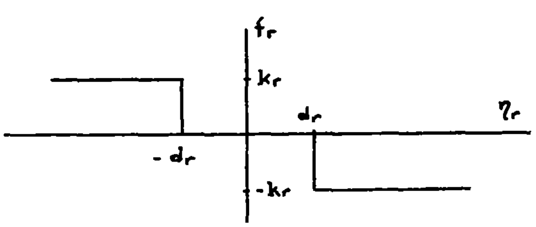
$$f_r(t) = \begin{cases} -k_r \ , & |\hat{u}_r| \geq d_r \text{ and } \hat{u}_r > 0 \\ 0 \ , & -d_r < \hat{u}_r < d_r \\ k_r \ , & |\hat{u}_r| \geq d_r \text{ and } \hat{u}_r < 0 \end{cases}$$

For simulation purposes, the response is available in closed form both for rigid-body and elastic modes.

Actuator locations immaterial when IMSC is used.

$$\underset{\sim}{F}(t) = B^{-1}\underset{\sim}{f}(t) \qquad F_j(t) = \sum_{r=1}^{4} (B^{-1})_{jr} f_r(t) \quad j=1,2,3,4$$

Actual control forces are a combination of on-off functions of the type



Actuators available at LARC have four components, which can be assigned to each modal control force. As a result, each actuator command becomes a linear combination of 4 modal on-off control forces.

THE LANGLEY BEAM EXPERIMENT (CONT'D)

Sensors measure displacements alone. Ideally, for bending they should measure displacements and slopes (velocities and slopes of velocities too)

Divide the beam into four equal segments (elements) and measure displacements at the ends and middle point of the element, so that the nine sensors are spaced at equal intervals.

As interpolation functions use

$$\underset{\sim}{L}_j = [\zeta(2\zeta-1) \quad 4\zeta(1-\zeta) \quad 1-3\zeta+2\zeta^2]^T, \quad 0 < \zeta < 1$$

where ζ is a local coordinate related to the global coordinate x by

$\zeta = j - \frac{4}{L}x$, in which j is the segment number

The C matrix is assembled from $\underset{\sim}{I}_{rj}$ tensor as

$$C_{rp} = \sum_{j=1}^{s} \sum_{\ell=1}^{3} I_{rj\ell}\, \delta_{2j+\ell-2,p} \qquad \begin{array}{l} r = 1,2,\ldots,n\ ,\ n=4 \\ s = 4 \\ p = 1,2,\ldots,k,\ k = 9 \end{array}$$

ℓ = index denoting the interpolation function

j = index denoting the element number

Because velocity measurements are not available estimate velocities by using the relation

$$\hat{\dot{u}}_r(jT) = \frac{\hat{u}_r(jT) - \hat{u}_r(jT-T)}{T}$$

T = sampling time = 1/33 sec.

Or, one could use a modal Luenberger type observer. Because the controls are nonlinear, the convergence of the observer can only be determined by trial and error.

THE LANGLEY BEAM EXPERIMENT (CONT'D)

Parameters associated with the beam:
$L = 12$ ft. cross-section = 6 x 3/16 in
6061 aluminum: $\rho = 0.1$ lb/in^3 $E = 1 \times 10^7$ lb/in^2

The free-free, uniform beam admits a closed-form solution. The transcendental equation was solved numerically to yield the eigenvalues

