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STABILITY AND CONTROL OF A SUPERSONIC TRANSPORT AIRPLANE DURING LANDING APPROACH

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SUMMARY

Previous simulator studies have shown that a proposed supersonic transport airplane exhibits undesirable lateral motions during landing approach. Large adverse sideslip excursions and large peak lateral acceleration at the pilot's station occurred during rolling maneuvers of the unaugmented airplane.

In this study, modal control theory has been applied to determine feedback gains that provide desirable stability characteristics and satisfactory transient response to aileron deflection input. However, the peak value of lateral acceleration at the pilot's station does not satisfy a proposed criterion during a rolling maneuver.

Optimal regulator theory does not provide a significant reduction of the peak lateral acceleration. The weighting matrices in the performance index were given an extreme variation, but the effect on lateral acceleration remained insignificant. Open loop control provided the desired bank angle (30°) with the desired roll rate (10°/sec), and provided a satisfactory level of lateral acceleration. However, a large adverse sideslip angle was required. In addition, the yawing velocity was negative for about 2 seconds and lateral acceleration changed sign during the maneuver.

The problem persists and, as suggested in previous studies, perhaps relaxed criteria must be proposed. The criteria could allow any combination of a lower average rolling velocity, a larger adverse sideslip angle excursion, and a higher peak lateral acceleration at the pilot's station.

INTRODUCTION

A simulation (Ref. 1) has shown that, during landing approach, a supersonic transport airplane exhibits undesirable lateral motion characteristics. Primarily, high pilot ratings were attributed to large adverse sideslip excursions that occurred during rolling maneuvers. A stability and control augmentation system was designed so that lateral motion characteristics were considerably improved. In response to a step wheel input, however, large peak lateral accelerations at the pilot station were evident. A modified stability and control augmentation system successfully reduced the peak lateral acceleration, but the rolling velocity was also considerably reduced when a step wheel input was applied to the airplane. Another simulation (Ref. 2) of the same airplane with stability augmentation systems attributed high pilot ratings to extreme lateral accelerations that were produced at the pilot station during rolling maneuvers. Suggested ways to alleviate the undesirable peak lateral acceleration at the pilot's station are: relax the criterion on peak lateral acceleration; restrict rolling maneuvers; and allow some adverse sideslip.

A stability augmentation system has been designed analytically by use of modal control theory. The requirements of reference 3 are satisfied with use of the

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system. Optimal regulator theory does not provide significant reduction of lateral acceleration at the pilot's station. Open-loop control was applied, therefore, to find control inputs that could satisfy the criteria (Refs. 2 and 4) for peak lateral acceleration.

The airplane was assumed to be rigid and control inputs were aileron and rudder deflection angles.

SYMBOLS

Values are given in the International System of Units (SI). Dots over symbols denote differentiation with respect to time. All calculations are based on the airplane body axes.

A	matrix of coefficients in equation (1)
$\hat{A}$	matrix of augmented coefficients as in equation (A-9)
$(a_y)_{PS}$	lateral acceleration at the pilot's station, g units
$(a_y)_{cg}$	lateral acceleration at the center-of-gravity of the airplane, g units
B	control effectiveness matrix in equation (1)
C	matrix of coefficients in equation (2)
$\bar{c}$	mean aerodynamic chord, m
C_Y	side force coefficient
C_ℓ	rolling moment coefficient
C_n	yawing moment coefficient
$\underset{\sim}{C}_i$	matrix used in equation (A-8), (i=1,2,3,4)
$C_{()_{\delta a}}$	coefficient defined in table II
D	matrix of coefficients in equation (2)
F,S,G	matrices defined by equation (A-7)
g	free fall acceleration, m/sec^2
j	$\sqrt{-1}$
K	feedback gain matrix
M	matrix with elements m_{ii}, m_{ik}, m_{ki}, $1\leq i\leq n$, $1\leq k\leq n$
I_{n-m}	unity matrix having dimension $(n-m)\times(n-m)$
I_X, I_Y, I_Z	moment of inertia about X, Y, and Z body axes, respectively, kg-m^2
I_{XZ}	product of inertia, kg-m^2
p,r	rolling and yawing velocities, respectively, deg/sec
R^2, R^4	real space of 2 and 4 dimensions, respectively
t	time, sec
T_2	time to double amplitude, sec
$T_{1/2}$	time to half amplitude, sec
u	control vector
u_c	control vector of commanded inputs
V	modal matrix
V_o	airplane airspeed, m/sec

v_i	characteristic vectors (i=1,2,3,4)
$v_{11}, v_{22}, v_{23}, v_{32}, v_{33}, v_{44}$	elements of modal matrix in equation (8)
v_R, v_S	characteristic vectors associated with the roll mode and spiral mode, respectively
$v_{DR,R}, v_{DR,I}$	real and imaginary vectors, respectively, of the characteristic vector associated with the Dutch-roll mode
x	state vector defined after equation (1)
x(t),x(o)	state vector as a function of time and initial value of the state vector, respectively
$\bar{x}, \bar{z}$	longitudinal and vertical distances, respectively, from the airplane center-of-gravity to pilot station, m
z_i, w_i	component vectors of a characteristic vector (i=1,2,3,4)
α	angle of attack, deg
β	angle of sideslip, deg
δ_a	aileron deflection, positive for right roll command, deg
δ_{af}	flaperon deflection, deg
$\delta_{a,p}$	aileron deflection caused by pilot action, deg
δ_f	trailing-edge flap deflection, deg
δ_r	rudder deflection, deg
$\delta_{r,p}$	rudder deflection caused by pilot action, deg
δ_s	deflection of spoiler-slot and inverted spoiler-slot deflectors, deg
$\delta_1, \delta_2, \delta_3, \delta_4, \delta_5$	elements of modal matrix in equation (8)
ζ_d	Dutch roll mode damping ratio
λ_i	characteristic values (i=1,2,3,4)
λ_R	characteristic value associated with roll mode
λ_S	characteristic value associated with spiral mode
Λ	matrix of characteristic values defined in equation (A-10)
$\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5$	elements of modal matrix in equation (8)
τ_R	roll mode time constant, sec
ω_{nd}	undamped natural frequency of Dutch roll mode, rad/sec

PRELIMINARIES

Mathematical Model

The airplane considered in this report is the same as the baseline concept of reference 1. A sketch of the airplane is shown in figure 1. The mass and dimensional characteristics and aerodynamic characteristics (α=8°) are given in tables I and II, respectively. The linearized equations of lateral motion for a landing approach configuration (δ_{flaps}=40°, V_o=78.71 m/sec and Altitude=91.44 meters), in body-axes, are given by the following equation.

$$\dot{x} = Ax + Bu \tag{1}$$

where

$$x = \begin{bmatrix} p \\ r \\ \beta \\ \phi \end{bmatrix}; \quad u = \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix};$$

$$A = \begin{bmatrix} -.5676 & .8599 & -2.193 & 0 \\ -.0069 & -.1475 & .2301 & 0 \\ .1950 & -.9706 & -.0799 & .1234 \\ 1.0 & .1405 & 0 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} .8295 & .2106 \\ -.0135 & -.1159 \\ -.0065 & .0111 \\ 0 & 0 \end{bmatrix}$$

Additionally, the lateral acceleration at the pilot's station is given, in g units, by

$$(a_y)_{PS} = Cx + Du \tag{2}$$

where

$$C = [.1405 \quad -.0879 \quad -.6732 \quad 0]$$

$$D = [.2915 \quad -.3311]$$

The pilot's station is located 44.2 meters forward (x-direction) of the center-of-gravity (cg) and 4.78 meters above (z-direction) the cg. The effect of pilot displacement in the y-direction and the effect of pitch rate on lateral acceleration at the pilot's station have been neglected. In terms of distance from the cg to the pilot's station, the lateral acceleration is, with the above assumptions,

$$(a_y)_{PS} = (a_y)_{cg} + \frac{\dot{r}\bar{x} - \dot{p}\bar{z}}{g} \tag{3}$$

where $\bar{x}$ = 44.2 m, and $\bar{z}$ = -4.78 m.

Lateral-Directional Characteristics (Unaugmented Airplane)

The characteristic values for the supersonic transport airplane modelled in the preceding section are the following (See also table III):

Roll Mode: $\lambda_R = -.611$

Dutch Roll Mode: $\lambda_{DR} = -.077 \pm .821\ j$

Spiral Mode: $\lambda_S = -.031$

The roll mode time constant, $\tau_R=-1/\lambda_R=1.64$ sec, exceeds the requirement of reference 3 (See Table III). Damping of the Dutch-roll mode, moreover, is too light for this airplane. The effect of these characteristics is shown in the response of the airplane to a moderate aileron angle input ($\delta_a=15°$) (Fig. 2). Since the rolling velocity, p, is oscillatory, the Dutch-roll mode and roll mode are coupled, and the rolling velocity does not approach a steady-state value in a desired manner. The spiral mode, although stable, shows an undesirable lag in the response of the yawing velocity. Table III compares the airplane characteristics with those specified in reference 3.

At the pilot's station, the calculated lateral acceleration is quite large for the moderate control input (See Fig. 2(b)). The criterion given in references 2 and 4 would certainly be unsatisfied during the prescribed rolling maneuver. The criterion of reference 4 states in part "Lateral acceleration at the pilot station shall not exceed a level of ±0.075 g peak, and the critical passenger station shall not exceed ±0.05 g peak. These levels shall be met for all normal maneuvers including 30° bank and capture using an average roll rate of 5°/sec in cruise and 10°/sec at landing. If unpiloted time studies are conducted, the wheel input should be a 0.5 second ramp of magnitude sufficient to produce the specified average roll rates." In reference 2, a criterion was suggested as follows: "A lateral acceleration criterion for large transports in the landing approach was developed. The parameter $Ny_{pilot_{max}}/P_{max}$ correlates well with pilot rating. A value less than .012 g's/deg/sec was required for satisfactory ratings and .035 g's/deg/sec for acceptable ratings." The largest part of the lateral acceleration comes from the second term of equation (3) which depends on the location of the pilot with respect to the cg. The airplane characteristics that have been mentioned are essentially those given in reference 1 for the unaugmented airplane. Differences arise primarily from slight differences in modelling. The simulations of reference 1 lead to high pilot ratings. As stated, the major objections were (1) unacceptable large adverse sideslip excursions in turns; (2) easily excited, lightly damped Dutch roll mode; (3) poor roll and heading control; and (4) sluggish roll response with low roll damping. Also mentioned in reference 1 was that large adverse sideslip excursions occurred during rolling maneuvers.

