# LARGE SPACE STRUCTURES

# CONTROL ALGORITHM CHARACTERIZATION

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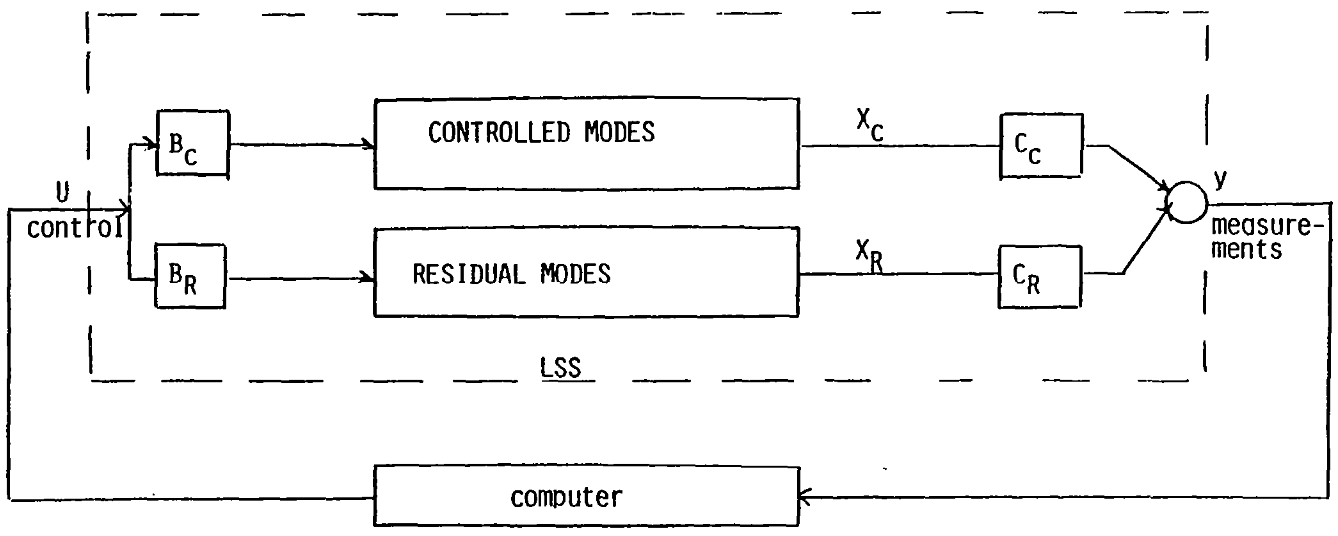
# Computation Consideration in
# LSS Control/Identification

- Algorithms
- Structures
- Computation considerations

Spillover Effect



## MODEL

$\dot{X} = AX + BU \qquad y = CX$

$A = \text{diag} \quad H_j \qquad H_j = \begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix}$

Separation to : controlled modes

$X_C$

: residual modes

$X_R$

$$X = \begin{bmatrix} X_C \\ X_C \end{bmatrix} \qquad \begin{cases} \dot{X} = \begin{bmatrix} A_C & \\ & A_R \end{bmatrix} X + \begin{bmatrix} B_C \\ B_R \end{bmatrix} U \\ y = C_C X_C + C_R X_R \end{cases}$$

## LAC/HAC

**LAC**: local feedback colocated sensor/actuator pairs

→ Augment damping

**HAC**: dynamic feedback to control a reduced order model

* frequency shaped K.F.

## HAC/LAC Control Algorithm

LAC: $U_L = \bar{G}\, y \qquad \bar{G} = \bar{G}^T > 0$

HAC:

