# OPTIMAL DAMPING FOR A TWO-DIMENSIONAL STRUCTURE

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## SYNTHESIS OF THE DAMPING MATRIX FOR SPECIFIED DAMPING RATIOS

$[M]\{\ddot{x}\} + [k]\{x\} = \{F\}$ $\{x\}$ is $n \times 1$

### CONTROL FORCES

$\{F\} = [B]\{u\}$ $\{u\}$ is $A \times 1$

$A$ = number of actuators

$\{u\} = -[D]\{\dot{x}\}$ $[D]$ is $A \times n$

or

$$\{F\} = -[C]\{\dot{x}\}$$

where

$$[C] = [B][D]$$

PROBLEM

Find $C_{ij}$ such that

$$J = \sum_{i,j} | C_{ij} |$$

is minimized subject to the constraints

$$\zeta_\ell > \zeta_{\ell p} \quad \text{for } \ell = 1 \text{ to } L$$

and

$$C_{ii} > 0 \quad , \quad C_{ij} = C_{ji}$$

$\zeta_{\ell p}$ = prescribed damping ratios for the $\ell$th mode

Eigenvalue Problem:

$$(s^2[M] + s \quad [C] + [K])\{\rho\} = \{0\}$$

with roots

$$s_\ell = -\zeta_\ell \omega_{\ell n} + j\omega_{\ell n} \sqrt{1 - \zeta_\ell^2}$$

DAMPED EIGENVALUE PROBLEM:

$$\{\dot{z}\} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}_{2n \times 2n} \{z\}$$

ALTERNATE EFFICIENT FORMULATION FOR CHARACTERISTIC EQUATION:

$$\det([I] + s[\hat{R}(s)][\hat{C}]) = 0$$

ASSUMING NO. OF DAMPERS << NO. OF D.O.F

$[\hat{R}]$ = SUBMATRIX OF $[R]$

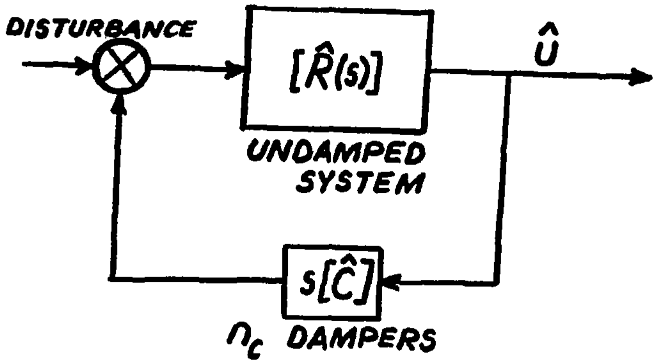
$$[R] = (s^2[M] + [K])^{-1}$$

SPECTRAL REPRESENTATION

$$R_{ik} = \sum_{l=1}^{n} \frac{\phi_{il}\,\phi_{kl}}{G_l(s^2 + \omega_l^2)}$$

$$\simeq \sum_{l=1}^{L} \frac{\phi_{il}\,\phi_{kl}}{G_l(s^2 + \omega_l^2)}$$

DERIVATION OF CHARACTERISTIC EQUATION :




$\therefore$ CHARACTERISTIC EQ. IS :

$$\det([I] + s[\hat{R}][\hat{C}]) = 0$$

SPECIAL CASE :