STATEMENT OF THE PROBLEM

From the preceding discussion, it is evident that a stability augmentation system is needed for the airplane. Also, a controller is needed to satisfy the criterion for lateral acceleration level during a rolling maneuver. In the simulations of references 1 and 2, several augmentation systems were used to satisfy the flying quality requirements in reference 3. However, although the lateral acceleration level was not sufficiently reduced to produce satisfactory handling qualities, it should be noted that the introduction of a first-order lag to the roll-rate command signal did produce acceptable handling qualities (6.5 > pilot rating > 3.5).

The objective of the present report is: (1) apply modal control theory in the design of SAS; and (2) apply optimal regulator theory to the combined airplane/SAS system and, thereby, obtain theoretical control inputs to augment stability and to alleviate lateral acceleration levels. This two step process was followed because modal control theory provides a means of calculating feedback gains that result in pole placement and partial mode separation. Optimal regulator theory provides the feedback gains that result in lower acceleration levels. Because both methods are regulator design methods, ultimately the gains from each method are effectively added in the feedback loop. Consequently, the optimal regulator will tend to negate the effect of the modal control regulator. If applied in the reverse order, the modal control regulator will tend to negate the effect of the optimal regulator. The overall result of the present application was considered best.

STABILITY AUGMENTATION

Analytical expressions for the gains that will produce any desired characteristic values of an augmented system are well known (Refs. 5, 6 and 7). The feedback gains are obtained as solutions of linear algebraic equations if the linear dynamical system has the property of being completely controllable using a single linear control function. When more than one linear control function is available, additional system characteristics can be obtained. Response of the dynamical system can be shaped by partial specification of the characteristic vectors of the augmented system (See the Appendix for an outline of the theory of Ref. 8). How much shaping of the responses can be obtained depends on the number of independent control inputs. Specifically, if n is the dimension of the state space and m is the dimension of the control space, nm elements of the matrix of characteristic vectors of the closed-loop system are arbitrary. As a design procedure, these elements may be specified. Consequently, linear algebraic equations are available to calculate feedback gains that assure that the dynamical system will have desired stability and response characteristics.

Characteristic Values

The choice of characteristic values must satisfy the criteria summarized in table III. Since the required stability parameters are given as inequalities, the choices come from a wide range of values. The spiral mode of the airplane is stable and, of course, satisfies the criterion of reference 3. Therefore, the characteristic value associated with this mode, $\lambda_S=-.031$, need not be changed.

On the other hand, the time constant of the roll mode does not satisfy the appropriate criterion. The largest acceptable value of the roll mode time constant is $\tau_R<1.4$ with the corresponding value of the characteristic value of $\lambda_R<-.714$. Obviously, a value of $\lambda_R=-.75$ should be satisfactory. For a 15° commanded aileron input (See Fig. 3.), the roll performance, as expressed by p and ϕ, is almost halved when λ_R is decreased from -.75 to -1.5. Also, $(a_y)_{PS}$ is almost halved. However, a choice of $\lambda_R=-1.5$ ($\tau_R=.67$) was made so that the corresponding characteristic vector would be more acceptable.

The Dutch-roll mode only needs more damping (That is, $\zeta_d\omega_{nd}$ needs to be doubled, at least). Consesquently, $\zeta_d=.182$, $\omega_{nd}=.825$, $\zeta_d\omega_{nd}=.15$ satisfies the criterion of reference 3. Figure 4 shows that the level of lateral acceleration at the pilot's station decreases with decreasing undamped frequency, ω_{nd}. So, $\zeta_d=.351$, $\omega_{nd}=.427$, and $\zeta_d\omega_{nd}=.15$ were chosen for the Dutch-roll mode. The corresponding characteristic value is $\lambda_{DR}=.15\pm0.4j$. Although feedback gains required to effect this change in damped frequency might be large, consideration of the lower lateral acceleration was considered to be overriding.

In summary, the following characteristic values were chosen as satisfactory.

$$\lambda_R = -1.5 \qquad (\tau_R = .67)$$

$$\lambda_S = -.031 \qquad (T_{1/2} = 22.4 \text{ sec.})$$

$$\lambda_{DR} = -.15 \pm .4j \qquad \begin{aligned} \zeta_d &= .351 \\ \omega_{nd} &= .427 \\ \zeta_d \omega_{nd} &= .15 \end{aligned}$$

Characteristic Vectors

Some definitions, and the theory necessary for understanding the following discussion, are given in the Appendix (see also Refs. 8 and 9). The solution of the closed-loop system

$$\dot{x} = (A+BK)x + Bu_c \tag{4}$$

In terms of the modal matrix V can be expressed as

$$x(t) = V \exp(\Lambda t)V^{-1}x(0) \tag{5}$$

when u_c=0 (See Appendix for definition of Λ). If $\tilde{x}$ is defined as

$$\tilde{x} = V^{-1}x(0), \tag{6}$$

then

$$x(t) = V \exp(\Lambda t)\tilde{x} \tag{7}$$

Although the entries of the modal matrix occur nonlinearly in the x terms, dominance arguments show that, if V for the airplane is the form

$$V = \begin{bmatrix} v_{11} & \varepsilon_1 & \varepsilon_2 & \varepsilon_3 \\ \delta_1 & v_{22} & v_{23} & \varepsilon_4 \\ \delta_2 & v_{32} & v_{33} & \varepsilon_5 \\ \delta_3 & \delta_4 & \delta_5 & v_{44} \end{bmatrix} \tag{8}$$

whenever ε and δ values are small compared with v values, then V^{-1} will be dominant and cross-coupling between modes will be minimal. Note that an nxn matrix, M, is a dominant matrix (Ref. 10) if

$$|m_{ii}| > \sum_{\substack{k=1 \\ k\neq i}}^{n} |m_{ik}|, \; 1 \leq i < n \text{ (row dominance)} \tag{9}$$

and

$$\left|m_{ii}\right| > \sum_{\substack{k=1 \\ k \neq i}}^{n} \left|m_{ki}\right|, \; 1 \leq i \leq n \text{ (column dominance)} \tag{10}$$

are satisfied. ($\left| \; \right|$ means absolute value.)

The modal matrix for the airplane is

$$V = \begin{bmatrix} .5209 & .6163 & .0 & -.0469 \\ .0023 & -.0636 & .0453 & .1137 \\ .0111 & -.1629 & -.2130 & .0561 \\ -.8535 & -.0606 & -.7338 & .9908 \end{bmatrix} \tag{11}$$

or

$$V = \left[v_R \quad v_{DR,R} \quad v_{DR,I} \quad v_S \right] \tag{12}$$

where v_R is the characteristic vector of the roll mode, $v_{DR,R}$ and $v_{DR,I}$ are the real and imaginary parts, respectively, of the characteristic vector of the Dutch-roll mode, and v_S is the characteristic vector of the spiral mode. The characteristic vectors are all assumed to be normalized by dividing each element of the vector by $\sqrt{v_i^* v_i}$ where v_i^* is the complex conjugate of v_i.

Also, the corresponding matrix of characteristic values for the airplane is

$$\Lambda = \begin{bmatrix} -.611 & 0 & 0 & 0 \\ 0 & -.077 & .821 & 0 \\ 0 & -.821 & -.077 & 0 \\ 0 & 0 & 0 & -.031 \end{bmatrix} \tag{13}$$

where

$$\begin{aligned} \lambda_R &= -.611 \\ \lambda_{DR} &= -.077 \pm j\,(.821) \\ \lambda_S &= -.031 \end{aligned} \tag{14}$$

The large entry in $v_{DR,R}$ indicates that the Dutch-roll mode interacts strongly with the roll mode. Figure 2 shows this interaction as previously mentioned. Not so evident, however, is the strong interaction of $v_{DR,I}$ and v_R on the spiral mode.

Specification of the z_i, $i=1,2,3,4$, vectors for any mode must satisfy equation A-6 for a given characteristic value, λ_i. For the roll mode, $\lambda_R=-.72$ is a satisfactory characteristic value. However, the corresponding characteristic vector is not satisfactory because it does not satisfy equation (10). Consequently, from calculation of w_i for many combinations of λ_i and z_i, the characteristic value for the roll mode was selected as λ_R=-1.5 with the corresponding characteristic vector of

$$v_R = \begin{bmatrix} .8300 \\ .0 \\ -.0705 \\ -.5533 \end{bmatrix} \qquad (15)$$

The Dutch-roll mode characteristic vector, for λ_{DR}=-.15±.4j, is

$$v_{DR,R} = \begin{bmatrix} .0 \\ .2691 \\ -.5800 \\ .0518 \end{bmatrix} \quad \text{and} \quad v_{DR,I} = \begin{bmatrix} .0 \\ .2691 \\ .7094 \\ -.1140 \end{bmatrix} \qquad (16)$$

The selected characteristic value, λ_S=-.031, yields a corresponding characteristic vector of

$$v_S = \begin{bmatrix} -.0458 \\ .1093 \\ .1376 \\ .9834 \end{bmatrix} \qquad (17)$$

Of course, many other characteristic vectors are available for λ_S=-.031. The v_S given, however, resulted in a more favorable feedback gain matrix than did most other values of v_S. The gain matrix resulted in small rudder angles initially in response to aileron angle input.

In summary, for the selected characteristic values, the resulting modal matrix is

$$V = \begin{bmatrix} .8300 & .0 & .0 & -.0458 \\ .0 & .2691 & .2691 & .1093 \\ -.0705 & -.5800 & .7094 & .1376 \\ -.5533 & .0518 & -.1140 & .9834 \end{bmatrix} \qquad (18)$$

The modal matrix, V, exhibits the required dominance and the corresponding matrix of characteristic values, Λ, satisfies the criteria of reference 3. The matrix of characteristic values may be written as

$$\Lambda = \begin{bmatrix} -1.5 & .0 & .0 & .0 \\ .0 & -.15 & .40 & .0 \\ .0 & -.40 & -.15 & .0 \\ .0 & .0 & .0 & -.03 \end{bmatrix} \qquad (19)$$

The resulting feedback gain matrix is

$$K = \begin{bmatrix} -1.176 & -1.172 & 2.569 & -.097 \\ -.016 & .508 & .252 & .055 \end{bmatrix} \qquad (20)$$

Response of the augmented airplane to an aileron angle input of 15° is presented in figure 5. Comparison with the response for the unaugmented airplane given in figure 2, shows that a more desired response has been achieved. However, the peak lateral acceleration at the pilot's station is still too large.

LATERAL ACCELERATION ALLEVIATION

The airplane, augmented as described in the preceding section is represented by

$$\dot{x} = (A+BK)x + Bu$$

or

$$\dot{x} = \hat{A}x + Bu \tag{21}$$

with

$$\hat{A} = A + BK \tag{22}$$

The controlled variable is lateral acceleration at the pilot's station and is represented as

$$(a_y)_{PS} = Cx + Du \tag{23}$$

(See Eq. (2) and (3).)