$$\dot{\varphi} = \Omega\,\varphi + M X_C$$

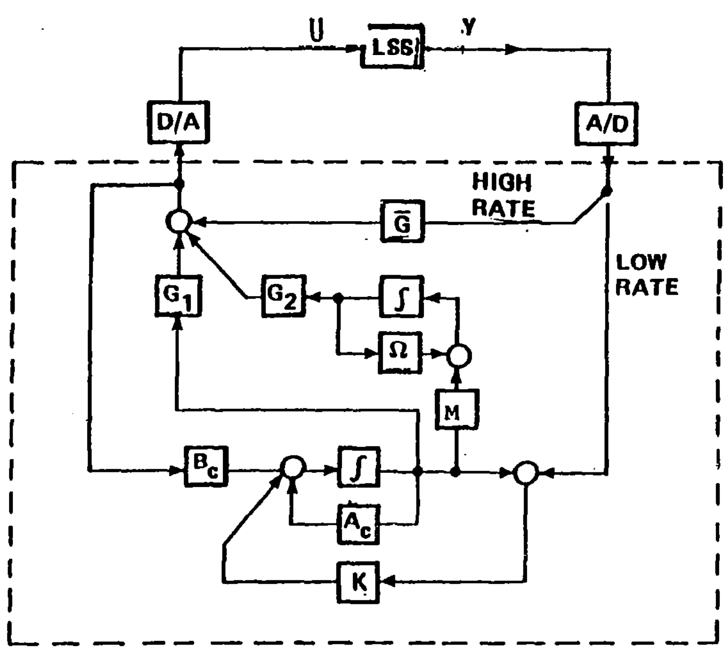
$$\dot{\hat{X}}_C = A_C \hat{X}_C + BU + K(y - C_C \hat{X}_C)$$

$$U_H = G_1 \hat{X}_C + G_2\,\varphi$$

$U = U_L + U_H$

Rate HAC rate = 1/2 LAC rate

## LAC/HAC BASED COMPUTATION REQUIREMENTS




COMPUTER

## Solution

$$\text{solve} \quad \text{LQG} \qquad \text{for } X_C$$

$$J = \int \left[ \| X_C \|_Q^2 + \| U \|_R^2 \right] dt$$

## Implementation

$$U = T K \hat{X}_C$$

$$\dot{\hat{X}}_C = A_C \hat{X}_C + B_C U + \bar{K} \bar{T} (y - C_C \hat{X}_C)$$

$$K = - R^{-1} B_C^T P$$

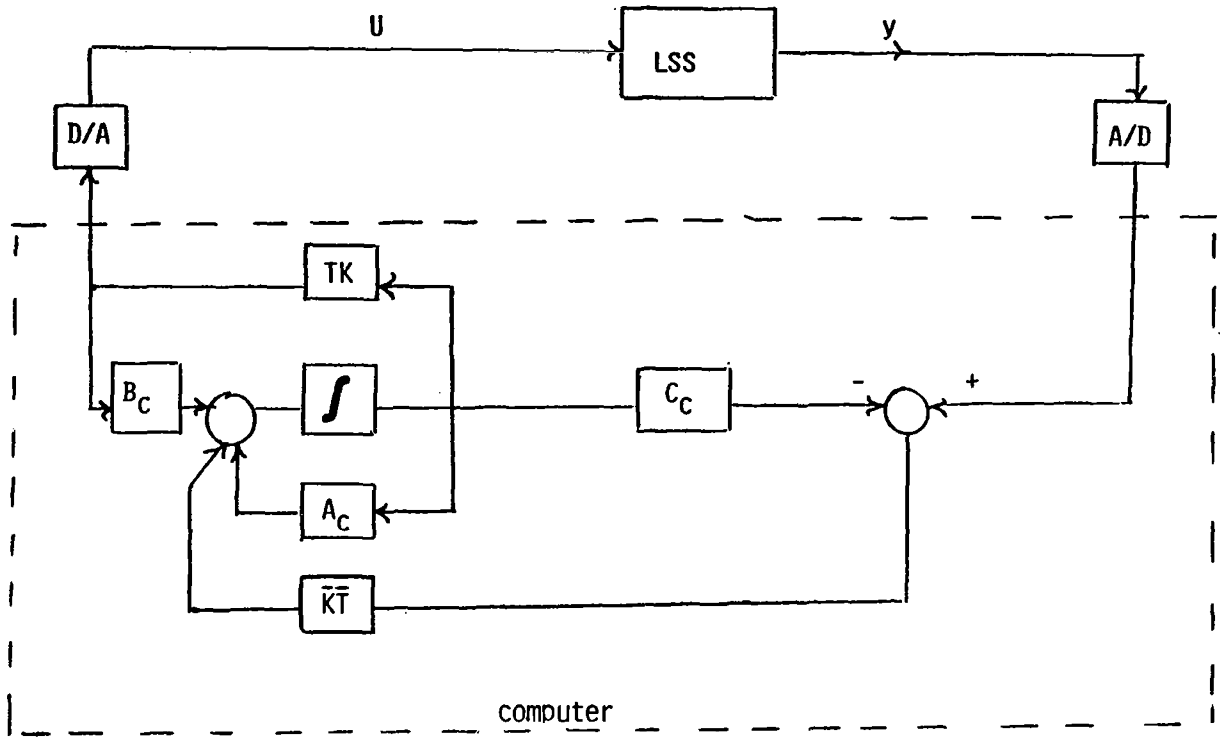
$$PA_C + A_C^T P + Q - PTB_C R^{-1} TB_C^T P = 0$$

