NASA Technical Memorandum 84354

Influence of Sampling Rate on the Calculated Fidelity of an Aircraft Simulation

James C. Howard

April 1983

NASA

National Aeronautics and
Space Administration

N83-23267 #

Influence of Sampling Rate on the Calculated Fidelity of an Aircraft Simulation

James C. Howard, Ames Research Center, Moffett Field, California

INFLUENCE OF SAMPLING RATE ON THE CALCULATED

FIDELITY OF AN AIRCRAFT SIMULATION

James C. Howard
Flight Systems and Simulation Research Division
NASA Ames Research Center
Moffett Field, California

ABSTRACT

One of the factors that influences the fidelity of an aircraft digital simulation is the
sampling rate. As the sampling rate is increased, the calculated response of the discrete
representation tends to coincide with the response of the corresponding continuous system.
Because of computer limitations, however, the sampling rate cannot be increased indefinitely.
Moreover, real-time simulation requirements demand that a finite sampling rate be adopted.
In view of these restrictions, a study was undertaken to determine the influence of sampling
rate on the response characteristics of a simulated aircraft describing short-period oscil-
lations. Changes in the calculated response characteristics of the simulated aircraft
degrade the fidelity of the simulation. In the present context, fidelity degradation is
defined as the percentage change in those characteristics that have the greatest influence
on pilot opinion: short-period frequency ω, short-period damping ratio ζ, and the product
$\zeta\omega$. To determine the influence of the sampling period on these characteristics, the equa-
tions describing the response of a DC-8 aircraft to elevator-control inputs were used. The
results indicate that if the sampling period is too large, the fidelity of the simulation
can be degraded. For example, it was determined that the motion characteristics of the simu-
lated aircraft, represented by ζ and $\zeta\omega$, as well as the corresponding fidelity, decreased
linearly with sampling period and vanished when the period reached 640 ms.

INTRODUCTION

The response of a dynamical system to control inputs may be stable or unstable, depending on
the characteristics of the system. When the response is being calculated by a digital com-
puter, the dynamical characteristics are modified by the need to use a discrete representa-
tion of the system. This involves the choice of a sampling rate, which influences the
fidelity of the simulation and determines the feasibility of simulating large aeronautical
systems, such as helicopters.

One way to determine the influence of a specific parameter on the response characteristics
of a continuous dynamical system is to map the eigenvalues on the complex plane and note
their migrations, in each mode, as the parameter is varied (1). When the loop is closed
around an open-loop system, and the feedback gain is varied, the conventional root loci are
obtained (2). These loci reveal the stability characteristics of the system, as the fre-
quency and the damping ratio vary with changes in the feedback gain.

By using the same procedure for a discrete representation of the system, the influence of
sampling rate on the response characteristics can be determined. Whereas a linear, continu-
ous dynamical system is described in terms of the Laplace transform, the discrete represen-
tation requires a formulation in terms of the Z-transform. When the Z-transfer function
corresponding to a given integration algorithm has been obtained as a function of the sam-
pling period, and the discrete eigenvalues determined, these can be plotted on the complex
z-plane to ascertain their location relative to the unit circle and thus determine the pos-
sibility of instability. To facilitate the interpretation of results, the discrete eigen-
values were transformed from the z-plane to the s-plane. By proceeding in this manner, and
using the equations describing the response of a DC-8 aircraft to elevator-control inputs,
the influence of sampling rate was determined.

1

ANALYSIS

Equations of Motion

The differential equation describing an aircraft's pitching motion, which is assumed to be linear and time invariant, is (from Ref. 3)

$$\left(\frac{d^4Y}{dt^4} + A_3 \frac{d^3Y}{dt^3} + A_2 \frac{d^2Y}{dt^2} + A_1 \frac{dY}{dt} + A_0 Y\right) = \left(\frac{d^2U}{dt^2} + B_1 \frac{dU}{dt} + B_0 U\right) \tag{1}$$

where Y is the pitch angle, U is the elevator-control input, and A_i, B_j are constants.