A control law, given by

$$u = R^{-1}[B^T P + S^T]x \tag{24}$$

minimizes a performance index

$$J = \int_0^T (a_y)^2_{PS} + u^T R_1 u)dt \tag{25}$$

or

$$J = \int_0^T (x^T Qx + 2x^T Su + u^T Ru)dt \tag{26}$$

where the matrix P satisfies the matrix Riccati equation

$$0 = -P\hat{A} - \hat{A}^T P - Q + (PB + S)(B^T P + S^T) \tag{27}$$

In equation (24) and equation (25),

$$Q = C^T C$$

$$S = C^T D$$

$$R = R_1 + D^T D$$

Lateral acceleration at the pilot's station of the supersonic transport airplane is much too large during a rolling maneuver. Optimal regulator theory (outlined above) was applied in an attempt to reduce the extreme acceleration peak. In principle, the theory worked very well. However, the reduction in lateral acceleration during the first 3 to 5 seconds of the rolling maneuver was almost inconsequential. Extreme variations in the weighting matrices, Q and R_1, of the performance index failed to change the results.

An open-loop control input was applied to alleviate the lateral acceleration. The aileron was deflected, at maximum rate, to its stop (30°), held for awhile, then returned to near zero deflection angle. From consideration of equation (2), an initial positive rudder deflection is required. The response to such control inputs is shown in figure 6. The control inputs are shown in figure 6(b). The airplane rolled to 30° in the required 3 seconds (See Ref. 4) and the lateral acceleration peak at the pilot's station was less than .075 g. Also, the peak occurred at about 2 seconds rather than immediately after the control input. However, lateral acceleration changed signs after 3 seconds. The responses of yawing velocity and sideslip angle, however, were not satisfactory. Sideslip angle increased to about 15° and yawing velocity was negative for 3 seconds.

The undesirable $(a_y)_{PS}$ seems to occur in the command response because of the sudden control input. Therefore, crossfeed and prefiltering, to slow up the input and coordinate the controls, seems to be required. Figure 6 illustrates the desirable crossfeed effects.

Two aileron and rudder interconnects were placed in the forward control path. The gain matrices were chosen to improve roll and yaw control by negating rolling moment caused by rudder deflections and by negating yawing moment caused by aileron deflections. The feedforward matrices are

$$G_1 = \begin{bmatrix} 1.0 & .0 \\ -.1161 & 1.0 \end{bmatrix}$$

and

$$G_2 = \begin{bmatrix} 1.0 & -.2538 \\ -.1161 & 1.0 \end{bmatrix}$$

Aileron deflections produced no yawing moment when G_1 was used. In addition, rudder deflections produced no rolling moment when G_2 was used.

Response of the airplane to the control input of figure 6(b) with G_1 and G_2 in the forward control path is shown in figure 7. Only minor improvements in the response were obtained by using G_1 or G_2 (compare Fig. 6 with Fig. 7). Furthermore, neither G_1 nor G_2 was superior.

CONCLUDING REMARKS

The results of this study show that modal control theory provides desired stability and desired transient response characteristics. Proposed criteria for the maximum allowable lateral acceleration at the pilot station was satisfied by using open-loop control inputs. However, large adverse sideslip angle excursions were required; and the yawing velocity was negative for about 2 seconds. Optimal regulator theory did not give a significant reduction of the lateral acceleration level during transient motion.

APPENDIX

Theory

The problem of selection of characteristic values and characteristic vectors is briefly outlined as follows (See Refs. 7 and 8).

Consider the controllable system given by equation (1). That is,

$$\dot{x} = Ax + Bu$$

where $x \epsilon R^4$, $u \epsilon R^2$, and A and B are properly dimensioned. Assume that B is full rank. The state variable feedback law takes the form

$$u = Kx + u_c \tag{A-1}$$

where K is the feedback matrix. The problem is to select K so that the closed-loop system matrix A+BK satisfies

$$[A+BK]\, v_i = \lambda_i v_i \quad (i=1,2,3,4) \tag{A-2}$$

where λ_i is the ith characteristic value and v_i is the corresponding characteristic vector.

Partition equation (A-2) as

$$\left[\begin{bmatrix} A_{11} & \vdots & A_{12} \\ \cdots & \vdots & \cdots \\ A_{21} & \vdots & A_{22} \end{bmatrix} + \begin{bmatrix} B_1 \\ \cdots \\ B_2 \end{bmatrix} [K_1 : K_2]\right] \begin{bmatrix} z_i \\ \cdots \\ w_i \end{bmatrix} = \lambda_i \begin{bmatrix} z_i \\ \cdots \\ w_i \end{bmatrix} \quad (i=1,2,3,4) \tag{A-3}$$

where A_{11}, B_1, and K_1 are 2x2 matrices and other matrices are compatibly dimensioned. Also, note that B_1, is nonsingular. z_i is a 2x1 vector, z_i and w_i are partitions of the characteristic vector, v_i, and $v_i^T = [z_i^T : v_i^T]$. Completing the multiplication of equation (A-3) yields

$$[A_{11} + B_1K_1]z_i + [A_{12} + B_1K_2]w_i = \lambda_i z_i \tag{A-4}$$

$$[A_{21} + B_2K_1]z_i + [A_{22} + B_2K_2]w_i = \lambda_i w_i \tag{A-5}$$

Solve equation (A-4) for $K_1z_i + K_2w_i$ and substitute into equation (A-5) to obtain the following constraining relationship

$$[\lambda_i I_{n-m} - F]w_i = [G + \lambda_i S]z_i \quad (i=1,2,3,4) \tag{A-6}$$

where $I_{n-m}=I_2$ is a 2nd order identity matrix and S, G, and F are matrices defined by

$$S = B_2 B_1^{-1}$$

$$G = A_{21} - SA_{11} \qquad \text{(A-7)}$$

$$F = A_{22} - SA_{12}$$

Equation (A-6) constitutes a set of n-m=2 linear equations in the n=4 unknown elements of each characteristic vector. Thus, if λ_i is not a characteristic value of F, at most m elements corresponding to the z_i vector can be arbitrarily chosen. Then the remaining n-m characteristic vector elements corresponding to the w_i vector can be computed from equation (A-6) as

$$w_i = C_i z_i \qquad (i=1,2,3,4) \qquad \text{(A-8)}$$

where $C_i=[\lambda_i I_{n-m}-F]^{-1}[G+\lambda_i S]$ is defined as the "modal coupling matrix" corresponding to λ_i.

The preceding analysis shows that all n characteristic values and up to n•m characteristic vector elements can be arbitrarily assigned through the use of state variable feedback. The resulting modal matrix

$$V = [v_1 \vdots v_2 \vdots \ldots \vdots v_n]$$

however, must be nonsingular.

With a nonsingular modal matrix, V, and characteristic value matrix, Λ, chosen subject to the constraints of equation (A-6), the closed-loop system matrix A=A+BK is uniquely determined by

$$\hat{A} = \begin{bmatrix} \hat{A}_{11} & \vdots & \hat{A}_{12} \\ \cdots & \vdots & \cdots \\ \hat{A}_{21} & \vdots & \hat{A}_{22} \end{bmatrix} = V\Lambda V^{-1} \qquad \text{(A-9)}$$

The matrix Λ is given by

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \alpha & \beta & 0 \\ 0 & -\beta & \alpha & 0 \\ 0 & 0 & 0 & \lambda_4 \end{bmatrix} \qquad \text{(A-10)}$$

where λ_1 and λ_4 are real and $\lambda_2=\alpha+j\beta$ and $\lambda_3=\alpha-j\beta$.

The required feedback matrix, K, which yields this closed-loop matrix, A, can be easily computed by using the relations

$$K_1 = B_1^{-1}[\hat{A}_{11} - A_{11}]$$

$$K_2 = B_1^{-1}[\hat{A}_{12} - A_{12}] \tag{A-11}$$

and

$$K = [K_1 : K_2] \tag{A-12}$$

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TABLE I.- MASS AND DIMENSIONAL CHARACTERISTICS OF SUPERSONIC TRANSPORT AIRPLANE

Weight, N	1 924 479
Reference wing area, m^2	784.75
Wing span, m	38.66
Wing leading-edge sweep, deg (see Fig. 1)	74.00/70.84/60.00
Reference mean aerodynamic chord, m	27.00
Center-of-gravity location, percent c	56
Static margin, percent	-3.9
I_X, kg-m^2	6 887 550
I_Y, kg-m^2	67 994 260
I_Z, kg-m^2	72 902 230
I_{XZ}, kg-m^2	-2 833 660
Maximum control surface deflections:	
δ_f, deg	0 to 40
δ_a, deg	±30
δ_{af}, deg	±22.5
δ_s, deg	±50
δ_r, deg	±35
Maximum control surface deflection rates:	
$\dot{\delta}_f$, deg/sec	±10
$\dot{\delta}_a$, deg/sec	±70
$\dot{\delta}_{af}$, deg/sec	±40
$\dot{\delta}_s$, deg/sec	±50
$\dot{\delta}_r$, deg/sec	±50
Horizontal tail:	
Gross horizontal-tail area, m^2	49.80
Mean aerodynamic chord, m	6.04
Distance from center-of-gravity to horizontal-tail $0.25\bar{c}$, m	32.90
Vertical tail:	
Exposed vertical-tail area, m^2	16.72
Mean aerodynamic chord, m	6.35
Distance from center-of-gravity to vertical-tail $0.25\bar{c}$, m	36.41

TABLE II.- AERODYNAMIC CHARACTERISTICS OF SUPERSONIC TRANSPORT AIRPLANE

Airspeed = 78.71 m/sec; Altitude = 91.44 m; α = 8°; δ_f = 40°

$C_{Y_p} = 1.1793\ \text{rad}^{-1}$	$C_{Y_r} = .4154\ \text{rad}^{-1}$	$C_{Y_\beta} = -.00723\ \text{deg}^{-1}$
$C_{\ell_p} = -.1389\ \text{rad}^{-1}$	$C_{\ell_r} = .1946\ \text{rad}^{-1}$	$C_{\ell_\beta} = -.00219\ \text{deg}^{-1}$
$C_{n_p} = -.0747\ \text{rad}^{-1}$	$C_{n_r} = -.2941\ \text{rad}^{-1}$	$C_{n_\beta} = .00160\ \text{deg}^{-1}$
$C_{Y_{\delta r}} = .00100\ \text{deg}^{-1}$		
$C_{\ell_{\delta r}} = .00017\ \text{deg}^{-1}$		
$C_{n_{\delta r}} = -.00119\ \text{deg}^{-1}$		
$C_{Y_{\delta s}} = -.00008\ \text{deg}^{-1}$	$C_{Y_{\delta af}} = -.00026\ \text{deg}^{-1}$	$C_{Y_{\delta a}} = -.00026\ \text{deg}^{-1}$
$C_{\ell_{\delta s}} = .00012\ \text{deg}^{-1}$	$C_{\ell_{\delta af}} = .00040\ \text{deg}^{-1}$	$C_{\ell_{\delta a}} = .00036\ \text{deg}^{-1}$
$C_{n_{\delta s}} = 0$	$C_{n_{\delta af}} = .00009\ \text{deg}^{-1}$	$C_{n_{\delta a}} = .00014\ \text{deg}^{-1}$

NOTE: The aileron, spoiler, and flaperon effectiveness terms were combined as follows:

$$\tilde{C}_{()_{\delta a}} = 1.667\ C_{()_{\delta s}} + .75\ C_{()_{\delta af}} + C_{()_{\delta a}}$$

since $\delta_s = 1.667\delta_a$ and $\delta_{af} = 0.75\delta_a$ for this report.