$$\left.\begin{array}{llll} T: & B_R T = 0 & B_C T \neq 0 \\ \bar{T}: & \bar{T} C_R = 0 & \bar{T} C_C \neq 0 \end{array}\right\} \begin{array}{l} \text{orthogonality} \\ \text{conditions} \end{array}$$

Closed loop:

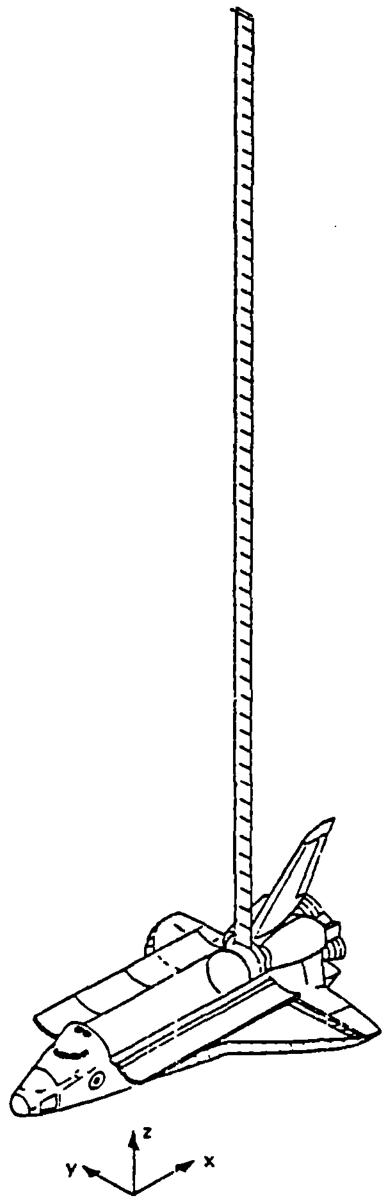
$$\frac{d}{dt} \begin{bmatrix} X_C \\ X_R \\ e \end{bmatrix} = \begin{bmatrix} A_C - B_C TK & 0 & B_C TK \\ 0 & A_R & 0 \\ 0 & 0 & A_C - \overline{KTC}_C \end{bmatrix} \begin{bmatrix} X_C \\ X_R \\ e \end{bmatrix}$$