INTRODUCE SINGLE DAMPER AT D.O.F. J

CHARACTERISTIC EQUATION:

$$1 + s c R_{JJ} = 0$$

OR

$$\frac{1}{c} = -\sum_{l=1}^{n} \frac{s\,\phi_{Jl}^2}{G_l(s^2+\omega_l^2)}$$

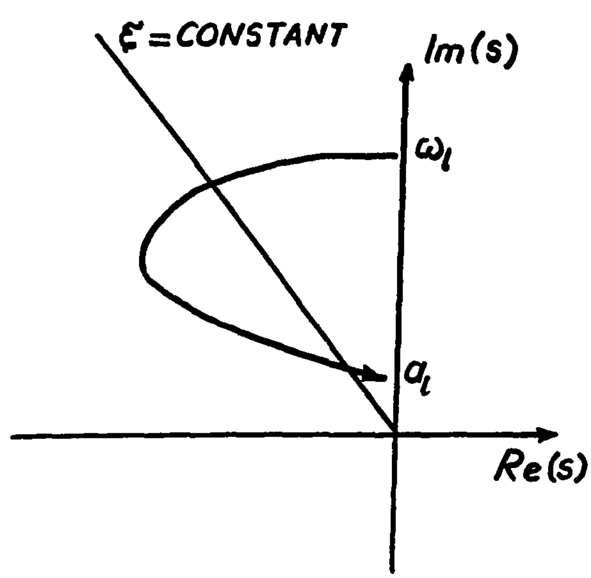
DEFINITION:

OPTIMAL DAMPER LOCATIONS FOR A PARTICULAR MODE ARE WHERE EITHER

MAXIMUM DAMPING CAN BE INTRODUCED

OR

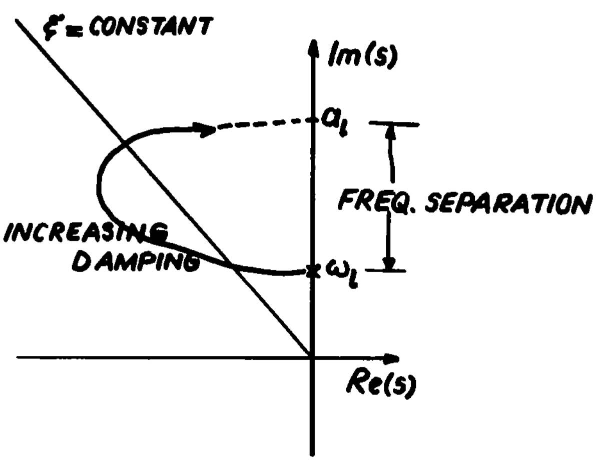
ACHIEVE GIVEN DAMPING WITH MINIMUM DAMPING CONSTANTS




TYPICAL ROOT LOCUS PLOT

# MINIMUM CONSTRAINED FREQUENCY CRITERION (MCFC)

THE OPTIMAL DAMPER LOCATION IS WHERE THE CONSTRAINED FREQUENCY IS A MINIMUM




TYPICAL ROOT LOCUS FOR INTRODUCING MORE DAMPERS THAN NO. OF RIGID BODY MODES

## MAXIMUM FREQUENCY SEPARATION CRITERION (MFSC)

THE OPTIMAL DAMPER LOCATION IS WHERE THE SEPARATION BETWEEN THE CONSTRAINED FREQUENCY AND THE NATURAL FREQUENCY IS MAXIMUM

## SYNTHESIS OF SINGLE DAMPER

CHARACTERISTIC EQUATION IS

$$f(s) = 1 + sR_{JJ}(s)c$$
$$= 1 + cP(s) + jcQ(s) = 0$$

SPECIFY $\xi$ (OR $\omega_{nl}$)

THEN

$$s_l = -\xi_l \omega_{nl} + j\sqrt{1-\xi_l^2}\,\omega_{nl}$$

SOLVE FOR $\omega_{nl}$ (OR $\xi_l$)

FROM $Q(s_l) = 0$

THE DESIRED DAMPING CONSTANT

$$c = -\frac{1}{P(s_l)}$$

$c > 0$ IF $s_l$ IS REALIZABLE

# SYNTHESIS OF MULTIPLE DAMPERS

CHARACTERISTIC EQUATION IS

$$f(s) = \det(I + s\hat{R}(s)\hat{c})$$
$$= P(s, \underline{c}) + jQ(s, \underline{c}) = 0$$