TABLE III.- COMPARISON OF AIRPLANE STABILITY AND REQUIRED STABILITY

	Airplane Stability	Required Stability
Roll Mode:	$\tau_R = 1.64$	$\tau_R < 1.4$
Spiral Mode:	$T_{1/2} = 22.4$ seconds	$T_2 > 20$ seconds
Dutch-Roll Mode:	$\zeta_d = .093$	$\zeta_d \geq .08$*
	$\omega_{nd} = .825$	$\omega_{nd} \geq .4$
	$\zeta_d \omega_{nd} = .077$	$\zeta_d \omega_{nd} \geq .15$*

*Whichever gives the largest value of ζ_d.

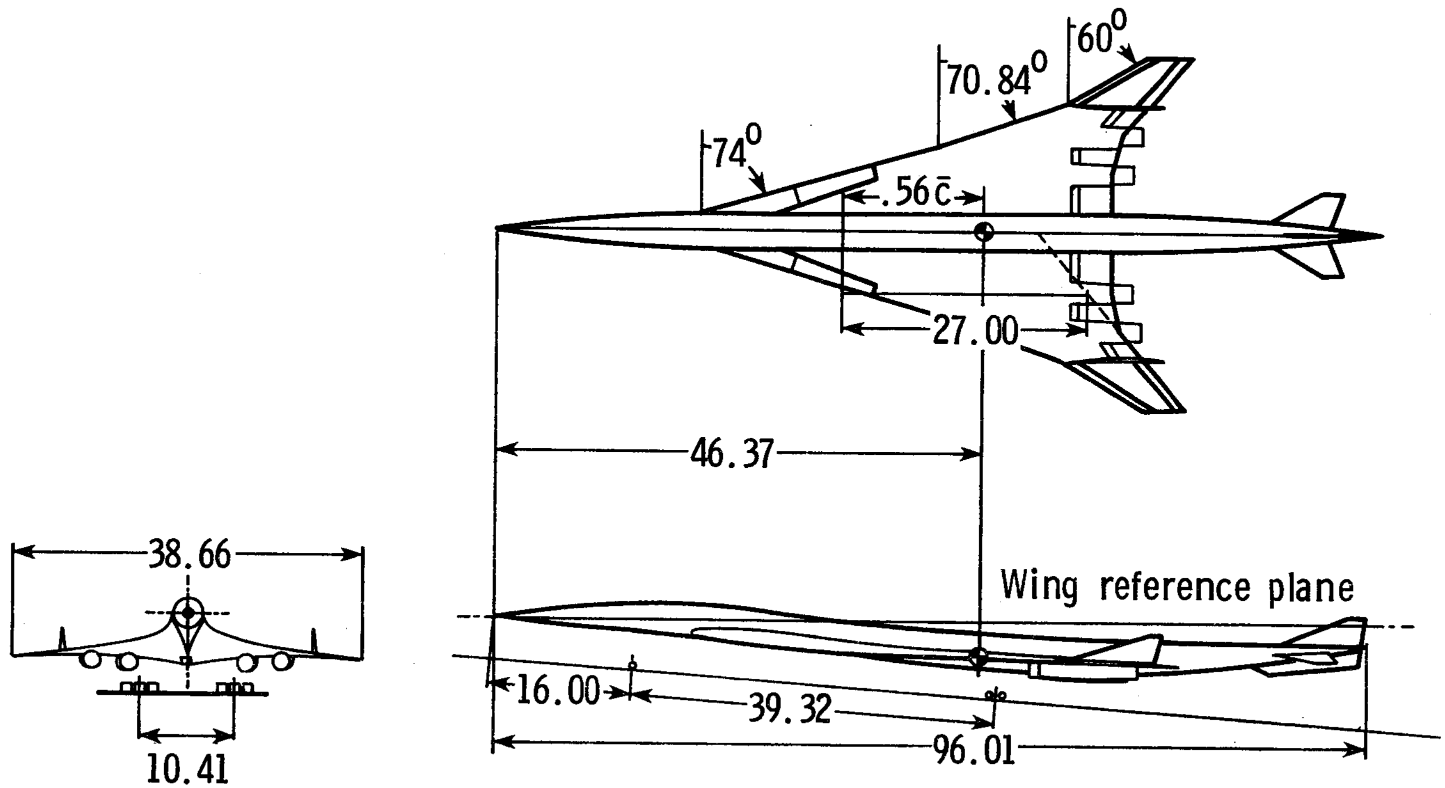
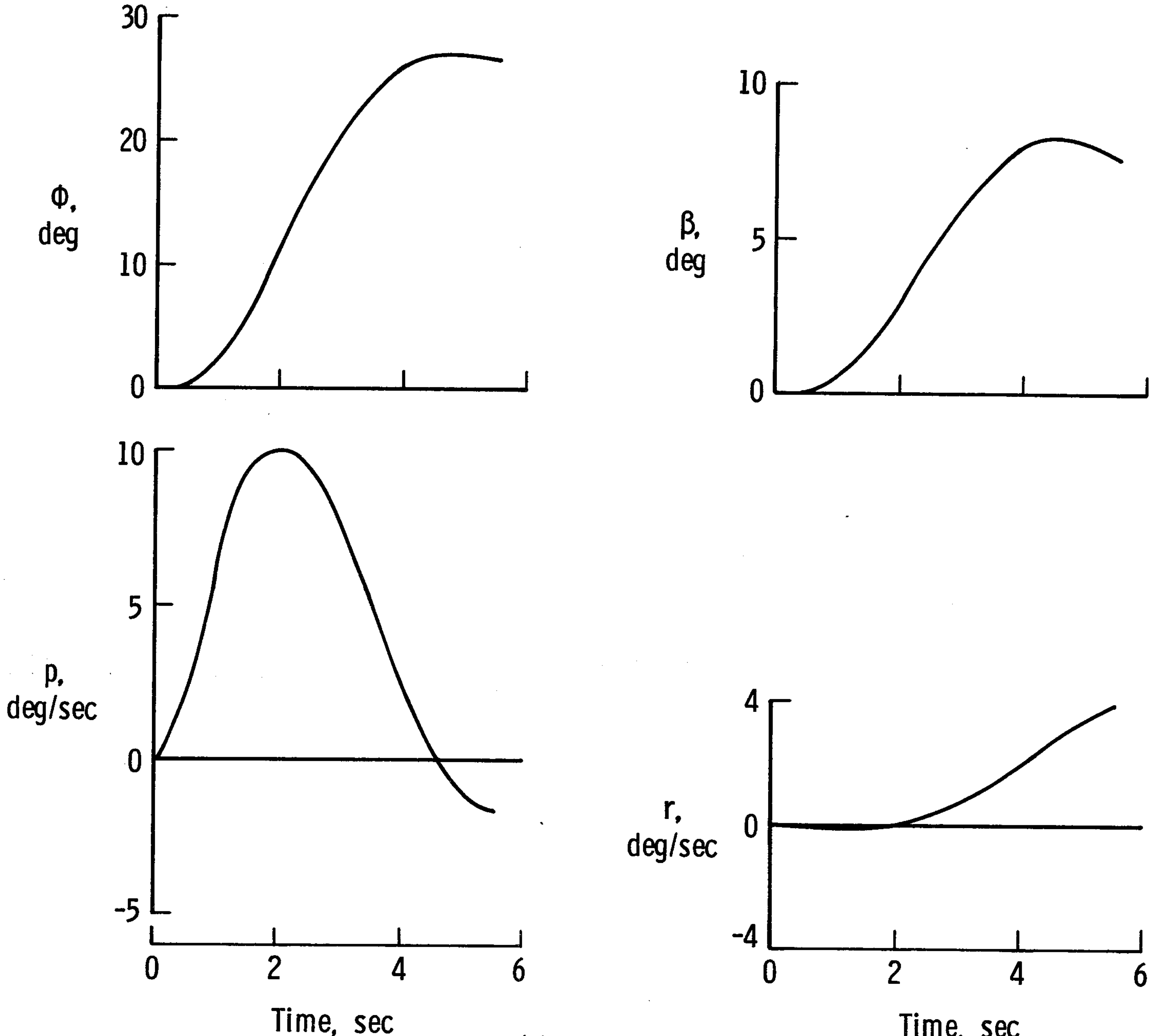
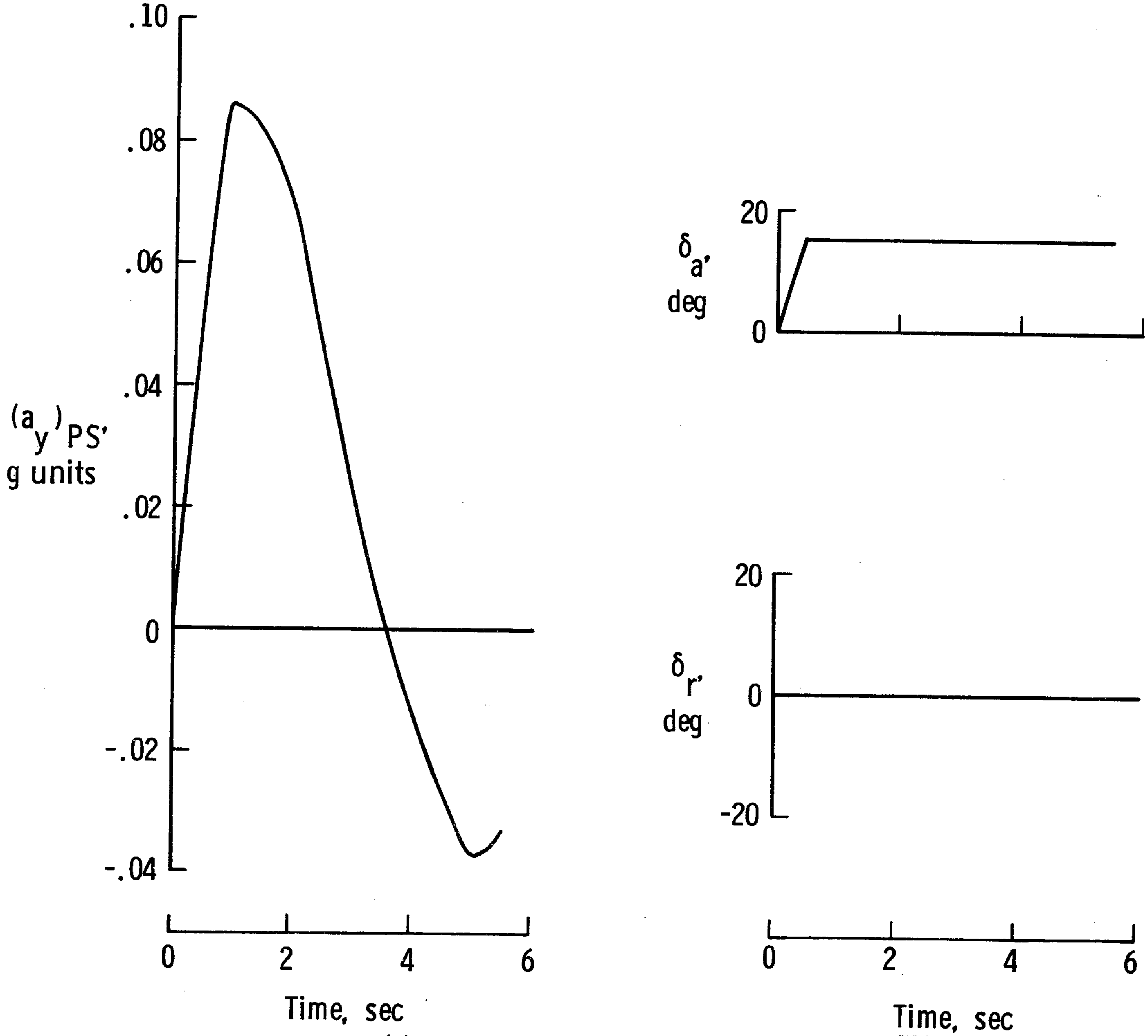