The formulation is simplified by specifying a state variable X which satisfies the differential equation

$$\frac{d^4X}{dt^4} + A_3 \frac{d^3X}{dt^3} + A_2 \frac{d^2X}{dt^2} + A_1 \frac{dX}{dt} + A_0 X = U \tag{2}$$

The remaining state variables can then be assigned as follows:

$$X = X_1$$
$$\dot{X}_1 = X_2$$
$$\dot{X}_2 = X_3 \tag{3}$$
$$\dot{X}_3 = X_4$$
$$\dot{X}_4 = -A_0 X_1 - A_1 X_2 - A_2 X_3 - A_3 X_4 + U$$

Using matrix notation, these equations assume the conventional form; that is,

$$\dot{X} = AX + BU \tag{4}$$
$$Y = C^T X \tag{5}$$

where

$$X = (X_1 \; X_2 \; X_3 \; X_4)^T \tag{6}$$

$$A = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -A_0 & -A_1 & -A_2 & -A_3 \end{vmatrix} \tag{7}$$

$$B^T = (0 \; 0 \; 0 \; K) \tag{8}$$

and

$$C^T = (B_0 \; B_1 \; 1 \; 0) \tag{9}$$

From Equation (4),

$$X(t) = e^{A(t-t_0)} X(t_0) + \int_{t_0}^{t} e^{A(t-\tau)} BU(\tau) d\tau \tag{10}$$

When the input is sampled with a zero-order hold after the sampler, the input is held constant for one sampling period (see Figure 1).

During this period the limits of integration are $t = (n+1)T$ and $t_0 = nT$, and Equation (4) becomes (from Ref. 4)

$$X(\overline{n+1T}) = e^{AT} X(nT) + \left(\int_{nT}^{(n+1)T} e^{A(\overline{n+1}T-\tau)} d\tau\right) BU(nT)$$

Therefore,

$$X\overline{(n + 1T)} = e^{AT}X(nT) + A^{-1}(e^{AT} - I)BU(nT) \tag{11}$$

By omitting the T from the arguments of X and U, this equation assumes a more compact form, namely,

$$X(n + 1) = \Phi x(n) + B_D U(n) \tag{12}$$

where $\Phi = e^{AT}$, and B_D is the discrete form of the control distribution matrix. That is,

$$B_D = A^{-1}(\Phi - I)B \tag{13}$$

It should be noted that Equation (13) is valid only if A is nonsingular.

Z-Transforms and Z-Transfer Functions

Assuming zero initial conditions, the Z-transform of the state vector is obtained from Equation (12):

$$X(Z) = (ZI - \Phi)^{-1}B_D U(Z) \tag{14}$$

The corresponding Z-transform of the output is

$$Y(Z) = C^T(ZI - \Phi)^{-1}B_D U(Z) \tag{15}$$

The Euler integration algorithm uses only the first two terms of the transition matrix e^{AT}:

$$e^{AT} = \Phi \doteq (I + AT) \tag{16}$$

Substituting this value into Equation (13) gives the Euler form of the control distribution matrix B_D,

$$B_D = A^{-1}(I + AT - I)B = TB \tag{17}$$

Substituting this result into Equation (15) gives the Z-transform of the output corresponding to the Euler integration algorithm:

$$Y(Z) = TC^T[(Z - 1)I - AT]^{-1}BU(Z) \tag{18}$$

The Z-transfer function relating the output Y(Z) to the input U(Z) is

$$Y(Z)/U(Z) = TC^T[(Z - 1)I - AT]^{-1}B \tag{19}$$

By introducing matrix A from Equation (7) and the column and row vectors from Equations (8) and (9), respectively, the required form of the transfer function is obtained:

$$\frac{Y(Z)}{U(Z)} = KT^2 \left(\frac{Z^2 + p_1 Z + p_0}{Z^4 + q_3 Z^3 + q_2 Z^2 + q_1 Z + q_0} \right) \tag{20}$$

where

$$T = \text{sampling period}$$

$$p_0 = B_0 T^2 - B_1 T + 1$$

$$p_1 = B_1 T - 2$$

$$q_0 = A_0 T^4 - A_1 T^3 + A_2 T^2 - A_3 T + 1$$

$$q_1 = A_1 T^3 - 2A_2 T^2 + 3A_3 T - 4$$

$$q_2 = A_2 T^2 - 3A_3 T + 6$$

$$q_3 = A_3 T - 4$$

RESULTS

The following values of the elements of the system dynamics matrix A and the control distribution matrix B were obtained from Teper (3).