CASE 1

SPECIFY $nc$ $\xi$'s

THEN SOLVE FOR

$$\underline{c} = [c_1, c_2, \ldots, c_{nc}]^T$$

AND

$$\omega_{n_1}, \omega_{n_2}, \ldots, \omega_{n_{nc}}$$

FROM THE $2nc$ EQS

$$\left.\begin{array}{l} P(s_l, \underline{c}) = 0 \\ Q(s_l, \underline{c}) = 0 \end{array}\right. \quad \text{FOR } l = 1 \text{ TO } nc$$

CASE 2

SPECIFY $E$ $\xi$'S WITH $E < nc$

THEN $\underline{c}$ AND $\omega_{n_1}, \omega_{n_2}, \ldots, \omega_{n_E}$
CAN BE FOUND FROM

$$\text{MINIMIZE } J = \sum_{i=1}^{nc} c_i + \sum_{l=1}^{E} |f(s_l)|$$

$$\text{WITH} \quad c_i \geqslant 0$$
$$\omega_n \geqslant 0$$

## RANK OF SOME TWO DAMPER LOCATIONS ACCORDING TO MCFC

| DAMPER LOCATIONS | CORRESPONDING LOWEST CONSTRAINED FREQUENCY | RANK |
|---|---|---|
| 1,88 | 4.5 8 RAD/SEC | 1 |
| 1,50 | 6.89 | 3 |
| 11,58 | 7.50 | 4 |
| 34,44 | 6.54 | 2 |

## DAMPING CONSTANTS REQUIRED TO ACHIEVE $\xi_4 = 0.6$

| DAMPER LOCATION | | REQUIRED | | |
|---|---|---|---|---|
| NO. 1 | NO. 2 | $C_1$ | $C_2$ | $C_1 + C_2$ |
| 1 | 88 | 0.296 | 0.136 | 0.432 |
| 1 | 50 | 0.262 | 1.024 | 1.286 |
| 11 | 58 | 0.311 | 1.130 | 1.441 |
| 34 | 44 | 0.498 | 0.565 | 1.064 |

## THREE DAMPER EXAMPLE

| DAMPER LOCATIONS | $a_4$ | $\lvert\omega_4 - a_4\rvert$ | $a_5$ | $\lvert\omega_5 - a_5\rvert$ |
|---|---|---|---|---|
| 1, 50, 88 | 0.62 | 13.13 | 11.56 | 4.81 |
| 1, 44, 78 | 6.40 | 7.35 | 8.48 | 7.89 |

MAXIMUM DAMPING ACHIEVED IN FIRST

TWO VIBRATORY MODES

(WITH SAME DAMPING CONSTANT AT ALL LOCATIONS)

| DAMPER LOCATIONS | $\xi_4$ | DAMPER VALUE | $\xi_5$ | DAMPER VALUE |
|---|---|---|---|---|
| 1, 50, 88 | 0.95 | 0.34 | 0.05 | 0.48 |
| 1, 44, 78 | 0.60 | 0.31 | 0.46 | 0.34 |

## CONCLUSIONS

- CRITERIA FOR SELECTION OF OPTIMAL DAMPER LOCATIONS PRESENTED
- DAMPING SYNTHESIS PROBLEM FORMULATED AND APPLIED TO NASA GRILLAGE MODEL