$$e = \overset{\Delta}{X}_C - \hat{X}_C$$




## STRUCTURES USED AS EXAMPLES

- 100-METER BEAM
- 50-METER REFLECTOR ANTENNA



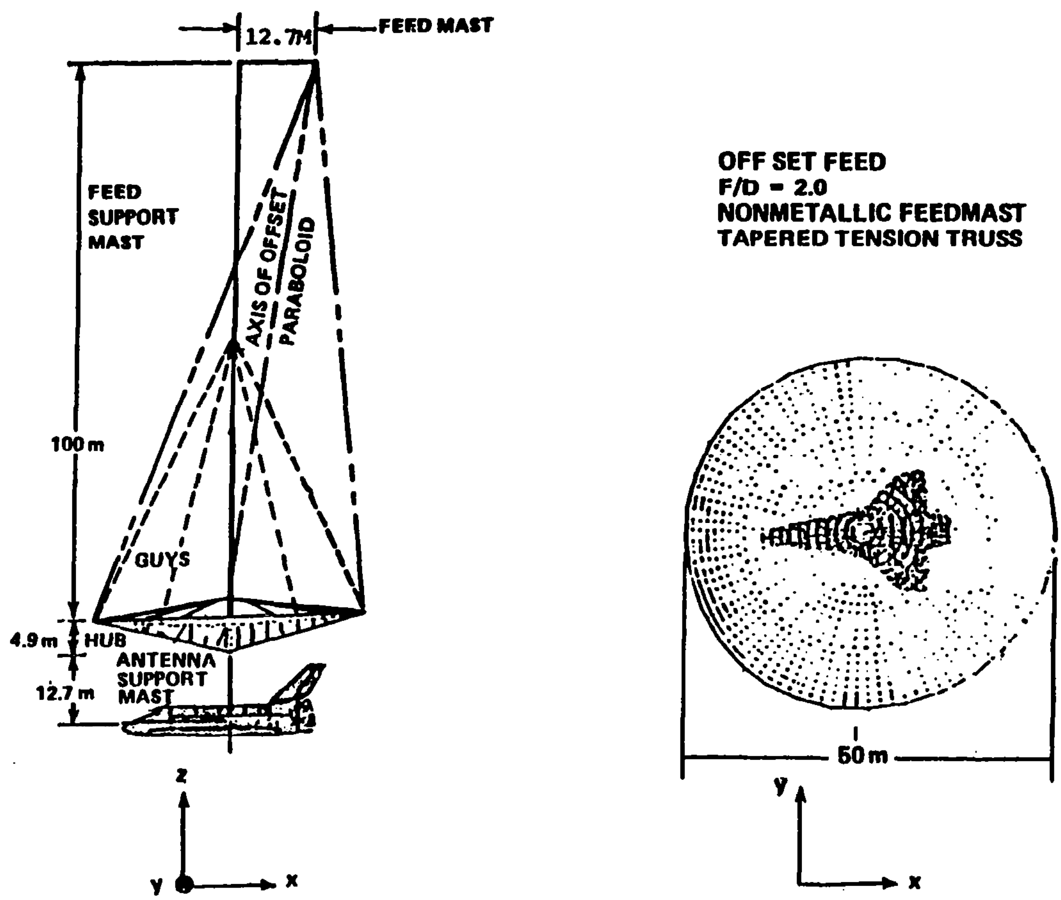

Beam Instrumentation

3 clusters of x y $\begin{cases}\text{accelerometer (sensor)} \\ \text{proof mass (actuator)}\end{cases}$

at: Top, Middle, Bottom

DIMENSION (M) OF SENSOR VECTOR AND ACTUATOR VECTOR = 3 x 2 = 6

**50m REFLECTOR**




Antenna Instrumentation (detail)

13 CLUSTERS OF COLOCATED SENSORS/ACTUATORS AS FOLLOWS:

1. MAST/ORBITER ATTACHMENT:

| | | |
|---|---|---|
| SENSORS | 2 DOF | ACCELEROMETER PKG (x, y) |
| ACTUATOR | 2 DOF | PROOF MASS |

2. REFLECTOR HUB (WHERE FEED SUPPORT MAST IS ATTACHED TO ANTENNA SUPPORT MAST)

| | | |
|---|---|---|
| SENSORS | 2 DOF | ACCELEROMETER PKG (x, y) |
| | 1 DOF | RATE GYRO (TORSION AXIS) |
| ACTUATOR | 2 DOF | PROOF MASS PKG |
| | 1 DOF | TORQUE WHEEL |

3. 8 CLUSTERS OF INSTRUMENTS AROUND RIM OF REFLECTOR:

| | | |
|---|---|---|
| SENSORS | 2 DOF | ACCELEROMETER (TANGENTIAL,+ z) TENSIOMETERS ON GUY WIRES |
| ACTUATORS | 2 DOF | PROOF MASS (tangential +Z), Guy Tensioner |