Figure 1.- Sketch of a supersonic transport.
All linear dimensions are in meters.



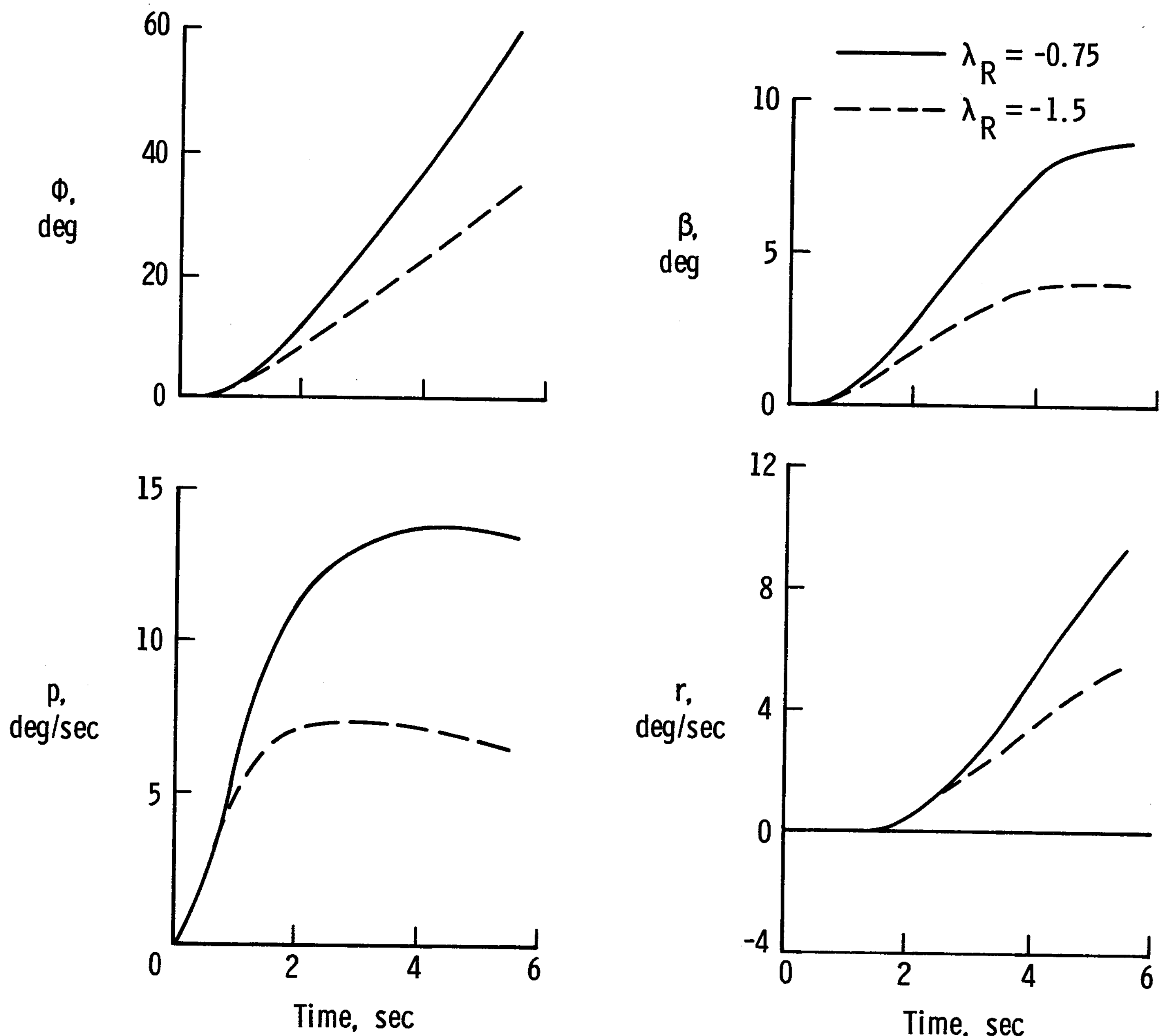
(a) State response.

Figure 2.- Response of unaugmented airplane to an aileron input.



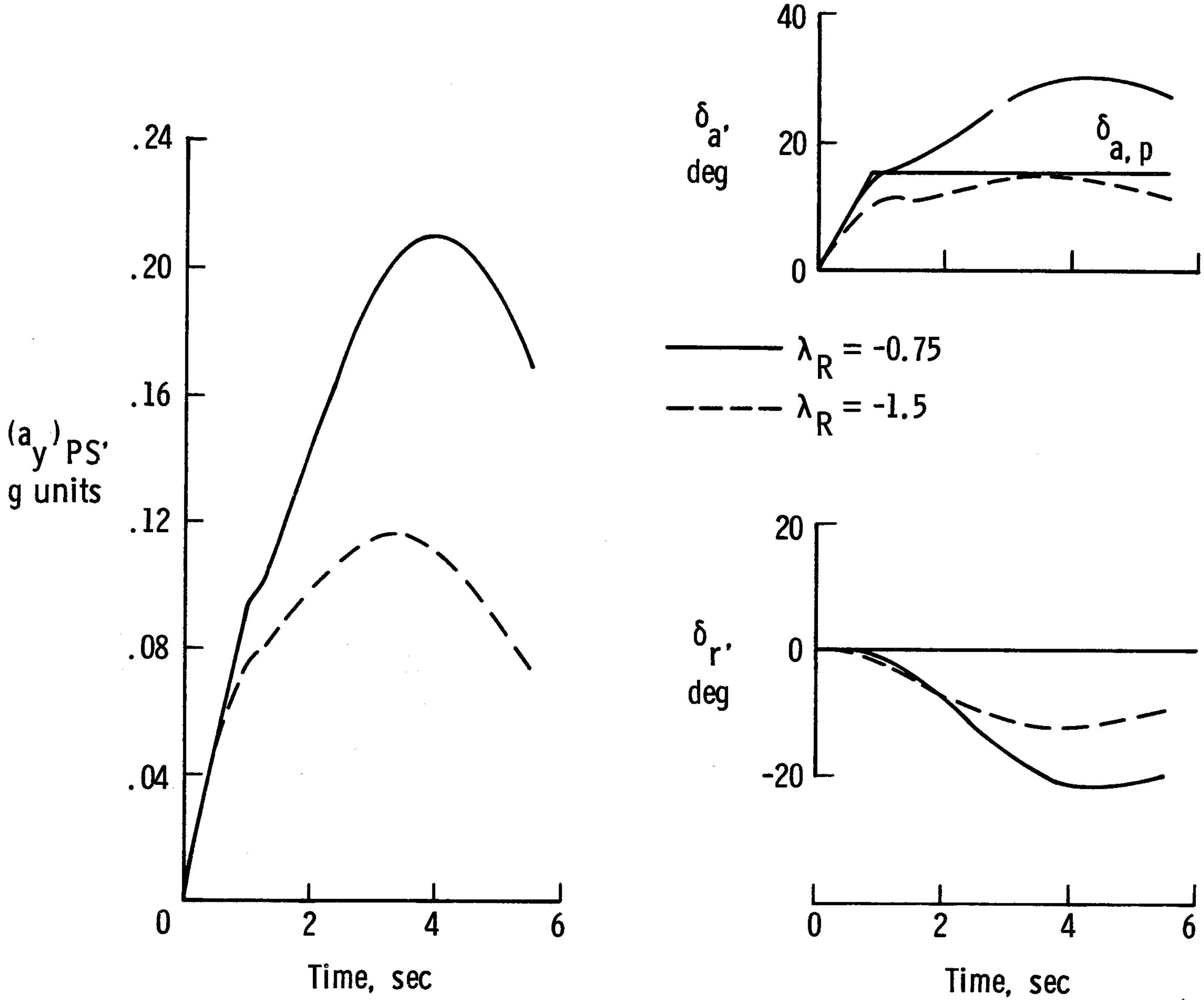
(b) Lateral acceleration and control deflections.

Figure 2.- Concluded.