## DAMPING SYNTHESIS EXAMPLE 1

DESIRED $\xi_4 = 0.6$

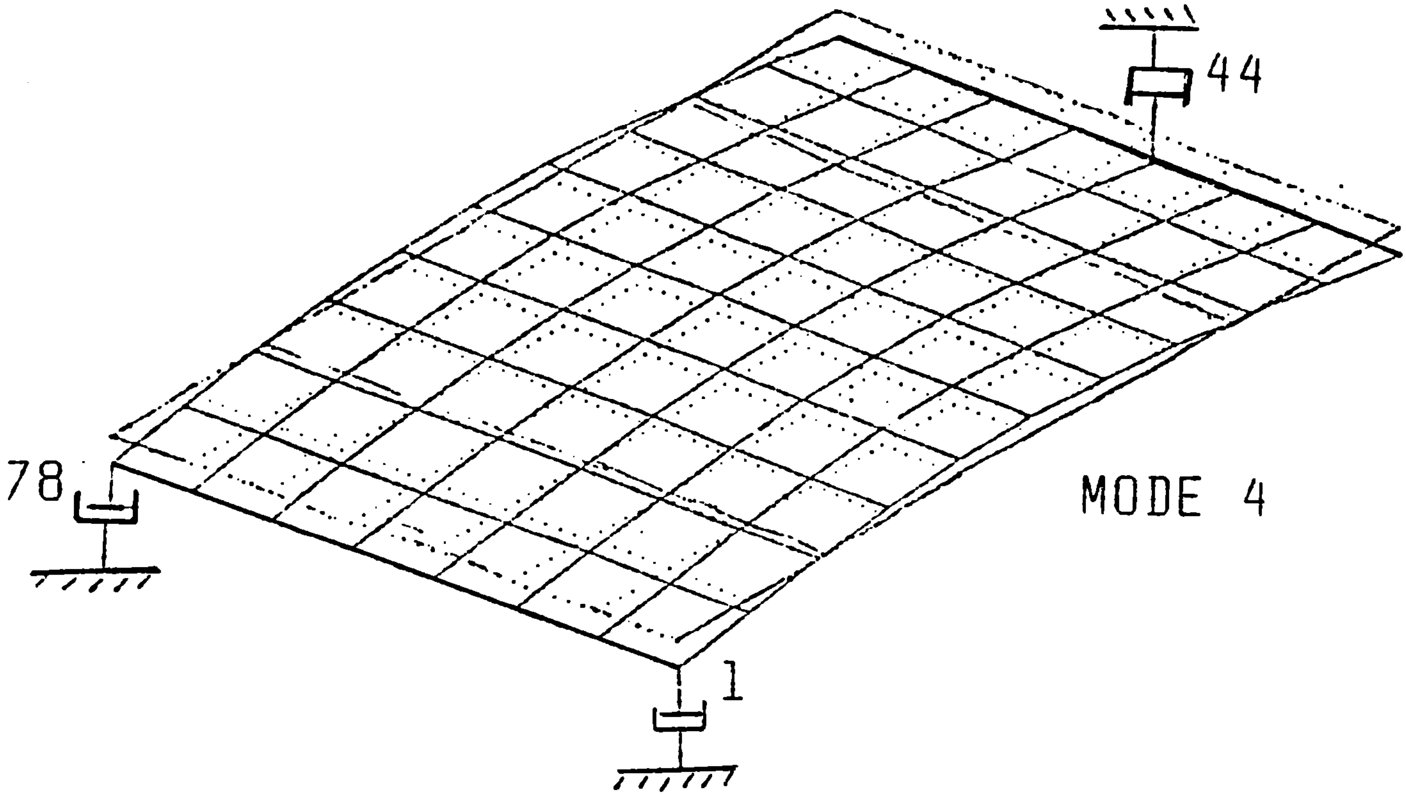
$\xi_5 = 0.5$

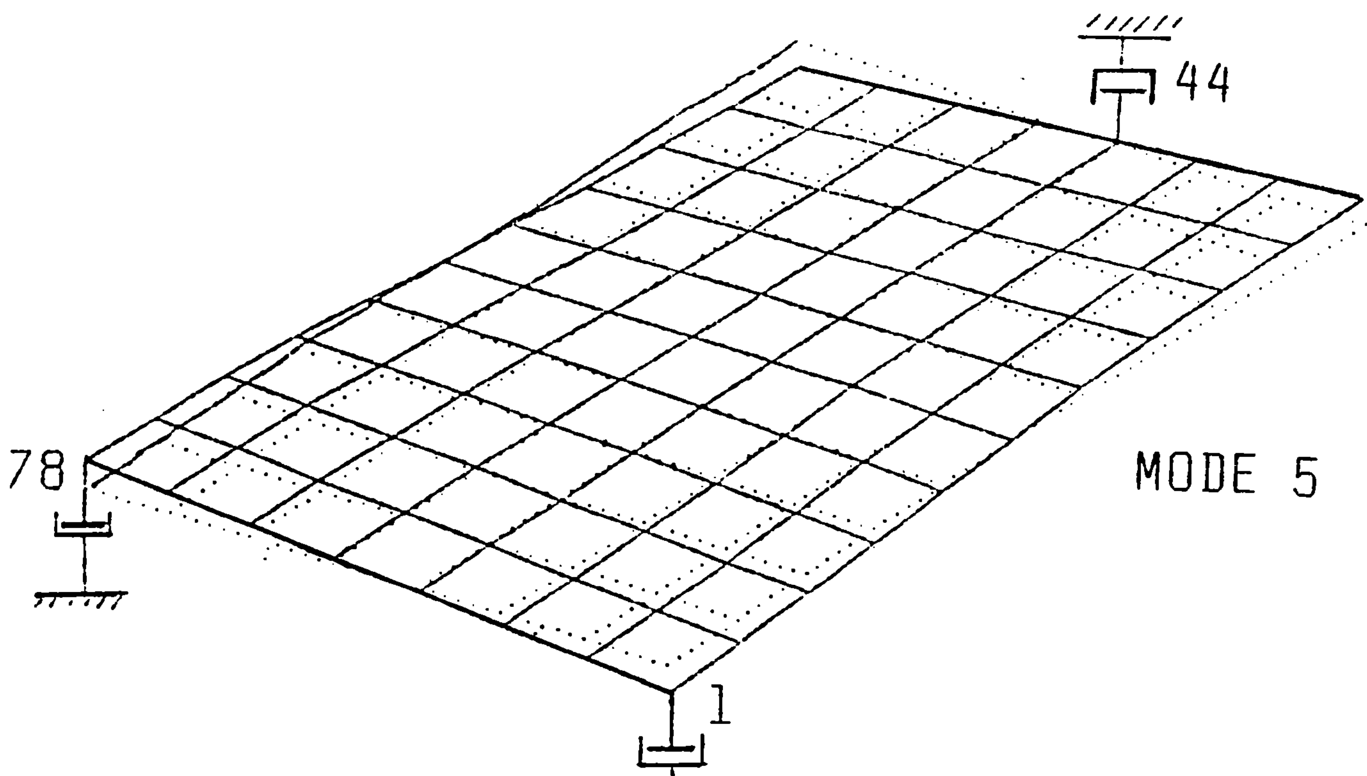
DAMPER LOCATIONS:

D.O.F. 1, 44, 78

RESULTS:

| LOCATION | GAIN |
|---|---|
| 1 | 0.272 |
| 44 | 0.514 |
| 78 | 0.269 |


44
78
1
MODE 4


44
78
1
MODE 5

## DAMPING SYNTHESIS EXAMPLE 2

DESIRED $\xi_4 = 0.7$

$\xi_5 = 0.6$

USE 6 DAMPERS AT D.O.F :

