ACTIVE CONTROL OF A FLEXIBLE BEAM

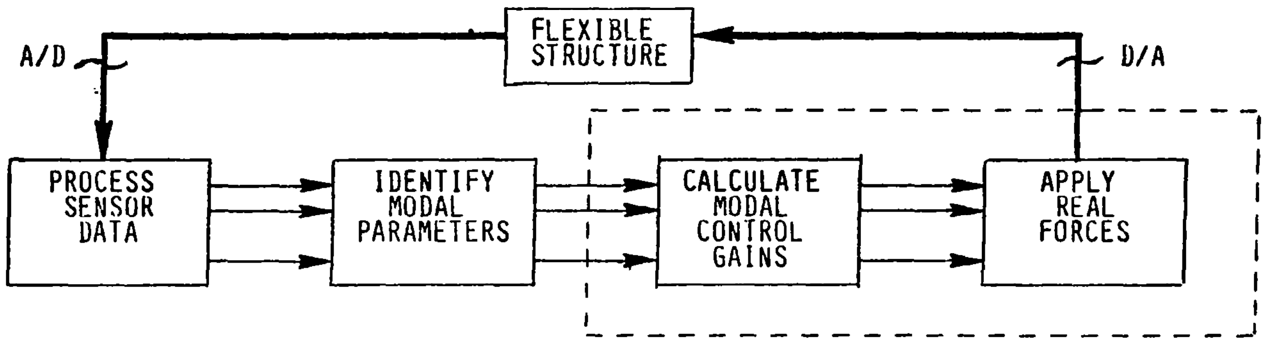
Jeffrey P. Williams
NASA Langley Research Center
Hampton, Virginia

ACTIVE CONTROL OF A FLEXIBLE BEAM

Because of inherent low damping and high flexibility, large space structures may require some form of active control of their dynamics. Because of the apparent inability to accurately model the dynamics of these structures, methods for parameter adaptive control are now being developed at Langley. The process currently being studied is shown in the block diagram below. This approach uses a digital computer to process discrete sensor data, identify modal parameters, calculate modal control gains, and then convert the modal forces to real forces. The last two blocks are the topic of this presentation. Some of the problems considered are: (1) the possibility that there may be many modes to control with limited amounts of hardware, and (2) the required accuracy of identified structural parameters.

BACKGROUND:

- NEED TO CONTROL FLEXIBLE MOTION OF LARGE SPACE STRUCTURES
- ABILITY TO ACCURATELY MODEL THE DYNAMICS OF THESE STRUCTURES IS UNCERTAIN
- THEORY NOW BEING DEVELOPED FOR PARAMETER ADAPTIVE CONTROL OF THESE STRUCTURES



PROBLEMS:

- POSSIBLY MANY MODES TO CONTROL
- LIMITED HARDWARE (COMPUTATION, SENSORS, ACTUATORS)
- HOW WELL STRUCTURAL PARAMETERS MUST BE IDENTIFIED FOR CONTROL

RESEARCH TASK

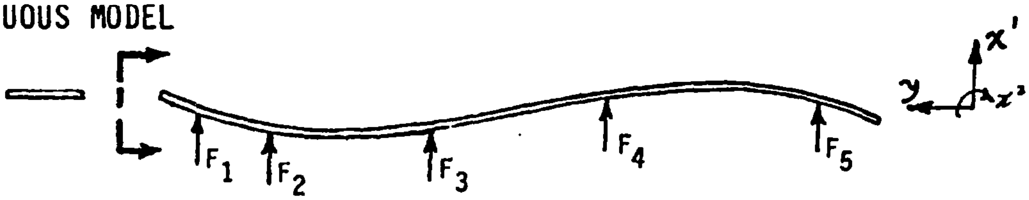
The specific research task was to design a digital control scheme to suppress vibration of a homogeneous free-free beam. A digital computer simulation algorithm was then used to test (1) the effects of controlling more modes than available actuators, and (2) the sensitivity to identified structural parameters.

