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ABSTRACT

The design of a controller for large space structures (LSS) based on the LQG theory requires the knowledge of the LSS parameters. Since apriori knowledge of the parameters is usually not reliable, the parameters must be identified prior to the controller synthesis, using methods such as the maximum likelihood technique. In this report, an expression is obtained for the Fisher information matrix for LSS, from which Cramér-Rao bounds can be obtained in order to determine the accuracy with which the parameters can be identified.

INTRODUCTION

The problems encountered in controlling large flexible space structures arise mainly because of (1) infinite number of structural modes, (2) numerous low-frequency structural modes, (3) extremely small structural damping ratios, and (4) closely-spaced structural mode frequencies. The controller design for large space structures (LSS) can be accomplished by using tools of modern control theory. Two types of control systems synthesis techniques were discussed in reference 1. The first technique, the "collocated controller technique," utilizes the stability and robustness properties of controllers employing positive definite feedback and collocated actuators and sensors. The second technique results in a reduced order Linear-Quadratic Gaussian (LQG) controller, where the order reduction is accomplished by methods such as those described in reference 2.

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In the course of the present grant, these two techniques were applied (ref. 1) for synthesizing control system for a large space antenna, and RMS (root-mean-square) performance, described by pointing, surface and feed misalignment errors, was obtained in the presence of sensor and actuator noise. The results were also reported in the preceding progress report of this grant. One of the outcomes of this study was that the LQG controller yielded markedly superior performance. However, the design of an LQG controller requires at least a reasonable knowledge of the parameters, i.e., natural frequencies, damping ratios and mode shapes (or slopes). It is extremely difficult, if not impossible, to accurately determine these parameters apriori, on the ground. Therefore, the parameter identification will probably have to be performed in orbit, and control system designed subsequently.

In the light of the above discussion, it is important to investigate the identifiability of the LSS parameters. In this report, expressions for the information matrix for LSS are derived using rate gyros as sensors. Cramér-Rao bounds, which indicate the best theoretical accuracy with which parameters can be identified, can be obtained by inverting the Fisher information matrix.

FISHER INFORMATION MATRIX FOR LSS

Consider a linear constant coefficient system described by

$$x = Ax + Bu + v_c \tag{1}$$

$$y = Cx + Du \tag{2}$$

$$z = y + w_c \tag{3}$$

where x, u, y and z represent $n \times 1$ state vector, $m \times 1$ input vector, $\ell \times 1$ output vector and $\ell \times 1$ observation vector; v_c and w_c represent process and observation noise (stationary, zero mean, white). A, B, C, and D are functions of a parameter vector p which has the dimension $n_p \times 1$.

The discrete-time, maximum likelihood identification problem can then be stated as follows:

Given observations $z(t)$ at $t = kT$, $k = 0,1,2,\dots,N$, for a known input $u(t)$, determine the estimate $\hat{p}$ of p which maximizes the conditional density $f(Z_N/p)$ where

$$Z_N = (z_o, z_1, \dots, z_N)$$

If the apriori density of p is known, then the conditional density (likelihood) above can be replaced by the unconditional density.

The discretized version of equations (1), (2), (3) is given by:

$$x(k+1) = F\,x(k) + Gu(k) + v(k) \tag{4}$$

$$y(k) = C\,x(k) + Du(k) \tag{5}$$

$$z(k) = y(k) + w(k) \tag{6}$$

If $x(o)$, $v(k)$ and $w(k)$ are Gaussian, the density $f(Z_N/p)$ is Gaussian. (If $f(p)$ is Gaussian, then the unconditional density $f(Z_N)$ is also Gaussian.)

$$f(Z_N/p) = c_1 \exp\left[-1/2\sum_{k=0}^{N}\left(z(k) - y(k)\right)^T W^{-1}\left(z(k) - y(k)\right)\right] \tag{7}$$

where c_1 is a constant and W is the covariance matrix of $w(k)$.
It can be proved that (ref. 3)

$$E[(p-\hat{p})(p-\hat{p})^T/p] > J^{-1} \tag{8}$$

where $\hat{p}$ is any absolutely unbiased estimate of p, and where

$$J = E\left\{\left[\frac{\partial}{\partial p}\ell nf(Z_N/p)\right]^T\left[\frac{\partial}{\partial p}\ell nf(Z_N/p)\right]\right\}/p$$

$$= -E\left\{\frac{\partial^2}{\partial p^2}\ell nf\,(Z_N/p)/p\right\} \tag{9}$$

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It can be proved from equations (7) and (9) that

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$$J = \sum_{k=0}^{N} \left\{\frac{\partial y(k)}{\partial p}\right\}^T W^{-1} \left\{\frac{\partial y(k)}{\partial p}\right\} \tag{10}$$

J is known as the "Fisher information matrix," and J^{-1} is the Cramér-Rao lower bound on the covariance of the parameter estimation error.