$$A_0 = 0.07006953$$

$$A_1 = 0.09712526$$

$$A_2 = 2.6813872 \qquad K = 1.338$$

$$A_3 = 1.700522$$

$$B_1 = 0.5955$$

$$B_0 = 0.032675$$

When these values were substituted into Equation (20), the open-loop eigenvalues corresponding to the phugoid and the short-period modes were obtained as functions of the sampling period T. To determine the influence of the sampling rate on the stability of the short-period motion, the discrete eigenvalues were plotted as a function of the sampling period, and their locations relative to the unit circle were noted. The results are plotted in Figure 2. It is seen that as the sampling period is increased, the eigenvalues remain inside the unit circle until T = 640 ms. At this point they lie on the unit circle, and further increases in T cause them to move into the region of instability.

In order to determine the changes in frequency and damping as functions of the sampling period, the discrete eigenvalues were transformed from the Z-plane to the s-plane. The migrations of the eigenvalues in the s-plane give a good indication of the influence of the sampling period on the frequency and the damping of the simulated system. The required transformation can be effected by using the equation of definition:

$$s = (\log_e Z)/T \tag{21}$$

Equation (21) was used to calculate the T-locus of open-loop eigenvalues for the following range of sampling periods:

$$0.02 \leq T \leq 0.80 \text{ s} \tag{22}$$

The results are plotted in Figure 3, where it is seen that the locus reaches the imaginary axis when the sampling period is 640 ms. This figure should be compared with Figure 2, where the locus of discrete eigenvalues reaches the unit circle for the same value of T. Moreover, the data used to plot Figure 3 can be used to calculate the modal frequency and the damping ratio as functions of the sampling period. The T-locus of frequency is plotted in Figure 4, and the locus of the corresponding damping ratios is shown in Figure 5. When T is increased from 20 to 640 ms, it appears that the damping ratio decreases almost linearly from its initial value to zero. The undamped natural frequency is shown plotted as a function of the damping ratio in Figure 6. The range of this curve indicates that as the sampling period is increased, unsatisfactory regions of the flying-qualities plot will be encountered (5). When simulation data become available, boundaries of acceptable flying qualities will be established. These boundaries assume the form shown in Figure 7, where acceptable and unacceptable flying qualities are related to the short-period frequency and to the short-period damping ratio (6).

Because the response of a simulated dynamical system to control inputs is calculated by a digital computer, the motion characteristics of the discrete representation are functions of the sampling rate; consequently, simulation fidelity is degraded. In the present context, fidelity degradation is defined as the percentage change in those characteristics that have the greatest influence on pilot opinion: short-period frequency ω, short-period damping ratio ζ, and the product $\zeta\omega$. For example, the fidelity loss owing to changes in the damping ratio is defined as

$$f_\zeta \triangleq -\left| \frac{\zeta_C - \zeta_D}{\zeta_C} \right| \times 100 \tag{23}$$

where f_ζ is the fidelity loss, ζ_C is the continuous damping ratio, and ζ_D is the damping ratio of the discrete representation. The f_ζ term is plotted as a function of the sampling period in Figure 8. As might be expected, f_ζ has a functional form similar to that of ζ, as shown in Figure 5.