- DESIGN A DIGITAL CONTROL SCHEME TO SUPPRESS VIBRATION OF A HOMOGENEOUS FREE-FREE BEAM
- EXAMINE EFFECT ON STABILITY OF:
  - FEWER ACTUATORS THAN CONTROLLED MODES
  - ERRORS IN STRUCTURAL MODEL PARAMETERS
- TEST WITH AN EXISTING SIMULATION ALGORITHM

MATHEMATICAL MODEL

The continuous beam was modeled by using the SPAR finite element algorithm which generates mode shapes and frequencies. These were used to write a modal representation of the beam dynamics which was used to design the control gains.

- CONTINUOUS MODEL



- FINITE ELEMENT MODEL OF BEAM
  - MODE SHAPES AND FREQUENCIES FOR 25 ELEMENT MODEL ARE GENERATED BY SPAR

$$[M]\ddot{\underline{x}} + [K]\underline{x} = \underline{F}$$

- MODAL REPRESENTATION

$$x = [E]\, q$$

$$\begin{bmatrix} \ddots & & \\ & m_i & \\ & & \ddots \end{bmatrix} \ddot{\underline{q}} + \begin{bmatrix} \ddots & & \\ & k_i & \\ & & \ddots \end{bmatrix} \underline{q} = \underline{u} \; ; \quad \underline{u} = [E]^T \underline{F}$$

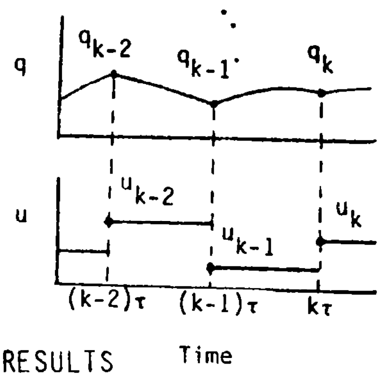
  - SET OF 50 UNCOUPLED 2ND-ORDER SYSTEMS

- CONTROL SYSTEM DESIGNED USING THIS MODAL REPRESENTATION OF THE STRUCTURAL DYNAMICS

DISCRETE TIME MODEL

In order to simplify the digital simulation and prepare for eventual digital implementation, the modal equations of motion were discretized. This results in the scalar equation which shows the present modal amplitude to be a function of the past two amplitudes and the past two controls. There is a discrete time transformation analogous to the Laplace transform which results in a characteristic polynomial in z. The roots of this polynomial may be plotted in the complex plane with stability represented by magnitudes of less than 1. Analysis of the control system will be done primarily in this z-plane.

- NECESSARY FOR:
  - DESIGN OF DIGITAL SIMULATION
  - EVENTUAL DIGITAL IMPLEMENTATION



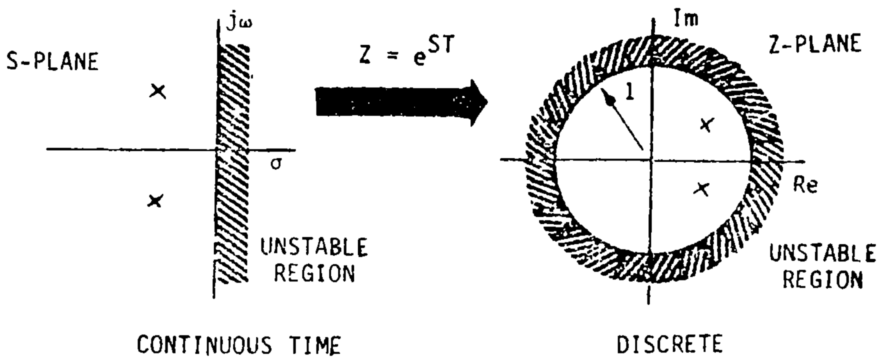
- DISCRETE EQUATION OF MOTION:

$$q_{(k)} = A_1\, q_{(k-1)} + A_2\, q_{(k-2)} + B_1\, u_{(k-1)} + B_2\, u_{(k-2)},$$

where $A = f(w, \tau)$, τ = sampling interval,

$B = f(w, \underline{e}, \tau)$, $\underline{e}$ = mode shapes.