4. MIDDLE OF FEED SUPPORT:

| | | |
|---|---|---|
| SENSORS | 2 DOF | ACCELEROMETER PKG (x,y) |
| ACTUATORS | 2 DOF | PROOF MASS (x,y) |

5. FEED MAST/SUPPORT MAST ATTACHMENT:

| | | | |
|---|---|---|---|
| SENSORS: | 2 DOF | ACCELEROMETER | (x,y) |
| | 1 DOF | RATE GYRO | (TORSION) |
| ACTUATORS | 2 DOF | PROOF MASS | (x,y) |
| | 1 DOF | TORQUE WHEEL | (TORSION) |

6. AT FEED:

| | | | |
|---|---|---|---|
| SENSORS | 2 DOF | ACCELEROMETER | (y,z) |
| ACTUATORS | 2 DOF | PROOF MASS | (y,z) |

- DIMENSION OF SENSOR/ACTUATOR VECTORS (M) = 2+3+2+3+2+8x3 = 36

## LQG AND HAC/LAC COMPUTATIONAL SIZING

- THESE ALGORITHMS HAVE BEEN SIZED IN TERMS OF
  - FLOATING POINT OPERATION (FLOP) DEMANDS
  - STORAGE FOR VARIABLES
  - INPUT/OUTPUT DATA FLOW
- FLOP SIZING (PER CONTROL CYCLE) DONE AS A FUNCTION OF THE NUMBER OF CONTROL STATES AND THE NUMBER OF SENSOR/ACTUATOR PAIRS
- STORAGE FOR VARIABLES AND I/O SIZING DONE FOR SPECIFIC STRUCTURE EXAMPLES

## Input/Output Data Flow Rates

<u>Assumption</u>
- Control bandwidth 50 Hz
- Accuracy - 2 byte/word

→ { Sampling frequency 250 Hz<br>Command frequency

<u>Data Flow rate</u> per { sensor / actuator } 500 [Bytes/sec]

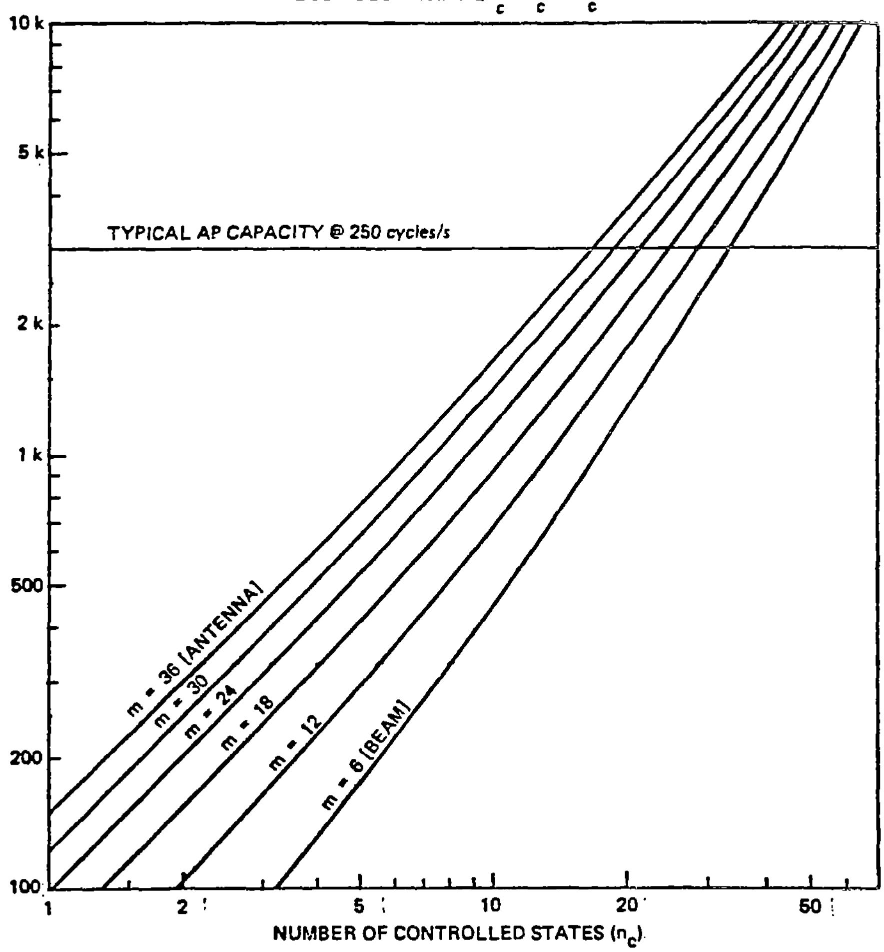
| | | | |
|---|---|---|---|
| <u>Total</u>: | Beam: | 3,000 | [Bytes/sec] |
| | Antenna: | 18,000 | [Bytes/sec] |
| 1553B bus capacity | | 48,000 | [Bytes/sec] |