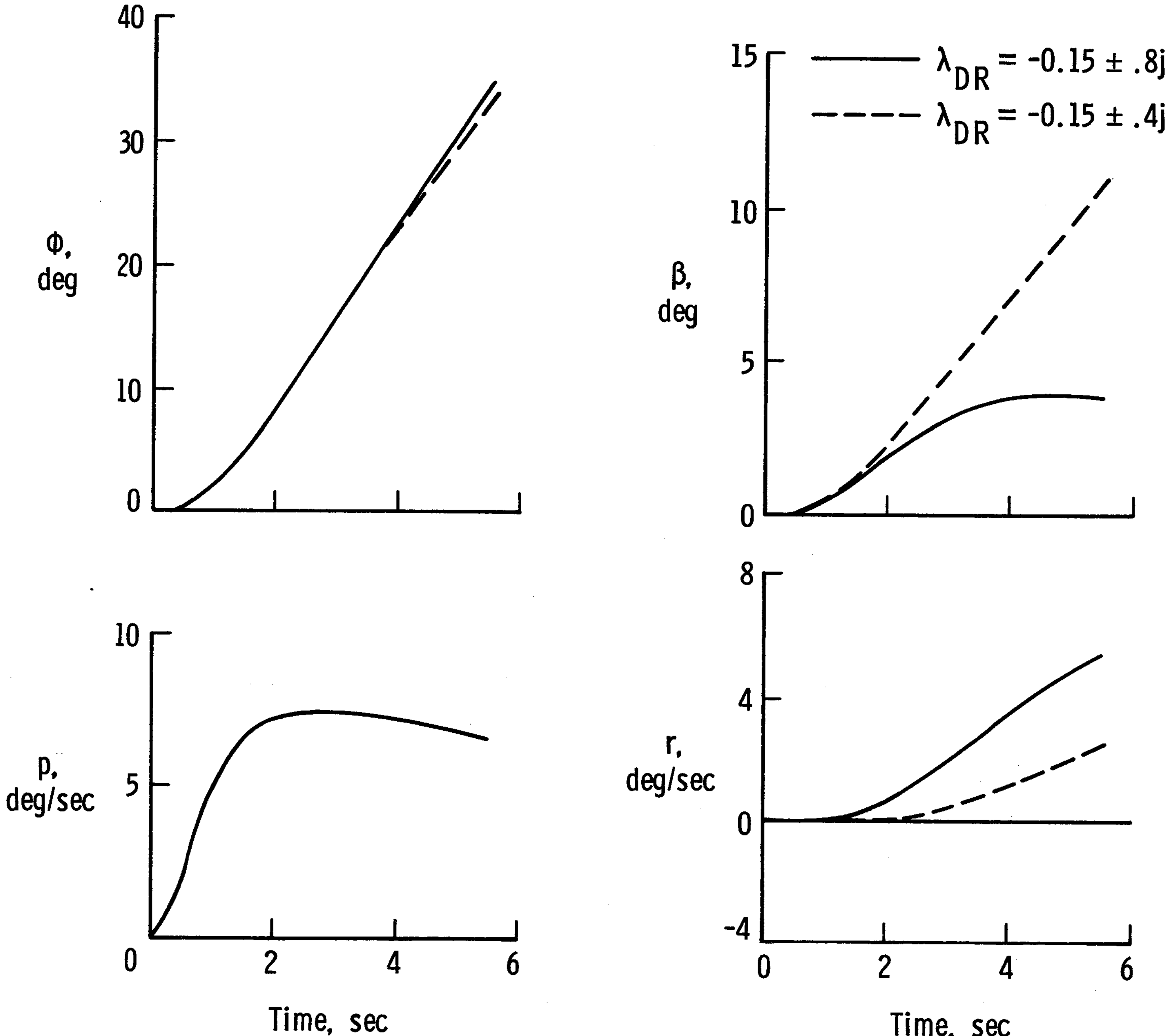
(a) State response.

Figure 3.- Effect of roll mode characteristic value on the response of the augmented airplane (λ_{DR}=-.15±.8j and λ_S=-.031).



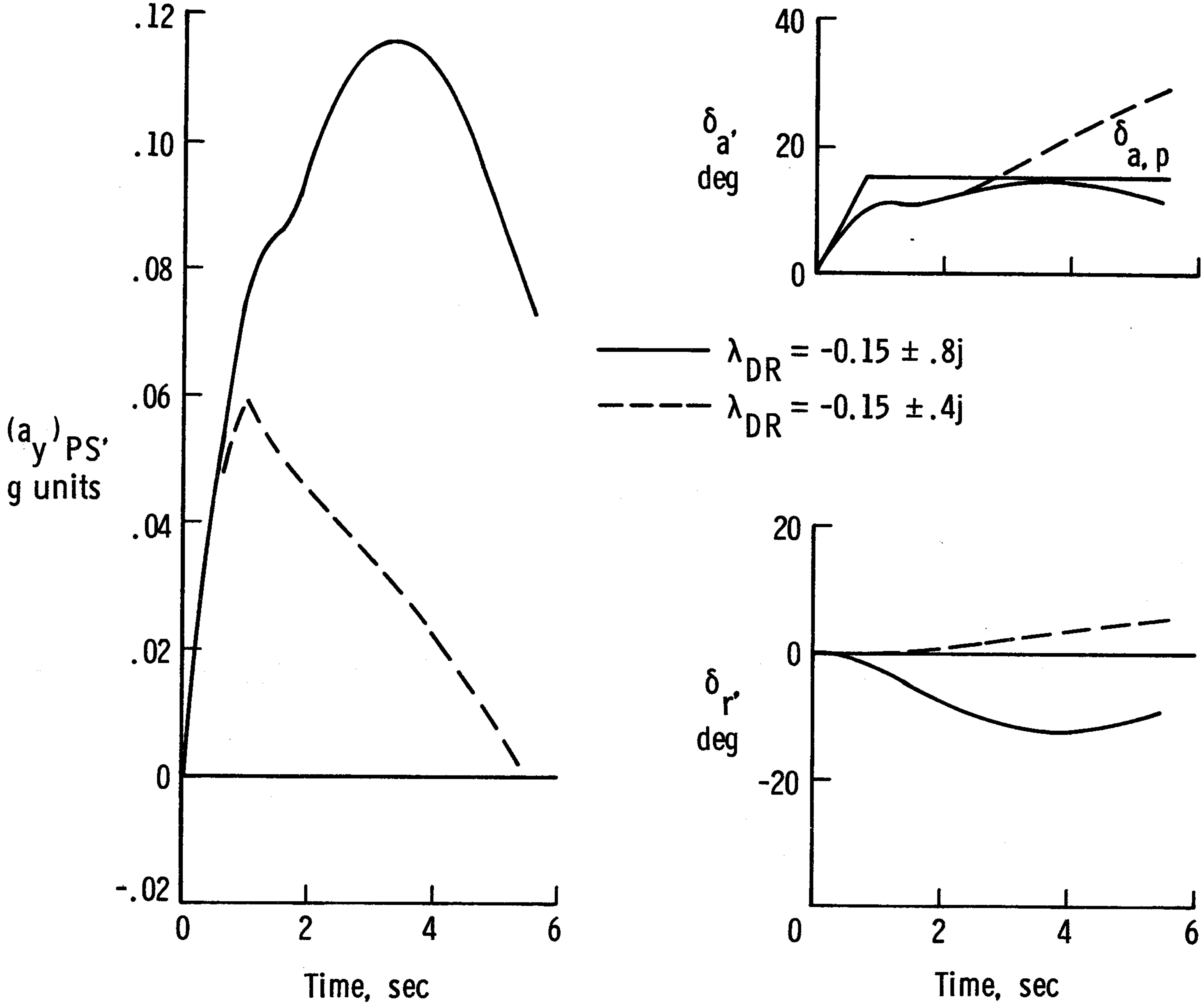
(b) Lateral acceleration and control deflections.

Figure 3.- Concluded.



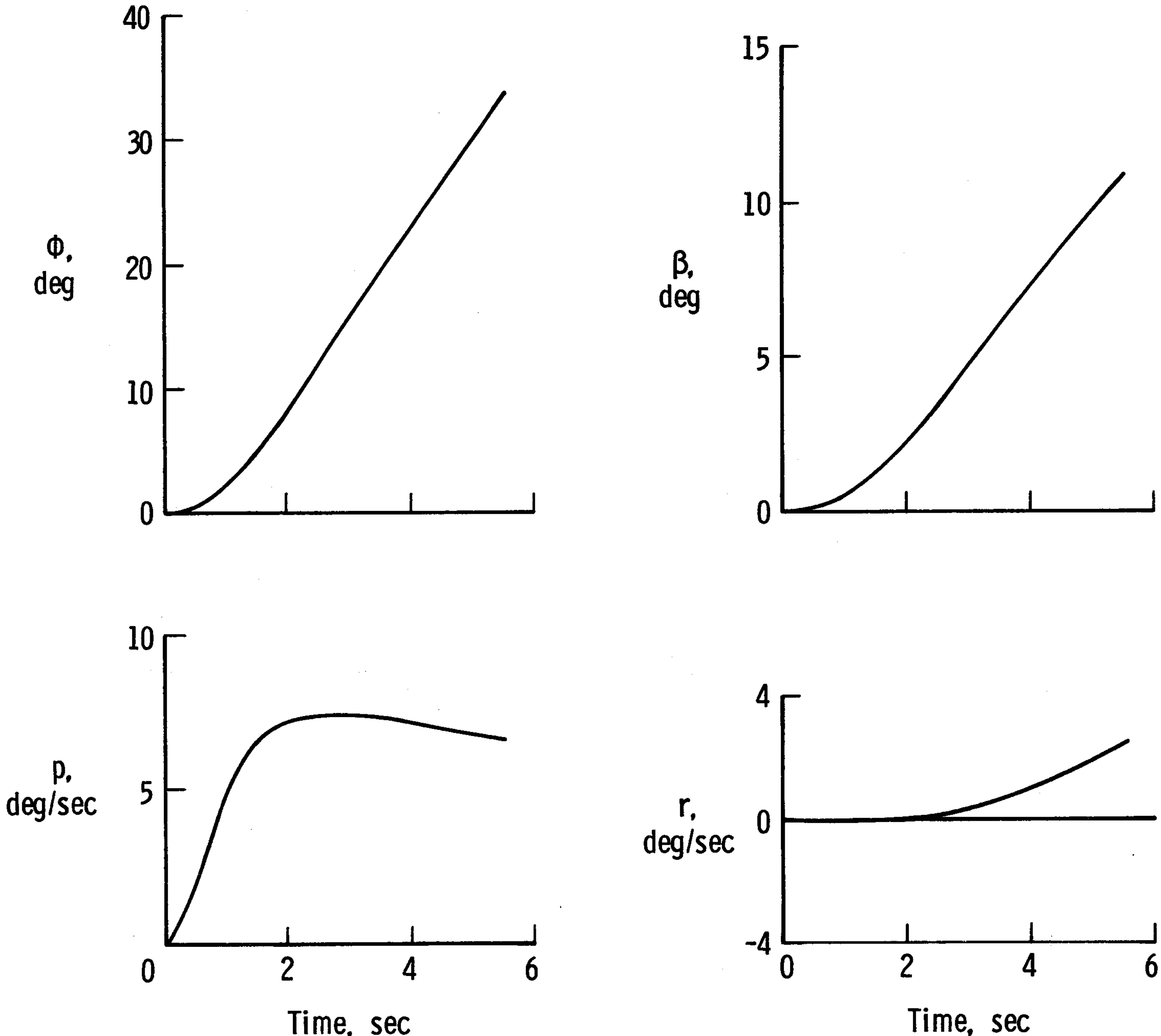
(a) State response.