- DISCRETE TIME TRANSFORM (ANALOGOUS TO LAPLACE TRANS.) RESULTS IN A CHARACTERISTIC POLYNOMIAL IN Z: $0 = \sum_{i=1}^{n} c_i\, z^{i-1}$



CONTROL DESIGN APPROACH

The modal control design approach is to choose desired closed-loop roots from which the modal controller gains can be calculated. The modal control forces may be calculated directly, and the actual control forces can be calculated using a pseudo-inverse.

- CHOOSE DESIRED CLOSED LOOP ROOTS FOR EACH MODE
- CALCULATE MODAL CONTROLLER GAINS
- CALCULATE MODAL CONTROL FORCES
- CONVERT MODAL FORCES TO ACTUATOR FORCES

DIGITAL CONTROL OF ONE MODE

The control of one mode is achieved by using the minimum order control law required for pole placement. This is of the same form as the modal equation of motion. The closed loop controller has a fourth-order characteristic equation as shown. The coefficients of this equation are determined from the desired closed-loop roots and are functions of the mode and control coefficients in the plant and control modal equations. The control objective is to achieve the desired roots defined by (a,b,c,d) by solving for the controller gains (C_1, C_2, D_1, D_2).

- MINIMUM ORDER CONTROL LAW FOR POLE PLACEMENT

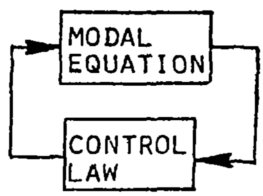
  $u_{(k)} = C_1\, q_{(k-1)} + C_2\, q_{(k-2)} + D_1\, u_{(k-1)} + D_2\, u_{(k-2)}$

  - NOTE SAME FORM AS PLANT MODEL

  $[q_{(k)} = A_1\, q_{(k-1)} + A_2\, q_{(k-2)} + B_1\, u_{(k-1)} + B_2\, u_{(k-2)}]$

  - C's & D's ARE FOUR CONTROL GAINS

- CLOSED LOOP CONTROLLER



  - HAS DISCRETE TIME CHARACTERISTIC EQUATION

  $$z^4 + a\,z^3 + b\,z^2 + c\,z + d = 0$$

  where $\{a, b, c, d\} = f(A, B, C, D)$.

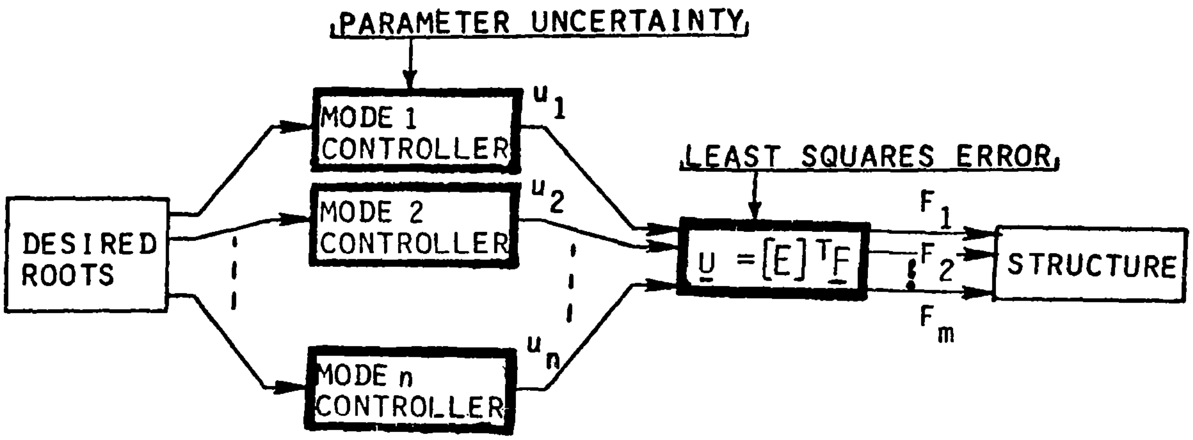
- CONTROL OBJECTIVE:

  - ACHIEVE DESIRED CLOSED LOOP ROOTS (AS DEFINED BY a,b,c,d) BY CALCULATING THE CONTROL GAINS.