The loss of fidelity that results from natural frequency changes is defined similarly:

$$f_\omega \triangleq -\left| \frac{\omega_C - \omega_D}{\omega_C} \right| \times 100 \tag{24}$$

4

where f_ω is the fidelity loss, ω_c is the natural frequency of the continuous system, and ω_D is the natural frequency of the discrete representation. As shown in Figure 8, the influence of natural frequency changes on system fidelity, as measured by Equation (24), is not so pronounced as the influence of the damping ratio. The third measure considered assesses the loss of fidelity owing to the characteristic $\zeta\omega$. The loss of fidelity caused by changes in $\zeta\omega$ is defined as

$$f_{\zeta\omega} \triangleq -\left| \frac{\zeta_c \omega_c - \zeta_D \omega_D}{\zeta_c \omega_c} \right| \tag{25}$$

where again the subscripts c and D denote continuous and discrete representations, respectively. This characteristic brings out the influence of the real component of the natural frequency vector on simulation fidelity. Since that component determines the damping ratio, the loss of fidelity, as defined by Equation (25), is seen to follow the same trend as f_ζ. The results are plotted in Figure 9.

CONCLUSIONS

One of the factors that influences the fidelity of an aircraft simulation is the sampling rate. As the sampling rate is increased, the calculated response to control inputs, of the discrete representation, tends to coincide with the response of the continuous system. Because of computer limitations, however, the sampling rate cannot be increased indefinitely, and simulation fidelity is degraded. In the present context, fidelity degradation is defined as the percentage change in those characteristics that have the greatest influence on pilot opinion: short-period frequency ω, short-period damping ratio ζ, and the product $\zeta\omega$. In order to determine the influence of the sampling period on these characteristics, the equations describing the response of a DC-8 aircraft to elevator-control inputs were used. Results indicate that if the sampling period is too large, the fidelity of the simulation can be degraded significantly. For example, it was determined that the motion characteristics of the simulated aircraft, represented by ζ and $\zeta\omega$, as well as the corresponding fidelity, decreased linearly with sampling period and vanished when the sampling period reached 640 ms.

REFERENCES

1. McRuer, Duane; Ashkenas, Irving; and Graham, Dunstan: Aircraft Dynamics and Automatic Control. Princeton University Press, Princeton, N.J., 1973.

2. Truxal, J. G.: Control System Synthesis. McGraw-Hill Book Company, Inc., New York, 1955.

3. Teper, Gary L.: "Aircraft Stability and Control Data," Report 176-1, Systems Technology, Hawthorne, Calif., Apr. 1969.

4. Gupta, C. S.; and Masdorff, L.: Fundamentals of Automatic Control. John Wiley & Sons, Inc., New York, 1970.

5. Kolk, W. R.: Modern Flight Dynamics. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1961.

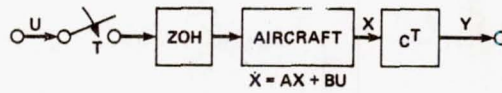6. O'Hara, F.: "Handling Criteria," J. Royal Aero. Soc., Vol. 71, No. 676, 1967, pp. 271-291.

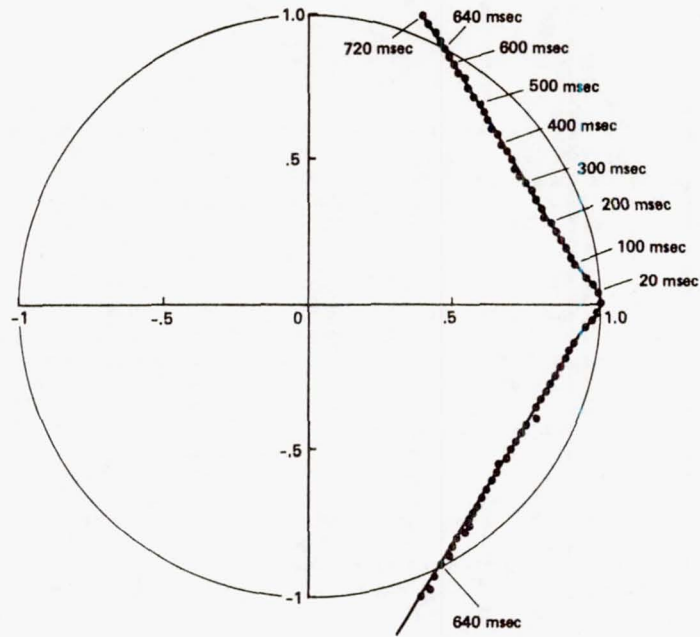Figure 1 — State Variable Sampled-Data System.