1, 11, 39, 50, 78, 88

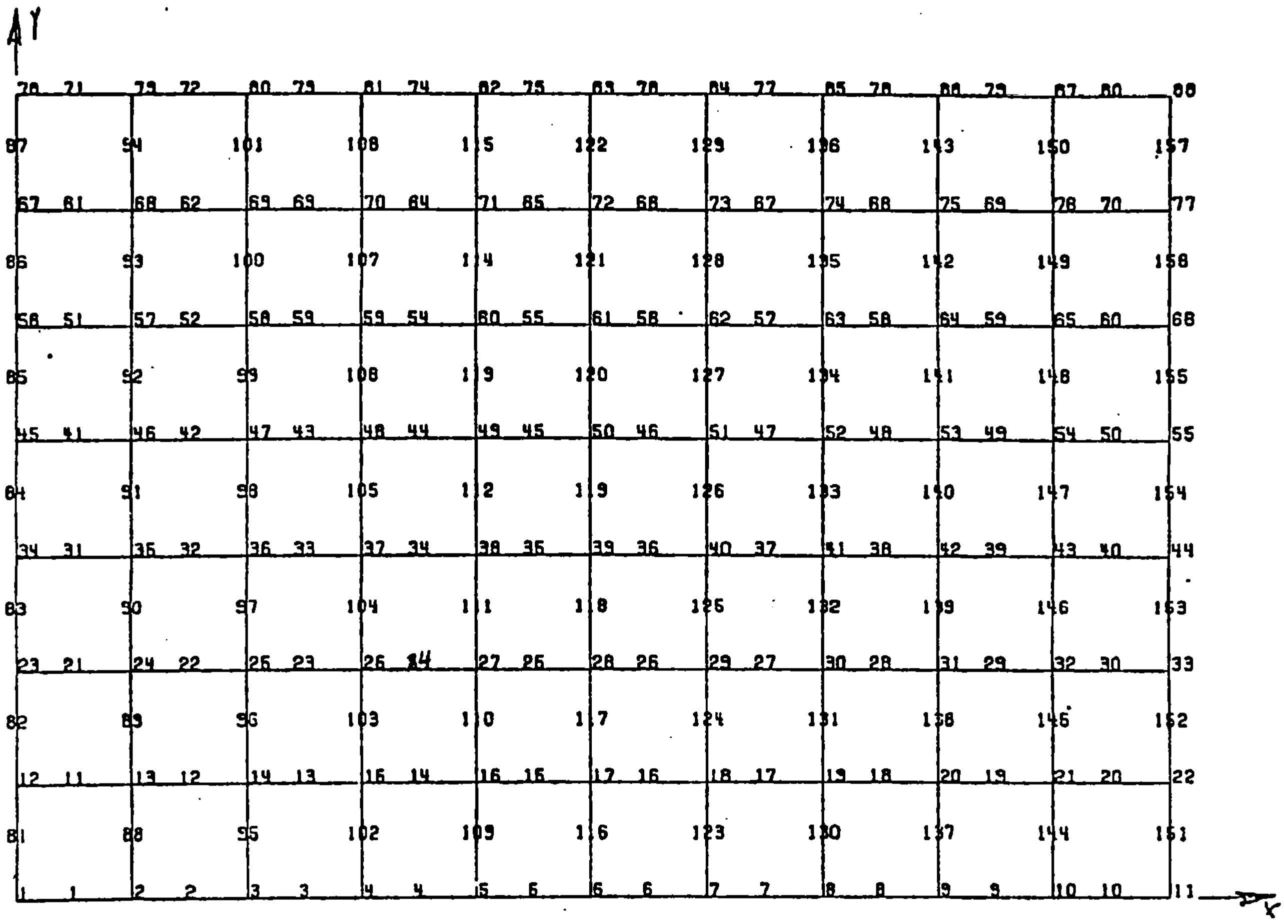
RESULTS :

| LOCATION $i$ | $c_i$ | $c_i$ |
|---|---|---|
| 1 | 0.21 | 0.240 |
| 11 | 0.25 | 0.240 |
| 39 | 0.20 | 0.131 |
| 50 | 0.20 | 0.131 |
| 78 | 0.20 | 0.247 |
| 88 | 0.21 | 0.245 |

ACHIEVED DAMPING

$\xi_4 = 0.71$ $\xi_4 =$

$\xi_5 = 0.59$ $\xi_5 =$





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