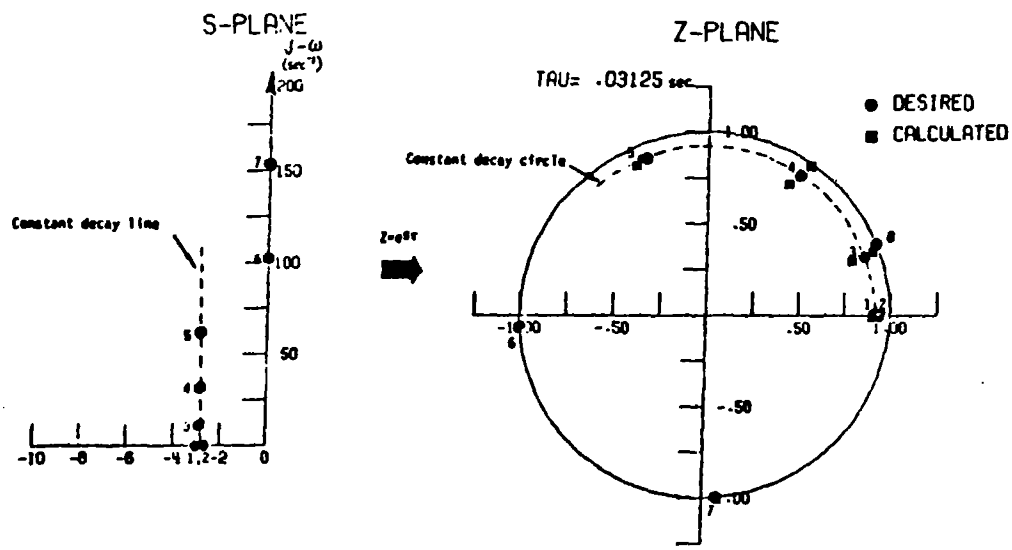
CLOSED-LOOP CHARACTERISTICS

The control problem is shown in the block diagram below. The parameter uncertainties affect the calculation of the modal controller gains and a pseudo-inverse results in a least squares type error in the actual forces applied to the beam. A typical set of closed-loop roots is shown in each of the two complex plane plots. The design criterion is to place the roots of the controlled modes on a constant damping line in the s-plane. This line maps onto a constant-radius circle on the complex z-plane, with uncontrolled roots on the unit circle.