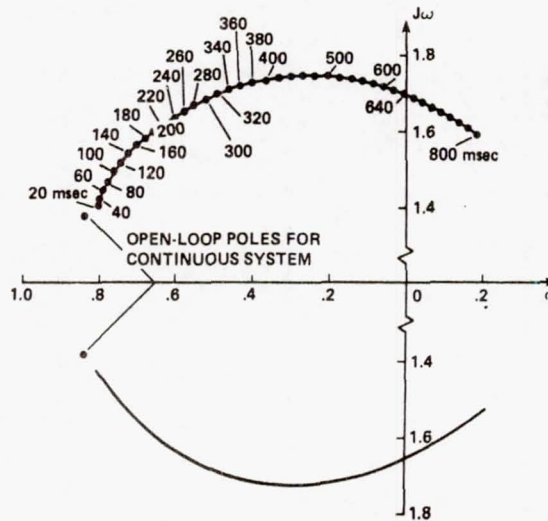Figure 2 — T-Locus for Solution of Equation (20) by Euler Integration.

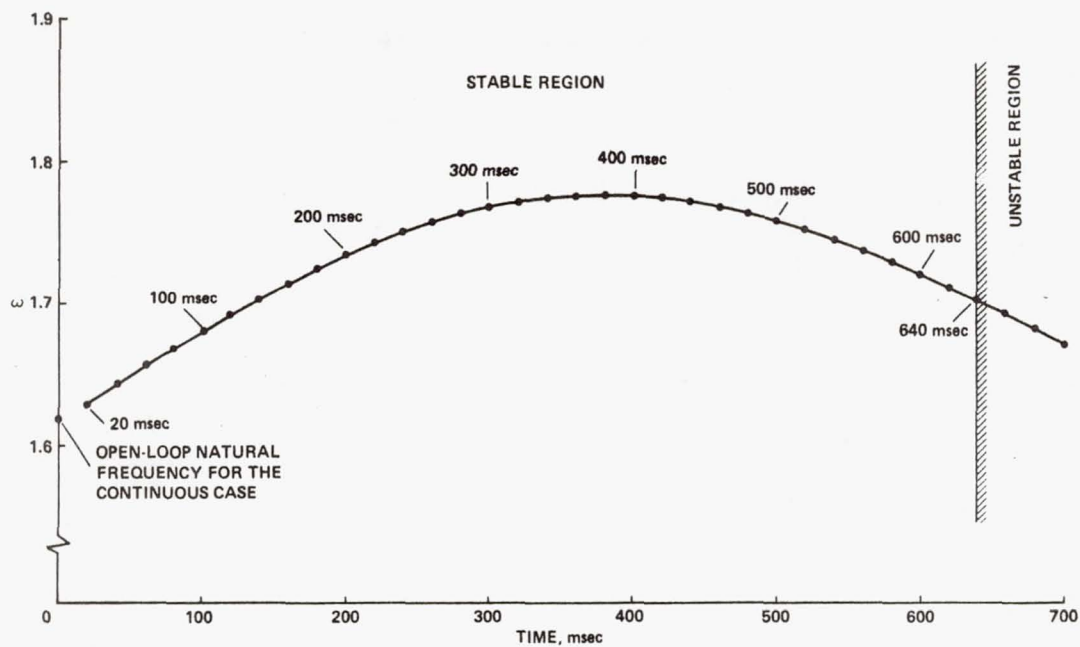Figure 3 — T-Locus of Open-Loop Poles for the Short-Period Mode.

6

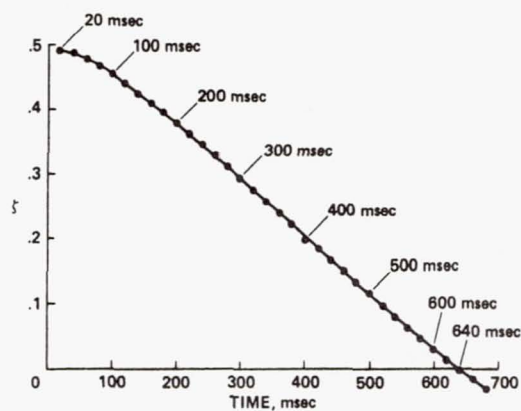Figure 4 — T-Locus of Natural Frequencies for the Short-Period Mode.