Figure 4.- Effect of Dutch-roll mode frequency on the response of the augmented airplane (λ_R=-1.5 and λ_S = -.031).



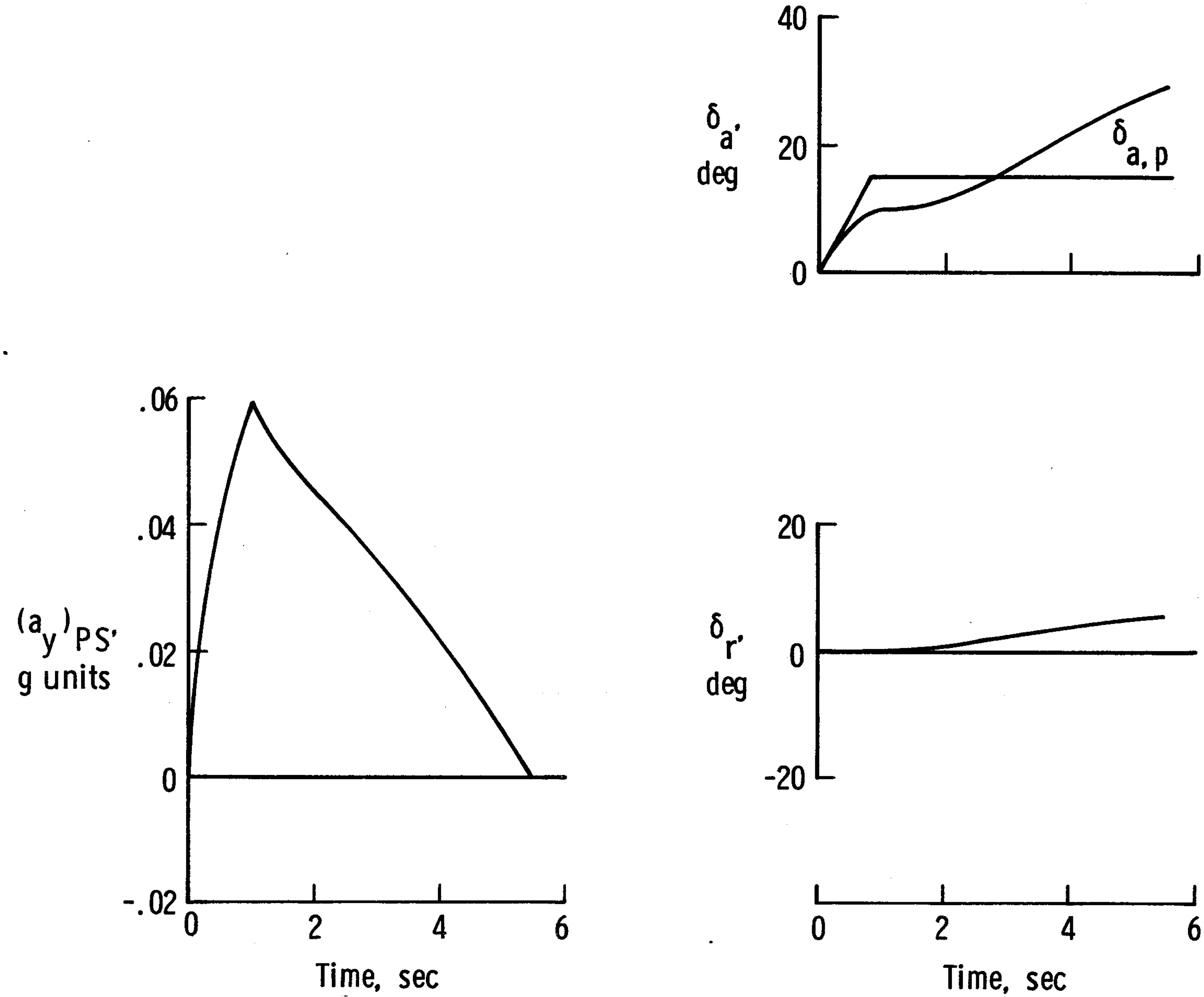
(b) Lateral acceleration and control deflections.

Figure 4.- Concluded.



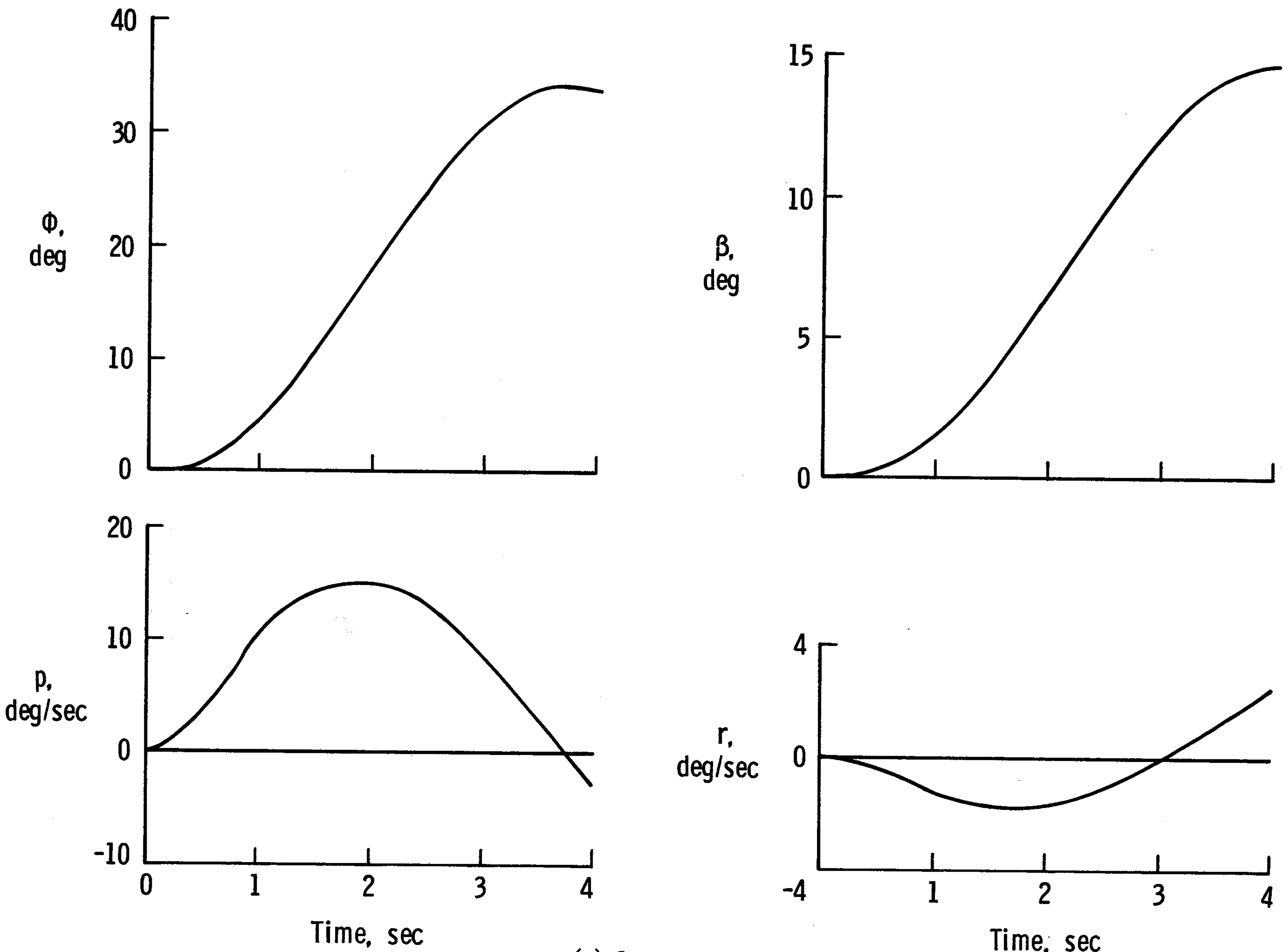
(a) State response.

Figure 5.- Response of augmented airplane to an aileron input.



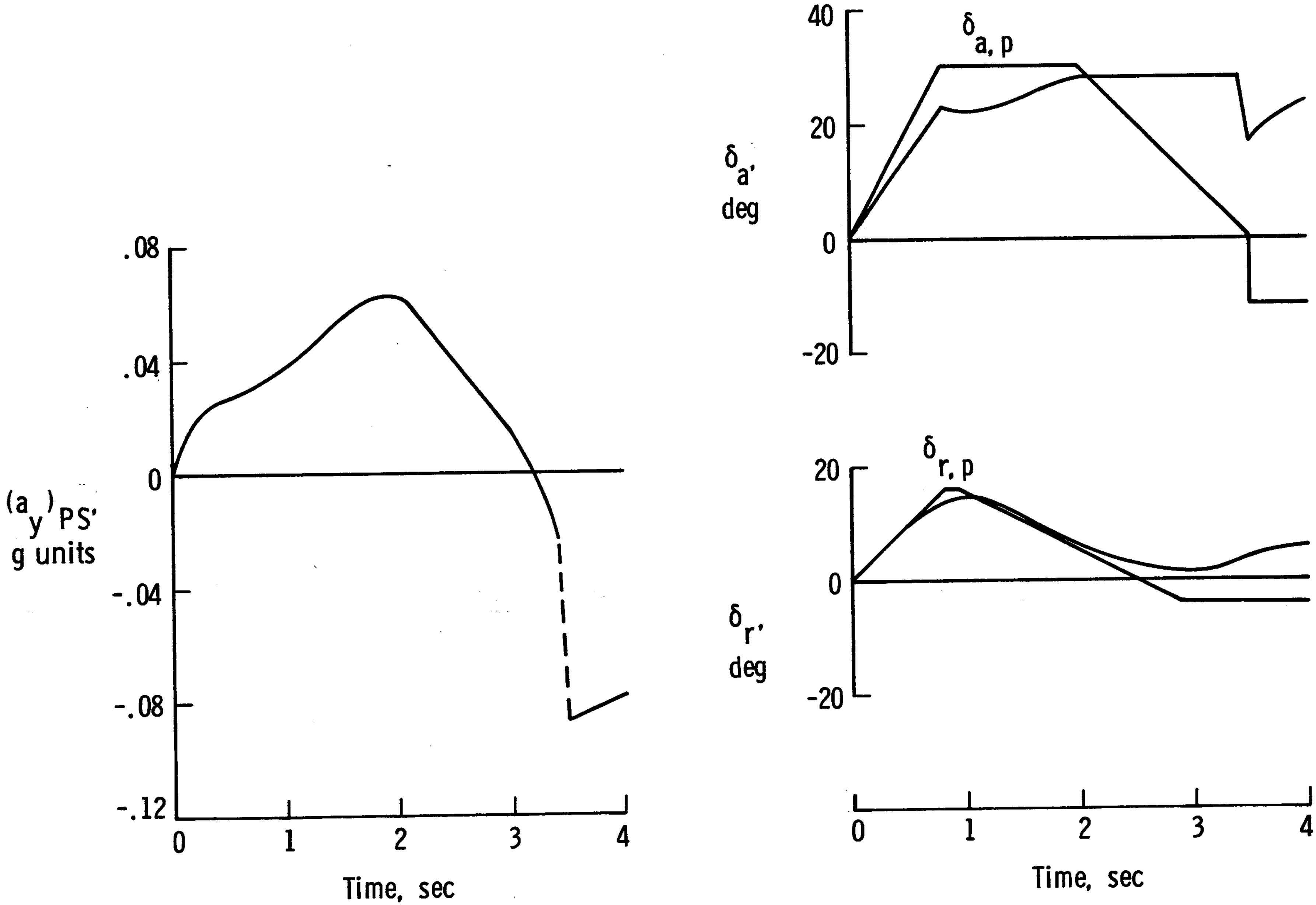
(b) Lateral acceleration and control deflections.