o CONTROL PROBLEM



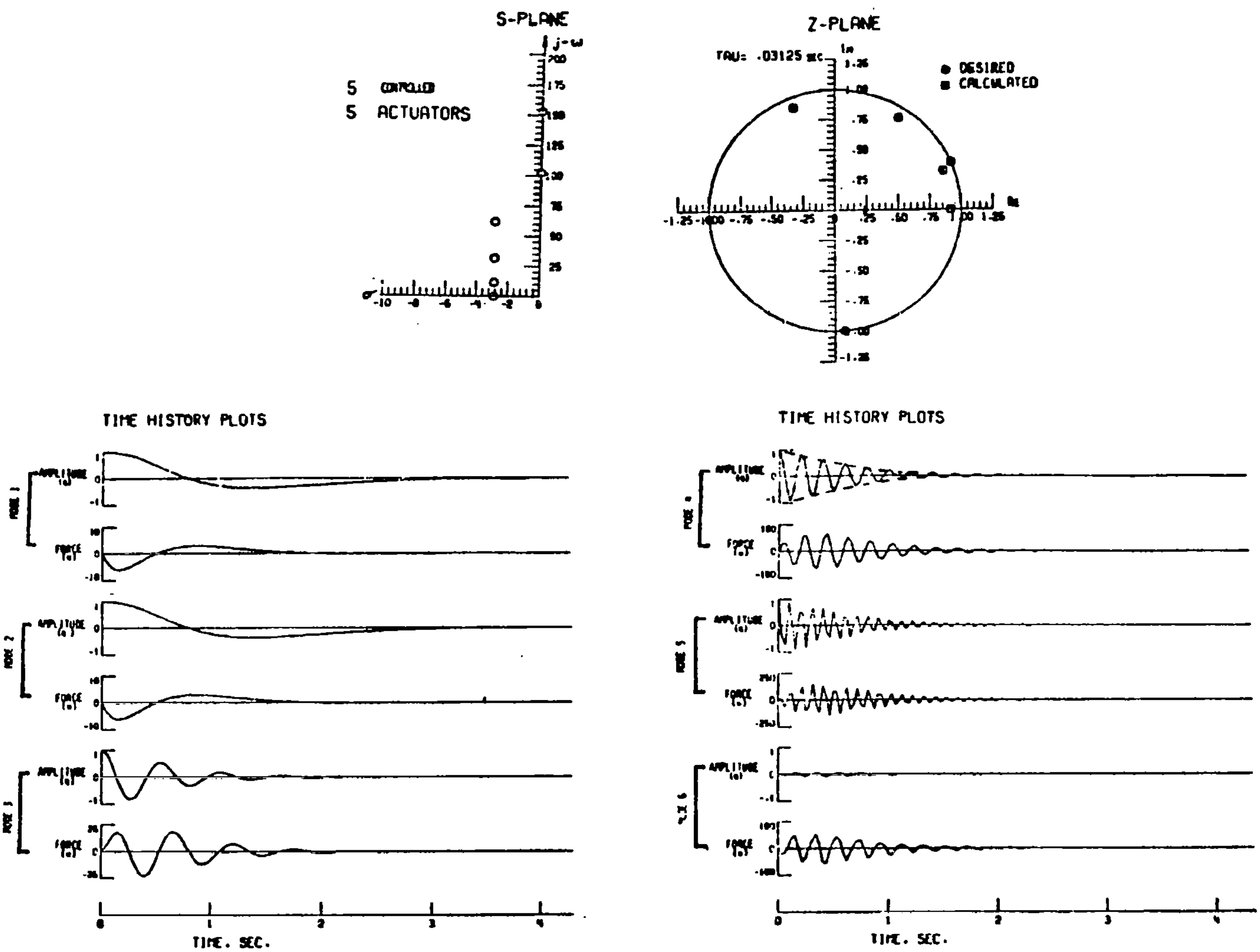
o REPRESENTATION OF CLOSED LOOP DYNAMICS



RESULTS - CASE 1

A baseline set of results is shown below. Exact parameters are used to calculate the control gains and five modes are controlled with five actuators. Note that all roots are calculated exactly so that all modal amplitudes have the same decay envelope. Also, mode six, which is not controlled, does show "minor" excitation and continues to "ring" after control to the other modes is stopped. This is not evident from the figure.