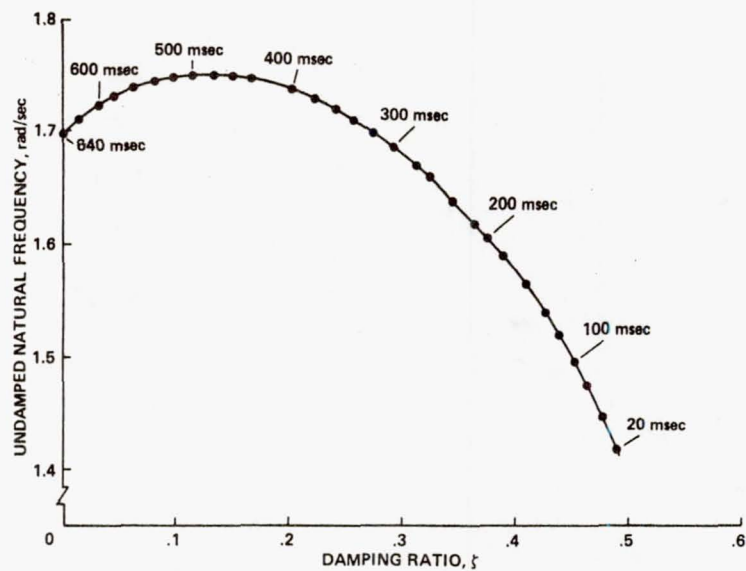Figure 5 — T-Locus of Open-Loop Damping Ratios for the Short-Period Mode.

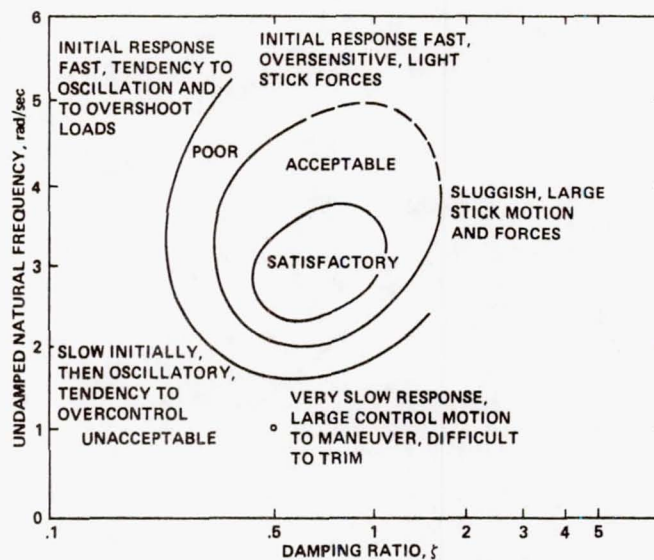Figure 6 — T-Locus of Undamped Natural Frequency versus Damping Ratio.

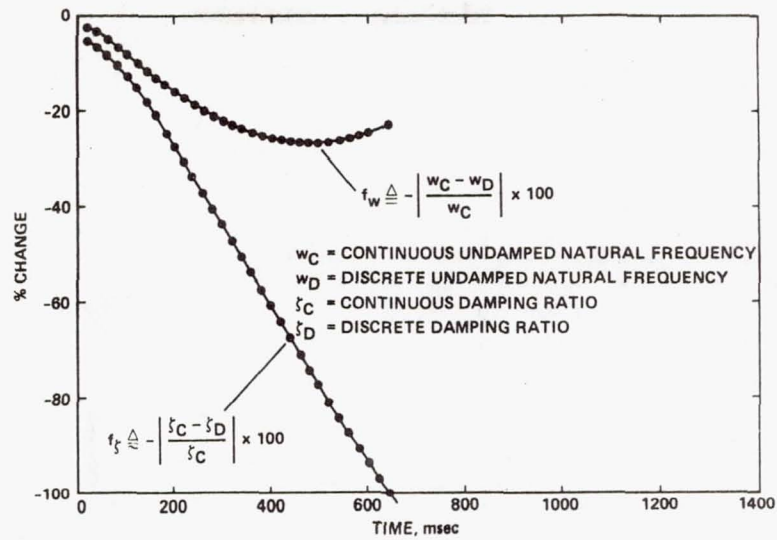Figure 7 — Flying-Qualities Boundaries.