## LQG SIZING

| | BEAM | ANTENNA |
|---|---|---|
| SENSOR/ACTUATOR PAIRS (m) | 6 | 36 |
| CONTROL STATES ($n_c$) | 20 | 20 |
| FLOP PER CYCLE* | 1420 | 4420 |
| VARIABLES** | 752 | 2312 |
| I/O PER CYCLE | 12 | 72 |

*INCLUDES SENSOR COMPENSATION FLOP (120 FOR BEAM, 720 FOR ANTENNA)

**INCLUDES SENSOR COMPENSATION VARIABLES (60 FOR BEAM, 360 FOR ANTENNA)



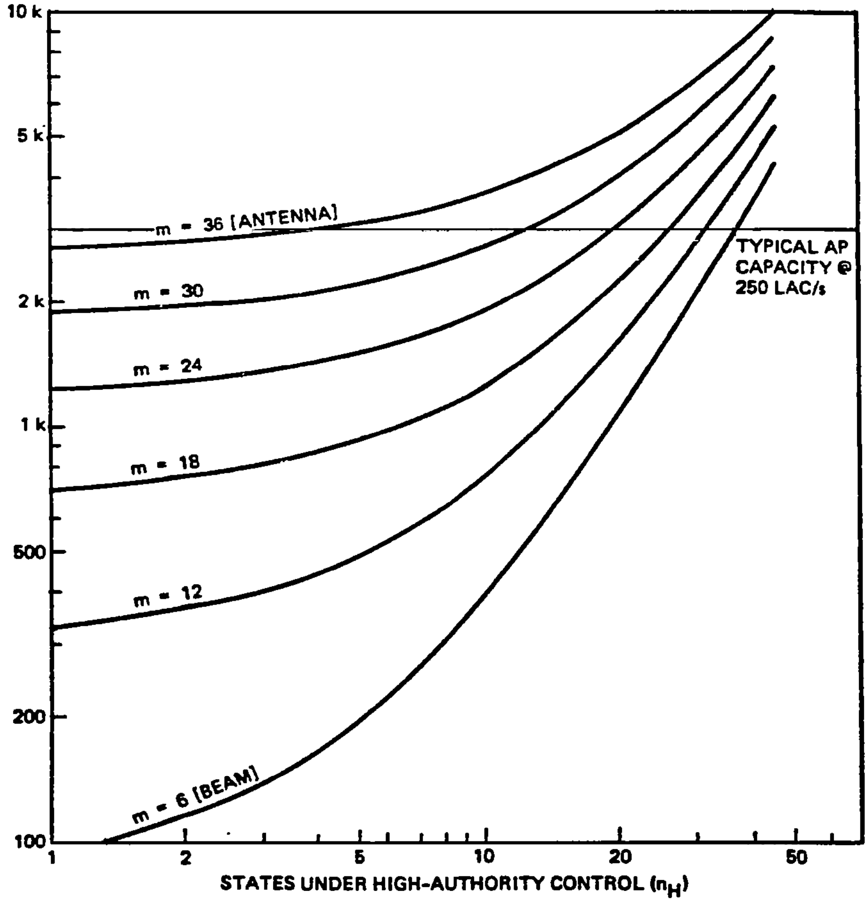
# HAC/LAC SIZING

| | BEAM | ANTENNA |
|---|---|---|
| SENSOR/ACTUATOR PAIRS (m) | 6 | 36 |
| CONTROL STATES ($n_C$) | 12 | 12 |
| FLOP PER CYCLE* | 633 | 4608 |
| VARIABLES** | 570 | 3060 |
| I/O | 12 | 72 |