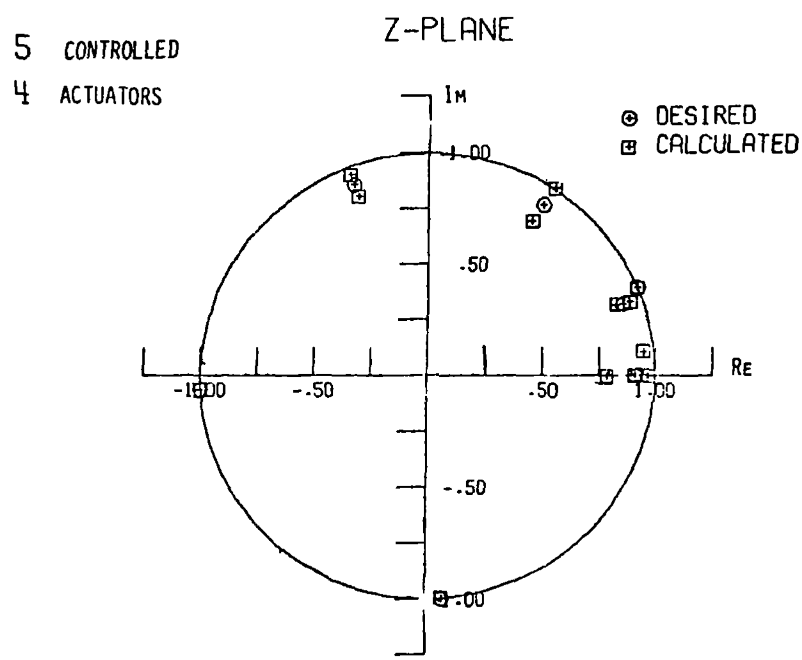
• CASE I: EQUAL NUMBER OF ACTUATORS AND CONTROLLED MODES

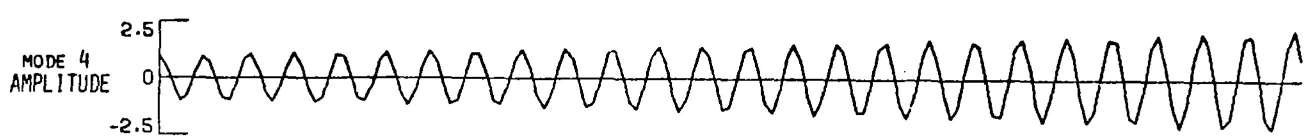


RESULTS - CASE II

This next case illustrates an attempt to control more modes than available actuators. Here it is no longer possible to solve exactly for the desired roots, and one root does become unstable, as shown by the increasing amplitude of the fourth mode.