Application to LSS.- The unforced modal equations of motion for a flexible LSS are given by:

$$\ddot{q}_i + 2\rho_i\omega_i\dot{q}_i + \omega_i^2 q_i = 0 \quad (i = 1, 2, 3, \ldots, n_m) \tag{11}$$

where n_m is the number of modes. The output equation is:

$$y = \sum_{i=1}^{n_m} \phi_i q_i \tag{12}$$

where q_i, ρ_i, ω_i and ϕ_i are the modal amplitude, damping ratio, natural frequency, and $(\ell x1)$ mode shape (or mode slope) matrix for the ith structural mode. y is the $\ell x1$ output vector. (The rigid-body equations are not given because they do not play a role in the identification of the structural parameters. It is assumed that the mass and inertias of the LSS are known). The parameters to be estimated for the ith mode are:

$$p_i = [\rho_i, \omega_i, \phi_{1i}, \phi_{2i} \ \ldots, \phi_{\ell i}] \tag{13}$$

Thus there are $(\ell+2)$ parameters per mode, or a total of $n_m(\ell+2)$ parameters to be estimated.

The evaluation of the Fisher information matrix in equation (10) requires the computation of $\frac{\partial y}{\partial p}$.

Since the parameters to be identified appear explicitly in the continuous model, it is more convenient to first write the continuous equations for the sensitivity states $(\partial q_i/\partial p)$, and then to discretize them. The sensitivity state equations are (from Eq.11):

$$\frac{d^2}{dt^2}\left(\frac{\partial q_i}{\partial \rho_i}\right) + 2\rho_i\omega_i \frac{d}{dt}\left(\frac{\partial q_i}{\partial \rho_i}\right) + \omega_i^2 \left(\frac{\partial q_i}{\partial \rho_i}\right) + 2\omega_i \dot{q}_i = 0 \tag{14}$$

$$\frac{d^2}{dt^2}\left(\frac{\partial q_i}{\partial \omega_i}\right) + 2\rho_i\omega_i \frac{d}{dt}\left(\frac{\partial q_i}{\partial \omega_i}\right) + \omega_i^2 \left(\frac{\partial q_i}{\partial \omega_i}\right) + 2\rho_i \dot{q}_i + 2\omega_i q_i = 0 \tag{15}$$

If rate gyros are used for rate measurements,

$$y = \sum_{i=1}^{n_m} \phi_i \dot{q}_i \tag{16}$$

where ϕ_i is the $\ell \times 1$ mode-slope matrix for the ith mode. (There would be three y vectors corresponding to each axis; only one is given here to retain simplicity). $\partial y/\partial \rho_i$, $\partial y/\partial \omega_i$, $\partial y/\partial \phi_{ji}$ $(j=1,2,..,\ell)$ can be obtained from (14) (15) and (16). The evaluation of $\partial y/\partial p$ then requires the discretization of the coupled continuous differential equations (11), (14) and (15), which represents a sixth-order system (for each mode). Therefore the total order of the system is $6n_m$. However, since the equations for each mode are not coupled with the other modes, discretization for each mode can be carried out separately.

The sampling period is next selected, so that the sampling frequency is higher than the Nyquist frequency based on the highest mode estimated. The estimation time (N in Eq. 10) is determined so that the final time is at least equal to the largest $1/(\rho\omega)$ (i.e., the longest decay time). Nonzero initial conditions are created by applying torques through the torquers (e.g. on the mast of the hoop/column antenna). The applied torques could consist of sine waves, or pulses, etc. The torques are turned off after the modes are sufficiently excited (as indicated by the deflections), and Fisher information matrix J is computed as the unforced motion continues. The Cramér-Rao standard deviations for each parameter are then the square roots of the appropriate diagonal elements of J^{-1}.

CONCLUDING REMARKS

In order to investigate the accuracy with which LSS parameters can be identified, expressions were derived for obtaining the Cramér-Rao bounds. A computer program has been written to compute the bounds, and is presently being used to investigate the identifiability of the parameters of the 122m hoop/column antenna. The results will be reported in a future report.