Figure 5.- Concluded.



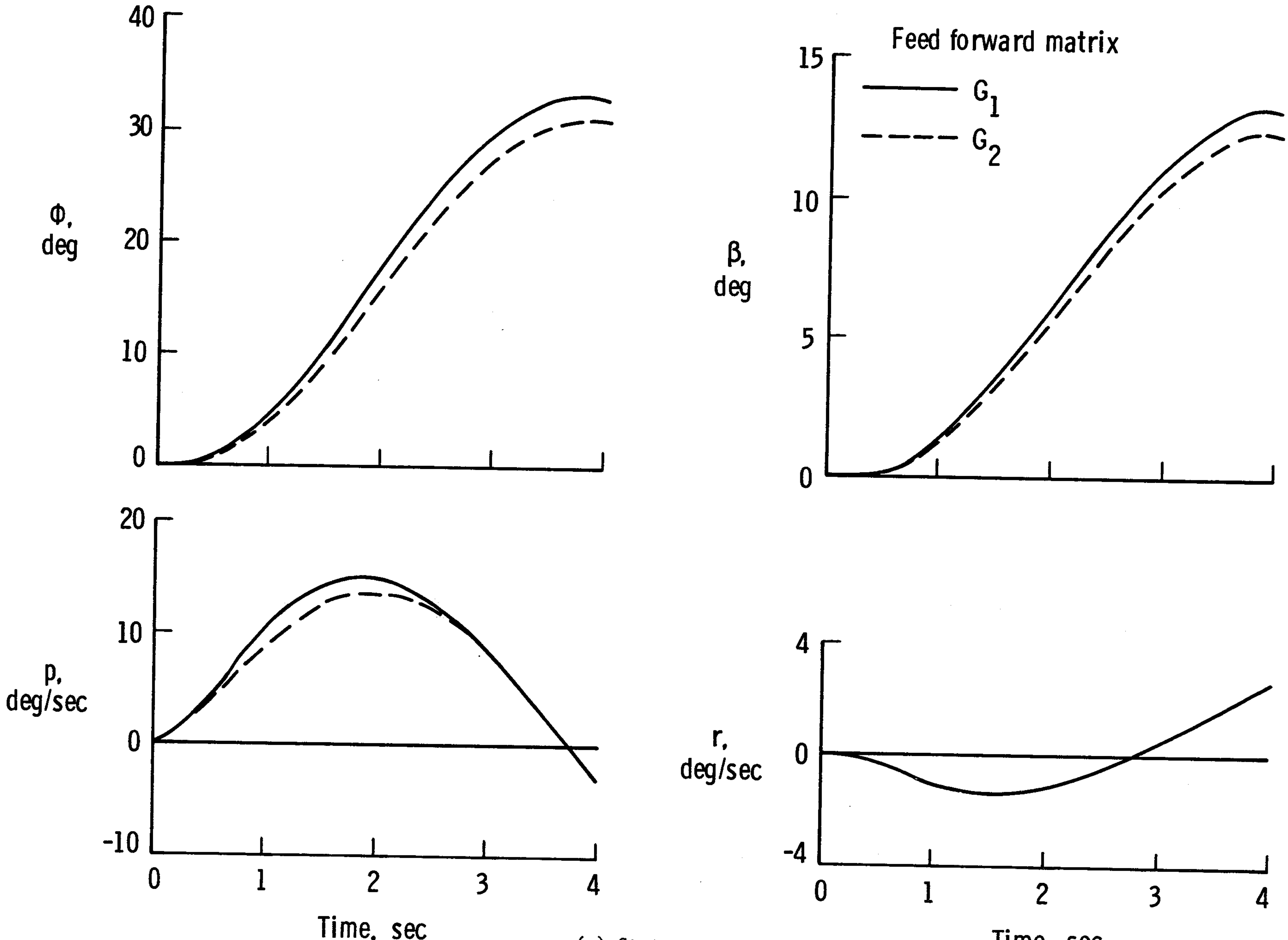
(a) State response.

Figure 6.- Response of augmented airplane to simultaneous aileron and rudder inputs.



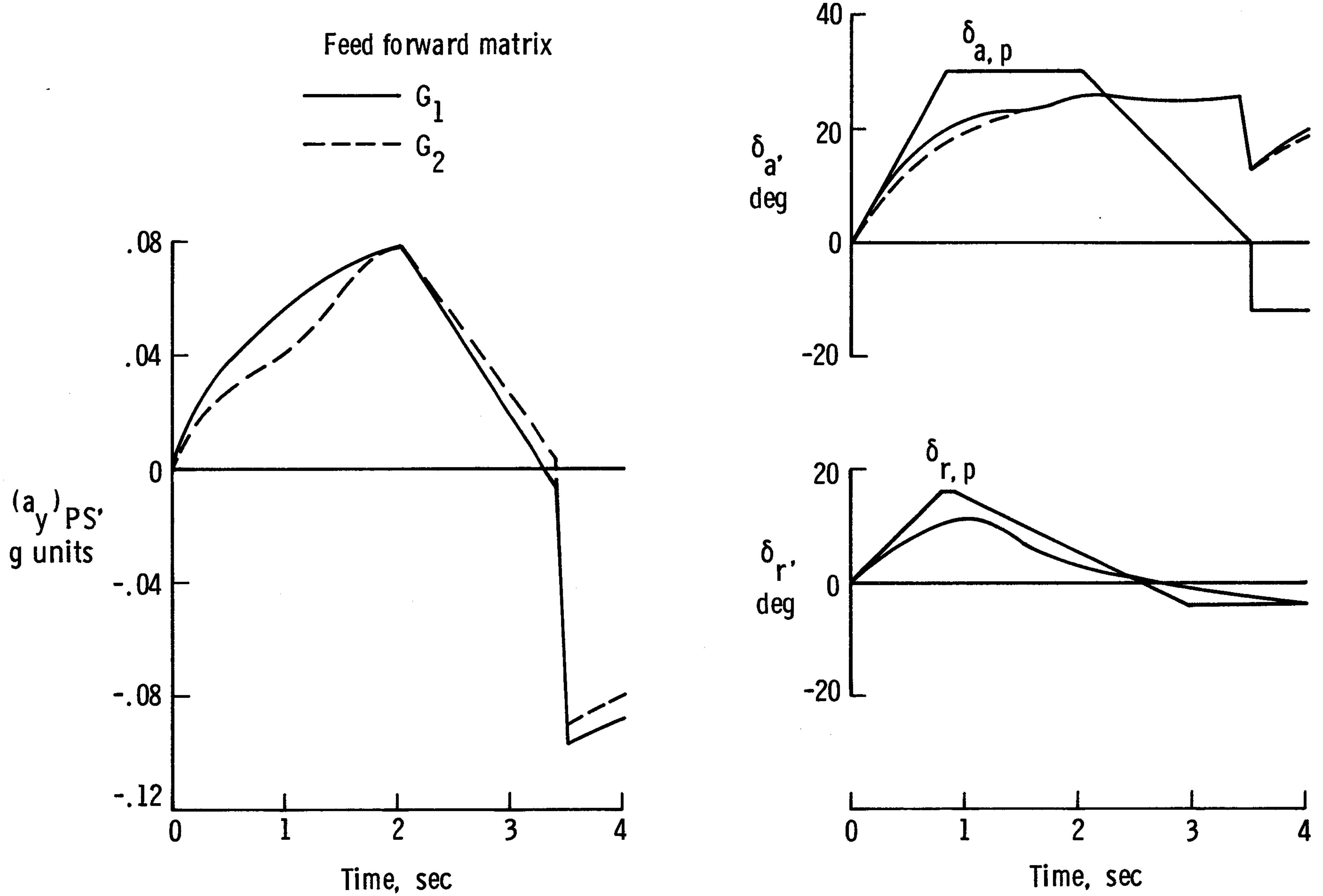
(b) Lateral acceleration and control deflections.

Figure 6.- Concluded.



(a) State response.

Figure 7.- Effect of control interconnects on the response of the augmented airplane.



(b) Lateral acceleration and control deflections.

Figure 7.- Concluded.

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15. Supplementary Notes

16. Abstract

Previous simulator studies have shown that a proposed supersonic transport airplane exhibits undesirable lateral motions during landing approach. Large adverse sideslip excursions and large peak lateral acceleration at the pilot's station occurred during rolling maneuvers of the unaugmented airplane.

In this study, modal control theory has been applied to determine feedback gains that provide desirable stability characteristics and satisfactory transient response to aileron deflection input. However, the peak value of lateral acceleration at the pilot's station does not satisfy a proposed criterion during a rolling maneuver.

Optimal regulator theory was then applied to the closed loop design provided by the modal procedure in an attempt to reduce the acceleration peak. However, this did not provide a significant reduction of the peak lateral acceleration. Subsequent experimentation with various open loop rudder and aileron control inputs provided the desired bank angle (30°) with the desired roll rate (10°/sec), and provided a satisfactory level of lateral acceleration. However, a large adverse sideslip angle was required.

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