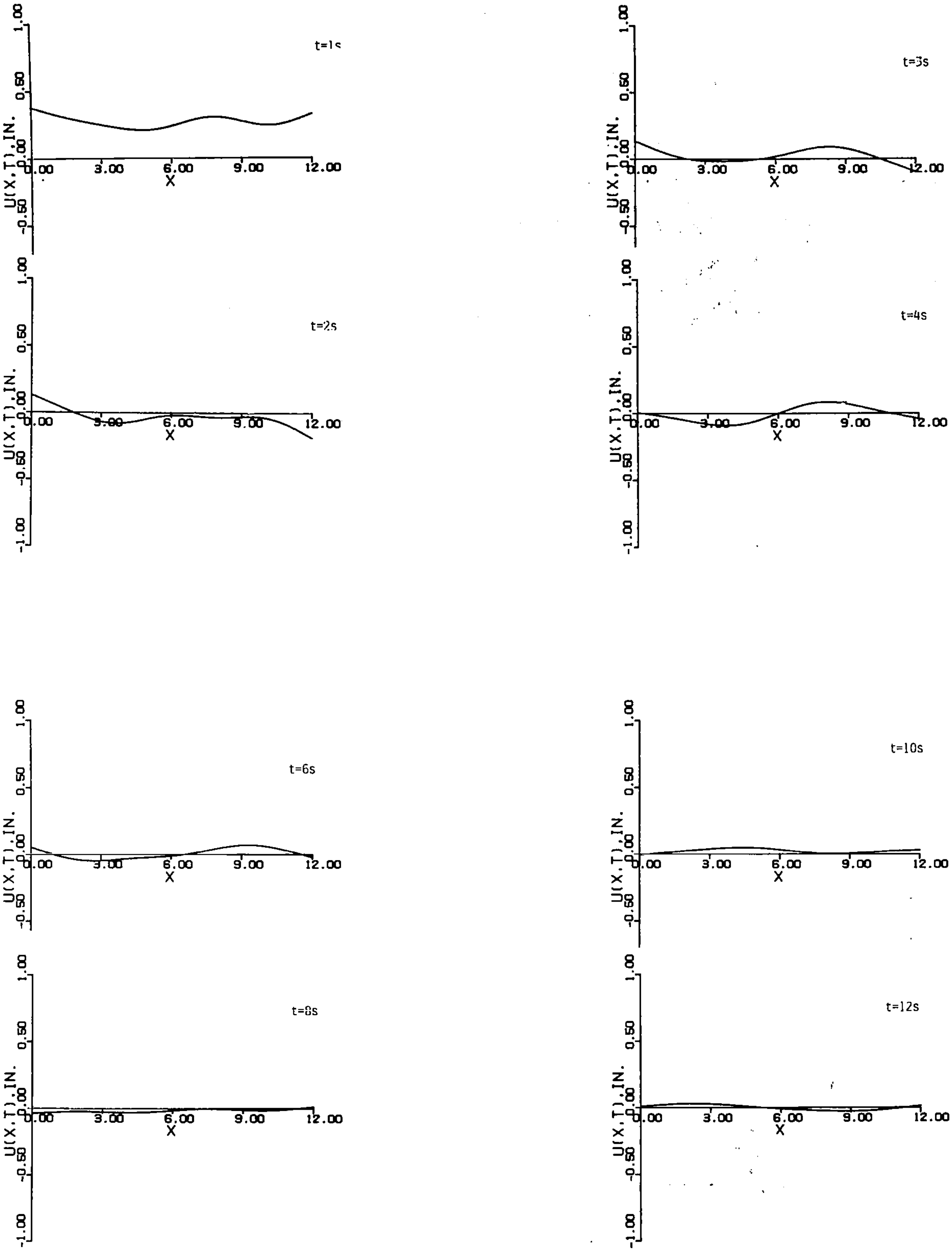
$\omega_1 = 0$
$\omega_2 = 0$
$\omega_3 = 11.47979$ rad/s
$\omega_4 = 31.64450$ rad/s
$\omega_5 = 62.03586$ rad/s
$\omega_6 = 102.5484$ rad/s
$\omega_7 = 153.1897$ rad/s

Simulation of the Beam motion:

The first 7 modes are included in the simulation: 4 controlled + 3 residual modes

Control Gain Parameters:
$d_1 = d_2 = 0.002$, $d_3 = d_4 = 0.0005$
$k_1 = k_2 = 0.3$, $k_3 = 0.12$, $k_4 = 0.03$
$\varepsilon_1 = \varepsilon_2 = 0.01$
$\eta_r = |\hat{u}_r| + |\hat{\dot{u}}_r| /10$, $r=1,2$
Sampling time = 1/33 sec.

Viscous damping was added to each flexible mode
Damping factor $\zeta_r = 0.002$, $r=3,4,5,6,7$
Disturbance of the beam was taken in the form of a unit impulse of magnitude 1/12 lb applied at $x_0 = 0.67L$.
The displacement of the beam cannot exceed 1 in at any time because of the location of the sensors and actuators in the experimental setup.



RESPONSE OF THE LANGLEY BEAM

THE LANGLEY BEAM EXPERIMENT (CONT'D)

Results

- The main contribution to the response is from the rigid-body modes.
- The second elastic mode shows noticeable participation. This participation will eventually disappear due to internal damping.
- Control of the second elastic mode can be enhanced by sensing velocities, or estimating velocitites via a Luenberger observer.
- Observation spillover (which may arise from the need of more sensors) was found to be negligible. So was the control spillover into the residual modes. Simulations of the beam with and without the residual modes indicated that spillover effects are infinitesimal. The reasons for this are:

    1) IMSC is used
    2) Modal filters are used
    3) Residual modes have very high frequencies

Conclusion

The IMSC method in conjunction with on-off modal controls does a good job in controlling the motion of the beam, where the motion consists primarily of the rigid-body modes.