*INCLUDES SENSOR COMPENSATION FLOP (120 FOR BEAM, 720 FOR ANTENNA)
**INCLUDES SENSOR COMPENSATION VARIABLES (60 FOR BEAM, 360 FOR ANTENNA)

FLOP/LAC CYCLE

(LAC FLOP + ½ HAC FLOP): $\frac{16}{9}n_H^2 + \frac{7}{6}n_H + \frac{7}{3}n_H m + \frac{m}{2} + 2m^2$



## COMPUTATION LOAD

| Structure | Algorithm | m | n | $n_C$ ($n_H$) | rate K Flops/sec | % GPC capacity | % typical AP capacity |
|---|---|---|---|---|---|---|---|
| Beam | LQG | 6 | 12 | 6 | 55 | 67 | 7 |
| | | 6 | 16 | 10 | 112 | 135 | 14 |
| | HAC/LAC | 6 | 12 | 6 | 34. | 41 | 4 |
| | | 6 | 16 | 10 | 57 | 69 | 7 |
| Antenna | LQG | 36 | 42 | 6 | 260 | 300 | 36 |
| | HAC/LAC | 36 | 42 | 6 | 750 | 900 | 100 |

m = # of sensors/actuators
n = # of modes in model

$\left.\begin{matrix} n_C \\ n_H \end{matrix}\right\}$ = # of controlled modes

## SYSTEM IDENTIFICATION COMPUTATIONAL SIZING

- ARMA-LEAST SQUARES ALGORITHM SIZED FOR FLOP AS FUNCTION OF MODEL ORDER (N) AND NUMBER OF SENSOR/ACTUATOR PAIRS (m)
- FLOP REQUIREMENTS FOR THIS ALGORITHM ARE SO LARGE THAT IMPLEMENTATION IN A FLIGHT SYSTEM OR ITS GTF ANALOG IS PRECLUDED
- EVEN IMPLEMENTATION IN GROUND-BASED COMPUTERS IS CONSIDERED QUESTIONABLE, BUT THIS STUDY ASSUMES A GROUND-BASED IMPLEMENTATION

- NOTE: SOME OTHER SYSTEM IDENTIFICATION ALGORITHM MAY BE IMPLEMENTABLE IN A FLIGHT SYSTEM

- Algorithm Assessed - Least Squares

  Motivation for choosing LS

  - Relative high spectral resolution
  - Comparable to other algorithm in computation complexity

    e.g.: Covariance algorithm
    : Maximum Entropy
  - "Better" algorithms - considerably more complicated
  - Less complex algorithms - considerable penalty in performance
  - LS - robust to order reduction
  - Useful for - control design
    - self tuning regulators

## Identification Algorithm Sizing

Assume the ARMA model

$$y_k = \sum_{i=1}^{N} A_i y_{k-i} + \sum_{i=0}^{N-1} B_i U_{k-i}$$

where $y_k$ = vector of measurements (sensors) at cycle k

$U_k$ = vector of control influence at cycle k

we can write

$$y_k = \left[ A_1 \,\vdots\, . \; . \; . \,\vdots\, A_n \,\vdots\, B_0 \,\vdots\, \cdots \; B_{n-1} \right] \begin{bmatrix} y_{k-1} \\ \vdots \\ y_{k-n} \\ U_k \\ \vdots \\ U_{k-n+1} \end{bmatrix}$$

$$= \alpha \cdot z_k$$

Use least squares identification

## SYSTEM IDENTIFICATION ALGORITHM FLOP REQUIREMENTS

| | BEAM | ANTENNA |
|---|---|---|
| SENSOR/ACTUATOR PAIRS (m) | 6 | 36 |
| MODES MODELLED (N) | 12 | 42 |
| OFF-LINE MEGAFLOP FOR 4000 CYCLES | 354.2 | 297,779 |
| OFF-LINE FLOP/CYCLE | 88,552 | 74,444,881 |
| OFF-LINE MEGAFLOPS (@ 250 CPS) | 22.1 | 18,611 |
| ON-LINE FLOP/CYCLE | 169,784 | 73,601,174 |
| ON-LINE MEGAFLOPS (@ 250 CPS) | 42.5 | 18,400 |