• CASE II: FEWER ACTUATORS THAN CONTROLLED MODES - UNSTABLE

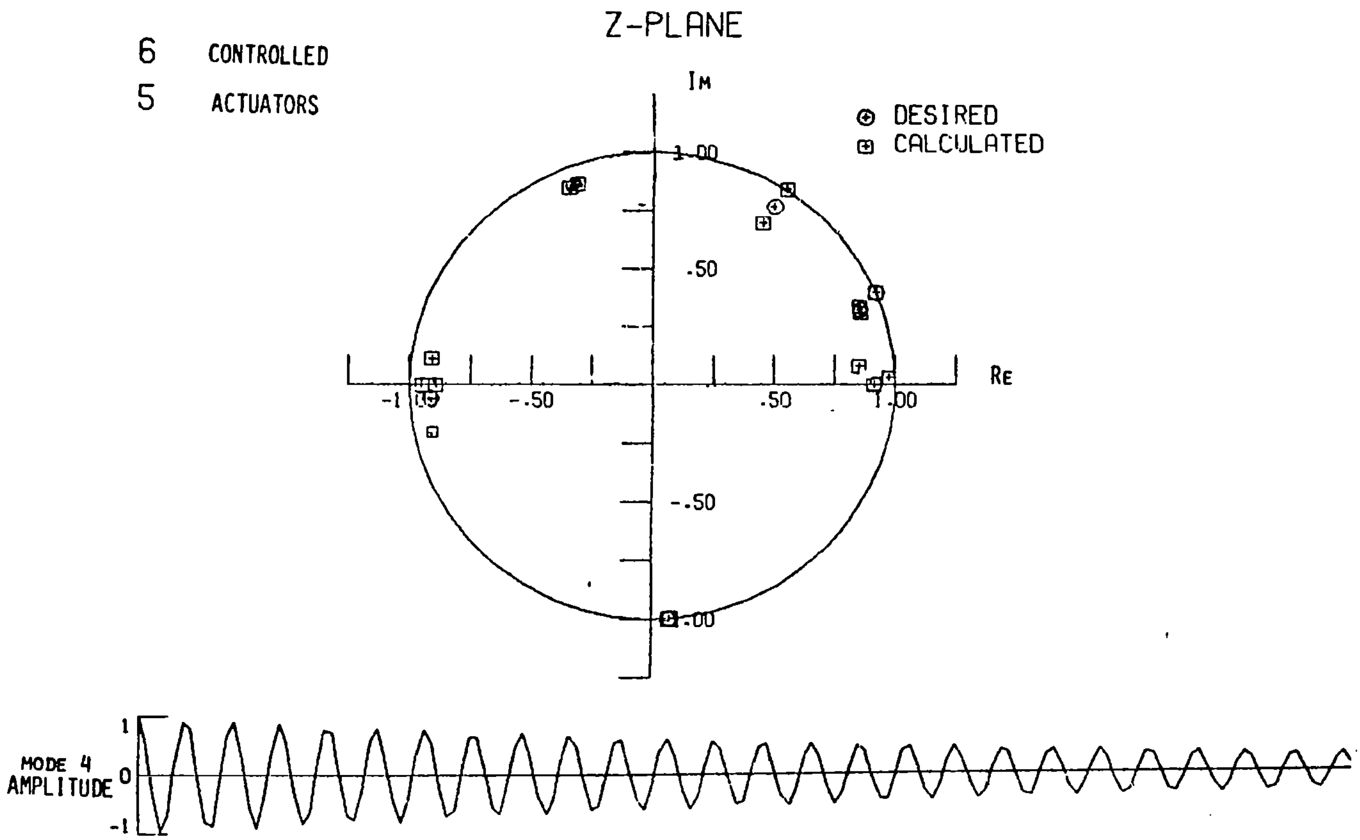




RESULTS - CASE III

In the third case it is shown that attempting to control more modes than available actuators does not necessarily mean the system will be unstable. Note here that the calculated roots are closer to the desired roots and that, while the fourth mode is near the unit circle, it now is slightly stable. This can be explained by considering that the additional actuator and mode provide one more data point for the least squares fit.

• CASE III: FEWER ACTUATORS THAN CONTROLLED MODES - STABLE



PARAMETER ERROR SENSITIVITY

The results of the parameter error sensitivity study are shown below. Error was placed on the modal frequency and damping parameters of the modes selected for control. Experimental results show that the parameter estimator may have errors on the order of ±5 percent. The locus of roots calculated using a parameter error range of ±20 percent is plotted below. The magnitude of the error at which the modes became unstable is summarized in the table. Note that as the mode number increases, the sensitivity decreases for the first three modes. The high sensitivity of mode six in case III is unexplained at this time.

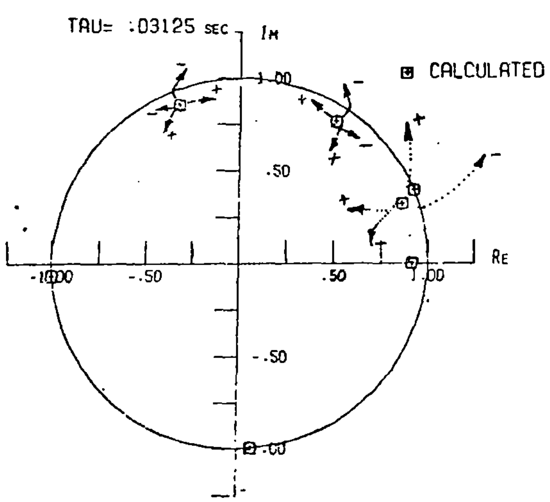
- ERROR ON MODAL FREQUENCY AND DAMPING PARAMETERS ONLY

$$q^i_{(k)} = \underline{A^i_1}\, q_{(k-1)} + \underline{A^i_2}\, q_{(k-2)} + B^i_1\, u_{(k-1)} + B^i_2\, u_{(k-2)}$$

- NO ERROR ON RIGID BODY PARAMETERS
- BASED ON EXPERIMENTAL PARAMETER ESTIMATE ERRORS OF ± 5%

5 CONTROLLED
5 ACTUATORS

Z-PLANE



	PARAMETER ERROR AT INSTABILITY			
CASE	MODE 3	MODE 4	MODE 5	MODE 6
I 5 CONT. 5 ACT.	-10 % +8 %	-12% —	-14% —	
II 5 CONT. 4 ACT.	-6 % +14%	0.0 % —	-14 % —	
III 6 CONT. 5 ACT.	-8 % +8 %	-12 % —	-16% —	-2 % +2 %

CONCLUSIONS

1. LIMITED ACTUATORS:

   - THIS DESIGN PROCESS YIELDS UNDESIRABLE CLOSED LOOP DYNAMICS WHEN THE NUMBER OF CONTROLLED MODES EXCEEDS THE NUMBER OF AVAILABLE ACTUATORS.

2. PARAMETER ERROR:

   - ERRORS WITHIN THE RANGE OF EXPERIMENTAL RESULTS CAN CAUSE INSTABILITY.

   - CONTROL SYSTEM MUST BE MADE MORE TOLERANT OF PARAMETER ERROR.