8

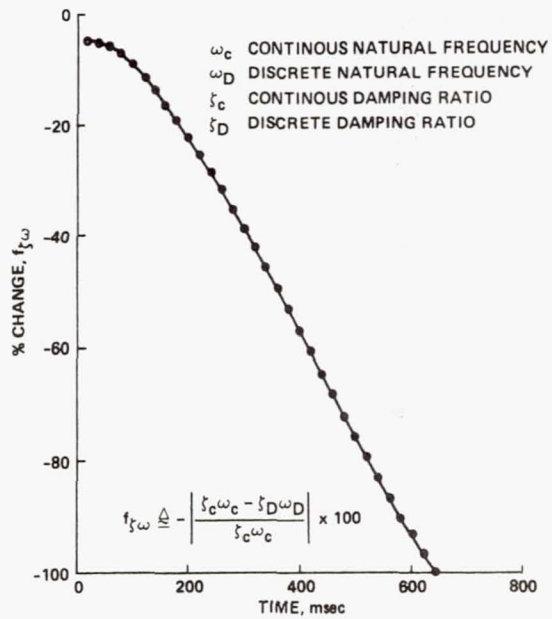Figure 8 — Loci of Fidelity Measures for DC-8 During Landing Approach.

Figure 9 — Locus of Fidelity Measures for DC-8 Aircraft in Approach Condition.

9

1. Report No. NASA TM-84354	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle INFLUENCE OF SAMPLING RATE ON THE CALCULATED FIDELITY OF AN AIRCRAFT SIMULATION		5. Report Date April 1983
		6. Performing Organization Code
7. Author(s) James C. Howard		8. Performing Organization Report No. A-9293
		10. Work Unit No. T-5232
9. Performing Organization Name and Address NASA Ames Research Center Moffett Field, Calif. 94035		11. Contract or Grant No.
		13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		14. Sponsoring Agency Code 505-35-31

15. Supplementary Notes

Presented at the ISA 14th Annual Modeling and Simulation Conference, April 21-22, 1983, Pittsburg, Mass. Point of Contact: James C. Howard, NASA Ames Research Center, MS 243-9, Moffett Field, Calif. 94035 (415) 965-5165 or FTS 448-5165.

16. Abstract

One of the factors that influences the fidelity of an aircraft digital simulation is the sampling rate. As the sampling rate is increased, the calculated response of the discrete representation tends to coincide with the response of the corresponding continuous system. Because of computer limitations, however, the sampling rate cannot be increased indefinitely. Moreover, real-time simulation requirements demand that a finite sampling rate be adopted. In view of these restrictions, a study was undertaken to determine the influence of sampling rate on the response characteristics of a simulated aircraft describing short-period oscillations. Changes in the calculated response characteristics of the simulated aircraft degrade the fidelity of the simulation. In the present context, fidelity degradation is defined as the percentage change in those characteristics that have the greatest influence on pilot opinion: short-period frequency ω, short-period damping ratio ζ, and the product $\zeta\omega$. To determine the influence of the sampling period on these characteristics, the equations describing the response of a DC-8 aircraft to elevator-control inputs were used. The results indicate that if the sampling period is too large, the fidelity of the simulation can be degraded. For example, it was determined that the motion characteristics of the simulated aircraft, represented by ζ and $\zeta\omega$, as well as the corresponding fidelity, decreased linearly with sampling period and vanished when the period reached 640 ms.

17. Key Words (Suggested by Author(s)) Simulation Sampling rate Simulation fidelity	18. Distribution Statement Unlimited Subject Category - 01		
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 12	22. Price* A02